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On Fixed and Variable Fiscal Surplus Rules

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African Department

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Abstract

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Fiscal rules are being increasingly used by both emerging and developed economies. This paper analyzes two alternative fiscal policy rules in terms of their impact on debt sustainability: a rule that fixes the ratio of primary surplus to GDP (“fixed surplus rule”) and one that sets the primary surplus as a linear function of debt to GDP ratio (“variable surplus rule”). A simple debt dynamics equation, incorporating real shocks, is constructed, and the probability of exceeding the critical debt level is simulated using Monte Carlo techniques. The results show that the variable surplus rule performs better than the simple fixed surplus rule, by reducing debt sustainability concerns and the necessary medium-term primary surplus. This result hinges on the government’s ability to make a credible commitment to the variable surplus rule in the medium run.

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I. INTRODUCTION

Public debt levels have remained a significant macroeconomic issue for developing countries and international investors at least since the debt crises of the 1980s, especially in countries with a weak reputation for fiscal prudence.² These countries range from relatively higher income countries, such as some Latin American countries and Turkey, to lower income countries such as those in Africa.³

Policies adopted in response to high debt levels and reactions by the international community vary. The international community adopted the enhanced Heavily Indebted Poor Countries (HIPC) initiative to eliminate the negative effect of the debt overhang on the development of lower income countries. In some relatively higher income countries with market access but lacking a record of fiscal prudence, fiscal rules have been adopted as a useful tool for fostering economic stability and growth.^{4, 5}

In countries with high debt-to-GDP ratios, such as Brazil and Turkey, the fixed primary surplus rule, which fixes the ratio of primary budget surplus to GDP, is being used as one form of fiscal rule, either to signal policy credibility, or to limit public debt levels, or both. In this paper we run numerical simulations (Monte Carlo technique) to compare the performance of the fixed primary surplus rule to an alternative, the variable primary surplus rule, under which the primary budget surplus is explicitly defined as an increasing function of the debt-to-GDP ratio. As the criterion of comparison, we use the probability of exceeding the sustainable debt level at the end of the simulation horizon starting from a given initial debt level. The results indicate that, provided the government can credibly commit to a variable surplus rule, it performs better than the fixed surplus rule as it reduces debt sustainability concerns as well as the necessary level of the medium-term primary surplus compared to the fixed surplus rule.

The paper is organized as follows. Section II develops the model, and the analytical definitions of the two fiscal rules and debt sustainability. The results of the numerical simulations are presented in Section III. Section IV presents the conclusions.

² Public debt and size of government are concerns for policymakers in developed countries as well (Daban and others, 2003).

³ In some of these countries, the public debt is composed mostly of external debt, while in others domestic debt is also involved.

⁴ Kopits (2001).

⁵ Whether based on explicit fiscal rules or not, prudent fiscal policy is generally correlated positively with lower debt levels (Wyplosz, 2002).

II. MODEL

A. Debt Dynamics Equation

The public sector debt dynamics of a country can be expressed as⁶

$$D_{t+1} = (1 + R_t) (D_t - PS_t), \quad (1)$$

where, D_t , PS_t and R_t denotes respectively the debt stock⁷ at the beginning of period t, the primary surplus in period t (assumed to occur instantaneously at the beginning of period t) and the real interest rate in period t. Both the debt stock and the primary surplus are in real terms. The primary surplus is the difference between gross budgetary revenues and non-interest budgetary expenditures, i.e. $PS_t = T_t - G_t$, where, T_t is the real budgetary revenues, and G_t is the real non-interest budgetary expenditures. Therefore, the primary surplus has a decreasing effect on end-of-period debt stock.

The path of real output is given by

$$Y_{t+1} = (1 + g_t) Y_t, \quad (2)$$

where g_t is the real GDP growth rate.

Defining, the debt to GDP ratio as $d_t = D_t / Y_t$ and combining (1) and (2),

$$d_{t+1} = [(1 + R_t) / (1 + g_t)] (d_t - ps_t), \quad (3)$$

where $ps_t = PS_t / Y_t$.

Assuming R_t and g_t have random components, we define the random variable, $r_t + \varepsilon_t$, the growth adjusted real interest rate, through the following decomposition:

$$1 + r_t + \varepsilon_t = \frac{(1 + R_t)}{(1 + g_t)}, \quad (4)$$

⁶ Note that this assumes no financing of the contemporaneous budget deficit by issuance of high powered money.

