WP/06/264

IMF Working Paper

Banks as Coordinators of Economic Growth

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INTERNATIONAL MONETARY FUND

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IMF Working Paper

Research Department

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Authorized for distribution by Eswar Prasad

November 2006

Abstract

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This paper formally identifies an important role of banks: Banks competitively internalize production externalities and facilitate economic growth. I formulate a canonical growth model with externalities as a game among consumers, firms, and banks. Banks compete for deposits to seek monopoly profits, including externalities. Using loan contracts that specify price and quantity, banks control firms' investments. Each bank forms a firm group endogenously and internalizes externalities directly within a firm group and indirectly across firm groups. This unique equilibrium requires a condition that separates competition for sources and uses of funds. I present a realistic institution that satisfies this condition.

JEL Classification Numbers: C72, D51, G21, O16, O41

Keywords: Bank-oriented financial system, bank control, firm group, economic growth

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[1] I would like to thank Maitreesh Ghatak, Raghuram G. Rajan, Philip J. Reny, and especially Robert M. Townsend, for many helpful comments. I also benefited from comments made by Christian Ahlin, Xavier Giné, Boyan Jovanovic, Joseph Kaboski, and Nobuhiro Kiyotaki, Ernesto Revilla, Martin Schindler, as well as by seminar participants at the Federal Reserve Bank of Richmond, the International Monetary Fund, and the Universities of Chicago, Kyoto, Osaka, Rochester, Tokyo Metropolitan, Tsukuba, and Virginia.

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I. Introduction

Historical case studies identify many characteristics of bank-oriented financial systems.[2] Although heterogenous across countries, the following two features are common among them: (i) banks promoted massive industrial development across broad sectors; and (ii) banks not only provided loans, but they also monitored and controlled firms' operational decisions. Looking at the history of the United States, *The House of Morgan* (Chernow, 1990, p. xvi) describes a prototypical example:[3]

> During the pre-1913 Baronial Age of Pierpont Morgan, bankers were masters of the economy, or "lords of creation," in author Frederick Lewis Allen's phrase. They financed canals and railroads, steel mills and shipping lines, supplying the capital for a nascent industrial society. In this age of savagely unruly competition, bankers settled disputes among companies and organized trusts to tame competition. As the major intermediaries between users and providers of capital, they oversaw massive industrial development. Because they rationed scarce capital, they were often more powerful than the companies they financed and acquired increasing control over them.[4]

Salient features of industrial development in its early stage are production externalities across firms. This is described by many authors, from Marshall (1920) to Murphy, Shleifer, and Vishny (1989). Obvious examples include providers of business infrastructure such as railroads and shipping lines. These are quasi-public goods which apparently increase productivity among each other as well as raise the profitability of manufacturing firms. Externalities may be present even among firms that do not provide quasi-public goods. Among, say, automobile companies, the investment of a company induces invention of better tools and components that can be used by other companies, thereby creating a spillover effect. A similar effect also occurs when the investment creates a larger demand for a particular job, allowing individual workers to specialize more (e.g., automotive engineers) and consequently attain higher productivity.

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[2] Several terminologies have been used to distinguish financial systems in which banks dominate from ones in which stock and bond markets dominate. I use Allen and Gale's (2000) terminology: the bank-oriented financial systems as opposed to market-oriented financial systems.

[3] According to Chernow (1990), the power of J.P. Morgan remained strong until it had to spin off Morgan Stanley in 1935 because of the Glass-Steagall Act of 1933. See also a more rigorous study by Cantillo Simon (1998).

[4] For other countries, Gerschenkron (1962, p. 14) argues that banks promoted industrial development "with the closest relationship with industrial enterprises," especially in countries like Germany, where the Industrial Revolution happened later. Cameron and others (1967) compare several European countries and Japan, and conclude that banks play a vital role in allocating scarce capital to productive projects and that competitiveness of the banking sector explains differences in success in economic growth. Similar arguments are also pointed out in a more recent detailed review by Guinnane (2002) on the German banking system, by Aoki and Patrick (1994) on the Japanese banking system, and by Allen and Gale (2000) on historical comparisons of financial systems in France, Germany, Japan, the United Kingdom, and the United States.

This paper asks whether there is a specific role for banks to play in the economic growth process with production externalities. Indeed, it is identified as a decentralized mechanism to internalize production externalities among firms and thereby to facilitate economic growth. To clarify this role, I shut down the other roles of banks[5] in the model. In other words, the model assumes away any exogenous stochastic shocks, any informational problems, any illiquid projects, and any transaction costs. Of course, these other roles are important, as suggested by many theoretical and empirical studies, and this paper should be viewed as a complementary study to the existing literature, offered with the hope that it will take us one step further in understanding the complex role of banks in economic growth.

The model is a canonical growth model with production externalities, similar to Romer (1986), except that banks compete with each other in deposit and loan markets.[6] Romer's paper (1986) is a seminal paper that succinctly formalizes the economic development process with positive production externalities among firms.[7] However, the original model lacks a key historical characteristic of economic development, as it implicitly assumes that a competitive equilibrium—abstraction of anonymous security markets—prevails in capital markets and that banks neither intermediate capital nor promote growth.

From a technical point of view, this paper can be regarded as an extension of the literature on strategic intermediation to a general equilibrium growth model with production. This literature has attempted to replace the Walrasian auctioneer with strategic firms or middlemen. Townsend (1983), Stahl (1988), and Yanelle (1998) study the strategic competition of middlemen in a frictionless economy.[8] Their common concern is whether strategic intermediaries achieve the Walrasian equilibrium. Results are mixed. Townsend (1983) shows positive results in an exchange economy.[9]

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[5]The microeconomic banking literature so far has explained banks' roles as mitigation of informational problems (Townsend, 1979, and Diamond, 1984), coalition for project selection (Boyd and Prescott, 1986), economization of transaction costs, provision of liquidity (Diamond and Dybvig, 1983, and Diamond and Rajan, 2001), and diversification of risks.

[6]I maintain the dynamics of the standard growth model. In the literature, with the exception of Greenwood and Jovanovic (1990), the dynamics typically are derived from overlapping generations, which make savings behaviors—one of the immediate consequence of financial sector development—difficult to compare with the standard growth model.

[7]This growth process is originally considered by Shell (1966). After Romer (1986), subsequent growth models are more targeted to explain the growth of advanced economies that allow monopolistic competition with differentiated goods by the patent system. However, free spillover effects appear to be a key characteristic of the early stage of modern economic development. Intellectual properties were less protected then, and still are in many developing countries. Also, as described in the above-mentioned historical case studies, quasi-public goods are often provided privately.

[8]Townsend (1978) addresses a similar issue, but in an economy with transaction costs, and points out that intermediaries emerge as they economize transaction costs. Also see Yanelle (1997) for an analysis with transaction costs associated with private information.

[9]Using the same structure as Townsend (1983), Acemoglu and Zilibotti (1997) show that introducing banks in their model does not make any difference from the market-based allocation, which is not the first best due to incomplete markets assumed in their model.

In a partial equilibrium framework, given traditional demand and supply functions, Stahl (1988) shows mixed results that depend on specification of the game, and Yanelle (1998) reports a negative result; that is, the allocation is inefficient. The effects on growth of strategic banking behavior have been unclear, as strategic intermediation has not been used as the core concept of the equilibrium of growth models.[10] In sum, the intermediaries in this literature have so far delivered an allocation that is the same or inferior to the Walrasian equilibrium. This has made it difficult to explain why a bank-oriented financial system may offer any advantage over a market-oriented system.

To compare a bank-oriented financial system with a market-oriented financial system, I first formulate the Romer growth model as a game in which consumers, firms, and an auctioneer interact strategically, similar to the setup of Arrow and Debreu (1954). I show in Section II that the Walrasian equilibrium in this economy is not Pareto optimal—the same result as Romer (1986) describes. As a firm does not take into account the externalities of its investment, the Walrasian interest rate is equal to the private marginal product of capital, which is lower than the social marginal product.

I replace an auctioneer with several banks in Section III. Banks strategically intermediate capital, as they compete in both deposit and loan markets. Banks are assumed to be technologically more sophisticated as they specify both prices and quantities in their contracts; in particular, loan rates and amounts are both written in loan contracts. As such, banks can force firms to invest more than is suggested by the private marginal product of capital. Hence, potentially, the equilibrium allocation may differ from the Walrasian outcome.

Indeed, the Walrasian rate or any rates lower than the social optimal return cannot be the equilibrium deposit rate. To see this, consider what a monopoly lender would do. A monopoly lender would specify the loan contracts to receive all rents from all firms including any external effects among firms. Apparently, given the collected deposits, the capital allocation by a monopoly lender would internalize the externalities. In the deposit market, to become a monopolist lender and obtain the monopoly rents, banks bid up deposit rates to the return that a monopoly lender would obtain. As a result, no bank becomes a monopoly lender and banks' profits are zero. The deposit and loan rates and the savings and investment amounts are all set at the levels that internalize externalities. This allocation is socially optimal in the case of the commonly used production function exhibiting constant returns to accumulated capital.

There is a caveat, however. At the social optimal rate, the private marginal return is less than the social optimal rate so that firms want to invest less than the social optimal amount and obtain extra profits by free riding on external productivity gains created by investments of other firms. Here, a clear deviation strategy exists. Specifically, a bank could limit the deposit amount and lend a smaller amount of funds to a firm than the other banks that try to be a monopoly lender; then, the deviant bank-firm pair would share higher profits by investing less than others. In sum, although banks compete for the deposit share by bidding up deposit rates to the social optimal return, at this rate, banks would like to limit the deposit amounts. As a consequence, no Nash equilibrium exists in the economy in which an auctioneer is simply replaced by banks.

[10]One exception may be the second of two models described in Greenwood and Smith (1997), though it is more general model, not specific to the financial sector. They analyze a game of market formation for each specialized intermediary goods by market makers, who act like the Walrasian auctioneer.

The feature that monopoly profits can be achieved when a bank obtains a full share of deposits is robust to any institutional settings as long as there is no government intervention. As such, the problem lies in discontinuity of banks' profit function: at the social optimal rate, banks suddenly have to worry about their fund positions being too large and limit deposit amounts. Therefore, the only remedy to support an equilibrium in a decentralized economy is to introduce a further institutional setting that allows banks' loan market behaviors to be independent from their collected deposits (a *weak link between sources and uses of funds*).[11]

Assuming a *weak link between sources and uses of funds*, the unique equilibrium candidate is indeed identified—this is the first main result of this paper. Banks compete for deposits aggressively without worrying about their fund positions in the loan market. Resulting deposit rates and amounts dictate a set of possible equilibrium loan contracts. The equilibrium is characterized by the Pareto-optimal allocation in which banks internalize externalities without any explicit coordination.

It is necessary to investigate if a realistic institution provides a *weak link between sources and uses of funds*. Section IV shows an example. Apparently, introducing the interbank market[12] is necessary to break the constraint that forces each bank's loan to be strongly tied to its collected deposits. However, just introducing an interbank market is not enough, because the free-riding strategy of limiting deposit amounts still delivers a higher profit to a deviating bank so that banks would still worry about their fund positions in the loan market. Note that the interbank market does not clear when there is a deviation: When other banks stick to the contracts that would deliver the social optimal allocation, the residual supply of the interbank capital for a deviating bank-firm pair would be equal to the social optimal amount of investment, but the deviating bank-firm pair would invest less than that.

Here, an additional institutional setting, a price adjustment mechanism, is necessary for the interbank market to clear.[13] I propose a simple realistic mechanism in which banks are allowed to have a *free-recontracting* opportunity in the loan market, so that they can adjust quotes on loan terms. For example, if there were two sessions in a day (morning and afternoon), banks could freely change their morning offers of loan contracts in the afternoon, before settlement at the end of the day.

The second main result of the paper shows that this mechanism is sufficient to support the identified equilibrium candidate. Because investment by a deviant is always lower than the socially efficient level, the interbank market would not clear with any deviation. If the interbank market did not clear in the morning, the free-recontracting opportunity allows banks to change their strategy in the

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[11]This concept may be described traditionally by the term *indirect finance*, as opposed to *direct finance*—firms raise capital directly from consumers by issuing corporate bonds. With direct finance, firms take the uses of capital into account. Demand for capital, then, depends on the marginal product of capital. Indirect finance breaks this strong link.

[12]While they compete in the loan market, banks strategically choose the interbank loan rate and amount.

[13]In game-theoretic terms, this economy is described as a discontinuous game, where payoff functions are too discontinuous to support a Nash equilibrium. In this literature, finding a condition to support a Nash equilibrium is the main theme, not refining it (e.g., subgame perfection and trembling-hand perfection). This paper suggests that a slight modification of the strategy space can support a reasonable equilibrium.

afternoon session to one that gives zero profits for all banks, including the deviant. Expecting such recontracting after a deviation, at the outset no bank has an incentive to deviate from the proposed equilibrium strategy. Note that this procedure is not far from reality: it is an abstract description of possible negotiations over loan terms, while deposit contracts are settled instantly. Section V discusses the robustness of the results, and Section VI concludes.

Note that, to date, banks' specific role in economic development with production externalities has not been clearly delineated in formal theories, as previous theoretical studies on finance and growth[14] are based on banks' roles as defined in microeconomic banking theories. For example, in Greenwood and Jovanovic (1990), banks provide insurance and better project selection. With a fixed cost of participation, participation rates are determined by the wealth level of each agent, and more participation by agents in the financial system generates higher aggregate growth. In Bencivenga and Smith (1991) and Greenwood and Smith (1997), banks provide liquidity that the market cannot, thereby allowing agents to select illiquid but more profitable investments.[15] Da Rin and Hellmann (2002) is most similar to this paper in spirit. They review historical studies and argue that a big push was necessary to start industrialization and that banks helped a big push. They also propose a simple model—a static coordination game of investments (either 0 or 1 unit) by many firms—but without formulating a general equilibrium growth process.[16]

II. Model Setting and Characteristics

A. Financial System

Investment must be financed by savings. If an individual owns a firm exclusively, and only she invests in it, then no financial activity is involved. However, users and producers of capital are typically different, and converting consumers' savings to producers' capital is the fundamental role of finance.

I focus on two basic financial sector arrangements. The first is the corporate bond market. Following the typical abstraction of a market in accordance with the general equilibrium theory, I assume that

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[14]On the empirical side, many studies support the positive role of financial deepening on economic growth based on aggregate variables (King and Levine, 1993) and in industry-level studies (Rajan and Zingales, 1998a), although the true causational link is difficult to identify (Townsend and Ueda, 2006). For more references of theoretical and empirical studies, see recent reviews, for example, by Levine (2005).

[15]In their models, the production functions also exhibit positive externalities. However, the externality is not a reason for the emergence of banks.

[16]Even with a possible dynamic extension, since a bank's role is depicted in their model as a catalyst for a one-time big push, it would be difficult to explain why a bank-oriented financial system survived for many years in Germany or Japan after the industrialization process began. Moreover, in their model, ad hoc assumption of bank's market power is necessary for a bank to bring better outcome than a market, and a similar ad hoc assumption of firm's market power, if it were made, would achieve the same result.

the bond market is cleared by a Walrasian auctioneer, as illustrated below:

$$Savings \longrightarrow Auctioneer \longrightarrow Capital.$$

The other involves intermediation by banks. I assume that many banks, labeled $\{1, 2, \cdots, H\}$, strategically clear the capital market. They compete both in deposit and loan markets, as illustrated below:

$$Savings \longrightarrow \boxed{\begin{matrix} 1 \\ \vdots \\ H \end{matrix}} \longrightarrow Capital.$$

In the real world, there are several types of financial service providers such as money lenders, wealthy financiers, and large finance departments in manufacturing firms. However, according to my model, financial activity must be clearly distinguished from firms' manufacturing activity and consumers' savings decisions. If an entity borrows and lends funds, I label it a bank.[17] In this paper, financial decisions by a household are confined to deciding how much to save in available financial products. Similarly, a financial decision by a firm is to determine which financial contracts to take, among those available.

B. Demography, Technology, and Preference

Demography

The economy is populated by consumers, indexed as $i \in \{1, \cdots, I\}$, and firms, indexed as $j \in \{1, \cdots J\}$. A consumer maximizes an infinite sum of discounted period-utility. A firm borrows capital from consumers at the beginning of each period, invests it in the production process, and returns outputs equivalent to capital and interest to consumers at the end of each period. In other words, firms are established and dismantled during each period. This assumption forces firms to decide their investments period-by-period. Note that, depending on the institutional setting, additional agents, an auctioneer and banks, will be added.

Technology

Production technology is almost the same as in Romer (1986). Firms have identical technology, which exhibits Marshallian externalities; that is, the productivity of each firm depends on the average

[17] In this regard, the empirical counterpart to banks in this paper include finance departments of large manufacturing firms and conglomerates, as manufacturing firms sometimes lend funds to other manufacturing firms in the real world. This paper regards the inner teams of these firms dealing with such indirect financial services as financial intermediaries. In other words, theoretically, a firm in this paper is a production unit and is not allowed to lend capital to other production units. This functional distinction is similar to what the standard microeconomic theory does between consumers and producers—production of traded goods by a household typically is viewed as an activity of a producer, not a consumer.

capital level of the economy. I write firm j's capital as $k_j \in \overline{\mathbb{R}}_+$. Let the set of active firms at date t be Ψ_t, in which firms invest positive amounts of capital. I also define the set of active firms that is not the jth firm as $-j_t \equiv \{l : l \in \Psi_t, l \neq j\}$, and the number of firms in the set Ψ_t as $\#\Psi_t$. The population average capital for firm j is defined[18] as

$$K_{jt} \equiv \frac{1}{\#\Psi_t - 1} \sum_{l \in -j_t} k_{lt}. \tag{1}$$

Let y_{jt} denote output of firm j at date t. Given the population's average capital, firm j produces its output from capital k_{jt} at date t as

$$y_{jt} = f(k_{jt}, K_{jt}) \equiv A K_{jt}^{\eta} k_{jt}^{\alpha}, \tag{2}$$

where $A \in \mathbb{R}_+$ is the total factor productivity. Technological parameters are assumed to be on the unit line, $\alpha \in [0, 1]$ and $\eta \in [0, 1]$. Let R_{jt} denote the average gross borrowing rate of capital that firm j pays at date t. The profit function of firm j, then, is defined as $\pi^f : \overline{\mathbb{R}}_+^3 \to \overline{\mathbb{R}}$, such that

$$\pi_{jt}^f = \pi^f(k_{jt}, K_{jt}, R_{jt}) \equiv A K_{jt}^{\eta} k_{jt}^{\alpha} - R_{jt} k_{jt}. \tag{3}$$

I focus on the case of $\eta = 1 - \alpha$. This is the case of constant returns to accumulated capital because, for the social planner who treats each firm equally, each firm's capital level is viewed as the average, $k = k_{jt} = K_{jt}$, and the production function becomes linear in capital, Ak. As Romer (1986) noted, this is the only case that delivers perpetual growth and, thus, has been the main interest of the literature.[19]

Preferences

All consumers are identical in preferences. Let $\beta \in (0, 1)$ be the discount rate, $c_{it} \in \overline{\mathbb{R}}_+$ individual i's consumption in period t, and $u : \overline{\mathbb{R}}_+ \to \overline{\mathbb{R}}$ the period-utility function. To obtain internal solutions, the period utility function u is assumed to be twice continuously differentiable with the properties $u' > 0$ and $u'' < 0$, and to satisfy Inada conditions, $\lim_{c_{it} \to 0} u'(c_{it}) = \infty$ and $\lim_{c_{it} \to \infty} u'(c_{it}) = 0$.

[18]To avoid circularity associated with the finite number of firms, it is assumed that the average quantity of capital from the remainder of the firms, $-j$, is taken. Also, I treat $\overline{K}$ as the average, not aggregate level of capital, because the aggregate capital level of a country in no way affects its growth. A similar logic is used in Lucas (1993) on the scale effect of population: "[the scale effect] carries the unwelcome implication that a country like India should have an enormous growth advantage over a small country like Singapore."

[19]Jones and Manuelli (1990) show that growth models based on differentiated goods can be transformed into the Cobb-Douglas production function with externalities. As in the case of increasing returns to accumulated capital, $\eta > 1 - \alpha$, the life-time utility will explode, and thus I do not analyze this case. As for the case of decreasing returns to accumulated capital, $\eta < 1 - \alpha$, the economy has a steady state and does not grow perpetually (see Section V).

I denote a sequence of consumption as $c_i = \{c_{it}\}_{t=1}^{\infty}$. Given an initial wealth m_{i1}, consumer i's lifetime utility is given by a discounted sum of period utilities, $U : \overline{\mathbb{R}}_+^{\infty} \to \overline{\mathbb{R}}$, defined as

$$U(c_i) \equiv \sum_{t=1}^{\infty} \beta^t u(c_{it}). \tag{4}$$

Consumer i who has wealth $m_{it} \in \overline{\mathbb{R}}_+$ at the beginning of period t makes decisions on consumption c_{it} and savings s_{it} in period t. Consumption and savings must satisfy the budget constraint within each period:

$$c_{it} + s_{it} \leq m_{it}. \tag{5}$$

I assume that initial wealth, m_{i1}, is equal for all consumers, and that ownership of firms is allocated equally to all consumers at the initial date. For simplicity, I assume that the ownership structure remains unchanged over time.[20]

Let $\psi_{ij}^f \in [0, 1]$ be the ownership of the jth firm by the ith consumer in period t. The associated feasibility condition is, for all $j \in J$,

$$\sum_{i=1}^{I} \psi_{ij}^f = 1. \tag{6}$$

Let w_{it} denote the total profit income of consumer i at date t. It is defined as

$$w_{it} \equiv w_i(\{k_{jt}, R_{jt}\}_{j=1}^{J}) \equiv \sum_{j=1}^{J} \psi_{ij}^f \pi^f(k_{jt}, K_{jt}, R_{jt}). \tag{7}$$

The wealth of consumer i in the beginning of period $t+1$, m_{it+1}, consists of the profit income and the gross return on savings at date t:

$$m_{it+1} = r_{it} s_{it} + w_{it}, \tag{8}$$

where $r_{it} \in \overline{\mathbb{R}}_+$ denotes the average gross rate of return on savings for consumer i in period t.

Assumption 1 below ensures that, when the (gross) return is in the limited range, $r_{it} \in [\alpha A, A]$, the maximum exists in a consumer's problem; that is, there is a consumption sequence that maximizes $U(c_i)$, defined in (4), subject to the budget constraint (5) (see Appendix II).

Assumption 1. *(i) There exists $\overline{\sigma} < \infty$ and $\overline{c} < \infty$ such that, for $c_{it} > \overline{c}$, $d \ln u(c_{it})/d \ln c_{it} \leq 1 - \overline{\sigma}$ and $\beta A^{1-\overline{\sigma}} < 1$.*[21]
(ii) $\beta \alpha A > 1$.

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[20] Homogeneous consumers and no technological shocks provide no incentive for consumers to trade these ownership shares.

[21] The asymptotic elasticity, $d \ln u(c_{it})/d \ln c_{it}$ for large c_{it}, is less than the upperbound, $1 - \overline{\sigma}$, which restricts the highest return A to a certain range. See Brock and Gale (1969).

Economywide Constraints

The economywide resource constraints are as follows:
(i) capital must be converted from savings in each period t,

$$\sum_{j=1}^{J} k_{jt} \leq \sum_{i=1}^{I} s_{it}; \tag{9}$$

(ii) consumption and savings in period t are bounded by total output at $t-1$, and

$$\sum_{i=1}^{I} (c_{it} + s_{it}) \leq \sum_{j=1}^{J} y_{jt-1}; \quad \text{and} \tag{10}$$

(iii) equation (1), known as a fixed-point condition in the literature, applies for all $j = 1, 2, \cdots, \Psi_t$.

To focus on the allocation of the financial system, produced consumption goods y_{jt-1} are assumed to be distributed to consumers as interest income $r_{it-1}s_{it-1}$ and profit income $\psi_{ij}^{f}\pi^{f}(k_{jt-1}, K_{jt-1}, R_{jt-1})$. In a decentralized economy, it is equivalent to assume the Walrasian competitive equilibrium in the product market; but, for the sake of simplicity I assume here that consumers either consume c_{it} or save s_{it} the consumption goods without selling or purchasing them in the product market. When consumers save consumption goods, they convert them into capital goods. The only active market is the capital market, where consumers rent capital goods, possibly via intermediaries, to firms who then invest them in the production process.

C. Pareto-Optimal Allocation and Walrasian Equilibrium

To compare welfare among different institutional settings, a natural benchmark is the symmetric first-best solution (i.e., the social optimal allocation with equal treatment of all households). A detailed analysis is reported in Appendix III. The summary results are as follows: (i) a benevolent social planner internalizes externalities by setting each firm's capital at the same level, $k_t = k_{jt} = K_{jt}$, so that the planner will face the linear Ak production technology; and (ii) capital is accumulated under the social marginal return A.

In the Walrasian economy, investment is financed by bonds, issued in competitive bond markets where an auctioneer matches supply and demand for capital by altering coupon rates (see again Appendix III for a detailed analysis). Each firm determines its investment level based on its private marginal return, as it does not take into account the spillover effect on others. The equilibrium coupon rate, which is the return for bond investors, turns out αA. This is lower than the social marginal return A, so that capital will be accumulated at slower speeds in the Walrasian equilibrium than in the Pareto-optimal growth path. With a constant relative risk aversion (CRRA) utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma \in \mathbb{R}_{++}$, the closed form solution can be obtained. Specifically, the growth rate in the Walrasian equilibrium is $(\beta\alpha A)^{1/\sigma}$, smaller than $(\beta A)^{1/\sigma}$ in the Pareto-optimal allocation. These results are essentially the same as in Romer (1986).

III. Unique Equilibrium Candidate with Strategic Intermediation

Without an auctioneer, the capital market in this economy inherently suffers from theoretical difficulties studied in the discontinuous game literature. If there is no organized bond market or no intermediation, firms would have to finance their inputs by issuing bonds privately and directly to consumers. In this regime, because the first-order conditions are the same, only the Walrasian interest rate αA supports the fixed point condition $k_j = K_j$ for all firms. However, it cannot be an equilibrium interest rate, because a firm would be better off by offering a coupon rate a little higher than αA and capturing all funds to become a monopolist who can internalize externalities and realize returns as much as A. Hence, no Nash equilibrium exists when bonds are issued privately.[22]

Apparently, some form of financial intermediation is needed to clear the market. Here, I formulate the economy as a game among consumers, firms, and banks in the style of Arrow and Debreu (1954). Banks compete in both the deposit and loan markets. Although, traditionally, strategic competition has been analyzed in the form of either Bertrand or Cournot competition, this paper considers a more general concept of competition: competition in both price and quantity. In essence, banks can tailor deposit contracts for depositors and loan contracts for client firms.

