

# Cyclical Implications of Changing Bank Capital Requirements in a Macroeconomic Framework

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INTERNATIONAL MONETARY FUND

## IMF Working Paper

## European Department

## Cyclical Implications of Changing Bank Capital Requirements in a Macroeconomic Framework

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## Authorized for distribution by Philip Gerson

August 2005

## Abstract

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There is a widespread view that bank capital requirements should be loosened during recessions and tightened during expansions to avoid excessive credit and output swings. This view is based on a partial analysis that ignores the effects of capital requirement policies on the saving decisions of households, and, through this channel, on bank loans and output. We present an intertemporal general equilibrium framework that accounts for such effects and evaluate the optimal responses to loan supply and productivity (loan demand) shocks. In contrast to the standard view, we show that, when loan supply is reduced, increasing the capital requirement policy. When productivity (loan demand) is reduced, lowering the capital requirement facilitates households' dissaving and amplifies the output decline, but enhances welfare. Finally, we show that if productivity reductions are anticipated—rather than unanticipated—by regulators, lowering the capital requirement preemptively enhances welfare through greater intertemporal smoothing of households' consumption and deposit holdings.

# JEL Classification Numbers: E58, E32, E44, G28

Keywords: Capital requirements, business cycles, regulation, deposit insurance.

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## I. INTRODUCTION

The view that bank regulators should loosen capital requirements during recessions and tighten them during expansions is gaining support among academic economists and policymakers, who advocate such a policy as a way to dampen credit and output swings. This view is reflected in standard critiques of the constant capital requirement policy defined in the 1988 Basel Accord, and the consultations and discussions leading to the Basel II Accord, including recent studies by Kashyap and Stein (2003 and 2004); Goodhart, Hoffmann, and Segoviano (2004); Pennacchi (2004); Borio (2003); and Danielsson and others (2001).

The main argument that supports this view goes as follows. During recessions, loan defaults cause bank capital write-offs that, in turn, force banks to raise new capital or withdraw maturing loans and accumulate cash assets, in order to satisfy the required risk-weighted asset ratio. As raising new capital is typically difficult in bad times, banks tend to satisfy the requirement through loan supply reductions, which amplify the credit crunches and the recessions. These amplification effects can be avoided by lowering the capital requirements at the beginning of recessions.

Though appealing, this argument overlooks the fact that the banking system's lending capacity is determined, to a large extent, by the households' willingness to provide savings in the form of bank deposits and equity holdings. The literature is missing an intertemporal general equilibrium framework that accounts for the effects of capital requirement policies on the consumption-saving decisions of households and, through this channel, on output. In this paper, we provide such a framework and address the following questions.<sup>2</sup> First, how should bank regulators set capital requirements in different phases of the business cycle? Second, should the policy response depend on whether the expansion or recession is triggered by loan supply or by productivity (loan demand) shocks? Third, should regulators respond differently when productivity (loan demand) shocks are anticipated rather than unanticipated?<sup>3</sup>

In regard to the first and second questions, we show that bank regulators should increase capital requirements in response to negative loan supply shocks, such as those associated with loan defaults. From a dynamic perspective, these adverse loan supply shocks reduce the economy's stock of loans and output below their steady state levels. Increasing capital requirements provides households with stronger incentives to save and allows a more rapid recovery of bank loans and aggregate output than a flat capital requirement policy. These stronger incentives to save arise because the increase in capital requirements widens the equity-deposit return spread,

<sup>&</sup>lt;sup>2</sup> The power of domestic savings to affect the banking system's lending capacity is particularly evident in a closed economy. This is why we present a closed-economy model in Section II. However, the results obtained in this paper can also be applied to open economies with imperfect capital mobility—as long as domestic and foreign savings are functioning as imperfect substitutes.

<sup>&</sup>lt;sup>3</sup> We do not study the effects of anticipated loan write-offs triggered by defaults, as in those cases, dynamic provisioning, rather than capital requirement policies, must be used.

thus reducing the households' willingness to hold deposits and consume—liquid deposits are used to pay for consumption, and thus, deposits and consumption are complementary for households.

This result contrasts, but is not incompatible, with that of the standard view. In our framework, lowering the capital requirement in response to negative loan supply shocks—as the standard view suggests—backfires, as it amplifies the credit and output contractions that would have occurred under a constant capital requirement policy. These different policy implications arise because we focus on the dynamic effects of capital requirement policies on savings and, through this channel, on bank lending, whereas the standard view focuses on how capital requirement policies can be used to prevent immediate, second-round loan supply reductions.

We follow Kashyap and Stein (2003 and 2004) and separate the partial effects of loan write-offs from those associated with productivity (loan demand) reductions, albeit both are present in most recessions.<sup>4</sup> We show that, when productivity (loan demand) is reduced, the capital requirement should be lowered. Such a response amplifies the output decline but enhances welfare by releasing deposit liquidity, thus facilitating households' dissaving during times of low productivity. These stronger incentives to dissave arise because the reduction in capital requirements narrows the equity-deposit return spread, thus increasing the households' willingness to hold deposits and consume.

In regard to the third question, we find that bank regulators should lower the capital requirement preemptively in response to an anticipated and negative productivity shock, so as to avoid the larger reduction of the requirement that would be warranted in the unanticipated case. This preemptive response enhances welfare by allowing greater intertemporal smoothing of households' consumption and deposit holdings.

These questions have attracted wide attention in the literature. Kashyap and Stein (2003) provides an excellent summary—along with new insights—and shows that the capital requirement policies of the standard view are optimal in the sense of maximizing social welfare in a one-period, stochastic model. Our approach differs from theirs in that we evaluate capital requirement policies in a dynamic general equilibrium model.<sup>5</sup> Pennacchi (2004) points out that

<sup>&</sup>lt;sup>4</sup> Even though Kashyap and Stein (2003 and 2004) assume that the supply-side effects of loan write-offs dominate those of productivity reductions in a typical recession, we can envisage particular cases in which productivity reductions are dominant. More precisely, Kashyap and Stein interpret the empirical literature on bank capital crunches—Peek and Rosengren (1995 and 1997), van den Heuvel (2002), and Bernanke and Lown (1991)—as supporting the notion that the (shadow) value of bank capital increases during recessions. However, the empirical evidence is still scanty and does not allow us to generalize across episodes and countries. Accordingly, particular cases in which productivity reduction effects dominate loan write-off effects cannot be ruled out on the basis of such evidence.

<sup>&</sup>lt;sup>5</sup> Our paper is more closely related, in its technical approach, to Edwards and Végh (1997) and Díaz-Gimenez and others (1992), in the sense that we develop a simple, but rigorous macro model that includes a meaningful role for banks but does not aim to fully "explain" the existence of banks.

Kashyap and Stein's analysis does not account for the deposit insurance losses associated with lower capital requirements. The policy of reducing capital requirements in recessions—as the standard view suggests—increases expected bank insolvencies and deposit insurance losses which must then be internalized by some agents in the economy. To avoid implicit deposit insurance subsidies, we include a self-financed and risk-based deposit insurance system in our framework.

We organize the rest of this paper as follows. In Section II, we present the "unrestricted" model, which allows for cyclical variations of capital requirements. In Section III, we present a "restricted" version of the model that constrains capital requirements to remain constant over time, as in the 1988 Basel Accord, and use it as a benchmark to evaluate the unrestricted model. Our goal in comparing the two models is to understand the macroeconomic consequences of regulators' failure to adjust the requirements over business cycles. In Section IV, we present the dynamic responses of the unrestricted and restricted economies to negative loan supply and productivity shocks, both anticipated and unanticipated. In Section V, we conclude.

## **II. UNRESTRICTED MODEL**

Consider a closed economy populated by households, firms, banks, deposit insurers, and the government. Households own the banks, consume the single storable good, and supply labor, bank capital, and deposits. Firms produce the single good using labor and bank loans.<sup>6</sup> Banks receive deposits and raise capital from households, provide loans to firms, and purchase deposit insurance from the insurers. Deposit insurers offer deposit insurance contracts to banks, collect insurance premiums, and pay back the deposits of failed banks. Finally, the government imposes full deposit insurance and capital requirements on banks.

#### A. Households

The lifetime utility of the representative household is given by

$$W = \int_{0}^{\infty} u(c_t, d_t^h) e^{-\beta t} dt , \qquad (1)$$

where  $c_t$  denotes consumption of the single good and  $d_t^h$  denotes liquid bank deposits at time *t*. We assume that the instantaneous utility function u(.) is homogeneous of degree one and strictly increasing and concave in both  $c_t$  and  $d_t^h$ , and  $\beta > 0$  is the subjective discount rate.

<sup>&</sup>lt;sup>6</sup> Readers may want to think that firms produce output using labor and physical capital and that investments in physical capital are fully financed with bank loans. For simplicity, we assume in Subsection B that bank loans enter into the firms' production function directly.

The household is endowed with one unit of labor, which is supplied inelastically in competitive labor markets. Thus, if we let  $n_t^h$  denote the household's supply of labor at time t, then  $n_t^h = 1$  for all *t*.

The household holds a portfolio of assets  $b_t^h$ , composed of bank equity (capital)  $k_t^h$ , and bank deposits  $d_t^h$ . Thus,

$$b_t^h = k_t^h + d_t^h. (2)$$

The household's flow constraint is given by

$$b_{t} = r_{t} b_{t}^{h} + w_{t} - (r_{t} - r_{t}^{d}) d_{t}^{h} - c_{t} + \Omega_{t}^{b}, \qquad (3)$$

where a dot over a variable indicates the time derivative of the variable,  $r_t$  is the real rate of return on bank equity,  $r_t^d$  is the real deposit interest rate,  $w_t$  is the real wage per unit of labor service, and  $\Omega_t^b$  denotes dividends from the banks. As we explain in Subsection B, bank equity holdings are subject to idiosyncratic risks, but such risks can be fully diversified because they are independent and the number of banks is large. Specifically, the household optimally holds equal equity positions in all banks, and thus the rate of return on the total household's equity,  $r_t$ , is riskless. The household is born at time t=0 with some (nonnegative) initial endowment of assets  $b_0^h$ .

The household's problem is to choose the paths of consumption and asset holdings  $\{c_t, k_t^h, d_t^h\}$  to maximize its lifetime utility (1) subject to constraints (2) and (3), taking as given the time paths of the rates of return, wages, and dividends  $\{r_t, r_t^d, w_t, \Omega_t^b\}$ . The current-value Hamiltonian is given by

$$H = u(c_t, d_t^h) + \lambda_t \cdot \{r_t b_t^h + w_t - (r_t - r_t^d) d_t^h - c_t + \Omega_t^b\}, \qquad (4)$$

where  $\lambda_t$  is the costate variable.<sup>7</sup> The first-order conditions and the law of motion for the costate variable are given by<sup>8</sup>

functions of the ratio  $\frac{c_t}{d_t^h}$ , as follows:  $u_c(c_t, d_t^h) = u_c(\frac{c_t}{d_t^h}, 1), u_d(c_t, d_t^h) = u_d(\frac{c_t}{d_t^h}, 1)$ .

<sup>&</sup>lt;sup>7</sup> Along the solution path of the household's problem,  $\lambda_t$  can be interpreted as the marginal value (measured in utility terms) of the household's wealth at time t.

<sup>&</sup>lt;sup>8</sup> The assumption that the instantaneous utility function is homogeneous of degree one implies that the marginal utility functions  $u_c(c_t, d_t^h)$  and  $u_d(c_t, d_t^h)$  are homogeneous of degree zero. Therefore, we can write them as

$$u_c(\frac{c_t}{d_t^h}, 1) = \lambda_t, \qquad (5)$$

$$u_d(\frac{c_t}{d_t^h}, 1) = \lambda_t \cdot (r_t - r_t^d), \qquad (6)$$

$$\dot{\lambda}_{t} = \lambda_{t} \cdot (\beta - r_{t}) . \tag{7}$$

According to (5), the household equates the marginal utility of instantaneous consumption to the marginal value of wealth at every instant t. According to (6), the household equates the marginal utility to the marginal cost of holding deposits. The latter is the marginal value of wealth multiplied by the equity-deposit spread.<sup>9</sup>

Combining (5) and (6), it is evident that the optimal consumption-deposit ratio,  $\frac{c_t}{d_t^h}$ , is uniquely determined by the equity-deposit spread,  $r_t - r_t^d$ . Assuming  $u_{d1}(.) > 0$ , which indicates complementarity between consumption and deposits, the optimal consumption-deposit ratio,  $\frac{c_t}{d_t^h}$ , is strictly increasing in the spread  $r_t - r_t^d$ .<sup>10</sup> Conditions (5) and (6) also define implicitly a deposit liquidity or 'money' demand function of the form  $d_t^h = \delta(r_t - r_t^d) \cdot c_t$ , where  $\delta(\cdot)$  is strictly decreasing in the spread  $r_t - r_t^d$ .<sup>11</sup>

inequality holds:

$$\frac{\partial(\frac{c_t}{d_t^h})}{\partial(r_t - r_t^d)} = \frac{u_c(\frac{c_t}{d_t^h}, 1)}{u_{d1}(\frac{c_t}{d_t^h}, 1) - u_{c1}(\frac{c_t}{d_t^h}, 1)(r_t - r_t^d)} > 0$$

<sup>&</sup>lt;sup>9</sup> The equity-deposit spread  $r_t - r_t^d$  is, in equilibrium, positive. Although both bank equity and deposits allow the household to store value, the former does not provide liquidity services, and therefore, the latter yields a lower return.

<sup>&</sup>lt;sup>10</sup>  $u_{c1}(\frac{c_t}{d_t^h}, 1) > 0$  from the concavity of the instantaneous utility function. If  $u_{d1}(\frac{c_t}{d_t^h}, 1) > 0$ , the following

<sup>&</sup>lt;sup>11</sup> Notice that the household increases its demand for liquid deposits—supplies more funds—when the deposit rate increases, and that the household's opportunity cost of holding liquid deposits—rather than bank equity—is given by the equity-deposit spread. In our model, we derive the deposit demand through a deposit-in-the-utilityfunction formulation rather than through a cash- or deposit-in-advance constraint because the former yields a demand that is 'elastic' with respect to deposit and equity rates.

#### **B.** Firms

Firms are indexed by *i*, produce output  $y_{it}$  by employing bank loans  $l_{it}$  and labor  $n_{it}$ , and are subject to idiosyncratic productivity shocks  $A_{it}$ . The production function is given by

$$y_{it} = A_{it} \cdot f(l_{it}, n_{it}), \qquad (8)$$

where f(.) is strictly increasing and concave in both arguments. Firm-specific productivity

shocks  $A_{it}$  are represented by two states: the high-productivity state,  $A_{it} = \overline{A}_t = \frac{A_t}{n}$ , and the

low-productivity state,  $A_{it} = 0$ , which occur with probabilities p and 1 - p, respectively. Thus, the expected productivity of any firm i is  $E(A_{it}) = A_t$ . Firms are uniformly distributed in the interval [0,1], and, by the "law of large numbers," the fraction of firms with high productivity is (ex-post) p.

Each firm *i* receives a loan from bank *i* in the amount  $l_{ii}$ , and bank *i*'s loan return is contingent on the productivity state of firm *i*. We assume that bank lending is specialized and, for simplicity, each bank *i* lends to a single firm *i*, while firm *i* only borrows from bank  $i^{12}$ . In this environment, we can interpret that firms act as banks' agents, and free entry of firms ensures that the return on bank *i*'s loan is maximized. The firm chooses the optimal amount of labor  $n_{ii}$ , conditional on the realization of the productivity shock  $A_{ii}$ , taking as given the loan  $l_{ii}$  and the market wage rate  $w_t$ . In the *high*-productivity state, the return on bank *i*'s loan is  $\overline{A_t} \cdot f(l_{ii}, n_{ii}) - w_t n_{ii}$ , and the firm's first-order condition is given by

$$w_t = A_t \cdot f_n(l_{it}, n_{it}), \qquad (9)$$

which implicitly defines firm *i*'s demand for labor as  $n_{it}^* = n^*(\overline{A}_t, l_{it}, w_t)$ . Plug (9) and  $n^*(.)$  into the objective function, and apply Euler's theorem to obtain the indirect return per unit of bank *i*'s loan, which is equal to the marginal product of loans in firm *i*, that is,  $\overline{A}_t \cdot f_l[l_{it}, n^*(\overline{A}_t, l_{it}, w_t)]$ .

In the *low*-productivity state, firm *i*'s demand for labor and the return on bank *i*'s loan are 0. Thus, the state-contingent labor demand of firm *i* and the loan return of bank *i*,  $1 + r_{it}^{l}$ , are given by

$$n_{it}^{*} = \begin{cases} n^{*}(\overline{A}_{t}, l_{it}, w_{t}) & \text{if } A_{it} = \overline{A}_{t} \\ 0 & \text{if } A_{it} = 0 \end{cases}; \quad 1 + r_{it}^{l} = \begin{cases} \overline{A}_{t} \cdot f_{l}[l_{it}, n^{*}(.)] & \text{if } A_{it} = \overline{A}_{t} \\ 0 & \text{if } A_{it} = 0 \end{cases}.$$
(10)

<sup>&</sup>lt;sup>12</sup> Our view is that this specialization arises from banks' expertise in monitoring certain industries or activities and transaction costs of diversification. This assumption allows us to introduce a meaningful deposit insurance scheme, as banks with zero revenue realizations are unable to pay back deposits.

### C. Banks

Bank *i* holds a portfolio of loans  $l_{it}$ , capital  $k_{it}$ , and deposits  $d_{it}$ , and its balance sheet satisfies  $l_{it} = k_{it} + d_{it}.$ (11)

Bank *i*'s loan and equity returns  $1 + r_{it}^{l}$  and  $1 + r_{it}$  are state contingent, whereas its deposit return  $1 + r_{t}^{d}$  is market determined and riskless, as all deposits are fully insured. Bank *i* enters into a fairly priced, full-deposit insurance contract with the insurer. According to the contract, the bank pays the insurer a premium per unit of loan  $\tau_{it} = \tau(k_{it}, l_{it}, r_{t}^{d}, p)$  in the high-revenue state, and the insurer assumes the deposit liabilities of the bank in the zero-revenue state. The premium  $\tau(.) \cdot l_{it}$ 

is decreasing in bank *i*'s capital and increasing in bank *i*'s assets, that is,  $\frac{\partial \tau(.) \cdot l_{it}}{\partial k_{it}} < 0$ ,

 $\frac{\partial \tau(.) \cdot l_{it}}{\partial l_{it}} > 0$ . Let  $\Omega_{it}^{b}$  denote bank *i*'s profits, which are paid as dividends to households and are

contingent on the state of bank *i*'s revenue, that is, on the productivity state of firm *i*. Bank *i*'s expected profit function,  $E(\Omega_{ii}^b)$ , is given by

$$E(\Omega_{it}^{b}) = p \cdot \overline{A}_{t} \cdot f_{l}[l_{it}, n^{*}(.)] \cdot l_{it} - E(1 + r_{it}) \cdot k_{it} - p \cdot (1 + r_{t}^{d}) \cdot d_{it} - p \cdot \tau(.) \cdot l_{it}.$$
 (12)

Bank *i*'s problem is to choose the stocks  $k_{it}$ ,  $l_{it}$ , and the state-contingent equity returns  $1 + r_{it}$  so as to maximize its expected profits (12), subject to its balance sheet constraint (11) and the equity-holder participation constraint,  $E(1+r_{it}) = 1+r_t$ , taking as given the rates of return  $r_t$ ,  $r_t^d$  and the wage rate  $w_t$ . Households are able to diversify away the specific risk of holding bank *i*'s capital, and the participation constraint ensures that bank *i* can raise capital as long as its expected return,  $E(1+r_{it})$ , is equal to the market-determined return,  $1+r_t$ . The first-order conditions of bank *i*'s optimization problem are given by

$$\overline{A}_{t} \cdot [f_{l} + f_{n} \cdot n_{l}^{*}] - w_{t} \cdot n_{l}^{*} = 1 + r_{t}^{d} + \frac{\partial [\tau(.) \cdot l_{it}]}{\partial l_{it}}, \qquad (13)$$

$$1 + r_t = p \cdot [1 + r_t^d - \frac{\partial [\tau(.) \cdot l_{it}]}{\partial k_{it}}].$$
(14)

Equation (13) is bank *i*'s first-order condition with respect to  $l_{ii}$ . The bank equates the expected marginal benefit and the expected marginal cost of financing new loans with deposits (the amount of bank capital remains constant). The expected marginal benefit is given by the increased production of firm *i* in the high-productivity state. In such a state, additional lending boosts production directly,  $(\overline{A}_t \cdot f_l)$ , and indirectly, by increasing the productivity of labor,  $(\overline{A}_t \cdot f_n \cdot n_l^*)$ . The latter benefit is not fully internalized by the bank because firm *i* pays a larger

wage bill  $(w_t \cdot n_t^*)$ . The expected marginal cost is the sum of the deposit return,  $1 + r_t^d$ , and the increase in the deposit insurance premium paid in the high-productivity state,  $\frac{\partial [\tau(.) \cdot l_{it}]}{\partial l_{it}}$ .

