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## Evolution of the Relative Price of Goods and Services in a Neoclassical Model of Capital Accumulation

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IMF Institute

### Evolution of the Relative Price of Goods and Services in a Neoclassical Model of Capital Accumulation

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#### Abstract

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This paper provides an explanation for the secular increase in the price of services relative to that of manufactured goods that relies on capital accumulation rather than on an exogenous total factor productivity growth differential. The key assumptions of the two-sector, intertemporal optimizing model are relatively high capital intensity in the production of goods and limited cross-border capital mobility, allowing the interest rate to vary. With plausible parameterization, the model also predicts a decline in the employment share of the goods sector over time.

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## I. INTRODUCTION

An increase in the price of services relative to that of manufactured goods is a well-documented feature of economic development (Baumol and Bowen, 1966; Obstfeld and Rogoff, 1996). Because manufactured goods are tradable across borders while services are largely not, one may observe a secular increase in the price of nontradables relative to that of tradables in an open economy – the celebrated Balassa-Samuelson effect.<sup>2</sup>

The explanation for this phenomenon relies largely on the difference in the growth of labor productivity over time between the two sectors.<sup>3</sup> As labor productivity goes up in manufacturing, wages will increase in that sector. In the presence of intersectoral labor mobility, upward pressure will be put on the wages in services. If labor productivity growth lags behind in the services sector, unit labor cost will rise in that sector, and the price of services will have to go up for their provision to remain profitable.

Labor productivity growth can be decomposed into contributions from capital deepening (increase in the capital-labor ratio) and total factor productivity (TFP) growth. While early models were agnostic about the source of labor productivity differential (e.g., Baumol, 1967), lately the emphasis has been on TFP. This is particularly true of open-economy models (e.g., Obstfeld and Rogoff, 1996), where it is customary to assume the domestic interest rate to equal a given world interest rate, the parity being maintained by perfect capital mobility. As technological parameters and the rate of interest completely determine capital-labor ratios and relative prices, TFP growth becomes the only possible source of a change in the relative price of the two sectors' outputs.

This approach rests on three postulates: (1) labor productivity growth is driven primarily by TFP growth; (2) the rate of TFP growth is faster in manufacturing than in services; and (3) the domestic interest rate equals the world interest rate when expressed in terms of tradable (manufactured) goods. Definitely, none of these propositions are universal truths, and their applicability should be assessed on a case-by-case basis. These presumptions have been challenged even in some cases where strong priors existed in their favor. One of the most vivid examples is the controversy generated by Young's (1995) accounting exercise on the sources of growth in East Asia, where he downplayed the role of TFP growth. Regarding proposition (2), Triplett and Bosworth (2003) have established that TFP in the services sector

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<sup>2</sup> Of course, the mapping of manufactured goods into tradables and services into nontradables is less than perfect, given the presence of other sectors in the economy and the fact that some services may be tradable while some manufactured goods may be rendered nontradable by policy measures. For that reason, empirical support for the Balassa-Samuelson effect is not as strong as for the Baumol-Bowen effect (Froot and Rogoff, 1995; Harberger, 2003).

<sup>3</sup> Canzoneri et al. (1999) present empirical evidence of this link. Other possible reasons – such as a shift of consumer demand from goods to services – appear to play a relatively minor role.

has been growing as fast as that in manufacturing in the United States since 1995.<sup>4</sup> And, of course, evidence abounds that even nowadays and even among the most open economies capital mobility is far from perfect.

Increases in the relative price of services may occur, and have occurred, in circumstances where propositions (1)-(3) do not apply. While TFP growth drives economic progress in the long run, developments in the medium term may be determined by capital accumulation. This is particularly true of economies that have high level of human capital and access to advanced technology, but relatively low physical capital stock. This description fits well post-war economies, like western European countries or Japan after World War II. Other examples could include the Asian “tigers” at the beginning of their takeoff and, arguably, transitional economies. One can also note that fast growth and change in relative prices occurred in many of these economies when cross-border capital movements were quite restricted, so that the assumption of an exogenously given interest rate was not applicable.

