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Perspectives on Low Global Interest Rates

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Abstract

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This paper looks at the dramatic decline in global real interest rates in recent years from a historical perspective and examines the various factors that may account for this trend. We show that current levels of real interest rates on long-term bonds in advanced economies are not low by historical standards and that it is the real long bond rates of the early 1980s through much of the 1990s that look anomalous. We also find that current global long-term interest rates are roughly in line with what one would predict given current price-earnings (P/E) ratios and under reasonable assumptions about the equity risk premia and the expected rate of growth of earnings in advanced countries. Finally, we provide econometric evidence that global long-term interest rates are significantly affected by commodity prices, expected productivity growth, and fiscal consolidation in advanced countries.

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I. INTRODUCTION

Long-term real interest rates in advanced economies, whether measured in nominal or in real terms, have declined to their lowest levels since the late 1950s. The declining trend is evident in the behavior of two measures of the real yield of the U.S. 10-year Treasury bond: one derived using the current rate of consumer price inflation and the other from a measure of expected inflation (Figure 1).² Although the measure derived with the current index has a greater variance than the other, both tell a similar story about the long swings in real long bond yields.

These trends are not unique to U.S. treasuries. Similar trends hold for the sovereign debt of other advanced countries regardless of whether one uses either expected or actual CPI inflation measures, or median vs. GDP-weighted measures to compute the representative global rate (Figures 2 and 3). Coupled with the fact that by mid-2005 the global yield gap had also dropped some 40 basis points below its post-war average of 1.4 percent (see Figures 4 and 5), this amounts to an unprecedented boom in the valuation of long-dated fixed-income assets.³

The low level of long-term interest rates has struck many economists as a conundrum, to use former Fed Chairman Alan Greenspan's famous term. In fact, low rates raise two basic questions. The first is whether they are explainable by the current economic and financial environment. Some economists take the view that low rates are not such a great puzzle. They maintain that inflationary expectations are well contained; that the global economy, although growing robustly despite the increase in oil prices, is not overheating; and that investment remains comparatively low. In these circumstances, low real interest rates are not surprising.⁴

² For the period since 1979, this measure of expected inflation consists of an average of the consensus inflation forecast. For the prior period, the measure is constructed from the fitted values of a regression over the more recent period of current expected inflation on 2-year and 10-year lags of actual inflation. The coefficients thus estimated are used to derive estimates of expected inflation. It was found that this specification yielded the best fit for the U.S. inflation expectations equation (with an R^2 equal to 0.92).

³ In most cases (and notably for the United Kingdom and the United States) the yield gap is measured as the differential between the 3-month Treasury bill and the (non-inflation-indexed) 10-year bond. For some countries, like Australia, we used the 15-year bond as the benchmark.

⁴ While some of these macroeconomic benefits are obvious, low interest rates and associated lower returns on assets can also be a source of stress to defined-benefit pension funds if the current rate of interest (and rate of return on assets) has fallen below the rate at which funds remain fully funded.

However, this argument begs the second question: whether current interest rate levels are themselves sustainable, and not simply a temporary blip—caused, say, by excess liquidity or an abnormal “savings glut.” In light of the abundant historical evidence that periods of abnormally low interest rates are typically characterized by aggressive chasing after assets with a higher return, a phenomenon that would tend to raise systemic financial risk (Kindleberger 1978), this is surely a cause for concern. In the present international setting, this sustainability issue is relevant for both highly indebted emerging markets and some advanced countries with high current account deficits such as the United States. Both groups would be vulnerable to an abrupt turnaround.

The main purpose of this paper is to put some of the current debate into a broader macroeconomic and historical perspective. As noted by some key policy makers and market observers, there are a variety of demographic and institutional factors (such as pension fund regulations affecting the composition of fixed-income versus equity instruments) that have a bearing on bond yields.⁵ Rather than aiming at dissecting and meticulously quantifying these various factors, the main aim of this note is to evaluate how far a standard macroeconomic approach coupled with supporting econometric and historical evidence can go in providing a rationale for recent developments.

⁵ See the Federal Reserve Board, “Testimony of Chairman Alan Greenspan Before the Committee on Financial Services, U.S. House of Representatives, July 20, 2005.” See also Packer and Wooldridge (2005).

Figure 1. United States' 10-Year Bond Rate
(Percent a year)

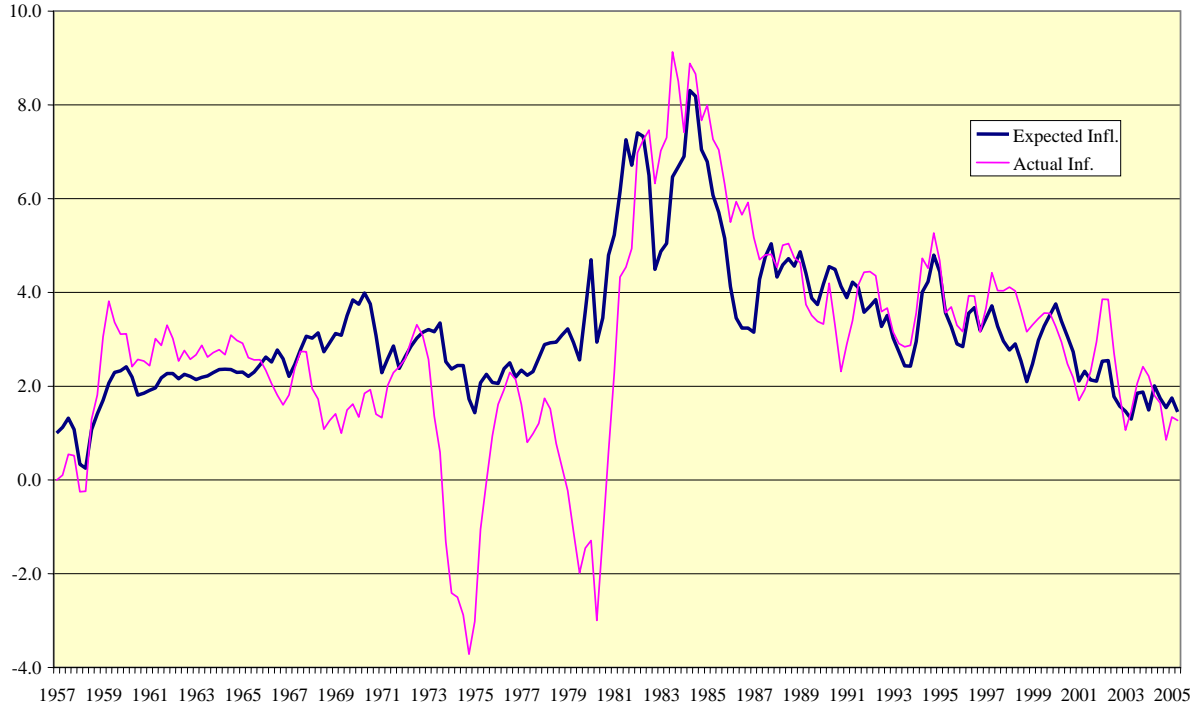


Figure 2. Advanced World's Long Real Interest Rate
(Percent a year)