The dynamics of the model is defined recursively in the style of Prescott and Mehra (1980). More precisely, the equilibrium concept is stationary Markov equilibrium, closely following analysis in Duffie and others (1994). In other words, I concentrate on the case in which consumers adopt the same strategies over time, conditional only on state variables. Since firms and banks emerge and are dissolved in each period as in Prescott and Mehra (1980), their strategies naturally become stationary and Markov. For consumers, this approach puts a restriction on the strategy space, but it does not lose much generality.[23] As strategies are stationary, I drop subscript t from all variables when it does not create confusion.

I formulate the within-period economy, without loss of generality, as a two-stage game:[24] Banks compete for deposits in the first stage and for loans in the second stage. A typical analysis would search over a subgame perfect equilibrium, starting with a Nash equilibrium in the second-stage game. However, without an additional institutional setting, this economy still suffers from lack of an equilibrium.[25] Hence, I take a different approach: I first identify a condition, a *weak link between sources and uses of funds*, to support the unique deposit market outcome, and then characterize the unique equilibrium candidate for the whole game within a period. The existence problem will be dealt with in the next section.

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[22]See the proof in Appendix V. Free-riding opportunity makes firms' payoff functions severely discontinuous, which violates an assumption needed for the existence of a Nash equilibrium. See Appendix VI for further discussion related to the discontinuous game literature.

[23]See a general discussion in Duffie and others (1994) and a discussion specific to the model presented here in subsection B below. Also note that Duffie and others (1994) assume a continuous payoff function but this paper deals with a discontinuous payoff function.

[24]See footnote 42 for more discussions.

[25]See detailed discussion in subsection D.

A. Banking Sector

There are H banks in the economy. The number of banks is assumed, without loss of generality, to be smaller than the number of firms. A deposit contract of bank $h \in \{1, \cdots, H\}$ offered to consumer i consists of deposit rate $r^b_{hi} \in \overline{\mathbb{R}}_+$ and recommended savings amount $s^b_{hi} \in \overline{\mathbb{R}}_+$. A loan contract of bank h offered to firm j consists of loan rate $R^b_{hj} \in \overline{\mathbb{R}}_+$, and recommended loan amount $k^b_{hj} \in \overline{\mathbb{R}}_+$. To the interbank market, bank h submits interbank loan B^b_h with interbank rate ρ^b_h.

Bank h must balance its balance sheet. I use k_{hj} to refer to the loan amount agreed between bank h and firm j, and k_h for all agreed loans; that is, $k_h \equiv \sum_{j=1}^J k_{hj}$. This is the asset side of bank h's balance sheet. Similarly, for the liabilities side, I use s_{hi} to refer to the deposit amount agreed on between bank h and consumer i and s_h for all deposits collected by bank h; that is, $s_h \equiv \sum_{i=1}^I s_{hi}$. Also, interbank borrowing, if any, may appear on the liability side. The equilibrium interbank loan for bank h is denoted by $B_h \in \overline{\mathbb{R}}$. This can be negative if the bank is a lender in the interbank market. In sum, the balance sheet constraint is described as:

$$k_h \leq s_h + B_h. \tag{11}$$

Although I describe consumers' and firms' behaviors in detail later, I introduce several additional notations here. I use z^b_{Dh} to denote bank h's strategy in the deposit market and z^b_{Lh} to denote bank h's strategy in the loan and interbank markets. I let $z^b_D \equiv \{z^b_{D1}, \cdots, z^b_{DH}\}$ denote a set of all banks' strategies in the deposit market and $z^b_L \equiv \{z^b_{L1}, \cdots, z^b_{LH}\}$ denote a set of all banks' strategies in the loan and interbank markets. Consumer i has a strategy z^c_{hi} for bank h, and a set of all consumers' strategies is denoted as $z^c \equiv \{z^c_{11}, \cdots, z^c_{HI}\}$. Firm j shows bank h its strategy z^f_{hj} and I write $z^f \equiv \{z^f_{11}, \cdots, z^f_{HJ}\}$ as a set of all firms' strategies. As an equilibrium outcome, the deposit rate r_{hi}, the deposit amount s_{hi}, the loan rate R_{hj}, the loan amount k_{hj}, the interbank market rate ρ_h, and the interbank borrowing B_h are realized.[26]

Bank h's objective is to maximize its profit, defined by:

$$\pi^b_h = \pi^b_h(z^c, z^b, z^f) \equiv \sum_{j=1}^J R_{hj}k_{hj} - \rho_h B_h - \sum_{i=1}^I r_{hi}s_{hi}. \tag{12}$$

The ownership of a bank is assumed to be allocated equally to consumers and is left unchanged over time, as is the case with firms. The ownership of the hth bank by the ith consumer is denoted by ψ^b_{hi}. The sum must be one, $\sum_{i=1}^I \psi^b_{hi} = 1$, for any bank $h \in H$. In this case, consumer i's profit income changes from (7) to (13):[27]

$$w_i(z^c, z^b, z^f) \equiv \sum_{j=1}^J \psi^f_{ij}\pi^f_j(z^b, z^f) + \sum_{h=1}^H \psi^b_{hi}\pi^b_h(z^c, z^b, z^f). \tag{13}$$

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[26]Equilibrium outcomes are denoted by the same symbols but without any superscripts.

[27]To save notation, I abuse the notation that is already used, as long as it is not confusing.

B. Deposit Market

In the first stage, the deposit market, the strategy of bank h consists of deposit rates r^b_{hi} and recommended deposit amounts s^b_{hi} to consumer i, for $i = 1, \cdots I$, and is denoted as $z^b_{Dhi} \equiv (r^b_{hi}, s^b_{hi})$. Note that this is a part of the whole strategy z^b_{hi}, which includes the deposit, loan, and interbank market strategies. A bank does not have to specify these values. In other words, "not specified" can be taken as a strategy, and it is abbreviated as "$N.S.$" Hence, the strategy set is defined as:[28]

$$Z^b_D \equiv (\overline{\mathbb{R}}_+ \cup \{N.S.\})^2. \tag{14}$$

Let z^b_{Di} denote the vector of deposit market strategies of all banks for consumer i and z^b_{Dh} denote the vector of the deposit market strategy of bank h to all consumers.

A consumer i's strategy is denoted by $z^c_i = (\{r^c_{hi}\}^H_{h=1}, \{s^c_{hi}\}^H_{h=1})$, chosen from the strategy set $Z^c \equiv \overline{\mathbb{R}}^{2H}_+$. However, this strategy set is constrained by banks' strategies z^b_{Di}. The constrained choice set of consumer i is written as $G^c_i(z^b_{Di})$. Let $G^c_{hi}(z^b_{Dhi})$ be an element of $G^c_i(z^b_{Di})$ corresponding to the constrained choice set of consumer i facing bank h's offer. I assume that zero deposit in bank h, (i.e., $s^c_{hi} = 0$) is always in the choice set:

$$\begin{aligned} G^c_{hi}(z^b_{Dhi}) &\equiv r^b_{hi} \times \overline{\mathbb{R}}_+ \quad \text{if bank } h \text{ specifies } r^b_{hi} \text{ only,} \\ &\equiv \overline{\mathbb{R}}_+ \times (s^b_{hi} \cup \{0\}) \quad \text{if bank } h \text{ specifies } s^b_{hi} \text{ only, and} \\ &\equiv r^b_{hi} \times (s^b_{hi} \cup \{0\}) \quad \text{if bank } h \text{ specifies both } r^b_{hi} \text{ and } s^b_{hi}. \end{aligned} \tag{15}$$

Note that the choice set of the last case is either (r^b_{hi}, s^b_{hi}) or $(r^b_{hi}, 0)$; that is, a consumer replies either to "*accept*" or "*reject*" the offer. The set constrained by all offers is defined as the Cartesian product of G^c_{hi} over $h \in H$:

$$G^c_i(z^b_{Di}) \equiv G^c_{1i}(z^b_{D1i}) \times G^c_{2i}(z^b_{D2i}) \times \cdots \times G^c_{Hi}(z^b_{DHi}). \tag{16}$$

Wealth in the next period, m^+_i, is a function, g, of all consumers' strategies over this period, $z^c \equiv \{z^c_i\}^I_{i=1}$, given all banks' strategies z^b and all firms' strategies z^f:[29]

$$m^+_i = g(z^c, z^b, z^f) = \sum_{h=1}^{H} r_{hi} s_{hi} + w_i(z^c, z^b, z^f). \tag{17}$$

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[28] Equity-type contracts whose returns depend on outcome are not worth considering, because they will be driven away by debt contracts that promise to repay the expected return of equity-type contracts. Since households are risk averse and banks are risk neutral, households apparently prefer debt contracts to equity-type contracts at the same expected return and banks are indifferent between them. Note that profit and loss of a bank are distributed to shareholders (households) as in a typical general equilibrium analysis.

[29] Banks' and firms' strategies include loan market strategies, which are introduced below.

The feasible set from which a consumer chooses her savings today is a combination of the constrained choice set and the budget set,

$$B_i = B(m_i, z_{Di}^b) \equiv \left\{ z_i^c : z_i^c \in G_i^c(z_{Di}^b) \quad \text{and} \quad \sum_{j=1}^{J} s_{hi}^c \in [0, m_i] \right\}. \tag{18}$$

Because the consumer's strategy is confined within this feasible set $z_i^c = (r_{hi}^c, s_{hi}^c) \in B_i(m_i, z_{Di}^b)$, the equilibrium outcome of the deposit market for consumer i facing bank h's offer is equal to the consumer's strategy; that is, $(r_{hi}, s_{hi}) = (r_{hi}^c, s_{hi}^c)$. I write an outcome of the first stage for ith consumer and h-th bank pair simply as $z_{Dhi} = (r_{hi}, s_{hi})$ without any superscript and let $z_{Di} = (z_{D1i}, z_{D2i}, \cdots, z_{DHi})$ denote the outcome vector for consumer i and $z_D = (z_{D1}, z_{D2}, \cdots, z_{DI})$ denote the outcome vector for all consumers.

Since a main concern is the existence of an equilibrium, the strategy space needs to be expanded to include mixed strategies. I define $\mathcal{B}(X)$ as a Borel σ-algebra of X,[30] and $\Lambda(X)$ as a space of probability measure on the measurable space $(X, \mathcal{B}(X))$. To save space, when it is not confusing, I simply use λ_{Dh} to denote bank h's mixed strategy in the deposit market, μ_i for a depositor i's mixed strategy, λ_{Lh} for bank h's mixed strategy in the second stage, and q_j for firm j, as well as λ_D, μ, λ_L, and q to denote a vector of individual strategies (e.g., $\mu = (\mu_1, \cdots, \mu_I)$).

Specifically, bank h's mixed strategy in the deposit market, λ_{Dh}, is a probability measure over bank h's pure strategies and a function of its expectation on other agents' strategies (i.e., depositors, firms, and other banks) conditional on the wealth distribution: $\lambda_{Dh}(z_{Dh}^b | m, \lambda_{D,-h}, \mu, \lambda_L, q) \in \Lambda(Z_D^b)$. Note that other banks' strategy $\lambda_{D,-h}$ is a vector of mixed strategies of all banks except bank h. Similarly, household i's mixed strategy is a probability measure over pure strategies and a function of the banks' offers as well as its expectation on other agents' strategies conditional on the wealth distribution: $\mu_i(z_i^c | m, z_D^b, \mu_{-i}, \lambda_D, \lambda_L, q) \in \Lambda(B_i)$. Although each consumer's strategy μ_i is mutually dependent on each other through the μ_{-i} element, all consumers' strategies as a whole are a function of banks' and firms' strategies only, $\mu(z^c | m, z_D^b, \lambda_D, \lambda_L, q) \in \Lambda(B)$. Note that this strategy is contingent on z_D^b, a realized specific element (a pure strategy) in support of banks' mixed strategies in the current-period deposit market, in addition to banks' stationary Markov strategies λ_D for future-period deposit markets. Bank h's mixed strategy in the second stage is a probability measure and a function of the deposit market outcome, as well as its expectation on other banks' and firms' strategies in the second stage, $\lambda_{Lh}(z_{Lh}^b | z_D, \lambda_{L,-h}, q)$. Similar to the case with consumers' strategies, the vector of all banks' strategies can be expressed as $\lambda_L(z_L^b | z_D, q)$. Finally, firm j's mixed strategy in the second stage is a probability measure and a function of the banks' offers and its expectation on other firms' strategies, $q_j(z_j^f | z_L^b, q_{-j})$.[31] Again, the vector of all firms' strategies can be expressed as $q(z^f | z_L^b)$. I add $+$ in the superscript for a next-period pure strategy, which is an element in support of a mixed strategy.

[30]To be consistent with the stochastic dynamic programming, I assume measurability of maximands (universal measurability). See the detailed discussion in Stinchcombe and White (1992) and its application to stochastic dynamic programming, for example, in Townsend and Ueda (2001).

[31]As banks move before firms, firms can adopt their mixed strategies contingent on realized support (pure strategies) of banks' mixed strategies.

Consumer i's problem can now be expressed as a functional equation to maximize her lifetime utility.[32] Given her own wealth, m_i, other consumers' wealth, m_{-i}, and her expectation on other agents' stationary strategies, she chooses her best mixed strategy μ_i for each specific realization z_D^b of banks' mixed strategy λ_D:

$$\begin{aligned} V(m_i|m_{-i},\lambda_D,\mu,\lambda_L,q) = \int_{\mathcal{B}(Z_D^b)}\int_{\mathcal{B}(B_{-i})}\Bigg\{\max_{\mu_i\in\Lambda(B_i)}\int_{\mathcal{B}(B_i)}\Bigg[u(m_i-s_i) \\ +\beta\int_{\mathcal{B}(Z_D^b)}\int_{\mathcal{B}(B_{-i})}\int_{\mathcal{B}(Z_L^b)}\int_{\mathcal{B}(Z^f)} V(m_i^+|m_{-i}^+,\lambda_D,\mu,\lambda_L,q) \\ q(dz^{f+})\lambda_L(dz_L^{b+})\mu_i(dz_i^{c+})\mu_{-i}(dz_{-i}^{c+})\lambda_D(dz_D^{b+})\Bigg]\mu_i(dz_i^c)\Bigg\}\mu_{-i}(dz_{-i}^c)\lambda_D(dz_D^b). \end{aligned} \tag{19}$$

A consumer chooses the individual savings function for all possible realization of the current deposit contract offers, Z_{Di}^b, which constitute the support of mixed equilibrium strategies of banks, and hence her value function has an integral over λ_D outside of the maximization. Similarly, other consumers' strategy μ_{-i} is placed outside of the maximization, as its realized value affects the next period wealth distribution m^+ and a consumer has to choose her strategy before she knows the specific realization of other consumers' strategies. Note that past consumer behaviors are not a state variable and do not enter the individual savings function. This is because consumers are not able to coordinate themselves by utilizing history of actions, as it is impossible to affect a deviating consumer's future deposit contracts, which will be offered by banks, not by other consumers.[33]

Since banks maximize within-period profits, they are only interested in the current-period reaction to their offers in the deposit market, as is clear from a bank's expected profit function:[34]

$$\begin{aligned} E\pi_h^b = \int_{\mathcal{B}(B)}\int_{\mathcal{B}(Z_{D,-h}^b)}\int_{\mathcal{B}(Z_{Dh}^b)}\int_{\mathcal{B}(Z_{L,-h}^b)}\int_{\mathcal{B}(Z_{Lh}^b)}\int_{\mathcal{B}(Z^f)} \pi_h^b(z^c,z_{Dh}^b,z_{D,-h}^b,z_{Lh}^b,z_{L,-h}^b,z^f) \\ q(dz^f)\lambda_{Lh}(dz_{Lh}^b)\lambda_{L,-h}(dz_{L,-h}^b)\lambda_{Dh}(dz_{Dh}^b)\lambda_{D,-h}(dz_{D,-h}^b)\mu(dz^c). \end{aligned} \tag{20}$$

Thus, to analyze the deposit market within a period, it is useful to view savings as a function of current deposit contract offers from banks. Indeed, the consumer's maximization problem within a period can be written as a function of the deposit market strategies of banks, and simultaneous moves by other consumers conditional on wealth distribution, given a specific expectations on the

[32]For the optimality and uniqueness of the value function in a perpetually growing economy, see Townsend and Ueda (2001), for example.

[33]Banks and firms cannot be punished depending on past period activities, because they emerge and are dissolved in each period.

[34]Note that a firm's expected profit function $E\pi_j^f$ is defined similarly.

future strategies of all agents, which are denoted by $\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L$, and $\hat{q}$:

$$\begin{aligned}\hat{V}(m_i|m_{-i}, \lambda_D, \mu_{-i}, \hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q}) = \int_{\mathcal{B}(Z^b_D)} \int_{\mathcal{B}(B_{-i})} \Bigg\{ \max_{\mu_i \in \Lambda(B_i)} \int_{\mathcal{B}(B_i)} \Bigg[u(m_i - s_i) \\ + \beta \int_{\mathcal{B}(Z^b_D)} \int_{\mathcal{B}(B_{-i})} \int_{\mathcal{B}(Z^b_L)} \int_{\mathcal{B}(Z^f)} \hat{V}(m^+_i|m^+_{-i}, \hat{\lambda}_D, \hat{\mu}_{-i}, \hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q}) \\ \hat{q}(dz^{f+})\hat{\lambda}_L(dz^{b+}_L)\hat{\mu}_i(dz^{c+}_i)\hat{\mu}_{-i}(dz^{c+}_{-i})\hat{\lambda}_D(dz^{b+}_D) \Bigg] \mu_i(dz^c_i) \Bigg\} \mu_{-i}(dz^c_{-i})\lambda_D(dz^b_D).\end{aligned} \tag{21}$$

In the original problem (19), a consumer must figure out the optimal strategy function μ_i considering any future effects on her own and other agents' strategies. Here, in (21), temporarily fixing her own future strategy function $\hat{\mu}_i$ and her expectations on other agents' strategies $\hat{\lambda}_D, \hat{\mu}_{-i}, \hat{\lambda}_L$, and $\hat{q}$, a consumer maximizes her utility by adjusting current behavior μ_i only, given the banks' current offers, λ_D, and taking into account simultaneous moves of other consumers, μ_{-i}. Thus, in this within-period problem, the individual savings strategy is a function of both current and future strategies: $\mu_i(z^c_i|m, z^b_D, \mu_{-i}, \hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q}) \in \Lambda(B_i)$.[35] Of course, in the stationary Markov equilibrium, the optimal current-period strategy must coincide with the expectations on equilibrium future strategies. Put differently, if future stationary strategies $\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L$, and $\hat{q}$ are indeed equilibrium strategies, then the Nash equilibrium strategies in today's deposit market must be the same as future stationary equilibrium strategies.

To search an equilibrium over the infinite period, it is useful to consider a game in which agents choose only current strategies, given arbitrary expectations on second-stage and future equilibrium strategies. Here, without loss of generality, second-stage and future equilibrium strategies can be temporarily assigned from a set of any arbitrary stationary Markov strategies satisfying: (i) $V(m_i|m_{-i}\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q}) \in \mathbb{R}_{++}$; and (ii) $\partial V/\partial m_i > 0$.[36]

Definition 1. *Given the wealth distribution m and arbitrary second-stage and future equilibrium strategies $\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L$, and $\hat{q}$, let $\Gamma_1(m, \hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q})$ denote the deposit market game, the first stage within a period. It consists of $(H + I)$ agents (banks and consumers), their constrained strategy space, the utilities of consumers, and the current wealth level and its law of motion:*

$$\Gamma_1(m|\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q}) \equiv (H + I, (\Lambda(Z^b_D), \Lambda(B_i)), (E\pi^b_h, V), (m, g)). \tag{22}$$

The individual strategy is a function of current wealth, deposit contract offers from banks, and simultaneous moves of other consumers, given that the expectation for future equilibrium strategies are omitted from the conditions. In the equilibrium, consumers' optimal strategies must satisfy the fixed point; that is, for any i, $\mu^*_i(z^c_i|m, z^b_D, \mu^*_{-i}, \hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q})$. The pure strategy version of this is the

[35] I apologize for the obvious abuse of notation for the sake of readability.

[36] These two properties are standard results of the growth theory showing perpetual growth with Assumption 1. See Appendix II for condition (i). For condition (ii), note that with larger wealth, a consumer can replicate all consumption plans under the current wealth m_i, but also is able to consume extra amounts today.

individual savings function, $S_i(m, z_D^b, S_{-i})$—future strategies are omitted here only to save space.[37] The sum of the individual savings functions is the aggregate savings function, denoted by $S(m, z_D^b) \equiv \sum_{i=1}^{I} S_i(m, z_D^b, S_{-i})$.[38]

C. Second Stage with a Monopoly Bank

I now formally define the loan market. Obviously, the game and strategies depend on what happened in the deposit market, and thus they are defined conditional on realized deposit market strategies. Various institutional assumptions can be made for the second stage; but it is impossible to deny that a bank becomes a monopoly lender if it captures all savings. As such, I start with characterizing the outcome of the monopoly loan market.

Let $M \in \{1, 2, \cdots, H\}$ denote a bank that captures all savings in the first stage. This monopoly bank M's strategy in the loan market is the loan contract to firm j that consists of the loan rate $R_{Mj}^b \in \overline{\mathbb{R}}_+$ and the loan amount $k_{Mj}^b \in \overline{\mathbb{R}}_+$, and defined as $z_M^b \equiv (R_{Mj}^b, k_{Mj}^b)$. The strategy set is defined as $Z_M \equiv (\overline{\mathbb{R}}_+ \cup \{N.S.\})^{2J}$; that is, loan rates and loan amounts must be nonnegative or left unspecified.

Firm j's strategy when it faces an offer from a monopoly bank is to choose $z_{Mj}^f \equiv (R_{Mj}^f, k_{Mj}^f)$ from its strategy set $Z_M^f \equiv \overline{\mathbb{R}}_+^2$. However, this strategy set is constrained by the bank's offer z_M^b. The constrained correspondence is written as $G_M^f(z_M^b)$. I assume that no borrowing, $k_j = 0$, is always in the choice set:

$$
\begin{aligned}
G_M^f(z_M^b) &\equiv R_{Mj}^b \times \overline{\mathbb{R}}_+ \quad \text{if the bank specifies } R_{Mj}^b \text{ only,} \\
&\equiv \overline{\mathbb{R}}_+ \times (k_{Mj}^b \cup \{0\}) \quad \text{if the bank specifies } k_{Mj}^b \text{ only, and} \\
&\equiv R_{Mj}^b \times (k_{Mj}^b \cup \{0\}) \quad \text{if bank } M \text{ specifies both } R_{Mj}^b \text{ and } k_{Mj}^b.
\end{aligned} \tag{23}
$$

Note that the choice set of the last case is either (R_{Mj}^b, k_{Mj}^b) or $(R_{Mj}^b, 0)$; that is, a firm has the choice to "*accept*" or "*reject*" the offer.

The best response of a firm is defined similarly to the case with the Walrasian economy:

$$
BR_{Mj}^f(z_M^b, k_{-j}^f) = \arg \max_{z_M^f \in G_M^f(z_M^b)} \pi^f(k_j^f, K_j^f, R_j^f). \tag{24}
$$

---

[37]For a consumer, the sources of future wealth are interest income from her bank deposits in banks and profits income from firms and banks. The profits income depends on firms' investment, which must be equal to aggregate savings. Thus, the aggregate savings affect the profits income, implying that other consumers' strategies μ_{-i} and banks' strategies $z_{D,-i}^b$ have an influence on the current strategy of consumer i. However, in the case of a large number of consumers, each consumer's savings has approximately no effect on aggregate savings. In this case, given other consumers' wealth distribution m_{-i} and her stationary expectations on other consumers' strategies $\hat{\mu}_{-i}$, firms' $\hat{q}$, and banks' $\hat{\lambda}_D$ and $\hat{\lambda}_L$, a consumer's savings decision would become a function of her own current wealth m_i and current deposit contract offered to her by banks z_{Di}^b only.

[38]When all elements of the vector of deposit contracts are the same, for the sake of simplicity, I will write only one element in place of the vector of deposit contract offers z_D^b.

I write elements of BR^f_{Mj} as the borrowing rate $R^{f*}_{Mj}(z^b_M, k^f_{-j})$ and the borrowing amount $k^{f*}_{Mj}(z^b_M, k^f_{-j})$. Naturally, both are functions of the banks' offer z^b_M and other firms' strategies k^f_{-j}.

The monopoly bank maximizes profit $\pi^b_M(z^b_M|z_D)$ by choosing the loan contracts they will offer to all firms,[39] given an outcome of the deposit market z_D, namely, the deposit amount s_{Mi} per consumer and its deposit rate r_{Mi}:

$$\max_{z^b_M \in Z_M} \pi^b_M(z^b_M|z_D) \equiv \sum_{j=1}^{J} R^{f*}_{Mj}(z^b_M, k^f_{-j}) k^{f*}_{Mj}(z^b_M, k^f_{-j}) - \sum_{i=1}^{I} r_{Mi} s_{Mi}, \tag{25}$$

subject to the resource constraint[40]

$$\sum_{j=1}^{J} k^{f*}_{Mj}(z^b_M, k^f_{-j}) \leq \sum_{i=1}^{I} s_{Mi}. \tag{26}$$

Definition 2. *Given a deposit market outcome z_D, the second stage with a monopoly bank is the game Γ_M, which consists of one bank and J firms, their strategy sets, and their profit functions:*

$$\Gamma_M(z_D) \equiv (1 + J, (\Lambda(Z^b_M), \Lambda(G^f_M)), (E\pi^b_M, E\pi^f_j)). \tag{27}$$

Lemma 1. *The Nash equilibrium of Γ_M is characterized by: the optimal decision of monopoly bank h, $z^*_M = (R^{b*}_{Mj}, k^{b*}_{Mj})$, that satisfies*

$$\sum_{j=1}^{J} k^{b*}_{Mj} = \sum_{i=1}^{I} s_{Mi}, \tag{28}$$

$$k^{b*}_{Mj} = \frac{\sum_{i=1}^{I} s_{Mi}}{J}, \quad \textit{and} \tag{29}$$

$$R^{b*}_{Mj} = A; \tag{30}$$

and the optimal decision by firms, which is to "accept."

Proof is provided in Appendix I. Intuitively, equation (28) states that all funds should be utilized; equation (29) states that symmetric allocation of funds among firms is the best (due to concavity of the production function); and equation (30) states that the monopolist bank obtains the return as much as the social optimal return. Note that Lemma 1 has a strong prediction. In equilibrium, at least one bank must offer a deposit rate more than or equal to A. Otherwise, a bank will offer a rate slightly higher than the prevailing deposit rate and will become a monopolist to enjoy a strictly positive profit.

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[39] Since a pure strategy dominates any mixed strategy due to the concavity of the production function (see Lemma 1 below), for the sake of simplicity, the profit function here is defined with pure strategies only.