This paper proposes an explanation for the change over time of the relative price of services and manufactures, or nontradables and tradables, that does not rely on TFP differential. The driving force in the model is capital accumulation, which leads to an increase in the relative price of services under the assumption that this sector is relatively less capital intensive than manufacturing. According to Obstfeld and Rogoff (1996), this assumption reflects reality.

While our result seems intuitive and has a familiar counterpart in trade theory, we have not been able to trace this particular application in the literature. Brock (1994) provides a rare example of an intertemporal optimizing model that avoids the use of exogenous technological change as an explanation of changes in relative prices. Brock’s objective is to emphasize the importance of investment, but as he assumes perfect capital mobility, he has to introduce a very complicated production structure (three factors of production and three goods, with two alternative technologies with different capital intensities available for the production of one of them) to generate the effect. Elsewhere in the literature the possibility that uneven capital accumulation may be responsible for changes in relative prices has been mentioned (e.g., Lipschitz et al., 2002), but no formal treatment has been provided.

Our model dismisses the assumption of perfect capital mobility to allow interest rate changes and gradual capital accumulation. To highlight the contrast with perfect capital mobility models, capital cannot move across borders in our model. However, one can readily see that the dynamics will be essentially similar in a system with imperfect capital mobility, where the differential between the domestic and the world interest rate declines over time as the economy develops.

The model also sheds additional light on the issue of “deindustrialization” – a decline in the share of labor employed in manufacturing that accompanies high labor productivity growth

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<sup>4</sup> A different, but not unrelated issue is a finding by Bernard and Jones (1996) that convergence among OECD economies both in terms of labor productivity and in terms of TFP can be found in services but not in manufacturing.

in that sector. While this phenomenon has been well documented (Baumol et al., 1989; Rowthorn and Ramaswamy, 1999), a model that assumes faster TFP growth in manufacturing than in services and a constant interest rate would predict an increase in manufacturing employment share for any reasonable parameter values (Obstfeld and Rogoff, 1996). In contrast, our model predicts a shift of labor from manufacturing into services over the course of development for the benchmark case of Cobb-Douglas preferences and production functions.

## II. MODEL

The economy produces goods  $G$  and services  $S$  using capital  $K$  and labor  $L$  by means of neoclassical production functions  $F$  and  $H$ . We assume uniform labor-augmenting technological progress in both sectors:

$$\begin{aligned} Q_G &= F(K_G, e^{xt} L_G) \equiv e^{xt} L_G f(\hat{k}_G), & \hat{k}_G &\equiv \frac{K_G}{e^{xt} L_G} \\ Q_S &= H(K_S, e^{xt} L_S) \equiv e^{xt} L_S h(\hat{k}_S), & \hat{k}_S &\equiv \frac{K_S}{e^{xt} L_S} \end{aligned} \quad (1)$$

In our notation, capital letters will be used to denote variables in levels (i.e., economy-wide aggregates), small letters will be used for per worker/per capita quantities (as well as prices, wages and interest rates), and “hats” represent variables per unit of effective labor (i.e. per worker variables divided by the efficiency factor  $e^{xt}$ ). The technology for producing goods is more capital intensive than that for services, so that we always have  $k_G > k_S$ , where

$$k_G \equiv \frac{K_G}{L_G} \quad \text{and} \quad k_S \equiv \frac{K_S}{L_S}.^5$$

Factors of production are freely mobile between the two sectors, but cannot move across the border. Neither can the residents of the country borrow abroad – the capital account of the balance of payments is closed. The current account could be closed as well, or we could allow trade in goods, but not in services, across the border. In the latter case, the ratio of services to goods prices can also be interpreted as the relative price of nontradables and tradables, or the real exchange rate. Of course, the financing constraint would impose balanced trade.

The infinitely lived households maximize the present discounted value of a logarithmic Cobb-Douglas utility scaled at each moment of time by the number of household members. Population is initially normalized to one and is assumed to grow exponentially at a rate  $n$ .

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<sup>5</sup> Since the “hat” variables are obtained by scaling their “no-hat” counterparts by the same factor, it is clear that  $k_G > k_S$  implies  $\hat{k}_G > \hat{k}_S$ .

$$U = \int_0^{\infty} (\alpha \log c_G + (1 - \alpha) \log c_S) e^{-(\rho-n)t} dt. \quad (2)$$

To guarantee that attainable utility is bounded, the rate of discount is assumed to exceed the rate of population growth:

$$\rho > n. \quad (3)$$

We will normalize the price of the consumption good to unity, so that all the prices and wages are measured in units of consumption good.

The household supplies inelastically  $e^{nt}$  units of labor to the market. Each worker receives wage  $w$  regardless of the sector in which they are employed. The household can also hold assets of two kinds – physical capital, which is convertible into the consumption good and back, so its unit price is one, and a riskless bond, denominated in the units of consumption good. The agents view both types of assets as perfect substitutes, so the interest rate on the bond  $r$  equals the rental rate of capital.<sup>6</sup> As all agents are identical and the economy is closed, no bonds will be issued in equilibrium, and household assets will consist only of capital. The household will maximize its utility (2) subject to an asset accumulation equation:

$$\dot{K} = rK + we^{nt} - C_G - pC_S, \quad (4)$$

where  $p$  is the price of services.

It is straightforward to rewrite the objective function and the accumulation equation in “efficiency units.” The household will maximize<sup>7</sup>

$$\tilde{U} = \int_0^{\infty} (\alpha \log \hat{c}_G + (1 - \alpha) \log \hat{c}_S) e^{-(\rho-n)t} dt \quad (5)$$

subject to the constraint

$$\dot{\hat{k}} = (r - n - x) \hat{k} + \hat{w} - \hat{c}_G - p \hat{c}_S. \quad (6)$$

From the household optimization problem we can derive an intratemporal condition

$$\frac{\hat{c}_G}{p \hat{c}_S} = \frac{\alpha}{1 - \alpha} \quad (7)$$

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<sup>6</sup> We ignore capital depreciation for notational simplicity.

<sup>7</sup> The maximand in (5) differs from that in (2) by a constant term that can be ignored.

and an intertemporal Euler equation

$$\frac{\dot{\hat{c}}_G}{\hat{c}_G} = r - x - \rho . \quad (8)$$

We will find it convenient to introduce total household expenditure in terms of goods:

$$C \equiv C_G + pC_S . \quad (9)$$

With this notation, the Euler equation and the capital accumulation equation take the following form:

$$\frac{\dot{\hat{c}}}{\hat{c}} = r - x - \rho \quad (10)$$

$$\dot{\hat{k}} = (r - n - x)\hat{k} + \hat{w} - \hat{c} . \quad (11)$$

Profit maximization implies equality between the value marginal products of capital and labor in both sectors and the prices of these factors – the rental rate of capital and the wage rate. These four marginal conditions determine four variables,  $\hat{k}_S$ ,  $r$ ,  $\hat{w}$ , and  $p$ , as functions of  $\hat{k}_G$ . If we fully differentiate the system, we obtain the following responses of these variables to changes in  $\hat{k}_G$ :

$$d\hat{k}_S = \frac{1}{p^2} \frac{f(\hat{k}_G)}{h(\hat{k}_S)} \frac{f''(\hat{k}_G)}{h''(\hat{k}_S)} d\hat{k}_G \quad (12)$$

$$dr = f''(\hat{k}_G) d\hat{k}_G \quad (13)$$

$$d\hat{w} = -\hat{k}_G f''(\hat{k}_G) d\hat{k}_G \quad (14)$$

$$dp = \frac{\hat{k}_S - \hat{k}_G}{h(\hat{k}_S)} f''(\hat{k}_G) d\hat{k}_G \quad (15)$$

Next we combine market clearing conditions for capital, labor, and services and obtain an equation that links  $\hat{k}$ ,  $\hat{c}$ , and  $\hat{k}_G$ :



$$\hat{k} = \hat{k}_G - \left( \hat{k}_G - \hat{k}_S(\hat{k}_G) \right) \frac{(1-\alpha)\hat{c}}{\hat{w}(\hat{k}_G) + r(\hat{k}_G)\hat{k}_S(\hat{k}_G)}. \quad (16)$$