⁷ We do not distinguish between domestic and external debt in this paper.

where r_t is the deterministic component of the real growth adjusted interest rate, and ε_t is a zero mean independently and identically distributed (iid) random variable which represents the interest rate, and growth shocks.⁸

Next, we assume that the deterministic component of the growth adjusted mean real interest rate, $r(d_t)$ is an increasing function of the debt to GDP ratio.⁹

$$r_t = r(d_t) \text{ with } r'(d_t) > 0, \quad (5)$$

where $r'(d_t)$ represents the first derivative of $r(d_t)$

Combining (3), (4), and (5), we obtain,

$$d_{t+1} = (1 + r(d_t) + \varepsilon_t) (d_t - ps_t), \quad (6)$$

where d_t denotes debt to GDP ratio at the beginning of period t , and ps_t denotes the ratio of primary surplus to GDP in period t . We assume that growth adjusted mean real interest rate, $r(d_t)$ is an increasing function of the debt to GDP ratio.

B. Alternative Fiscal Policy Rules

We consider two alternative fiscal policy rules:

Rule 1: Fixed primary surplus. The fixed primary surplus rule is equal to a constant s percent of GDP at every period: $ps_t = s$ for all t .

Rule 2: Variable primary surplus. Primary surplus is defined as a linearly increasing function of the debt stock; a higher fiscal surplus (a tighter fiscal policy) is set as the debt stock increases: $ps_t = \sigma d_t$ for all t , $\sigma > 0$.

⁸ In the more generalized case where distinction is made between the domestic and external debt, the random term would also cover the real exchange rate shocks.

⁹ Hu and others (2001) provide evidence for a significant negative relationship between debt to GDP ratios and credit ratings, and thus a positive one with interest rates. Likewise, Cantor and Packer (1996) also show that lower external debt is consistent with higher credit ratings.

C. Critical Debt Ratio and Sustainability

We define the **critical debt ratio** (d_c) as the steady state of the deterministic version of the debt ratio as follows:

$$E[d_{t+1}] = d_t = d_c \quad (7)$$

The critical debt level can be shown to be an unstable equilibrium.¹⁰ We use this property to define the debt level as *sustainable* when it is below the critical debt level.

Critical debt ratio can be computed for both fixed and variable rules by using relation (7). For simplicity, we will define.¹¹

$$r(d_t) = \rho d_t \text{ for all } t, \quad (8)$$

where, $0 < \rho < 1$.

For the fixed surplus rule ($ps_t = s$), setting $E[d_{t+1}] = d_t = d_c$, equation (6) becomes,

$$\rho d_c^2 - s \rho d_c - s = 0, \quad (9)$$

In this case, critical debt level can be calculated as,

$$d_c = \frac{(s\rho + \sqrt{(s\rho)^2 + 4s\rho})}{2\rho}, \quad (10)$$

as $s, \rho > 0$ so that $s\rho < \sqrt{(s\rho)^2 + 4s\rho}$.

For the variable surplus rule ($ps_t = \sigma d_t$) for all t , critical debt level, d_c' , becomes,

$$d_c' = \sigma / \rho (1 - \sigma), \quad (11)$$

Note that the critical debt ratio is policy dependent under both rules.

¹⁰ See proposition 3 for the proof.

¹¹ See Catao and Kapur (2004) for a kinked version of the real interest rate function. Since we limit the analysis here to the neighborhood of the critical debt levels in the context of a hypothetical emerging country, we assumed a linear function for simplicity. Other nonlinear monotonic functions of debt ratio for the growth adjusted real interest rate do not change the main conclusions. We also assume that real interest rate is independent of the fiscal rule adopted.

D. Properties of Fiscal Policy Rules

Let d_c and d_c' denote the critical debt levels of the constant surplus rule, and the variable surplus rule respectively.

Proposition 1: Given any initial debt ratio, d_0 , and any fixed surplus rule, $ps_t=s$ for all t that makes expected debt dynamics sustainable, there exists an alternative linear surplus rule, the variable surplus rule, $ps_t=\sigma d_t$, that dominates the former by alleviating sustainability concerns, i.e., $d_c < d_c'$.