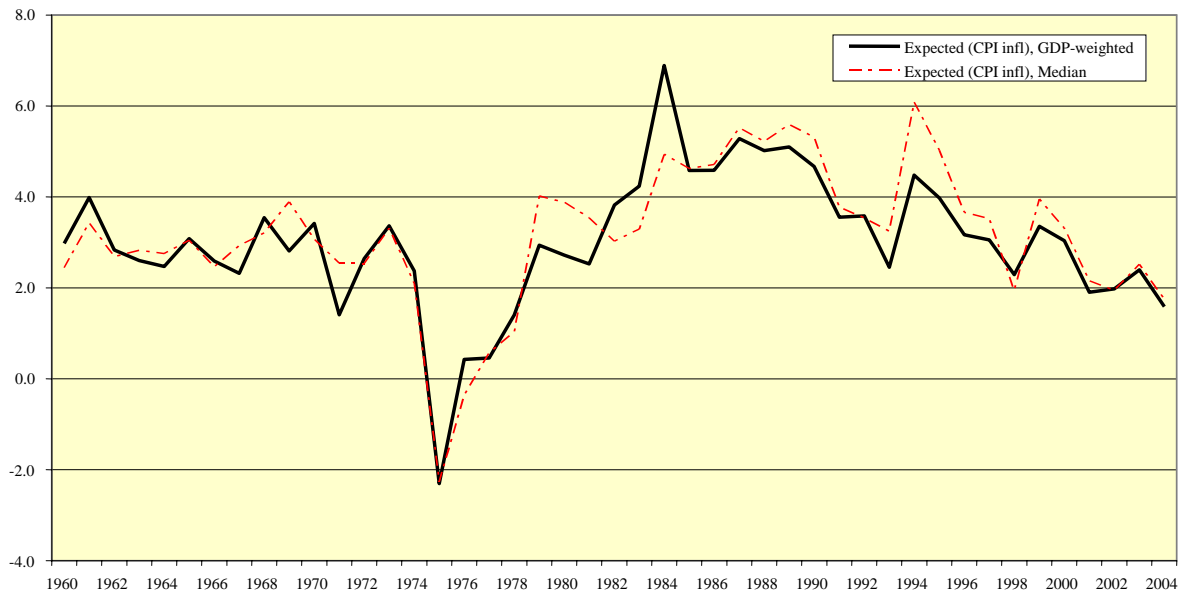
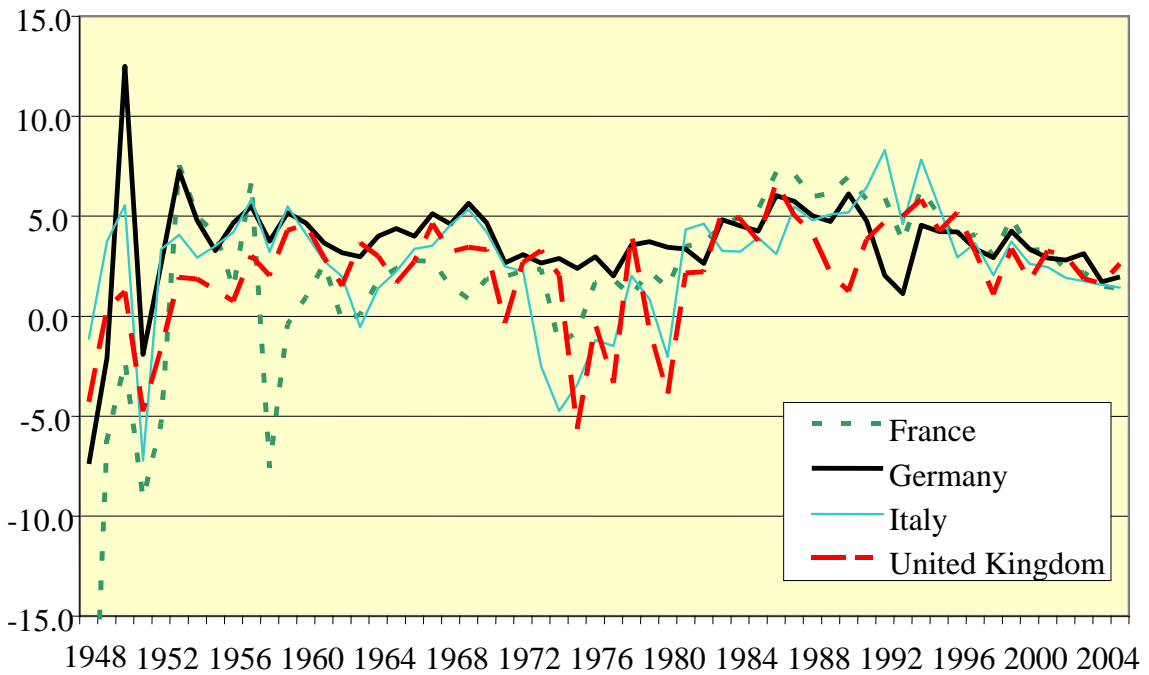
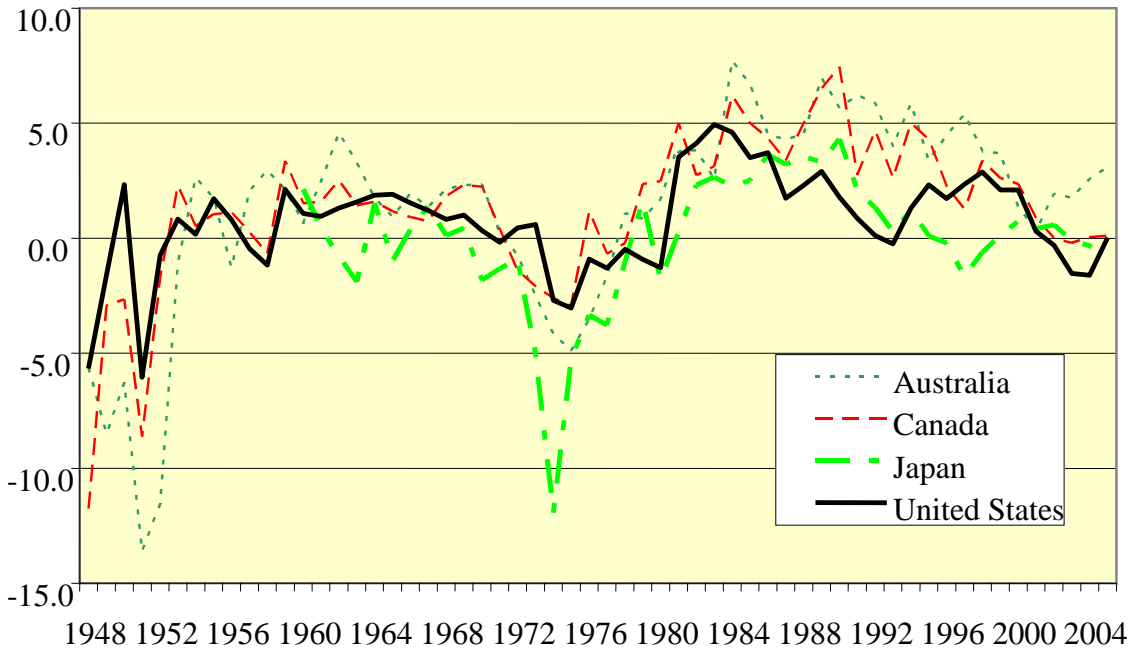
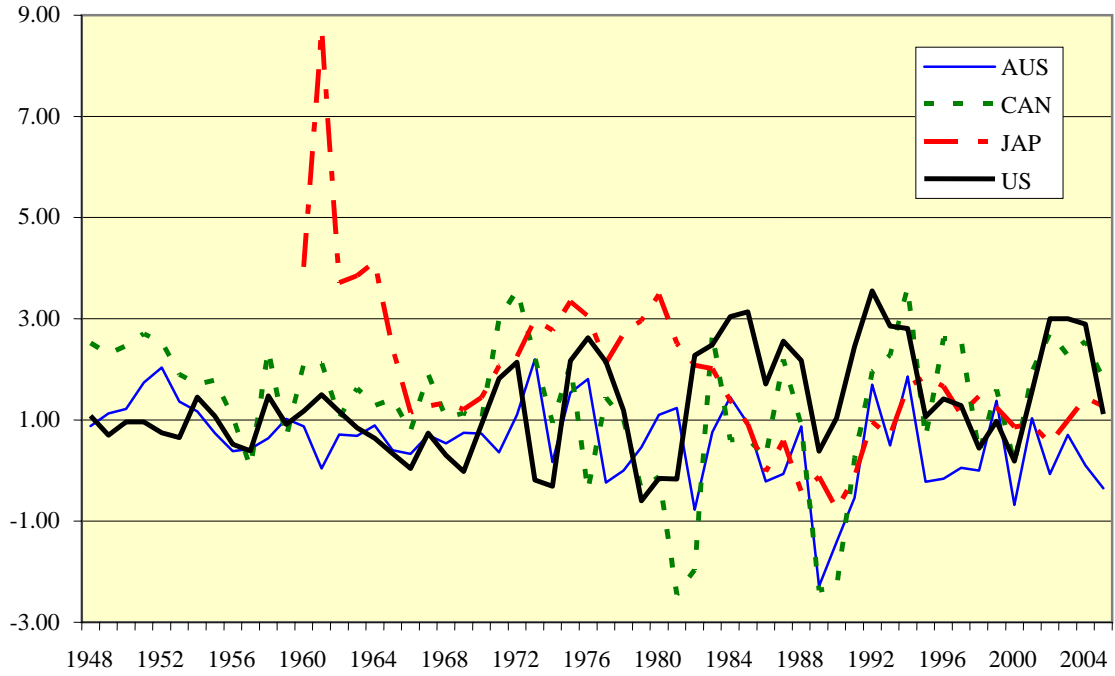


Figure 3. Individual Countries' Real Long Interest Rate
(Deflated by current CPI, percent a year)