Equation (14) is the bank's first-order condition with respect to  $k_{ii}$ . The bank equates the expected marginal benefit and the expected marginal cost of substituting deposits for capital to finance its loans (the amount of loans remains constant). The expected marginal benefit is the sum of the deposit return,  $p \cdot (1+r_t^d)$ , and the reduction in the deposit insurance premium associated with a higher capital-asset ratio,  $-p \cdot \frac{\partial [\tau(.) \cdot l_{it}]}{\partial k_{it}}$ . The expected marginal cost is the expected return on equity,  $E(1+r_i)$ .

Given these conditions, bank *i*'s optimal demands for deposits  $d_{it}$  and equity  $k_{it}$ , and its loan  $l_{it}$ , can be determined as functions of  $r_t$ ,  $r_t^d$ ,  $w_t$ ,  $A_t$ , and *p*. Accordingly, we specify the solution to bank *i*'s problem as follows:

$$\vec{d}_{it}^* = d^*(r_t, r_t^d, w_t, A_t, p), \ k_{it}^* = k^*(r_t, r_t^d, w_t, A_t, p), \ l_{it}^* = l^*(r_t, r_t^d, w_t, A_t, p).$$
(15)

Similarly,  $\mathbf{x}_{it}^* = \frac{k^*(.)}{l^*(.)} = x^*(r_t, r_t^d, w_t, A_t, p)$ , where  $x_{it}$  denotes bank *i*'s capital-asset ratio. Free

entry ensures zero expected profits in the banking industry. As bank *i*'s profit is obviously zero in the zero-revenue state, it must also be zero in the high-revenue state. Therefore, we can write bank *i*'s state-contingent equity return  $1 + r_{ii}$  as follows:

$$1 + r_{it} = \begin{cases} \overline{A}_t \cdot f_l[l^*(.), n^*(\overline{A}_t, l^*(.), w_t)] - (1 + r_t^d) \cdot d^*(.) - \tau[x^*(.), r_t^d, p] \cdot l^*(.) & \text{if } A_{it} = \overline{A}_t \\ 0 & \text{if } A_{it} = 0 \end{cases} . (16)$$

#### **D.** Deposit Insurers

The representative deposit insurer collects fair insurance premiums from banks with positive revenue realizations and pays the deposits of banks with zero revenue realizations. In addition, the insurer incurs operational costs  $C(d_t, l_t)$  when bank *i* fails, where  $d_t$  and  $l_t$  are the aggregate stocks of deposits and loans in the banking system. The function C(.) is homogeneous of degree one and strictly increasing and convex in both  $d_t$  and  $l_t$ . Notice the presence of cost externalities in the insurance industry, whereby bank *i*'s insurance premium depends not only on its own

expected losses but also on those of other banks.<sup>13</sup> The insurer's zero-expected-profit condition is given by

$$p \cdot \tau_{it} \cdot l_{it} - (1-p) \cdot [d_{it} \cdot (1+r_t^d) + C(d_t, l_t)] = 0, \qquad (17)$$

where the first term,  $p \cdot \tau_{it} \cdot l_{it}$ , is the expected revenue,  $(1-p) \cdot d_{it} \cdot (1+r_t^d)$  is the expected payout to depositors, and  $(1-p) \cdot C(d_t, l_t)$  is the expected operational cost. We can write the function

C(.) as follows: 
$$C(d_t, l_t) = l_t \cdot c(x_t)$$
, where  $c(x_t) = C(\frac{d_t}{l_t}, 1)$  and satisfies  $c'(.) < 0$ ,  $c''(.) > 0$ .<sup>14</sup>

#### E. Government

The cost externalities in the insurance industry imply that government intervention aimed at forcing banks and deposit insurers to internalize the external effects of their decisions can improve upon the decentralized, free market equilibrium. Specifically, in the absence of government intervention, banks have an incentive to hold less capital per unit of asset than is socially optimal. Equation (17) implies that bank *i*'s marginal insurance costs do not include the external effects, and are given by

$$\frac{\partial[\tau(.)\cdot l_{it}]}{\partial l_{it}} = (\frac{1-p}{p})\cdot(1+r_t^d), \quad \frac{\partial[\tau(.)\cdot l_{it}]}{\partial k_{it}} = -(\frac{1-p}{p})\cdot(1+r_t^d). \tag{18}$$

By contrast, the inclusion of the external effects yields the following marginal insurance costs:<sup>15</sup>  $\frac{\partial [\tau(.) \cdot l_{it}]}{\partial l_{it}} = (\frac{1-p}{p}) \cdot [1+r_t^d + c(x_{it}) - c'(x_{it}) \cdot x_{it}], \quad \frac{\partial [\tau(.) \cdot l_{it}]}{\partial k_{it}} = -(\frac{1-p}{p}) \cdot [1+r_t^d - c'(x_{it})]. \quad (19)$ 

<sup>&</sup>lt;sup>13</sup> Our view is that the function C(.) represents the costs of verifying that the loans of a failed bank are indeed in a state of default and assessing their residual values, as well as the administrative costs of dealing with depositors. According to (17), we assume that individual insurers perceive such costs as "fixed" and independent of the insured bank's balance sheet. However, these costs are increasing in the expected insurers' losses vis-á-vis the whole banking system, due to industry-specific factors that are in high demand and short supply at times of systemic stress (such as bank auditors). Notice that the aggregate payout from insurers to depositors per unit of loan,  $(1 - x_t) \cdot (1 + r_t^d) + c(x_t)$ , increases as the aggregate bank capital-asset ratio decreases. The banking literature typically justifies the imposition of bank capital requirements on the basis of cross-bank externalities throughout the payments system (see Berger, Herring, and Szego (1995)). We assume for convenience, given our framework, that cross-bank externalities are imposed through the deposit insurance system.

<sup>&</sup>lt;sup>14</sup> These properties of c(.) are obtained directly from the properties of C(.).

<sup>&</sup>lt;sup>15</sup> We are assuming at this point that all banks are equal, which is indeed the case in the equilibrium, as we show in Subsection F.

The exclusion of the external effects understates both the costs of increasing loans and the benefits of increasing capital. Therefore, the banking equilibrium without government intervention implies lower than socially optimal capital-asset ratios.

The first-best equilibrium could be attained through a system of taxes and lump-sum transfers, so that the government collects zero net revenue. Alternatively, the government could impose capital requirements on individual banks. To do so, the government solves bank *i*'s optimization conditions (13) and (14) using the marginal insurance cost functions, which include the external effects (19). The resulting capital-asset ratio,  $x_{ii}$ , is the minimum capital-asset ratio requirement that must be imposed on bank *i*.<sup>16</sup> Henceforth, we assume that the government is using bank capital requirements to eliminate cross-bank externalities, and refer to the minimum required capital-asset ratio simply as "the capital requirement."

#### F. Equilibrium Conditions

Let  $d_t^b = \int_0^1 d_{it}^* \cdot di = d^*(.)$  and  $k_t^b = \int_0^1 k_{it}^* \cdot di = k^*(.)$  be the aggregate demands for deposits and equity from the banking system. Let  $l_t = \int_0^1 l_{it}^* \cdot di = l^*(.)$  be the aggregate stock of loans, and  $n_t^f = \int_0^1 n_{it}^* \cdot di = p \cdot n^*[\overline{A}_t, l^*(.), w_t]$  the aggregate demand for labor in the economy. An equilibrium in this economy satisfies the following market-clearing conditions:

$$d_t^h = d_t^b = d_t , \qquad (20)$$

$$k_t^h = k_t^b = k_t , \qquad (21)$$

$$n_t^h = n_t^f = n_t = 1. (22)$$

Notice that (2), (11), (15), (20), and (21) imply that, in equilibrium, the aggregate stock of household's assets  $b_t^h$  is equal to the aggregate stock of bank loans  $l_t$ . Hence,  $\lambda_t$  is, in equilibrium, the economy's "shadow value" of loans, that is, the marginal value (in the representative household's utility terms) of the aggregate stock of loans at time *t*.

<sup>16</sup> The insurance premium per unit of loan that incorporates the external effects

 $\tau_{it} = \tau(x_{it}, r_t^d, p) = -(\frac{1-p}{p}) \cdot [(1-x_{it}) \cdot (1+r_t^d) + c(x_{it})]$  is a decreasing and convex function of bank i's

$$\frac{\partial \tau_{it}}{\partial x_{it}} = -(\frac{1-p}{p}) \cdot [1+r_t^d - c'(x_{it})] < 0, \quad \frac{\partial^2 \tau_{it}}{\partial x_{it}^2} = (\frac{1-p}{p}) \cdot c''(x_{it}) > 0,$$
$$\frac{\partial \tau_{it}}{\partial p} = -\frac{1}{p^2} \cdot [(1-x_{it}) \cdot (1+r_t^d) + c(x_{it})] < 0, \quad \frac{\partial^2 \tau_{it}}{\partial p^2} = -\frac{2}{p} \cdot \frac{\partial \tau_{it}}{\partial p} > 0.$$

capital-asset ratio,  $x_{it}$ , and the probability of a high-revenue state *p*. Specifically, the first- and second-order derivatives of  $\tau(.)$  are given by

The equilibrium must also satisfy the condition that the household allocates bank capital evenly across banks to fully eliminate bank capital risk, that is,  $k_t^h = k_{it}$ . It also follows from (15) and (17) that all banks pay the same insurance premium per unit of loan,  $\tau_t = \tau_{it}$ .