This equation implicitly determines the capital intensity in the goods sector as an increasing function of consumption expenditure and the stock of capital in efficiency units:

$$\hat{k}_G = \hat{k}_G(\hat{k}, \hat{c}). \quad (17)$$

$$\frac{\partial \hat{k}_G}{\partial \hat{k}} = \frac{1}{l_G - l_S \frac{(\hat{k}_G - \hat{k}_S)^2 f''(\hat{k}_G)}{ph(\hat{k}_S)} + l_S \frac{f(\hat{k}_G)}{ph(\hat{k}_S)} \frac{d\hat{k}_S}{d\hat{k}_G}} > 0 \quad (18)$$

$$\frac{\partial \hat{k}_G}{\partial \hat{c}} = \frac{(1-\alpha)(\hat{k}_G - \hat{k}_S)}{ph(\hat{k}_S)} \frac{\partial \hat{k}_G}{\partial \hat{k}} > 0$$

With this, we obtain a system of two differential equations in two unknowns:

$$\frac{\dot{\hat{c}}}{\hat{c}} = r(\hat{k}_G(\hat{k}, \hat{c})) - x - \rho \quad (19)$$

$$\dot{\hat{k}} = \left[ r(\hat{k}_G(\hat{k}, \hat{c})) - n - x \right] \hat{k} + \hat{w}(\hat{k}_G(\hat{k}, \hat{c})) - \hat{c}. \quad (20)$$

We can apply standard techniques to solve the dynamic system. The steady state (which corresponds to the balanced growth path of the economy) will be found at the intersection of the loci satisfying the conditions  $\dot{\hat{c}} = 0$  and  $\dot{\hat{k}} = 0$ . The former is given by the equation

$$f'(\hat{k}_G^*) = x + \rho. \quad (21)$$

The locus of points  $\hat{k}_G(\hat{k}, \hat{c}) = \hat{k}_G^*$  is a downward sloping straight line with the slope equal

$$\frac{d\hat{c}}{d\hat{k}} \Big|_{\dot{\hat{c}}=0} = - \frac{\partial \hat{k}_G}{\partial \hat{k}} \Big/ \frac{\partial \hat{k}_G}{\partial \hat{c}} = - \frac{\hat{w}^* + (\rho + x)\hat{k}_S^*}{(1-\alpha)(\hat{k}_G^* - \hat{k}_S^*)}. \quad (22)$$

To the right of that line  $\hat{k}_G > \hat{k}_G^*$ , so  $r$  is smaller than  $\rho + x$ , and consumption per unit of effective labor is declining. To the left of that line  $\dot{\hat{c}}$  is positive.

The constancy of capital stock per unit of effective labor requires

$$\left[ r(\hat{k}_G) - n - x \right] \hat{k} + \hat{w}(\hat{k}_G) - \hat{c} = 0. \quad (23)$$

The points satisfying this equation are located on an upward sloping line<sup>8</sup> with a slope

$$\frac{d\hat{c}}{d\hat{k}} \Big|_{\hat{k}=0} = \frac{r - n - x - (\hat{k}_G - \hat{k}) f'' \frac{\partial \hat{k}_G}{\partial \hat{k}}}{1 + (\hat{k}_G - \hat{k}) f'' \frac{\partial \hat{k}_G}{\partial \hat{c}}} = \frac{r - n - x - (\hat{k}_G - \hat{k}_S) f'' \frac{\partial \hat{k}_G}{\partial \hat{k}}}{1 + (\hat{k}_G - \hat{k}_S) f'' \frac{\partial \hat{k}_G}{\partial \hat{c}}}. \quad (24)$$

The stock of capital is increasing to the right of this line and decreasing to its left.

These findings are presented in the graphical form in the phase diagram in Figure 1. The system will evolve along the stable arm of the saddle path, starting from the point where  $\hat{k}$  equals the initial stock of capital and converging over time to the balanced growth path.