Proof: The initial debt ratio is below the critical debt level of constant surplus rule ($d_0 < d_c$). In the variable surplus rule, let $\sigma = s / d_0$ so that primary surplus is determined as $ps_t = \sigma d_t$. Critical debt levels for constant and variable surplus rules are given by equations (10) and (11). In this construction, $d_c' = \frac{s}{\rho(d_0 - s)}$. We need to show $d_c < d_c'$.

Suppose not, $d_c \geq d_c'$, then,

$$d_c \geq \frac{s}{\rho(d_0 - s)} . \quad (12)$$

After rearranging (12) with the assumption ($d_0 > s$),

$$d_0 \geq \frac{s}{\rho d_c} + s . \quad (13)$$

By (13) and $d_0 < d_c$,

$$d_c > \frac{s}{\rho d_c} + s , \quad (14)$$

which implies

$$\rho d_c^2 - s \rho d_c - s > 0 , \quad (15)$$

which contradicts equation (9).

Q.E.D.

Proposition 1 shows that the fixed primary surplus levels may be improved upon by a variable surplus rule as it increases the critical debt ratio and alleviates sustainability concerns. This implicitly assumes that the government can credibly commit to a variable surplus rule.

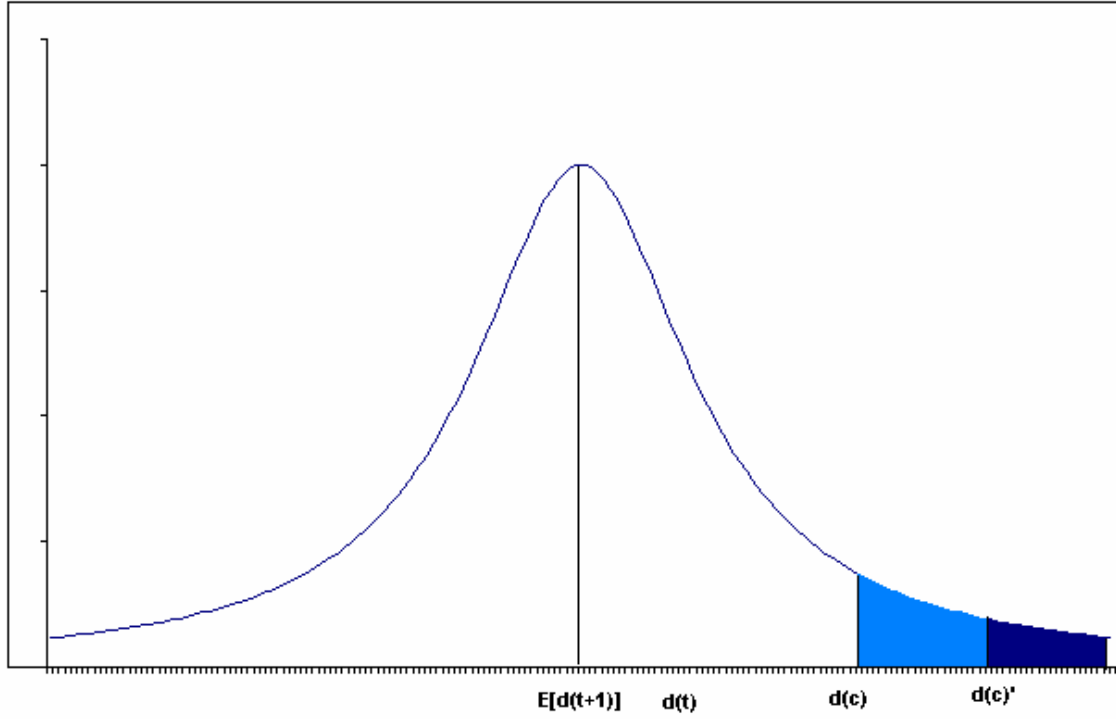


Figure 1. Probability Distribution of Exceeding the Critical Debt Levels at d_t and $E[d_{t+1}]$.
 ($d(c)$ and $d(c)'$ denote, respectively, the critical debt level for the fixed surplus rule and the variable surplus rule)

Proposition 2: Given any initial debt ratio, d_0 , and any fixed surplus rule, $ps_t = s$ for all t that makes expected debt dynamics sustainable, there exists an alternative variable surplus rule, $ps_t = \sigma d_t$, that dominates the former by reducing the average primary surplus that corresponds to the fixed rule in the medium term.

Proof: Trivial.

Proposition 3: For both the fixed and variable primary surplus rules, the critical debt ratio is an unstable equilibrium.