Condition (22) implies that the labor employed in a high-productivity firm is  $n^*(\overline{A}_t, l_t, w_t) = \frac{1}{p}$ , which implicitly defines the wage rate in terms of  $l_t$ ,  $A_t$ , and p. Condition (22) also implies

$$n_l^*(\bar{A}_l, l_l, w_l) = 0.$$
 (23)

Plug the insurer's zero-expected-profit condition (17) and the equilibrium conditions into bank *i*'s expected profit function (12), and integrate profits over all banks to obtain the banking system's profit,  $\Omega_t^b$ , which is certain by the law of large numbers and equal to the expected profit of each bank *i*:

$$\Omega_t^b = E(\Omega_{it}^b) = A_t \cdot f(l_t, \frac{1}{p}) - w_t - (1 + r_t) \cdot k_t - (1 + r_t^d) \cdot d_t - (1 - p) \cdot c(x_t) \cdot l_t.$$
(24)

The aggregate flow constraint is obtained from the flow constraint of the representative household, (3), the aggregate bank profit function, (24), and the equilibrium conditions, (20)–(22):

$$\overset{\bullet}{b}_{t}^{h} = \overset{\bullet}{l}_{t} = A_{t} \cdot \tilde{f}(l_{t}, p) - c_{t} - \xi(x_{t}, p) \cdot l_{t}, \qquad (25)$$

where  $\tilde{f}(l_t, p) = f(l_t, \frac{1}{p})$  and  $\xi(x_t, p) = 1 + (1-p) \cdot c(x_t)$ . The function  $\xi(.)$  is decreasing and convex in  $x_t: \xi_x = (1-p) \cdot c'(x_t) < 0$ ,  $\xi_{xx} = (1-p) \cdot c''(x_t) > 0$ . According to (25), the economy's instantaneous saving flow is given by the output minus the household's consumption and minus the operational cost of the deposit insurance industry.<sup>17</sup>

#### G. Solution

Plug (19), (23), and the equilibrium conditions into bank i's first-order conditions (13) and (14) to write them as follows:

$$E(1+r_{it}^{l}) = A_{t} \cdot \tilde{f}_{l}(l_{t},p) = r_{t}^{d} + \xi(x_{t},p) - x_{t} \cdot \xi_{x}(x_{t},p), \qquad (26)$$

$$E(r_{it}) - r_t^d = r_t - r_t^d = -\xi_x(x_t, p).$$
(27)

<sup>&</sup>lt;sup>17</sup> A bank's insurance premium embeds two components: one corresponds to transfers that are received by depositors (households), and the other corresponds to real operational costs. Only the latter are social costs and thus are reflected in the aggregate flow constraint (25).

Notice that the loan-deposit and equity-deposit spreads,  $r_t^l - r_t^d$  and  $r_t - r_t^d$ , are decreasing in the capital-asset ratio  $x_t$ , where  $r_t^l = E(r_{it}^l)$ .<sup>18</sup> This implies that, in equilibrium, an increase (decrease) in  $x_t$  can only be associated with narrower (wider) spreads.

From equations (5) and (20) we can solve for the ratio  $\frac{c_t}{d_t}$  as a function of  $\lambda_t$ . Denote this

function  $z(\lambda_t)$ , as follows:

$$\frac{c_t}{d_t} = z(\lambda_t), \qquad (28)$$

where  $z'(\lambda_t) < 0$ .<sup>19</sup> Combine equations (6) and (27), and impose (28) and the equilibrium condition (20) to obtain

$$u_d[z(\lambda_t), 1] = -\lambda_t \cdot \xi_x(x_t, p), \qquad (29)$$

which implicitly defines the capital requirement  $x_i$  in terms of  $\lambda_i$ . Denote this function  $\chi(\lambda_i, p)$ , as follows:

$$x_t = \chi(\lambda_t, p) , \qquad (30)$$

where  $\chi_{\lambda}(\lambda_t, p) > 0$ .<sup>20</sup> From equations (26), (27), (29), and (30), we can express  $r_t$  in terms of  $\lambda_t$ ,  $l_t$ , and  $A_t$ :

$$r_t = A_t \cdot \tilde{f}_l(l_t, p) - \xi[\chi(\lambda_t, p), p] + [1 - \chi(\lambda_t, p)] \cdot \frac{u_d[z(\lambda_t), 1]}{\lambda_t} .$$
(31)

<sup>18</sup> From (13) and (14), the derivatives with respect to  $x_i$  are the following:

$$\frac{\partial [r_t^l - r_t^d]}{\partial x_t} = \frac{\partial [\xi(x_t, p) - 1 - \xi_x(x_t, p) \cdot x_t]}{\partial x_t} = -x_t \cdot \xi_{xx}(x_t, p) < 0$$
$$\frac{\partial [r_t - r_t^d]}{\partial x_t} = \frac{\partial [-\xi_x(x_t, p)]}{\partial x_t} = -\xi_{xx}(x_t, p) < 0.$$

<sup>19</sup> 
$$z'(\lambda_t) = \frac{d_t}{\partial \lambda_t} = \frac{1}{u_{c1}[z(\lambda_t, 1)]} < 0.$$
  
<sup>20</sup>  $\chi_{\lambda}(\lambda_t, p) = \frac{u_{d1}[z(\lambda_t), 1] \cdot z'(\lambda_t) + \xi_x(x_t, p)}{-\lambda_t \cdot \xi_{xx}(x_t, p)} > 0.$ 

 $\partial \underline{c_t}$ 

We can express  $c_t$  in terms of  $\lambda_t$  and  $l_t$ :  $c_t = z(\lambda_t) \cdot d_t$  and  $d_t = (1 - x_t) \cdot l_t$ . Thus,

$$c_t = z(\lambda_t) \cdot [1 - \chi(\lambda_t, p)] \cdot l_t .$$
(32)

The two equations that characterize the dynamic equilibrium behavior of this economy for any initial aggregate stock of assets  $b_0^h = l_0$  can be expressed in terms of  $\lambda_t$ ,  $l_t$ , and  $A_t$ , and are the following:

$$\lambda_t = \lambda_t \cdot \{\beta - A_t \cdot \tilde{f}_l(l_t, p) + \xi[\chi(\lambda_t, p), p]\} - [1 - \chi(\lambda_t, p)] \cdot u_d[z(\lambda_t), 1], \qquad (33)$$

$$\tilde{l}_{t} = A_{t} \cdot \tilde{f}(l_{t}, p) - \xi[\chi(\lambda_{t}, p), p] \cdot l_{t} - z(\lambda_{t}) \cdot [1 - \chi(\lambda_{t}, p)] \cdot l_{t} .$$
(34)

We obtain the first differential equation (33) by plugging (31) into (7), and the second differential equation (34) by plugging (30) and (32) into (25).

Consider a constant path of the productivity parameter  $A_t = A$ . Now, (33) and (34) form a system of differential equations in  $\lambda_t$  and  $l_t$ . Let  $(\lambda^*, l^*)$  denote the steady state values of  $\lambda_t$  and  $l_t$ . Such steady state values are implicitly defined by the following equations:

$$A \cdot \tilde{f}_l(l^*, p) = \beta + \xi[\chi(\lambda^*, p), p] - [1 - \chi(\lambda^*, p)] \cdot \frac{u_d[z(\lambda^*), 1]}{\lambda^*} , \qquad (35)$$

$$\frac{A \cdot f(l^*, p)}{l^*} = \xi[\chi(\lambda^*, p), p] + z(\lambda^*) \cdot [1 - \chi(\lambda^*, p)] .$$
(36)

The system defined by (33) and (34) for a constant productivity path  $A_t = A$ , when linearized around the steady state  $(\lambda^*, l^*)$ , exhibits saddle-path stability. Appendix I shows the dynamic properties of the system, and Figure 1 shows the corresponding phase diagram.

Along a perfect foresight equilibrium path with constant productivity ( $A_t = A$ ), and for any arbitrary initial level of bank loans  $l_0$ , Figure 1 shows how the economy determines the initial value of  $\lambda_t$  at the corresponding point on the saddle path ( $SP^U$ ). Then, the economy travels over time along the saddle path until it converges to the steady state 1. Notice that in this model  $\lambda_t$  is a jumping variable, whereas  $l_t$  is predetermined.<sup>21</sup>

<sup>&</sup>lt;sup>21</sup> We justify our modeling of  $l_t$  as a nonjumping variable as follows. Typically, banks hold liquid assets as well as long-maturity loans, which, to a large extent, cannot be liquidated or extended further immediately after the realization of shocks. Thus, we interpret that, at every instant, the stock of bank loans is predetermined. This interpretation, in turn, allows us to simplify our analysis by ignoring the liquid assets that banks typically hold.

#### III. A RESTRICTED CASE: EXOGENOUS AND CONSTANT CAPITAL REQUIREMENTS

In this section, we consider a constrained version of the economy analyzed in Section II, in which the government sets the capital requirement at some arbitrary level  $\overline{x} \in (0,1)$ , that is,  $x_t = \overline{x}$  for all  $t \ge 0$ . In this version, therefore, the government does not allow cyclical variations of the capital requirement, and thus, its equilibrium solution may be suboptimal. Our goal is to understand how the government's failure to adjust the capital requirement over the business cycle affects the performance of the economy.

The differential equation system in  $\lambda_i$ ,  $l_i$ , and  $A_i$  that describes the dynamic equilibrium behavior of this economy is given by

$$\lambda_{t} = \lambda_{t} \cdot [\beta - A_{t} \cdot \tilde{f}_{l}(l_{t}, p) + \xi(\overline{x}, p)] - (1 - \overline{x}) \cdot u_{d}[z(\lambda_{t}), 1] , \qquad (37)$$

$$\hat{l}_{t} = A_{t} \cdot \tilde{f}(l_{t}, p) - z(\lambda_{t}) \cdot (1 - \overline{x}) \cdot l_{t} - \xi(\overline{x}, p) \cdot l_{t} .$$
(38)

For a constant productivity path  $A_t = A$ , (37) and (38) form a system of differential equations in  $\lambda_t$ ,  $l_t$ . Let  $(\overline{\lambda}, \overline{l})$  denote the steady state of the system, which is implicitly defined by

$$A \cdot \tilde{f}_l(\bar{l}, p) = \beta + \xi(\bar{x}, p) - \frac{(1 - \bar{x}) \cdot u_d[z(\lambda), 1]}{\bar{\lambda}} , \qquad (39)$$

$$\frac{A \cdot \tilde{f}(\bar{l}, p)}{\bar{l}} = z(\bar{\lambda}) \cdot (1 - \bar{x}) + \xi(\bar{x}, p) .$$
(40)

The system defined by (37) and (38), when linearized around  $(\overline{\lambda}, \overline{l})$ , displays saddle-path stability, as shown in Appendix II. For the sake of comparing equilibrium trajectories of the restricted and unrestricted models meaningfully, and to sharpen our focus on cyclicality issues, we henceforth assume that the initial steady state's capital requirement satisfies  $\overline{x} = x^* = \chi(\lambda^*, p)$ . Figure 1 shows the corresponding phase diagram and the saddle path of the restricted economy  $(SP^R)$ . Appendix III also proves that the restricted economy's saddle path is steeper than the unrestricted economy's saddle path.

## IV. CYCLICAL IMPLICATIONS OF CHANGING CAPITAL REQUIREMENTS

In this section, we study the dynamic response of the unrestricted and restricted economies to productivity and loan supply shocks, and in this context draw lessons for bank capital requirement policies.

## A. Unanticipated and Permanent Reductions in Productivity

Suppose that the economy is initially at steady state 1 in Figure 2. Consider an unanticipated and permanent reduction in productivity  $A_t$  at t = 0, from the initial level  $A_1$  to the new level  $A_2$  ( $A_2 < A_1$ ). Steady state 2 corresponds to the new permanent value of productivity.<sup>22</sup>

In the unrestricted economy, the marginal value of loans  $\lambda_t$  jumps down immediately after the shock and then travels along the saddle path  $SP_2^U$  until it converges to steady state 2.

Figure 3 shows the time paths of selected variables. The capital requirement  $x_t$  jumps down on impact (at t = 0) and increases over time, returning to its steady state level in the long run. The stock of bank loans  $l_t$  decreases monotonically as the economy converges to steady state 2, whereas output  $y_t$  jumps down on impact due to the discrete fall in productivity and then decreases monotonically toward its (lower) long-run level. Deposits  $d_t$ , consumption  $c_t$ , and the consumption-deposit ratio  $\frac{c_t}{d_t}$ , jump up on impact and decrease smoothly during their transitions

to lower steady state levels.

Intuitively, the dynamic response of the economy is as follows.

On impact, the decline in productivity reduces the marginal productivity of loans and the wage rate. The lower productivity of loans triggers a reduction in lending, deposit, and equity rates of return, which, in turn, induces households to dissave by increasing consumption despite their lower wage income. Households are willing to complement the increased consumption with additional deposits, and, therefore, the supply of funds in the form of deposits expands at given interest rates. This shift in deposit supply leads in equilibrium to a lower deposit rate and a wider equity-deposit spread.

Consumption increases as the household weighs the utility gain from current consumption  $(u_c)$  against the future gain associated with wealth accumulation  $(\lambda)$ , and the latter falls due to the

<sup>22</sup> In Figure 2, we assume for simplicity that the production function is Cobb-Douglas. Appendix I shows that in such a case  $\frac{\partial \lambda^*}{\partial A} = 0$ . As  $x_t = \chi(\lambda_t, p)$ , it follows that the steady state capital-asset ratio  $x^*$  does not change when the productivity parameter  $A_t$  changes. This is not always true for more general production functions.

lower deposit and equity rates. In other words, the intertemporal substitution effect caused by the lower deposit and equity rates dominates the wage income effect.

As banks cannot change their loans on impact, the discrete increase in household deposits implies an equal reduction in bank equity, from which it follows that the capital-asset ratio falls. As the equity-deposit spread widens, banks rebalance their capital structures by demanding more deposits and less capital.

*During the transition* to the final steady state, interest rates rise and output and consumption fall as loans decline.

Figures 2 and 3 also show the dynamic response of the restricted economy. The reduction in the shadow value of loans at t = 0 in the restricted economy is larger than in the unrestricted economy. Similarly, as the banking system is unable to take increased household's deposits due to the flat capital requirement, the increases in the consumption-deposit ratio and the equity-deposit spread are larger in the restricted economy. Notice also that loans and output decline more slowly in the restricted economy during low-productivity times, as shown in Appendix IV.<sup>23</sup>

The main advantages and welfare gains of the unrestricted economy vis-à-vis the restricted one arise from the following features. The unrestricted economy allows households to increase their liquidity (deposits), so as to complement the higher consumption that results from the lower value of wealth. Welfare gains arise from such high liquidity, and these gains are only partially

<sup>23</sup> Notice that the unrestricted economy converges faster to the final steady state than the restricted economy. This stems from the condition satisfied by the characteristic roots  $\theta_1^U < \theta_1^R < 0$ , and the dynamic equilibrium

Notice also that the jump in consumption on impact in the unrestricted model is larger than in the restricted model in Figure 3. A sufficiently strong complementary between consumption and deposits in household utility guarantees this result. Intuitively, if consumption and deposits were almost perfect complements, the jump in consumption in the restricted model would be negligible, as deposits cannot jump on impact. In contrast, consumption would jump substantially in the unrestricted model. Analytically, these results follow from the fact that, as the cross derivative  $u_{d1}$  increases, the sensitivity of the consumption-deposit ratio with respect to changes in  $\lambda_t$ ,  $\chi_\lambda(.)$ , is not affected, whereas the sensitivity of the capital requirement with respect to changes in  $\lambda_t$ ,  $\chi_\lambda(.)$ , increases.

Finally, notice that the jump in the equity-deposit spread on impact is larger in the restricted model.

Analytically, this follows from  $\frac{\partial(\frac{c_t}{d_t^h})}{\partial(r_t - r_t^d)} > 0$ , and from the restricted model's larger jump in the consumptiondeposit ratio.

equations for  $l_t$  (see Appendixes III and IV).

offset by the rising operational costs of the deposit insurance industry. Thus, in balance, lowering the capital requirement improves welfare.

In sum, these results show that capital requirements should be lowered in response to negative and permanent productivity shocks. Contrary to conventional views, such a response, although it amplifies the output decline, enhances welfare because it releases liquidity that complements higher household consumption.

# B. Unanticipated Reductions in Loan Supply

The unrestricted economy is initially at the steady state shown in Figure 4. At t = 0, an unanticipated and negative loan supply shock, possibly reflecting loan write-offs, reduces the household's assets and the stock of loans from  $l_0$  to  $l_{0+}$  ( $l_0 > l_{0+}$ ). The shadow value of loans  $\lambda_t$  jumps up to the corresponding equilibrium path, declines along the saddle path  $SP^U$ , and ends in the steady state.

Figure 5 shows the time paths of selected variables. The capital requirement  $x_t$ , jumps up on impact and declines over time, whereas the amount of bank loans,  $l_t$ , increases motonically after the initial shock as the economy returns to the steady state. Output  $y_t$  jumps down on impact due to the discrete fall in loans and then rises monotonically toward its long-run level as the stock of loans is restored. Deposits and consumption  $d_t$  and  $c_t$ , as well as the consumption-deposit ratio

 $\frac{c_t}{d_t}$  and the equity-deposit spread  $r_t - r_t^d$ , jump down on impact and then increase smoothly

during the transition to the steady state.

Intuitively, the adjustment process is as follows.

*On impact*, as loans and output decline, the marginal productivity of loans and the lending, deposit, and bank equity rates increase. The spike in rates of return and the lower wage income induce households to reduce consumption and deposits. The smaller deposit supply increases in equilibrium the deposit rate and narrows the equity-deposit spread. This decline in the return of bank equity relative to deposits induces, in turn, banks to finance their loans with more equity relative to deposits, thereby raising the capital-asset ratio.

*During the transition*, the stock of loans is gradually rebuilt, output rises, and interest rates and the capital requirement decline. Accordingly, consumption and deposits increase while the economy returns to the steady state.

In the restricted case, the impact response of the shadow value of loans is larger than in the unrestricted case. As banks are not allowed to change the composition of their liabilities, they reduce deposits and equity in equal proportions, and, therefore, deposits decline less than in the unrestricted case. The impact decline in the equity-deposit spread is larger in the restricted case,

reflecting the less stringent capital requirements, and thus, the weaker demand for capital from banks.

However, as capital requirements are lower in the restricted economy than in the unrestricted, deposit insurance premiums and the aggregate operational costs of insurers are higher during the transition, and, hence, the set of intertemporal consumption possibilities is smaller.

The welfare gains in the unrestricted economy stem from the more rapid response of savings and the faster restoration of the stock of loans.<sup>24</sup> The unrestricted economy allows a larger reduction in deposits, which complements consumption and thus provides households with stronger incentives to reduce consumption and save more. Finally, the higher capital requirements of the unrestricted economy boost savings further through lower insurers' operational costs.

In sum, capital requirements should be raised in response to adverse loan supply shocks. Contrary to conventional views, such a response allows credit and output to recover more rapidly, because the household's willingness to cut consumption and save is higher the easier it is to lower deposits, which is the case when capital requirements are increased.

# C. Anticipated and Permanent Reductions in Productivity

Consider an anticipated and permanent reduction in productivity  $A_t$  in the unrestricted economy, which is initially at steady state 1 in Figure 6. At t = 0, the household learns that at t = T productivity will fall from the initial level  $A_1$  to the new level  $A_2$  ( $A_2 < A_1$ ). Steady state 2 corresponds to the new permanent value of productivity.<sup>25</sup> On impact, the marginal value of loans  $\lambda_t$  jumps down while the stock of loans  $l_t$  remains constant. During the transition, the economy travels along a dynamic equilibrium trajectory that intersects the saddle path associated with steady state 2 ( $SP_2$ ) exactly at time t = T, with the stock of loans and its shadow value declining meanwhile. Once the productivity shock has been realized at t = T, the economy moves along the saddle path until it converges to steady state 2, with the stock of loans declining and its shadow value increasing over time.<sup>26</sup>

<sup>&</sup>lt;sup>24</sup> The proof that loans recover faster in the unrestricted model, i.e.  $l_{0+}^U > l_{0+}^R$ , follows the same logical steps of Appendix IV:  $\theta_1^U < \theta_1^R$ ,  $l_{0+}^U = (l_{0+} - \overline{l}) \cdot \theta_1^U > (l_{0+} - \overline{l}) \cdot \theta_1^R = l_{0+}^R$ .

<sup>&</sup>lt;sup>25</sup> As in Figure 2, we assume for simplicity that the production function is Cobb-Douglas.

<sup>&</sup>lt;sup>26</sup> The marginal utility of wealth  $\lambda_t$  cannot jump when an anticipated event, such as the productivity reduction at time t = T, is realized. Furthermore, (30) implies that the capital requirement  $x_t$  cannot jump at time t = T either.

Figure 7 shows the time paths of selected variables. The capital requirement  $x_t$  jumps down on impact and decreases over time to reach its minimum level when the productivity shock has been realized (at t = T). Thereafter, it increases and returns to its steady state level in the long run. The stock of bank loans  $l_t$  decreases motonically as the economy converges to steady state 2. Output decreases smoothly until t = T, jumps down discretely at this time due to the discrete fall in productivity, and then, once again, decreases smoothly toward its (lower) long-run level. Deposits  $d_t$  and consumption  $c_t$  jump up on impact and decrease smoothly during their transitions to lower steady state levels.

Intuitively, the dynamic response of the economy is as follows.

*On impact*, the decline in future productivity reduces the shadow value of loans leading households to dissave and to increase consumption and deposits through an intertemporal substitution effect. The wage rate is unchanged at t = 0, and thus no income effect occurs. As the deposit supply expands, a lower deposit rate and a wider equity-deposit spread bring the economy to equilibrium. Accordingly, as banks cannot change their loans on impact, the discrete increase in household deposits implies a reduction in bank equity and in the capital-asset ratio.

