WP/03/75

# IMF Working Paper

# Job-Specific Investment and the Cost of Dismissal Restrictions—The Case of Portugal

*Hajime Takizawa*

INTERNATIONAL MONETARY FUND

 WP/03/75

# IMF Working Paper

European I Department

# Job-Specific Investment and the Cost of Dismissal Restrictions—The Case of Portugal

Prepared by Hajime Takizawa[1]

Authorized for distribution by Thomas Krueger

April 2003

## Abstract



Using a search and matching labor market equilibrium model, this paper quantifies lost labor productivity and consumption per worker that emerges from the restrictions on dismissals. Dismissal restrictions hamper the efficient reallocation of workers, with workers remaining longer in jobs. But the restrictions also tend to induce job-specific investments. A calibration exercise applied to Portugal suggests that the restrictions on dismissal slow the pace of worker reallocation and cause substantial losses of labor productivity and consumption. Although lower worker mobility induces job-specific investment that offsets part of the labor productivity and consumption losses, the size of this offsetting effect is, at most, modest.

JEL Classification Numbers: E24, J23, J32, J41, J63

Keywords: Dismissal restrictions, worker mobility, job-specific investment, and welfare.

Author's E-Mail Address: htakizawa@imf.org

---

[1] I thank Thomas Kruger, Kenichi Ueda, Luisa Zanforlin, and seminar participants at the IMF for their helpful comments. All remaining errors are my own.

Contents Page

# I. Introduction

Strictness of employment protection measures—dismissal restrictions in particular—and labor mobility are highly correlated, with Portugal representing the firm end of this strong correlation. The OECD ranks the Portuguese labor market as the most restrictive in employment protection among OECD countries, with average job tenure in Portugal among the longest.[2]

Dismissal restrictions are likely to cause labor productivity and welfare losses. Existing theoretical studies argue that low labor mobility due to dismissal restrictions adversely affects labor productivity and welfare by impeding the reallocation of labor to its most productive uses. Existing empirical studies tend to support this theoretical argument, although the size of the labor productivity and welfare losses varies widely. Blanchard and Portugal (2001), for example, report a substantial output loss (14 percent), resulting from the dismissal restrictions in Portugal. Millard and Mortensen (1997) quantify the welfare loss of a dismissal tax (measured in consumption per participating worker) in the United Kingdom and the United States as 2 percent and less than 1 percent, respectively. Both types of studies, however, have largely ignored productivity-enhancing, job-specific investment that is likely to be higher when labor is less mobile.

The two main purposes of this paper are (i) to analyze how job-specific investment affects the size of labor productivity and welfare losses due to dismissal restrictions and (ii) to quantify those losses in Portugal. In sum, the analysis finds large labor productivity and welfare losses and a small offsetting effect of job-specific investment. The estimated labor productivity and welfare losses due to dismissal restrictions amount to about 35 percent and 31 percent, respectively, where the losses are defined as the lost labor productivity and consumption that would have been available had there been no restrictions. The offsetting effect of additional job-specific investment is limited to 7 percent of the labor productivity loss and 3 percent of the welfare loss.

The analysis proceeds in three steps. The first is to develop an equilibrium model of the labor market characterized by search and matching frictions. The second step retrieves key parameter values by calibrating the model, where the calibration matches the labor market performance implied by the model to the performance observed in the data for Portugal. The third step computes the steady state equilibrium of the model and measures the welfare loss.

The reason behind the large estimated labor productivity and welfare losses is that dismissal restrictions slow the pace at which workers in unproductive jobs are dismissed and placed into high-productivity jobs. The worker reallocation process, therefore, is the key mechanism that generates the labor productivity and welfare losses. Job-worker matches facing uncertainties about future idiosyncratic productivity are dissolved when sufficiently low idiosyncratic productivity levels are realized and thus expected profits and incomes from the job fall below what employers and workers can expect from separating. Dismissal

[2] Evidence is reported in OECD (1999).

restrictions discourage this type of separation by raising employers' cost of separation and thus lowering the threshold productivity level below which job-worker matches separate. With many job-worker matches remaining with low idiosyncratic productivity under dismissal restrictions, the aggregate labor productivity is lower than it would be without restrictions.

The important mechanism underlying the offsetting effect of job-specific investment is the way labor mobility affects job-specific investment that determines productivity of individual jobs. Even under no dismissal restriction, employers and workers may make a job-specific investment to raise productivity, as long as both parties find it optimal not to immediately walk away from their respective partners. The longer the expected job duration is, the higher is the expected return of job-specific investment. The resulting additional job-specific investment offsets at least part of the output loss associated with lower labor mobility.

This paper is organized as follows. Section II develops an equilibrium search and matching model that incorporates a dismissal restriction and job-specific investment. Section III calibrates the model and then quantifies the welfare loss associated with dismissal restrictions in Portugal. Section IV concludes the analysis.

## II. An Equilibrium Labor Market Model

The analysis in this paper builds on the search and matching equilibrium labor market model developed by Mortensen and Pissarides (1994). The model developed in this paper augments the original model by incorporating dismissal restrictions and job-specific investment.[3]

Search and matching frictions in labor markets (labor market frictions) are essential to analyze the welfare implications of dismissal restrictions and job-specific investment. Limited labor mobility due to labor market frictions gives rise to worker flows between employment and unemployment. In contrast, any model without limited labor mobility fails to properly define worker flows. Since analyzing the effect of dismissal restrictions entails examining the size of worker flows, the model needs to build on search and matching frictions. Labor market frictions also justify analyzing job-specific investment. Only under limited labor mobility can individual workers be thought of as filling particular job positions. This concept of a match between a worker and a job forms the basis of job-specific investment.