Proof: When $ps_t = s$ and $r(d_t) = \rho d_t$ are substituted in the fixed surplus rule, the dynamics becomes,

$$E[\Delta d_{t+1}] = \rho d_t^2 - s - s \rho d_t . \quad (16)$$

By (10) and (16), $E[\Delta d_{t+1}] > 0$ when $d_t > d_c$ and $E[\Delta d_{t+1}] < 0$ when $d_t < d_c$; the critical debt level is an unstable equilibrium for the fixed surplus rule.

For the variable surplus rule ($ps_t = \sigma d_t$), the dynamics can be written as

$$E[\Delta d_{t+1}] = d_t(\rho d_t - \sigma \rho d_t - \sigma). \quad (17)$$

By (11) and (17), debt ratio diverges from the equilibrium point (d_c'); the critical debt level is an unstable equilibrium for the variable surplus rule.

Therefore, when $d_t > d_c$, debt level blows up, and it tends to decline when $d_t < d_c$ under both fiscal rules.

III. SIMULATION RESULTS

We used the results in the previous sections to conduct simulations for both fiscal rules using Monte Carlo technique, for initial debt ratios (d_0) ranging from 20 percent of GDP to 100 percent of GDP. First, the debt ratio at the next period was calculated with normally distributed random shocks by using equation (6); subsequently, quantitative results have been obtained for 1000 Monte Carlo trials, repeating the same shocks under both fiscal rules. Arithmetic averages of these trials are used in the quantitative analysis.

In the simulations, ρ is set at 10 percent so that a debt ratio of, say, 60 percent corresponds to 6 percent growth adjusted real interest rate.¹² Other levels for ρ ¹³ for the growth adjusted real interest rate do not change the main conclusions from the analysis. Shocks to debt dynamics, ε_t , assumed to be iid with a normal distribution, is set to have zero mean, and 5 percent variance.

For the numerical simulations, we set the parameters for the two policy rules as follows:

Fixed surplus rule: $ps_t = 0.04 \quad \forall t$ (corresponding to $d_c = 0.6528$).

Variable surplus rule: $\sigma = 0.0667$ and thus $ps_t = 0.0667d_t \forall t$ (corresponding to $d_c' = 0.7143$).

These variable and fixed surplus rules require the same primary surplus at 60 percent initial debt ratio, which is below the critical level of the fixed rule. The initial debt level in the simulations is, thus, below the critical debt ($d_0 < d_c$). In our set up, the probability of

¹² With a real GDP growth of, say, 5 percent, this translates into a real interest rate of 11 percent from Equation 5.

¹³ The conclusions from the simulations are not effected by different values of ρ provided that the policy rule for the variable surplus rule (σ) is adjusted appropriately.

exceeding the critical debt ratio corresponds to one minus the probability of maintaining the debt sustainability.¹⁴

The results of the numerical simulations show that the variable surplus rule dominates the fixed surplus rule for both horizons; the cumulative probability of explosion in the next period or over the medium term is higher for the fixed surplus rule than for the variable surplus rule. The results are displayed in Figures 2 and 3 where the horizontal and the vertical axis show the initial debt ratio, and the cumulative probability of explosion from the simulations, respectively.

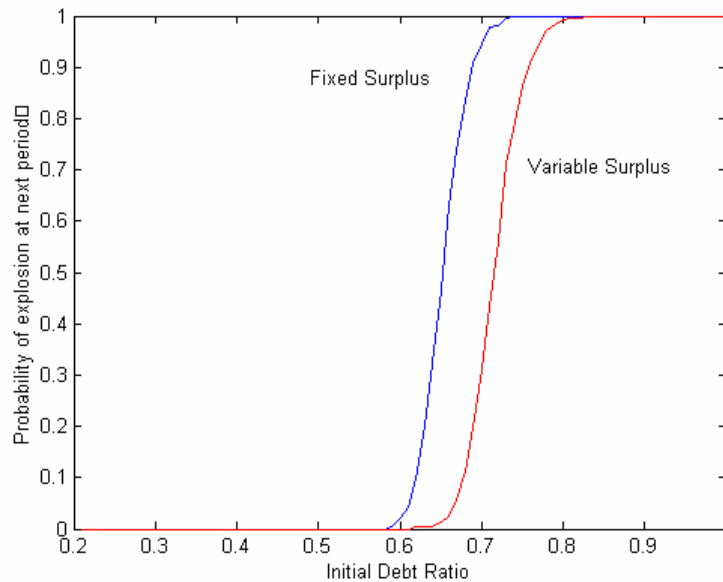


Figure 2. Cumulative Probability of Exceeding the Critical Debt Ratio at the End of the Next Period.