[40] The set of equilibria here does not include the case where the aggregate demand for capital is larger than aggregate savings. This assumption is not restrictive. It can be formulated in such a way that the monopoly bank rations credit to meet the resource constraint, when the aggregate demand is larger than the aggregate supply of funds.

D. A Weak Link between Sources and Uses of Funds

Without further institutional settings, no equilibrium exists in this economy either, as it is still isomorphic to the economy in which firms finance funds by directly issuing bonds to consumers. For any proposed equilibrium strategies, if it can offer a better contract than other banks, a bank can obtain all marginal profit of a firm by picking a firm and tailoring its loan contract. Apparently, a bank faces the firm's profit function, implying that this economy has essentially the same incentive structure as an economy without intermediaries. The inherent lack of an equilibrium comes from the dilemma that banks face: Two opposing strategies are profitable. First, a bank wants to compete aggressively in the deposit market to become a monopolist. Second, a bank wants to collect a small amount of deposits and invest it in a firm to free ride on other firms' investments. In both strategies, banks' profits rely on other banks' actions. On the one hand, if other banks offer deposit rates less than the monopoly loan rate, the first strategy brings higher profit. On the other hand, if other banks offers deposit rates as high as the monopoly loan rate, the second strategy brings higher profit. A combination of the two strategies creates a dilemma: Until the deposit rates are bid up to the monopoly loan rate, banks want to compete aggressively, but at that rate, banks want to shrink their size relative to others. This fundamental discontinuity stems from production externalities and does not rely on the formulation of the deposit and loan markets, either two-stage or simultaneous games.

Either of the two opposing strategies must be eliminated for banks to be able to make a clear decision on their deposit market strategies. However, it is difficult to discourage banks from taking the first strategy (seeking more deposits), as the monopoly loan rate is always higher than the Walrasian rate. The only exception is the strict interest rate control at the Walrasian rate, while market share restriction would still give the same incentive for banks to internalize some externalities within the restriction. In contrast, without any government intervention, banks may be easily discouraged from taking the second strategy (limiting deposit intake), if banks can adjust their fund positions using an interbank market. If so, the unique equilibrium strategy is determined, at least in the deposit market. Moreover, the deposit market outcome dictates the overall equilibrium candidate.

In summary, a key assumption here is that banks do not worry about consequences in loan market competition when they compete for deposits. This assumption, a *weak link between sources and uses of funds*,[41] is formally described as Assumption 2 below. Obviously, Assumption 2 places a restriction on equilibrium strategies and needs to be justified. Indeed, in the next section, I will show an example of a realistic institution that satisfies Assumption 2. Before doing so, however, I show an institution-free results under Assumption 2.

I now define the additional setup for the institution-free result, though I must admit that it is somewhat courageous to characterize the equilibrium of the whole game without describing it completely. Several characteristics, however, can be imagined easily. For example, the set of active banks in the competitive second stage may be less than H, which is the pool of potential entrants to the banking sector. The set of active banks should be a function of strategies in the first stage. It is defined as $D(z_D) \equiv \{h \in H : \sum_{i=1}^{I} s_{hi} > 0\}$ and the number of active banks is denoted as $\#D$. If

[41] This assumption is obviously consistent with banks' behavior in the real world. Banks do not limit deposits based on how much they can lend, but rather adjust their fund size using the interbank market, so that deposit amounts do not strictly restrict lending operations.

$D(z_D)$ is a singleton (i.e., $\#D = 1$), the loan market is monopolized, and otherwise (i.e., $\#D \geq 2$) it is competitive.[42] The competitive second stage is defined as follows.

Definition 3. *Given a deposit market outcome z_D, the competitive second stage is the game Γ_C, which consists of $\#D$ banks and J firms, their strategy sets, and their profit functions:*

$$\Gamma_C(z_D) \equiv (\#D + J, (\Lambda(Z_C^b), \Lambda(G_C^f)), (E\pi_h^b, E\pi_j^f)). \tag{31}$$

The strategy sets (Z_C^b, G_C^f) are left ambiguous here, because this section describes results in the second stage free from a specific institutional setting.

Definition 4. *A second stage is the following game, given a deposit market outcome z_D:*

$$\begin{aligned} \Gamma_2(z_D) &\equiv \Gamma_M(z_D) \quad \textit{if } D(z_D) \textit{ is a singleton, and} \\ &\equiv \Gamma_C(z_D) \quad \textit{otherwise}. \end{aligned} \tag{32}$$

Definition 5. *A strategically intermediated economy within a period is the game Γ, given wealth distribution m and arbitrary future equilibrium strategies $\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L$, and $\hat{q}$. It consists of the following elements:*

- *the first stage, $\Gamma_1(m|\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q})$;*
- *the set of all possible histories for the second stage, which is all possible strategies in the first stage, $(Z_D^b)^H \times (Z^c)^I$; and*
- *the second stage, $\Gamma_2(z_D)$.*

In sum,

$$\Gamma(m|\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q}) \equiv (\Gamma_1(m|\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L, \hat{q}), (Z_D^b)^H \times (Z^c)^I, \Gamma_2(z_D)). \tag{33}$$

Definition 6. *An equilibrium of the strategically intermediated economy within a period is a combination of a Nash equilibrium of the first stage game Γ_1 and a Nash equilibrium of the second-stage game Γ_2. Specifically:*

- *a consumer's equilibrium strategy μ_i^* maximizes the consumer's lifetime utility $\hat{V}$ defined in (21), given second-stage and future equilibrium strategies $\hat{\lambda}_D, \hat{\mu}, \hat{\lambda}_L$, and $\hat{q}$, as well as other*

[42] Simultaneous opening of both the deposit and loan markets are similarly analyzed. In this case, loan market strategy must be decided before knowing how much deposits are collected by each bank. Thus, instead of formulating the subgame conditional on realized deposit market strategies, the loan market should be formulated conditional on expectations for all possible equilibrium deposit market strategies. This approach can also be applied to the two-stage game in which the loan market is open in the first stage but banks and firms cannot commit to honor contracts when banks fail to raise sufficient funds. Note that Stahl II's (1988) result is different, as it assumes that a firm-bank pair must honor the loan contract in any circumstances without knowing the banks' fund positions. As such, there is no motivation for banks to collect all deposits (and make profits by squeezing other banks), once loan contracts are made in the loan-first two-stage game in Stahl II (1988).

consumers' current equilibrium strategies μ^*_{-i} and banks' z_D^{b*}, which is a pure strategy in the support of banks' equilibrium mixed strategies λ^*_D;

- *bank's equilibrium strategies λ^*_{Dh} and λ^*_{Lh} maximize the bank's profit $E\pi^b_h$ defined in (20), given consumers' current equilibrium strategies μ^*, firms' q^*, and other banks' $\lambda^*_{D,-h}$ and $\lambda^*_{L,-h}$;*
- *firm's equilibrium strategies q^*_j maximizes the firm's profit $E\pi^f_j$, given consumers' current equilibrium strategies μ^*, banks' λ^*_D and λ^*_L, and other firms' q^*_{-j};*
- *a bank's equilibrium profit is nonnegative, $E\pi^b_h \geq 0$; and*
- *a firm's equilibrium profit is nonnegative, $E\pi^f_j \geq 0$.*

Note that consistency of consumers' expectations within a current period as well as over future periods are not required; that is, consumers' expectations on second-stage strategies by banks $\hat{\lambda}_L$ and by firms $\hat{q}$ do not necessarily coincide with the equilibrium strategies λ^*_L and q^*, respectively. Obviously, the consistency in expectations for future strategies are required for the equilibrium over the infinite period.

Definition 7. *An equilibrium of the strategically intermediated economy over the infinite period is a stationary Markov equilibrium of the game Γ. It is a within-period equilibrium that also satisfies the consistency of expectations: $\mu^* = \hat{\mu}$, $\lambda^*_D = \hat{\lambda}_D$, $\lambda^*_L = \hat{\lambda}_L$, and $q^* = \hat{q}$.*

To distinguish it from z^b_{Lh}, the strategy of bank h for the whole within-period second stage, I use z^b_h to denote bank h's strategy for the competitive second stage only and $\lambda_h \in \Lambda(Z^b_C)$ to denote the corresponding mixed strategy. Moreover, $\lambda^{l\backslash h}$ denotes a mixed strategy in which banks h and l exchange strategies, keeping the order of firms fixed:
$\lambda^{l\backslash h} \equiv (\lambda_1, \cdots, \lambda_{h-1}, \lambda_l, \lambda_{h+1}, \cdots, \lambda_{l-1}, \lambda_h, \lambda_{l+1}, \cdots, \lambda_H)$. Furthermore, $\lambda^{l\backslash h}_h$ denotes the h-th element of this strategy vector. Also, let $q^{l\backslash h}$ denote a set of mixed strategies in which all firms' strategies for bank h are exchanged by their strategies for bank l and $q^{l\backslash h}_j$ denote its jth element. The assumption of a *weak link between sources and uses of funds* can now be described formally as follows.

Assumption 2. ***[Weak Link between Sources and Uses of Funds]*** *In the competitive second stage, there exists an equilibrium in which the probability distribution over second-stage strategies is the same for all active banks, regardless of each bank's own performance in the deposit market. Specifically, the probability of adopting a specific equilibrium mixed strategy depends only on aggregate savings $\overline{S}$; that is, for any active pair of banks (l, h),*

$$\lambda^*_h(z^b_h | z_D, \lambda^*_{-h}, q^*) = \lambda^{*l\backslash h}_l(z^b_l | \tilde{z}_D, \lambda^{*l\backslash h}_{-l}, q^{*l\backslash h}), \tag{34}$$

where z_D and $\tilde{z}_D$ are any pair of deposit market outcomes that deliver the same active banks and the same amount of aggregate savings $\overline{S}$.

This assumption implies that competition in the deposit market and in the loan market are independent of each other, except that banks still maximize overall profits and need to balance their balance sheets. Note that, if there is no externalities (i.e., $\alpha = 1$), this assumption is obviously satisfied. In this case, similar to the results in Townsend (1983) and Stahl II (1988), strategic intermediation brings an allocation equal to the Walrasian equilibrium, which is Pareto optimal.

Before proceeding further, one more assumption is needed for the competitive second stage. It may be obvious, but it is assumed that banks are not discriminated against by firms. More specifically, if two banks offer the same loan contract, the probability of its acceptance by a firm should be the same. This equal footing condition can be spelled out formally. Let $z^{b,l\backslash h}$ denote a set of pure strategies of banks, in which bank h and l exchange their pure strategies.

Assumption 3. *In the competitive second stage, the best responses of firms to a set of banks' strategies should not discriminate against any active banks. In other words, a set of firms' equilibrium strategies must satisfy the following: for any firm j and for any active pair of banks $(l, h) \in D(z_D)^2$,*

$$q_j(z_j^f|z^b, q_{-j}) = q_j^{l\backslash h}(z_j^f|z^{b,l\backslash h}, q_{-j}^{l\backslash h}). \tag{35}$$

Assumption 3 is not so restrictive because it states that only profit motives matter for firms to choose offers from banks. Hence, unlike Assumption 2, Assumption 3 is taken for granted throughout the paper.

Under Assumption 2, banks decide their loan market strategies conditional only on aggregate savings, not their own fund positions. Hence, the joint probability of a set of equilibrium strategies (z_h^b, z_j^f) is also conditional only on aggregate savings. Using the Bayes rule, it is defined as

$$Q^*(z^b, z^f|\overline{S}) \equiv \lambda^*(z^b|z_D, q^*)q^*(z^f|z^b), \tag{36}$$

with the deposit market outcome z_D satisfying $\sum_{i=1}^I s_i = \overline{S}$. Let $Q_{hj}^*(z^b, z^f|\overline{S})$ denote its (h, j) element. It is an equilibrium probability distribution over the rectangle consisting of bank h's strategies toward firm j and firm j's strategies toward bank h.

The search for an equilibrium in this section is limited to the probability space that satisfies Assumptions 2 and 3. Apparently, expected revenues of bank h and l from loans are unchanged for the *permutated strategy*, in which bank h and l exchange their loan market strategies; accordingly firms also exchange strategies for bank h and l. Indeed, the joint probability of the permutated strategy is the same as that of the original equilibrium strategy:

$$Q_{lj}^{*l\backslash h}(z^b, z^f|\overline{S}) = Q_{hj}^*(z^b, z^f|\overline{S}), \tag{37}$$

where

$$Q_{lj}^{*l\backslash h}(z^b, z^f|\overline{S}) \equiv \lambda^{*l\backslash h}(z^b|z_D, q^{*l\backslash h})q^{*l\backslash h}(z^f|z^{b,l\backslash h}), \tag{38}$$

with the deposit market outcome z_D satisfying $\sum_{i=1}^I s_i = \overline{S}$. Condition (37) implies that loan market competition does not depend on results in the deposit market and thus banks do not have to worry about collecting too large a deposit. Consequently, they compete aggressively in the deposit market, placing a restriction on a candidate for a Nash equilibrium for the whole game within a period.

The first main theorem below characterizes the properties of a candidate for a Nash equilibrium, as a result of competition in the deposit market under any institutional settings that create a *weak link between sources and uses of funds* (the formal proof is provided in Appendix I). Intuition is as follows. Banks' expected profits for the competitive second stage are, at most, the monopoly bank's profit. Hence, all banks try to become monopolist by bidding up the deposit rate. In the end, the deposit rate is bid up to A, the monopolist's return from loans. As arbitrage opportunity cannot exist between the deposit market and the loan market in an equilibrium, the equilibrium loan rate must be A, the same as the deposit rate. To repay this loan rate, firms must invest a symmetric amount of capital, utilizing all deposits. Banks engage in this aggressive competition, because they do not worry about the disadvantage of having funds that are too large in the loan market.

Theorem 1. ***[Unique Equilibrium Outcome]*** *If an equilibrium exists for an economy with a competitive second stage satisfying Assumptions 2 and 3, an equilibrium outcome of the strategically intermediated economy within a period is a Pareto-optimal allocation with aggressive competition by many active banks. It is characterized further as follows: (i) although there may be numerous Nash equilibrium strategies for banks in the deposit market, the equilibrium offers always specify that the deposit rate be equal to A; (ii) both not specifying deposit amounts (i.e.,* $s^b_{hi} = \{N.S.\}$*) and specifying a depositor's willingness to supply at rate A (i.e.,* $s^b_{hi} = S(m, (A, N.S.))$*) are dominant strategies;*[43] *(iii) depositors face the deposit rate offer* A *and deposit their willingness to supply at* A*, so that* $S(m, (A, N.S.))$ *represents the aggregate savings; (iv) the equilibrium loan offer by any banks is* $(A, S(m, (A, N.S.))/J)$*, the same as the monopolist's; and (v) firms accept this offer.*

IV. EXISTENCE OF AN EQUILIBRIUM WITH FREE RECONTRACTING OPPORTUNITY

Theorem 1 shows the unique equilibrium candidate under Assumption 2 that enables banks to compete in the deposit market without worrying about their fund positions in the loan market. Without Assumption 2, banks have two concerns: (i) collected deposits may be too large to lend out; and (ii) the equilibrium loan rate may be lower than the deposit rate, which is bid up to A. For the first concern, apparently, an interbank market needs to be introduced for banks to trade excess deposits. For the second concern, the only solution is to have an equilibrium loan market rate higher than or equal to the deposit rate A. However, having a loan market rate higher than the Walrasian rate αA in an equilibrium is difficult, as the loan rate exceeds firms' private marginal return. Indeed, under such a loan rate, any investment less than others would be profitable, which in turn creates aggregate excess deposits and prevents the interbank market from clearing.

Here, I introduce a specific mechanism as an example of possibly many mechanisms that satisfy Assumption 2 and ensure the existence of an equilibrium. Specifically, banks are allowed to have *free recontracting* opportunities to adjust the price, in addition to an interbank market to adjust funds among banks. This whole mechanism follows the spirit of the Walrasian *tâtonnement* process[44] in which economic agents try to find the right price to clear the market. Because the *tâtonnement* process for the Walrasian equilibrium is not based on strategic behaviors, I define a similar process that is, nonetheless, consistent with strategic moves, and name it *strategic tâtonnement*.[45]

[43] This property implies that competition in the deposit market is likely to become *à la Bertrand*, competition in price only.

[44] See Arrow and Hahn (1971) for a formal definition and Negishi (1987) for its historical origin.

[45] As a result, this procedure somewhat resembles the Groves-Clarke mechanism to finance public goods, but the problem here is not public goods provided by a government but private goods with externalities provided by many private agents. Moreover, *strategic tâtonnement* is a decentralized implementation to internalize externalities, not a centralized one.

In *strategic tâtonnement*, banks have one or more chances to alter contracts freely. In its simplest form, two sessions each day, morning and afternoon, are open for the loan and interbank market. Banks can use the afternoon session as a punishment phase to support any target contract between the technologically feasible highest return and the lower Walrasian interest rate. More generally, analysis is almost the same for any number of recontracting sessions as long as it is more than or equal to two. A finite number of sessions is enough to obtain existence and uniqueness of a Nash equilibrium, though an infinite number of sessions is necessary to achieve a subgame perfect Nash equilibrium within *strategic tâtonnement*.[46]

A. Definition of *Strategic Tâtonnement*

In a *strategic tâtonnement*, there are substages or sessions that are repeated many times, possibly infinitely. In each session $\tau \in \{1, 2, \cdots, T\}$, there are five phases as below:[47]

1. each bank $h \in D(z_D)$ offers a tentative loan contract $(R^b_{\tau hj}, k^b_{\tau hj})$ to each firm $j \in J$;
2. firms submit their tentative decisions on offered contracts to banks;
3. banks submit tentative interbank rates and net borrowing amounts $(\rho^b_{\tau h}, B^b_{\tau h})$ to the interbank market, and a tentative match of demand and supply is undertaken;[48]
4. if a bank satisfies its profit based on tentative matches in the loan and the interbank market, it sends a confirmation letter to each firm to accept the firm's response and finalize transactions, denoted as $d^b_{\tau hj} = 1$ (otherwise $d^b_{\tau hj} = 0$); and
5. a firm responds to the confirmation letter from a bank, denoted as $d^f_{\tau hj} = 1$, if it accepts, and $d^f_{\tau hj} = 0$, if it rejects the letter or did not receive the letter.

An outcome of session τ is status of agreements between bank h and firm j, $d_{\tau hj} \in \{0, 1\}$, defined as $d_{\tau hj} = 1$ if $d^b_{\tau hj} = d^f_{\tau hj}$ and otherwise $d_{\tau hj} = 0$. If all banks and firms agree, the *strategic tâtonnement* ends; otherwise, the next session begins. Once a bank and firm accept a contract, they

[46]In the proposed process, loan contracts are finalized only after banks negotiate with firms and make sure their balance sheet match, while deposit contracts are assumed to be finalized immediately. This asymmetry of contracting process between deposit and loan market is in line with casual observation. While they usually finalize a deposit contract instantly when a consumer puts money in his account, banks often gather information about the financial needs of firms and negotiate loan terms.

[47]Since the unique equilibrium outcome is given by pure strategies, I focus pure strategy equilibria in the rest of the paper.

[48]If all banks balance their balance sheets, the interbank market would clear, but it is not guaranteed that all banks can always balance their balance sheet.

must honor it. In later sessions, no matter what alternatives, both parties are assumed to submit the agreed contract to each other and repeat confirmation and acceptance.[49]

To allow banks to adjust price and quantity, I assume that at least two sessions, $T \geq 2$, exist. Here, $T = 2$ suggests that morning and afternoon sessions exist in the interbank market.[50] Finite T suggests a longer, but similar situation. Infinite T implies that banks and firms talk continuously all day. Even if T is infinite, it still is contained in one period. After the *strategic tâtonnement* ends, transactions are made based on agreed contracts, and profits of banks and firms are realized. If a bank-firm pair does not reach an agreement, then there is no transaction of capital within the pair.

In each session τ, the strategy of bank h is defined as

$$z^b_{\tau h} \equiv \{\tilde{z}^b_{\tau h}, d^b_{\tau h}, (\rho^b_{\tau h}, B^b_{\tau h})\}, \tag{39}$$

where $\tilde{z}^b_{\tau h}$ is a bank's strategy in terms of the loan contract (i.e., $(R^b_{\tau hj}, k^b_{\tau hj})^J_{j=1}$), and $d^b_{\tau h}$ is a set of bank h's confirmation strategies toward all firms (i.e., $(d^b_{\tau hj})^J_{j=1}$). Thus, the strategy set is

$$Z^b \equiv (\mathbb{R}_+ \cup \{N.S.\})^{2J} \times \{0, 1\}^J \times (\mathbb{R}_+ \cup \{N.S.\})^2. \tag{40}$$

Note that this strategy set is the same for all banks h and sessions τ. I also use the vector notation $z^b_\tau \equiv \{z^b_{\tau h}\}^H_{h=1}$ for all the strategies in session τ and $\tilde{z}^b_\tau \equiv \{\tilde{z}^b_{\tau h}\}^H_{h=1}$ for the strategies on loan contracts.

The interbank market is assumed to be a multilateral clearing system among banks, without an auctioneer or a central bank, where each bank picks a specific interest rate $\rho^b_{\tau h} \in \mathbb{R}_+$ and indicates its willingness to borrow $B^b_{\tau h} \in \mathbb{R}$—note that negative value means supply of funds.[51] The interbank market must clear in an equilibrium. I define the set of active banks that submit the interbank market rate ρ as $\tilde{D}_\tau(\rho) \equiv \{h \in D | \rho^b_{\tau h} = \rho\}$. Using this notation, aggregate net borrowing at each interbank market rate $\rho \in \mathbb{R}_+$ can be defined as

$$\tilde{B}_\tau(\rho) \equiv \sum_{h \in \tilde{D}_\tau(\rho)} B^b_{\tau h}. \tag{41}$$

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[49] This assumption is not restrictive as an abstract description of negotiation over loan terms. It is made only because of notational simplicity compared with an alternative assumption that a specific bank-firm pair withdraws the process when they agree. In either specification, banks and firms face essentially the same decision. Note that banks and firms have a choice not to agree to a specific loan contract, if they think there would be a better opportunity later in the session.

[50] If there is only one chance $T = 1$, the economy is again isomorphic to the economy without any intermediaries: $R = \alpha A$ is the only candidate for equilibrium loan rates in the second stage by a similar argument in the Walrasian equilibrium, but it cannot be an equilibrium for the whole game, because there is incentive for a bank to become a monopolist by offering higher interest rates in the deposit market.

[51] Instead of picking one specific interest rate, it could be formulated that each bank submit a borrowing function for all the possible interest rates in the real value. It could also be formulated with a bilateral clearing system in which contracts can be indexed by lenders' and borrowers' identities denoted by subscripts. These changes would make the model more complex but would not affect the main results.

The interbank market–clearing condition is now written as follows. For all $\rho \in \mathbb{R}_+$, there exists $\tau^* \leq T$ such that

$$\tilde{B}_{\tau^*}(\rho) = 0. \tag{42}$$

Because it is assumed that all agreed bank-firm pairs repeat their agreed contracts, the interbank market always clears in any session once it clears in session τ^*; that is, for any $\hat{\tau} \geq \tau^*$, $\tilde{B}_{\hat{\tau}}(\rho) = 0$, for all $\rho \in \mathbb{R}_+$.

Firm j's strategy, when it faces offers from banks in session τ, is to choose[52] $z^f_{\tau j} \equiv (R^f_{\tau hj}, k^f_{\tau hj}, d^f_{\tau hj})_{h \in H}$ from its strategy set $Z^f \equiv \mathbb{R}^{2H}_+ \times \{0,1\}^H$. However, this strategy set is constrained by banks' strategies z^b_τ. The constrained correspondence is written as $G^f_j(z^b_\tau)$. I assume that $k^f_{\tau hj} = 0$ is always in the choice set. Let $G^f_{hj}(z^b_{\tau hj})$ be an element of $G^f_j(z^b_\tau)$ corresponding to the constrained choice set of firm j with respect to bank h:

$$G^f_{hj}(z^b_{\tau hj}) \equiv \tilde{G}^f_{hj}(z^b_{\tau hj}) \times \{0,1\}, \tag{43}$$

where $\tilde{G}^f_{hj}(z^b_{\tau hj})$ is the constrained choice set in the loan market for the loan contract $\tilde{z}^f_{\tau hj} \equiv (R^f_{\tau hj}, k^f_{\tau hj})$:

$$\begin{aligned} \tilde{G}^f_{hj}(\tilde{z}^b_{\tau hj}) &\equiv R^b_{\tau hj} \times \mathbb{R}_+ \quad \text{if bank } h \text{ specifies } R^b_{\tau hj} \text{ only,} \\ &\equiv \mathbb{R}_+ \times (k^b_{\tau hj} \cup \{0\}) \quad \text{if bank } h \text{ specifies } k^b_{\tau hj} \text{ only, and} \\ &\equiv R^b_{\tau hj} \times (k^b_{\tau hj} \cup \{0\}) \quad \text{if bank } h \text{ specifies both } R^b_{\tau hj} \text{ and } k^b_{\tau hj}. \end{aligned} \tag{44}$$

The constrained choice set of each firm is defined as the Cartesian product of G^f_{hj} over $h \in H$:

$$G^f_j(z^b_{\tau h}) \equiv G^f_{1j}(z^b_{\tau 1j}) \times G^f_{2j}(z^b_{\tau 2j}) \times \cdots \times G^f_{Hj}(z^b_{\tau Hj}). \tag{45}$$

Now I formally define a session. Before doing so, a strategy history must also be defined, because strategies in session τ can be conditional on their history. Let $z_\tau \equiv (z^b_\tau, z^f_\tau)$, the strategies of all agents in session τ. The strategy set for z_τ is $Z \equiv Z^b \times Z^f$. Similarly, $z_0 \equiv (z^b_D, z^c)$ denotes the strategy set for the deposit market and $Z_0 \equiv Z^b_D \times Z^c$ denotes its strategy set.

Definition 8. *A history $z^{\tau-1}$, for $\tau = 1, 2, \cdots, \infty$, denotes a strategy sequence before session τ: $(z_0, z_1, z_2, \cdots, z_{\tau-1})$. The space of history is denoted as $\Omega_{\tau-1}$ for $\tau = 1, 2, \cdots, \infty$,*

$$\begin{aligned} \Omega_{\tau-1} &\equiv Z_0 \quad \textit{for } \tau = 1, \\ &\equiv Z_0 \times Z^{\tau-1} \quad \textit{for } \tau \geq 2. \end{aligned} \tag{46}$$

A *session* τ, $\tau = 1, 2, \cdots, T$, with a history $z^{\tau-1}$ is similar to an extensive game with perfect information and simultaneous moves, though it is not a game because the payoff functions are not defined until all sessions are completed. It consists of the following elements:

- $(H + J)$ agents (banks and firms);

[52] For simplicity, I hereafter focus on the case in which all the banks are active, $\#D = H$. I will make clear those situations when the case $\#D < H$ should be treated carefully.