This section first discusses individual decision problems in a labor market with frictions, followed by the discussion of the wage-setting mechanism. It then defines the concept of a steady state labor-market equilibrium and various equilibrium outcomes.

[3] Mortensen and Pissarides (1999) work out the model incorporating dismissal restrictions. The extension of this paper from their work is the introduction of job-specific investment into their framework.

### A. Frictional Labor Markets and Individual Decision Problems

This section discusses decision problems faced by employers and workers in a frictional labor market.

Several assumptions are made. Time is continuous. The economy is populated by a continuum of homogenous workers whose lives are infinite. The measure of workers in the economy is constant and is normalized to one. The economy is also populated by a large number of firms (henceforth interchangeably referred to as employers). Workers and employers maximize the expected present value of incomes and profits, respectively, taking aggregate variables in the economy as given.

**Formation of job-worker matches and job-specific investments**

The production technology is simple. A job filled by a worker produces an output whose amount depends on a common technology level $A$ and a job-specific investment $k$. $A$ is a constant. The worker is paid a wage, and the employer obtains the residual value of the output. However, the labor market is characterized by search and matching frictions. Hence, when employers create a job, the job is initially vacant. To engage in a production activity that generates profits, employers need to successfully recruit a worker in the labor market to fill the vacant job. Similarly, some workers are unemployed while others are employed. To become employed, the unemployed need to successfully find a job. For simplicity, it is assumed that employed workers are not able to search for a job.

The search and matching process is modeled as a matching function that determines the rates of meeting as a function of search efforts by employers and workers. Underlying this way of modeling is an idea that the matching process is an economic activity that requires economic resources. Employers looking for a worker to fill their vacant jobs post a vacancy by incurring a flow cost $c$. Recruitment intensity is assumed to be constant. Unemployed workers may search for a job without incurring any cost, but only at a fixed intensity. Hence, the unemployed always search for a job. The aggregate search effort by employers is represented by the number of vacancies, $v$, and the aggregate search effort by workers is represented by the measure of the unemployed, $u$. The ratio $v/u$ represents the tightness of the market in which potential employers and job-seeking workers randomly meet. Henceforth, the tightness $v/u$ is denoted by $\theta$. The rate at which a job-seeking worker finds a job is a function of $\theta$ and is denoted by $m(\theta)$. Constant returns to scale property of the matching function—as often assumed and estimated in the literature[4]—implies that the rate at which a vacancy is filled by a job-searching worker is $m(\theta)/\theta$.

---

[4] For estimating a matching function, see Blanchard and Diamond (1989), Pissarides (1986), and Van Ours (1991).

While unemployed, workers enjoy leisure whose value is denoted by $b$. Unemployed workers search for a job and accept a job offer if, and only if, doing so yields the present value of expected incomes higher than what they could expect by remaining unemployed.

After a match is formed, the employer and the worker decide how much job-specific investment $k$ to make. An example of job-specific investment is an investment in a worker's skill and knowledge that can be effectively used only with a particular employer. For simplicity, investment is assumed to remain fixed, to permanently raise productivity, and to be made and to become effective immediately after the job-worker match is formed. As will become clear later, the employer and the worker end up sharing fixed fractions of the return and the cost of investment, regardless of who pays the cost. In what follows, it is assumed that employers pay all the cost and optimally choose job-specific investment. A $k$ unit investment costs $t(k)$ and yields a return $k^{\alpha}$ (where $\alpha$ satisfies $0<\alpha<1$) that multiplicatively raises the output of the job. Hence, a match with $k$ unit job-specific investment produces $Ak^{\alpha}$ unit output. Marginal cost $t'(k)$ is increasing while marginal return $\alpha k^{\alpha-1}$ is decreasing. It becomes clear later that job-specific investment levels are the same across all jobs in the economy.

The behaviors of employers and workers described so far imply that the value of unemployed search to a worker and the value of a vacant job to an employer satisfy asset pricing equations (Bellman equations). The value of unemployed search is denoted by $U$. By letting $r$ denote interest rate, the flow value of unemployed search $rU$ is written as the sum of the flow value of leisure and the option value of securing a job in the future:

$$rU = b + m(\theta)\max\left\langle W_o - U, U\right\rangle, \tag{1}$$

where $W_o$ is the value of a new match to a worker that is defined later.

The flow value of a vacant job is the sum of the flow cost of posting a vacancy and the capital gain that would be realized when the vacant job is filled. The pricing equation for the value of a vacant job, $V$, can be written as

$$rV = -c + \frac{m(\theta)}{\theta}\max\left\langle \max_k J_o - V - t(k), V\right\rangle, \tag{2}$$

where $J_o$ is the value of a new job to an employer that is defined later. Here, the cost of job-specific investment is subtracted from the capital gain because the employer incurs the cost of investment upon forming a match.

**Separation of job-worker matches**

While workers are assumed to be homogenous, the model allows heterogeneity in job-worker match productivity. