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Pricing Growth-Indexed Bonds

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Abstract

Growth-indexed bonds have been suggested as a way of reducing the procyclicality of emerging-market countries' fiscal policies and the likelihood of costly debt crises. Investor attitude surveys suggest that pricing difficulties are seen as a considerable obstacle. In an effort to reduce such concerns, this article presents a simple way of pricing growth-indexed bonds. As a pleasant by-product, the analysis tracks the quantitative implications of an increase in the share of growth-indexed bonds in total debt, measuring the ensuing decline in the probability of default and the reduction in the spreads at which standard bonds can be issued.

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I. Introduction

The way in which countries structure their public debts has important implications for their susceptibility to costly crises, the degree of procyclicality in their fiscal policies, and their economic performance more broadly. Emerging market countries in particular often rely to a large extent on short-term debt or foreign currency debt. When economic growth weakens, investors often become more concerned about the borrowing country's ability to meet its external obligations: as a result, its currency tends to depreciate, and any new debt can only be issued at a higher interest rate. Both factors increase the local-currency cost of servicing the debt while the country is experiencing an economic downturn. In an effort to maintain a low overall fiscal deficit and persuade investors of its creditworthiness, a country will then tend to cut noninterest expenditures or raise taxes exactly at the time when this seems least desirable from the cyclical point of view. Indeed, several studies have found fiscal policies to be more procyclical in emerging market countries than in advanced countries (Gavin and Perotti, 1997; International Monetary Fund, 2003, Ch. 3; Talvi and Vegh, forthcoming).

To alleviate the cost of servicing the debt in times of weak growth, and to reduce the likelihood of default, countries could consider issuing growth-indexed bonds, that is, bonds providing a return indexed to the issuing country's real GDP growth rate. By requiring a higher-than-usual payment in years of strong growth, and a lower-than-usual payment in years of weak growth, growth-indexed bonds would act as an automatic stabilizer, helping reduce the need for procyclical fiscal policies.

Of course, financial engineering is no substitute for sound institutions and good policies. These can improve economic performance and reduce the risk of crises both directly and by making it possible for countries to ameliorate their debt structures. For example, a clear commitment to stable macroeconomic policies facilitates the issuance of long-term, domestic currency bonds. Nevertheless, a reasonable case can be made in favor of growth-indexed bonds as part of a broader package of measures to make policies less procyclical and reduce emerging market countries' vulnerabilities while the credibility of its macro policies is consolidated.

Despite their potential advantages and a long intellectual history, growth-indexed bonds have not been used on a large scale in the past, owing to a number of obstacles to their introduction. As with other types of financial innovation, there are first-mover costs for individuals on both the investor side and the issuer side. Individual market participants are reluctant to incur the initial costs related to writing a new type of contract, marketing it, and pricing it, and to run the risk that the secondary market would be too thin.

In this paper, we seek to alleviate one specific obstacle, which surveys of investors' opinions have identified (somewhat to our surprise) as significant, namely, the potential difficulties in pricing growth-indexed bonds (Borensztein and others, 2004). Such surveys point to difficulties in quantifying factors such as "novelty" and liquidity premia; but they also emphasize the analytical complexity involved in estimating what market participants refer to as the "theoretical price" of a GDP-indexed bond, that is, a model-based estimate of the expected payoff. Our exercise will focus on estimating such a "theoretical price," abstracting from the other, perhaps more subjective, factors; while these may be a large obstacle to the emergence of new markets, prices may converge to their theoretical expected values as markets develop.

We propose a simple method to price growth-indexed bonds. Our contention is that the really difficult part in this exercise is pricing standard, "plain-vanilla" bonds—something that most people take for granted and is done every day on existing markets. The approach we use to extract information from plain-vanilla bond prices, and subsequently price them under different scenarios, is grounded in standard reasoning that follows common market practice. Nevertheless, the exact procedure we use to price plain-vanilla bonds is not crucial; rather, the point we wish to emphasize is that if one uses a consistent method to price both plain-vanilla bonds and growth-indexed bonds, the additional complication involved in pricing growth-indexed bonds is minimal. Specifically, the difficult part is estimating whether a country will default and the implied loss for investors, a challenge that also applies to the pricing of plain-vanilla bonds.[2]

While the benefits of growth-indexed bonds could be substantial for advanced countries as well (especially those with fiscal rules or limits on the overall deficit, such as those involved in the European Union's Stability and Growth Pact), in this paper we focus on parameter values that are relevant for emerging markets, where macroeconomic fluctuations are more pronounced and the possibility of default is usually considered to be greater (as reflected in bond spreads).[3] Indeed, the exercise in pricing growth-indexed bonds is numerically more interesting for those countries where the probability of default is considerable.

In a nutshell, we proceed as follows. Risk-neutral investors holding the bonds issued by an emerging market must expect that, on average, they will obtain the same return as on safe U.S. Treasury bonds of similar duration.[4] From the observed yields on the emerging market and U.S. bonds we extract reasonable combinations of the probability of default for the emerging market and recovery rates on defaulted bonds. By extracting that information from observed market spreads we minimize the need to make assumptions on the default probability and recovery rates. Based on past data, we estimate the joint distribution of the key macroeconomic variables (real GDP growth, the primary balance, and the real exchange rate) for the emerging market.

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[2] For the purposes of this paper, default is defined to include instances of debt restructuring below net present value at market prices.

[3] Another reason why issuing growth-indexed bonds may be more attractive for emerging markets is that their growth risks are more easily diversifiable in a portfolio held by international investors. The comovement between GDP growth in an emerging market and different measures of the "market portfolio" is typically very small: CAPM-type regressions yield low estimates for β and R^2 coefficients (Borensztein and Mauro, 2004).

[4] The assumption of risk-neutrality is for simplicity. A risk premium could be introduced with essentially no impact on the main messages of the paper.

We use those statistical properties to simulate 250,000 paths for the economic variables and the debt/GDP ratio. We then extract a default trigger level for the debt/GDP ratio and the recovery rate that would yield the expected repayments implicit in the spreads. While our simulations assume an implicit default trigger for the debt/GDP ratio, they still allow for defaults to take place from an initially moderate debt/GDP ratio if large adverse shocks occur, such as a sharp real depreciation.[5] Finally, using the debt/GDP ratio default trigger and the simulated paths for the economic variables, we compute the corresponding payoff for both the growth-indexed bonds and the standard plain-vanilla bonds. We repeat the exercise for different shares of growth-indexed bonds in total debt and different underlying assumptions.

Our main objective in presenting this set of simple numerical exercises is to demonstrate that pricing growth-indexed bonds is only marginally more complicated than pricing standard bonds. In addition, our numerical exercises have two pleasant by-products. First, they make it easy to track the consequences of a greater share of growth-indexed debt for the probability of default, and the spreads at which standard bonds can be issued. Indeed, a simple but important point that becomes transparent in the numerical exercises is that a full assessment of the desirability of increases in the share of growth-indexed debt requires tracking its implications for standard bonds as well. In our simulations, growth-indexed debt can lower the default frequency by 1/4 to 1/3 of its initial level. Second, we show that when the share of indexed debt rises, both types of bonds become less sensitive to "surprises" in the growth rate and its volatility. The reason is that growth indexation attenuates the effect of those surprises on the probability of default.[6]

The remainder of the paper is as follows. In Section II, we provide a succinct overview of the stylized facts about emerging market debt. In Section III, we give a quick primer on growth-indexed bonds, outlining their advantages but also the obstacles to their introduction. In Section IV, we present a simple method to price growth-indexed bonds. In Section V we conclude.

II. EXISTING DEBT STRUCTURES

Existing debt structures seem to have been associated with frequent and costly crises, especially in emerging market countries, where the shares of short term debt and foreign currency debt are substantially higher than in advanced countries. On average, the share of foreign currency debt in emerging markets was almost one half in 2001, compared with approximately 5 percent in advanced countries (Table 1). Long-term domestic currency debt represented three quarters of

[5] While our simplifying assumption that default occurs if and only if the debt/GDP ratio exceeds an estimated trigger level may not do full justice to defaults triggered by illiquidity, distinctions between illiquidity and solvency, or the role of the maturity composition of the debt, our approach seems to be based on an empirically relevant and observable variable.

[6] Growth surprises that lower the average growth path will disproportionately harm growth-indexed bond returns. But even then, part of the effect is attenuated by the lower prevalence of defaults. Of course, growth surprises that raise the average growth path will disproportionately benefit growth-indexed bond holders.

Table 1. Currency Composition of Central Government Debt in 2001
(in percent of total)

	Foreign-Currency Debt[1]	Long-Term Domestic-Currency Debt[1]	Total Debt as a Ratio to GDP (in percent)
Emergin market countries	48.0	32.4	50.4
Lain America	67.9	15.2	37.0
Argentina	96.8	...	53.7
Brazil	43.8[2]	3.3[3,4]	66.2[2,5]
Chile	92.7	0.0	15.6
Mexico	35.6	57.5[6]	22.6
Venezuela	70.6	0.0	27.0
Asia	28.5	44.2	62.8
China	17.7	82.3	24.0
India	14.5	69.9[7]	65.1[7]
Indonesia	46.0	51.0[6]	90.9
Malaysia	16.7	0.0	69.2
Philippines	47.4	17.6[8]	64.9
Others	47.6	17.4	51.3
Poland	34.8	34.8	39.3
Russia	90.3	...	50.0
South Africa	14.4	61.2[3,4,8]	46.8
Hungary	30.1	44.0[9]	52.1
Turkey	68.2	0.0	68.5
Advanced Economies[6]	5.6	75.9	51.8

Sources: IMF staff, OECD, *Central Government Debt Yearbook 1992-2001*; and websites of the country authorities.

[1] In percent of total central government debt for emerging market coutnries and total central government marketable debt for advanced economies.

[2] Includes debt held by the Central Bank.

[3] Based on residual maturity.

[4] Only marketable domestic-currency bonds.

[5] Consolidated government debt.

[6] Includes debt indexed to inflation and domestic interest rates.

[7] Includes debt owed to National Small Savings Funds.

[8] Includes debt with maturities of three years or more.

[9] Data for 2002.

total debt in advanced countries, whereas it amounted to one third in emerging markets. There is also considerable variation within the group of emerging markets. For example, the share of long-term local currency debt is far higher for Asian emerging markets than Latin American emerging markets.

The data presented above refer to the composition of countries' total debt. Interesting patterns emerge considering internationally-issued debt and domestically-issued debt separately (this refers to the location of issuance, rather than currency nomination). The structure of international debt is fairly uniform across countries: it has typically taken the form of foreign currency debt with a maturity at issue of 5–10 years (though this has risen somewhat in the current environment of relatively abundant global liquidity). More interesting variation is observed focusing on domestically issued debt. Here, large differences are observed across countries in the ability to issue long-term, domestic currency bonds. Moreover, various emerging markets have taken considerably different approaches to dealing with their difficulties issuing long-term, domestic currency bonds: some have relied instead on foreign currency debt, others on inflation-indexed debt, others still on debt indexed to domestic interest rates (Table 2). Incidentally, this also shows that indexed debt, including debt indexed to a macroeconomic variable measured by agencies that often report to the issuing government, namely inflation-indexed debt, is commonplace in many emerging market countries.[7]

Crisis-prone debt structures in emerging markets are unlikely to be solely the result of historical accident. Rather, they are probably, to a considerable extent, an equilibrium response by issuers and investors to underlying features of the issuing country. In particular, lack of credibility of monetary and fiscal policies may help explain the difficulties experienced by many countries in issuing long-term debt in their own currency. Other variables—some of which clearly lie outside the control of the government, especially in the short run—are likely to play a role. Such variables include institutional quality (political stability, rule of law...), and the size of the domestic investor base (which in turn depends on factors such as the size of the population, per capita income levels, the type of pension system, financial regulations, and capital controls).

A few countries have improved their debt structures considerably within a few years, through credible inflation stabilization and a few well-chosen and well-implemented institutional reforms establishing commitment to stable policies, often combined with lengthening the debt structure first with the issuance of inflation-indexed bonds. During the 1980s and 1990s emerging market countries such as Chile, Israel, Mexico, and Poland combined fiscal stabilization and a reduction in inflation to the single digits with reforms in the monetary area, such as the introduction of central bank independence or inflation targeting, as well as reforms in the fiscal area, such as pension reforms; as a result, these countries were able to improve their

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[7] Furthermore, the wide cross-country variation in the use of and types of indexation, with no obvious regular patterns, is consistent with the view that policy makers can play a significant role in fostering financial innovation (see also Borensztein and Mauro, 2004; and Borensztein and others, 2004).

Table 2. Structure of Domestically Issued Government Bonds at End-2001
(in percent of total)

	Domestic Government Bonds/GDP (in percent)	Domestic-Currency-Denominated Bonds				Foreign-Currency-Denominated Bonds
		Not indexed		Indexed to		
		Long term[1]	Short term[1]	Domestic interest rate	Inflation	
Emerging market countries	28.2	41.5	18.6	26.4	7.2	6.3
Latin America	24.0	5.6	13.7	50.8	16.6	13.4
Brazil	52.1	9.5	0.0	53.0	7.0	30.5
Chile[2]	...	0.0	21.0	0.0	55.8	23.2
Mexico	11.0	12.8	23.6	60.1	3.5	0.0
Venezuela	9.0	0.0	10.0	90.0	0.0	0.0
Asia	26.6	52.4	16.5	22.2	7.8	1.1
India	27.0	81.6	18.4	0.0	0.0	0.0
Indonesia	34.0	24.6	0.0	30.9	38.9	5.6
Malaysia	36.3	0.0	19.8	80.2	0.0	0.0
Thailand	13.8	91.5	8.5	0.0	0.0	0.0
Philippines	22.0	64.2	35.8	0.0	0.0	0.0
Europe and others	32.4	56.5	23.6	13.6	0.4	5.9
Czech Republic	...	41.1	58.9	0.0	0.0	0.0
Hungary	27.0	56.0	23.0	21.0	0.0	0.0
Poland	16.0	62.6	26.5	10.9	0.0	0.0
Slovak Republic	29.0	86.8	13.2	0.0	0.0	0.0
South Africa	35.0	92.4	5.2	0.0	2.4	0.0
Turkey	55.0	0.0	14.5	49.9	0.0	35.6

Sources: IMF staff estimates; and JPMorgan, *Guide to Local Markets* (2002).
[1]Short term is defined as an initial maturity of less than one year, and long term is defined as an initial maturity of more than one year.
[2]For Chile, the shares refer to bonds issued by the central bank. The amount of bonds issued domestically by the central government is negligible.

debt structures, making them less reliant on short-term or foreign-currency debt, and to reduce their cost of borrowing significantly, within a limited number of years (Borensztein and others, 2004, p. 20).

This brief review of existing debt structures suggests that, although a few countries have been able to improve their debt structures significantly using existing instruments, most emerging market countries will likely retain debt structures that make them vulnerable to crises and prone to procyclical fiscal policies. Thus, it seems worth exploring the potential advantages of new financial instruments.

III. Advantages of Growth-Indexed Bonds and Related Obstacles

In this section, we briefly review the intellectual history, the precedents, and advantages of growth-indexed bonds, as well as the obstacles to their introduction. These topics are developed in further detail in Borensztein and Mauro (2004) and Borensztein and others (2004).

A. Potential Advantages and Intellectual History of the Proposal

The potential advantages of growth-indexed bonds have solid analytical foundations. Barro (1995) shows that, while the optimal scheme in a tax-smoothing model of debt management is to index to consumption and government expenditure, growth-indexed bonds are a more practical alternative. They also have a long intellectual history. An initial wave of interest in indexing bond payments to GDP, exports, or commodity prices originated in the aftermath of the debt crisis of the 1980s (Bailey, 1983; Lessard and Williamson, 1985). At that time, however, emphasis was placed on concerns regarding the possibility that growth-indexation would reduce countries' incentives to grow rapidly, and seemed instead to prefer indexation to more exogenous variables such as commodity prices (Krugman, 1988). A second wave of interest is associated with the influential work by Shiller (1993, 2003). Following a number of emerging market crises in recent years, several authors have recently revived the case in favor of growth-indexed bonds or related instruments (Athanasoulis, Shiller, and van Wincoop, 1999; Borensztein and Mauro, 2004; Drèze, 2000a and 2000b; and Obstfeld and Peri, 1998).

Growth-indexed bonds are clearly feasible in practice, as evidenced by several precedents for growth-indexation clauses in debt instruments, mostly issued in the context of debt restructurings. These include the GDP-based Value Recovery Rights in the relatively small restructurings for Bosnia, Bulgaria, and Costa Rica.[9] The most significant example to date is

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[9] Value Recovery Rights linked to commodity prices were somewhat more common, particularly in the context of the Brady deals. An interesting precedent is also provided by a small restructuring by the city of Buenos Aires, which included indexation to tax revenues. Closer to the notion of pure "Shiller" securities, a small market for options on economic statistics such as U.S. nonfarm payroll has occasionally operated in the past few years under the auspices of Deutsche Bank and Goldman Sachs.

provided by the GDP warrants attached to every bond in the debt exchange recently undertaken by Argentina (early 2005).

As mentioned above, growth-indexed bonds offer three main advantages over plain-vanilla bonds. First, lower likelihood of default and costly debt crises. Second, less pro-cyclical fiscal policy—an advantage of relevance also for advanced countries, especially those with fiscal rules or limits on the overall deficit. Third, greater international risk sharing: when growth in the issuing country is rapid, international investors obtain a portion of the benefits, but when growth is slow international investors take part in the loss.

B. Potential Obstacles

Despite their long intellectual history and potential advantages, growth-indexed bonds have been issued to date in limited quantities, suggesting that there may be a number of serious obstacles to their introduction. Some obstacles are common to all financial innovation: these may be summarized under the umbrella of externalities and coordination problems, and the need to ensure "critical mass" for the potential new instruments (Allen and Gale, 1994). Individual issuers (or individual investors) need to incur a number of costs to write a new type of contract, explain its features to market participants, price it, and so on. They are also often concerned about uncertainties related to the novelty of the instruments, especially the possibility that there will be no active secondary market and thus no way of selling the instruments to other market participants should it become desirable to do so. For individuals, it may not be worth incurring these costs (and risks), though it is possible that starting a new market would be desirable for all market participants jointly. In addition, there may also be political-economy factors underlying governments' reluctance to issue new instruments: governments with limited horizons may be unwilling to pay even a small premium for an insurance whose benefits would largely accrue to a successor government.

Other potential obstacles may be related more specifically to growth-indexed bonds issued by sovereign governments. It has been argued (typically by economists, though seldom by participants in financial markets) that growth-indexed bonds might reduce the government's incentives to pursue growth-enhancing policies. More plausibly, it has also been argued that the issuing country might intentionally underestimate growth in an attempt to reduce its payments to foreign investors.

In our opinion, none of these obstacles are insurmountable. (A more thorough analysis of obstacles and proposed solutions is in Borensztein and Mauro, 2004; and Borensztein and others, 2004.) Take, for example, the most serious potential concern—namely, that the country would intentionally underestimate growth. Five considerations suggest that this concern should not be overplayed. First, consider the political incentives: it is high growth, not low growth that gets governments reelected.[10] Second, a substantial share of the bonds issued by an emerging market are often held by its residents. This would constitute a powerful domestic lobby opposed to underestimating growth. Third, markets adapt to and survive episodes of crass misreporting, as shown by recent accounting scandals on the U.S. stock market and elsewhere. In the context of emerging market bonds, a revealing episode relates to the case of inflation-indexed bonds in Brazil, where in the 1970s and 1980s the government repeatedly altered the official inflation indices. The distrust this engendered led the private sector to set up an alternative inflation measure tracked by an independent foundation, which was later used to compute the payments arising from the inflation indexed securities issued by the government. Fourth, it is not clear by how much countries could cheat, repeatedly, over the long duration of the bonds, without harming the country's reputation in international markets and ability to issue in the future, thus having greater financial costs than the immediate savings on growth-indexed bonds outstanding. Fifth, a number of policy measures could help ensure the reliability of countries' official statistics: greater independence for the statistical agencies; outside monitoring of data quality in accordance with international standards.

In a systematic survey and more informal contacts with a wide range of international investors (hedge funds, dedicated emerging market bond investors, etc.), the obstacles that were mentioned most frequently related to the verifiability of the GDP data, the need for liquidity on the secondary markets and, somewhat to our surprise, the difficulty in pricing growth-indexed bonds. (The full results are reported in the Appendix to Borensztein and others, 2004.)

Of course, difficulties in pricing might stem from two sources. The first is the sheer novelty of the instrument, uncertainties about its future liquidity, possible concerns about the integrity of the growth data, and so on. The second is the analytical difficulty in pricing an instrument that is somewhat more complicated than a plain-vanilla bond. Both interviews with market participants and the phrasing of the survey used to solicit market participants' views suggest to us that the second factor, namely the analytical complexity in pricing growth-indexed bonds, is a significant factor weighing on potential investors' minds.

This may be surprising, given that financial market participants routinely price (whether explicitly or implicitly) instruments that are far more complex than growth-indexed bonds. Nevertheless, we learned that the investor community is somewhat segmented and that specialists in bonds favor simple, easy to price instruments, and that a generally accepted pricing method or formula would help improve market acceptance of new instruments. We thus accept market participants' views at face value and develop a simple method to price growth-indexed bonds.

[10] Note also that mismeasurement concerns would apply even more strongly to inflation-indexed bonds, where the political and financial incentives are aligned: underestimating inflation would both yield political benefits and reduce the interest burden. Nevertheless, inflation-indexed bonds are widely used in many countries.

C. Is a Pricing Model Really Required, and Who Should Develop it?

Before moving to the technical section which presents our proposed method to price the bonds, we briefly raise three, perhaps paradoxical, questions. First, is a pricing model really needed for a financial market to operate? Second, should academics or practitioners develop such a model? And third, should one develop the model before the concrete instruments are issued, or will the market develop the model just at the time when the instruments are issued?

1. *Is a pricing model needed for a financial market to operate?* It seems clear that no formal pricing model, and certainly not a generally accepted model, is a prerequisite for a financial market to operate. Joseph de la Vega's (1688) aptly titled *Confusion de Confusiones* provides a colorful description of a thriving market for not only stocks and futures, but also options (calls, puts, and straddles) in Amsterdam in the 17th century, well before Luis Bachelier's 1900 thesis addressed Brownian motion and option pricing, and certainly before the Black-Scholes formula became part of standard software.[11] Nor is there a universally accepted pricing formula for commonly traded, less exotic assets such as stocks, real estate or, even more relevant for our purposes, plain-vanilla sovereign bonds. This said, of course, we agree that a standard pricing model or methodology could certainly help start up a market.