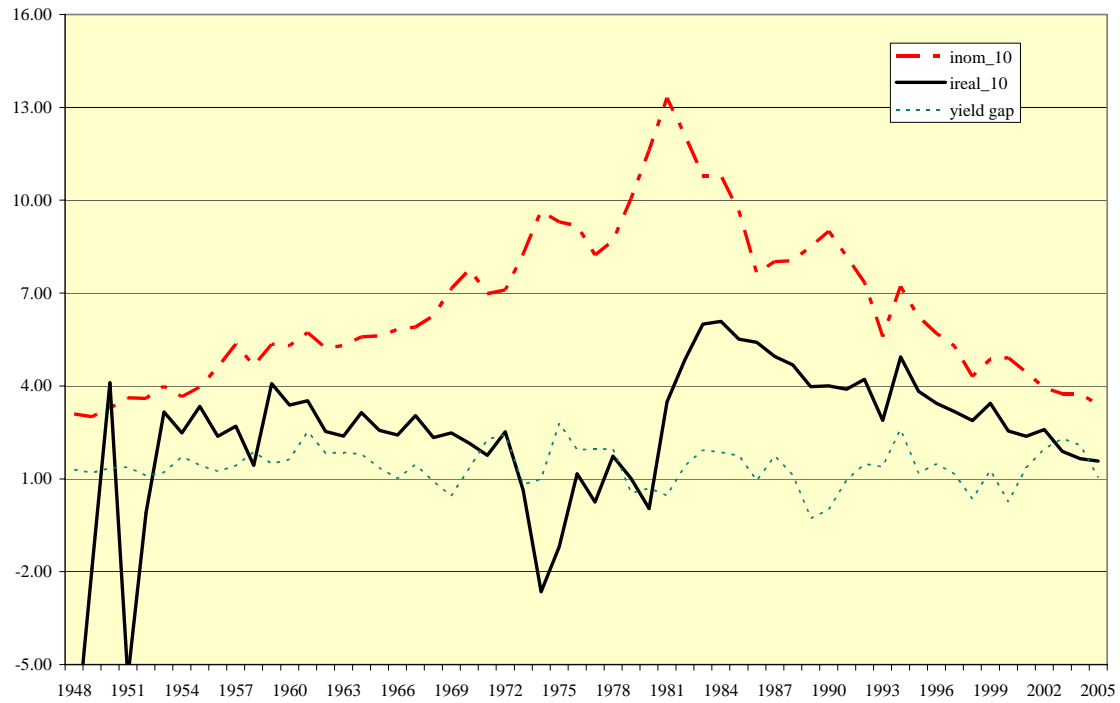
Source: IMF staff calculations.

Figure 4. The Yield Gap Between 10-Year and 3-Month Government Bonds (Percent)



Source: IMF staff calculations.

Figure 5. G-8 Averages of Long-Term Interest Rates and the Yield Gap (Percent, 1990 PPP weights)



II. SOME HISTORICAL PERSPECTIVE

Table 1 provides a historical overview of the behavior of real long-term rates from the early days of financial globalization around the 1870s through the eve of World War II.⁶ Over the classical gold standard era (1870–1913), the weighted average of the long-term real interest rate among the eight most advanced economies at the time was 3.5 percent. Breaking down the pre-WWI period into its two main phases—the first, from 1870 to 1895, which was marked by the great 19th century depression that started with the 1873 stock market crash through the Barings crisis of the early 1890s; and the second, the long global economic boom through the eve of WWI—suggests that the relatively high average real rate of 3.5 percent owes a great deal to the influence of the deflationary period of 1870–95. Since we know from Friedman and Schwartz (1982) about the existence of a “Gibson paradox” implying that the nominal interest rate adjusts rather slowly to price level changes, higher real interest rates for the period were mainly the result of lagging adjustment of nominal rates to price developments.⁷ During the Edwardian boom starting in the late 1890s through the eve of World War I, the average real rate of interest was 2.2 percent.

The historical evidence also shows that the difference between the real rate of interest and the growth rate was generally small, and sometimes negative. Moreover, this relationship was not a special feature of the classical gold standard. Even in 1919–39, the global real interest was a whisker lower than the global growth rate. This finding is in striking contrast with the assumption of many theoretical models that the real rate of interest should be higher than the economy’s growth rate.⁸

Although this apparent anomaly partly reflects the highly negative real interest rate associated with the German hyperinflation of 1922–23, France, Australia, and Japan also witnessed a negative gap between the real interest rate and the economy’s growth rate during

⁶ Two watershed developments that provided the basic foundation for the concept of a global economy were the first transatlantic cable in 1866 and the dissemination of a hegemonic monetary regime (the gold standard) among core advanced economies in the 1870s. Kindleberger (1978) describes the 1873 stock market crash in continental Europe and the United States as the first global financial crisis.

⁷ The Gibson paradox implies that rational expectations versions of the Fisher equation have a hard time in explaining these historical data. This has been subject of a large literature. Barsky and Delong (1991) note how hard it is to explain such long lags in the behavior of price expectations. They document that, given the contemporary knowledge of monetary theory at the time, it would have been easy to predict rising world prices following the gold discoveries of the 1890s. Yet interest rates completely failed not only to anticipate but also to keep pace with the subsequent rise in world prices.

⁸ This assumes that the dynamic efficiency condition will in practice be satisfied.

a different but also relatively long subperiod. In both the pre-1914 and the interwar era, it is noteworthy that high long-term real interest rates were typically associated with periods of growth slowdown (as in 1870–95) or absolute output declines (as in the 1930s). There is less frequent evidence of causality running from higher growth to higher real interest rates.

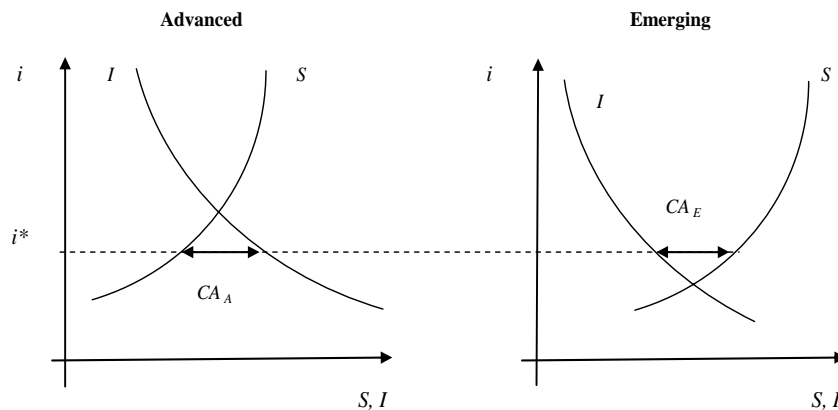
In short, current levels of long-term real interest rates of around 2 percent among advanced countries appear low from the vantage point of the 1980s and 1990s, but not so low when compared with earlier epochs. In addition, periods when long-term real interest rates are below the economy’s growth rate—as they have been recently in some advanced economies (notably the United States)—are not exceptional. This was clearly the case in the last two decades of the classical gold standard, the interwar period as a whole, and the early postwar period through the 1960s.

III. SIMPLE ANALYTICS OF GLOBAL INTEREST RATE DETERMINATION

A. A Basic Model

National interest rates are known to be affected by a myriad of factors. Nonetheless, the common tendencies in the various countries’ rates displayed in Figures 2 and 3 suggest the existence of a global common factor. One basic model of interest rate determination would have a global interest rate determined by the intersection of global saving and investment schedules, rather than by national schedules. The annex develops a simple two-country two-period model in which the implications of changes to the parameters that determine saving and investment can be explored (Figure 6). This section presents a heuristic derivation and summary of the annex’s main results.

Figure 6. The Two-Region World Economy



Since it will be useful in the discussion that follows, we set out the equation for the simple production function that the model uses, which is given by

$$Y = AK^\alpha. \tag{1}$$

Table 1. Long-Term Real Interest Rates and Real GDP Growth Rates 1/
(Percent per annum)

	1870–1913		1870–1895		1896–1913		1919–1939		1919–1929		1930–1939	
	interest rate	GDP	interest rate	GDP	interest rate	GDP	interest rate	GDP	interest rate	GDP	interest rate	GDP
Global	3.53	2.71	4.43	2.45	2.22	3.11	2.25	2.43	1.78	3.37	2.54	0.23
Australia	3.88	3.38	5.15	2.92	1.94	4.10	3.49	2.58	4.04	3.23	1.58	1.93
Canada	2.20	3.94	4.29	2.59	1.05	5.99	4.16	2.40	4.04	4.27	3.79	0.55
France	3.76	1.62	4.23	1.43	2.72	1.90	-1.00	3.12	-2.39	5.95	4.69	0.36
Germany	3.33	2.77	4.39	2.52	2.22	3.17	3.70	4.46	-16.98	5.29	4.17	3.63
Italy	4.48	1.92	6.55	0.87	2.86	3.43	3.12	1.90	3.12	1.68	4.87	2.12
UK	2.94	1.88	3.24	1.92	2.02	1.85	5.04	1.42	5.09	1.04	2.67	1.80
Japan	3.32	2.41	5.22	2.46	1.35	2.38	3.46	3.57	7.03	2.41	0.18	4.75
US	3.89	3.86	5.22	3.80	2.30	4.02	4.00	1.84	4.71	3.48	3.92	0.23

1/ Median of annual observations. Global interest rates and growth rates are weighted averages according to each country's GDP in constant 1990 US\$ dollars, taken from Maddison (2003). The interest rate variable was constructed as the nominal yield on 10-year bonds (or closest benchmark) for each country deflated by the respective current

where Y is output, A is the productivity parameter, K is the stock of capital, and $\alpha < 1$. Assuming for the moment that the risk-free rate of interest is given, and equal to r^* , and that the equity risk premium is given, and equal to ρ then the marginal product of capital (MP_K) is given by

$$MP_K = \alpha AK^{\alpha-1} = r^* + \rho \quad (2)$$

The recent behavior of price-earnings (P/E) ratios in advanced countries, and particularly in the United States after the 2000–01 market crash, suggests that the expected profitability of investment may have declined. Using a simple constant-valuation asset-pricing model (often known as the Gordon equation), a decline in the P/E ratio combined with a constant equity risk premium (as discussed more fully below) and a lower risk-free interest rate implies that the expected growth of future earnings must have declined.⁹

As the annex discusses in more detail, this decline can be readily captured by a downward shift in the productivity parameter A of equation (1). A fall in A in either or both countries has the effect of reducing the amount of desired investment at any given interest rate (because MP_K varies directly with A). Consequently the global investment schedule shifts to the left, and assuming the global saving schedule is not affected, the rate of interest will decline.

