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Probabilistic Sustainability of Public Debt: A Vector Autoregression Approach for Brazil, Mexico, and Turkey

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Probabilistic Sustainability of Public Debt:
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Abstract

This paper examines the sustainability of fiscal policy under uncertainty in three emerging market countries, Brazil, Mexico, and Turkey. For each country, we estimate a vector autoregression (VAR) that includes fiscal and macroeconomic variables. Retrospectively, a historical decomposition shows by how much debt accumulation reflects unsustainable policy, adverse shocks, or both. Prospectively, Monte Carlo techniques reveal the primary surplus that is required to keep the debt/GDP ratio from rising in all but the worst 50 percent, 25 percent, and 10 percent of circumstances. Such a value-at-risk approach presents a clearer menu of policy options than currently used frameworks.

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I. INTRODUCTION

A *sustainable* fiscal policy is often defined as one that can be continued into the future without modification—no adjustment of the primary surplus and no default (by inflation or otherwise). At a minimum, the intertemporal solvency criteria must be satisfied. More often, sustainability has come to mean that the debt stock (or its ratio to output) does not rise.

In developing countries, it may be difficult to design fiscal policy in such a way that no further modification will ever be necessary. Despite the best intentions of policymakers to stabilize the debt, there is some chance that adverse movements of interest rates, exchange rates, output, and other key variables may cause persistent increases in government debt. In this sense, sustainability is *probabilistic*. Accordingly, fiscal authority may seek to reduce the probability that such adjustments will be necessary, through further increases of the primary surplus. A primary surplus that reduces the probability of future adjustments to less than 50 percent will, on average, *reduce* the debt.

This paper examines the sustainability of fiscal policy under uncertainty in three emerging market economies, namely Brazil, Mexico, and Turkey. Sustainability is assessed both *retrospectively* ("If historical policies were to be continued into the future, would fiscal policy be sustainable—or will a modification of policies be required?"), and *prospectively* ("What policies should be undertaken today in order to prevent the need for further adjustments in the future?")

Other retrospective assessments of fiscal sustainability do not emphasize uncertainty. The *fiscal gap* calculations of Blanchard and others (1990) and Talvi and Végh (2000) (see also Croce and Juan-Ramón (2003)) tell us how high a primary surplus must be in order to ensure sustainability. Such accounting-based (not econometric) calculations typically assume full knowledge about certain key variables, namely long-run GDP growth and interest rates.

Econometric *solvency tests* introduced by Hamilton and Flavin (1986) (later extended by Wilcox (1989), Trehan and Walsh (1990, 1991), Hakkio and Rush (1991), and others) are also retrospective. They tell us whether the historical trajectory of fiscal data can be sustained into the future—but not how shocks to key variables will affect debt accumulation.

Prospectively, sustainability assessments have most often been implemented with accounting (not econometric) frameworks. Such approaches introduce uncertainty in a rudimentary way. For example, the International Monetary Fund (2003) advocates a stress test approach that examines outcomes of a fiscal program in the event of adverse shocks certain key variables in isolation (interest rates or growth economic growth).

However, for a prospective assessment of fiscal sustainability under uncertainty, an econometric approach may be better suited than an accounting one to model the interactions of key variables—and thus their *joint* impact on debt accumulation. For this reason, several authors (see, for example Hoffmaister and others (2001); Garcia and Rigobon (2004); Hostland and Karam (2005); Celasun, Debrun, and Ostry (2006); and Penalver and Thwaites

(2006) have proposed the use of multivariate stochastic simulations of future debt behavior (potentially based on an econometric model)).
Our approach to sustainability, both retrospective and prospective, differs from previous work. Retrospectively, beginning in some base period, the evolution of debt is attributed to either a baseline policy or accumulated shocks—movements of certain key explanatory variables that were not anticipated at the base period. A simple (near) vector autoregression (VAR) model yields such *historical decompositions*. Hence, if under the baseline forecast, the debt/GDP ratio does not rise, fiscal policy is sustainable in the way discussed by Blanchard and others (1990) and Talvi and Végh (2000). To the extent that adverse (beneficial) shocks to *nonpolicy variables* (those not directly controlled by the fiscal authority) contribute to debt increases (decreases), the country is said to be "unlucky" ("lucky"). Likewise, shocks to *fiscal policy itself* (that is, the primary deficit) may be thought of as departures from an implicit fiscal policy rule (possibly including discretionary policy).

For the prospective analysis, we then simulate this model. This aspect of our work is similar to papers by Garcia and Rigobon (2004) and Celasun, Debrun, and Ostry (2006), Penalver and Thwaites (2006). However, their work is positive in nature. They present forecasts of debt—the mean value along with upper- and lower-confidence intervals that increase with time ("fan charts").

However, our work takes a more normative tack. We link such "fan chart" forecasts to an objective function (similar to Tanner and Carey (2005)) that summarizes the maximum fiscal adjustment that a country is willing to target in order to avoid further increases in debt—probabilistically, over a given horizon. Thus, we thus calculate the average primary surplus required to stabilize debt with probability 90 percent (and, where applicable, 75 percent) for one- to five- year horizons.[2]

Of course, when policies change, so might the behavior of market participants—in ways not captured by the econometric model. This is the "Lucas critique." For example, the level of the primary surplus may itself affect interest rates (both level and volatility) in a way not captured in the data. Such issues may be addressed on an ad-hoc basis; this defect is also inherent in other extant debt sustainability frameworks. While it would be better to address the Lucas critique in a full-fledged general equilibrium model of debt sustainability, such models are still in their infancy.

The paper is organized as follows. In Section II, we review previous work on fiscal sustainability. In Section III, we provide an overview of our methodology. In Sections IV, V, and VI, we analyze the cases of Brazil, Mexico, and Turkey. Section VII concludes.

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[2] Hoffmaister and others (2001) and Koeva (2005) present similar policy tradeoffs for Costa Rica and the United Kingdom, respectively. Our work may be thought of as a value-at-risk approach to fiscal policy. For other work in this vein, see Kopits and Barnhill (2003) and Adrogué (2005).

II. Fiscal Sustainability: Some Previous Work

In this section, we briefly discuss some previous work on fiscal sustainability. In Section A, basic identities are presented. In Section B. previous work on retrospective sustainability is discussed. In Section C, previous and current work on prospective sustainability is discussed.

A. Basic Identities

Any notion of fiscal sustainability should begin with public sector's budget constraint. In any period, this is:

$$b_{t-1}(1+r) + \gamma_t - \tau_t = b_t \tag{1}$$

where b is real government debt, γ is noninterest expenditures, and τ are tax revenues. Forward substitution of budget constraint over an infinite horizon (1) yields the intertemporal constraint:

$$b_{-1}(1+r) - E\{\sum_{t=0}^{\infty} ps_t/(1+r)^t\} = E\{\lim_{t\to\infty} (b_t/(1+r)^{t-1}\} \tag{2}$$

where $ps_t = \tau_t$-γ_t is the expected primary surplus. The "no-Ponzi game" condition is:

$$E\{\lim_{t\to\infty} b_t/(1+r)^{t-1}\} = 0 \tag{3}$$

Now, assume that interest expenditures ($\theta_t = rb_{t-1}$) in any period have both a deterministic and a time-varying component:

$$\theta_t = \theta^P + z(\theta)_t \tag{4}$$

The primary surplus can be decomposed into three elements: one that tracks interest expenditures θ^P, a "tax gap" (Blanchard and others, 1990) and an "own" shock $z(ps)_t$.

$$ps_t \equiv \tau_t - \gamma_t = \theta^P - \kappa + z(ps)_t \tag{5}$$

In any period, the deficit is thus:

$$b_t - b_{t-1} \equiv \gamma_t + \theta_t - \tau_t \equiv \kappa + z(\theta)_t - z(ps)_t \equiv \kappa + z_t \tag{6}$$

B. Intertemporal Solvency and Tax Smoothing

Arguably, the least restrictive notion of fiscal sustainability is *intertemporal solvency*: satisfaction of conditions (2) and (3). Note that the government remains solvent even if $\kappa > 0$. This point was emphasized by McCallum (1984), who showed that a government can run a constant deficit (inclusive of interest payments) forever—and still remain solvent. Here, debt growth is less than the interest rate (r).

However, continued debt accumulation may not be desirable: to service an ever rising debt stock, the government must correspondingly increase the primary surplus over time. As one benchmark, a benevolent, far-sighted planner would wish to distribute the burden of the primary surplus evenly over time. Such a planner might set $\kappa = 0$—a policy of *debt stabilization* (Blanchard and others (1990), Talvi and Végh (1999), International Monetary Fund (2003a), Burnside (2005)).

Formally, debt stabilization closely resembles another well-known policy, namely tax smoothing (discussed by Barro, 1979). Sargent (1987, pp. 385–88) noted that, under a tax-smoothing regime, the government minimizes a quadratic loss function based on collection costs:

$$\Phi(\tau_t) = \Phi\,\tau_t^2 \tag{7}$$

subject to budget constraint (2) and no-Ponzi game condition (3). In this case, for $\varphi = r$, the Euler equation yields a well-known result, namely that taxes follow a random walk:

$$\tau_t = E_t\,(\tau_{t+1}),\ \forall t. \tag{8a}$$

However, substituting the Euler condition into budget constraint (2) yields another (and perhaps more important) implication of tax smoothing emphasized by Barro: taxes τ_t move one-to-one with total expenditures ($\gamma^P + \theta^P$) over the long run:[3]

$$\tau^P = \gamma^P + \theta^P \tag{8b}$$

where τ^P is the long-run tax rate. Note that (8b) is equivalent to the *debt stabilization* policy (κ=0). Thus, such a policy has an implicit objective function that is similar to Barro (1979) and Sargent (1987, pp. 385–88).

C. Retrospective Sustainability

A test for retrospective sustainability seeks to answer the following question: "If historical policies were to be continued into the future, would fiscal policy be sustainable—or will a modification of policies be required?" Table 1 presents a summary of past work on this topic.

Naïvely, one might simply ask whether debt growth (in real terms or relative to GDP) was positive between some initial period M and a future date $M+j$, $j>0$. If so, one might say that the fiscal policy was "unsustainable" between M and $M+j$.

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[3] After Barro's (1979) paper, other authors focused on the random-walk implication. However, Barro's own focus is the one-to-one relationship between taxes and expenditures; the random walk hypothesis is mentioned only in passing. Subsequent literature (for example Chari, Christiano, and Kehoe (1994) and Aiyagari, Marcet, Sargent, and Sepälla (2002)) indicates that tax rates will follow a random walk only if certain restrictive assumptions hold.

However, such a (tautological) conclusion ignores some important features of fiscal policy. Typically, fiscal policy (as gauged by the structural primary balance) is adjusted at intervals that are measured in years—not weeks or months. Adverse shocks that were not anticipated at some initial date *M* and are not under the control of the policy maker (interest rates, GDP growth, oil prices, etc.) may render an otherwise well-intentioned fiscal policy unsustainable. In the absence of such shocks (given the forecasts for growth, interest rates and other variables based *only on information available at period M*) might policies initiated at period *M* have been sustainable? Essentially, Blanchard and others (1990) and Talvi and Végh (1999) attempt to answer such a question. They calculate a *fiscal gap*: the difference between a country's primary surplus and that primary surplus required to stabilize the debt (real interest payments, adjusted for economic growth). If the gap at period *M* was zero—if a tax smoothing policy was then in place—we would have expected that the debt remain constant from *M* onward, *in the absence of shocks, forecast errors, and policy changes.*

Time-series econometrics methods have also been used to examine retrospective sustainability. Specifically, Hamilton and Flavin (1986), and later Trehan and Walsh (1990,1991), Hakkio and Rush (1991), and others tested for the *stationarity* of the deficit $\Delta b_t = \kappa + z_t$. Consistent with McCallum (1984), stationarity of the interest-inclusive deficit implies solvency.