If initially the capital stock is below its equilibrium value, as one would expect for an emerging market, over time  $\hat{k}$  and  $\hat{c}$  will rise, as can be seen from the phase diagram. Being an increasing function of  $\hat{k}$  and  $\hat{c}$  (see equation (17)), the capital intensity in the goods sector  $\hat{k}_G$  will rise as well. This means that the capital intensity in the services sector  $\hat{k}_S$  will also go up (according to equation (12)), and so will the relative price of services  $p$  (equation (15)). This establishes the main result of the paper. The wage rate per unit of effective labor  $\hat{w}$  will increase in terms of both goods and services. The interest rate, on the other hand, will go down, as the marginal product of capital declines.

These simple considerations do not establish the direction of the evolution of several other macroeconomic variables. In particular, the way in which employment will shift between the two sectors cannot be established in the general case. The Appendix looks further into this issue and derives more definite results for certain special cases. We establish that labor will shift from the goods sector into the services sector in the course of development if the production technology in both sectors is Cobb-Douglas. This fits the pattern observed in many countries across the world. In contrast, an open economy model with perfect capital mobility, which assumes a constant interest rate and relies on TFP growth differential to generate change in relative prices, predicts an *increase* in manufacturing employment as a result of an increase in manufacturing productivity if consumer preferences are Cobb-Douglas (which is arguably a reasonable baseline) and would require a very low elasticity of

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<sup>8</sup> To be precise, the slope is guaranteed to be positive to the left of the line  $\dot{\hat{c}} = 0$  (where  $r > x + \rho > x + n$ ), which is the relevant region.

substitution between goods and services in combination with other extreme parameter values to reverse the result (Obstfeld and Rogoff, 1996, p. 224).

### III. CONCLUSION

This paper has presented a model in which an increase in the price of services relative to goods is generated by capital accumulation under the assumption that goods production technology is relatively more capital intensive. While the idea that changes in relative price can be driven by capital accumulation has been mentioned in the literature, no formal optimizing models of that process have been developed. Traditionally, the secular decline in the price of goods relative to that of services has been accounted for by a differential in total factor productivity growth. In reality, these two mechanisms complement each other, but the capital channel has been given short shrift.

The relative importance of TFP and capital accumulation channels depends on the magnitude of TFP growth differential, the gap in capital intensity (as reflected in capital income shares), and the rate of capital deepening. The capital accumulation channel will be particularly important for economies whose stock of capital has been depleted by wars or natural disasters, or was far below potential because of government policies, which start growing fast, primarily through high rates of investment, once impediments have been removed.

The pattern of relative price evolution derived in the model is quite general. We did not specialize the production functions other than imposing the standard neoclassical properties. The choice of the utility function was more restrictive, but it is easy to see that it is not critical for obtaining the secular increase in the relative price of services. Indeed, the relative price will go up over time as capital is accumulated as long as the goods sector is relatively more capital intensive. The relative capital intensity condition is therefore critical for the result.<sup>9</sup>

The evolution of sectoral output and employment does depend on the choice of the utility function. We have shown that in the baseline case of Cobb-Douglas preferences and production functions, labor will shift from manufacturing into services as the country develops. This prediction accords with the pattern observed across the world. It is worth noting that models that assume perfect international capital mobility and inter-industry TFP differential make the opposite prediction for this baseline case.