2. *Should academics or practitioners develop a pricing model?* Certainly financial market participants are best placed and have the necessary expertise to price complicated instruments. Nevertheless, there are many instances in which this fails to happen, especially for instruments that do not yet exist. The most famous and brilliant example of a new pricing formula for a financial asset—the Black and Scholes formula for option pricing—was developed by academics rather than financial market participants. Moreover, this happened after options had been in existence for a very long time, and helped foster a major increase in the size of the market.

3. *When should a pricing model be developed?* It is interesting to note that while we were (slowly) writing this paper, a number of financial market participants (presumably with greater and more immediate financial incentives) rapidly developed a method to price the Argentine GDP warrants, as soon as it became clear that such warrants would be included in the Argentine debt restructuring.[12] This shows that when a new financial instrument becomes available, financial market participants quickly strive to develop a systematic way of coming up with a price at which they feel confident trading the instrument. At the same time, improvements in pricing techniques would arguably still be needed and might help increase investor interest in the instruments.

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[11] de la Vega lived in Amsterdam but wrote in Spanish as was customary for a Sephardic Jew at the time.

[12] See, for example, Deutsche Bank's "calculator" on their web site.

Despite the recent development of pricing methods for the specific case of the Argentine GDP warrants, the method we present below may have significant value added. Indeed, there are a number of differences between our own pricing method and those applied to the Argentine warrants. The two most notable are the following. First, we explicitly model the conditions for a default, whereas the models related to Argentina only incorporate default considerations indirectly through spreads, without addressing endogenous changes in default probabilities. Second, and related, we trace the implications of the introduction of growth–indexed bonds for the pricing of plain-vanilla bonds.

IV. PRICING GROWTH-INDEXED BONDS

This section presents a simple approach to pricing growth-indexed and plain-vanilla bonds, based on simple simulations of the debt dynamics and resulting payoffs over a ten year horizon. Monte Carlo simulations for the debt dynamics have been used in previous studies for different purposes (for example, IMF 2003; Garcia and Rigobon, 2004; Mendoza and Oviedo, 2004). One way in which we depart from standard practice adopted in previous studies is that we use such simulations to recover a default-trigger rule and then estimate the effect of different debt compositions (different shares of indexed versus non-indexed debt) on the probability distribution of defaults.[13]

Our pricing approach is illustrated for a hypothetical country, which we call Emergingland. The parameters for Emergingland are constructed based on averages for Brazil, Mexico and Turkey, the three largest emerging market borrowers not experiencing repayment difficulties in recent years. The exercise could just as easily be conducted using data from a single country. Throughout the exercise, we assume that investors are risk-neutral, and value the bonds at the expected net present value of their payment streams.

[13] The approach based on a default-trigger rule for the debt/GDP ratio is primarily for analytical simplicity, but is also broadly in the spirit of Reinhart, Rogoff, and Savastano (2003), who note a tendency for emerging markets to be "debt-intolerant" and default when their debt/GDP ratio exceeds a certain value.

A. Debt Dynamics

The country's debt dynamics evolve according to a standard equation:[14]

$$\frac{D_t}{Y_t} = \frac{D_{t-1}(\alpha(e_t/e_{t-1})(1+i^{dollar}) + (1-\alpha)(1+i^{local})}{Y_t} - pb_t$$
$$= \frac{D_{t-1}}{Y_{t-1}} \frac{(\alpha(e_t/e_{t-1})(1+i^{dollar}) + (1-\alpha)(1+i^{local})}{(1+g_t)(1+\pi)} - pb_t$$

where: D_t/Y_t is the debt to GDP ratio, α is the share of dollar debt, e_t is the nominal exchange rate (measured in terms of local currency units per dollar), i^{dollar} and i^{local} are the interest rates on the dollar and local currency debt respectively, g_t is the real GDP growth, π the change in the GDP deflator and pb_t is the primary balance as a percentage of GDP. Let ε_t denote the change in the real exchange rate, defined as:

$$1+\varepsilon_t = \frac{e_t}{e_{t-1}} \frac{(1+\pi^*)}{(1+\pi)},$$

where π^* is the change in the GDP deflator in the United States. Substituting this expression into the debt equation and rearranging terms yields:

$$\frac{D_t}{Y_t} = \frac{D_{t-1}}{Y_{t-1}} \left(\frac{1}{1+g}\right)\left(\frac{\alpha(1+\varepsilon_t)(1+i^{dollar})}{(1+\pi^*)} + \frac{(1-a)(1+i^{local})}{1+\pi}\right) + pb_t$$

For simplicity, we assume the real interest rate is the same for both dollar and local currency debt:

$$\frac{(1+i^{dollar})}{(1+\pi^*)} = \frac{(1+i^{local})}{1+\pi},$$

Finally, we now consider two types of debt: plain-vanilla debt paying a fixed interest i_{pv} and growth-indexed debt paying interest i_{ind} (both measured in dollar terms).[15] If a share θ of the debt is growth-indexed, the debt dynamics will evolve according to:

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[14] Setting α, the share of dollar debt, to zero yields a debt equation that is even more standard and familiar.