In an alternative formulation, Bohn (1998, 2005), International Monetary Fund (2003b, Chapter 3), and Tanner and Ramos (2003), test for a positive relationship between the primary surplus and debt by running the regression $ps_t = \kappa + \alpha b_{t-1}$; solvency is assured if $\alpha > 0$.[4]

However, neither test constrains κ to be zero. Thus, while tests such as these may indicate that the government is solvent (either Δb is stationary or $\alpha > 0$), the debt (or its ratio to GDP) will still be growing over time if κ exceeds zero; in this case, ever-increasing primary surpluses are required to offset ever-increasing interest payments. A more stringent test might be for a one-to-one *cointegrating* relationship between tax revenues τ and interest inclusive expenditures $\gamma + rb$ (with no intercept). Several authors including Trehan and Walsh (1990,1991), Hakkio and Rush (1991), Bohn (1991) and Tanner and Liu (1994), also use such tests. Such a test is viewed as a more precise characterization of the government's effective fiscal rule– the long-run linkage between expenditures and revenues.

[4] Note that, at a minimum, for solvency, α need only be positive; tax smoothing with positive output growth requires that κ=0 and $\alpha = (r-\lambda)/(1+\lambda)$, where λ = output growth.

Table 1. Approaches to Retrospective Sustainability:
"*If historical policies were to be continued into the future, would fiscal policy be sustainable—or will a modification of policies be required*?"

Description of Approach	Citations	Remarks
1. Fiscal gap (noneconometric)	Blanchard and others (1990); Talvi-Végh (2000)	Evaluates historical primary surplus against debt stabilizing benchmark $ps^{*}=\{(r-\lambda)/(1+\lambda)\}b$, λ = GDP growth.
2. Stationarity of deficit (time-series econometric).	Hamilton and Flavin (1986); Wilcox (1989), Kremers (1989), Trehan and Walsh (1990,91), Corsetti and Roubini (1991), Hakkio and Rush (1991); Tanner (1995); extensions include Feve and Henin (2000), Martin (2000), Uctum and Wickens (2000), Arestis and others (2004).	Solvency guaranteed by stationarity of real deficit Δb, but debt may still rise (since deficit may fluctuate around non-zero mean).
3. Cointegration of revenues and expenditures (time-series econometric).	Hakkio and Rush (1991); Bohn (1991), Haug (1991), Tanner and Liu (1994), Ahmed and Rogers (1995), Quintos (1995), Telatar and others (2005), Leachman and others (2005)	Solvency guaranteed by cointegration of primary expenditures, revenues, and debt (b), but debt may still rise unless vector of coefficients is $[1,-1,r]$ with no constant or trend.
4. Link between primary surplus and debt (econometric model).	Bohn (1998, 2005), International Monetary Fund (2003b), Tanner and Ramos (2003).	Solvency guaranteed by positive relationship between primary surplus and debt ($ps_t=\kappa+\alpha b_{t-1}$, $\alpha>0$).
5. Historical Decomposition used to distinguish "policy" from "luck" (shocks) (time-series econometric).	This paper only.	For details, see text; evolution of "baseline" debt ratio corresponds to approach (1) above.

D. Prospective Sustainability

For policy makers, the question of prospective sustainability is of critical interest: "What policies should be undertaken today in order to prevent the need for further adjustments in the future?" Previous approaches to this issue are summarized in Table 2.

Here again, debt stabilization is frequently used as a benchmark. Following Blanchard and others (1990) and Talvi and Végh (2000), the government should target the primary surplus to a level that stabilizes the debt—conditional on its forecasts of GDP growth and interest rates.

Such an approach is well-suited to an economic environment with no uncertainty. However, the question of fiscal sustainability has increasingly emphasized the importance of uncertainty: exogenous shocks to nonpolicy variables that impact the evolution of public debt. For example, the IMF's Sustainability Framework (2003) includes a (noneconometric) stress test that shows how high the debt would be (relative to a baseline scenario) if the economy suffered certain adverse shocks (two standard deviations in magnitude).

More recently, several authors have proposed stochastic simulations of debt accumulation: see, for example Hoffmaister and others (2001), Garcia and Rigobon (2004), Hostland and Karam (2005), and Celasun, Debrun, and Ostry (2006). An important feature of such simulations is that the forecast variance of debt increases with the forecast horizon; this gives rise to a "fan chart" similar to those frequently used in the forecast of monetary aggregates. At this juncture, the objective function of the authority becomes crucial: the normative implications a "fan chart" for policy decisions will reflect that objective function. If the authorities' objectives are characterized by the quadratic objective function (7), such a "fan chart" would be of little interest.

However, if the objective function emphasizes the avoidance of undesirable outcomes—that is, if it is an objective function with a third moment that is, "prudence;" see Tanner and Carey (2005), the "fan chart" is of greater interest. In this case, the upper tails of the "fan chart" illustrate precisely the undesirable outcomes that the authority would want to avoid.[5] Such an objective function is discussed in greater detail below.

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[5] In a similar vein, Mendoza and Oviedo (2004) model the fiscal authority as a "tormented insurer" whose maximum sustainable debt ratio depends on the variance of growth, interest rates, and an upper limit on the attainable primary surplus. Their analysis thus tells authorities when they should restructure debt, but (in contrast to this paper) it provides little guidance regarding the magnitude of a fiscal adjustment.

Table 2. Approaches to Prospective Sustainability:
"*What policies should be undertaken today in order to prevent the need for further adjustments in the future?*"

Description of Approach	**Citations**	**Remarks**
1. Fiscal gap (noneconometric).	Blanchard and others (1990); Talvi-Végh (2000).	Primary surplus should be equal to debt stabilizing benchmark $ps^{*}=\{(r-\lambda)/(1+\lambda)\}b$, λ = GDP growth.
1a. Fiscal gap (noneconometric).	Croce and Juan-Ramon (2003).	Like approach (1) but permits gradual adjustment.
2. Stress test (noneconometric).	IMF (2003a) and subsequent country reports.	Targeted primary surplus typically aims at debt reduction. Alternative scenarios for two standard deviation shocks to interest rates, growth, etc.
3. Value-at-Risk (noneconometric).	Kopits and Barnhill (2003, applied to Ecuador).	Examines main sources of shocks to net worth.
4. Value-at-Risk (econometric).	Adrogué (2004, applied to Central American countries).	Forecasts deficit, debt.
5. Simulated debt projections, baseline policies (econometric or other stochastic model).	Celasun, Debrun, and Ostry (2006), and Garcia and Rigobon (2004); see also Hostland and Karam (2005).	Projects debt accumulation under uncertainty (means and confidence intervals, including "fan charts").
6. Simulations debt projections, baseline and adjustment policies (econometric).	This paper; see also Hoffmaister and others (2001), Guerson (2004), Koeva (2005), Penalver and Thwaites (2006).	Uses projections similar to (5), includes alternative policies, consistent with objective to avoid further adjustment for all but the worst w-percent of cases.

III. Overview of Our Methodology

The recent emphasis of uncertainty in the fiscal sustainability literature highlights the role of both deliberate policies and exogenous, non-policy shocks in the process of debt accumulation. Accordingly, our methodology to assess sustainability—both retrospectively and prospectively—is based on a simple near-vector autoregression (VAR) model of fiscal policy that helps quantify the role of policy and luck. The vector of variables ***X*** will include both the key fiscal variables (the real debt, b, and the real primary deficit, pd) and nonpolicy variables (interest rates, exchange rates, industrial output, oil prices).[6]

As summarized in Figure 1 and detailed in an appendix, our time series model is developed through several familiar steps, including unit root tests of individual variables in the model, determination of optimal lag length, an analysis of issues related to model structure and identification, and other diagnostic procedures.

To isolate distinct economic regimes, we rely mainly on prior knowledge of the country; in some cases, certain statistical tests supplement such knowledge. In some cases it was possible to lengthen our dataset by accounting for regime changes with discrete intercept shifts (dummy variables chosen using prior knowledge.) Alternative permutations of sample and dummy may yield similar results; we feature our preferred choice in the text, leaving discussion of alternatives for an appendix.

A. Retrospective Sustainability

Our methodology features a well-known representation of a VAR model, namely the *historical decomposition*. We also present the (more familiar) variance decomposition. Both tell us which shocks were most important for debt accumulation.[7] However, only the historical decomposition reveals *when* such shocks occurred and whether such shocks *increased or decreased* debt.

In a historical decomposition, each element of X is expressed as the sum of: (i) a baseline projection of that variable, conditional on all information available in the base period M; plus (ii) the (orthogonal) impacts of shocks from all variables thereon, accumulated from M+1 forward. Thus, in any period $M+j$ (j=1,2,3,4...J) the change in the debt (that is, the deficit) Δb_{M+j} is:

$$\Delta b_{M+j} = \Delta b(\text{base})_{M+j} + z^*_{b1j} + z^*_{b2j} + \ldots.. z^*_{bIj} \tag{9a}$$

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[6] The system is a near (rather than full) VAR because oil prices, determined in world markets, are not affected by country-specific variables. However, results from a simple VAR yielded similar results.

[7] While less well-known than the impulse response function or the variance decomposition, the historical decomposition has been in use for some time. One of the first papers to feature this representation is Burbidge and Harrison (1985).

where $\Delta b(\text{base})_{tB+k}$ incorporates all information about the evolution of deficit that is available before time M+1, and z^*_{bij} represent the impacts of the i^{th} variable (i = 1,2,3,..I) on the deficit, accumulated from M+1 through M+j. The variables corresponding to z^*_{bij} are both policy and nonpolicy; they are discussed below. Thus, a country's debt level at the end of period M+j is:

$$(9b) \qquad b_{M+j} = b_{M+j-1} + \Delta b(\text{base})_{M+j} + z^*_{b1j} + z^*_{b2j} + \ldots.. z^*_{bIj}$$

Absent shocks, fiscal policy is sustainable over the period M+1 through M+j if:

$$(10) \qquad b(\text{base})_{M+\text{j}}/\text{GDP}_{M+j} \leq b_M/\text{GDP}_M.$$

That is, a country's policy is "unsustainable" if the debt stock rises under certainty (the baseline projection); otherwise, policy is "sustainable."

Note that observed and baseline debt need not be equal at end of the sample. It would be tempting to conclude that, since all shocks will have impacts that have a zero mean, all shocks should eventually cancel out, thus implying that $b(\text{base})_{M+J} = b_{M+J}$, where J is the end-of-sample period. If this were the case, sustainability criteria would only be of interest for periods *prior to M+J*.