An increase in the price of energy can also be captured by a fall in A , since by reducing the amount of energy that will be used with the existing capital stock, the productivity of the stock has to fall. This is more readily grasped if we include energy consumption, E , as a separate argument in the production function. The marginal product of capital is then given by equation (3), which shows that a decline in the use of energy, other things equal has to reduce profitability at the margin:

$$MP_K = \alpha AK^{\alpha-1} E^\beta . \quad (3)$$

In addition to their impact on the investment schedule, a decline in productivity and an increase in energy cost might cause a shift in the saving schedule. For example, if a decline in productivity means that the income households expect to receive in the future will decline, they may save more now to smooth the pattern of consumption over time. This would reinforce the effect of a decline in productivity on the interest rate. On purely theoretical

⁹ Note that this is not at all inconsistent with the fact that the share of profits in U.S. national income is now at about its highest levels since the 1960s (see the recent U.S. Article IV report—IMF, 2005). This is so partly because current profitability is not always strongly correlated with higher expected future profit growth, and partly because of the compression in wage earnings, itself a function of a number of technological and institutional variables. In addition, the *share* of profits does not invariably move in the same direction as the *ex post rate* of profit.

grounds, however, the impact on saving cannot be determined, because the substitution effect entailed by the decline in interest rates increases the cost of postponing consumption. This tends to offset to some degree the income effect.

B. Other Factors

While the one-good/one-factor model has the advantage of fleshing out some key determinants of global interest rates in a simple and intuitive manner, reality is more complex. Other plausible factors must be also considered.

An obvious additional factor is the role of labor supply and expected wage developments. An important development of the past two decades has been greater real wage flexibility in mature economies, notably in the United States and the United Kingdom, which has been accompanied by falling shares of labor compensation in national income in the industrial world. In the United States, in particular, the share of labor compensation in national income has declined from a peak of 58 percent around 1980 to 51 percent, the lowest share in 40 years. As discussed elsewhere,¹⁰ this phenomenon can be explained by a multitude of factors, including not only changes in variables such as total factor productivity (TFP) and oil prices as discussed above, but also trade openness and institutional developments that have weakened labor's market power.

Further declines in this share appear unlikely. From a cyclical perspective, the real wage gap should be expected to narrow as the current economic recovery gathers momentum. How much some of this tightening in labor markets will be permanent, thus affecting long-run factor shares, is uncertain. Nonetheless, in an environment of high and increasing energy costs and lower productivity growth, real wage pressures emanating from tighter labor markets coupled with historically low labor participation shares would depress profitability in the medium term. To the extent that term arbitrage along the yield curve is sufficiently strong, these developments could lower short-term real rates.¹¹

The other factor not directly contemplated in the above model is the desire for reserve accumulation in the developing region. While reserve accumulation is one side of higher

¹⁰ Guscina, Anastasia, 2005, "Explaining Labor's Share in National Income," paper prepared for the 2005 U.S. Article IV Consultation (IMF, 2005).

¹¹ The expectations hypothesis embodied in the Fisher equation implies that the two-period real rate will be an average of the current and the expected one-period-ahead real rate. See Estrella (2004) for a more detailed discussion. Obviously, the recent monetary tightening in the United States is a pull in the opposite direction, so the question is which effect predominates in the determination of the period-ahead real rate. If higher oil prices, lower productivity, and tighter labor markets combined impart a sufficiently depressive effect on future real rates, then it is obviously possible that the long rate falls while the current rate rises, potentially giving rise to an inverted yield curve.

public saving, it tends to have a greater impact on long global bond yields when such saving are channeled toward foreign government bond instruments. Scaled by either import coverage or key monetary aggregates, developing countries' international reserves (typically held in either U.S.-dollar or euro-denominated bonds) have risen sharply over the past five years. This development undoubtedly reflects precautionary saving: the emerging market crises of the 1990s have brought into sharp relief the need for precautionary reserve accumulation in a world of lower capital account restrictions where "sudden stops" in capital inflows can have devastating effects on economic activity and financial sector stability.

The phenomenon is not new. During the heyday of world capital mobility of the classical gold standard, emerging market countries held large stocks of the main convertible currency (mostly gold or sterling claims on London), averaging 63 percent of currency in circulation and 22 percent of broad money (Catão, 2006). By comparison, the (unweighted) worldwide median ratio of net central bank reserves to broad money reached 32 percent at the end-2004, up from 7 percent in 1990 and 18 percent on the eve of the first main emerging market crisis of the 1990s (Mexico).

The flip side of this large reserve accumulation is the resistance by many emerging markets to let their currencies appreciate, since this would lead to deteriorating current accounts and thus greater vulnerability to capital flow reversals and disruptive currency depreciations. From this perspective, current policies in several emerging markets can be interpreted as precautionary and geared toward avoiding large current account deficits that typically preceded financial and debt crises in the past. Letting variations in reserves bear the burden of the adjustment to changes in international liquidity is of course not new: during the pre-war gold standard, exchange rate stability was accompanied by periods of large accumulation and decumulation of international reserves, except among a subset of peripheral countries that were unable to accumulate sufficient reserves during periods of abundant international liquidity (often due to highly procyclical fiscal and financial policies), and thus had to resort to large periodic exchange rate adjustments.

Finally, while the model aggregates private and public saving, it is important to distinguish between the two to the extent that public saving is driven by a distinct (and likely more exogenous) set of factors. Figure 7a plots the GDP-weighted average of public saving ratios for the advanced and emerging regions, respectively. Public saving ratios in the advanced region have declined markedly over the past five years (largely reflecting U.S. trends), but this has been partly compensated by a marked recovery in developing country public saving. At the same time, a nonnegligible part of fiscal consolidation in emerging markets has been accomplished by cuts in public investment, rather than public consumption (Figure 7b). To the extent that these cuts in public investment do not crowd in private investment on a one-to-one (or higher) basis, and hence tend to depress aggregate investment in this group of countries, this in turn tends to depress world interest rates.

Figure 7a. Public Saving
(Percent of GDP)