- phases $p = 1, 2, 3, 4, 5$;
- a player function $P(p)$ that assigns agents to phases p as follows:
  - $P(1) = \{1, 2, \cdots H\}$ (banks offer loan contracts),
  - $P(2) = \{H + 1, H + 2, \cdots, H + J\}$ (firms submit demands),
  - $P(3) = \{1, 2, \cdots H\}$ (banks offer interbank contracts),
  - $P(4) = \{1, 2, \cdots H\}$ (banks' send confirmation letters), and
  - $P(5) = \{H + 1, H + 2, \cdots, H + J\}$ (firms' replies);
- Constrained strategy spaces $G(p)$ for players $P(p)$ as follows:
  - $G(1) = (\mathbb{R}_+ \cup \{N.S.\})^{2J}$ (for a bank's offer of loan contracts $\tilde{z}^b_{\tau hj}$ to firms),
  - $G(2) = \tilde{G}^f_{hj}(\tilde{z}^b_{\tau hj})$ (for a firm's choice $\tilde{z}^f_{\tau hj}$),
  - $G(3) = (\mathbb{R}_+ \cup \{N.S.\})^{2J}$ (a bank's offer to the interbank market (ρ^b_h, B^b_h)),
  - $G(4) = \{0, 1\}$ (a bank's confirmation $d^b_{\tau hj}$), and
  - $G(5) = \{0, 1\}$ (a firm's confirmation $d^f_{\tau hj}$);
- agreed status $d_{\tau hj} \in \{0, 1\}$.

In summary, I write each session τ as

$$\Phi_\tau(z^{\tau-1}) \equiv (H + J, p, P, G, d_{\tau hj}), \tag{47}$$

for any history $z^{\tau-1} \in \Omega_{\tau-1}$.

All the necessary notations have been introduced to define the *strategic tâtonnement* as follows.

Definition 9. *The strategic tâtonnement is a settlement procedure represented as T-times (possibly infinitely) repeated sessions. More specifically, it consists of the set of all possible histories and each corresponding session as a function of histories,*

$$\Phi^T \equiv \{\Omega_T, \{\Phi_\tau(z^{\tau-1})\}^T_{\tau=1}\}. \tag{48}$$

The competitive second stage of the game consists of this procedure and associated profit functions for banks and firms:[53]

$$\Gamma_C \equiv (\Phi^T, (\pi^b_h, \pi^f_j)). \tag{49}$$

Definition 10. *An equilibrium of the competitive second stage is a set of strategies $\{z^*_\tau\}^T_{\tau=1}$ that is a Nash equilibrium of the game Γ_C and clears the interbank market; that is, there exists $\tau^* \leq T$ that satisfies (42).*

[53]To be consistent with the rest of the paper, it might be better to formulate the *strategic tâtonnement* process in a strategic form, where at the outset firms and banks pick their strategies for all sessions, $\tau = 1$ to T, for all possible realizations of histories of strategies. However, it is defined in an extensive form, because the description of the game is simpler and intuitive. Besides, any extensive form game can be converted into a strategic form.

An equilibrium $\{z_\tau^*\}_{\tau=1}^T$ is a set of strategies, not outcome values. As in the deposit market, equilibrium outcome values are written without any superscripts and, here, also without session subscript τ. If the loan contract between bank h and j is agreed on at session τ^* (i.e., $d_{\tau^* hj} = 1$), then the equilibrium outcome is

$$(R_{hj}, k_{hj}) = (R^b_{\tau^* hj}, k^b_{\tau^* hj}) = (R^f_{\tau^* hj}, k^f_{\tau^* hj}). \tag{50}$$

Similarly, if all banks and firms agree at session τ^*, then the interbank market contracts at session τ^* represent an equilibrium outcome; that is, for all $h \in D(z_D)$, if $d_{\tau^* hj} = 1$ for all $h \in D(z_D)$ and $j \in (1, 2, \cdots, J)$,

$$(\rho_h, B_h) = (\rho^b_{\tau^* h}, B^b_{\tau^* h}). \tag{51}$$

B. Equilibrium in a Loan Market with *Strategic Tâtonnement*

I do not intend to describe all possible equilibrium strategies, because the objective is to show an example that ensures the existence of an equilibrium. To identify this, I use a "guess and verify" method. The intuition behind the guess is as follows. Because banks have incentives to exploit as much revenue from firms as possible, they prefer to offer a take-it-or-leave-it contract that specifies both loan rate and amount. Some banks would take into account spill-over effects among their client firms and force them to invest more than suggested by the private marginal product of capital.[54] However, other banks would take advantage of the externalities and make profits by offering a small amount of investment with a slightly higher loan rate to a firm. This free-riding strategy dramatically lowers the return from the former, large-investment strategy. Apparently, banks want to detect the deviation. The interbank market can be used as a detection mechanism: this market will not clear with any free-riding deviations, because deviants always use less capital than others. Banks punish such deviations by changing their offers after they observe an uncleared interbank market. Banks' strategies over the *strategic tâtonnement*, then, should consist of a target contract, a detection mechanism, and a punishment contract.

I define two contracts in the loan market here, as a candidate for a punishment strategy and as a candidate for a target strategy, and call them the *Walrasian contract* and the *Pareto-optimal contract*, respectively. The Walrasian contract in the loan market is denoted by:

$$z_w \equiv (\alpha A, N.S.), \tag{52}$$

where the first element is the loan rate and the second is the loan amount. The loan rate is the same as the Walrasian equilibrium rate, and the amount is not specified so that the loan market always clears at this rate. The Pareto-optimal contract is defined similarly as

$$z_p \equiv (A, \frac{\overline{S}}{J}), \tag{53}$$

where the loan rate A is the same as the social planner's return from the aggregate production function, and the loan amount $\overline{S}/J \equiv \sum_{h=1}^H s_h/J$ is the collected deposit per firm.

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[54] As a consequence, equilibrium contracts are exclusive. Although banks do not prohibit firms from acquiring other loans, a bank's offer specifies a large enough loan amount for a firm to decline any other loan offers.

With the Walrasian contract, banks and firms can achieve the same outcome as the Walrasian equilibrium, where the private marginal product of capital is equal to the loan rate. It, thus, is neither surprising nor difficult to show that the Walrasian contract is an equilibrium contract. Specifically, from the outset $\tau = 1$ or after any session $\underline{\tau}$ in which no banks and firms reach agreement, it is a Nash equilibrium strategy for a bank to repeatedly offer the Walrasian contract if all other banks do so. As long as all banks stick to this strategy, it is also a Nash equilibrium strategy for a firm to accept one of the offered Walrasian contracts. Lemma 2 below formally describes this equilibrium strategies.

Lemma 2. *Repeatedly offering the Walrasian contract constitutes a Nash equilibrium. Specifically, in consecutive sessions from the outset ($\tau \geq 1$) or after any history of disagreed sessions ($\tau > \underline{\tau}$), an equilibrium is the following:*
(i) all banks $h \in D(z_D)$ offer the Walrasian contract to all firms $j = 1, 2, \cdots, J$,

$$\tilde{z}^b_{\tau hj} = z_w; \tag{54}$$

(ii) given these banks' strategies, a firm's best response is determined independent of other firms' strategies, as[55]

$$\begin{aligned} \tilde{z}^f_{\tau hj} &= (\alpha A, \frac{\overline{S}}{J}), \quad \textit{picking one bank, say } h\textit{, and accepting the offer, and} \\ \tilde{z}^f_{\tau lj} &= (\alpha A, 0), \quad \textit{rejecting offers from (submitting no demand for) the other banks } l \neq h; \end{aligned} \tag{55}$$

(iii) when all banks adopt the loan market strategy (54) *and all firms adopt the demand submission strategy (55), an equilibrium strategy in the interbank market is to offer the interbank market rate that is the same as the loan rate, with the interbank borrowing filling any gap between the lending and deposit amounts:*

$$(\rho^b_{\tau h}, B^b_{\tau h}) = (\alpha A, \sum_{j=1}^{J} k^f_{\tau hj} - s_h); \tag{56}$$

(iv) when all banks adopt the loan market strategy (54)*, all firms adopt the demand submission strategy (55), and all banks adopt the interbank market strategy (56), it is an equilibrium strategy for all banks in the confirmation phase to confirm the transaction:*

$$d^b_{\tau hj} = 1; \quad \textit{and} \tag{57}$$

(v) finally, when all banks adopt the loan market strategy (54)*, all firms adopt the demand submission strategy (55), and all banks adopt the interbank market strategy (56) and, subsequently, the confirmation strategy (57), it is an equilibrium strategy for all firms in the confirmation phase to confirm the transaction:*

$$d^f_{\tau hj} = 1. \tag{58}$$

Although the Walrasian contract may prevail in an equilibrium in the loan market as shown in Lemma 2 above, a bank can use a different strategy in the *strategic tâtonnement* to achieve a higher

[55] An equilibrium can be also supported by symmetric borrowing other than as specified below—for example, borrowing the same small amounts of capital from all banks.

return. With a target loan rate ϕA, I define a target loan contract as

$$z_\phi \equiv (\phi A, \frac{\overline{S}}{J}). \tag{59}$$

For now, the loan rate is assumed to be somewhere between the Walrasian and the Pareto-optimal rate; that is, $\phi \in (\alpha, 1]$. The loan amount is the same as in the Pareto-optimal allocation.

Using the Walrasian contract as the punishment strategy, when the interbank market does not clear, any target contract with loan rates between αA and A can be supported as an equilibrium (see Lemma 3 below). Intuitively, if all banks offer the same target contract at all sessions, firms are forced to accept it. It might appear profitable for a bank to deviate by offering a higher interest rate and a smaller loan amount, but with the deviation the interbank market would not clear. Consequently, other banks would not confirm their offers and will adopt the Walrasian contract in the subsequent sessions as punishment, which itself constitutes a Nash equilibrium (Lemma 2). Expecting this future decline in the loan rate, firms also would not confirm their submitted demands. As a result, the punishment phase would begin and all banks, including the deviant, would receive lower revenue than with the target contract: The deviation would not be profitable.

Lemma 3. *The following strategies constitute a Nash equilibrium for $T < \infty$ and a subgame perfect equilibrium for $T = \infty$ in the competitive second stage:*
(i) every bank offers the same target contract unless some banks deviate in previous sessions; that is,

$$\begin{aligned} \tilde{z}^b_{\tau hj} &= z_\phi \quad \textit{if } \tau = 1 \textit{ or if } \tau > 1 \textit{ with } \tilde{z}^b_{\tau-1lj} = z_\phi \textit{ for all } l \in -h, \\ &= z_w \quad \textit{otherwise}; \end{aligned} \tag{60}$$

(ii) when all banks adopt the loan market strategy (60), only two cases happen: either (a) a firm receives only z_ϕ offers from all banks and in this case the firm's best response is

$$\begin{aligned} z^f_{\tau hj} &= (\phi A, \frac{\overline{S}}{J}), \quad \textit{accepting the offer from one bank (e.g., bank } h\textit{), and} \\ z^f_{\tau lj} &= (\phi A, 0), \quad \textit{rejecting the offers from other banks } l \neq h; \end{aligned} \tag{61}$$

or (b) a firm receives at least one offer of the Walrasian contract z_w (e.g., from bank h) and in this case the firm's best response is

$$\begin{aligned} z^f_{\tau hj} &= (\alpha A, \frac{\overline{S}}{J}), \quad \textit{accepting the Walrasian contract offered from bank } h\textit{, and} \\ z^f_{\tau lj} &= (R^b_{lj}, 0), \quad \textit{rejecting the offers from other banks } l \neq h; \end{aligned} \tag{62}$$

(iii) when banks and firms adopt loan market strategies described above (60)–(62), an equilibrium strategy in the interbank market for bank h is

$$\begin{aligned} (\rho^b_{\tau h}, B^b_{\tau h}) &= (\phi A, \sum_{j=1}^{J} k^f_{\tau hj} - s_h) \quad \textit{when } \tilde{z}^b_{\tau hj} = z_\phi, \\ &= (\alpha A, \sum_{j=1}^{J} k^f_{\tau hj} - s_h) \quad \textit{otherwise (when } \tilde{z}^b_{\tau hj} = z_w\textit{)}; \end{aligned} \tag{63}$$

(iv) when banks and firms adopt the loan market and interbank market strategies described above (60)–(63), an equilibrium strategy for a bank in the banks' confirmation phase is to confirm only when the interbank market clears; that is,

$$\begin{aligned} d^b_{\tau hj} &= 1 \quad \text{if } \tilde{B}_\tau(\rho) = 0 \text{ for all } \rho \in \mathbb{R}_+, \\ &= 0 \quad \text{otherwise}; \quad \text{and} \end{aligned} \tag{64}$$

(v) finally, when banks and firms adopt the strategies described above (60)–(64) in the loan market, the interbank market, and the banks' confirmation phase, an equilibrium strategy for a firm in the firms' confirmation phase is to confirm only when the interbank market clears; that is,

$$\begin{aligned} d^f_{\tau hj} &= 1 \quad \text{if } \tilde{B}_\tau(\rho) = 0 \text{ for all } \rho \in \mathbb{R}_+, \\ &= 0 \quad \text{otherwise}. \end{aligned} \tag{65}$$

The set of equilibrium strategies in Lemma 3 produces immediate clearing of the loan and interbank markets. Note that the target loan rate range $(\alpha A, A]$ has been assumed; but, indeed, this must be true with the Nash equilibrium strategies described in Lemma 3. First, $\phi A > A$ is not feasible as a target contract, because A is the highest loan rate technologically possible when firms invest the same amount of capital as specified in the target contract. Second, $\phi A < \alpha A$ does not work either. It cannot be an equilibrium, because the punishment strategy is αA.

The equilibrium outcome is summarized as follows:
(i) no arbitrage of the loan and interbank rates: for all h and j, $R_{hj} = \rho_h = \phi A$;
(ii) upper and lower bounds of the equilibrium interest rate: $\alpha A \leq \phi A \leq A$;
(iii) symmetrical capital allocation for each firm, but not necessarily among banks: $k_{hj} = \overline{S}/J$, for some h, and $k_{lj} = 0$, for $l \neq h$; and
(iv) the interbank market clears:[56] for all $\rho \in \mathbb{R}_+$, $\tilde{B}(\rho) = 0$.

Because the target loan rate ϕA can be any number between αA and A, Lemma 3 implies that, when two or more banks are active, many Nash equilibrium outcomes exist in the second stage for $T < \infty$, as well as many subgame perfect equilibrium outcomes for $T = \infty$. Moreover, there can be many strategies other than those described in Lemma 3 to support the same equilibrium outcome in the loan market, and any mixed combinations of these strategies can constitute equilibria. The punishment loan rate may be different from αA or a more complex scheme can work as well as the simple trigger strategy scheme in Lemma 3. Again, however, the objective is not to list all the possible equilibria in the competitive second stage, but to show the existence of one equilibrium that is consistent with the institution free result of Theorem 1.

C. Equilibrium for the Whole Game

The last question is whether an equilibrium exists so that the unique equilibrium outcome described in Theorem 1 can be realized. But Lemma 3 with $\phi = 1$ is indeed sufficient to support an

[56] Here, bank h's net borrowing from the interbank market is given by $B_h = \sum_{j=1}^{J} k_{hj} - s_h$ at the interbank rate $\rho = \phi A$. For other interest rates, there is neither demand nor supply of funds in the interbank market.

equilibrium for the whole game. Because the *strategic tâtonnement* prevents free riders and loan rates will be A, banks have no fear of competing aggressively for monopolist profits in the deposit market and offering A as the deposit rate. Although there are multiple equilibrium outcomes in the loan market as described in Lemma 3 (i.e., ϕ can be any number between α and 1), competition in the deposit market selects unique Nash equilibrium outcomes in terms of savings, investment, and prices[57] for the whole game within a period, which is apparently consistent over infinite periods. Note that Lemma 3 clearly shows that Assumption 2 is satisfied, as a bank's equilibrium lending strategy does not depend on its deposit share.[58] Finally, the existence theorem is as follows (proof is outlined above and thus omitted).

Theorem 2. ***[Existence of an Equilibrium]*** *With strategic tâtonnement, an equilibrium exists for the strategically intermediated economy* Γ. *The equilibrium outcome is uniquely determined. It is a Pareto-optimal allocation with many active banks with the following properties:*
(i) there is no arbitrage between the deposit rate, the loan rate, and the interbank rate, and these rates are equal to the technologically highest rate; that is, for all i, h, *and* j,

$$r_{hi} = R_{hj} = \rho_h = A; \tag{66}$$

(ii) investment is equal to symmetrically allocated savings at the equilibrium rate

$$k_j = \frac{S(m, (A, N.S.))}{J}, \quad \textit{for all } j; \tag{67}$$

(iii) each firm j *borrows funds from only one bank,*

$$\begin{aligned} k_{hj} &= k_j, \\ k_{lj} &= 0, \quad \textit{for other banks } l \neq h; \textit{ and} \end{aligned} \tag{68}$$

(iv) the interbank market clears

$$\tilde{B}(\rho) = 0. \tag{69}$$

V. Discussion

The interbank market with adjustment process plays an important role in the model. Without it, a strategy similar to Lemma 3 does not work. Consider the following strategy: unless other banks or firms deviate, a bank offers the Pareto-optimal contract; otherwise, the Walrasian contract. This strategy works only when the deposit amount of each bank is assumed to be always equal for the same deposit rate. In general, it does not work, because a fortunate bank with less deposits can offer firms a lower capital level and a slightly higher loan rate to free ride on externalities without fear of punishment. For the same reason, this strategy does not work in an economy with a one-shot interbank market without any price adjustment process.

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[57] Multiple Nash equilibria exist, only because the deposit and loan market shares of each bank are not uniquely determined.

[58] Recall that Assumption 3 is always taken for granted. Assumption 2 restricts the set of equilibrium in the competitive second stage and thus it needs to be satisfied by a proposed equilibrium in the competitive second stage.

If firms are also allowed to issue bonds directly to consumers, in addition to bank intermediation with *strategic tâtonnement*, no Nash equilibrium exists. The reason is the same as in the economy in which the only financial transaction is private placement of bonds (Appendix V). Specifically, consider the case in which, in the first stage, both firms and banks compete for savings and, in the second stage, if banks collect a positive amount of deposits, banks competitively lend to firms while adjusting their funds in the interbank market. A strategy similar to Lemma 3 does not work here either, because the strategy cannot prevent a deviation by firms in the first stage. Firms can raise capital and free ride on others by offering bonds to consumers with a coupon rate slightly higher than A on limited bond issues, when banks offer deposit rate A with no restriction on the deposit amount. Here, including consumers in the *strategic tâtonnement* does not work either, because there is a clear incentive for a consumer to take advantage of a deviating firm's offer with a higher coupon rate.[59]

A Walrasian corporate bond market cannot coexist in an equilibrium, either. This case can be analyzed by replacing Bank 1 with an auctioneer, thereby, restricting the strategy space of Bank 1 to price only. It is easy to see that Banks 2 to H can act the same way as before and, hence, the Pareto-optimal contract can be realized. Bank 1, the auctioneer, must then offer interest rate $r = A$ to obtain any positive deposit. However, if firms can choose the investment amount at that rate, firms free ride on each other and end up not investing at all. Thus, Bank 1 cannot take this strategy. As it would also result in negative profits to offer a higher rate, only remaining strategies are to offer interest rates lower than A. But, by offering a lower rate, Bank 1 would not obtain any deposit and become inactive.[60]

The Pareto optimality of the equilibrium in the benchmark model is specific to the case of constant returns to accumulated capital. As Romer (1986) and Jones and Manuelli (1990) point out, however, it is the only case that allows an economy to grow perpetually. Appendix IV describes a generalized version of Theorem 2. Specifically, a Nash equilibrium interest rate of a strategically intermediated economy is equal to the monopoly loan rate, and the equilibrium savings and investment amount are determined by consumers' optimal choice at the monopoly loan rate. As a result, the equilibrium outcome is not always the same as the social optimal outcome, but often better than the Walrasian equilibrium outcome, when there is sizable production externalities among firms. The proof is exactly the same as above.

Pareto optimality is also peculiar to the production function without labor inputs. Appendix IV also shows another generalized version of Theorem 2, in the case of a production function with labor inputs. Of course, the result is exactly the same as in the benchmark model if the utility function does not exhibit disutility of labor, so that workers are willing to work for no wages. When the utility function exhibits disutility of labor, however, positive wages must be paid to attract workers and,

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[59]The deposit rate would be lowered to αA after the deviation, but this lower rate would not hurt the deviating consumer, as long as the consumer does not want to save more than her investment to the deviating firm. This is likely to happen, because typical savings amount by a consumer is smaller than a typical investment amount by a firm, so that a consumer can invest all of the optimal savings amount in the deviating firm's bond.

[60]For an active Walrasian corporate bond market to coexist with the proposed mechanism, an additional contract space is necessary. For example, a "main bank" clause could work well, if it allows a bank to monitor the aggregate capital level of a firm while permitting a firm to borrow funds from others.

thus, even the monopolist bank cannot obtain all the revenue from firms. Still, the monopolist bank would be able to choose the best wage to maximize its share of firms' revenue, while taking the elastic labor supply into account. As in the benchmark model, competitive banks can mimic this allocation. The equilibrium outcome is not Pareto optimal, but always Pareto superior to the Walrasian equilibrium. The smaller the elasticity of labor supply is, the closer it is to the Pareto-optimal allocation.

The equilibrium loan contract produces an industrial organization resembling a trust, a cartel, or a conglomerate, in the way banks control and coordinate client firms' investment decisions with taking into account externalities, explicitly within a firm group and implicity across firm groups. As such, this paper successfully explains a root of expansion of bank control and formation of competing firm groups, which are identified by various researchers as salient features of industrial development as well as of contemporary financial systems in many countries.[61] Of course, the quest for size of banks themselves and their client firms may well be a result of seeking monopoly rents from consumers.[62] Indeed, in the United States, this criticism led to the introduction of the Glass-Steagall Act, which dissolved the bank-oriented financial system in the United States. However, it seems difficult to evaluate the claim, as such policy action is rare outside the United States and Cantillo Simon (1998) reports *bona fide* values created by banks before the Glass-Steagall Act.

Externality in production is a key aspect in which a bank-oriented financial system outperforms a market-oriented system. Put differently, as industrial development matures, a bank-oriented financial system loses its advantage. This prediction seems in line with contemporary Japanese and German experiences which have shown weakening of the bank-oriented financial system.[63] This phenomenon could also be explained by typical theories, as a consequence of lower informational problems and transaction costs. However, policy implications for developing countries are different between two explanations. If internalizing production externalities is more important as a bank's role in promoting growth, then even with improved information flows, say, by adoption of a better disclosure system, the bank-oriented financial system thrives and should not be viewed as an inferior regime. On the other hand, if mitigation of informational problems is more important, improved information flows should help a U.S.-like market-oriented financial system to emerge in any country, regardless of its development stage.

VI. Concluding Remarks

In an era of massive industrial development in the United States, Europe, and Japan, as well as in contemporary emerging market economies, several bankers, financiers, and

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[61]Bank control is one of the main issues when comparing financial systems (Allen and Gale, 2000) and when comparing corporate governance (Becht, Bolton, and Röell, 2003).

[62]For example, Rajan and Zingales (2003) argue that big financiers and large companies typically are protected under relationship-based corporate governance and that they tend to alter institutions via their powerful political influence. Recall that, to clarify the banks' role in the capital market, this paper implicitly assumes that the Walrasian competitive equilibrium prevails in the product market, leaving out the possibility of monopoly rents there.

[63]See Aoki and Patrick (1994) and Hoshi and Kashyap (2001) for the Japanese case and Krahnen and Schmidt (2004) for the German case.

industrialists-turned-investors became famous for the aggressive expansion of their business and organized control of many affiliated companies. They raised funds from the public and invested heavily in businesses that became the foundation of modern industry, such as shipping lines, railroads, and steel mills. There is no doubt that these businesses raised the productivity of many firms, both affiliated and nonaffiliated.

This paper is the first theoretical study to formalize the active development role played by banks. Banks competitively internalize production externalities and facilitate economic growth. In a canonical growth model with production externalities, banks compete for deposits to obtain potential monopoly profits, taking externalities into account. Using loan contracts that specify both price and quantity, banks control firms' investment decisions. By doing so, each bank forms a firm group endogenously, and internalizes externalities directly within a firm group and indirectly across firm groups.

It is not straightforward to identify an equilibrium, as this economy inherently lacks a Nash equilibrium without some institutional setup. Hence, I first identified the unique equilibrium candidate under a general condition called a *weak link of sources and uses of funds*, which allows banks to compete for deposits without worrying about their fund positions in the competitive loan market. Second, I presented an example of institutions that satisfy the condition and support the equilibrium. The example—*strategic tâtonnement*—is not far from reality: banks should be able to negotiate loan terms (*free recontracting*) with firms and adjust their fund positions in an interbank market.

The equilibrium allocation is Pareto optimal for the conventional case of constant returns to accumulated capital. As such, government intervention is unnecessary, even during an economic development process with positive production externalities, which is a main reason for subsidies and industrial policies that often conceal corruption and generate monopoly rights. This policy implication contrasts with the traditional view that banks and firm groups should be regulated to limit their monopolistic behaviors.

The presented theory complements existing theories of banks such as alleviation of informational problems and transaction costs. However, to understand banks' role in economic development further, future research is warranted to assess how well each model explains actual economic growth paths. The presented framework, as it formulates strategic behavior of a financial sector interwoven with a standard growth model, should be suited to include other roles of banks and to compare both qualitative and quantitative predictions of competing theories.

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APPENDIX I. PROOFS

A. Proof of Lemma 1

Proof. The monopolist bank always specifies both the loan rate and amount for each firm, because by doing so it earns the highest revenue, as long as firms accept the contracts. A firm accepts the contract if it delivers nonnegative profit; in other words, firm j's best response can be replaced by the participation constraint,

$$A\left(\frac{\sum_{l\in -j} k^b_{Ml}}{J-1}\right)^{1-\alpha}(k^b_{Mj})^{\alpha} - R^b_{Mj}k^b_{Mj} \geq 0. \tag{A1}$$

Here, the monopolist bank's problem (25) can be restated as

$$\max_{R^b_{Mj},k^b_{Mj}} \sum_{j=1}^{J} R^b_{Mj}k^b_{Mj} - \sum_{i=1}^{I} r_{Mi}s_{Mi}, \tag{A2}$$

subject to the resource constraint

$$\sum_{j=1}^{J} k^b_{Mj} \leq \sum_{i=1}^{I} s_{Mi} \tag{A3}$$

and the participation constraint (A1).