However $b(\text{base})_{M+J}$ and b_{M+J} are not always identically equal. To see this, first note that the total deficit has two components: interest payments and the primary balance: $\Delta b_t = pd_t + \underline{\text{rb}}_{t\text{-}1}$. In any period t, the baseline is: $\Delta b(\text{base})_t = pd(\text{base})_t + r(\text{base})_t * b(\text{base})_{t\text{-}1}$. The observed primary deficit pd_t fluctuates about a fixed mean, namely pd(base). Positive shocks to the interest rate – $r_t > r(\text{base})_t$—will raise the debt *in the absence of an offsetting movement in the primary balance*. More importantly, cases are easily found for which interest rate shocks have impacts on the debt that do not cancel out—even though the interest rate itself fluctuates about mean value r(base).

Thus, consider a simple example in which bad interest rate shocks happen before good ones. This helps boost the debt. At the end of the horizon (period J) observed debt may exceed baseline debt. Two factors are at work: (i) the baseline (mean) primary surplus is may not be high "enough;" (ii) period-by-period responses of the primary balance to interest rate shocks may not be sufficient to neutralize debt increases.

The framework permits alternative definitions of baseline fiscal policy. Typically, pd(base) might be estimated as a fixed mean; around which fluctuations in pd reflect both exogenous shocks and responses to shocks in other variables.[8] However, such a definition might be

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[8] Regimes in which the primary surplus is invariant to shocks from other variables have been termed "fiscal dominant" or "non-Ricardian." The recent literature on the "fiscal theory of the price level" suggests that, under such a policy, monetary policy is subordinate to fiscal policy since inflation is required to balance the budget over time. However, the current

(continued)

unnecessarily restrictive. We may have prior knowledge that the mean primary surplus changed during the sample—although not as a systematic response to shocks from other variables. Such prior information might be reflected in a time trend or dummy variable.

The framework permits analysis of the effects of individual shocks. We also analyze the effect of individual shocks on the debt. We calculate the counterfactual value of the deficit at time M+j if the shocks to variable i *had not* occurred:

(11a) $$\Delta b(\text{omit } i)_{M+j} = \Delta b_{M+j} - z^{*}_{bij}$$

For any period $M+j$, the debt purged of the impact of shocks to variable i thus evolves according to:

(11b) $$b(\text{omit } i)_{M+j} = b(\text{omit } i)_{M+j-1} + \Delta b_{M+j} - z^{*}_{bij}$$

With respect to the i^{th} shock to a *non-policy variable* a country may be said to be "lucky" if, in the absence of that shock the debt would have been higher ($b(\text{omit } i)_{M+j} > b_{M+j}$); or "unlucky" if, in the absence of that shock, the debt would have been lower ($b(\text{omit } i)_{M+j} < b_{M+j}$).

Table 3. Taxonomy: Fiscal Policy and Shocks			
		Shocks	
		"Lucky"—Beneficial shocks help reduce debt	"Unlucky"—Adverse shocks help increase debt
Policy	"Sustainable"—Debt constant or falling under baseline projection	Sustainable and lucky: debt does not rise	Sustainable but unlucky
	"Unsustainable"—Debt rising under baseline projection	Unsustainable but lucky	Unsustainable and unlucky: debt rises

This taxonomy is informally summarized in Table 3. A country's policy is "unsustainable" if the debt stock rises under certainty (the baseline projection); otherwise, policy is "sustainable." Whether the country is "lucky" or "unlucky" depends on the impact of *nonpolicy* shocks on the debt level.

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analysis suggests otherwise: the intertemporal budget can be balanced even if the primary surplus is exogenous—so long its average value is high enough to keep debt from rising.

Of course, there will also be shocks to policy variables. Insofar as such shocks are departures from an estimated policy reaction function, they may be thought of as containing both discretionary policy (similar to Fatás and Mihov (2003)) and other random shocks to the primary surplus process (spending, tax collection). Counterfactually, if an expansionary (contractionary) shock had not occurred, the debt would have been lower (higher).

B. Prospective Sustainability/Objective Function

To prospectively assess fiscal sustainability, *simulations* of the VAR system with randomly generated shocks are presented. Specifically, the simulated value of the debt b(sim) for any period $t > J$ is:

$$b(\text{sim})_t = b(\text{sim})_{t-1} * (1 + r(\text{sim})_t) + pd(\text{sim})_t \quad \text{(11c)}$$

where simulated values of the interest rate and primary deficit, r(sim) and pd(sim) respectively, are:

$$r(\text{sim})_t = \zeta_{r0} + \zeta^*_{r1t} + \zeta^*_{r2t} + \ldots\ldots \zeta^*_{rIt} \quad \text{(11d)}$$

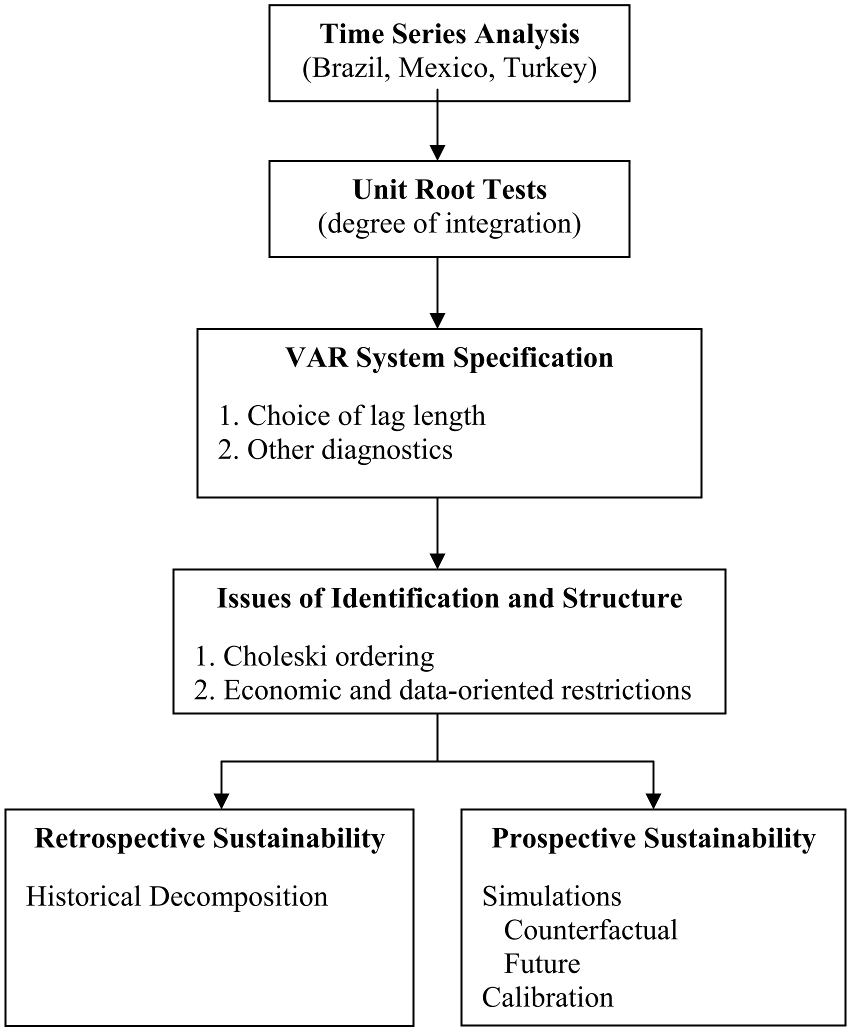
$$pd(\text{sim})_t = \zeta_{p0} + \zeta^*_{p1t} + \zeta^*_{p2t} + \ldots\ldots \zeta^*_{pIt} \quad \text{(11e)}$$

where ζ_{r0} and ζ_{p0} are the assumed mean levels of the real interest rate and primary surplus and the terms ζ^*_{rit} and ζ^*_{pit} are *simulated* impacts of shocks to variable i on the real interest rate and primary deficit, respectively.

Papers that present simulations similar to these include Garcia and Rigobon (2004) and Celasun, Debrun, and Ostry (2006). However, these papers are primarily positive in nature. Their simulations yield forecasts for the mean debt level over some horizon, conditional on a preexisting fiscal rule. Their analysis features “fan charts” designed to display a forecast variance that increases with horizon length.

By contrast, our emphasis is more normative in nature. We consider an authority whose objective would be to cushion the country’s residents from future adjustments. As an illustrative example, Tanner and Carey (2005) discuss a constant elasticity of risk aversion (CARA) / exponential objective function, specifically $\Phi(\tau_t) = -1/\phi \exp(-\phi\tau_t)$, where $\phi \leq 0$ is a “prudence” parameter. They show that minimizing such a collection cost function typically yields a long-run (expected) relationship between tax rates, expenditures, and debt:

Figure 1. Econometric Methodology Underlying Fiscal Sustainability Analysis



(8c) $$\tau_t^* = \kappa_t^* + \gamma_t + rb_{t-1}$$

where $\kappa_t^* \equiv -\phi\sigma_t^2/2 \geq 0$. Thus, for $\phi < 0$, equation (8c) describes a government with a precautionary motive: its primary surplus at any time is positively related to the variance of the tax burden ($\gamma+rb$). Of course, for $\phi < 0$, the debt ratio itself should fall over time. (That is, κ_t^* is time-varying: the variance of the tax burden ($\gamma+rb$) falls as the debt ratio falls.) And, note that, if $\phi = 0$, (8c) collapses to (8b)—the debt stabilization benchmark proposed by Blanchard and others.

The prudence parameter $-\phi$ has an alternative, common-sense interpretation, consistent with the value-at-risk approach: it summarizes the willingness to pay for precautionary cushion. If $-\phi$ equals or exceeds some critical value $-\phi(z)$, $0<z<1$, the government will be willing to levy taxes today sufficient to "cover itself" with probability ($1-z$).

Thus, for a baseline scenario, the simulations report the means, standard deviations, and fractiles (median, 75th, and 90th percentiles) of simulated debt/GDP ratios. But, we are also able to depart from the baseline scenario by modifying the mean primary surplus ζ_{p0}. Such simulations thus yield a *menu* of policy options: they reveal what primary surplus is required to keep the debt ratio constant for all but the worst *w*-percent of cases (w = 50 percent, 75 percent, 90 percent)—*over a given horizon*.[9]

Note that debt reduction is a by-product of this objective function ($\phi < 0$). Also, the horizon itself is a choice variable. *A longer horizon implies a less stringent adjustment than a shorter horizon.* That is, trying to prevent a bad outcome after (say) five years of continual debt reduction is easier (lower primary surplus) than trying to prevent that same outcome after just one year—precisely because the benefits of debt reduction are cumulative.

To inform current policy making, data should be as recent as possible. However, it may be also appropriate to present a *counterfactual* prospective analysis whose data end (and whose simulations begin) at some previous date $M+J^*$, $J^* < J$. Such an exercise thus provides a "rear view mirror" for policy makers. It asks: "If we had used our model at some date in the past—*using only information available at that date*—might we have reached different conclusions about policy?"

IV. Brazil, 2000–05

In Brazil, public debt has grown from about 30 percent of GDP during the 1990s to about 51 percent in 2005.[10] This happened in the context of several dramatic changes in economic policies, including the Real Plan of 1994 (when inflation fell dramatically), increased exposure to global market shocks (spillovers from the Mexican, Asian, and Russian crises of 1995, 1997, and 1998, respectively), and again in 1999, when a pegged exchange rate regime (within a narrow band) ended in crisis and devaluation.

The period 2000–05 is well suited for sustainability analyses like those proposed in this paper. In 2000, in the aftermath of the crisis and while both an IMF program and an inflation targeting regime were being inaugurated, the primary surplus was substantially increased, from about zero to just over 3 percent of GDP. At that time the debt/GDP ratio was about 49 percent, but it was envisaged (as reflected in IMF Staff Reports) that the fiscal adjustment would be large enough to gradually reduce the debt ratio to about 46 ½ percent of GDP by 2005.

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[9] Such adjustments are typically modest in magnitude. Hoffmaister and others (2001) perform a similar normative analysis for the case of Costa Rica, as does Guerson (2004) for Uruguay. Also, in some case, we include small adjustments on other variables, including average interest rates or economic growth. In so doing, we simply pool our own prior information regarding the means of such variables with the data's information regarding the volatility thereof.

[10] All Brazilian debt data are corrected for contingent liabilities—the "skeletons." For details, see Tanner and Ramos (2003).

However, this hope was dashed. Instead, between 2000 and 2004, the debt ratio rose. Notwithstanding additional primary adjustments during this period, electoral uncertainties helped boost the real interest rate, real depreciation, and hence debt (whether denominated in domestic or foreign currency). In 2002, the debt ratio peaked at about 60 percent of GDP. By 2004, however, both output and the Real recuperated, helping the debt ratio to fall—but not enough to meet the initial projections.

A. Econometric Preliminaries

The analysis of Brazil focuses on a vector $\boldsymbol{X}_t$ of endogenous variables:

(12) $$\boldsymbol{X}(\text{Brazil})_t = [ip_t, pd_t, \varepsilon_t, r_t]$$

where *ip* is the industrial production index, *pd* is the primary deficit, ε is real depreciation (bilaterally, against the U.S. dollar), and *r* is the (implicit) average real interest factor calculated directly from the budget constraint, namely $r_t = ([b_t+pd_t]/b_{t-1})-1$, a measure of the real cost of government borrowing. Exogenous variables include the change in oil prices and discrete intercept dummy variables, discussed below.

The frequent, dramatic regime changes in Brazil make it difficult to choose the sample period for estimation. Our principal estimates (see Table A.2, Appendix) use the period from mid-1995 to mid-2005. To account for regime shifts, we include two dummy intercept variables: one to isolate the exchange rate crisis (D=1 for t =1999:1–1999:4 and $D = 0$ otherwise), and one to isolate the more recent floating rate period (D=0 for t =1995:5–1999:4, D = 1 thereafter). We also include a time trend in the estimates (thus capturing increases in the primary surplus during 2000–05). Standard tests (Aikaike, Schwarz) suggest that 4 lags should be included (see Appendix).

We also discuss alternative estimates in the Appendix. These include results using data only post-1999 data. While these options do not yield identical results, they do confirm that the upsurge in debt from 2001–03 was largely the result of innovations to exchange rates and interest rates, rather than baseline policy.

B. Retrospective Analysis, 2000–05

Table 4a presents results of a retrospective analysis whose baseline begins in mid-2000 (M = 2000:5) and ends in mid-2005. At the beginning of this period, Brazil's debt ratio was about 50 percent of GDP. Over the period, the debt ratio rose and then fell dramatically; it returned to its initial GDP ratio by the end-period.

Movements in exchange rates and interest rates (reflecting shifts in investor sentiment) jointly explained over 97 percent of variation in the debt ratio over this period. Figure 2 illustrates this: for most of the period, the debt level purged of these two shocks (b(omit ε+r)) is close to the baseline level (b(base). When the debt peaks in September 2002, the cumulative (adverse) impact of these shocks ($b_t - b(\text{omit } \varepsilon+r)_t > 0$) was about 13 percent of GDP.

4a. Brazil: Retrospective Analysis, 2000:5–2005:6

Historical Decomposition		**Percent of GDP**
Initial Debt (b_M)		50.5
End Period Debt (b_{M+J})		50.8
Baseline Projection ($b(\text{base})_{M+J}$)		50.7
Shock Component (b_{M+J}-($b(\text{base})_{M+J}$)		0.2
Shocks: Var. and Historical Decomposition	**Percent of total variation in debt**	**$b(\text{omit})_{M+J}$ Percent of GDP**
Oil Price (*poil*)	0.75	51.2
Exchange rate+int rat (ε+r)	97.2	50.2
Industrial Production (*ip*)	1.9	51.0
Primary Deficit (*pd*)	0.13	50.9
Initial primary surplus (*ps*(base))		3.5
Debt stabilizing constant primary surplus		4.0

Source: Central Bank of Brazil and author's estimates.

Thereafter, pressures subsided and the Real recuperated. By mid-2005 (end sample) the gap ($b_t - b(\text{omit } \varepsilon+r)_t < 0$) fell to about 0.6 percent of GDP.[11]

As Figure 3 shows, innovations in output (industrial production) also affected the debt accumulation process—but to a much smaller extent than exchange rates and interest rates (under 2 percent of total variation). In 2003, debt levels purged of the output shocks are higher than observed levels ($b(\text{omit } ip)_t - b_t > 0$)—that is, during this period, industrial production shocks helped reduce the debt. In August 2003, the gap between $b(\text{omit } ip)_t$ and b_t is about 1.3 percent of GDP.

As mentioned above, there were several fiscal adjustments between 2000 and 2005: the primary surplus ratio rose from about 3.5 percent of GDP at the end-2000 to over 6 percent by mid-2005 (4.8 percent end-year). From a tax-smoothing perspective, it might have been preferable if the initial adjustment in 2000 had been somewhat larger, thus avoiding the need for subsequent adjustments. As an illustrative calculation (Table 4a, bottom) if the primary surplus had initially been adjusted to about 4 percent of GDP (rather than 3.5 percent), the debt ratio would have been stabilized—without the need for further increases in the primary surplus (relative to GDP).[12]

[11] That is, end-period values for b_t and $b(\text{base})_t$ are very close. In an appendix, we note that such is not the case if the "crisis dummy" (D=1 for t =1999:1–1999:4 and $D = 0$ otherwise) is omitted.

[12] As mentioned in the text, fiscal adjustments from 2000–05 are captured with a time trend. However, the counterfactual is derived from estimates that do not include a time trend. Note

(continued)

Figure 2. Brazil: Real Public Debt Purged of Exchange Rate and Interest Rate Shocks
(b(omit ε,r); Units are Millions of Constant *Reais* (1995=100))

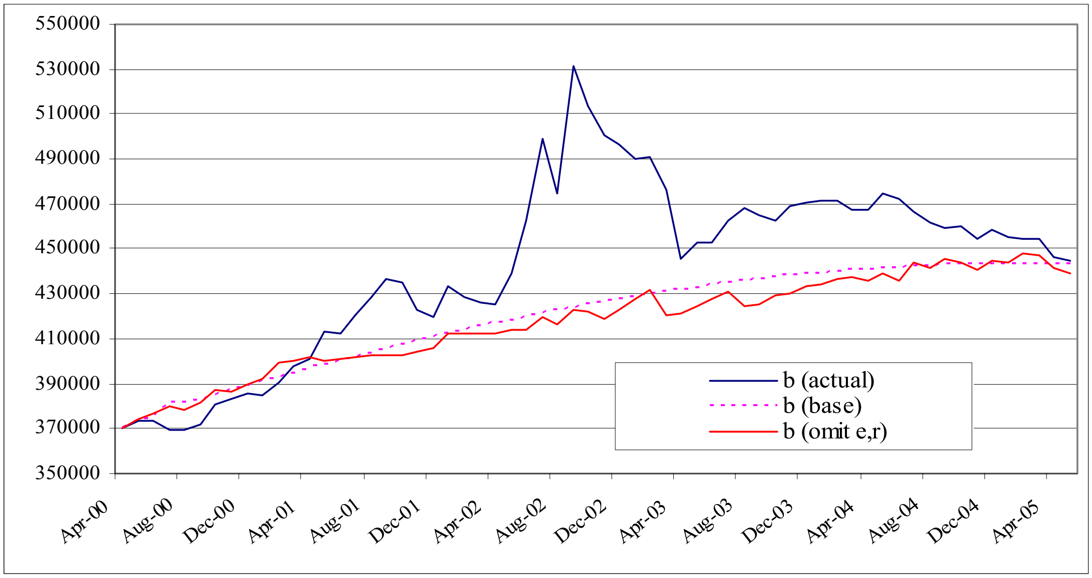
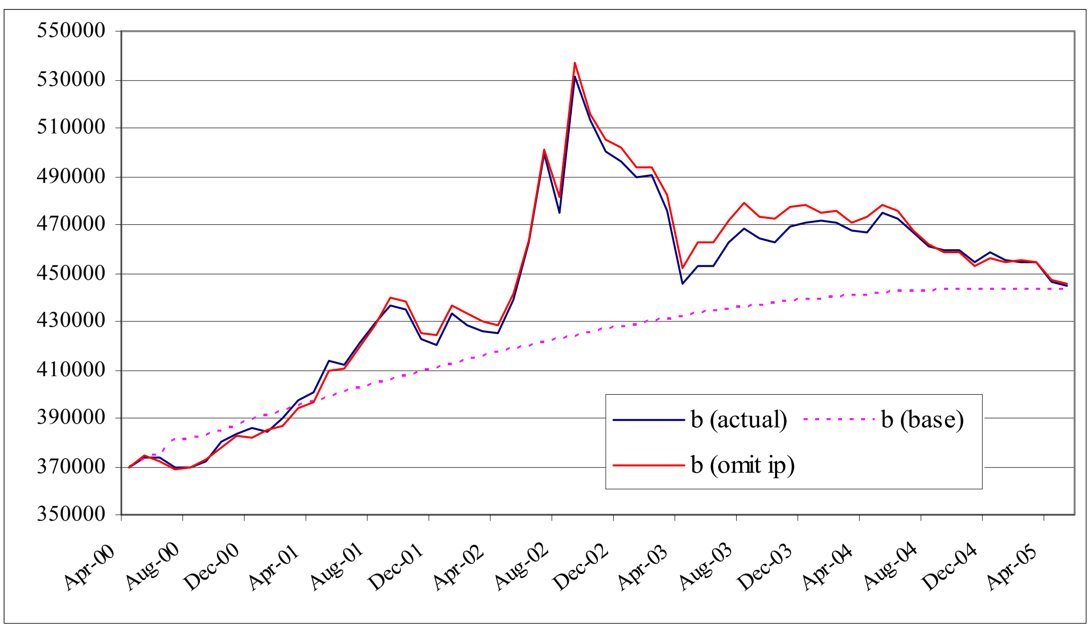


Figure 3. Brazil: Real Public Debt Purged of Industrial Production Shocks
(b(omit ip); Units are Millions of Constant *Reais* (1995=100))



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that the counterfactual mean primary surplus is calculated in constant price Reais; accordingly, the *ratio* of the primary surplus to GDP will fall somewhat over time.

C. Prospective Analysis from 2005 Onward

Prospective analyses are presented in Tables 4b(i) and 4b(ii). The upper portion of each table shows a baseline scenario: the primary surplus for the initial year of the simulation (2005:6) is designed to roughly correspond to the actual policy: a primary surplus of about 4 ½ percent of GDP. The second portion of both tables presents the menu alternatives for probabilistic debt sustainability.

Table 4b(i), presents a scenario in which the real interest factor is assumed to 12.8, in line with recent history. Mean GDP growth is assumed to be just under 4 percent; this reflects the optimistic side of current market assessments (for example, Deutsche Bank (2006)).[13]

Simulations (1000 draws) reveal a modest increase in the mean debt/GDP ratio, from 51.4 percent in 2005 to 54.7 percent in 2010. They also show the probability of less desirable outcomes: by 2010, the debt ratio exceeds 64.2 percent and 79.3 percent with probabilities of 25 percent and 10 percent, respectively.

The second portion Table 4b(i) presents a menu of policy alternatives. The first line shows that government must run a primary surplus of at least 5 percent of GDP if it wishes to stabilize the debt on average, over any horizon. The third line shows the surplus that would be required to keep the debt from rising with probability 75 percent. Note that, as the horizon increases, the required primary surplus falls. For a one-year horizon, such an objective would require a 9.3 percent primary surplus; on average, the debt ratio would fall to 47.3 percent of GDP (line 4). By contrast, applying this objective over a 5-year horizon requires a primary surplus of only 6.7 percent of GDP; on average, the debt/GDP ratio falls to 42.8 percent in 2010. *This finding demonstrates that the benefits of debt reduction accumulate over time (as mentioned above).* The fifth line shows the surplus that would be required to keep the debt from rising with probability 90 percent. Applying this objective over a 5-year horizon implies that the primary surplus must be 8.6 percent of GDP; on average, the debt/GDP ratio falls to 31.4 percent in 2010.

Alternatively, Table 4b(ii) presents a more moderate scenario. Interest rates are lower, about 8½ percent (average) over the period, reflecting survey data from 2005. Also, in this scenario, GDP growth is also lower—about 3 ½ percent (consistent with recent IMF estimates). Under these assumptions, by 2010, average debt falls to 39½ percent. By end-of-horizon, the probability that debt does not rise exceeds 90 percent. The table also shows the adjustment required to stabilize debt with a 90 percent probability for shorter horizons. For a three-year horizon, for example, the required initial primary surplus is 5 percent of GDP.

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[13] Note that, since expectations can diverge from ex-post realizations, the primary surplus required for debt stabilization for a given level of debt may differ between retrospective and prospective analyses.

A caveat should be placed on these results. Under Brazil's recent debt management strategy, the fraction of debt that is denominated or indexed to the U.S. dollar has dropped substantially. This may reduce exchange rate risk, but only to the extent that exchange rate shocks are not transmitted to interest rates (that is, deviations from uncovered parity). In fact, some (unreported) simulations wherein exchange rate shocks were omitted yielded results very close to those presented in Table 4.

D. Counterfactual Prospective, 2000–05

Table 4c provides another "rear view mirror:" it summarizes what the model might have said if it had been used in 2000—using only the data then available.[14] The upper part of the table indicates that the assumed primary surplus (3.5 percent of GDP for the initial year and slightly less thereafter) might not have been perceived to stabilize the debt beyond the initial year.

Table 4b(i). Brazil: Prospective Sustainability from 2005:6 Onward; Higher Interest Rate
First year primary surplus/GDP *ps* = 4.5 %

	Time Horizon				
Statistics	**1 Year**	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Mean	52.19	52.51	53.05	53.65	54.72
Standard Deviation	6.64	9.95	13.39	16.08	19.35
Median	51.82	51.93	51.41	51.28	51.73
75th Percent	56.63	58.55	61.29	62.30	64.18
90th Percent	60.79	65.98	70.38	74.87	79.31

	Time Horizon				
	1 Year	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Stabilizing debt (*b)* with probability:					
50 %; Requires initial *ps* of:	5.44	5.17	5.14	5.14	5.20
▸ average debt ratio, end of horizon	51.40	51.40	51.40	51.40	51.40
75 % Requires initial *ps* of;:	9.31	7.78	7.41	6.85	6.66
▸ average debt ratio, end of horizon	47.30	45.75	43.85	43.57	42.79
90 %; Requires initial *ps* of :	12.92	10.55	8.75	8.92	8.60
▸ average debt ratio, end of horizon	43.52	39.79	39.41	34.13	31.38

[14] The dummy variable in the equation here applies to 1999:1–1999:3—the exchange rate crises and its immediate aftermath only. Also, in the simulations, mean output growth is adjusted to roughly conform to expectations held at that time.

Table 4b(ii). Brazil: Prospective Sustainability from 2005:6 Onward; Lower Interest Rate
First year primary surplus/GDP *ps* = 4.5 %

	Time Horizon				
Statistics	**1 Year**	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Mean	48.5	46.3	44.1	41.7	39.5
Standard Deviation	3.8	5.3	6.7	7.5	8.4
Median	48.4	45.9	43.6	41.3	38.9
75th Percent	51.0	49.6	48.6	46.1	44.5
90th Percent	53.5	53.7	53.1	51.7	50.2

	Time Horizon				
	1 Year	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Stabilizing debt (*b*) with probability:					
90 %; Requires initial primary surplus of:	6.6	5.5	5.0	4.6	...
► average debt ratio, end of horizon	46.4	44.2	42.5	41.6	...

Rather, over a 5-year horizon (2000 through 2005) the mean debt/GDP ratio rises from 50.4 percent to 55.3 percent. More importantly, the simulations suggest more dramatic increases in the debt ratio might have been foreseen: simulated debt exceeds 64.2 percent and 79.3 percent with probabilities of 25 percent and 10 percent, respectively.

The bottom part of the table suggests that somewhat more stringent fiscal adjustment would have been necessary to insure against sharp increases in the debt/GDP ratio that did occur. For a 5-year horizon (by 2005), primary surpluses of about 4.2 percent, 5.4 percent and 6.5 percent would have stabilized the debt with probabilities of 50 percent, 75 percent, and 90 percent respectively.

Table 4c. Brazil: Counterfactual Prospective Sustainability from 2000:5 Onward
First year primary surplus/GDP *ps* = 3.5

	Time Horizon				
Statistics	**1 Year**	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Mean	50.43	51.23	52.27	53.54	55.26
Standard Deviation	4.37	6.41	8.48	10.03	12.12
Median	50.22	51.15	52.15	53.11	54.33
75th Percent	53.47	55.23	57.73	59.44	62.08
90th Percent	56.29	59.66	63.44	66.76	70.82

	Time Horizon				
	1 Year	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Stabilizing debt (*b*) with probability:					
50 %; Requires primary surplus of:	3.21	3.81	4.01	4.10	4.18
▸ average debt ratio, end of horizon	50.76	50.13	50.66	50.93	51.42
75 %; Requires primary surplus of:	6.21	5.62	5.54	5.38	5.36
▸ average debt ratio, end of horizon	47.62	46.74	45.60	45.12	44.49
90 %; Requires primary surplus of:	8.65	7.43	7.00	6.62	6.50
▸ average debt ratio, end of horizon	45.08	42.84	40.76	39.47	37.77

V. MEXICO

Mexican public debt contains both a traditional component and a sizable augmentation. Over the past two decades, the traditional component from almost 100 percent of GDP in the late 1980s to around 18 percent of GDP in 2005. This latter element, which includes liabilities associated with bank bailouts and development borrowing, became important after the crisis of 1994–95. When summed, the two elements reached about 45.3 percent of GDP in 2005. Since only the traditional component of public debt is available on a monthly basis, the retrospective analysis covers only this measure. However, the prospective analysis highlights the augmented measure.[15]

A. Econometric Preliminaries

Estimates reported in Table A.3 (Appendix) use monthly data from mid-1997 to mid-2005. (This period thus omits both the 1994 crisis and its aftermath.) For Mexico, the vector $\boldsymbol{X}(\text{Mexico})_t$ is defined:

(13) $$\boldsymbol{X}(Mexico)_t = [ip_t, pd_t, \varepsilon_t, \Delta b, r_t]$$

[15] Note that we have omitted estimates of implicit debt of public social security systems. For further details, see Bauer (2002).

where Δb_t is the real operational deficit (that is, the change in real debt), and r_t is the real interest rate as traditionally defined $(1+i)/(1+\pi)-1$. As detailed in the Appendix, 6 lags are included. This specification incorporates the familiar discrepancy between overall balance ("above the line") and the change in government debt ("below the line"). (This discrepancy is also present in data from Turkey data but not from Brazil). Of course Δb_t contains information about other variables included in the VAR, namely the primary deficit and real interest rates. But, the VAR filters out precisely these effects. Since shocks to Δb_t are orthogonal to both exchange rates and real interest rates, they are treated as additional (error) component of the primary deficit. Hence, the impacts of discretionary movements in primary and operational deficits are combined (Table 5a, Figure 5).

B. Retrospective Analysis, 1998–2005

In the base period (M = 1999:5), the debt/GDP ratio was about 19 percent. By the end of the sample, the baseline forecast debt ratio was 17.7 percent (Table 5a). Thus, in the absence of shocks, the debt ratio in Mexico would have fallen slightly over the 1999–2005 period. Hence, looking at the traditional measure in isolation, fiscal policy was *sustainable*.

Table 5a. Mexico: Retrospective Sustainability, 1999:5–2005:4

Historical Decomposition		**Percent of GDP**
Initial Debt (b_M)		19.70
End Period Debt (b_{M+J})		18.10
Baseline Projection ($b(\text{base})_{M+J}$)		17.69
Shock Component (b_{M+J}-($b(\text{base})_{M+J}$)		0.41
Shocks: Var. and Historical Decomposition	**Percent of total variation in debt**	**$b(\text{omit})_{M+J}$ Percent of GDP**
Exchange rate+int rat (ε+r)	*15.5*	18.23
Deficit (Δb, includes pd)	*79.6*	17.40
Industrial Production (*ip*)	*2.2*	18.29
Oil Price (poil)	*2.8*	18.08

Source: Central Bank of Mexico and author's estimates.

According to Table 5a, over the entire period, the impacts of shocks to interest rates and real exchange rates (combined) and industrial production helped to slightly reduce the debt ratio, while oil price shocks had a small but positive impact on the debt, suggesting that oil windfalls were spent and not saved. Furthermore, Figure 4 suggests that the beneficial impacts of interest rate reductions and a stronger Peso were most important in 2001–02.

Figure 4. Mexico: Real Public Debt Purged of Exchange Rate and Interest Rate Shocks
(b(omit ε,r); Units are millions of 1995 Pesos)

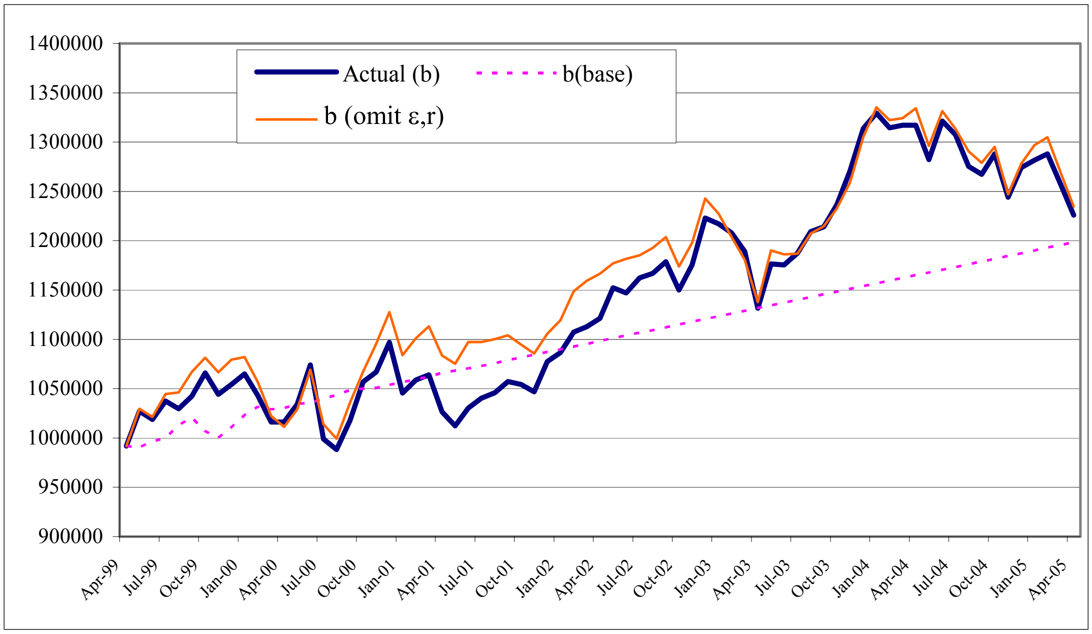
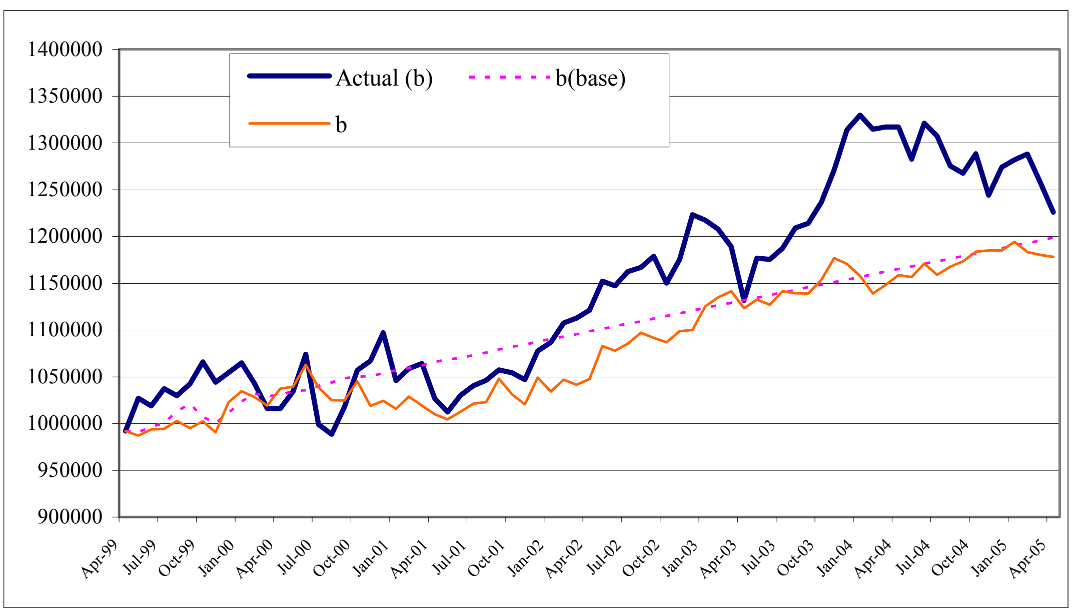


Figure 5. Mexico: Real Public Debt Purged of Deficit Shocks
(b(omit ps, Δb); Units are millions of 1995 Pesos)



However, the discretionary shocks represented a slightly positive impact on the debt. As Table 5a shows, omitting these shocks would have further reduced the debt about one-half percent of GDP. As Figure 5 shows, such shocks were especially important during 2000–01 and again in 2004–05; there were discretionary expansions of fiscal policy during these periods. Table 5a reveals that, even while revenues from the state run oil company (PEMEX) are a substantial source of public sector revenue, oil price changes played an only minor role in debt accumulation. This suggests that oil price changes were transmitted both to revenues and to spending in a way that left the primary balance unchanged.[16]

C. Prospective Analysis from 2005 Onward

For a prospective analysis, the upper part of Table 5b presents a scenario that is comparable to one presented in a recent IMF Staff Report insofar as similar interest rates, GDP growth rates, and average overall surpluses are assumed in both. The key feature of both scenarios is that a modest primary surplus—around 2.1 percent of GDP—will reduce the debt over a five year horizon from 45.0 percent of GDP in 2005 to just under 38 percent in 2010.

However, the table presents a somewhat different picture of the risks associated with such a policy. By 2010, there is a 10 percent chance that debt will exceed 46 percent.

The bottom part of Table 5b presents a menu of policy options for Mexico. For example, to stabilize the debt at the current value of 45.3 percent of GDP with a 75 percent probability, over a five-year horizon, an initial primary surplus of about 1.5 percent of GDP would be required; to stabilize the debt with a 90 percent probability requires a primary surplus of about 2.3 percent of GDP. As before, a longer horizon implies a less stringent adjustment.

Some caveats should be made regarding debt sustainability for governments that obtain revenues from nonrenewable resources or temporary bonanzas (in Mexico, petroleum). Typically, spending from a windfall should be spread out over time. Our estimations suggest, however, that this has not happened in Mexico. Rather, recent high oil revenues have helped the authorities to boost spending—*not* the surplus.

Therefore, if oil revenues fall, in order to maintain the same primary surplus, the Mexican authorities will have to (symmetrically) cut *spending*. Conversely, if the Mexican authorities wished to retain both current levels of expenditures and the primary surplus, it would have to raise additional (non-oil) revenues.

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[16] It should be noted that, if entered in *levels*, either the price of oil or real PEMEX revenues did display larger impacts on the debt than those presented here. However, since both variables are level nonstationary (unlike the other variables), such a finding is potentially spurious, and hence not emphasized.

Table 5b. Mexico: Prospective Sustainability from 2005:6 Onward
Initial period primary surplus/GDP *ps* = 2.1%

	Time Horizon				
Statistics	**1 Year**	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Mean	45.0	43.5	41.9	40.2	38.7
Standard Deviation	2.1	3.4	4.2	4.9	5.6
Median	44.9	43.3	41.5	40.0	38.1
75th Percent	46.5	45.7	44.6	43.1	42.0
90th Percent	47.7	47.9	47.6	46.6	46.0

	Time Horizon				
	1 Year	**2 Years**	**3 Years**	**4 Years**	**5 Years**
Stabilizing debt with probability:					
75 % requires primary surplus of:	3.2	2.3	1.9	1.6	1.5
► average debt ratio, end of horizon	42.7	43.1	42.6	42.3	41.9
90 % requires primary surplus of:	4.4	3.2	2.8	2.4	2.3
► average debt ratio, end of horizon	42.7	41.2	39.9	39.1	38.0

VI. TURKEY

Until recently, Turkey has suffered from chronic high inflation and fiscal imbalances. In the currency crises of 1994 and especially in 2000–01, the recognition of unrecoverable assets in the banking sector, along with defensive interest rate hikes, substantially increased debt.

However, more recently, under an IMF program that featured a primary surplus exceeding 6 ½ percent of GDP. This adjustment, along with higher GDP growth and a real appreciation of the Turkish Lira, helped to reduce the debt ratio from over 90 percent of GDP in 2000 to about 55 ½ percent in 2005–06.

A. Econometric Preliminaries

Estimations (Table A.4, Appendix) use monthly data from mid-1994 to mid-2005. For Turkey, vector $\boldsymbol{X}_t$ is defined exactly as for Mexico (see equation (13)); 8 lags are included. To account for the extraordinary nature of the currency crisis in 2001, the estimates include a crisis dummy that equals unity for the crisis period 2001:2–2001:6 and zero otherwise.

B. Retrospective Analysis

The retrospective analysis suggests that, prior to the 2001 currency crisis, fiscal policy was unsustainable. As Table 6a shows, in mid-1996, the initial debt/GDP ratio was about 43.2 percent. Note that shocks to fiscal policy itself accounts for 60 percent of the total variation in debt. In turn, most of this variability—about three-fourths—is due to shocks from below the line, including the assumption of financial sector obligations. (*In the prospective analysis below, such shocks are omitted*). Between 1996 and 1998, expansionary deficit shocks helped boost the debt. Thereafter, policy tightened somewhat. From mid-1996 through late 2000, if the deficit shocks are omitted, the debt is higher than baseline by about 1.9 percent of GDP. However, this effect was largely offset by shocks to industrial production; omitting such shocks reduces the debt by about 1.9 percent of GDP. After the 2001 crisis, volatility in exchange rates and interest rates helped increase debt levels (see Figure 6); omitting such shocks leaves debt lower than baseline by about 1 percent of GDP.

Table 6a. Turkey: Retrospective Analysis, 1997:5–2000:9

Historical Decomposition		**Percent of GDP**
Initial Debt (b_M)		43.2
End Period Debt (b_{M+J})		48.8
Baseline Projection ($b(\text{base})_{M+J}$)		49.1
Shock Component (b_{M+J}-($b(\text{base})_{M+J}$)		-0.2
Shocks: Var. and Historical Decomposition	**Percent of total variation in debt**	**$b(\text{omit})_{M+J}$ Percent of GDP**
Oil Price (*poil*)	8.3	48.1
Exchange rate + interest rate (ε+r)	20.4	49.3
Deficit (Δb, includes *pd*)	60.0	50.9
Industrial Production (*ip*)	11.3	47.1

Source: Central Bank of Turkey and author's estimates.

C. Prospective Analysis from 2005 Onward

Prospective simulations permit cautious optimism. The upper portion of Table 6b presents a scenario that broadly reflects recent assumptions about Turkey: the mean primary surplus over is assumed to be 6.5 percent of GDP in 2006 and afterwards; average economic growth is just under 5 percent and the average real interest rate is about 8 percent per annum. Importantly, the prospective scenario, like the retrospective, includes shocks to the primary balance (pd) and the real interest payments. However, as mentioned above, "below the line" shocks—such as public assumption of financial sector obligations—are prospectively assumed to be zero.