[15] Since we assume the real interest rates are the same in dollar and local currency terms, the corresponding nominal interest rates in local currency are i_{pv} and i_{ind} multiplied by $(1+\pi)/(1+\pi^*)$.

$$\frac{D_t}{Y_t} = \frac{D_{t-1}}{Y_{t-1}} * \frac{(\alpha(1+\varepsilon_t)+(1-\alpha))(\theta(1+i_{ind})+(1-\theta)(1+i_{pv}))}{(1+g_t)(1+\pi^*)} - pb_t$$

We assume the terms of the growth-indexed bond contract to be such that it pays an interest rate equal to the maximum between zero or the interest rate in the plain-vanilla debt adjusted for "excess" growth:

$$i_{ind} = \max(0, i_{pv} + g_t - \overline{g})$$

where $\overline{g}$ is the contractually specified growth rate above or below which adjustments to the indexation component are made. Note the indexation formula will never imply negative interest rates.

B. Calibration

In order to illustrate how this exercise could be applied to an actual country, the parameter values for our fictitious country, Emergingland, are calibrated using average data for Brazil, Mexico and Turkey at the time of writing (summer 2005). These countries are the largest emerging market borrowers that have not defaulted in recent years. Moreover, they have historically experienced substantial volatility, thus providing an interesting setting to study the hedging benefits of growth-indexation. For simplicity, the parameters α, θ, π^* and i_{pv} are assumed to be constant, while $(g_t, \varepsilon_t, pb_t)$ are joint-normally distributed random variables. We set the initial debt to GDP ratio to 60 percent and the share α of dollar debt to ½, which are close to the recent averages for these three countries. The baseline interest rate i_{pv} is set to 6¾ percent based on their average EMBI spreads. Future payments are discounted at the 10 year risk-free rate, set to 4.0 percent based on the 10-year U.S. yield. The U.S. GDP deflator π^* is set to 2.0 percent, close to current projections for the medium term. The joint-normal distribution of $(g_t, \varepsilon_t, pb_t)$ is based on the historical sample for these three countries in 1981-2004.[16] The expected values for the realization of g_t and pb_t are set to the historical sample averages: $E_{t-1}g_t = 3.0\%$ and $E_{t-1}pb_t = 2.1\%$. The expected real depreciation is set to zero: $E_{t-1}\varepsilon_t = 0$.[17] The covariance matrix is obtained from the sample moments after subtracting

[16] Data coverage for the primary balance begins in 1986 for Turkey and 1990 for Brazil.

[17] Although the real exchange rate depreciated on average by 1.4 percent a year in the historical sample, a zero expected real depreciation seems more reasonable going forward, owing to factors (such as those envisaged by Balassa-Samuelson reasoning) that would even point to a long-run real exchange rate appreciation for emerging markets.

the country-specific mean for each variable.[18,19] We assume the growth-indexed bond contracts specify $\overline{g}$ at 3 percent. Note that this parameter can in principle be contracted at any value. The higher $\overline{g}$ the lower the interest payments on the growth-indexed bond, which would be reflected in its price.[20]

The parameter values for the expected probability of default and the amount recovered in the event of a default can be extracted from the prices of plain-vanilla bonds observed in the current environment of no indexation. (Once estimated, the paramerters will be held constant and used in exercises where the share of indexed debt in total debt is allowed to be greater than one.) The parameters are estimated by imposing a no-arbitrage constraint that the expected return on Emergingland's plain-vanilla bond equals the return on a U.S. bond with similar duration. This condition is based on the scenario where all its debt is plain-vanilla. There are multiple pairs of the default probability and recovery rate that can satisfy this no-arbitrage constraint. We consider two scenarios, one where the recovery rate is 25 percent and one where it is 50 percent. To derive the corresponding probability of default for each scenario, we simulate 250,000 joint paths for the stochastic variables over a 10-year horizon. We assume that defaults take place as soon as the debt/GDP ratio increases beyond a trigger level along the simulated paths.[21] That trigger level is chosen so that the resulting probability distribution of defaults implies that the plain-vanilla bond will trade at par. Note that it is possible for a default to take place from an

[18] The standard deviations (in percentage points) are: σ_g=3.8, σ_ε=16.1 and σ_{pb}=3.3. The correlations are: $\rho_{g,\varepsilon}$=-0.63, $\rho_{g,pb}$=-0.34 and $\rho_{\varepsilon,pb}$=0.16.

[19] One could extend the model to allow for serial correlation in these variables, for example by computing conditional means and covariances. VAR estimates for these variables yield very noisy and implausible results (probably owing to crisis episodes in the sample which may alter some of the relationships between these variables). Since the focus of our paper is on illustrating our pricing algorithim, we chose, for simplicity, to use the unconditional means and convariance matrix. Similarly, one could consider policy response rules—for example, a potential relationship between the lagged debt ratio and the primary surplus. Again, historical relationships are difficult to estimate in a reliable manner; as a result, one might argue that investors are unlikely to take such response rules into consideration.

[20] In this paper, we focus on pricing the bonds given $\overline{g}$: estimating the level of $\overline{g}$ that would achieve the optimal level of insurance is beyond the scope of this paper.

[21] In practice, other factors—notably liquidity and rollover difficulties—also contribute to determining defaults. Such factors, especially those related to investor expectations and multiple equilibria would be far more difficult to model. Our simplifying strategy is to focus on the debt/GDP ratio as a key fundamental variable that is both observable and empirically tractable. Yet another approach would be to use models that relate default probabilities to economic fundamentals (for example, Detragiache and Spilimbergo, 2001; Manasse and Roubini, 2005). However, the predictive power of these models is limited.

initially moderate debt/GDP ratio in the previous period if strong adverse shocks occur (for example, a sufficiently large real depreciation). The resulting probabilities of default within 10 years are 27.8 percent when the recovery rate is 25 percent, and 35.9 percent when the recovery rate is 50 percent.

C. Simulation and Pricing

We now have all the necessary ingredients to move to pricing. We simulate 250,000 joint paths for the stochastic variables over a 10-year horizon. We do this for three different debt compositions: (i) virtually all debt is plain-vanilla; (ii) half the debt is plain-vanilla and half is GDP-indexed; and (iii) virtually all debt is GDP-indexed. We hold i_{pv} and the contracted i_{ind} formula constant across these scenarios (even though the scenarios with more indexed debt are less risky). The default trigger is held constant at the value used to calibrate the baseline probability of default.