I now solve this constrained optimal problem. First, given the liability $\sum_{i=1}^{I} r_{Mi}s_{Mi}$ committed in the first stage, the monopoly bank's profit π^b_M increases in loan amount k^b_{Mj} for any given loan rate R^b_{Mj}. Hence, the monopoly bank lends out all deposits and resource constraint (A3) is satisfied with equality. Second, because the monopoly bank earns the highest profits when firms' profits are zero, participation constraint (A1) is satisfied with equality.

By replacing the first term of (A2) by the participation constraint (A1) at equality, the optimal problem can be written as, for any firm $j \in \{1, 2, \cdots, J\}$,

$$\max_{R^b_{Mj},k^b_{Mj}} \sum_{j=1}^{J} A\left(\frac{\sum_{l\in -j} k^b_{Ml}}{J-1}\right)^{1-\alpha}(k^b_{Mj})^{\alpha} - \sum_{i=1}^{I} r_{Mi}s_{Mi}. \tag{A4}$$

Because of the concavity of the production function, given resource constraint (A3), the sum of profits from all firms is maximized by assigning an equal amount of capital to all firms; that is, for any firm $l \in -j$ and $j \in \{1, 2, \cdots, J\}$,

$$k^b_{Ml} = k^b_{Mj} = \overline{k} \equiv \frac{\sum_{i=1}^{I} s_{Mi}}{J}. \tag{A5}$$

To clarify this point, let $k^b_{Mj} = \gamma_j \overline{k}$. Because $\sum_{j=1}^{J} k^b_{Mj} = J\overline{k}$, there is a natural constraint on scaler γ_j,

$$\sum_{j=1}^{J} \gamma_j = J. \tag{A6}$$

Using the same arithmetic as in (A7), the revenue can be written as

$$\sum_{j=1}^{J} A \left(\frac{\sum_{l\in -j} k^b_{Ml}}{J-1} \right)^{1-\alpha} (k^b_{Mj})^{\alpha} = \sum_{j=1}^{J} A\overline{k} \left(\frac{1}{J-1} \gamma_j^{\alpha} (J-\gamma_j)^{1-\alpha} \right). \tag{A7}$$

Then, the monopolist faces the following constrained maximum problem:

$$L(\gamma_1, \gamma_2, \cdots, \gamma_J, \lambda) = \sum_{j=1}^{J} \left(\gamma_j^{\alpha} (J-\gamma_j)^{1-\alpha} \right) + \lambda \left(J - \sum_{j=1}^{J} \gamma_j \right). \tag{A8}$$

This is exactly the same as the social planner's problem explained in Appendix III. Therefore, symmetric lending $\gamma_j = 1$ for all firms is the unique maximizer of the Lagrangean (A8).

As in the social planning problem, the output of firm j becomes $A\overline{k}$, because all firms invest the same amount. Hence, the highest interest rate that the bank can charge firms is A. Note that, at this loan rate, each firm has zero profit, satisfying the participation constraint (A1). *Q.E.D.*

B. Proof of Theorem 1

The proof consists of six lemmas below. First, Lemma 4 shows that expected revenues from the loan market is the same for all banks. Second, Lemma 5 shows that the highest return is achieved when banks mimic the loan allocation by a monopoly lender. Third, Lemma 6 shows that banks do not discriminate against depositors and there would be no arbitrage opportunities between the deposit and loan markets in an equilibrium. Fourth, Lemma 7 shows that competition in the deposit market drives up the deposit rate to A. Fifth, Lemma 8 shows that equilibrium loan contracts mimic the monopolist's. Finally, Lemma 9 shows that banks compete for deposits essentially in price only.

Lemma 4. *Under Assumptions 2 and 3, expected revenue in the competitive loan market is the same for all active banks regardless of performance in the deposit market.*

Proof. This Lemma is trivially true, if the equilibrium strategies are pure and symmetric and the associated outcomes (market shares) are the same. Indeed, Assumption 3 implies that, if the equilibrium strategies of banks and firms are pure and symmetric, market shares must be the same.

When the equilibrium strategies are pure but asymmetric, outcomes can be asymmetric. In this case, some banks free ride on others to achieve higher returns than others. However, condition (37) states that, if bank h adopted bank l's strategy and bank l adopted bank h's while all firms exchanged their strategies toward bank h with those toward bank l, then bank h would earn bank l's revenue and bank l would earn bank h's revenue. Moreover, the probabilities of these two scenarios occurring (i.e., the original strategies and the exchanged ones) are the same in an equilibrium.

Hence, given an asymmetric equilibrium for bank h, any permutation of its equilibrium strategy with other banks, together with associated changes in firms' strategies toward banks, would constitute equilibria. In other words, there exists a set of equilibria that is generated by permutating an equilibrium. Let it be called as *an equilibrium group generated by permutations of an equilibrium*. For example, in a two-bank and two-firm economy, if

$$\{z^b, z^f\} = \{(z^b_{11}, z^b_{12}), (z^b_{21}, z^b_{22}), (z^f_{11}, z^f_{12}), (z^f_{21}, z^f_{22})\} = \{(\gamma, \delta), (\xi, \psi), (\Gamma, \Delta), (\Xi, \Psi)\}$$

is an equilibrium, so is

$$\{z^{b,2\backslash 1}, z^{f,2\backslash 1}\} = \{(\xi, \psi), (\gamma, \delta), (\Xi, \Psi), (\Gamma, \Delta)\}.$$

These two equilibria constitute the equilibrium group generated by permutations of an equilibrium $\{(\gamma, \delta), (\xi, \psi), (\Gamma, \Delta), (\Xi, \Psi)\}$. This notion is expressed similarly using mixed strategies—for example, by defining that λ^*_{11} has a mass of one at γ and zero elsewhere, that λ^*_{12} has a mass of one at δ and zero elsewhere, and so on.

If $\#D$ banks are active, there are only $\#D$ numbers of equilibrium outcomes for a particular bank in the equilibrium group generated by an equilibrium. This is because bank h's outcome is determined only by bank h's strategy, $(z^b_{h1}, z^b_{h2}, \cdots)$, and firms' strategies toward bank h, $(z^f_{h1}, z^f_{h2}, \cdots)$, exchanging strategies among bank-firm pairs that does not involve bank h would not affect bank h's profits.

Given an equilibrium strategy group, condition (37) assures that a bank faces the same chances to realize any outcomes among $\#D$ possibilities.[64] If the equilibrium strategy group is generated by permutations of an equilibrium that gives (R^*_{hj}, k^*_{hj}), then a bank's expected revenue is

$$\frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R^*_{hj} k^*_{hj}, \tag{A9}$$

which is the average of revenues from all strategies in the equilibrium strategy group.

Moreover, multiple equilibrium strategy groups may exist. In this case, condition (37) implies that all active banks face the same probability of selecting a specific equilibrium strategy vector and, thus, a specific equilibrium strategy group. Therefore, expected revenue in the competitive loan market is just a linear combination of (A9) and is the same for all active banks. Let $e_1, e_2, \cdots, e_N$ are equilibrium strategy groups and $\Psi(e_n)$ denote equilibrium probability of realization of the n-th equilibrium strategy group. The expected revenues are the same for all active banks,

$$\sum_{n=1}^{N} \left(\frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R^*_{hj} k^*_{hj} \right) \Psi(e_n), \tag{A10}$$

[64] This $\#D$ number of equilibrium outcomes could be the same, if an original equilibrium were pure and symmetric. For the sake of simplicity, I still treat it as a distinct $\#D$ number of outcomes.

and so are the expected returns,

$$R^e \equiv \sum_{n=1}^{N} \left(\frac{1}{\#D} \sum_{h \in D(z_D)} \frac{\sum_{j=1}^{J} R^*_{hj} k^*_{hj}}{\sum_{j=1}^{J} k^*_{hj}} \right) \Psi(e_n). \tag{A11}$$

Q.E.D.

Lemma 5. *A bank's expected revenue is the highest if active banks together mimic a loan allocation assigned by a monopolist. In this case, expected return R^e is equal to A, the technologically highest return under symmetric investment.*

Proof. Lemma 4 shows that all active banks face the same expected revenue, given an equilibrium strategy group. But, as in the proof for Lemma 1, the sum of profit income is highest when the same amount of capital is allocated among firms; that is, for any equilibrium group,

$$\frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R^*_{hj} k^*_{hj} \le \frac{1}{\#D} \sum_{j=1} A K_{Mj}^{1-\alpha} k_{Mj}^{\alpha} = \frac{1}{\#D} A k_{Mj}. \tag{A12}$$

When asymmetric multiple equilibrium strategy groups exist, (A12) is true for each equilibrium strategy group and overall expected revenue is a linear combination of (A12). Hence, it has the same upper bound:

$$\sum_{n=1}^{N} \frac{1}{\#D} \sum_{h \in D(z_D)} \sum_{j=1}^{J} R^*_{hj} k^*_{hj} \Psi(e_n) \le \sum_{n=1}^{N} \frac{1}{\#D} \sum_{j=1} A K_{Mj}^{1-\alpha} k_{Mj}^{\alpha} \Psi(e_n) = \frac{1}{\#D} A k_{Mj}. \tag{A13}$$

Therefore,

$$R^e \le A. \tag{A14}$$

Q.E.D.

Lemma 6. *Equilibrium deposit rates of a bank are nondiscriminatory among depositors; that is, $r_{hi} = r_h$, for all $i = 1, 2, \cdots, I$. Moreover, no arbitrage condition holds in an equilibrium: $r_h = r = R^e$.*

Proof. There are two claims in this proof. The first claim is that deposit rates may vary among depositors, but, in an equilibrium, the weighted average of deposit rates must be equal to the expected return,

$$\frac{\sum_{i=1}^{I} r^*_{hi} s^*_{hi}}{\sum_{i=1}^{I} s^*_{hi}} = R^e, \tag{A15}$$

in a pure strategy equilibrium and in a support of a mixed equilibrium.[65]

Here is the proof for the first claim. If the left-hand side of (A15) is strictly greater than the right-hand-side, bank h would have negative profit and cease to operate. If the left hand side is

[65]Lemma 4 has shown that R^e is equal for all banks in an equilibrium.

strictly smaller than the right hand side, an apparent arbitrage opportunity by rival banks would exist, offering slightly higher interest $r_{hi} + \epsilon$ for a depositor. With this strategy, it would be feasible for rival banks to collect the same deposit amount as bank h[66] and they would expect positive profits,

$$R^e - \frac{\sum_{i=1}^{I}(r_{hi} + \epsilon)s_{hi}}{\sum_{i=1}^{I} s_{hi}} > 0. \tag{A16}$$

Therefore (A15) must hold in an equilibrium.

The second claim is that banks would not discriminate against depositors. Proof is given by contradiction. Assume bank h discriminates against same depositors in an equilibrium. Condition (A15) then implies that some depositors are offered higher-than-average interest $r_{hi} > R^e$ and at least one of the others, say the m-th household, faces an offer with lower-than-average interest $r_{hm} < R^e$. Here, however, a rival bank has an arbitrage opportunity, something impossible in an equilibrium. To see the arbitrage opportunity, note that a rival bank can earn positive profits by offering a slightly higher deposit rate $r_{hm} + \epsilon$ to the m-th household, only by specifying the same deposit amount. Since the expected loan rate is the same, the profit is positive; that is, $(R^e - (r_{mh} + \epsilon))s_{mh} > 0$.

In summary, deposit rates offered by a bank must be nondiscriminating among depositors in an equilibrium. Hence, i subscript of r_{hi} can be omitted as r_h. But, condition (A15) implies $r_h = R^e$ and, thus, the deposit rate must be the same for all active banks in an equilibrium. Accordingly, subscript h can be also dropped, so that $r_h = r = R^e$. *Q.E.D.*

The next two lemmas show that banks bid up their deposit rates until $r = A$ in an attempt to capture monopoly profits and that banks have to charge at least this rate in the loan market $R \geq A$ to meet their own nonnegative profit condition.

Lemma 7. *The deposit rate r is equal to A in a Nash equilibrium of the whole game, if it exists.*

Proof. Lemma 6 shows that an equilibrium deposit rate is the same for all banks and depositors. Lemmas 5 and 6 implies $r = R^e \leq A$. However, a deposit rate $r = \underline{A} < A$ cannot be an equilibrium rate. because, if it prevailed in an equilibrium, a bank could become a monopolist by deviating to offer a slightly higher deposit rate $\underline{A} + \epsilon$ with specifying the same deposit amount under $\underline{A}$. The deviating bank then earns a positive profit $(A - \underline{A} - \epsilon)S(m, (\underline{A}, N.S.))$. There always exists $\epsilon > 0$ for some banks to make this deviation profitable.

To clarify this, let us consider two cases, assuming the prevailing deposit rate is $\underline{A} < A$. First, consider a case in which some banks do not collect any deposits. If one of these unsuccessful banks deviates and offers a slightly higher deposit rate $\underline{A} + \epsilon$, it instantly increases its profit from zero to $(A - \underline{A} - \epsilon)S(m, (\underline{A}, N.S.))$. Obviously, any $\epsilon < A - \underline{A}$ works well.

Second, consider a case in which all banks collect some deposits. Let $\gamma_h \in (0, 1)$ denote the deposit market share of bank h. For deviation to be profitable, it must be the case that profit without

[66] The deviating firm can specify deposit amount as can bank h. With a higher interest rate, this specified deposit might not be the optimal amount for households. However, any household would prefer this deviant's offer, because the deposit rate is higher for the same amount of deposits.

deviation is less than the profit with deviation. Since the expected loan rate is $R^e \leq A$, profit without deviation has the natural upperbound,

$$\gamma_h(R^e - \underline{A})S(m, (\underline{A}, N.S.)) \leq \gamma_h(A - \underline{A})S(m, (\underline{A}, N.S.)). \tag{A17}$$

Here, take $\epsilon \equiv (A - \underline{A}) - \gamma_h(A - \underline{A}) - \delta$ with some small $\delta > 0$. Because $0 < \gamma_h < 1$ (all banks collect some deposits), there always exists such $\epsilon > 0$. Using this ϵ, it becomes clear that the right-hand-side of (A17) is less than the profit if deviated,

$$\gamma_h(A - \underline{A})S(m, (\underline{A}, N.S.)) < (A - \underline{A} - \epsilon)S(m, (\underline{A}, N.S.)). \tag{A18}$$

Q.E.D.

Note that Lemma 6 ($R^e = r$) and Lemma 7 ($r = A$) imply that, in an equilibrium, the expected loan rate must be equal to the deposit rate determined by competition in the deposit market. Namely,

$$R^e = r = A. \tag{A19}$$

Lemma 8. *If an equilibrium exists, equilibrium contracts generate a Pareto-optimal allocation. More specifically, each firm faces only one type of offer, which is $(A, S(m, (A, N.S.))/J)$ and accepts one bank's offer while rejecting offers from other banks. Accordingly, each firm invests the same amount $S(m, (A, N.S.))/J$ and repays them with gross loan rate A.*

Proof. As shown in the proof of Lemma 1, the aggregate production, given total funds, is uniquely maximized by symmetric capital allocation among firms. Therefore, symmetric capital allocation among firms is necessary for all banks to achieve the expected loan rate $R^e = A$; that is,

$$\sum_{h=1}^{H} k_{hj} = \frac{S(m, (A, N.S.))}{J}. \tag{A20}$$

Because banks' expected return is $R^e = A$, the weighted average loan rate must be A; that is,

$$\frac{\sum_{h=1}^{H} R_{hj} k_{hj}}{\sum_{h=1}^{H} k_{hj}} = \frac{\sum_{h=1}^{H} R_{hj} k_{hj}}{S(m, (A, N.S.))/J} = A. \tag{A21}$$

Note that the two conditions above, (A20) and (A21), do not exclude the possibility that loan contracts are different among bank-firm pairs in an equilibrium. However, this is not the case.

To clarify this, I first show that only one loan rate prevails in an equilibrium and I then show that all banks specify the same amount of loans.

Suppose that different loan rates exist in an equilibrium. Then, at least one bank must offer a loan rate greater than A to firm j, as (A21) describes that the weighted average of equilibrium loan rates must be A. Sorting banks according to the loan rate to firm j from low to high, the last, H-th bank is assumed to offer the highest loan rate $R_H > A$ without loss of generality. Because bank H's offer is accepted by a firm in an equilibrium, the marginal cost for the firm of accepting the offer must be lower than the marginal revenue. The marginal cost is $R_H k_H$ and the marginal revenue is the difference between investing the equilibrium amount $\overline{K}$ and the amount without the last bank's fund k_H:

$$A\overline{K}^{1-\alpha}\overline{K}^{\alpha} - A\overline{K}^{1-\alpha}(\overline{K} - k_H)^{\alpha}. \tag{A22}$$

The marginal revenue minus the marginal cost then is

$$\begin{aligned} &A\overline{K}^{1-\alpha}\overline{K}^{\alpha} - A\overline{K}^{1-\alpha}(\overline{K} - k_H)^{\alpha} - R_H k_H, \\ <&A\overline{K} - A\overline{K}^{1-\alpha}(\overline{K} - k_H)^{\alpha} - Ak_H, \\ =&A(\overline{K} - k_H) - A\overline{K}^{1-\alpha}(\overline{K} - k_H)^{\alpha}. \end{aligned} \tag{A23}$$

However, this is always negative, because the first term is less than the second term, as is easily shown:

$$\begin{aligned} A(\overline{K} - k_H) <&A\overline{K}^{1-\alpha}(\overline{K} - k_H)^{\alpha}, \\ 1 <&\left(\frac{\overline{K}}{\overline{K} - k_H}\right)^{1-\alpha}. \end{aligned} \tag{A24}$$

Obviously, a nonspecified option for the loan amount cannot alter this result. Different loan rates therefore cannot exist in an equilibrium.

It is now clear that loan rates are the same A for any bank-firm pairs in an equilibrium. At this loan rate, firms want to borrow less than others. Hence, a firm will pick a loan contract that specifies the least loan amount and reject all other offers. But, in an equilibrium, as shown above, (A20) must be satisfied and thus the least amount of loan offered to a firm must be the Pareto-optimal amount, $S(m, (A, N.S.))/J$. In an equilibrium, it must also be the case that the sum of loans over J firms is equal to $S(m, (A, N.S.))$ to clear the loan market. Therefore, symmetrical loan amount $S(m, (A, N.S.))/J$, together with loan rate A, is the only equilibrium offer by each bank to each firm. *Q.E.D.*

Lemma 9. *In an equilibrium, a depositor faces at least one unspecified offer or, if all are specified, she must be able to combine offers to replicate her willingness to supply capital at A. For a bank, not specifying deposit amounts $s^b_{ih} = \{N.S.\}$ and specifying the depositor's willingness to supply $s^b_{ih} = S(m, (A, N.S.))/I$ are two dominant strategies. Other specification may be a Nash equilibrium strategy as long as the combination of offers from several banks replicates each depositor's willingness to supply capital at A.*

Proof. If any combination of offers does not fulfill each depositor's willingness to supply at deposit rate A, then an apparent arbitrage opportunity exists, so that these offers cannot constitute an equilibrium. Indeed, if any combination of offers limits a savings amount less than a depositor's willingness to supply at deposit rate A, a bank will deviate to offer a lower rate $A - \epsilon$, at which the bank is still able to collect the same deposit amount as at A. Using this strategy, this bank would earn extra profit, because the equilibrium expected loan return is A by Lemma 8. On the other hand, if any offers from banks specify a savings amount larger than a depositor's willingness to supply at A, a bank will deviate to offer a nonspecified amount with a slightly lower interest rate $(r_h = A - \epsilon, s_h = \{N.S.\})$ and could capture the entire deposit market and earn the monopoly rent.

Therefore, in an equilibrium, at least one combination of offers must give each depositor exactly the same as his willingness to supply capital at deposit rate A. Because bank's strategies $(A, N.S.)$ and

$(A, S(m, (A, N.S.))/I)$ always satisfy these conditions, these two strategies are always equilibrium strategies. Note that since these equilibrium strategies are not conditional on other banks' (as well as consumers' and firms') strategies, they are dominant strategies.

However, any specified amounts with deposit rate A may be a Nash equilibrium conditional on other banks' strategies, because only a combination of offers concerns depositors. For example, when some banks offer $(A, S(m, (A, N.S.))/3I)$ to a depositor, other bank's strategy $(A, 2S(m, (A, N.S.))/3I))$ to the same depositor is an equilibrium strategy, as these strategies enable the depositor to make deposits equal to her willingness to save, $S(m, (A, N.S.))/I$. *Q.E.D.*

The proof for Theorem 1 is complete.

C. Proof of Lemma 2

Proof. There exists no profitable deviation: When other banks stick to the Walrasian contract, the best strategy is for a bank to also adopt the Walrasian contract. Under this equilibrium, bank h's revenue is $\alpha A s_h$. I will show that this is the highest revenue that a bank could obtain when other banks stick to the Walrasian contract. In the following, I consider all potential deviations, which are classified into four cases.

First, consider the case where a deviant offers a lower loan rate than the Walrasian contract to firms but keeps the same strategy in the interbank market as in the Walrasian contract; that is, for $\epsilon > 0$, $\{(R^b_{\tau hj} = \alpha A - \epsilon, k^b_{\tau hj} = N.S.)^J_{j=1}, (\rho_{\tau h} = \alpha A, B_{\tau h} = k^f_{\tau h} - s^c_h)\}$. Obviously, the deviant attracts all demands for funds, at least the same aggregate demand $\overline{S}$ for the Walrasian contract. Thus, loan revenue is equal to or greater than $(\alpha A - \epsilon)\overline{S}$. The deviant needs to finance the difference between the loan amount and collected deposits from the interbank market at rate αA. Hence, the net revenue from this operation is, at most, $\alpha A s_h - \epsilon\overline{S}$, which is less than $\alpha A s_h$, the revenue the bank could earn if it did not deviate. The result would not change if the deviant restricts the loan amount to less than $\overline{S}$.

Second, consider the case where a deviant offers a loan rate lower than the Walrasian contract to firms as well as to the interbank market; that is, for $\epsilon > 0$ and $\xi > 0$, $\{(R^b_{\tau hj} = \alpha A - \epsilon, k^b_{\tau hj} = N.S.)^J_{j=1}, (\rho_{\tau h} = \alpha A - \xi, B_{\tau h} = k^f_{\tau h} - s^c_h)\}$. Because the interbank market rate is different from other banks, the interbank market will never clear at $\rho = \alpha A - \xi$ when the deviant participates in the interbank market. Hence, the revenue from this strategy is, at most, $(\alpha A - \epsilon)s_h$, when firms agree to obtain loans from the deviating bank. The potential profit from deviation is less than $\alpha A s_h$, what the bank could get if it did not deviate.

Third, consider the case where the deviant offers a higher loan rate to firms than the Walrasian contract, but keeps the same strategy in the interbank market as in the Walrasian contract, such that $\{(R^b_{\tau hj} = \alpha A + \epsilon, k^b_{\tau hj} = N.S.)^J_{j=1}, (\rho_{\tau h} = \alpha A, B_{\tau h} = k^f_{\tau h} - s^c_h)\}$. Because the interbank market rate is the same as the Walrasian contract, other banks borrow the funds. Because the loan rate from the deviating bank is higher than others, firms do not borrow from the deviating bank (see the proof for this claim below). In this case, the deviant earns revenue solely from the interbank market, at most $\alpha A s_h$, what the bank could earn if it did not deviate.

Finally, consider the case where a deviant offers a higher interest rate than the Walrasian contract to firms, as well as to the interbank market; that is, for $\epsilon > 0$ and $\xi > 0$, $\{(R^b_{\tau hj} = \alpha A + \epsilon, k^b_{\tau hj} = N.S.)^J_{j=1}, (\rho_{\tau h} = \alpha A + \xi, B_{\tau h} = k^f_{\tau h} - s^c_h)\}$. Again, firms do not have a positive demand for loans from the deviant offering $\alpha A + \epsilon$ rate (see again the proof for this claim below). Instead, a firm's best reaction is to submit demand $(\overline{S} - s_h)/J$ to other banks.[67] As a consequence, the deviating banks cannot lend out any funds to firms. Moreover, because the deviant asks for a higher interest rate and other banks do not need the deviant's capital to clear the interbank transactions at rate αA, the deviating bank cannot lend out its funds in the interbank market either. Therefore, the deviant cannot earn any revenue.

The remaining task here is to prove the claim that firms do not have positive demand for the loan offer from the deviant with the loan rate at $\alpha A + \epsilon$. This can be seen by comparing a firm's profit under the Walrasian contract and its profit under the deviating bank's contract. Given any level of other firms' investment K_j, firm j's profit with loan rate R_j can be written as

$$\pi^f_j = AK_j^{1-\alpha} k_j^\alpha - R_j k_j. \tag{A25}$$

Using the marginal revenue $\partial f/\partial k_j$, it is

$$\pi^f_j = \left(\frac{\partial f/\partial k_j}{\alpha} - R_j \right) k_j. \tag{A26}$$

Because a firm is facing loan offers (both Walrasian and deviant) that do not specify loan amount, firm j can chose the optimal level of investment by equating the private marginal revenue to the loan rate,

$$\frac{\partial f}{\partial k_j} = R_j. \tag{A27}$$

This first-order condition determines firm j's optimal investment, given loan rate R_j and other firms' average investment K_j, as follows:

$$k_j = \left(\frac{\alpha A}{R_j} \right)^{\frac{1}{1-\alpha}} K_j. \tag{A28}$$

Substituting (A28) and (A27) into (A26), the privately maximized profit is described as

$$\left(\frac{1}{\alpha} - 1 \right) R_j \left(\frac{\alpha A}{R_j} \right)^{\frac{1}{1-\alpha}} K_j. \tag{A29}$$

Hence, the maximized profit under the Walrasian contract is

$$\left(\frac{1}{\alpha} - 1 \right) \alpha A K_j. \tag{A30}$$

Similarly, the maximized profit with the deviant's offer, given the (same) other firms' average investment K_j, can be calculated by setting loan rate R_j equal to the deviating bank's offer, which is

[67] As long as symmetric investment by each firm is maintained, demand submission to each bank could be anything; for example, submitting demand $(\overline{S} - s_h)/J(H-1)$ to all banks provided that other firms do the same. Another example is to submit all the demand $(\overline{S} - s_h)/J$ to one bank and no demand to other banks.

$\alpha A + \epsilon$. It becomes

$$\left(\frac{1}{\alpha} - 1\right)(\alpha A + \epsilon)\left(\frac{\alpha A}{\alpha A + \epsilon}\right)^{\frac{1}{1-\alpha}} K_j. \tag{A31}$$

Here, I claim that profit with the Walrasian contract (A30) is larger than profit with the deviant's offer (A31); that is, after canceling out K_j and $(1/\alpha - 1)$,

$$\alpha A > (\alpha A + \epsilon)\left(\frac{\alpha A}{\alpha A + \epsilon}\right)^{\frac{1}{1-\alpha}}. \tag{A32}$$

This can be simplified to

$$\left(\frac{\alpha A}{\alpha A + \epsilon}\right)^{\frac{\alpha}{1-\alpha}} < 1, \tag{A33}$$

and it is easy to see that this is always true for any $\epsilon > 0$ given $\alpha \in (0, 1)$.