Figure 6. Turkey: Real Public Debt Purged of Exchange Rate and Interest Rate Shocks
(*b*(omit *ε*,*r*); Units are millions of 2001 Turkish Lira)

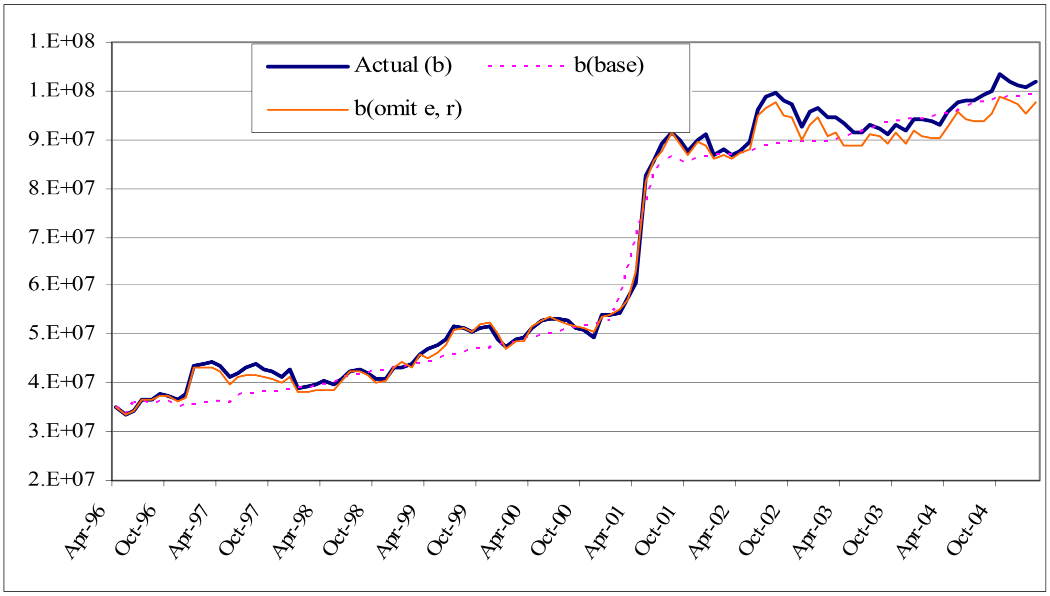

Figure 7. Turkey: Real Public Debt Purged of Deficit Shocks
(b(omit ps, Δb); Units are millions of 2001 Turkish Lira)

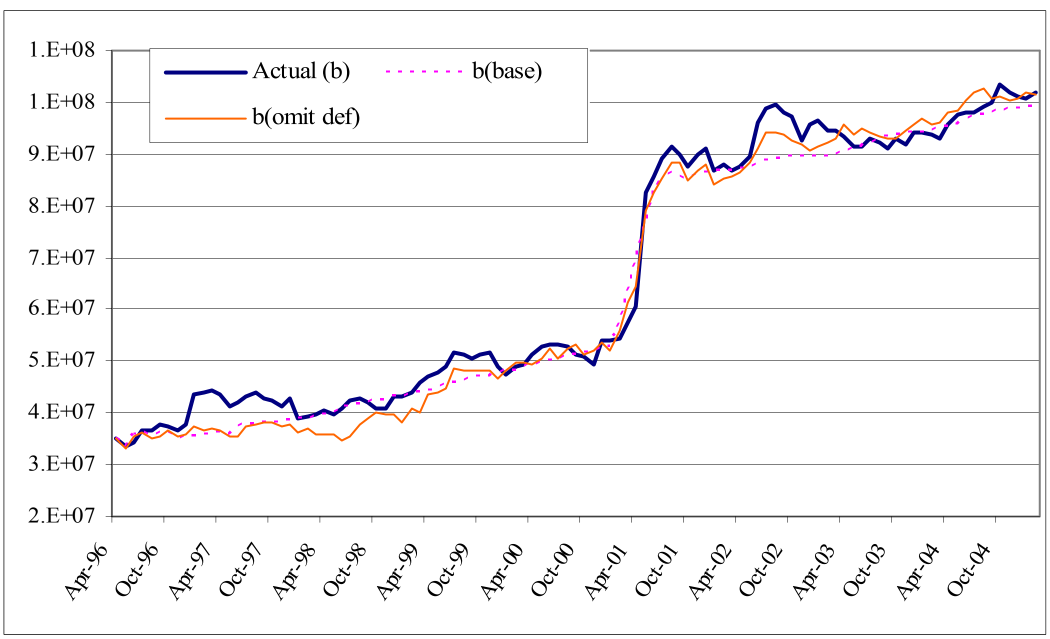

Table 6b. Turkey: Prospective Sustainability from 2005:5 Onward
Average primary surplus/GDP *ps* ≈ 6.5 Percent (Years 1 through 5)

Statistics	Time Horizon 1 Year	2 Years	3 Years	4 Years	5 Years
No Shock Scenario	49.6	45.5	40.8	36.2	31.7
Mean	51.3	49.2	46.0	42.8	39.4
Standard Deviation	6.3	9.7	11.7	13.5	15.1
Median	50.9	48.0	44.8	40.4	37.3
75th Percent	55.3	55.0	52.9	50.9	47.2
90th Percent	59.9	61.9	61.3	60.8	58.9

	Time Horizon 1 Year	2 Years	3 Years	4 Years	5 Years
Stabilizing debt with probability:					
90 %; Requires first year primary surplus of:	10.8	9.5	8.5	8.1	7.5
▸ average primary surplus/GDP, years 1–5	10.3	9.1	8.0	7.6	7.1
▸ average debt ratio, end of horizon	47.4	43.8	41.2	38.2	36.5

The Turkish case illustrates how macroeconomic shocks do not always cancel out for debt accumulation (see discussion in Section III.A). In the absence of any shocks ("No shock scenario," first line of Table 6b), the debt/GDP ratio falls from 55.5 percent of GDP to about 32 percent of GDP by 2010. However, when random shocks are included, the mean debt ratio falls, less dramatically, to about 39 percent of GDP. Regarding risks, the table reveals that over the five year horizon, there is a 10 percent probability that the debt/GDP ratio will rise to at least 59 percent of GDP.

The bottom part of Table 6b shows the primary surplus that is required to stabilize the debt with a 90 percent probability—for some horizon. As with Brazil, a longer horizon implies a smaller adjustment. For a one-year horizon, such an objective would require a primary surplus of just under 11 percent of GDP in the first year (declining thereafter, about 10 percent over the entire horizon). By contrast, for a five-year horizon, the required primary surplus must average about 7 percent of GDP over the five-year horizon.

VII. Summary and Conclusions

This paper has examined the sustainability of fiscal policy under uncertainty in three emerging market economies—namely Brazil, Mexico, and Turkey—both retrospectively and prospectively.

Our retrospective assessment differs from previous work in the way that it decomposes the effects of a baseline policy, policy shocks, and other shocks. Our prospective approach to debt sustainability has at least two advantages over currently used ones. First, an econometric framework like ours uses data to inform the policy process in a richer and more sophisticated way than accounting frameworks. Second, we believe that our framework communicates a clearer menu of options for policymakers than other frameworks that are currently used.

As a next step, the analysis in this paper might be placed into general equilibrium model. Doing so would permit a true welfare analysis of the costs and benefits to further fiscal adjustment. The optimal primary surplus and debt reduction path would, in this context, depend on the specific technology and preferences for a country.

APPENDIX

ECONOMETRIC METHODOLOGY AND ESTIMATES

A. Model Setup and Modeling Steps

The VAR system is:

(A1) $$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + (\alpha + Z_t \theta) + u_t \text{ , } E(u_t u_t^{'}) = \Sigma_u$$

where (y_t) is the vector of endogenous variables, Z_t is the vector of deterministic factors including trend, and dummy variables, and u_t is a vector of error terms. As discussed in the text, elements of (y_t) differ across country. In all cases, the vector includes the change in industrial production index (*ip*), the real primary surplus (*ps*), the percent change of the real bilateral exchange rate (ε), and the real interest rate (r):

$$y_t = (ip, pd, \varepsilon, r)^{'};$$

For Mexico and Turkey, to capture below-the-line adjustments, the real operational deficit (that is, the change in real debt Δb_t) is also included.

If the vector of endogenous variables follow a $I(1)$ process, the lag polynomial contains a unit root that can be extracted; this leaves the system with all its eigenvalues inside the unit circle.[17] Therefore, the remaining lag polynomial of the first difference matrix is expressed in a moving-average form:

(A2) $$\Delta y_t = C(L)(\alpha + X_t \beta + u_t) = \sum_{i=0}^{+\infty} \phi_i (\alpha + Z_{t-i} \theta + u_{t-i}) = \sum_{i=0}^{+\infty} \phi_i (\alpha + L^i X_t \theta + L^i u_t) \text{ ,}$$

where L is the lag operator. Both sides of (A2) are stationary and can be written as:

(A3) $$\Delta y_t = \beta + \sum_{i=0}^{q} L^i Z_t \theta + \sum_{i=0}^{+\infty} \phi_i L^i u_t$$

If the vector of endogenous variables are all stationary, a similar representation of (3) can be obtained in levels.

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[17] The VAR(p) process has unit roots if $\det(I_K - (\sum_{i=1}^{p} \phi_i z^i)) = 0$ for $z = 1$.

Thus, as a precursor for the estimations, unit root tests (Augmented Dickey-Fuller) are performed on the individual variables. *Results are presented in Table A.1.* These tests confirm that, all elements of y_t , and oil price growth (an element of Z_t) are stationary.

Estimates of the (raw) VAR are summarized for Brazil In Table A.2, Mexico in Table A.3, and Turkey in Table A.4. As in any VAR exercise, issues regarding identification must be addressed. Our initial model used a simple Choleski decomposition, with ordering: ip, ps, ε, r. Such an ordering, similar to those used in the monetary policy literature, reflects the fact that production and fiscal policy decisions (*ip*, *pd*) typically respond to other variables with a lag, but not contemporaneously. By contrast, exchange rates (ε) and interest rates (r) adjust more rapidly to a shock that occurs in a given period. In addition to this decomposition, we also tried a structural approach that imposed additional zero-restrictions; this approach yielded results similar to that of the Choleski ordering.

B. Historical Decomposition for Retrospective Sustainability Analysis

Next, to conduct a retrospective analysis of sustainability, the following historical decomposition is performed:

$x_t = \begin{cases} y_t & if \quad y_t \sim I(0) \\ \Delta y_t & if \quad y_t \sim I(1) \end{cases}$; the vector y_t is defined above.

Let M and j, are such that: $0 \le M \le M + j \le T$ where T is the total number of observations.

(A4)
$$x_{M+j} = \left[Z_{M+j} + \sum_{i=j}^{+\infty} \phi_i u_{M+j-i} \right] + \left[\sum_{i=0}^{j-1} \phi_i u_{M+j-i} \right]$$

Since $E(u_t u_t^{'}) = \Sigma_u$, for the general case, the innovations may not be uncorrelated. Thus, consider the matrix Ω which satisfies:

(A5)
$$\Omega v_t = u_t$$

Taking expectation of both sides of (A5), imposing orthogonality and normality conditions on $E(v_t v_t^{'})$ one obtains:

(A6)
$$\Omega E(v_t v_t^{'}) \Omega^{'} = \Sigma_u = \Omega \Omega^{'}$$

That is $E(v_t v_t^{'}) = I_K$, vs. are orthogonal innovations.[18]

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[18] Note also that Ω is such that $\Omega^{-1} \Sigma_u \Omega^{'-1} = I_K$.

The $\Omega\Omega'$ matrix is symmetric with $K(K+1)/2$ unknown, and imposing normality conditions on its diagonal elements leads to $K(K-1)/2$ unknown. The factorization of (6) reflects the above-mentioned Choleski decomposition (whose results are reported in this paper). Structural decompositions (that is, Bernanke-Sims) were also performed but are not reported.