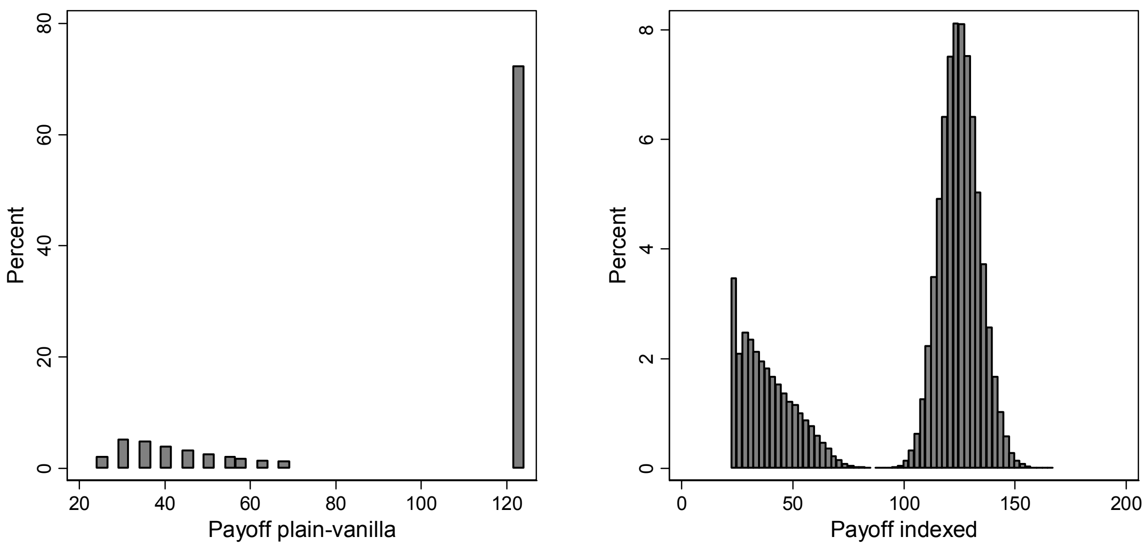
Based on the resulting distribution of defaults across these paths, we can compute the expected discounted payments on any marginal bond. We simulate the expected payoffs of a 10-year plain-vanilla bond with a coupon equal to 6¾ percent (the calibrated baseline yield) and an indexed bond whose coupon payment is:
$coupon_{ind} = \max(0, 6\tfrac{3}{4}\% + g_t - 3\%)$ and whose principal is not indexed.

Figure 1 shows the histogram of payoffs (in net present discounted value terms, using the interest rate on U.S. Treasuries as the discount rate) for a plain-vanilla bond and a growth-indexed bond on the basis of the 250,000 paths simulated in the case where all debt is plain-vanilla (except for a growth-indexed bond issue of negligible size), the recovery rate is ¼ and the probability of default is 27.8 percent.

The plain-vanilla bond pays the full amount (principal plus 10 years of coupons) 72.2 percent of the time; but only the coupons due up to the time of default (plus the recovery rate on the principal) in the 27.8 percent of cases when default occurs. For the cases of default, there are 10 discrete frequency spikes, corresponding to the payoffs for the cases of default in each of the 10 years. Defaults in the second year are more frequent than defaults in the first year, but defaults in following years become less and less frequent. Defaults in the first year are relatively rare because the economy starts far from the debt/gdp trigger level. After the second year, the frequency declines because as time goes by there are fewer and fewer adverse paths for which default has not yet occurred.

The growth-indexed bond shows a bell-shaped distribution for the non-default cases, with higher payments when growth turns out to be strong, and lower payments when growth turns out to be weak. The value depends on the growth rate also for default cases (thus providing a smoother distribution than for the plain-vanilla bond). The relative frequency of defaults again shows a declining pattern as the payoff in default increases, because higher payoffs correspond to more coupons being paid, and as years go by there are fewer and fewer "survival" cases of paths that eventually lead to a default.

Figure 1. Distribution of Payoffs for Plain-Vanilla and Growth-Indexed Bonds When Virtually all Debt is Plain-Vanilla.



Notes: The payoff is based on the present discounted value of the repayments in the scenario where only 0.0001 percent of the debt is growth-indexed, the probability of default within 10 years is 27.8 percent and the recovery rate following a default is 25 percent.

Table 3 presents the estimated value of such bonds under the different debt compositions, for the scenario where the recovery rate is ¼ and the baseline probability of default is 27.8 percent (within 10 years). Moving from no indexation to indexation of all debt reduces the default probability from 28 percent to 19 percent, that is, a reduction of the default risk by one third. This estimate is likely a lower bound because we kept the interest rates fixed, rather than letting interest rates decline as default risk falls. Moreover, we only analyzed the effects of indexation on preventing defaults: further welfare gains would result from less procyclical fiscal policies. Table 4 presents the results for the scenario where the recovery rate is 50 percent and the baseline probability of default is 35.9 percent. The higher recovery rate reduces the benefits of indexation slightly, because defaults are less costly to investors.

An alternative way to quantify the benefits of growth indexation is through the impact on borrowing costs. Tables 3 and 4 also indicate the coupon rate necessary to achieve an expected return on plain-vanilla debt similar to the one in the baseline scenario with (virtually) no indexed debt. The estimated savings are over 100 basis points. Note that the indexed bond in our example is always worth more in expectation than its plain-vanilla counterpart. This is a result of the asymmetric indexation (the coupon is bounded below at zero), which increases the average coupon payment.