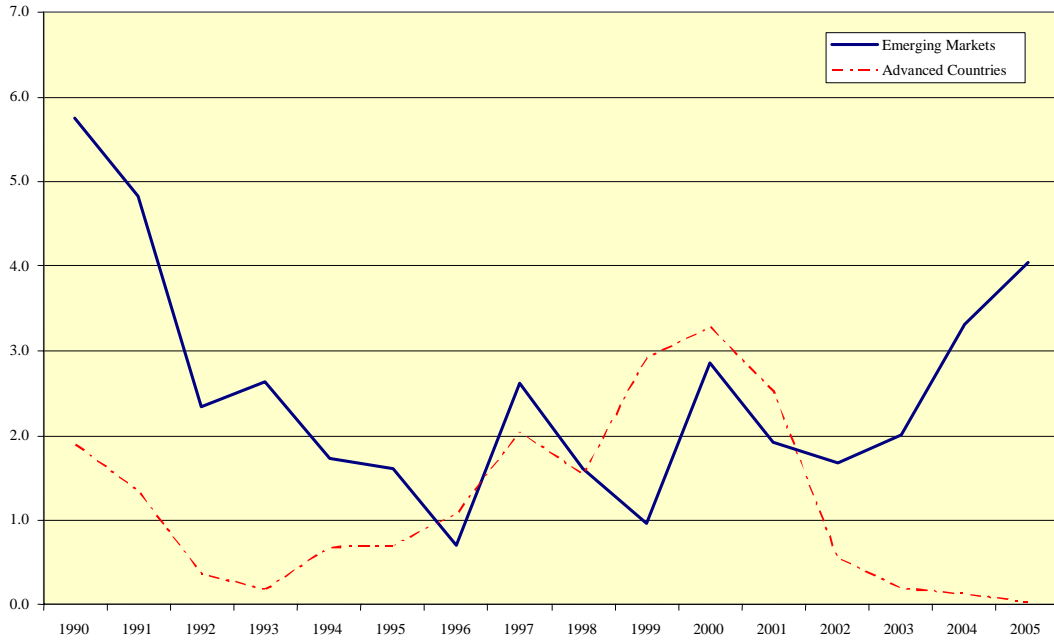
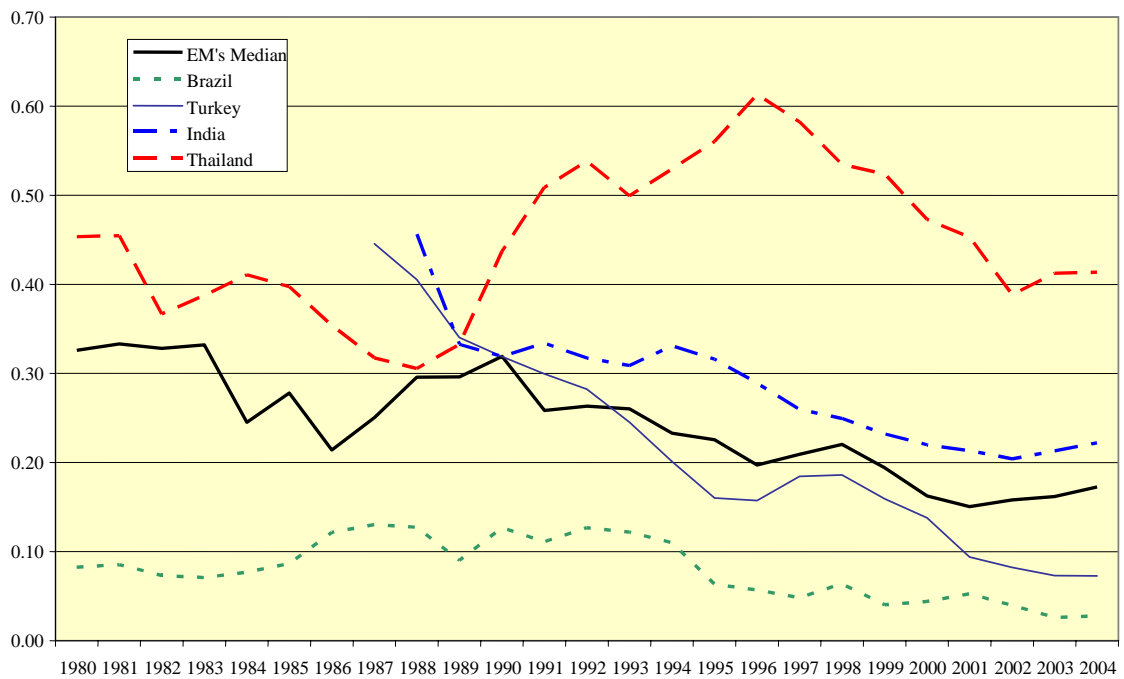


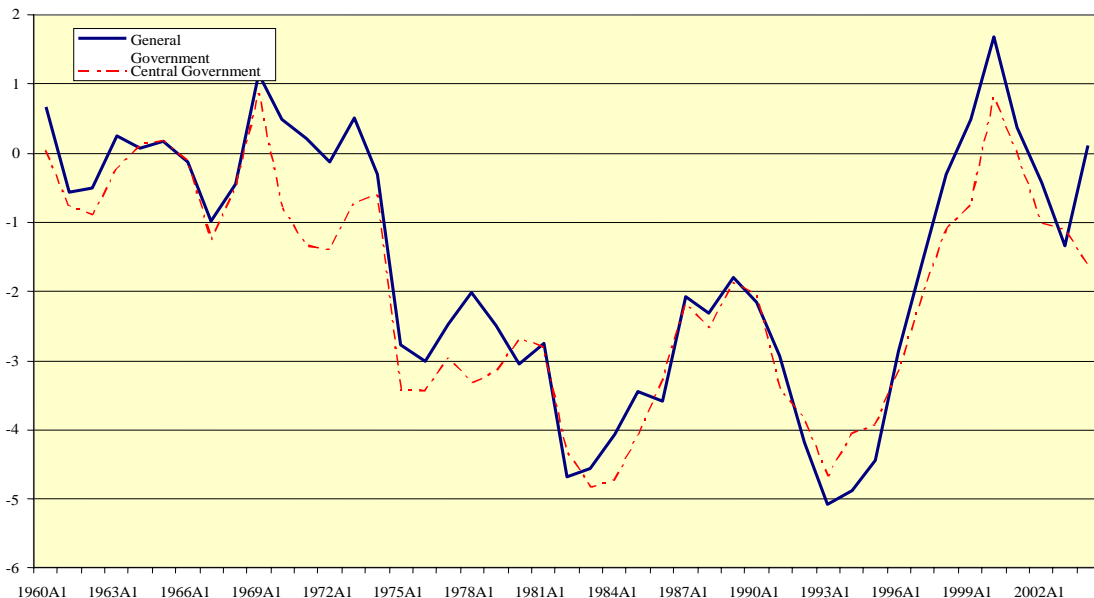
Figure 7b. Share of Fixed Capital Formation in General Government Expenditure
in Emerging Markets



Although this divergent behavior of public saving in two regions does not seem to do much to explain lower global interest rates, such a focus on public saving over the past 15 years overlooks the important fact that, despite some slippages in individual countries, fiscal discipline has risen overall relative to the 1970s and 1980s (Figure 8). To the extent that this has a permanent component (resulting from deeper-seated institutional arrangements, greater financial and trade openness and politicians' realization that markets "vote everyday," so that fiscal profligacy will be more severely and readily punished), it is likely to induce private sector saving by making it easier to channel that saving through domestic financial systems (due to lower risk of seigniorage, controls to capital repatriation, or other regulatory changes that impart a loss to savers). The flip side of such greater fiscal discipline is thus greater monetary deepening, which, in turn, has been found to be a strong determinant of private sector saving (Edwards, 1995). So, greater fiscal discipline relative to the past two decades seems to be another explanatory factor for lower global real interest rates.

Recent regulatory and institutional developments may also have helped depress bond yields. Regulatory changes and the bursting of the equity bubble in 2000 have increased the demand of both life insurance companies and pension funds for long-dated bonds. Similarly, increases in longevity have raised concern over duration mismatches in both classes of institutions. However, although the qualitative impact of these changes on interest rates is clear, the data are lacking for an assessment of their quantitative impact.

Figure 8. Fiscal Balances in OECD Advanced Economies
(Percent of GDP)



IV. AN ASSET-PRICING PERSPECTIVE

The Gordon equation, which was introduced earlier, can be used to gauge the mapping between the real interest rate on bonds and the rate of return on equity (equation (4)) :

$$P = \frac{\gamma E}{r^* + \rho - g} \therefore r^* = \frac{\gamma E}{P} - \rho + g, \quad (4)$$

where P/E is the price-earnings ratio, γ is the dividend payout ratio, ρ is the equity risk premium, and g is the real growth rate of earnings.¹²

Given current levels of the P/E ratio for the aggregate stock markets in advanced countries and once the expected growth rate of earnings (g) is determined by factors pertaining to technology and relative input prices (notably oil) as contemplated in the above model, we can back out r^* , the risk-free real rate of interest from equation (4), provided that we have some estimate of the equity risk premium. Existing estimates based on 1980–2005 data for the United States (by far the largest equity market in the advanced world) yield an implicit equity risk premium of close to 3 percent.¹³ While this estimate may seem high in light of both the reduction of transaction costs in equity holdings in recent years and of some existing estimates, it is still considerably lower than the estimates typically derived under the standard approach, which compares the ex post difference between real equity returns and real bond yields over longer time spans.¹⁴ Further, to the extent that there is evidence that the equity risk premium tends to rise in the wake of stock market crashes (as in the 1930s and 1970s),¹⁵ and that retiring baby boomers tend to favor bonds relative to stocks, 3 percent is not an implausible figure and is certainly consistent with (ex post) evidence over the past 30 years.

¹² Implicit in the derivation of this formula is the assumption of a stable payout ratio and that $r^* + \rho > g$. Whereas the stability of dividend payout ratios have fluctuated in the United States in recent years due to stock buybacks, this assumption is not too restrictive over the long-term. In the United States, the dividend-earnings (the so-called dividend payout) ratio has fluctuated in the 40 to 50 percent range before dropping to around 35 percent because of an intensity of equity repurchases during the stock market bubble of the late 1990s. Equity repurchases can only affect the time pattern of expected future dividend per share but do not invalidate equation (10) as an equilibrium, long-term relationship. See Campbell et al. (1997).

¹³ IMF, *World Economic Outlook*, May 2000, p. 82. Using a different methodology and time period (which includes the 1970s), Blanchard also obtains an average risk premium for advanced economies of 3 percent. See Blanchard (1993).

¹⁴ See Siegel (1998).

¹⁵ See Blanchard (1993).

Using equation (4), Table 2 provides estimates of r^* for other countries, most of which are in the same range.

Based on current levels of P/E ratios and dividend payouts for the United States, assuming an equity risk premium of 3 percent, and a long-term rate of real earnings growth of 3.4 percent (which are roughly in line with *World Economic Outlook* estimates of potential real GDP growth in the United States),¹⁶ these calculations yield a prospective estimate of r^* of 2¼ percent. This is only slightly higher than current levels.

Table 2. Implicit Equilibrium Real Interest Rate

	Price-Earnings Ratio	Payout Ratio	Dividend Yield	Equity Risk Premium	Real Earnings Growth	Real Interest Rate
Canada	17.6	37.1	2.1	1.4	2.9	3.7
France	13.9	38.6	2.8	2.2	1.8	2.5
Germany	12.2	27.9	2.3	0.1	1.4	3.6
Italy	17.7	56.8	3.2	1.7	1.2	2.7
Japan	27.9	34.0	1.2	1.4	1.5	1.3
United Kingdom	14.7	45.6	3.1	3.8	2.3	1.6
United States	19.1	33.0	1.7	2.9	3.4	2.2
Norway	14.7	45.3	3.1	2.3	1.8	2.6
Sweden	17.4	47.9	2.8	2.5	2.4	2.6
Spain	18.4	44.9	2.4	3.7	3.0	1.8
Netherlands	12.6	42.6	3.4	2.4	2.0	3.0
Portugal	16.4	52.6	3.2	2.5	1.5	2.2
Finland	14.8	52.6	3.6	2.5	2.4	3.4
Ireland	14.3	30.7	2.2	2.5	4.7	4.4
Belgium	14.0	47.0	3.4	1.7	2.0	3.7
Austria	19.9	35.4	1.8	2.5	2.0	1.3
Median	15.6	43.7	2.8	2.4	2.0	2.6

Estimates for other countries (also using mid-2005 values of the P/E ratio and dividend yields, current *World Economic Outlook* projections of potential real GDP growth, and historical estimates of the risk premium for 1980–2005) are also generally sensible.¹⁷ In

¹⁶ The underlying assumption behind using potential real GDP growth as an indicator of aggregate earnings growth is that of an stable participation of capital remuneration in national income. As discussed above, the fact that the labor remuneration share has now hit low historical levels suggests that further declines in this ratio are unlikely over the foreseeable future.

¹⁷ For Austria, Ireland, Portugal, and Sweden, time series of sufficient length were not available to estimate the equity risk premium, so this was set arbitrarily at 2.5, which is consistent with the average for the other countries for which such data exist.

particular, a median 2 percent dividend growth is consistent with the historical experience (see Blanchard, 1993), and the resulting median r^* of 2.6 percent for the world real long-term interest rate is, again, slightly higher than current levels but not far off the mark. In sum, this analysis suggests that there may be some current overshooting of long bond valuations at a global level, but not much.

V. ECONOMETRIC EVIDENCE

The investment and saving functions in the theoretical model summarized above and set out in the appendix give rise to a parsimonious reduced-form equation for the global interest rate. To see this, consider a linearized representation of the investment equation:

$$(I/Y)_t = \alpha_0 + \alpha_1 A_t - \alpha_2 r^* + \alpha_3 (I/Y)_{t-1} - \alpha_4 oil_t, \quad (5)$$

where oil stands for period changes in the relative price of oil and the lagged term on I/Y is a standard way of capturing dynamics stemming directly from the neoclassical production function, where current changes in the capital stock are a function of the past level of capital stock which in turn is a direct function of past investment levels. Likewise, the saving equation can be written as:

$$(S/Y)_t = \beta_0 + \beta_1 r_t^* + \beta_2 \Psi_t' + \beta_3 (S/Y)_{t-1}, \quad (6)$$

where Ψ is a vector of other determinants that may include fiscal variables as well as proxies for changes in time preference parameters that underlie private saving behavior, as discussed in the annex. The long-run equilibrium real interest rate is thus given by:

$$r_t^* = \lambda_0 + \lambda_1 A_t - \lambda_2 oil - \lambda_3 \Psi_t' + \sum_{i=1}^l \theta_{t-i} r_{t-i}^*, \quad (7)$$

where $0 < \theta_i < 1$.

The main challenge in estimating equation (7) lies in obtaining satisfactory proxies for future productivity and earnings growth (A) as well as for other factors driving world aggregate saving behavior besides the interest rate. After experimenting with the growth of total factor productivity, current levels of potential output, and changes in real stock prices, the latter proved to be by far the most significant proxy.¹⁸ In the case of Ψ , we considered four controls—namely, the extent of financial deepening in emerging markets (a key gauge for

¹⁸Blanchard and Summers (1984) and Barro and Sala-i-Martin (1991) use the same indicator in their analysis of global investment determinants. One limitation in the present context is the lack of a similar series for emerging markets, implying that the measure of the growth of real stock prices in the regressions is confined to a GDP-weighted average of that variable for each of the G-8 economies.

private saving behavior—see Edwards, 1995); the ratio of fiscal deficits to GDP in the advanced and emerging market regions;¹⁹ and the ratio of global non-oil commodity prices to manufacturer prices. The last seeks to capture a variety of effects, some of which potentially offset each other. These include lower borrowing needs by emerging market governments (which will tend to depress interest rate on long-dated international bonds) but also the effect of shifting world income to primary producing countries that have a higher propensity to spend out of income, thus depressing world saving and raising bond yields.²⁰ Finally, as with oil, a rise in non-oil commodity input prices tends to pull down profitability in advanced economies (to the extent such commodities cannot be readily substituted), thus depressing real bond yields.

Estimation results using an autoregressive distributed lag (ARDL) representation are reported in Table 3.²¹ As expected, oil price inflation, real growth in world stock prices, and the advanced countries' fiscal position are found to be significant determinants of the world real interest rate calculated on the basis of expected long-term inflation (as plotted in the bold line in Figure 2). The real fiscal deficit variable for emerging markets yields the right sign but is not significant at conventional levels and so was dropped from the regression. Also taking on a plausibly negative but statistically insignificant coefficient is the non-oil primary commodity terms of trade—the statistical insignificance of this variable possibly reflecting at least some of the offsetting factors discussed above. In contrast, the effect of fiscal consolidation in advanced countries on global interest rates is strong—a 1 percentage point improvement in the ratio of the fiscal balance to GDP leads to a reduction in the real long-term global interest rate between $\frac{1}{2}$ and $\frac{3}{4}$ percentage points, all else constant. Overall, the model is capable of explaining 76 percent of fluctuations in real world interest rates and passes with flying colors standard diagnostic tests. Plotting the actual against fitted values of the regression, Figure 9 shows that the model captures fairly well the recent trend decline in global rates.

¹⁹ Once again, obtaining an equivalent measure for emerging markets going back to at least 1960 proved impossible because of the dearth of data on domestic debt prior to the 1990s.

²⁰ See Rostow (1980) for a discussion and supporting historical evidence on this international transmission channel.

²¹ The optimum lag structure was selected according to the Akaike Information criterion subject to a maximum order length of 2 so as to conserve on degrees of freedom. Truncating the lag structure at 1 yields essentially the same point estimates but larger standard errors and significant residual autocorrelation. The global real interest rate measure was obtained by deflating the long-term bond yield for each of the eight advanced countries by a measure of the respective country's one-year-ahead expected inflation obtained from a Garch model, where in most cases only the first-order auto-regressive component was statistically significant. The aggregate global rate was obtained by weighing the various countries' real interest rates by their respective GDPs denominated in 1990 U.S. dollars.