¹⁴ Simulations for both the next-period (next year) and for the medium term (five years) were run for both rules. With the parameters selected as above for each rule, exceeding the critical debt ratio at the end of the next period (or the medium term) is not possible under both rules when the simulations start from an initial debt ratio of less than 50 percent of the GDP. Likewise, to have a sustainable debt ratio in the next period (or in the medium run) is practically not possible when the simulations are started with an initial debt ratio of higher than 80 percent (of GDP) for either rule. For an initial debt level higher than 80 percent of GDP, policy makers would need to further tighten the fiscal policy to raise the primary surplus, thus raising the critical debt level and rendering the debt dynamics sustainable.

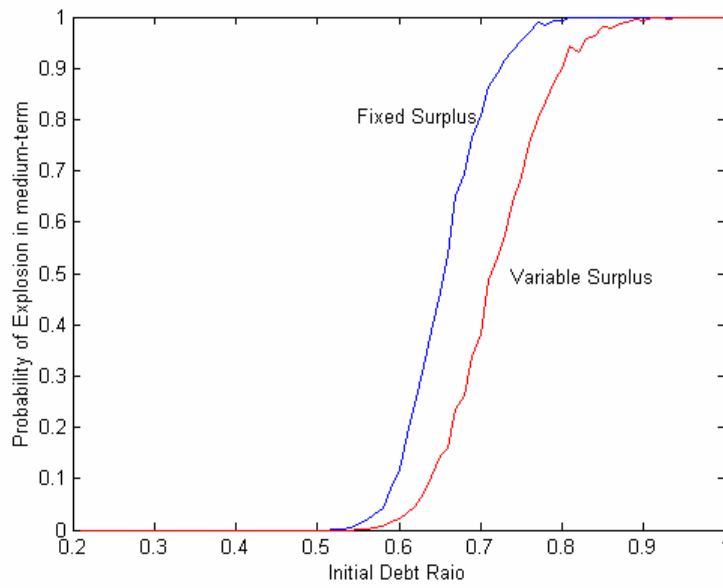


Figure 3. Cumulative Probability of Exceeding the Critical Debt Ratio in the Medium Term

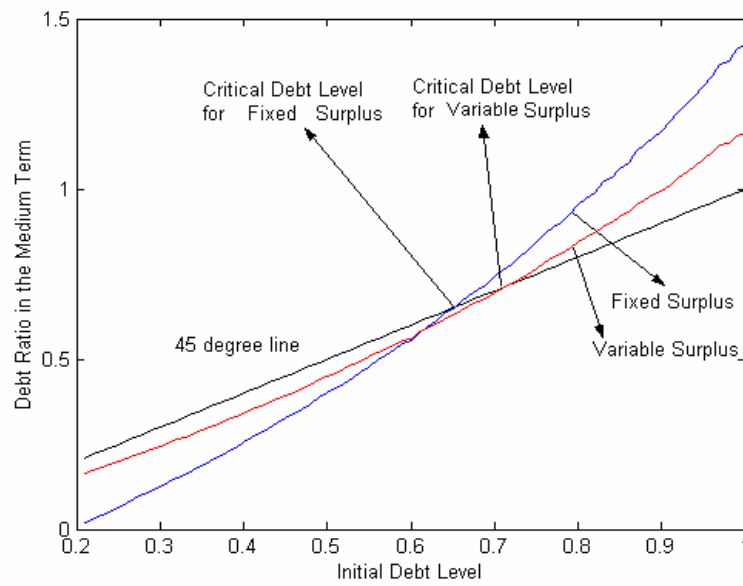


Figure 4. Average Debt Ratio in the Medium Term

In Figure 4, the horizontal axis and the vertical axis represent, respectively, the initial debt to GDP ratio, and the debt to GDP ratio at the end of the next period which is yielded by the simulation. Points under the 45 degree line show that the next period debt ratio is lower than the initial value.

As a corollary of the probabilistic domination of the variable surplus rule over the fixed surplus rule in terms of debt sustainability, the magnitude of the primary surplus generated by the variable rule is lower than that generated by the fixed rule at debt levels below the initial debt level, set at 60 percent of GDP in our simulations (Figure 5). This can be interpreted as the desirability of the variable rule over the fixed rule.