Our final set of exercises is intended to dispel fears that investors would face an excessive loss on growth-indexed bonds if they estimated the growth-generating process incorrectly. Table 5 shows the effect of a reduction in the mean growth rate by one percentage point on the price of both the growth-indexed and the plain-vanilla bonds. This is to gauge the loss incurred by investors if, say, the day after purchasing the bonds expected growth were to decline by one percentage point owing to a shock such as a sharp increase in oil prices. The loss is of course greater for growth-indexed bonds than for plain-vanilla bonds, owing to the direct effect on the indexation formula. However, the higher the share of indexed debt, the less this lower growth will translate into more frequent defaults, attenuating the loss (on both plain-vanilla and growth-indexed bonds). Increasing the share of growth-indexed bonds from zero to 100 percent reduces the loss from 6.74 percent to 0.81 percent for plain-vanilla debt and from 12.42 percent to 7.09 percent for growth-indexed bonds. (In fact, the expected loss on a plain-vanilla bond when all debt is plain-vanilla is comparable to the loss on a growth-indexed bond when all the debt is growth-indexed.) Table 6 performs a similar exercise for the case where growth becomes 1½ times more volatile than initially expected. That shock actually results in a lower loss for the indexed bond than for the plain-vanilla bond (because coupon payments are bound at zero, so the average coupon is increasing in the volatility). Again, the larger the share of indexed debt in total debt, the lower the loss implied by the shock, for both plain-vanilla and growth-indexed bonds. On the whole, these exercises suggest that difficulties in forecasting future growth tend to have a lower impact on bond prices, the higher the share of indexed debt in total debt. This is because growth-indexation attenuates the effect of growth surprises on the probability of default.

Table 3. Default Rates and Expected Discounted Payoffs Assuming a
25 Percent Recovery Value and a 27.8 Percent Baseline Default Frequency
(Implicit Default Trigger is Debt/GDP > 73.2 percent)

Share of Indexed Debt in Total Debt (in percent)	.0001	50	99.9999
Default Frequency (in percent) (within 10 years)	27.8	23.0	18.9
Price of Indexed Bond with Coupon=Max(6¾ % + growth – 3 %,0)	101.3	105.2	108.4
Price of Plain-Vanilla Bond with 6 ¾ % Coupon	100.0	104.1	107.6
Coupon (in percentage points) Required for Plain-Vanilla Bond to Sell at 100	6 ¾	6 ¾ - 0.58	6 ¾ - 1.03

Table 4. Default Rates and Expected Discounted Payoffs Assuming a
50 Percent Recovery Value and a 35.9 Percent Baseline Default Frequency
(Implicit Default Trigger is Debt/GDP > 69.5 percent)

Share of Indexed Debt in Total Debt (in percent)	.0001	50	99.9999
Default Frequency (in percent) (within 10 years)	35.9	31.4	26.9
Price of Indexed Bond with Coupon=Max(6¾ % + growth – 3 %,0)	101.5	104.3	107.0
Price of Plain-Vanilla Bond with 6 ¾ % Coupon	100.0	103.0	106.0
Coupon (in percentage points) Required for Plain-Vanilla Bond to Sell at 100	6 ¾	6 ¾ - 0.48	6 ¾ - 0.90

Table 5. Default Rates and Expected Loss if Growth is on Average 1 Percent Lower than Initially Expected, Under the Assumptions and Prices of Table 3

Share of Indexed Debt in Total Debt (in percent)	.0001	50	99.9999
Default Frequency (in percent) (within 10 years)	36.27	27.79	19.98
Expected Loss on Indexed Bond with Coupon=Max(6¾ % + growth – 3 %, 0)	12.42	9.61	7.09
Expected Loss on Plain-Vanilla Bond with 6 ¾ % Coupon	6.74	3.58	0.81

Table 6. Default Rates and Expected Loss if Growth is 1½ Times as Volatile as Initially Expected, Under the Assumptions and Prices of Table 3

Share of Indexed Debt in Total Debt (in percent)	.0001	50	99.9999
Default Frequency (within 10 years) (in percent)	33.47	27.97	22.93
Expected Loss on Indexed Bond with Coupon=Max(6 ¾ % + growth – 3 %, 0)	2.65	1.84	0.94
Expected Loss on Plain-Vanilla Bond with 6 ¾ % Coupon	5.04	4.04	3.06

V. Conclusions

In this paper, we have argued that many countries, emerging market countries in particular, have debt structures that render them vulnerable to crises and prone to procyclical fiscal policies. Sound macroeconomic policies and institutions are key to improving debt structures, and financial innovation alone is no quick fix. This said, growth-indexed bonds can play a helpful role as part of a broader package to improve debt structures and thereby reduce vulnerabilities and the degree of procyclicality in fiscal policies.

Despite their desirable features and long intellectual history, growth-indexed bonds only exist on a limited scale, pointing to the presence of a number of obstacles to their introduction. Formal surveys and informal discussions with potential market participants identified a number of obstacles, none of which are, in our view, insurmountable. One of the obstacles is the difficulty of pricing growth-indexed bonds.

This paper has shown a simple method for pricing growth-indexed bonds, which relies on extracting as much information as possible from the observed prices of plain-vanilla bonds, and applying that information to the case of growth-indexed bonds through a Monte Carlo exercise. Beyond the specifics of the method used in this paper, our main objective has been to show that growth-indexed bonds are not much more difficult to price than plain-vanilla bonds. As a pleasant by-product of the analysis, we have shown the quantitative importance of externalities from growth-indexed bonds to standard bonds. Large issuance of growth-indexed bonds reduces the probability of default, thereby reducing the required coupon on standard bonds as well. It also reduces the sensitivity of bond payoffs to surprises in growth averages or volatilities by reducing the extent to which such surprises affect default probabilities.

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