*During the transition*, output falls as loans decline, jumping down discretely when the productivity shock hits the economy. At this time (t = T), the wage rate, as well as lending, deposit, and equity rates, falls discretely. As the household had perfectly foreseen this shock, neither the capital requirement, the shadow value of loans, consumption, nor deposits jump. After t = T, loans, output, consumption, and deposits continue decreasing toward their lower long-run values.

Comparing the unrestricted economy's performance in response to anticipated and unanticipated shocks of identical size at t = T, Figure 7 shows that when the shocks are anticipated, the preemptive response of bank regulators and households allows for greater intertemporal smoothing of consumption and deposit holdings. If bank regulators anticipate a negative productivity shock, therefore, they should respond by lowering the capital requirement preemptively so as to avoid the larger reduction of the requirement that is called for in the unanticipated case. The smoother time paths of consumption and deposits reflect to some extent the smoother paths of the capital requirement, loans, and output.

In sum, these results show that lowering capital requirements in anticipation of future negative productivity shocks brings welfare benefits in terms of smoothing household consumption and deposits, while optimally inducing households to dissave in a forward-looking way.

# D. Unanticipated and Temporary Reductions in Productivity

Consider an unanticipated and temporary reduction in productivity  $A_t$  in the unrestricted economy, which is initially at steady state 1 in Figure 8. At time t = 0, the productivity parameter  $A_t$  decreases temporarily from  $A_1$  to  $A_2$  ( $A_2 < A_1$ ), returning to the initial level  $A_1$  at

time t = T. The temporary fall in productivity is unanticipated as of date t = 0, whereas the duration of the temporary shock is known with certainty. Steady states 1 and 2 correspond to productivity levels  $A_1$  and  $A_2$ , respectively. On impact, the economy moves down to a point such as 0+ and evolves over time so as to converge to the saddle path associated with steady state 1 ( $SP_1$ ) exactly at time t = T. The phase diagram in Figure 8 shows the equilibrium paths for short-(S) and long-(L) lasting temporary shocks. For a short-lasting temporary shock, the equilibrium trajectory intersects the saddle path  $SP_1$  at time  $t = T^S$ , not having crossed the line  $l_{t,2} = 0$ , whereas, for a long-lasting temporary shock, the equilibrium trajectory intersects  $SP_1$  at

time  $t = T^{L}$  after having crossed the line  $t_{t,2} = 0$  at some time  $T' < T^{L}$ .

Figure 9 shows the time paths of selected variables for the short-lasting case. The capital requirement  $x_t$  jumps down on impact and increases during the low-productivity times, reaching its maximum level at t = T and thereafter converging from above to its steady state value. Output  $y_t$  jumps down on impact due to the discrete fall in productivity, decreases until t = T as loans decline, jumps up at t = T owing to the discrete increase in productivity, and finally rises smoothly, following the trajectory of loans. Deposits  $d_t$  and consumption  $c_t$  jump up on impact, decrease during the low-productivity period to reach their lowest levels at t = T, and thereafter recover smoothly to return to their initial steady state levels.

Intuitively, this temporary productivity shock can be thought of as a combination of the shocks studied in Subsections A and C, that is, an unanticipated permanent decline in productivity and an anticipated and permanent increase in productivity. On impact, the shadow value of loans falls as the negative effect of the current productivity decline offsets the positive discounted effect of the future productivity increase. As time t approaches T, the positive discounted effect of the productivity reversal increases, becoming dominant before t = T. After t = T, the qualitative behavior of the economy is similar to what would be observed if a positive, unanticipated, and permanent productivity shock had occurred.

Notice that this logic implies that the reduction on impact of the capital requirement is necessarily larger when the unanticipated productivity decline is permanent rather than temporary. Furthermore, the shorter the low-productivity time interval, the smaller is the reduction of the capital requirement at  $t = 0^+$ .

The intuition for the trajectories of the remaining variables is consistent with that of the shadow value of loans. *On impact*, the lower value of loans and the lower rates of return induce households to dissave by raising consumption and deposit holdings, and the increase in the deposit supply leads to a wider equity-deposit spread. *During the transition*, as the shadow value of loans rises, households save to build up a larger stock of loans as they anticipate the future reversal of the productivity decline.

In sum, these results show that when there is an unanticipated decline in productivity for a temporary period, the capital requirement should be lowered at the beginning of the period and increased above its steady state level before the end of the period. Such a policy provides adequate incentives to households in the form of high levels of deposit liquidity when it is optimal to dissave (beginning of period), and of low levels of deposit liquidity when it is optimal to save (end of period).

## V. CONCLUSIONS

We addressed three fundamental economic and policy questions. First, how should regulators set bank capital requirements in different phases of the business cycle? In particular, should such requirements be loosened during recessions and tightened during expansions, as a growing literature suggests? Second, should the policy response depend on whether the expansion or recession is triggered by loan supply or by productivity shocks? Third, should regulators respond differently when shocks are anticipated rather than unanticipated?

Our answer to the first and second questions is the following. *Capital requirements should be tightened and deposit insurance premiums lowered when negative loan supply shocks occur, whereas capital requirements should be loosened and deposit insurance premiums increased when negative productivity shocks occur.* In the former case (negative loan supply shocks), the optimal policy compares favorably with a flat capital requirement policy because it provides households with stronger incentives to reduce consumption and save, thus allowing a more rapid recovery of bank credit and aggregate output. In the latter case (negative productivity shocks), the optimal policy amplifies the output decline but enhances welfare because it releases liquidity that complements the higher household consumption and, thus, optimally speeds up the process of dissaving during low-productivity times.

Our answer to the third question is that *regulators should lower bank capital requirements and increase deposit insurance premiums preemptively when productivity shocks are anticipated.* This response enhances welfare by allowing greater intertemporal smoothing of household consumption and deposit holdings.

Our last, but not least, important contribution is our emphasis on intertemporal welfare maximization. We believe that the excessive focus of the literature on output fluctuations is misleading. What is wrong if given bank capital standards amplify output fluctuations? We showed, for example, that lowering capital requirements in response to negative productivity shocks deepens recessions but improves welfare. Of course, we acknowledge that in most theoretical cases—and in practice—output fluctuations may lower welfare through different mechanisms. Employment fluctuations, for example, are one such mechanism absent in our framework that may further reconcile theory with popular views that output fluctuations reduce welfare. To the extent that households prefer to smooth leisure intertemporally, welfare gains will arise from dampening output and employment fluctuations.

We have taken the literature on the cyclical effects of capital requirements a step forward; however, remaining drawbacks should not be overlooked in future research. Although we did not

judge it necessary to set up a stochastic model to answer the questions that we posed, additional insights could be gained from such a model. Our framework, for example, has the limitation that the costs of banks' bankruptcies are fully borne by the banking system itself through the reserve-targeting, fairly priced deposit insurance system. It is well documented, however, that many of the banking crises in the last three decades were financed with general taxation. Hence, bank regulation and fiscal policy are intimately intertwined, and the challenge that lies ahead is to search for jointly optimal fiscal and capital requirement policies in stochastic general equilibrium environments. Let light be shed on these issues from future research, which we hope to motivate with this paper.

For policy purposes, we raise a red flag regarding the increasingly popular view that capital requirements should be relaxed during recessions. Our results suggest that policymakers should exercise great caution before implementing policies consistent with that view. Accordingly, we are less concerned than others about preliminary quantitative evaluations of the effects of Basel II, which point to more stringent effective capital requirements during downturns.

### I. UNRESTRICTED MODEL: STABILITY PROPERTIES OF THE SOLUTION

The linearization of differential equations (33) and (34) around the steady state  $(\lambda^*, l^*)$  for  $A_l = A$  yields the following Jacobian matrix  $J^*$ :

$$J^{*} = \begin{bmatrix} (\frac{\partial \lambda_{t}}{\partial \lambda_{t}})_{\lambda^{*}, l^{*}} & (\frac{\partial \lambda_{t}}{\partial l_{t}})_{\lambda^{*}, l^{*}} \\ (\frac{\partial l_{t}}{\partial \lambda_{t}})_{\lambda^{*}, l^{*}} & (\frac{\partial l_{t}}{\partial l_{t}})_{\lambda^{*}, l^{*}} \end{bmatrix}, \text{ where its elements are given by}$$

$$(\frac{\partial \lambda_{t}}{\partial \lambda_{t}})_{x^{*}, \lambda^{*}} = \beta - A \cdot \tilde{f}_{l}(l^{*}, p) + \xi[\chi(\lambda^{*}, p), p] - [1 - \chi(\lambda^{*}, p)] \cdot u_{d1}[z(\lambda^{*}), 1] \cdot z'(\lambda^{*}) > 0,^{27}$$

$$(\frac{\partial \lambda_{t}}{\partial l_{t}})_{x^{*}, \lambda^{*}} = -A \cdot \lambda^{*} \cdot \tilde{f}_{ll}(l^{*}, p) > 0,$$

$$(\frac{\partial l_{t}}{\partial \lambda_{t}})_{x^{*}, \lambda^{*}} = -z'(\lambda^{*}) \cdot l^{*} \cdot [1 - \chi(\lambda^{*}, p)] + l^{*} \cdot \chi_{\lambda}(\lambda^{*}, p) \cdot \{z(\lambda^{*}) - \xi_{x}[\chi(\lambda^{*}, p), p]\} > 0,$$

$$(\frac{\partial l_{t}}{\partial l_{t}})_{x^{*}, \lambda^{*}} = A \cdot [\tilde{f}_{l}(l^{*}, p) - \frac{\tilde{f}(l^{*}, p)}{l^{*}}] < 0.$$

The last inequality follows from the strict concavity of the function  $\tilde{f}$  (the marginal product of bank loans is lower than their average product) and the use of the steady state conditions. The previous inequalities imply that the determinant of the Jacobian matrix  $Det(J^*)$  is strictly negative. Let  $\theta_1$ ,  $\theta_2$  denote the eigenvalues of the matrix  $J^*$  that solve the quadratic equation  $\theta^2 - tr(J^*) + Det(J^*) = 0$ , where  $tr(J^*)$  denotes the trace of  $J^*$ . Because  $Det(J^*) < 0$ , the eigenvalues are of opposite signs, and thus, the solution of the dynamic system exhibits saddle-path stability.