Therefore, a firm always prefers the Walrasian contract over the deviating bank's offer with a higher loan rate and, as a result, the deviating bank would not get any loan demand from firms. *Q.E.D.*

D. Proof of Lemma 3

Proof. First I show $T = \infty$ case. Since all banks adopt the same target strategy z_ϕ, the target loan contract is the only contract offered to firms for all sessions τ. Knowing this, there is no gain in waiting. Hence firms accept banks' target contract z_ϕ immediately.

The target loan amount $\overline{S}/J$ and punishment strategy z_w are necessary to sustain the above-mentioned strategy as an equilibrium. With this strategy, no bank wants to deviate from the target strategy z_ϕ, because the revenue from firms would dwindle if it deviates. To clarify, consider a one-bank-and-one-firm deviation when $\phi = 1$. Bank 1 could think about offering Firm 1 a smaller amount of capital $\underline{\mathrm{k}}$ to share the potential profits with the client firm by free riding on other firms' investment. Note that this would be profitable, $AK^{1-\alpha}\underline{\mathrm{k}}^{\alpha} - A\underline{\mathrm{k}} > 0$ for $\underline{\mathrm{k}} < K$.

However, this scheme does not work under the other banks' strategy specified in Lemma 3. Because other banks are offering contract $z_\phi = (R = A, k_{lj} = \overline{S}/J)$ to $(J-1)$ firms, the net borrowing demand from the other banks in the interbank market is

$$\begin{aligned} \sum_{l=2}^{H} B_l &= \sum_{l=2}^{H} k_{lj} - s_l \\ &= \frac{J-1}{J}\overline{S} - (\overline{S} - s_1) \\ &= s_1 - \frac{1}{J}\overline{S}. \end{aligned} \tag{A34}$$

This is the difference between the deposit amount that Bank 1 collected and the average size of deposits in the banking system. In other words, it is the residual demand for Bank 1's fund in the interbank market. Depending on the size of residual demand and Bank 1's willingness to clear the interbank market, there are three scenarios.

1. If deposits in Bank 1 are equal to average deposits, $s_1 = \frac{1}{J}\overline{S}$, the residual demand/supply is equal to zero. In this case, there is no lending opportunity for Bank 1 in the interbank market. Then it is optimal for Bank 1 to lend out the whole fund s_1 to firms. This is not a deviation.

2. If Bank 1 has deposits different from the average deposit per firm, $s_1 \neq \frac{1}{J}\overline{S}$, but decides to clear the interbank market, Bank 1 must lend the average size of all deposits in the banking system and lend or borrow funds equal to the difference of the deposit amount that Bank 1 collected. This is not a deviation either.

3. If Bank 1 has deposits different from the average deposit per firm, $s_1 \neq \frac{1}{J}\overline{S}$, and prevents the interbank market from clearing, Bank 1 would lend less than the average deposit per firm.[68] Because the interbank market does not clear, this deviation is detected by other banks. According to the specified strategy, punishment from other banks would be triggered. That is, in the second session and later sessions, other banks would offer the Walrasian contract $z_w = (R^b_{lj} = \alpha A, k^b_{lj} = N.S.)$ in the loan market with the associated interbank market strategy $(\rho^b_l = \alpha A, B^b_l = \sum_{l=2}^{H} k_{lj} - s_1)$. Knowing this, in the first session, firms that received offers from Bank 1 would not submit any demand to Bank 1; or, even if they submit a demand, they would not sign the confirmation letter (i.e., $d_f = 0$) at the end of the first session. As a result, Bank 1 would have to engage in the second session. Moreover, in the second and later sessions, the deviating bank can no longer attract any clients with a higher loan rate, $R_1 > A$. Therefore, it is not profitable to deviate from the specified strategy.

Other cases, such as one-bank-and-two-firm deviations, are analyzed similarly. Because deviation is profitable only with less investment than others, if some banks deviate, the interbank market will not clear when other banks stick to the target contract, z_ϕ.

Since z_w constitutes a Nash equilibrium by Lemma 2, the specified strategy in Lemma 3 is a subgame perfect equilibrium for $T = \infty$. For the case with $T < \infty$, the only Nash equilibrium interest rate at the last session T is αA, as is the case with a one shot interbank market. Hence, when $T < \infty$, only the Walrasian contract survives through the *strategic tâtonnement* as a subgame perfect equilibrium.[69] This is because a part of the proposed strategy, "$\tilde{z}^b_{\tau hj} = z_\phi$ as long as $\tilde{z}^b_{\tau-1lj} = z_\phi$," is not a credible promise.[70] However, the banks' strategy specified in Lemma 3 still constitutes a Nash equilibrium from the viewpoint of the beginning of the *strategic tâtonnement*, in the "normal form" description of the game, where all strategy sequences are presented at the outset.[71] *Q.E.D.*

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[68] This is true even when bank 1's deposit is larger than the average deposit per firm, because bank 1 can raise extra revenue per firm only by lending less than other banks.

[69] In this subgame perfect equilibrium, firms would like to reject other offers and wait until session T to accept the Walrasian contract.

[70] It is still a credible threat that banks revert to the Walrasian contract if some banks deviate.

[71] Recall that the important issue when analyzing a discontinuous game is to find conditions for the existence of a Nash equilibrium, not to use refinement techniques to pick one among many.

APPENDIX II. EXISTENCE OF AN OPTIMAL PLAN

The first assumption (i) of Assumption 1 avoids the dilemma that the discounted sum of the period utilities could become $+\infty$; while, with the second assumption (ii) of Assumption 1, I restrict my focus to the economies that display perpetual growth in the Walrasian economy described in the next section. Lemma 10 below ensures the existence of an optimal path when the interest rate is between αA and A.

Lemma 10. *Under Assumption 1 and with $r_{it} \in [\alpha A, A]$ for all t, there exists an optimal plan c_i^* that attains the maximum of $U(c_i)$.*

Since the proof is fairly standard,[72] I provide only a sketch here.

The feasible set of a consumer's consumption c_{it} in period t is compact. By Tychonoff's theorem, the feasible set of a consumer's life-time consumption sequence c_i is compact in product topology. Second, since the period utility $u(c_{it})$ is continuous in c_{it}, the life-time utility $U(c_i)$ is upper semi-continuous in c_i in product topology. Therefore, by Weierstrass' theorem, an optimal plan c_i^* exists, and Claims **A** and **B** below assures the life-time utility $U(c_i)$ is bounded below from $-\infty$ and above from ∞.

Claim. ***A.*** *Under Assumption 1 (i) and with $r_{it} \leq A$ for all t, the life time utility is bounded from above, $U(c_i) < \infty$.*

Proof. By assumption 1 (i), the asymptotic elasticity is $1 - \overline{\sigma}$ and thus there exists constants $\overline{c}$, B, and D such that for $c_{it} > \overline{c}$,

$$u(c_{it}) \leq B\frac{c_{it}^{1-\overline{\sigma}}}{1-\overline{\sigma}} + D. \tag{A1}$$

Given the initial level of wealth m_{i1}, let τ be the first period in which the wealth exceeds $\overline{c}$ if the wealth grows at rate A. Since $c_{it} \leq m_{it} \leq m_{i1}A^{t-1}$,

$$\begin{aligned} \sum_{t=1}^{\infty} \beta^{t-1}u(c_{it}) &\leq \sum_{t=1}^{\infty} \beta^{t-1}u(m_{i1}A^{t-1}), \\ &= \sum_{t=1}^{\tau} \beta^{t-1}u(m_{i1}A^{t-1}) + \beta^{\tau} \sum_{t=\tau+1}^{\infty} \beta^{t}u(m_{i1}A^{t-\tau}). \end{aligned} \tag{A2}$$

But, by (A1), the right hand side is bounded from above by

$$\leq \sum_{t=1}^{\tau} \beta^{t}u(m_{i1}A^{t-1}) + \beta^{\tau} \sum_{t=\tau+1}^{\infty} \left(B\frac{m_{i\tau}^{1-\overline{\sigma}}}{1-\overline{\sigma}}(\beta A^{1-\overline{\sigma}})^{t-\tau} + \beta^{t-\tau}D \right). \tag{A3}$$

This upperbound converges to a finite number as $t \to \infty$ by Assumption 1, in particular, $\beta A^{1-\overline{\sigma}} < 1$. Therefore $U(c_t) < \infty$. *Q.E.D.*

Claim. ***B.*** *Under assumption 1 (ii) and with $r_{it} \geq \alpha A$ for all t, the lifetime utility is bounded from below, $U(c_i) > -\infty$. Also, the economy grows perpetually; that is, $c_{it+1} > c_{it}$ for all t.*

[72] See, for example, Becker and Boyd (1997) and Townsend and Ueda (2001).

Proof. Consider an economy where the return on savings is always $\beta\alpha A$. Let $\{c_{Rt}\}_{t=1}^{\infty}$ denote an optimal consumption sequence for individual i in this economy. Obviously, it satisfies the Euler equation:

$$u'(c_{iRt}) = \beta\alpha A u'(c_{iRt+1}). \tag{A4}$$

By assumption 1 (ii),

$$u'(c_{iRt}) > u'(c_{iRt+1}). \tag{A5}$$

Since $u'' < 0$,

$$c_{iRt} < c_{iRt+1}. \tag{A6}$$

In other words, consumption by individual i in this economy grows perpetually.

Given the initial level of wealth $m_{i1} > 0$, the optimal consumption level in the initial period, c_{Ri1}, must be positive by the Inada condition. Hence $u(c_{Ri1}) > -\infty$, implying

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_{Rit}) > \sum_{t=1}^{\infty} \beta^{t-1} u(c_{Ri1}) = \frac{u(c_{Ri1})}{1-\beta} > -\infty. \tag{A7}$$

Let $\{c_{it}^{*}\}_{t=1}^{\infty}$ denote the consumption sequence in the original economy where the return on savings r_{it} is greater than or equal to $\beta\alpha A$. Apparently, the life time utility delivered by $\{c_{it}^{*}\}_{t=1}^{\infty}$ is greater than or equal to that by $\{c_{Rt}\}_{t=1}^{\infty}$:

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_{it}^{*}) \geq \sum_{t=1}^{\infty} \beta^{t-1} u(c_{Rit}) > -\infty. \tag{A8}$$

Q.E.D.

Appendix III. Pareto-Optimal Allocation and Walrasian Equilibrium

A. Allocation by a Benevolent Social Planner

Definition 11. *A symmetric first-best Pareto-optimal allocation is a sequence of savings and capital, each as a function of wealth, $\{s^p(m_t^p), k^p(m_t^p)\}_{t=1}^{\infty}$, for all $i = 1, 2, \cdots, I$, and $j = 1, 2, \cdots, J$, that maximizes the equally weighted sum of the consumer's utility:*

$$\sum_{i=1}^{I} U(c_i), \tag{A1}$$

subject to technological constraints (2), and economywide resource constraints, (9), (10), and (1).

Note that it is possible to chose any number of firms less than or equal to J and define the optimal allocation accordingly. For the sake of simplicity, however, as long as generality is maintained, I concentrate hereafter on the case in which all J firms operate. Using a standard argument, it is easy to characterize the social optimal allocation.

Proposition 1. *A first-best Pareto-optimal allocation is characterized by the following:*
(i) the return on investment is A;
(ii) household wealth evolves, given the initial level of wealth m_{i1}, as

$$m^p_{it+1} = As^p(m^p_{it}); \tag{A2}$$

(iii) the optimal household savings function $s^p(m^p_{it})$ satisfies the fixed point of the Euler equation,

$$u'(m^p_{it} - s^p(m^p_{it})) = \beta A u'(As^p(m^p_{it}) - s^p(As^p(m^p_{it}))); \quad \textit{and} \tag{A3}$$

(iv) optimal investment by a firm is determined by

$$k^p(m^p_{it}) = \frac{\sum_{i=1}^{I} s^p(m^p_{it})}{J}. \tag{A4}$$

Proof. Because of the concavity of the production function, it is optimal to allocate capital equally among firms; that is, $k_{jt} = \overline{k}_t \equiv \sum_{i=1}^{I} s_{it}/J$. To see this clearly, let $k_{jt} = \gamma_{jt}\overline{k}_t$. Because $\sum_{j=1}^{J} k_{jt} = J\overline{k}_t$, there is a natural constraint on the relative allocation γ_{jt}:

$$\sum_{j=1}^{J} \gamma_{jt} = J. \tag{A5}$$

Note that the average investments of other firms can be expressed using firm j's own investment level and the average investment including firm j; that is,

$$\begin{aligned} \overline{K}_{jt} &= \frac{\sum_{l\in -j} k_{lt}}{J-1}, \\ &= \frac{J\overline{k}_t - k_{jt}}{J-1}, \\ &= \frac{(J-1)\overline{k}_t - (k_{jt} - \overline{k}_t)}{J-1}, \\ &= \overline{k}_t - \frac{(k_{jt} - \overline{k}_t)}{J-1}. \end{aligned} \tag{A6}$$

Now, the aggregate output can be written as

$$\begin{aligned} \sum_{j=1}^{J} A\left(\overline{k}_t - \frac{(k_{jt} - \overline{k}_t)}{J-1}\right)^{1-\alpha} \gamma_{jt}^{\alpha}\overline{k}_t^{\alpha} &= \sum_{j=1}^{J} A\left(1 - \frac{(\gamma_{jt} - 1)}{J-1}\right)^{1-\alpha} \overline{k}_t^{1-\alpha}\gamma_{jt}^{\alpha}\overline{k}_t^{\alpha}, \\ &= \sum_{j=1}^{J} A\left(\frac{J - \gamma_{jt}}{J-1}\right)^{1-\alpha} \gamma_{jt}^{\alpha}\overline{k}_t, \\ &= \sum_{j=1}^{J} A\overline{k}_t\left(\frac{1}{J-1}\right)^{1-\alpha} \gamma_{jt}^{\alpha}(J - \gamma_{jt})^{1-\alpha}. \end{aligned} \tag{A7}$$

Using γ_{jt} and $\overline{k}_t$, the resource constraint (9) with any aggregate savings $\overline{S}_t \equiv \sum_{i=1}^{I} s_{it}$ in period t

can be written as

$$\sum_{j=1}^{J} \gamma_j \overline{k}_{jt} = \overline{S}_t, \tag{A8}$$

or, equivalently,

$$\overline{k}_t = \frac{\overline{S}_t}{J}. \tag{A9}$$

This is the very definition of $\overline{k}_t$ and thus the resource constraint does not need to be considered explicitly.

Hence, the optimal relative allocation of capital γ_{jt}, given any level of aggregate savings $\overline{S}_t$, is a solution of the following Lagrangean formula, maximizing revenue (A7) normalized by $A\overline{S}_t/J$ with constraint (A5):

$$L(\gamma_{1t}, \gamma_{2t}, \cdots, \gamma_{Jt}, \lambda_t) \equiv \sum_{j=1}^{J} \left(\gamma_{jt}^{\alpha}(J-\gamma_{jt})^{1-\alpha}\right) + \lambda_t \left(J - \sum_{j=1}^{J} \gamma_{jt}\right). \tag{A10}$$

Specifically, the optimal γ_{jt} satisfies constraint (A5) and the first-order condition, for each firm j,

$$\alpha\gamma_{jt}^{\alpha-1}(J-\gamma_{jt})^{1-\alpha} - (1-\alpha)\gamma_{jt}^{\alpha}(J-\gamma_{jt})^{-\alpha} = \lambda_t. \tag{A11}$$

Because the shadow price λ_t for the constraint is not firm specific, symmetric lending $\gamma_{jt} = 1$, for all j and t, is the solution, implying that $k_{jt} = K_{jt} = \overline{k}_{jt} = \overline{S}_t/J$ is socially optimal for any aggregate savings $S_t \in \mathbb{R}_+$.

The economy-wide output then becomes

$$\sum_{j=1}^{J} Ak_{jt} = \sum_{j=1}^{J} A\overline{k}_{jt} = JA\overline{k}_{jt} = A\overline{S}_t. \tag{A12}$$

It is easy to see that the return on aggregate savings is A. By Lemma 10, an optimal consumption plan then exists to attain maximum of $U(c_i)$. Hence, analyzing the Euler equation (A3) is enough to characterize optimal consumption plans.

A remaining feature of the Pareto-optimal allocation (A2) is the equilibrium law of motion of wealth distribution, which simply restates wealth formation (7) with zero profit income at optimal return A.

Note that the Bordered Hessian matrix for the Lagrangean formula (A10) is negative definite, implying that the Lagrange formula is globally strictly concave and, thus, the first-order condition uniquely determines the maximum of the Lagrangean formula (A10). Specifically, the $(J+1) \times (J+1)$ Bordered Hessian has the structure such that
(i) elements of the first column ($\partial^2 L/\partial\lambda_t\partial\gamma_{jt}$) and the first row ($\partial^2 L/\partial\gamma_{jt}\partial\lambda_t$) are -1, except that $(1,1)$ element ($\partial^2 L/\partial\lambda_t^2$) is zero;
(ii) for the remaining $J \times J$ submatrix starting $(2,2)$, all the nondiagonal elements ($\partial^2 L/\partial\gamma_{jt}\gamma_{lt}$) are

zero; and
(iii) diagonal elements of the $J \times J$ submatrix has strictly negative diagonal elements:

$$\phi_{jjt} \equiv \frac{\partial^2 L}{\partial \gamma_{jt}^2} = -\alpha(1-\alpha)\left(\gamma_{jt}^{\alpha-2}(J-\gamma_{jt})^{1-\alpha} + \gamma_{jt}^{\alpha}(J-\gamma_{jt})^{-\alpha-1}\right) < 0. \tag{A13}$$

In sum, the Bordered Hessian is

$$\begin{pmatrix} 0 & -1 & -1 & 0 & \cdots & 0 \\ -1 & \phi_{11t} & 0 & 0 & \cdots & 0 \\ -1 & 0 & \phi_{22t} & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -1 & 0 & 0 & 0 & \cdots & \phi_{JJt} \end{pmatrix}.$$

The determinants of any first $l \times l$ elements can be calculated as

$$det_{lt} = -\sum_{i=1}^{l}\sum_{j=1}^{l}\Phi_{jit}, \tag{A14}$$

where Φ_{jit} is the cofactor for ϕ_{jit}. It is the determinant of the matrix created by eliminating the j-th row and i-th column of the $J \times J$ submatrix starting $(2,2)$ element. But, because all the nondiagonal elements of the $J \times J$ submatrix are zero, the matrix created by eliminating the j-th row and i-th column of $J \times J$ submatrix has a row and a column whose elements are zero everywhere, if it is asymmetrically created (i.e., $j \neq i$). Hence, only the symmetric cofactor has nonzero value, so that

$$det_{lt} = -\sum_{i=1}^{l}\Phi_{iit} = -\sum_{i=1}^{l}\prod_{j\neq i}^{l}\phi_{jjt}. \tag{A15}$$

Because ϕ_{jj} is negative for any j (A13) and they are multiplied by each other $(l-1)$ times before the summation of these products is taken, the sign of det_{lt} is positive if l is even, and is otherwise negative. Therefore, the condition for negative definite of the Bordered Hessian is always satisfied, so that, for any $l \leq J$,

$$(-1)^l det_{lt} > 0. \tag{A16}$$

Q.E.D.

For a CRRA utility function, $u(c) = c^{1-\sigma}/(1-\sigma)$ with $\sigma \in \mathbb{R}_{++}$, I articulate the result further based on the Euler equation (A3).

Corollary 1. *With the CRRA utility function, for all wealth level $m_{it} \in \mathbb{R}_+$, the growth rate in the first-best Pareto-optimal allocation is $(\beta A)^{1/\sigma}$ in each period, and the associated savings rate is $\beta^{1/\sigma}A^{(1-\sigma)/\sigma}$.*

B. Allocation by a Walrasian Corporate Bond Market

As in the standard microeconomic theory, a competitive bond market is considered here as an abstract of the actual corporate bond market. As in the economy with banks, I consider a Walrasian equilibrium of the bond market as a game among consumers, firms, and an auctioneer and its dynamics is defined recursively.

In each period, the auctioneer's strategy is to set the interest rate $r_t \in \overline{\mathbb{R}}_+$, so as to maximize her payoff, $\pi_a : \overline{\mathbb{R}}_+^{1+I+J} \rightarrow \overline{\mathbb{R}}_+$, which is the value of excess demand in the capital market:

$$\pi^a(r_t, \{s_{it}\}_{i=1}^I, \{k_{jt}\}_{j=1}^J) = r_t \left(\sum_{j=1}^J k_{jt} - \sum_{i=1}^I s_{it} \right). \tag{A17}$$

Since the auctioneer acts the same way in each period, I drop the time subscript hereafter. The auctioneer's best response correspondence is defined as $BR_a : \overline{\mathbb{R}}_+^{I+J} \rightarrow \overline{\mathbb{R}}_+$, such that

$$BR^a(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J) = \arg\max_{r \in \overline{\mathbb{R}}_+} r \left(\sum_{j=1}^J k_j - \sum_{i=1}^I s_i \right). \tag{A18}$$

This can be described as

$$\begin{aligned} BR^a(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J) &= 0 \quad \text{if} \sum_{j=1}^J k_j - \sum_{i=1}^I s_i < 0, \\ &= \overline{\mathbb{R}}_+ \quad \text{if} \sum_{j=1}^J k_j - \sum_{i=1}^I s_i = 0, \\ &= \infty \quad \text{if} \sum_{j=1}^J k_j - \sum_{i=1}^I s_i > 0. \end{aligned} \tag{A19}$$

Firms solve the same problems in each period, and thus I drop the time subscript. In each period, the strategy for firm j is investment $k_j \in \overline{\mathbb{R}}_+$, and its objective is to maximize its profit, given $R_j = r$.[73] Firm j's best response correspondence is defined as $BR_{fj} : \overline{\mathbb{R}}_+^J \rightarrow \overline{\mathbb{R}}_+$, such that

$$BR_j^f(\{k_j\}_{l \in -j}, r) = \arg\max_{k_j \in \overline{\mathbb{R}}_+} AK_j^{1-\alpha} k_j^\alpha - rk_j, \tag{A20}$$

which can be simplified to

$$BR_j^f(K_j, r) = \left(\frac{\alpha A}{r} \right)^{\frac{1}{1-\alpha}} K_j. \tag{A21}$$

K_j is the average of other firms' investment; but investment by each of the other firms is also a function of the interest rate and the average of all firms' investment excluding its own. As such, the economy-wide investment vector can be expressed as a function of the interest rate r only:

$$BR^f(r) \equiv (BR_1^f(\{k_j\}_{l \in -1}, r), BR_2^f(\{k_j\}_{l \in -2}, r), \cdots, BR_J^f(\{k_j\}_{l \in -J}, r)). \tag{A22}$$

Because the auctioneer's and firms' problems are identical for every period, a consumer's expectations regarding equilibrium interest rates should be identical for all periods, $E_t[r_{t+l}] = r$ for

[73]The profit function is the same as (3).

all $l \geq 0$. Similarly, a consumer's expectations regarding an equilibrium stationary strategy of firms should be identical, which is $BR^f(r)$, described above. A consumer then faces the same problem in each period, and I drop the time subscript but add superscript $+$ to denote variables in the next period. Specifically, given his own current wealth level m_i and conditional on the auctioneer's stationary strategies r and the firms' stationary strategies $BR^f(r)$, a consumer's problem can be written recursively using a value function $V_w : \overline{\mathbb{R}}^2_+ \to \overline{\mathbb{R}}$ as a value of lifetime utility, maximized with the optimal choice of savings s_i from the budget set $[0, m_i]$ (budget constraint (5)) in each period:[74]

$$V_w(m_i, r) = \max_{s_i \in [0, m_i]} u(m_i - s_i) + \beta V_w(m_i^+, r), \tag{A23}$$

subject to wealth formulation (7) for all $i \in I$, as follows:[75]

$$m_i^+ = r s_i + w_i(r). \tag{A24}$$

I denote the vector of wealth as follows:

$$m \equiv (m_1, m_2, \cdots, m_I). \tag{A25}$$

This recursive formulation of the consumer's problem enables us to describe the dynamic economy as if it were a static game, as formally defined below.

Definition 12. *A recursive Walrasian economy is the game $\Gamma(m)$, which is conditional on wealth distribution $m \in \mathbb{R}^I_+$ and consists of $I + J + 1$ agents (I consumer, J firms, and one auctioneer), typical strategy sets (savings, investments, and the interest rate), and typical utilities (V_w, π^f, and π^a):*

$$\Gamma_W(m) \equiv (I + J + 1, ([0, m_i], \overline{\mathbb{R}}_+, \overline{\mathbb{R}}_+), (V_w, \pi^f, \pi^a)). \tag{A26}$$

Consumer i's best response, $BR_i^c : \overline{\mathbb{R}}_+ \to \overline{\mathbb{R}}_+$, is the first-order condition of the value function (A23), which is a function of the interest rate, conditional on her wealth m_i:

$$BR_i^c(r|m_i) = \arg \max_{s_i \in [0, m_i]} u(m_i - s_i) + \beta V_w(m_i^+, r). \tag{A27}$$

Let the best response correspondence, $BR : \overline{\mathbb{R}}_+^{I+J+1} \to \overline{\mathbb{R}}_+^{I+J+1}$, for the game $\Gamma_W(m)$ denote a Cartesian product of each best response, given the current wealth vector m in the economy:

$$\begin{aligned} &BR(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J, r|m) \\ &= \Pi_{i=1}^I BR_i^c(r|m_i) \times \Pi_{j=1}^J BR_j^f(K_j, r) \times BR^a(\{s_i\}_{i=1}^I, \{k_j\}_{j=1}^J). \end{aligned} \tag{A28}$$

[74] More precisely, a consumer's value function is also conditional on firms' strategy of the current and next periods, $\{k_j\}_{j=1}^J$ and $\{k_j^+\}_{j=1}^J$, respectively. However, a consumer knows the stationary strategies of firms and thus those strategies can be replaced by $BR^f(r)$. Since this is a function of the interest rate r only, it is not necessary to write the value function conditional on firms' strategy.

[75] I apologize here for abusing the notation. If I strictly follow (7), the profit income should be written as $w_i(\{k_j, r\}_{j=1}^J)$. However, again, a consumer here knows firms' stationary strategies $BR^f(r)$, and thus the profit income becomes just a function of interest rate r.

Definition 13. *An equilibrium of the recursive Walrasian economy is a Nash equilibrium of the game $\Gamma_W(m)$ that satisfies economywide constraints (9), (10), and (1). Here, a set of strategies $(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w|m)$ conditional on current wealth is a Nash equilibrium for the game $\Gamma_W(m)$ if it is a fixed point of the best response correspondence for the game:*

$$(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w|m) \in BR(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w|m). \tag{A29}$$

The resulting equilibrium is the same as described by Romer (1986). Note that first-order conditions for firms and consumers are the same as in Romer's nongame formulation.