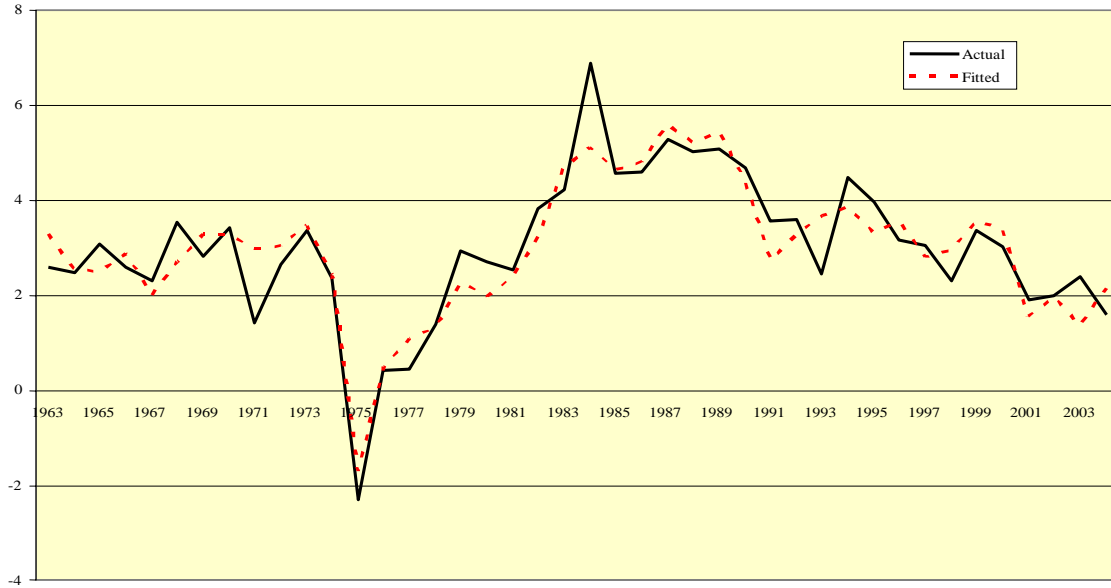
Table 3. Global Real Interest Rate: ARDL Estimates
(Estimation period: 1963–2004)

	Coefficient	t-ratio
a) Long run:		
$\Delta \ln(\text{Poil/US WPI})$	-0.02	-2.31
$\Delta \ln(\text{Real Stock Prices})$	0.11	4.18
Adv. Countries' (T-G)/Y	-0.75	-4.27
b) Short run:		
$\Delta \ln(\text{Poil/US WPI})_{t-1}$	-0.02	-2.50
$\Delta \ln(\text{Real Stock Prices})_t$	0.03	2.56
$\Delta \ln(\text{Real Stock Prices})_{t-1}$	0.01	1.11
$\Delta \ln(\text{Real Stock Prices})_{t-2}$	0.03	2.92
Adv. Countries' ((T-G)/Y) _{t-1}	-0.51	-2.91
ireal_{t-1}	0.33	2.49
$R^2 = 0.76$	Auto $\chi(1) = 0.01$	
DW = 2.01	Normal $\chi(1) = 0.51$	

We also tested the robustness of the results to the inclusion of three additional variables. The first is the variability of world inflation, measured by either 10-year or 5-year window standard deviations of the *World Economic Outlook* world inflation series—neither of which is significant at conventional levels. This is in a sense not surprising: lower inflation volatility tends to reduce the demanded risk premium on bond holdings but, on the other hand, also reduces saving under sufficient risk aversion, so the two effects may cancel out in the aggregate.

Second, we experimented with the inclusion of a variable that seeks to capture an increased preference for future over current consumption in emerging markets—the median ratio of M2 to GDP. This also proved to be insignificant at conventional levels, probably due to similar offsetting effects at play: on the one hand, higher financial development facilitates saving by reducing the cost of holding long-term deposits and other fixed instruments but, on the other, also spurs credit demand (for instance by reducing bank intermediation spreads), thus fostering consumption and investment spending.

Figure 9. Global Long-Term Interest Rate: Actual and Model-Fitted Values
(Percent a year)



Third, we added the constant price version of the world ratio of investment to GDP using whatever investment deflator was available from the IMF's International Financial Statistics and World Economic Outlook databases. Lagged investment has been used by previous researchers (see Barro and Sala-i-Martin, 1991) in an attempt to capture the interest rate-investment dynamics, which may not be fully captured by the inclusion of lagged real interest rates among the regressors. The signs, relative magnitudes, and statistical significance of all other explanatory variables are maintained, but the I/Y ratio is not statistically significant at any conventional level, both in the short- and long-run specifications. While this may simply be a result of measurement errors and the poor country representativeness of the real I/Y series for the 1960s, the statistical insignificance of this variable suggests that the inclusion of lagged real interest rates among the explanatory variables is satisfactorily capturing the dynamics of the saving-investment adjustment.

Overall, these econometric results lend credible support to the theoretical framework laid out in this paper. This is particularly so in light of the parsimonious nature of the model, which deliberately abstracted from some institutional factors, policy announcements, and other short-run developments that, although they have a bearing on the long rate through the yield curve, are noisy and often hard to measure.

VI. CONCLUSIONS

Current levels of long-term world interest rates appear unduly low from the perspective of the 1980s and 1990s. This paper has sought to show, however, that they are neither out of line with the longer-term historical experience nor incongruous with what one might predict on the basis of a reasonably specified macro model or simple tabulations based on historical estimates of the equity risk premium and asset market arbitrage relations.

In many ways, the 1980s and 1990s are outliers from a broader historical perspective: long-term real yields on bonds issued by key sovereigns have been historically very low relative to other financial instruments, and sometimes lower than the real trend growth of GDP in these countries for long stretches of time. The fact that the high real interest rates of the 1980s and much of the 1990s were anomalous from a broad historical perspective does not imply, however, that some key recent developments have not contributed to restraining on long-term interest rates—notably high real oil prices, the relatively subdued performance of global stock markets, and higher emerging market demand for international reserve assets and lower borrowing needs which, in turn, result from a variety of factors including higher commodity prices. Hence, and as our empirical evidence suggests, not until trends in these variables are dramatically reversed, should we expect a substantial rise in long-term global interest rates.

This evidence poses a number of interesting and important policy questions that are clearly beyond the scope of this paper. Chief among them is the question of why investors have been historically willing to hold government bond instruments that have yielded not only relatively low but also highly volatile ex post real rates of return. This source of cheap finance can clearly give governments in advanced countries considerable latitude in postponing needed fiscal adjustment and ultimately shift the burden of future adjustment to holders of such long-term instruments—pension funds and baby boomers among them. Given the well-known inverse relationship between the bond rates and equity returns, such historically low real rates of return on advanced countries' long-term bonds also suggests that, by and large, the “equity risk premium puzzle” effectively boils down to a low treasury bond rate puzzle. Fully explaining the latter thus could well be a critical task for analytical and policy-related work in international finance in the years to come.

I. THEORETICAL FRAMEWORK

As discussed in the main text, what determines global interest rates levels can be viewed as a question about what shapes the saving-investment schedules in the two regions. To study those, we start from a simple inter-temporal optimizing framework in the spirit of the new open macroeconomics literature (cf. Obstfeld and Rogoff, 1996). Consider a two period, one-good world where production in the two economies be given by the following production functions:

$$Y_t = A_t K_t^\alpha; \quad Y_t^* = A_t^* K_t^{\alpha^*},$$

where A and A^* are time-varying productivity parameters and asterisks denote the emerging region. Letting $\alpha, \alpha^* < 1$ implies decreases returns to scale; strict concavity in these production functions ensures that the investment schedules in Figure 6 are downward sloping.

As standard, consumption behavior is characterized by the intertemporal Euler equation:

$$u'(C_1) = (1+r)\beta u'(C_2), \quad (8)$$

where the r is the real interest rate and $\beta = 1/(1+\delta)$ is the discount factor, where δ is the subjective discount rate. So, higher values of β imply that the country is more “patient.” A similar counterpart expression hold for the other region. For each region, consumption in the two periods has also to respect the intertemporal budget constraint:

$$Y_1 - C_1 - I_1 + K_2/(1+r) + Y_2/(1+r) - C_2/(1+r) = 0, \quad (9)$$

where K_2 is the stock of capital in the second period which can be assumed to be consumed at the end of that period. Similar counterpart expressions hold for the foreign (or emerging) country region. For simplicity, we assume for now that capital depreciation between the two periods is negligible in both regions.

Once we attribute a functional form to the utility functions in the two regions, $u(C), u^*(C)$, take K_1, K_1^* as given initial endowments as well as A_1, A_1^* as pre-determined, some interesting comparative static exercises can be undertaken. We let the utility functions assume the

standard functional form $u = \frac{C^{1-1/\sigma}}{1-1/\sigma}$ so that:

$$C_1 = C_2 / \beta^\sigma (1+r)^\sigma, \quad (10)$$

where σ is the elasticity of intertemporal substitution between any two periods, or equivalently is the inverse of the consumer’s coefficient of risk aversion ρ , i.e. $\sigma = 1/\rho$. The lower the ρ (i.e. the higher the σ), the less willing agents will be to give up present

consumption for future consumption in response to a change in interest rate (or any other factor that lowers the cost of future consumption relative to present consumption). In other words, the higher ρ , the flatter the consumption path the agent will choose. Thus, equations (9) and (10), together with an investment equation relating investment to interest rates (see below), pin down period 1 saving and current accounts in the two-region world economy:

$$CA_1 = S_1 - I_1 = Y_1 - C_1 - I_1; \quad CA_1^* = S_1^* - I_1^* = Y_1^* - C_1^* - I_1^* \quad (11)$$

$$CA_1 + CA_1^* = 0. \quad (12)$$

We are now in a position to characterize how the global interest rate moves in response to a variety of shocks. In what follows, we consider three types of shocks that have prominently featured in the current debate, namely:

- A productivity shock in either the advanced or the emerging region, as well as a common global shock that hits the two regions symmetrically;
- A positive exogenous shock to world oil prices;
- A positive shock to emerging region's saving which, inter alia, can be traced to a decline in the subjective discount rate or a rise in "patience" in the emerging region.