An alternative formulation of equation (A5) is:

(A7) $$x_{M+j} = \left[x_{M+j}(base)\right] + \left[\sum_{i=0}^{j-1} \phi_i u_{M+j-i}\right] \text{ or:}$$

(A8) $$x_{M+j} = \begin{bmatrix} x_{M+j,1}(base) \\ . \\ . \\ . \\ x_{M+j,K}(base) \end{bmatrix} + \begin{bmatrix} x_{M+j,1}(shock) \\ . \\ . \\ . \\ x_{M+j,K}(shock) \end{bmatrix}$$

The first term in bracket is the forecast of x_{M+j} based upon information available at time M; while the component $x_{M+j,k}(shock)$ of the second term is the accumulated impact of shock to the K variables. $x_{M+j,k,k}(shock)$ can be interpreted as the impact on the k^{th} variable of unanticipated shock to variable k. Thus, the historical decomposition permits us to purge from any variable the effects of a single shock or group of selected shocks.

Table A.1. Brazil, Mexico, and Turkey: Model Specifications and Diagnostic Tests [1/]

Univariate Stationarity Tests, Endogenous Variables

	Brazil		**Mexico**		**Turkey**	
Dates	95:5–05:5		97:8–05:5		94:5–05:3	
Lags	4		4		4	
	ADF	PP	ADF	PP	ADF	PP
ip	-5.61**	-16.46**	-2.99*	-10.97**	-8.05**	-17.87**
pd	-1.51	-7.89**	-6.71**	-11.89**	-5.19**	-11.40**
ε	-4.21**	-7.21**	-4.59**	-8.42**	-5.55**	-7.48**
r	-4.20**	-10.88**	-2.84	-5.53**	-4.80**	-9.47**
od			-4.57**	-10.31**	-4.86**	-9.72**

ADF: Augmented Dickey Fuller (1979); T-test; PP: Phillips-Perron (1988) Tests.
Asterisks: "**",'*' indicate rejection of null hypothesis of nonstationarity at 99 percent and 95 percent levels, respectively, according to MacKinnon's (1991) critical values (performed in RATS package). Variables: *ip* = monthly percent change, industrial production; *pd* = real primary deficit; *e* = monthly percentage in bilateral real exchange rate (against US$); *r* = real interest rate; *od* = real operational deficit (monthly percentage change in real debt).

Table A.2. Brazil: Summary of Estimation, Near VAR 1995:5–2005:5

Degrees of Freedom = 99		**Lags = 4**			
		Dependent Variable			
		ip	***pd***	***ε***	***r***
F-Statistic:	*ip* (lagged)	2.33	0.67	1.54	0.77
	pd (lagged)	2.01	1.82	1.41	1.54
	ε (lagged)	0.84	0.41	2.92	2.54
	r (lagged)	0.61	0.82	18.99	1.38
	poil (lagged)	0.37	1.61	0.83	1.57
T-statistic:	DUM99	0.10	-6.52	1.98	2.06
R(bar)2		0.03	0.40	0.50	0.03

Table A.3. Mexico: Summary of Estimation, Near VAR 1995:5–2005:5

Degrees of Freedom = 59		**Lags = 6**				
		Dependent Variable				
		ip	***pd***	***ε***	***od***	***r***
F-Statistic:	*ip* (lagged)	1.65	0.82	2.47	1.85	0.28
	pd (lagged)	1.69	3.84	2.40	2.58	3.01
	ε (laqgged)	0.37	0.74	1.08	0.38	0.45
	od (lagged)	0.47	0.78	4.56	0.64	0.37
	r (lagged)	1.26	2.98	1.74	1.25	5.10
	poil (lagged)	1.21	0.60	0.28	0.65	1.01
$R(bar)^2$		0.05	0.26	0.30	0.04	0.36

Table A.4. Turkey: Summary of Estimation, Near VAR 1994:5-2005:3

Degrees of Freedom = 81		**Lags = 8**				
		Dependent Variable				
		ip	***pd***	***ε***	***od***	***r***
F-Statistic:	*poil* (lagged)	1.26	0.99	0.53	1.88	0.47
	ip (lagged)	5.97	0.65	1.26	1.65	1.00
	pd (lagged)	3.42	1.08	1.43	0.90	2.34
	ε (lagged)	1.48	0.48	4.02	1.13	1.27
	od (lagged)	0.85	0.98	2.20	0.65	0.63
	r (lagged)	2.55	1.74	2.32	2.61	1.71
T-Statistic	Crisis Dummy	-2.19	-0.34	4.94	0.38	-0.76
R(bar)2		0.52	0.08	0.47	0.38	0.27

C. Brazil: Alternative Estimates

As mentioned in the text, frequent and dramatic regime changes in Brazil present challenges in choosing the sample period for estimation. As reported above, the principal estimates use the period from mid-1995 to mid-2005; these estimates include a two dummy intercept variables: exchange rate crisis (*D*=1 for *t* =1999:1-1999:4 and *D* = 0 otherwise), floating rate period (*D* = 0 for *t* =1995:5–1999:4, *D* = 1 thereafter), and a time trend (to capture increases in the primary surplus during the 2000-05).

However, as summarized in Table A.5, in addition to the reported estimates (version (i)) the four alternatives are versions (ii) through (iv).

Table A.5. Brazil: Summary of Alternative Estimates (Versions (i)–(v))

Version	**Sample**	**Crisis Dummy 98:11–99:3**	**Flex Regime Dummy 99:4–05:6**	**Time Trend**
(i)*	95:5–05:6	Yes	Yes	Yes
(ii)	95:5–05:6	Yes	Yes	No
(iii)	95:5–05:6	No	Yes	No
(iv)	99:4–05:6	NA	NA	Yes
(v)	99:4–05:6	NA	NA	No

* Main version reported in text.

Figure A.1 suggests that version (iii) is substantially different from other versions, insofar as the baseline debt level as of end-2005 is substantially higher than in other versions. This probably reflects the fact that, for version (iii)—unlike versions (i) and (ii)—the dummy variable for the 1999 exchange rate crisis was omitted. (Versions (iv) and (v) use only post-crisis data). Thus, version (iii) yields a policy conclusion that differs dramatically from other versions: in the absence of appreciation of the real exchange rate that occurred after 2003, Brazil's fiscal policy would have been unsustainable (in the sense of equation (10)).

Figure A.2. shows debt levels purged of exchange rate and interest rate effects (b (omit e, r)) for versions (i)–(iv). This figure highlights the fact that, in all versions, upsurge in debt from 2001–03 was largely the result of innovations to exchange rates and interest rates—not baseline policy. However, as in Figure A.1, version (iii) is substantially different from other versions. Consistent with the version (iii) baseline in Figure A.1, this figure suggests that, by end-2005, the debt level would have been substantially higher—*had pressures on the Real not eased up*. However, such a conclusion must be considered with caution: in version (iii), the 1999 crisis is not treated as a *sui generis* event.

Figure A.1. Brazil: Observed and Baseline Debt, Alternative Estimates (Versions (i)–(v))

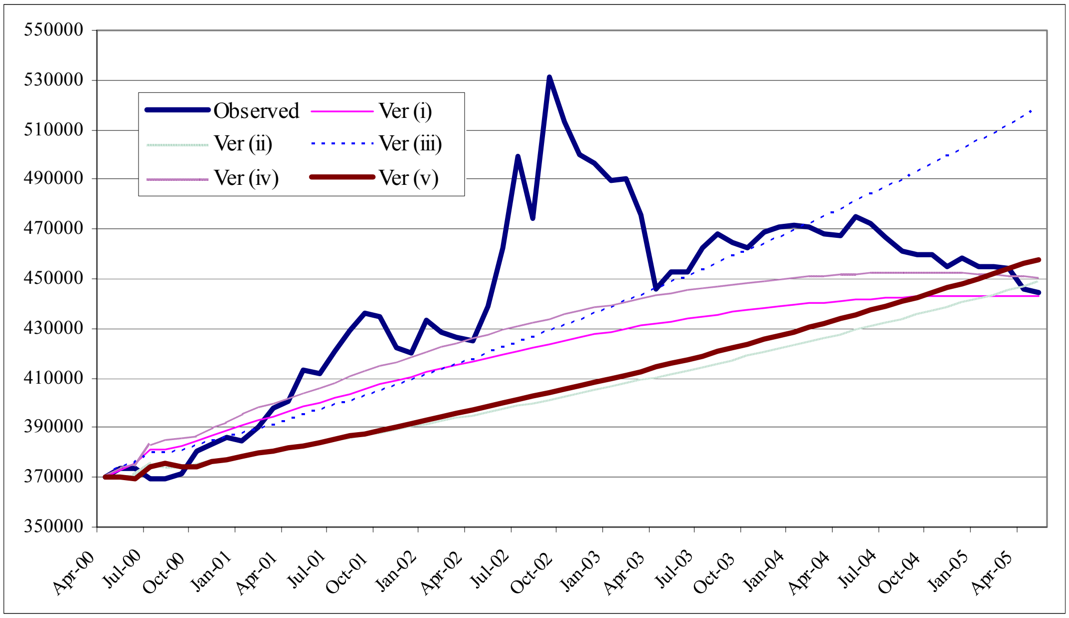
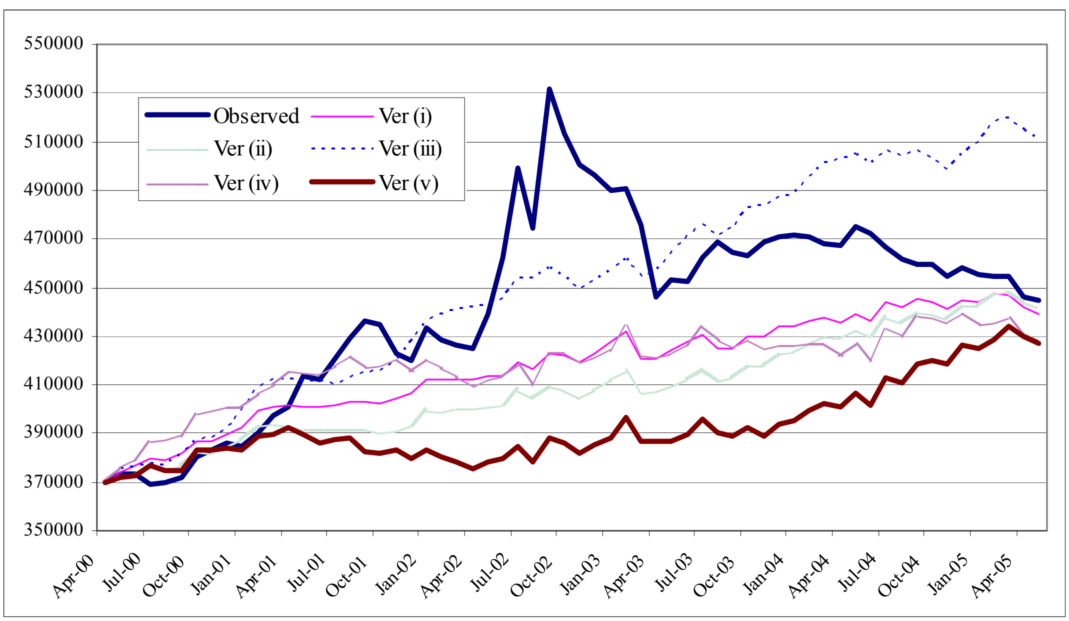


Figure A.2. Brazil: Observed Debt and "Purged" Debt b(omit ε, r) Alternative Estimates (Versions (i)–(v))



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