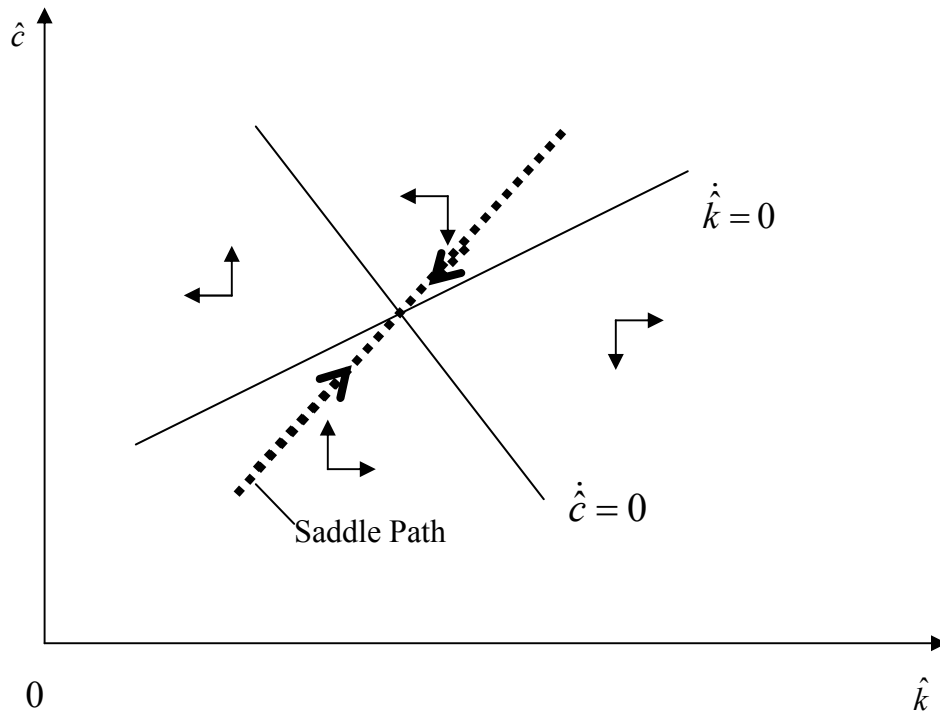
In the modern world, the assumption of a completely closed capital account of the balance of payments is arguably as unrealistic as the opposite assumption of perfect capital mobility. We certainly agree with that. We would emphasize, however, that as long as imperfect capital mobility creates room for domestic interest rate movements, capital accumulation will

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<sup>9</sup> Of course, the assumption of perfect factor mobility across sectors is important as well, but the models that rely on the TFP growth differential to explain changes in the relative price also make that assumption.

have an effect on the evolution of the relative price of goods and services along the lines drawn in our model.

Figure 1. Phase Diagram



### EVOLUTION OF SECTORAL OUTPUT AND EMPLOYMENT OVER TIME

As indicated in the main text, the direction of the evolution on the convergence path of the output of goods and services and the labor employment in the two sectors is in the general case ambiguous. We will explore the evolution of these variables in the vicinity of the steady state using the following approach. For notational simplicity, we will assume away population growth and technological progress.

We know that if the initial capital endowment is smaller than the steady-state value, consumption expenditure increases monotonically over time. Hence, a variable will increase over time if and only if its full derivative with respect to  $C$  is positive. For example, the production of services will increase if and only if  $\frac{dQ_s}{dC} > 0$ .

Now, as

$$Q_s = C_s = \frac{(1-\alpha)C}{p},$$

we can write

$$\frac{dQ_s}{dC} = \frac{(1-\alpha)}{p} \left[ 1 - \frac{C}{p} \frac{dp}{dC} \right] = \frac{(1-\alpha)}{p} \left[ 1 + \frac{C(k_G - k_S) f''}{ph} \frac{dk_G}{dC} \right].$$

According to equation (17),  $k_G$  is a function of  $K$  and  $C$ . In addition, along the convergence path  $K$  and  $C$  are related one to one. Hence,

$$\frac{dk_G}{dC} = \frac{\partial k_G}{\partial C} + \frac{\partial k_G}{\partial K} \frac{dK}{dC}.$$

The partial derivatives are given by equations (18). The inverse of the slope of the convergence line,  $\frac{dK}{dC}$ , cannot be found in the general case, as it would require solving the dynamic system. However, we can express this derivative in the vicinity of the steady state via the parameters of the model. To do that, we linearize the model around the steady state.

$$\frac{d}{dt} \begin{bmatrix} C - C^* \\ K - K^* \end{bmatrix} \approx \begin{bmatrix} C^* \frac{\partial k_G}{\partial c} f''(k_G^*) & C^* \frac{\partial k_G}{\partial K} f''(k_G^*) \\ - \left[ 1 + L_S^* (k_G^* - k_S^*) \frac{\partial k_G}{\partial c} f''(k_G^*) \right] & \left[ \rho - L_S^* (k_G^* - k_S^*) \frac{\partial k_G}{\partial K} f''(k_G^*) \right] \end{bmatrix} \begin{bmatrix} C - C^* \\ K - K^* \end{bmatrix}$$

The determinant of the matrix equals  $C^* f''(k_G^*) \left[ \frac{\partial k_G}{\partial K} + \rho \frac{\partial k_G}{\partial C} \right]$ . Since the partial derivatives of  $k_G$  with respect to  $K$  and  $C$  are positive, the determinant has a negative sign, which confirms the saddle point stability in the vicinity of the steady state.