Proposition 2. *The recursive Walrasian economy $\Gamma_W(m)$ has a unique equilibrium such that (i) the return on investment is*

$$r^w = \alpha A; \tag{A30}$$

(ii) given the initial level of wealth m_i, the wealth evolves as

$$m_i^{w+} = \alpha A s_i^w + w_i(\alpha A), \tag{A31}$$

where the equilibrium savings is the consumer's best response to the equilibrium interest rate,

$$s_i^w = BR_i^c(r^w|m_i); \quad \textit{and} \tag{A32}$$

(iii) the equilibrium investment is determined by[76]

$$k_j^w = \frac{\sum_{i=1}^I s_i^w}{J}. \tag{A33}$$

Proof. First, I will show that the proposed solution is a fixed point of the best response correspondence BR defined in (A28). Because aggregate investment equals aggregate deposits under the equilibrium strategy of firms $k_j^w = \sum_{i=1}^I s_i^w/J$, as in the middle case of (A19), the auctioneer's best response can be any number including $r^w = \alpha A \in BR^a(\{s_i^w\}_{i=1}^I, \{k_j^w\}_{j=1}^J, r^w)$. By substituting the auctioneer's equilibrium interest rate $r^w = \alpha A$ in (A21), the best response of firm j becomes $k_j^w = K_j^w = \sum_{i=1}^I s_i^w/J$. Also, by substituting $r^w = \alpha A$ in (A27), the best response of a consumer becomes $s_i^w = BR_i^c(\alpha A|m_i)$.

Next, I show that the equilibrium is unique. First, interest rate r cannot be smaller than αA in an equilibrium. This is because, if $r < \alpha A$, firm j's optimal investment level is higher than the average ($k_j > K_j$) by its best response BR_j^f defined in (A21); consequently, no fixed point would exist except ∞. But consumers' savings cannot be ∞ given $r < \alpha A$ and $m_i < \infty$.

Second, interest rate r cannot be larger than αA in an equilibrium. To see this, suppose $r > \alpha A$. Firm j's optimal investment level then becomes smaller than the average $k_j < K$ according to its best response (A21), and this is true for all firms. Hence, the average capital level must be at the corner solution, $k_j = K = 0$. However, because of the Inada condition, savings are always positive for a positive interest rate. This, in turn, implies that the auctioneer's best response must be $r = 0$, which contradicts assumption $r > \alpha A$. *Q.E.D.*

[76] I assume all firms are active. Even if this is not the case, the equilibrium is still the same once the denominator on the right hand side J is replaced by the number of active firms $\#\Psi$.

Note that $BR_i^c(r^w|m_i)$ in (A32) is the savings function. Moreover, condition (A32) suggests that the savings function must be the fixed point of the Euler equation, given the (expected) interest rate r^w:

$$u'(m_i - BR_i^c(r^w|m_i)) = \beta r^w u'(\alpha A BR_i^c(r^w|m_i) - BR_i^c(r^w|\alpha A BR_i^c(r^w|m_i)). \tag{A34}$$

Using this Euler equation, for a CRRA utility function $u(c) = c^{1-\sigma}/(1-\sigma)$, the result can be further specified.

Corollary 2. *For all wealth levels $m_{it} \in \mathbb{R}_+$, the growth rate in the Walrasian economy with the CRRA utility is $(\beta\alpha A)^{1/\sigma}$ in every period, and the associated savings rate is $\beta^{1/\sigma}(\alpha A)^{(1-\sigma)/\sigma}$.*

For the CRRA utility case, the growth rate of the Walrasian equilibrium $(\beta\alpha A)^{1/\sigma}$ (Corollary 2) is lower than that of Pareto-optimal level, $(\beta A)^{1/\sigma}$ (Corollary 1). Similarly, with the more general utility functions considered in this paper, consumption growth is lower in the Walrasian equilibrium than in the Pareto-optimal allocation. It is easy to see this by comparing the Euler equation for the social planner (A3) with that for the Walrasian equilibrium (A34). They essentially are $u'(c_t) = \beta A u'(c_{t+1})$ and $u'(c_t) = \beta\alpha A u'(c_{t+1})$, respectively. Because the marginal utility is decreasing, when today's consumption level is the same, the Pareto-optimal allocation gives higher consumption in the next period than the Walrasian equilibrium. Effect on wealth growth is not straightforward, as households could consume more in the next period with lower savings under a higher interest rate—the income effect may dominate the substitution effect. With the CRRA utility function, Corollaries 1 and 2 imply that the economy is always on a balanced growth path; that is, wealth and consumption grow at the same rate.[77]

APPENDIX IV. ALLOCATIONS UNDER OTHER PRODUCTION FUNCTIONS

A. Decreasing Returns to Accumulated Capital

I consider the case in which the production function exhibits decreasing returns to accumulated capital. As long as the degree of externalities is large enough, the allocation delivered by the banking sector is shown to be Pareto superior to the Walrasian outcome.

Romer (1986) classifies production functions with a Marshallian externality into three categories: constant returns, decreasing returns, and increasing returns to the accumulated factor. In a simple production function,

$$y_{hj} = AK_j^{\eta} k_j^{\alpha}, \tag{A1}$$

these correspond to the cases of $\eta = 1-\alpha$, $\eta < 1-\alpha$, and $\eta > 1-\alpha$. Since the case of constant returns is discussed in previous sections, this appendix questions whether results vary depending on η. I focus only on the case of decreasing returns, because Jones and Manuelli (1990) show that only constant returns to the accumulated factor are consistent with both perpetual growth and finite life-time utility; that is, $\sum_{t=1}^{\infty} \beta^t u(c_t) < \infty$.

[77] In the long run, utility functions satisfying Assumption 1 behave similarly to the CRRA utility function with the (asymptotic) constant relative risk aversion $\overline{\sigma}$. See Brock and Gale (1969) and the proof of Lemma 10 in Appendix II.

As is the case with constant returns to the accumulated factor, the interest rate in the Walrasian equilibrium is equal to the private marginal product. Together with the fixed point condition, $k_j = K_j = K$, the Walrasian interest rate is given by $\alpha AK^{\alpha+\eta-1}$. It is lower than the social marginal product, $(\alpha+\eta)AK^{\alpha+\eta-1}$, and thus attracts savings differently from the Pareto-optimal one. When banks intermediate capital in this economy, unlike the case with constant returns, the equilibrium interest rate (and thus savings) would be even higher than in the Pareto-optimal allocation as shown below.

Proposition 3. *When technology exhibits decreasing returns to accumulated capital, $\eta < 1-\alpha$, the equilibrium of the strategically intermediated economy is characterized by a higher interest rate r_m, investment K_m, and thus growth rate than the Pareto-optimal allocation, where (r_m, K_m) is determined by the unique fixed point of two equations:*

$$r_m = AK_m^{\alpha+\eta-1} = r_{hi}^c = R_{hj}^f = \rho_h, \quad \textit{for all } h, i, \textit{ and } j; \tag{A2}$$

and

$$K_m = \frac{S(m,(r_m, N.A.))}{J}, \quad \textit{for all } j. \tag{A3}$$

Proof. The proof is almost the same as before and I outline it below.

If a bank becomes a monopolist, it can obtain all the revenue of client firms under the take-it-or-leave-it offer. This implies that the monopolist would charge the average product of capital, $AK_m^{\alpha+\eta-1}$, as its loan rate r_m.

This monopolist interest rate r_m must be an equilibrium deposit rate. Anything smaller than r_m cannot be an equilibrium, because a deviating bank can offer a little higher rate to become a monopolist. Anything higher than r_m is not an equilibrium either. To achieve a higher loan rate, a deviating bank would need to limit the amount of savings and thus there would be an excess supply of funds. This deviation, however, requires smaller investment of client firms of the deviating bank than other firms. A similar strategy as described in Lemma 3 can prevent such deviation. Therefore, banks offer deposit rate r_m to consumers in an equilibrium, knowing that they would charge the same rate to firms in the loan market. *Q.E.D.*

Note that, to achieve the Pareto-optimal allocation, savings should equate the deposit rate with the social marginal product, which is $(\alpha+\eta)$ times smaller than the average product. Also note that, if externalities were not present, this equilibrium would be even worse than the Walrasian equilibrium, because the investment amount is more than that suggested by the private marginal product of capital. However, as long as the externality parameter η is large, close to $1-\alpha$, this equilibrium is superior to the Walrasian equilibrium.

B. Labor Inputs

I consider here the case in which the production function requires labor inputs as well as capital. Firms have to pay positive wage to produce output if the labor supply is elastic. In the strategically intermediated equilibrium, the wage turns out to be lower but the interest rate is higher than in the Walrasian market. In other words, more rewards are paid to investments than in the Walrasian equilibrium, but the amount of investment is less than the first best. Hence, the allocation is Pareto superior to the Walrasian outcome, but is not Pareto optimal.

Consider a Cobb-Douglas production function $F : \mathbb{R}^3_+ \to \mathbb{R}_+$ with labor inputs l_j,

$$y_j = F(k_j, l_j, K_j) \equiv AK_j^{\eta} k_j^{\alpha} l_j^{1-\alpha}. \tag{A4}$$

A consumer earns wage income and interest income on savings. I assume each consumer is endowed with one unit of time. Instead of the profit income in the benchmark model, it is wages that fill the gap between output and interest paid for rented capital in this economy. Hence, the allocation of revenue of a firm after the interest payments, ψ^f_{ij} in equation (6), should be interpreted as the portion of wage income that consumer i receives from firm j, proportional to her hours worked in firm j. It must satisfy the feasibility condition as in (6):

$$\sum_{j=1}^{J} \psi^f_{ij} = 1. \tag{A5}$$

Also, a household's income w_i, previously defined in equation (13), should now consist of consumer i's wage income from firms and profit income from banks:

$$w_i(z^c, z^b, z^f) \equiv \sum_{j=1}^{J} \psi^f_{ij} \pi^f_j(z^b, z^f) + \sum_{h=1}^{H} \psi^b_{hi} \pi^b_h(z^c, z^b, z^f). \tag{A6}$$

As long as a utility function does not exhibit disutility of labor, each consumer always spends one unit of endowed time in labor. As a result, the aggregate labor supply is inelastic. In this case, it is easy to see that results are the same as in the benchmark model. Note that in the strategically intermediated economy, all revenue of firms would be taken away by banks and the equilibrium wage would be zero.

When the utility function exhibits disutility of labor, firms have to pay some positive wages to hire workers. Consider the following period-utility function including disutility of labor in an additively separable form:

$$u(c_i) + v(1 - L_i), \tag{A7}$$

where the usual assumptions hold for the utility of leisure v: concave, twice continuously differentiable, satisfying the Inada conditions near zero and near one.

In the Walrasian equilibrium, the interest rate should be equal to the private marginal product of capital. The equilibrium savings equates this interest rate with the marginal utility of current consumption. The labor market is determined simultaneously by the marginal product of labor and the marginal disutility of labor. Without loss of generality, assume that the number of firms J is equal to the number of agents I and that agent j works for firm j, so that $L_j = l_j$. Also, note that

$k_j = K_j = K$ in an equilibrium. In sum, K, l_j, and r must solve the following system of equations simultaneously in the Walrasian equilibrium:

$$\begin{aligned} r &= \alpha A l_j^{1-\alpha}, \\ K &= S(m, (\alpha A l_j^{1-\alpha}, N.A.)), and \\ v'(1-l_j) &= (1-\alpha) A K l_j^{-\alpha}. \end{aligned} \tag{A8}$$

As in the benchmark model, the capital share is α and the remaining $(1-\alpha A)$ portion of the revenue of firms goes to workers.

In the Pareto-optimal allocation, the social planner assigns savings and investment to equate the social marginal product of capital to the marginal utility of current consumption. Since the social marginal product of capital $A l_j^{1-\alpha}$ is higher than the private one $\alpha A l_j^{1-\alpha}$, given the same labor inputs, savings and investment in the Pareto-optimal allocation are different from those in the Walrasian equilibrium—with the CRRA utility, they are higher. However, the Pareto-optimal labor supply is also different from—with the CRRA utility, higher than—that observed in the Walrasian equilibrium, because different amounts of capital are invested in the Pareto-optimal allocation and labor inputs are determined by the same condition, equating marginal product of labor to marginal disutility of labor, as in the Walrasian equilibrium.

The strategically intermediated equilibrium is different from both the Walrasian equilibrium and the Pareto-optimal allocation. This can be characterized by the equilibrium share of capital. Define the elasticity of labor supply with respect to the wage as ϵ_w^L. The equilibrium capital share can be expressed as

$$\hat{\alpha} = \frac{\alpha + 1/\epsilon_w^L}{1 + 1/\epsilon_w^L}. \tag{A9}$$

Proposition 4. *The equilibrium of the strategically intermediated economy with elastic labor supply is $r_{hi}^c = \rho = R_{hj}^f = \hat{\alpha} A$ and $k_j = K_j = S(m, (\hat{\alpha} A, N.A.))/J$.*

Proof. Since the monopoly bank can obtain all capital shares of the representative firm's revenues, it maximizes the capital share by considering only one trade-off: the labor supply function is decreasing in the capital share (increasing in wage). Hence, the problem that a monopoly bank would face is to choose wage to maximize the capital share:[78]

$$\max_w AKL(w)^{1-\alpha} - wL(w), \tag{A10}$$

where $L(w)$ is a labor supply function. The first order condition is

$$(1-\alpha) AKL^{-\alpha} L'(w) = L + wL'(w). \tag{A11}$$

[78] As before, I assume that J firms exist, but I omit subscript j because the monopoly bank faces the same problem with each firm. The game of the labor market can be described more formally. For example, in each period, the labor market game between households and firms starts after they finish the capital market game. Hence, in the capital market, players knows that the labor allocation may depend on the capital allocation. Simultaneous opening of two markets, however, would not affect the main results.

Dividing both sides by $L'(w)$ and multiplying both sides by L, the labor share becomes

$$wL = (1-\alpha)Y - \frac{L}{L'(w)}L. \tag{A12}$$

Using the elasticity of labor supply, ϵ_w^L,

$$wL = (1-\alpha)Y - \frac{wL}{\epsilon_w^L}. \tag{A13}$$

Hence,

$$wL = \frac{1-\alpha}{1+\frac{1}{\epsilon_w^L}}Y = (1-\hat{\alpha})Y. \tag{A14}$$

Then the capital share becomes

$$rK = \frac{\alpha+\frac{1}{\epsilon_w^L}}{1+\frac{1}{\epsilon_w^L}}Y = \hat{\alpha}Y. \tag{A15}$$

The remaining proof is the same as in the benchmark model. Competitive banking achieves this monopoly solution. *Q.E.D.*

This strategically intermediated equilibrium is not a Pareto-optimal allocation, but is Pareto superior to the Walrasian equilibrium. As long as the elasticity of labor supply is small, the equilibrium allocation is close to the case with inelastic labor supply. In other words, the less wages firms pay, the more socially efficient the investment is. Note that, as labor supply becomes inelastic $\epsilon_w^L \to 0$, the labor share approaches zero and the capital share goes to one, replicating the result seen in the economy without disutility of labor. On the other hand, as labor supply becomes highly elastic $\epsilon_w^L \to \infty$, the labor share approaches $(1-\alpha)$ and the capital share goes to α, replicating the Walrasian equilibrium allocation.

APPENDIX V. ECONOMY WITH PRIVATE DIRECT FINANCE

Here I consider the case where firms design and issue corporate bonds to consumers directly, not through an auctioneer. A financial contract to a consumer i from firm j at date t consists of a coupon rate and an issue amount, which may be unspecified. I consider only a set of contracts contingent on consumers' current wealth.[79] Again, I focus on stationary Markov strategies. I also assume a variant of Assumption 3: the same household is not discriminated against repeatedly by firms over time. When the set of optimal contracts is not a singleton, firms may take asymmetric strategies (and also mixed strategies) in which some consumers receive differential treatment. Since firms are assumed to die and be born in every period, a specific consumer cannot be discriminated by "j-th" firm over periods.

[79] Similar to the bank intermediated economy, contracts contingent on history of actions or future promises are not worth considering, because a firm's life is contained in one period.

I use $z_{ijt}^f \equiv (r_{ijt}^f, s_{ijt}^f)$ to represent a corporate bond,[80] where r_{ijt}^f denotes the coupon rate and s_{ijt}^f denotes the issue amount at date t from firm j to consumer i. Each element can be taken from the nonnegative real number or left unspecified, abbreviated as $N.S.$ (not specified), so that $z_{ijt}^f \in Z^f \equiv (\overline{\mathbb{R}}_+ \cup \{N.S.\})^2$.

A consumer maximizes her life-time utility by choosing savings amounts in each period. Consumer i's strategy given firm j's offer at date t is denoted as $x_{ijt} \equiv (r_{ijt}^c, s_{ijt}^c)$. It is chosen from the strategy set $X \equiv \mathbb{R}_+^2$. However, this strategy set is constrained by firms' strategies at date t, z_{ijt}^f. For example, if a firm specifies the coupon rate of the bond, consumers cannot change it. I also assume that a consumer can always refuse to buy the bond, so that $s_{ijt}^c = 0$ is always in the choice set. The constrained choice set of consumer i given firm j's offer z_{ijt}^f is then defined as:

$$\begin{aligned} G_{ij}^c(z_{ijt}^f) &\equiv r_{ijt}^f \times \overline{\mathbb{R}}_+ \quad \text{if firm } j \text{ specifies } r_{ijt}^f \text{ only}, \\ &\equiv \mathbb{R}_+ \times (s_{ijt}^f \cup \{0\}) \quad \text{if firm } j \text{ specifies } s_{ijt}^f \text{ only}, \\ &\equiv r_{ijt}^f \times (s_{ijt}^f \cup \{0\}) \quad \text{if firm } j \text{ specifies both } r_{ijt}^f \text{ and } s_{ijt}^f. \end{aligned} \tag{A1}$$

Note that the choice set of the last case is either "*accept*" (r_{ijt}^f, s_{ijt}^f) or "*reject*" $(r_{ijt}^f, 0)$. The whole constrained choice set for a consumer i is now defined as the Cartesian product of G_{ij}^c over all firms $j \in J$:

$$G_i^c(z_{it}^f) \equiv G_{i1}^c(z_{i1t}^f) \times G_{i2}^c(z_{i2t}^f) \times \cdots \times G_{iJ}^c(z_{iJt}^f), \tag{A2}$$

where $z_{it}^f = (z_{i1t}^f, z_{i2t}^f, \cdots, z_{iJt}^f)$, a vector of offers to consumer i from all firms. Given firms' offers z_{it}^f, consumer i chooses his strategy $x_{it} \equiv \{x_{ijt}\}_{j=1}^J$.

The initial wealth is a positive real number, $m_{i1} \in \mathbb{R}_{++}$, and savings cannot exceed wealth. Hence, the budget constraint in period t is,

$$\sum_{j=1}^J s_{ijt}^c \in [0, m_{it}]. \tag{A3}$$

The feasible set for consumer i at date t given particular offers from firms z_{it}^f is defined as a set of strategies x_{it}, which must belong to the constrained choice set and satisfy the budget constraint,

$$B_{it}(m_{it}, z_{it}^f) \equiv \left\{ x_{it} : x_{it} \in G_i^c(z_{it}^f) \quad \text{and} \quad \sum_{j=1}^J s_{ijt}^c \in [0, m_{it}] \right\}. \tag{A4}$$

A consumer i's strategies in period t can be chosen only from this set. Note that the economy-wide resource constraint (9) is always satisfied. Notice, also, that the feasible set is nonempty and compact valued. For example, even when a consumer receives offers specifying an issue amount larger than her wealth level, she can still choose zero, which is in the set B_{it}.

A consumer's feasible set (A4) depends on her wealth m_{it} and current offers from firms z_{it}^f. When she makes her decision on savings, however, she also needs to take into account her future wealth m_{it+1} and future offers from firms z_{it+1}^f. Her future wealth m_{it+1} depends not only on gross return on her savings $\sum_{j=1}^J r_{ijt}^c s_{ijt}^c$, but also on profit income $w_{it}(\{k_{jt}, R_{jt}\}_{j=1}^J)$ defined in (7). In

[80] I apologize for the abuse of notations here and below in this section. Note that equity-type contracts whose return depends on outcome is not worth considering, as discussed in footnote 28.

equilibrium, consumers' strategies (r^c_{ijt}, s^c_{ijt}) will be realized, because consumers make final decisions given firms' offers. Hence, a consumer's future wealth is a function of strategies of all the consumers $x_t \equiv \{x_{it}\}_{i=1}^I$ given all firms' offers $z_t^f \equiv \{z_{it}^f\}_{i=1}^I$ and consumer i's share on firms ψ_{ij}^f,

$$m_{it+1} = g(x_t, z_t^f) \equiv \sum_{j=1}^J r^c_{ijt} s^c_{ijt} + w_{it}(\{k_{jt}, R_{jt}\}_{j=1}^J). \tag{A5}$$

The definition of strategies and problems faced by consumers and firms are similar to those in bank intermediated economies. I use stochastic dynamic programming to describe the consumer's maximization problem, dropping time subscript t.[81] Given her own wealth m_i, wealth distribution of others $m_{-i} \equiv \{m_1, m_2, \cdots, m_{i-1}, m_{i+1}, \cdots, m_I\}$, and other consumers' stationary Markov mixed strategies $\mu_{-i}(x_{-i}|m, \mu_i, z^f) \in \Lambda(B_{-i})$ consumer i's problem is to choose her strategy $\mu_i(x_i|m, \mu_{-i}, z^f) \in \Lambda(B_i)$ for each specific realization z^f of a vector of all firms' strategies $q(z^f|m, \mu) \in \Lambda(Z^{fJ})$,

$$\begin{aligned} V_D(m_i|m_{-i}, \mu_{-i}, q) = \int_{\mathcal{B}(Z^{fJ})} \Bigg\{ \max_{\mu_i \in \Lambda(B_i)} \int_{\mathcal{B}(B_i)} \Bigg[u(m_i - s_i) \\ + \beta \int_{\mathcal{B}(B_{-i})} \int_{\mathcal{B}(Z^{fJ})} V_D(m_i^+|m_{-i}^+, \mu_{-i}^+, q^+) q(dz^{f+}) \mu_{-i}(dx_{-i}^+) \Bigg] \mu_i(dx_i^+) \Bigg\} q(dz^f) \end{aligned} \tag{A6}$$

Given wealth distribution m, the vector of all consumers' strategy $\mu(x|m, z^f)$, and other firms' strategy $q_{-j}(z_{-j}^f|m, \mu, q_j)$, firm j chooses its strategy $q_j(z_j^f|m, \mu, q_{-j})$ to maximize its expected profit, $E\pi^f : \Lambda(Z^f) \to \overline{\mathbb{R}}_+$, defined as:

$$E\pi^f(q_j|m, \mu, q_{-j}) \equiv \int_{\mathcal{B}(Z^f)} \int_{\mathcal{B}(Z^{f(J-1)})} \int_{\Pi_{i=1}^I \mathcal{B}(B_i)} \pi^f(k_j, K_j, R_j) \mu(dx) q_{-j}(dz_{-j}^f) q_j(dz_j^f). \tag{A7}$$

Definition 14. *A recursive economy with private direct finance is the game $\Gamma_D(m)$, which is defined over all $m \in \mathbb{R}_+^I$, and consists of $I + J$ agents, their typical strategy sets, and their typical utilities:*

$$\Gamma_D(m) \equiv (I + J, (\Lambda(B_i), \Lambda(Z^f)), (V_{Di}, E\pi^f)) \tag{A8}$$

Definition 15. *An equilibrium of a private direct finance economy is a stationary Markov equilibrium strategies (μ^*, q^*) of the game Γ_D.*

A stationary Markov equilibrium means the following: (i) for every consumer i, given firms' equilibrium strategies q^* and other consumers' equilibrium strategies μ_{-i}^*, consumer i's equilibrium strategy μ_i^* satisfies her value function (A6); and (ii) for every firm j, given consumers equilibrium strategies μ^* and other firms' equilibrium strategies q_{-j}^*, firm j's equilibrium strategy q_j^* maximizes its expected profit (A7).

A firm's profit suddenly changes when a firm becomes monopolist. This discontinuity of the profit function in this economy is too severe to support any existence theorems of a Nash equilibrium. To

[81] Recall that the next period strategies with superscript $+$ do not imply different rules from stationary strategies, but the same rule with the next period values.

analyze this economy, as in the bank intermediated economy, I focus on the game within a period given arbitrary expectations on equilibrium strategies from tomorrow on, as is the case with the bank intermediated economy. Obviously, if there is no Nash equilibrium in the game within a period, there is no equilibrium in this economy. I start with the case of pure strategies.

Lemma 11. *There exists no pure strategy equilibrium in an economy with private direct finance.*

Proof. Without loss of generality, I focus on the case with two firms. Moreover, for simplicity, suppose there exist only one (representative) household who ignores the wealth effect from profits income.

The following first-order condition determines the optimal investment level of Firm 1 together with the optimal interest rate r_1: Given k_2, the investment level of Firm 2,

$$k_1 = \left(\frac{\alpha A}{r_1}\right)^{\frac{1}{1-\alpha}} k_2. \tag{A9}$$

Suppose the investment level is symmetric. Then the fixed point condition says

$$k_1 = k_2. \tag{A10}$$

The interest rate that satisfies both conditions is $r_1 = r_2 = \alpha A$ only. Hence, in an equilibrium, if exists, investments by firms must be

$$k_1 = k_2 = \frac{S(m, (\alpha A, N.S.))}{2}. \tag{A11}$$

However, a firm has an incentive to deviate from this strategy by offering slightly higher interest rate $\alpha A + \epsilon$ with $\epsilon > 0$ and specifying the same aggregate savings under rate αA. This is because the deviating firm can become a monopolist and earn higher profits than the profits under the fixed point interest rate αA; that is,

$$(A - (\alpha A + \epsilon))S(m, (\alpha A, N.S.)) > (A - \alpha A)\frac{S(m, (\alpha A, N.S.))}{2}. \tag{A12}$$

Therefore, there is no symmetric pure strategy Nash equilibrium.