**Comparative Statics.** We show how the steady state of the system changes when the parameter *A* changes. The steady state equations (35)–(36) can be used to solve implicitly for  $\lambda^*$  and  $l^*$  as

 $(\frac{\partial \lambda_{l}}{\partial \lambda_{l}})_{x^{*},\lambda^{*}} = \beta - A \cdot \tilde{f}_{l}(l^{*},p) + \xi[\chi(\lambda^{*},p),p] - [1 - \chi(\lambda^{*},p)] \cdot u_{d1}[z(\lambda^{*}),1] \cdot z'(\lambda^{*}) + \chi_{\lambda}(\lambda^{*},p) \cdot \{u_{d}[z(\lambda^{*}),1] + \lambda^{*} \cdot \xi_{x}[\chi(\lambda^{*},p),p]\}.$  However, the last term,  $\{u_{d}[z(\lambda^{*}),1] + \lambda^{*} \cdot \xi_{x}[\chi(\lambda^{*},p),p]\} = 0$ , from (29).

<sup>27</sup> 

functions of *A*:  $\dot{\lambda}_t = 0 = G(\lambda^*, l^*, A)$ ,  $\dot{l}_t = 0 = H(\lambda^*, l^*, A)$ . To evaluate the change of the steady state point  $\lambda^*(A)$ ,  $l^*(A)$  when *A* changes, we must solve the following system:

		$\left( \partial \lambda^* \right)$	
$G_{\lambda}$	$G_l$	$\partial A$	$\left(-G_{A}\right)$
$\Big(H_{\lambda}$	$H_l$ .	$\partial A \\ \partial l^*$	$= \left(-H_A\right)^{\prime}$
		$\left( \overline{\partial A} \right)$	

where the partial derivatives  $G_{\lambda}, G_{l}, H_{\lambda}, H_{l}, G_{A}$  and  $H_{A}$  are evaluated at the initial steady state and are given by

$$\begin{split} G_{\lambda} &= (\frac{\partial \lambda_{t}}{\partial \lambda_{t}})_{x^{*},\lambda^{*}}, \ G_{l} &= (\frac{\partial \lambda_{t}}{\partial l_{t}})_{x^{*},\lambda^{*}}, \ H_{\lambda} = (\frac{\partial l_{t}}{\partial \lambda_{t}})_{x^{*},\lambda^{*}}, \ H_{l} = (\frac{\partial l_{t}}{\partial l_{t}})_{x^{*},\lambda^{*}}, \\ G_{A} &= -\lambda^{*} \cdot \tilde{f}_{l}(l^{*},p) < 0 \ , \ \mathbf{H}_{A} = \tilde{f}(l^{*},p) > 0 \ . \end{split}$$

Let |M| denote the determinant associated with the matrix of partial derivatives of G(.) and H(.) with respect to  $\lambda$  and l. It is straightforward to verify that |M| < 0 when evaluated at the steady state. Using Cramer's rule, we obtain

$$\frac{\partial \lambda^*}{\partial A} = \frac{-G_A \cdot H_l + G_l \cdot H_A}{|M|} = \frac{\lambda^* \cdot A}{|M|} \cdot \{\tilde{f}_l(l^*, p) \cdot [\tilde{f}_l(l^*, p) - \frac{\tilde{f}(l^*, p)}{l^*}] - \tilde{f}(l^*, p) \cdot \tilde{f}_{ll}(l^*, p)\} \stackrel{>}{_{<}} 0,$$
$$\frac{\partial l^*}{\partial A} = \frac{-G_\lambda \cdot H_A + G_A \cdot H_\lambda}{|M|} > 0.$$

Notice that, if we consider the Cobb-Douglas production function  $\tilde{f}(l, p) = l^{\gamma} \cdot (\frac{1}{p})^{1-\gamma}, \ 0 < \gamma < 1,$ 

it follows that  $\frac{\partial \lambda^*}{\partial A} = 0$ .

## II. RESTRICTED MODEL: STABILITY PROPERTIES OF THE SOLUTION

We proceed as in Section A, applying the logical steps described there to the restricted model's system of differential equations (37)–(38). We linearize the system around its steady state  $(\overline{\lambda}, \overline{l})$  and obtain the Jacobian matrix  $\overline{J}$ , whose elements are given by

$$\begin{split} (\frac{\partial \lambda_{t}}{\partial \lambda_{t}})_{\bar{\lambda},\bar{l}} &= [\beta - A \cdot \tilde{f}_{l}(\bar{l},p) + \xi(\bar{x},p)] - (1 - \bar{x}) \cdot u_{d1}[z(\bar{\lambda}),1] \cdot z'(\bar{\lambda}) > 0\\ &(\frac{\partial \lambda_{t}}{\partial l_{t}})_{\bar{\lambda},\bar{l}} = -\bar{\lambda} \cdot A \cdot \tilde{f}_{ll}(\bar{l},p) > 0\,, \end{split}$$

$$\begin{aligned} & (\frac{\partial l_{t}}{\partial \lambda_{t}})_{\overline{\lambda},\overline{l}} = -z'(\overline{\lambda}) \cdot \overline{l} \cdot (1-\overline{x}) > 0 , \\ & (\frac{\partial l_{t}}{\partial l_{t}})_{\overline{\lambda},\overline{l}} = A \cdot [\tilde{f}_{l}(\overline{l},p) - \frac{\tilde{f}(\overline{l},p)}{\overline{l}}] < 0 \end{aligned}$$

The determinant of  $\overline{J}$  is strictly negative, and, therefore, the solution of the dynamic system exhibits saddle-path stability.

**Comparative Statics.** The steady state equations (39)–(40) can be used to solve implicitly for  $\overline{\lambda}$  and  $\overline{l}$  as functions of *A* and  $\overline{x}$ :  $\dot{\lambda}_t = 0 = G'(\overline{\lambda}, \overline{l}, A, \overline{x})$ ,  $\dot{l}_t = 0 = H'(\overline{\lambda}, \overline{l}, A, \overline{x})$ . The partial derivatives of *G'*(.) and *H'*(.) with respect to  $\overline{\lambda}$ ,  $\overline{l}$ , and *A*, evaluated at the steady state, are

$$\begin{split} G_{\lambda}^{'} &= (\frac{\partial \lambda_{t}}{\partial \lambda_{t}})_{\overline{\lambda},\overline{l}}, \ G_{l}^{'} &= (\frac{\partial \lambda_{t}}{\partial l_{t}})_{\overline{\lambda},\overline{l}}, \ H_{\lambda}^{'} &= (\frac{\partial l_{t}}{\partial \lambda_{t}})_{\overline{\lambda},\overline{l}}, \ H_{l}^{'} &= (\frac{\partial l_{t}}{\partial l_{t}})_{\overline{\lambda},\overline{l}}, \\ G_{A}^{'} &= -\overline{\lambda} \cdot \tilde{f}_{l}(\overline{l},p) < 0 \ , H_{A}^{'} &= \tilde{f}(\overline{l},p) > 0 \ . \end{split}$$

Let |M'| denote the determinant of the matrix of partial derivatives of G'(.) and H'(.) with respect to  $\lambda$  and l. It is easy to verify that |M'| < 0. Using Cramer's rule, we obtain

$$\frac{\partial \overline{\lambda}}{\partial A} = \frac{-G'_{A} \cdot H'_{I} + G'_{I} \cdot H'_{A}}{|M'|} = \frac{\overline{\lambda} \cdot A}{|M'|} \cdot \{\tilde{f}_{I}(\overline{l}, p) \cdot [\tilde{f}_{I}(\overline{l}, p) - \frac{\tilde{f}(\overline{l}, p)}{\overline{l}}] - \tilde{f}(\overline{l}, p) \cdot \tilde{f}_{II}(\overline{l}, p)\} \stackrel{>}{_{<}} 0,$$
$$\frac{\partial \overline{l}}{\partial A} = \frac{-G'_{\lambda} \cdot H'_{A} + G'_{A} \cdot H'_{\lambda}}{|M'|} > 0.$$

If the production function is Cobb-Douglas,  $\tilde{f}(l) = l^{\gamma} \cdot (\frac{1}{p})^{1-\gamma}$ , then  $\frac{\partial \lambda}{\partial A} = 0$ .

# III. PROOF: COMPARING THE SLOPES OF THE SADDLE PATHS FOR THE RESTRICTED AND UNRESTRICTED MODELS

We prove that the slope of the restricted model's saddle path is stepper than the slope of the unrestricted model's saddle path in a plane with  $\lambda_i$  on the vertical axis and  $l_i$  on the horizontal axis. Comparing the Jacobian matrices of the constrained and unconstrained models, we observe that the following conditions are satisfied: 1) the traces of the Jacobian matrices are equal, that is,  $tr(J^*) = tr(\overline{J})$ ; 2) the determinant of the Jacobian matrix in the constrained model is greater than the corresponding determinant in the unconstrained model, and both determinants are negative, that is,  $Det(J^*) < Det(\overline{J}) < 0$ ; 3) the first rows of both Jacobian matrices are the same.

We have saddle-path stability in both models. It follows that the negative characteristic roots  $\theta_1^U$ ,  $\theta_1^R$  satisfy  $\theta_1^U < \theta_1^R < 0$ . To find the eigenvector associated with the negative root in each model, we must solve the following system of two linearly dependent (redundant) equations:

$$(J-\theta_1\cdot I)\cdot \begin{pmatrix} e_{11}\\ e_{12} \end{pmatrix} = \begin{pmatrix} 0\\ 0 \end{pmatrix}.$$

To determine one arbitrary eigenvector, set  $e_{12} = 1$  and solve for  $e_{11}$  using the first equation (first row of  $(J - \theta_1 \cdot I)$ ). As we said above, the first rows of the two Jacobians are equal. Thus,

$$e_{11}^{U} = \frac{A \cdot \lambda^{*} \cdot \tilde{f}_{ll}(l^{*}, p)}{\beta - A \cdot \tilde{f}_{l}(l^{*}, p) + \xi[\chi(\lambda^{*}, p), p] - [1 - \chi(\lambda^{*}, p)] \cdot u_{d1}[z(\lambda^{*}), 1] \cdot z'(\lambda^{*}) - \theta_{1}^{U}} + e_{11}^{R} = \frac{A \cdot \overline{\lambda} \cdot \tilde{f}_{ll}(\overline{l}, p)}{\beta - A \cdot \tilde{f}_{l}(\overline{l}, p) + \xi[\overline{x}, p] - [1 - \overline{x}] \cdot u_{d1}[z(\overline{\lambda}), 1] \cdot z'(\overline{\lambda}) - \theta_{1}^{R}}.$$

Thus, it immediately follows that  $e_{11}^R < e_{11}^U < 0$ . The saddle-path equations are given by  $\lambda_t - \overline{\lambda} = e_{11}^U \cdot (l_t - \overline{l})$ ,  $\lambda_t - \overline{\lambda} = e_{11}^R \cdot (l_t - \overline{l})$ . These equations clearly show that the restricted model's saddle path is steeper than the unrestricted model's saddle path, when evaluated in a plane with  $\lambda_t$  on the vertical axis and  $l_t$  on the horizontal axis.