The trace equals simply  $\rho$ , as

$$\begin{aligned} C^* \frac{\partial k}{\partial c} f''(k_G^*) - L_s^* (k_G^* - k_s^*) \frac{\partial k}{\partial K} f''(k_G^*) &= C^* \frac{\partial k}{\partial c} f''(k_G^*) \left[ 1 - \frac{L_s^*}{C^*} \frac{\frac{\partial k}{\partial K}}{\frac{\partial k}{\partial c}} (k_G^* - k_s^*) \right] = \\ &= C^* \frac{\partial k}{\partial c} f''(k_G^*) \left[ 1 - \frac{(1-\gamma)}{p^* h(k_s^*)} \frac{p^* h(k_s^*)}{(1-\gamma)(k_G^* - k_s^*)} (k_G^* - k_s^*) \right] = 0 \end{aligned}$$

The rate of convergence, equal to the modulus of the negative eigenvalue of the above matrix, can be expressed as  $\lambda = \frac{1}{2} \left[ \sqrt{\text{trace}^2 - 4 \times \det} - \text{trace} \right]$ .

The derivative  $\frac{dK}{dC}$  at the steady state can be expressed through the coefficients in the top row of the matrix of the linearized system and the convergence rate in the following fashion:

$$\frac{dK}{dC} = - \frac{C^* \frac{\partial k_G}{\partial c} f''(k_G^*) + \lambda}{C^* \frac{\partial k_G}{\partial K} f''(k_G^*)}.$$

This implies

$$\frac{dk_G}{dC} = \frac{\partial k_G}{\partial C} + \frac{\partial k_G}{\partial K} \frac{dK}{dC} = \frac{\partial k_G}{\partial C} - \frac{\partial k_G}{\partial K} \frac{C^* \frac{\partial k_G}{\partial c} f''(k_G^*) + \lambda}{C^* \frac{\partial k_G}{\partial K} f''(k_G^*)} = - \frac{\lambda}{C^* f''(k_G^*)}.$$

Combining all these results, we obtain in the vicinity of the steady state

$$\frac{dQ_s}{dC} = \frac{(1-\alpha)}{p^*} \left[ 1 + \frac{\lambda (k_G^* - k_s^*)}{p^* h(k_s^*)} \right]$$

Hence, the condition for the production of services to increase over time is

$$\lambda < \frac{p^* h(k_S^*)}{k_G^* - k_S^*}$$

The expression for  $\lambda$  is fairly cumbersome, and we will find it helpful to use the following easily derivable proposition:<sup>10</sup>

$$\frac{1}{2} \left[ \sqrt{\text{trace}^2 - 4 \times \det} - \text{trace} \right] < A \quad \Leftrightarrow \quad -\det < A \times (A + \text{trace})$$

In our case

$$A = \frac{p^* h(k_S^*)}{k_G^* - k_S^*}$$

and

$$A \times (A + \rho) = \frac{f(k_G^*) p^* h(k_S^*)}{(k_G^* - k_S^*)^2}$$

Hence, the output of services will increase over time if and only if the absolute value of the determinant is less than the above expression. Now, the determinant can be expressed in the following way through the steady state values of capital intensities and the relative price, which, in turn, depend on the production functions and the discount rate:

$$-\det = - \frac{f(k_G^*) p^* h(k_S^*) f''(k_G^*) \left[ 1 + \rho \frac{(1-\alpha)(k_G^* - k_S^*)}{p^* h(k_S^*)} \right]}{\alpha p^* h(k_S^*) - (1-\alpha) \frac{f(k_G^*)}{p^* h(k_S^*)} \left[ (k_G^* - k_S^*)^2 f''(k_G^*) - f(k_G^*) \frac{dk_S}{dk_G} \right]}$$