A. Effects of Lower Expected Productivity

In the context of the proposed model, this can be readily captured by a downward shift in the advanced region's productivity parameter A in the second period, i.e., $dA_2 < 0$.

Since under no capital market friction, $\frac{\partial Y}{\partial I} = r^*$, total differentiation of the latter with respect to A_2 yields:

$$\frac{dI}{dA_2} = \frac{-\alpha(K_1 + I_1)^{2\alpha-1}}{A\alpha(\alpha-1)}, \quad (13)$$

which is strictly positive if $\alpha < 1$. Thus, the investment schedule will shift downwards, thus tending to depress r^* .

What happens to saving in both regions is crucial, however, to determine the net effect on r^* . Three standard effects play out. One is a pure wealth effect arising from the fact that output in the second period will be discounted by a lower r^* , implying a wealth gain and hence lower current saving. This is further reinforced by a substitution effect associated with falling interest rates (as the investment schedule shifts down) which fosters current consumption. But there is also an income effect: if consumers are moderately risk averse (as most empirical estimates suggest), they will compensate for a lower A_2 (and hence lower Y_2) by saving more

in the first period.²² This implies that the saving schedule of the advanced region will tend to shift downward, further depressing r^* . Yet, if the negative productivity shock is limited to the advanced region (AR henceforth) and under the sensible assumption that the capital stock in period 2 is higher than output in period 1, the following result follows:

Proposition 1. *Under symmetric preferences and technology between the two regions, expected lower productivity in the advanced region implies a lower global interest rate. The effects on current accounts are, however, ambiguous.*

Proof: As discussed in the text, global equilibrium implies that:

$$0 = Y_1 - C_1 - I_1 + Y_1^* - C_1^* - I_1^* . \quad (14)$$

Using the consumer Euler equation (10) and the budget constraint (9), we can express current period's consumption as:

$$C_1 = \frac{1}{\beta^\sigma (1+r)^{\sigma-1} + 1} [Y_1 - I_1 + (Y_2 + I_1 + K_1)/(1+r)] , \quad (15)$$

where $Y_2 = A_2(I + K_1)^\alpha$, which follows from assuming zero depreciation of capital. An analogous expression holds for consumption in the emerging region, C_1^* .

Making use of the envelope theorem, which allows us to abstract the effects of changes in r on C_1 via its effects on I_1 when evaluated at the optimum, we can differentiate equation (15) to obtain:

$$\frac{dC_1}{dr} = \frac{(Y_1 - C_1 - I_1) - \sigma C_2 / (1+r)}{C_2 / C_1 + (1+r)} , \quad (16)$$

with an analogous expression holding for the counterpart emerging region. Substituting equation (15) into equation (14) and after total differentiation using equation (16), this yields

$$dr^* = \frac{A_2 K_2^\alpha + [1+r^* + (1+r^*)^\sigma \beta^\sigma] \partial I_1 / \partial A_2}{\frac{\sigma}{(1+r^*)} (C_1 + C_2) - [1+r^* + (1+r^*)^\sigma \beta^\sigma] \left(\frac{\partial I_1}{\partial r^*} + \frac{\partial I_1^*}{\partial r^*} \right)} dA_2 , \quad (17)$$

²² Empirical studies of the intertemporal elasticity of consumption estimate it in the 0.3–0.5 region, implying a coefficient of risk aversion greater than one with CES preferences. This implies that savings do not respond as much to current low interest rates as to lower income in the future.

which is unambiguously positive since $\frac{\partial I_1}{\partial r^*}, \frac{\partial I_1^*}{\partial r^*} < 0$. Consequently, a positive productivity shock in one of the regions will unambiguously rise global interest rates.

To establish the second result that the impact on the current account is ambiguous, recall that:

$$CA = Y_1 - C_1(r^*, W) - I(r^*, A_2), \quad (18)$$

where W is lifetime wealth. Differentiating equation (18) with respect to A_2 yields:

$$\frac{dCA_2}{dA_2} = -\frac{dC_1}{dr^*} \frac{dr^*}{dA_2} - \frac{dI_1}{dr^*} \frac{dr^*}{dA_2} - \frac{dI_1}{dA_2} - \frac{dC_1}{dA_2}, \quad (19)$$

where the last term measures the effect of a higher end-of-period capital stock (which is assumed to be consumed at the end of the period) on period 1 consumption. Inputting the respective functional forms and taking the derivatives yields:

$$\frac{\partial CA_1}{\partial A_2} = \left[-\frac{CA_1 - \sigma C_2 / (1+r^*)}{1+r^* + C_2 / C_1} - \frac{dI}{dr^*} \right] \frac{dr^*}{dA_2} - \frac{dI_1}{dA_2} - \frac{K^\alpha / (1+r^*)}{1+(1+r^*)^{\sigma-1} \beta^\sigma}. \quad (20)$$

Notice that, with the advanced region running current account deficits (i.e. $CA_1 < 0$), the term in brackets in equation (20) is unambiguously positive (since $dI/dr < 0$). Since dr/dA_2 is positive (as shown above) and the last two terms of equation (20) are negative, it is not possible to sign the direction of the effect on purely theoretical grounds. It is possible, for instance, that if the elasticity of global interest rate to the productivity shock (dr/dA_2) is sufficiently high, there will be a strong negative wealth effect on the advanced region (which runs current account deficits) which will induce a contraction in consumption greater than the outward shift in the investment schedule, thereby leading to a current account improvement. But this is not warranted, since the net effect will therefore depend on the relative elasticities.

B. Oil Shocks

Although this simple one-good setup does not allow us to explicitly model the effects of oil shocks, a similar intuition applies. Since the advanced region (AR) is a large net oil importer, an exogenous rise in world oil prices is equivalent to a negative shock to A_2 insofar as it represents an inward shift in the production possibility frontier (see Bruno and Sachs, 1985, for a three-factor model that yields this very result). While in a two period setup it is not possible to discern the temporary vs. permanent components of such shocks, most observers see at least part of the recent rise as permanent. So it effectively implies a negative wealth loss in present value terms. The main qualification, relative to the “pure” TFP shock above is that saving in oil producing countries (which can be suitably grouped as part of the ER

group) will rise and their current accounts typically improve, implying that the AR is likely to be allowed to run larger current account deficits. Thus, the following result obtains:

Proposition 2. *A higher global oil price implies a lower global interest rate. The effect on the current account of the advanced region remains ambiguous (being dependent on the various elasticities) but more likely to lead to a further current account deterioration.*

Proof: Direct analogy with proposition 1.

C. Time-Preference Shocks

The model is also suitable for us to look at the effect of time preference shocks that may be important independent drivers of the rise in the emerging region's (ER) saving. As discussed in the text, these shocks can ultimately stem from diverse channels. In the context of our model, they can be all captured at once by a corresponding shift in the impatience parameter δ^* (recall that $\beta^* = 1/(1 + \delta^*)$) in the emerging region. The following result obtains:

Proposition 3. *All else constant, a decrease in the subjective discount rate in the emerging region will lower global interest rates, boost consumption and widen the current account deficit in the advanced region, and improve further the current account balance in the emerging region.*

Proof: The displacement effect on the ER saving schedule can be easily derived from the above model by expressing S_1^* as a function of the exogenous parameters as well as the endogenously determined global interest rate r^* :

$$S_1^* = Y_1^* - \frac{1}{1 + \beta^{*\sigma} (1 + r^*)^{\sigma-1}} \left[Y_1^* + (1 - \gamma) K_1^* + \frac{1 - \alpha^*}{1 + r^*} \left(\frac{\alpha^*}{r^*} \right)^{\alpha^*/(1-\alpha^*)} A_2^{*1/(1-\alpha^*)} \right]. \quad (21)$$

Clearly, a time-preference shock where consumers in the emerging market region become more "patient," implies that β^* so S_1^* will shift outward on impact, putting a downward pressure at global rates. Since nothing happened to productivity, the investment schedules in the two countries will stay put. Yet, investment will increase in both countries as a lower global interest rate will move investment up along the respective schedule. Since nothing happened to changes in the time preference in AR, then the saving schedule will also stay put but saving will decline, as clear from the following expression:

$$\frac{dS_1^*}{dr^*} = - \frac{CA_1 - \sigma C_2 / (1 + r^*)}{(1 + r^*) + \beta^{*\sigma} (1 + r^*)^\sigma} > 0 \quad \text{if} \quad CA_1 < 0. \quad (22)$$

Since saving declines and investment increases in the advanced region, then its current account will widen. To maintain global equilibrium, it must be then that the ER's current account will improve, which in turn implies that saving will rise relative to investment.

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