Assume now that the investment level is asymmetric. Assume $k_1 > k_2$ and let $\hat{S}(m, z_1^f, z_2^f)$ denote the optimal savings by a representative household when Firm 1 offers z_1^f and Firm 2 offers z_2^f to the household. Again, the first-order conditions are

$$k_1 = \left(\frac{\alpha A}{r_2}\right)^{\frac{1}{1-\alpha}} k_2, \tag{A13}$$

and

$$k_2 = \left(\frac{\alpha A}{r_1}\right)^{\frac{1}{1-\alpha}} k_1. \tag{A14}$$

Combining the two first-order conditions (A13) and (A14),

$$k_1 = \left(\frac{\alpha A}{r_2}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha A}{r_1}\right)^{\frac{1}{1-\alpha}} k_1. \tag{A15}$$

This implies

$$r_1 r_2 = (\alpha A)^2. \tag{A16}$$

Let $(\hat{r}_1, \hat{k}_1)$, $(\hat{r}_2, \hat{k}_2)$ be the equilibrium strategies. Because $k_1 > k_2$, together with the two first-order conditions (A13) and (A14), the equilibrium condition (A16) implies that

$$\hat{r}_1 < \alpha A < \hat{r}_2. \tag{A17}$$

There are two cases. In the first case, the household may be indifferent between z_1^f and z_2^f. Namely, a low interest rate with large savings may bring the same utility level as a high interest rate with small savings. In this case, however, the same logic in the symmetric investment case applies: Firm 2 will increase or decrease its coupon rate slightly and take the monopolist share of savings, as it can earn higher profits. Hence, this case cannot be an equilibrium.

In the second case, either offer is strictly preferred to the other; for example, households prefer z_2^f to z_1^f. In this case, Firm 2 can reduce its coupon rate slightly to $\hat{r}_2 - \epsilon$, for some $\epsilon > 0$, without failing to issue the same amount of bonds $\hat{k}_2$. Apparently, the smaller coupon rate lowers the cost and thus brings a larger profit than $\hat{\pi}_2$ to Firm 2. Hence, this case cannot be an equilibrium.

In summary, there exists no asymmetric pure strategy Nash equilibrium with a representative household.

This result also holds for the case with nonrepresentative households. Consider the case of two households without loss of generality. It is essentially the same as the representative household case, if firms offer corporate bonds for both households without differential treatment. The same is true if firms competitively offer bonds for both households, because there is no intrinsic difference among households and firms. Only if firms somehow specialize specific households and receive monopolistic rents, the two household case may differ from the representative household framework. For example, in the two-firm-and-two-household case, Firm 1 offers bonds only for Household 1 and Firm 2 only for Household 2. However, if any monopolistic rents emerged from the Firm 1-Household 1 relationship, Firm 2 would take over Household 1's savings by offering a slightly higher coupon rate with the same quantity, and thereby becomes the monopolist to earn higher profits. Hence, it is impossible for firms to specialize and to obtain monopolistic rents in a noncooperative way. Therefore, it is sufficient to analyze only the case of the representative household.

Q.E.D.

Next, I expand the strategy space to mixed strategies. Intuitively, as in simple Bertrand competition, firms bid up from the lower end of the support of a mixed strategy, ending up with degenerated pure strategy or negative profit. Or, similar to Edgeworth cycle, without reaching a specific strategy, firms may revert back to interest rates at a lower level at some point and start bidding again.

Proposition 5. *There exists no mixed strategy equilibrium in an economy with private direct finance.*

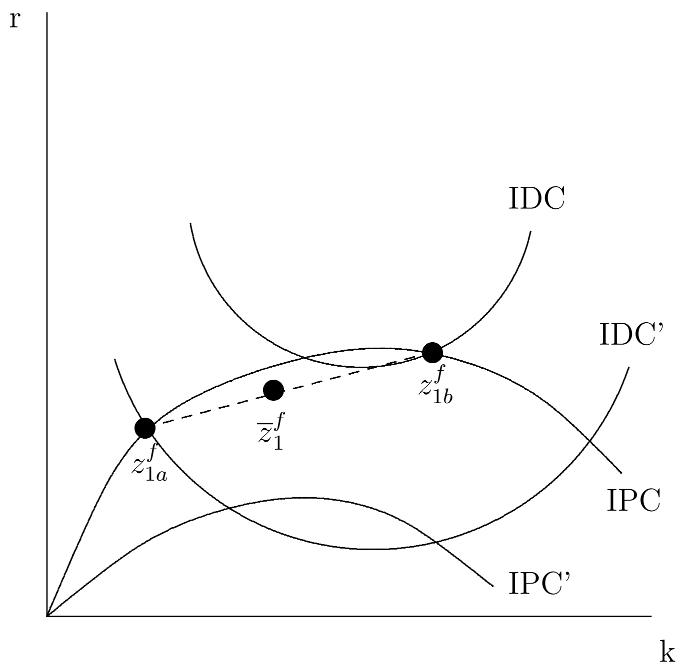

Figure 1. Iso-Profit Curve (IPC) and Indifference Curve (IDC)

Proof. I use contradiction to prove this proposition. Namely, if a nondegenerate mixed strategy is an equilibrium strategy for a firm, then I will show that there always exists another strategy that brings higher profit. Without loss of generality, I focus on the case with two firms.

Assume there exists a nondegenerate mixed strategy equilibrium. Let q_1^* denote a nondegenerate equilibrium mixed strategy of Firm 1 and q_2^* denote Firm 2's equilibrium mixed strategy (possibly degenerated). Note that, for q_1^* to be an equilibrium, given q_2^*, any pure strategies in the support of q_1^* should provide the same expected profit to Firm 1.[82] Hence, without loss of generality, analysis here focuses on any arbitrary two points in the support of Firm 1's equilibrium mixed strategy, namely $z_{1a}^f = (r_{1a}, k_{1a})$ and $z_{1b}^f = (r_{1b}, k_{1b})$. Both should lie on the same iso-profit curve defined on (k, r) plane (see Figure 1).[83]

Consider an equilibrium where Firm 1 adopts a nondegenerate mixed strategy q_1^* but Firm 2 offers a pure strategy $z_2^{f*} = (r_2, k_2)$. Take any arbitrary two points in the support of Firm 1's equilibrium mixed strategy, $z_{1a}^f = (r_{1a}, k_{1a})$ and $z_{1b}^f = (r_{1b}, k_{1b})$ with associated equilibrium probability q_{1a}^* and q_{1b}^*, respectively. For them to be in the support of an equilibrium mixed strategy, both q_{1a}^* and q_{1b}^* must be weakly preferred by consumers to Firm 2's strategy z_2^{f*}. Given Firm 2's strategy z_2^{f*}, Firm 1's expected profit from these two pure strategies in the support of the mixed strategy is a convex combination of underlying profits; that is,

$$E_{ab}^*\pi \equiv Q_{1a}^*\pi(k_{1a}, k_2, r_{1a}) + Q_{1b}^*\pi(k_{1b}, k_2, r_{1b}), \tag{A18}$$

[82]See a text book, for example, Osborne and Rubinstein (1994, pp. 33-34)

[83]Figure 1 shows the case of $r_{1a} < r_{1b}$ and $k_{1a} < k_{1b}$, but the same analysis applies to the case with $r_{1a} < r_{1b}$ and $k_{1a} > k_{1b}$.

where weights are conditional probabilities given by the equilibrium mixed strategies,[84] $Q^*_{1a} \equiv q^*_{1a}/(q^*_{1a}+q^*_{1b})$ and $Q^*_{1b} = 1 - Q^*_{1a}$.

Define $\bar{z}^f_1 = (\bar{r}, \bar{k})$ as a weighted average of z^f_{1a} and z^f_{1b} with Q^*_{1a} and Q^*_{1b} as the weights. This $\bar{z}^f_1$ faces positive demand. To see this, consider consumers' indifference curves on (k, r) plane (see Figure 1). Since a consumer prefers a higher coupon rate given her savings k, the utility level increases with r as long as Firm 1's offers are preferred to Firm 2's offer z^f_2. Given the coupon rate r, the consumer's maximization problem determines a unique optimal savings level k.[85] Hence, given a utility level, an indifference curve decreases toward the optimal k, and then increases for excessive k, because consumers demand a higher interest rate to compensate for nonoptimal savings k. In Figure 1, note also that the utility level is higher on the upper indifference curve (IDC) than the lower one (IDC'). Since the indifference curves at the different utility levels do not cross each other, the utility level of the weighted average offer $\bar{z}^f_1$ is strictly preferred to either z^f_{1a} or z^f_{1b}, whichever is less attractive to consumers. However, as even less attractive offers face positive demand from consumers in an equilibrium, the average offer $\bar{z}^f_1$ must attract positive demand.[86]

While z^f_{1a} and z^f_{1b} lie on the same iso-profit curve, the weighted average offer $\bar{z}^f_2$ lies inside the iso-profit curve (IPC)—it is easy to show that the iso-profit curves on the (k, r) plane is strictly concave and that it has a single peak (i.e., the unique optimal capital level exists, given a coupon rate). Also, profit is higher on the lower iso-profit curve (IPC') than the upper one (IPC). Hence, by Chebyshev's inequality, a pure strategy strictly dominates the equilibrium mixed strategy,

$$\pi(\bar{k}, k_2, \bar{r}) > E^*_{ab}\pi. \tag{A19}$$

This contradicts the assumption that a nondegenerate mixed strategy equilibrium exists.

Now, consider an equilibrium where Firm 1 adopts a nondegenerate mixed strategy q^*_1 and Firm 2 also offers a nondegenerate mixed strategy q^*_2. A similar analysis as above goes through. Again, both z^f_{1a} and z^f_{1b} are lying on the same iso-profit curve. The weighted average offer $\bar{z}^f_1$ generates a strictly higher profit for Firm 1, while it gives strictly higher utility to consumers than one of the support giving lower utility, say z^f_{1a}. Hence, $\bar{z}^f_1$ must be accepted by consumers with higher probability than the probability that z^f_{1a} is accepted, given the Firm 2's mixed strategy q^*_2. In other words, Firm 1 can obtain a strictly higher profit by rearranging its mixed strategy; namely, removing some positive probability mass on z^f_{1a} and reallocating it on $\bar{z}^f_1$. Therefore, there always exists another mixed strategy that strictly dominates an equilibrium nondegenerate mixed strategy. This is a contradiction. *Q.E.D.*

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[84]For the sake of simplicity and without loss of generality, z^{f*}_2 is assumed here to give strictly less utility than z^f_{1a}.

[85]As in the Walrasian economy, the value function $V_D(m_i|m_{-i}, \mu_{-i}, q)$ is increasing and strictly concave with respect to coupon rate r.

[86]In figure 1, z^f_{1a} is less attractive than z^f_{1b} and also than $\bar{z}^f_1$.

APPENDIX VI. RELATION TO DISCONTINUOUS GAME LITERATURE

There is no theorem that assures the existence of a Nash equilibrium in an economy without intermediaries or an auctioneer.[87] This is because the payoff function is seriously discontinuous. It is easy to check discontinuity of the payoff function with respect to a firm's own action.

Reny (1999) is the latest work that specifies a sufficient condition for existence of a Nash equilibrium in a game with a discontinuous payoff function. The condition is called *better-reply secure:*[88] *if for every nonequilibrium strategy z^{f*} and every payoff vector π^{f*} for which (z^{f*}, π^{f*}) is in the closure of the graph of the game's vector payoff function, some player j has a strategy yielding a payoff strictly above π_j^{f*} even if the others deviate slightly from z^{f*}.*

Assume that all funds are supplied by a representative household—as in the proof of Lemma 11, it is sufficient to analyze this case only. Consider[89] a nonequilibrium strategy that every firm offers the Pareto-optimal bond[90] $z_j^{f*} = (r_j^f = A, k_j^f = N.S.)$. Apparently, profits are zero, $\pi_j^{f*} = 0$. Suppose now that Firm 1 slightly deviates to offer a higher rate $(r_1^f = A + \epsilon, k_1^f = N.S.)$. Firm 1 would become the monopolist, but its profit would be negative, because the coupon rate is larger than the technologically highest return A. Here, other firms $j = 2 \cdots J$ remain at zero profits as they would not raise any funds. Hence, in this Bank 1's deviation at the specific point in the payoff functions, no other player has a strategy yielding a payoff strictly above $\pi_j^{f*} = 0$. Therefore, the game is not better reply secure.

Simon and Zame (1990) suggest that there always exists an equilibrium with an endogenous sharing rule under a general condition. If the sharing rule when firms offer the same price is chosen before competition, but not restricted to an equal share, there exists a sharing rule and associated allocation that constitutes an equilibrium. Although they call this sharing rule endogenous, it has to be exogenously given before firms compete. Yanelle (1998) uses Simon and Zame (1990)'s result and moves it further in a slightly different setting: if there exist public lotteries to select specific shares of consumers when firms set the same price, then there will be an equilibrium, in which one firm is chosen as a monopolist.

This paper so far has not specified a sharing rule. More specifically, if the interest rate and saving amount are specified, the offer is take-it-or-leave-it and, thus, households cannot chose to deposit less than the specified amount. Simon and Zame's (1990) mechanism or Yanelle's version could be introduced by assuming a specific sharing rule for the case with deposit market competition with nonspecified amounts. However, these techniques do not work well. To see this, consider a case in which only one firm will receive all funds when several firms offer the same rate—a firm will be assigned 100 percent share either a priori or by lottery. Even in this case, the equilibrium candidate coupon rate is A; otherwise a firm would offer a slightly higher, but less than A, interest rate to take

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[87]Of course, a discretized version of the game—a finite game—always possess at least one Nash equilibrium.

[88]The following definition is from Reny (1999) except that I use z^{f*} to denote the strategy instead of x^* and π^{f*} to denote the payoff instead of u^* in Reny (1999).

[89]I thank Philip Reny for suggesting this example.

[90]With a representative household, the household subscript is omitted.

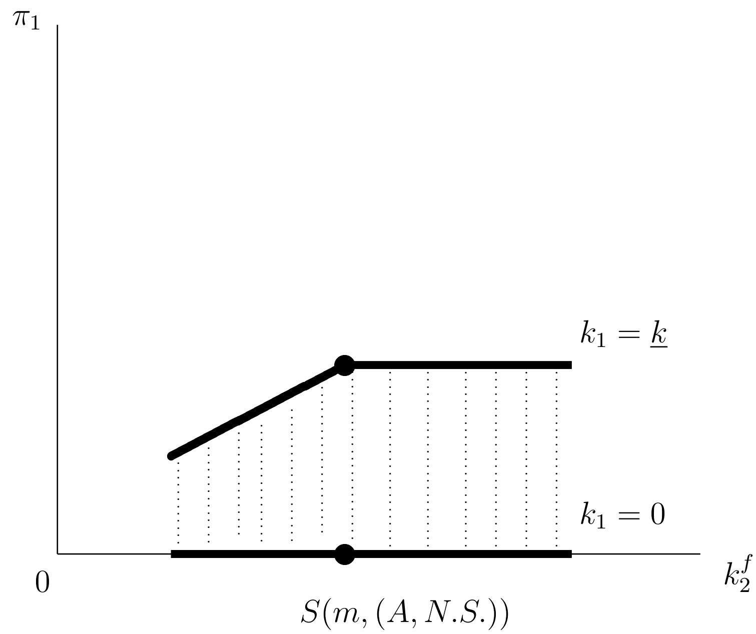

Figure 2. Profit of Firm 1

all the savings. But again, if the prevailing rate is A, a deviating firm would offer a slightly higher rate $A + \epsilon$ with a limited bond issue to free ride on others' investments and earn positive profits. This upsets the specified equilibrium with a 100 percent share assignment as a tie-breaking rule.[91]

It turns out that this game violates one of the conditions of Simon and Zame (1990), namely, the upper hemicontinuity of the payoff correspondence. This game also violates another condition always assumed in the literature: compactness of the strategy space. However, the nonexistence result is not caused by the unbounded strategy space (i.e., interest rate and investment level can be any real number). Even if I restrict the strategy space to be some (sufficiently large) compact subset of real numbers, no Nash equilibrium exists. I illustrate these arguments in detail below.

Consider the case with two firms, $j = 1, 2$ without loss of generality. The strategy space is assumed to be large but compact: $r_j^f \in [0, 2A]$ and $k_j^f \in [0, S(m, (4A, N.S.))]$, where r_j^f denotes the interest rate of j firm and k_j^f denotes the investment level of firm j. I focus on a neighborhood of the strategy with which Firm 1 tries to become a free rider and Firm 2 tries to become a monopolist at an interest rate A; that is,

$$\hat{z}^f = \Big((r_1^f = A, k_1^f = \underline{k}), (r_2^f = A, k_2^f = S(m, (A, N.S.)))\Big), \tag{A1}$$

where $\underline{k}$ is assumed to be at most $S(m, (A, N.S.))/2$. To analyze more general case, possible partial shares of firms 1 and Firm 2 are assumed. Specifically, Firm 1 may be assigned less than $\underline{k}$ and Firm 2 may be assigned less than $S(m, (A, N.S.))$.

The profit of Firm 1 with this strategy depends on the sharing rule. When Firm 1 has a positive share,

[91] Yanelle (1998) restricts the domain essentially bounded by A, and thus equilibrium exists, but apparently this restriction is too *ad hoc* for the analysis here.

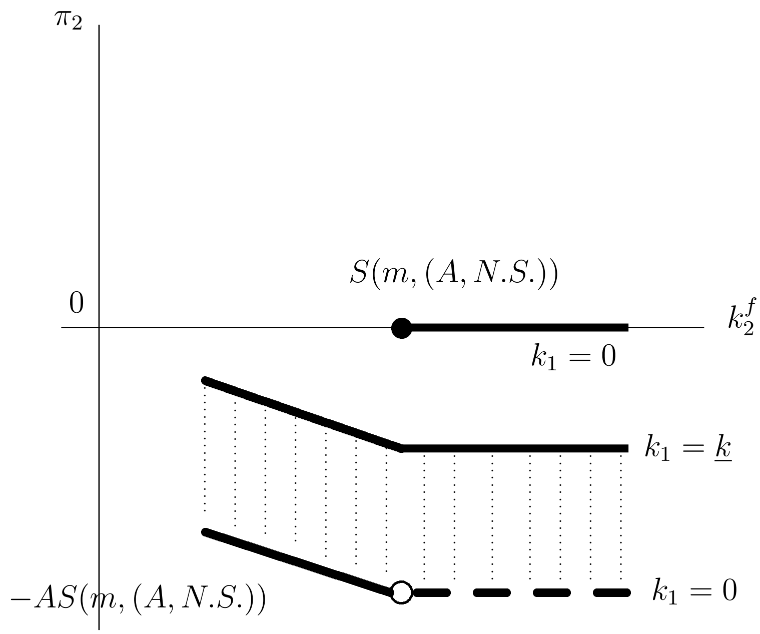

Figure 3. Profit of Firm 2

its profit can be written as[92]

$$Ak_2^{1-\alpha}k_1^{\alpha} - Ak_1. \tag{A2}$$

This takes the highest value when Firm 1 receives its full share (i.e., $k_1 = \underline{k}$). When Firm 1's share is zero, its profit is also zero. Apparently, when its share changes, the profit of 1 changes continuously. This case is drawn in Figure 2 at $k_2^f = S(m,(A,N.S.))$. Note that Figure 2 shows possible profits for Firm 1 in the neighborhood of the strategy vector $\hat{z}^f$ in the dimension of k_2^f.

In the right region of $\hat{z}^f$ (i.e., $k_2^f > S(m,(A,N.S.))$), the offer k_2^f by Firm 2 is larger than demand and thus the same profits are drawn for Firm 1 as at $k_2 = S(m,(A,N.S.))$ depending on its share. At the bottom line on the x-axis, Firm 2 receives all the funds and Firm 1's profits are zero. For a given aggregate demand and strategy $\hat{z}^f$, an optimal share exists for Firm 1. For the sake of simplicity and without loss of generality, assume that it is higher than $\underline{k}$. In Figure 2, the upper line shows the highest, but not optimal, profits of Firm 1. Apparently, all profit values between the upper line and the bottom line can be generated depending on the equilibrium share of Firm 1.

In the left neighborhood of $\hat{z}^f$ (i.e., $k_2^f < S(m,(A,N.S.))$), the profit of Firm 1 with specific k_1 is lower as the spillover effect from Firm 2 is less. Again, all values between the two lines are possible depending on the share.

Now look at the profit of Firm 2 in the k_2^f-dimension of the neighborhood of the strategy vector $\hat{z}^f$. When Firm 1 has some positive share, the profit of Firm 2 can be written as

$$\pi_2^f = Ak_1^{1-\alpha}k_2^{\alpha} - Ak_2. \tag{A3}$$

[92]As before, variables without superscript are equilibrium values.

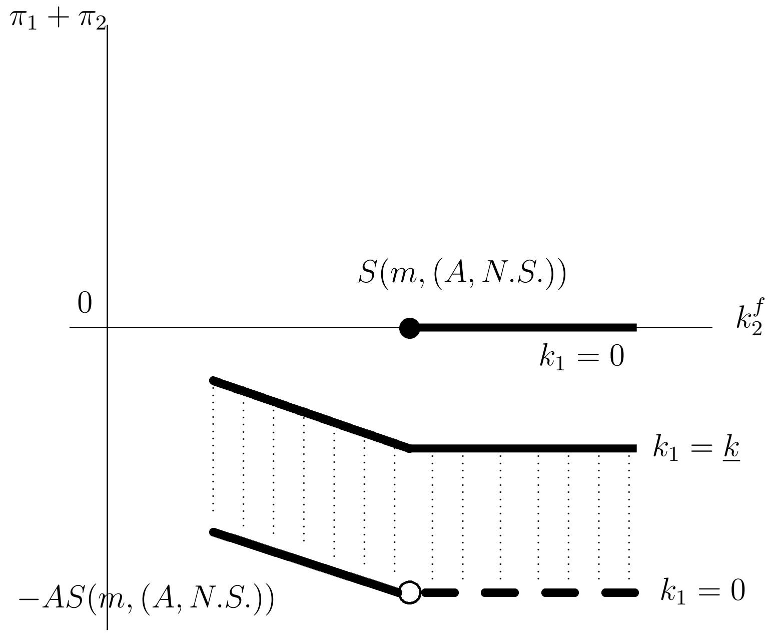

Figure 4. Sum of Two Firms' Profits

The profit of Firm 2 at the strategy $\hat{z}^f$ is zero when Firm 2 has a full share.[93]. This is because output is $AS(m, (A, N.S.))$ but capital cost is also $AS(m, (A, N.S.))$. If Firm 2 has a positive share but not a full share, then the maximum profit will be obtained when Firm 1 (the free rider) invests most, $\underline{k}$. Still, even at this maximum, Firm 2's profit is strictly less than zero. As Firm 1's share declines, Firm 2's profit declines and approaches $-AS(m, (A, N.S.))$. However, it never takes a value $-AS(m, (A, N.S.))$, because, as discussed, Firm 2's profit is zero at zero shares of Firm 1 (full shares to Firm 2). When its share changes, Firm 2's profit changes discontinuously and is not even closed in the graph.

In the right region of the point (i.e., $k_2^f > S(m, (A, N.S.))$), the offer k_2^f is larger than the demand; thus, the same profits are drawn for Firm 2 at $k_2^f = S(m, (A, N.S.))$ depending on its share. In the left neighborhood of the point (i.e., $k_2^f < S(m, (A, N.S.))$), even with full shares to Firm 2, Firm 2 cannot become a monopolist and Firm 1 will always take some share. Hence, the uppermost line on the x-axis disappears and the values on the bottom line increases and are possibly realized. In either region, all profit values between the middle and the bottom lines can be generated depending on the equilibrium share of Firm 2.

The sum of two firms' profits (Figure 4) inherits the shape of Figure 3. With the strategies $\hat{z}^f$, as Firm 1's share declines to zero, the sum declines to $-AS(m, (A, N.S.))$. However, it never takes that value and thus the graph is open at $-AS(m, (A, N.S.))$. When Firm 1's share is exactly equal to zero, the sum of two firms' profit suddenly becomes zero. This is because Firm 2 becomes a monopolist and its profit is zero. If both firms have a positive share, the sum of profits is strictly less than zero. This is because aggregate revenue is strictly less than that of a monopolist due to free-riding by Firm 1.

[93] See Figure 3 at $k_2^f = S(m, (A, N.S.))$

To check the continuity of the sum of the two firms' profits at strategy $\hat{z}^f$, it is necessary to examine how their profits vary with a slight change in each element of strategy $\hat{z}^f$. This is not an easy task because the dimension of the strategy space is four. So, I investigate one element at a time, taking the other three elements as fixed. It is easy to show that it is upper hemicontinuous in prices[94] r_1^f and r_2^f, but not in quantities k_1^f and k_2^f.

Suppose k_2 is changed from $S(m,(A,N.S.))$ to $S(m,(A,N.S.))+\epsilon$. This change does not affect profits in any way because k_2 was already equal to total savings and Firm 2 cannot take more capital from consumers than total savings $S(m,(A,N.S.))$.

Now suppose k_2 is changed from $S(m,(A,N.S.))$ to $S(m,(A,N.S.))-\epsilon$. Then the profit of Firm 1 could be larger than before if Firm 1 had a full share. It is now impossible for the profit of Firm 2 to take value zero (the monopolist's value), because Firm 2 offers less than the total savings at interest rate A. On the other hand, the losses of Firm 2 should be less than before. This is because Firm 2 is hurt less by Firm 1's free-riding behavior. In particular, the share of Firm 1 is bounded below by $\epsilon/S(m,(A,N.S.))$ so that Firm 2 now can attain the lower bound of its profit. This lower bound is strictly above, $-AS(m,(A,N.S.))$. The sum of the two firms' profits apparently inherits the shape and the graph is closed. In summary, as shown in Figure 4, the sum of the two firms' profits is not an upper hemicontinuous correspondence.[95]

Finally, it may be redundant to note conditions that appeared in Dasgupta and Maskin (1986a), since the better-reply secure condition (Reny, 1999) is a relaxation of their diagonal discontinuity and weak lower semicontinuity conditions.[96] However, I note these here to better understand the situation of discontinuity of payoff functions in this paper. Dasgupta and Maskin (1986b) discuss the Rothschild-Stiglitz insurance market, where the sum of payoff functions is not upper semicontinuous. They show that this discontinuity is only on the diagonal element (i.e., on a set of points whose dimension is less than strategy space); thus, by taking the limit of mixed strategies of the finite game, probability measures (in a mixed-strategy Nash equilibrium) on the discontinuous points are zero. Thereby, they show the existence of mixed Nash equilibrium. Here, the problem is much more severe. As I show above, discontinuity lies in off-diagonal[97] elements—the neighborhood of $(k_1=\underline{k},k_2=S(m,(A,N.S.)))$. It is not merely a point: the value of $\underline{k}$ to create this discontinuity can be anything below $S(m,(A,N.S.))/2$. Hence the technique of Dasgupta and Maskin (1986b) cannot be applied in our case.

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[94]This is why the Walrasian equilibrium exists.

[95]Note that the single-valued function, which is obtained by fixing the sharing rule, is not upper semicontinuous.

[96]They analyze a discontinuous game based on a single-valued payoff function.

[97]On the diagonal elements, the sum of the two functions is an upper hemicontinuous correspondence, because both are trying to become monopolists or free-riders and thus production functions show no drastic change at A.