Comparing the two expressions, we will arrive at the following criterion – the output of services increases over time along the convergence path if and only if

$$\alpha \left[ p^* h(k_S^*) + f''(k_G^*) (k_G^* - k_S^*)^2 \right] + (1-\alpha) \frac{f(k_G^*)^2}{p^* h(k_S^*)} \frac{dk_S}{dk_G} > 0.$$

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<sup>10</sup> This relies on the determinant being negative and the trace being positive, as is the case here.

This condition will not necessarily hold, since the negative term containing the second derivative may dominate the two positive terms, except in the limiting case where the two sectors have equal capital intensity.

Analogously, we can show that  $\frac{dL_S}{dC} > 0$  if and only if

$$\lambda < -\frac{p^* h(k_S^*) f''(k_G^*)}{\rho \frac{dk_S}{dk_G} - (k_G^* - k_S^*) f''(k_G^*)}.$$

Tracing the same steps as above, we can show this condition to be equivalent to the following:

$$\begin{aligned} & -\alpha f''(k_G^*) f(k_G^*) \left[ p^* h(k_S^*) + f''(k_G^*) (k_G^* - k_S^*)^2 \right] + \\ & + \left\{ \alpha \rho^2 p^* h(k_S^*) - f''(k_G^*) f(k_G^*) \left[ (1 + \alpha) p^* h(k_S^*) - 2\alpha f(k_G^*) \right] \right\} \frac{dk_S}{dk_G} - \\ & - \alpha \rho^2 f(k_G^*) \left( \frac{dk_S}{dk_G} \right)^2 > 0 \end{aligned}$$

Obviously, this is a more complicated criterion than the one for the output of nontradables. In the general case this criterion may or may not be satisfied.

We can get more definitive results in some special cases. In particular, we show below that if the production technology in both sectors is Cobb-Douglas, with the capital share and hence capital intensity higher in the goods sector, then both the output of services and employment in that sector increase over time.

A simple, though tedious, proof is obtained by writing out the production functions explicitly:

$$f(k_G) = Dk_G^\gamma \quad h(k_S) = Bk_S^\beta \quad \gamma > \beta.$$

From the first-order conditions for profit maximization, the following relationships are easy to establish:

$$k_S = \frac{1-\gamma}{\gamma} \times \frac{\beta}{1-\beta} k_G$$

and



$$ph(k_s) = \frac{w}{1-\beta} = \frac{1-\gamma}{1-\beta} f(k_G).$$

Now these expressions and the derivatives of the Cobb-Douglas functions can be plugged into the left-hand side expression of the employment criterion. The resulting expression can be shown to equal

$$f(k_G^*) D^2(k_G^*)^{2\gamma-2} \frac{(1-\gamma)^2}{(1-\beta)^2} \times \\ \times \{ \alpha(\gamma + \gamma\beta - \gamma^2 - \beta^2) + \beta(1 - \alpha - \gamma + 2\alpha\beta) - \alpha\beta^2 \}$$

The three terms in the braces correspond to the three terms in the criterion. Now the terms before the braces are positive and can be dropped, while the expression in the braces simplifies to

$$(1-\gamma) \{ \alpha\gamma + \beta(1-\alpha) \},$$

which is obviously positive. Hence, the criterion is satisfied, which means that in the case of Cobb-Douglas technologies the employment share of services will increase over time. Of course, a combination of increase in employment and capital deepening in the services sector means that the output of services will increase over time as well.

Another special case is one where no capital is used in the production of services. In that case the criteria for the increase in the service sector output and employment reduce to a fairly simple restriction:

$$-\frac{(k_G^*)^2}{w^*} f''(k_G^*) < 1.$$

This condition would be satisfied for a Cobb-Douglas production function,<sup>11</sup> but it would be violated, for example, for a CES function with a sufficiently low elasticity of substitution between capital and labor.

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<sup>11</sup> For the Cobb-Douglas function, the left-hand-side expression equals simply the capital share.

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