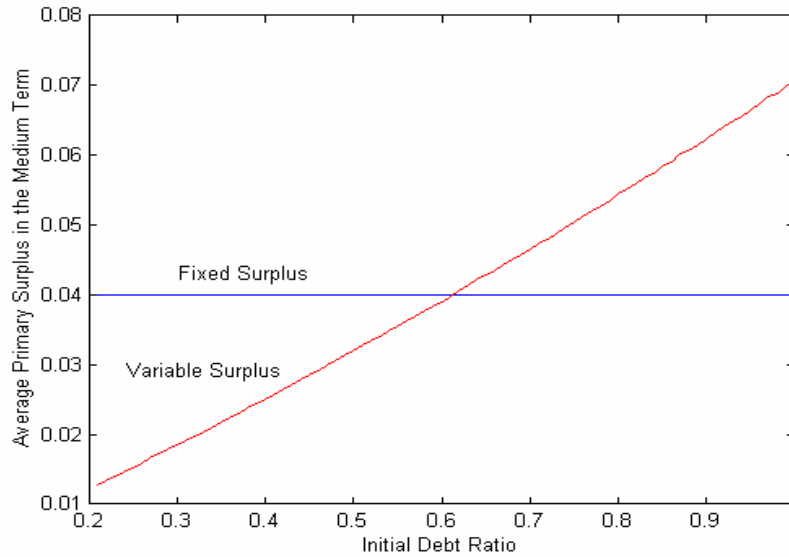


Figure 5. Primary Surplus in the Medium Term

IV. CONCLUDING REMARKS

Using a simple debt dynamics equation and numerical simulations, two alternative fiscal policy rules are analyzed in this paper. The results show that a variable primary surplus rule, defined as an increasing function of the debt ratio, performs better than a fixed primary surplus rule by reducing debt sustainability concerns (measured in this paper by the probability of exceeding the critical debt level) and reducing the necessary level of the medium-term average primary surplus.

The analysis of this study can be extended by distinguishing output, the real interest, the real exchange rate and budget execution shocks and by endogenizing the growth-adjusted real interest rate function. Game theoretic analysis of the credibility issues involved in the selection between the two rules (or among further alternatives) may be another avenue for further research.

References

- Cantor, Richard, and Frank Packer, 1996, "Determinants and Impact of Sovereign Credit Ratings," *FRBNY Economic Policy Review*, Vol. 2 (October), pp. 37–54.
- Catao, L., and Sandeep Kapur, 2004, "Missing Link: Volatility and the Debt Intolerance Paradox," IMF Working Paper 04/51 (Washington: International Monetary Fund).
- Daban, Teresa, and others, 2003, *Rules-Based Fiscal Policy in France, Germany, Italy, and Spain*, IMF Occasional Paper No. 225 (Washington: International Monetary Fund).
- Halpern, Laszlo, and Judith Nemenyi, 2002, "Fiscal Foundations of Convergence to European Union in the Pre-Accession Transition Countries," Discussion Paper 03/02, (Frankfurt: Economic Research Center, Deutsche Bundesbank).
- Hausman, Ricardo, 2002, "Good Credit Ratios, Bad Credit Ratings: the Role of Debt Denomination," (unpublished; Cambridge, Massachusetts: Harvard University).
- Hu, Yen-Ting, Rudiger Kiesel, and William Perraudin, 2001, "The Estimation of Transition Matrices for Sovereign Credit Ratings," *Journal of Banking and Finance*, Vol. 26, No. 7, 1353–1406.
- Kopits, George, 2001, "Fiscal Rules: Useful Policy Framework or Unnecessary Ornament," IMF Working Paper 01/145 (Washington: International Monetary Fund).
- _____, and Steven Symanski, 1998, *Fiscal Policy Rules*, IMF Occasional Paper No. 162 (Washington: International Monetary Fund).
- Larrain, Guillermo, Helmut Reisen, and Julia von Maltzan, 1997, "Emerging Market Risk and Sovereign Credit Ratings," OECD Development Centre Technical Paper No. 124 (Paris: OECD).
- Wyplosz, Charles, 2002, "Fiscal Policy: Rules or Institutions?" CEPR Discussion Paper 3238 (London: Centre for Economic Policy Research).