## IV. PROOF: THE STOCK OF LOANS DECREASES FASTER IN THE UNRESTRICTED MODEL THAN IN THE RESTRICTED

We show that  $l_{0+}^{U} < l_{0+}^{R} < 0$ , where 0+ is the time immediately after the shock is realized. Along the corresponding equilibrium paths, the dynamic equations for  $l_t$  are given by

$$l_t^U = \overline{l_2} + (\overline{l_0} - \overline{l_2}) \cdot e^{\theta_1^U \cdot t} , \qquad l_t^R = \overline{l_2} + (\overline{l_0} - \overline{l_2}) \cdot e^{\theta_1^R \cdot t} .$$

The corresponding time derivatives are given by

$$l_{0+}^U = (\overline{l_0} - \overline{l_2}) \cdot \theta_1^U , \qquad l_{0+}^R = (\overline{l_0} - \overline{l_2}) \cdot \theta_1^R .$$

Because  $\theta_1^U < \theta_1^R < 0$  from Section C, and  $\overline{l_0} - \overline{l_2} > 0$ , it follows that

$$\stackrel{\bullet}{l_{0+}^U} = (\overline{l_0} - \overline{l_2}) \cdot \theta_1^U < (\overline{l_0} - \overline{l_2}) \cdot \theta_1^R = l_{0+}^R$$

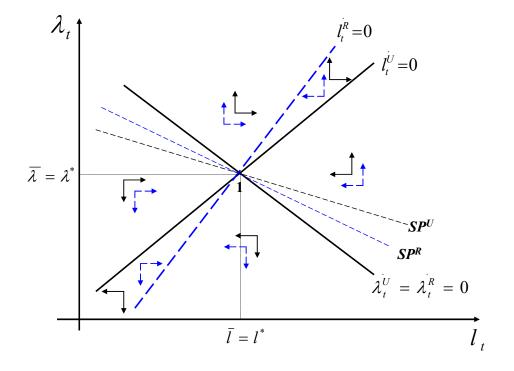
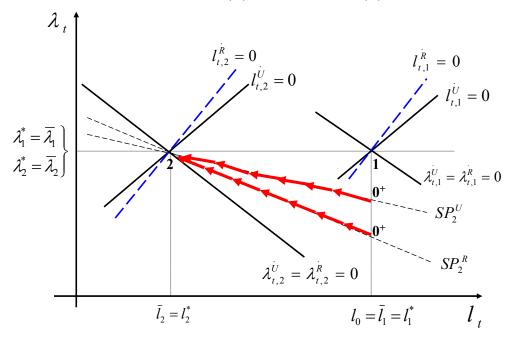
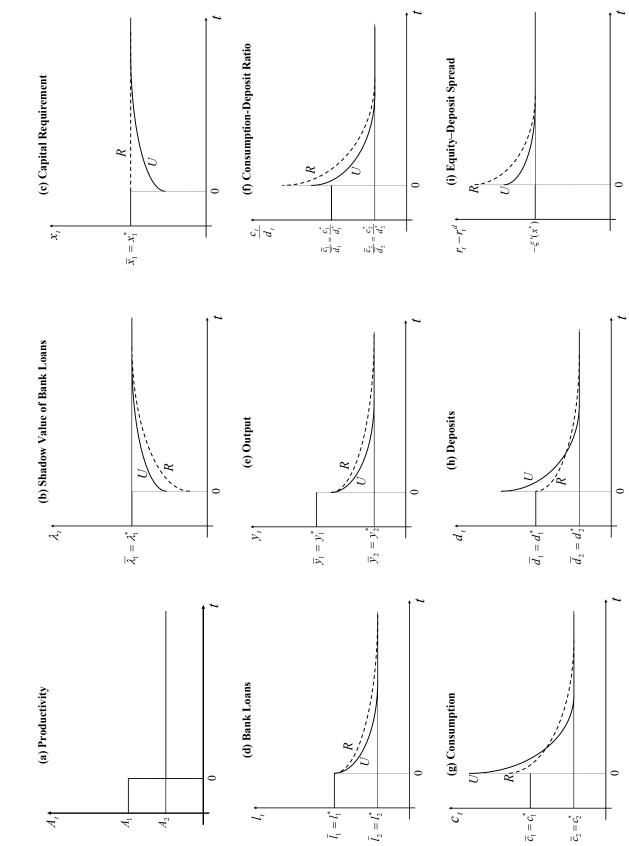


Figure 1. Phase Diagrams: Unrestricted (U) and Restricted (R) Models

Figure 2. Unanticipated and Permanent Reduction in Productivity in Unrestricted (U) and Restricted (R) Models





Unrestricted (U) and Restricted (R) Models: Time Paths of Selected Variables Figure 3. Unanticipated and Permanent Reduction in Productivity in

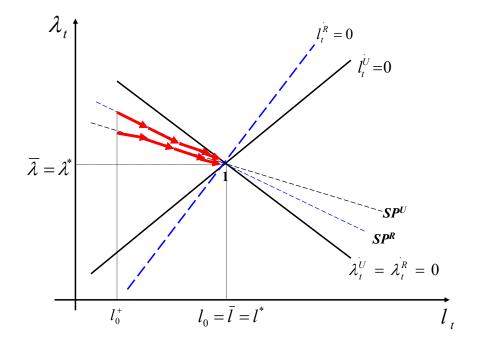
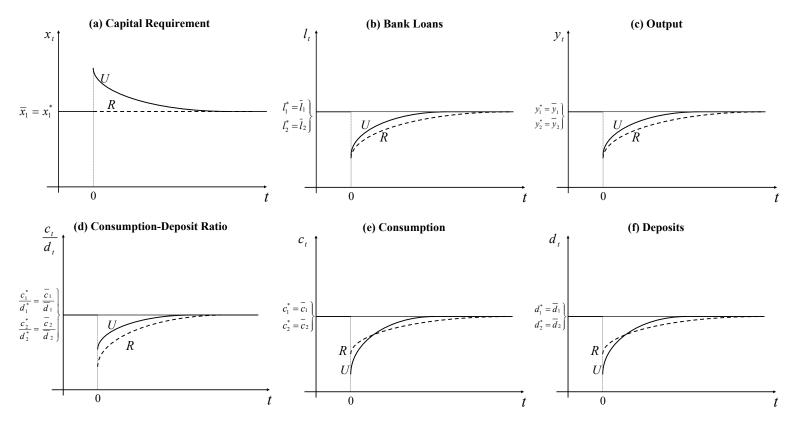


Figure 4. Loan Write-Offs in Unrestricted (U) and Restricted (R) Models

Figure 5. Loan Write-Offs in Unrestricted (U) and Restricted (R) Models: Time Paths of Selected Variables



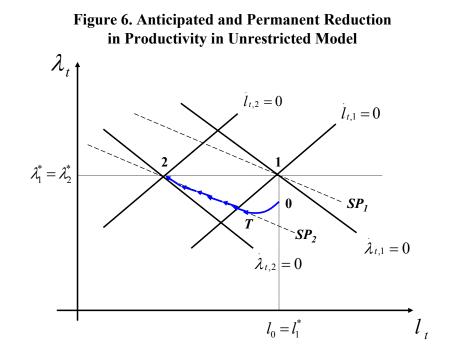
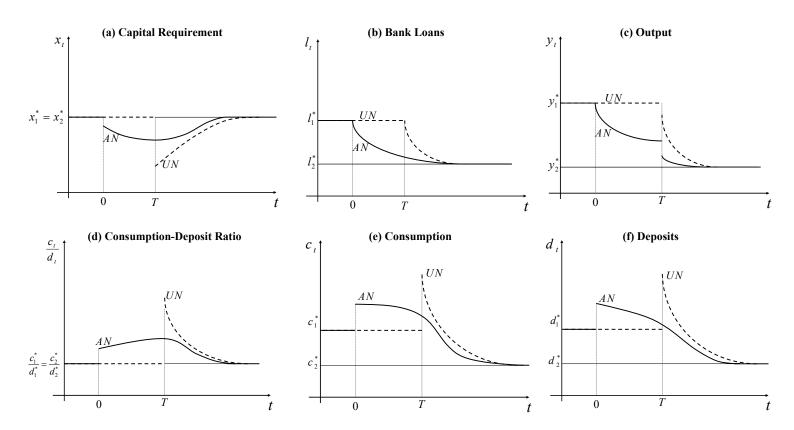
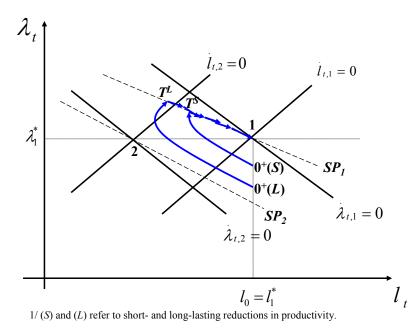


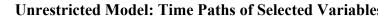
Figure 7. Anticipated (AN) and Unanticipated (UN) Permanent Reduction in Productivity in Unrestricted Model: Time Paths of Selected Variables

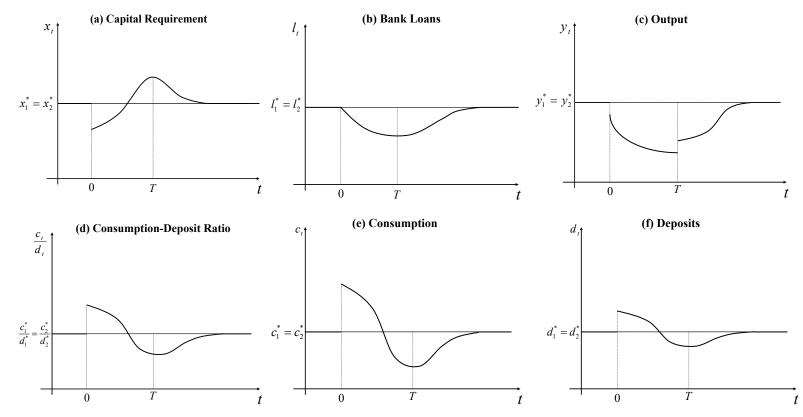




# Figure 8. Unanticipated and Temporary Reduction in Productivity in Unrestricted Model <sup>1/</sup>

Figure 9. Unanticipated and Temporary Reduction in Productivity in Unrestricted Model: Time Paths of Selected Variables <sup>1/</sup>





1/ The time paths shown correspond to a short-lasting reduction in productivity.

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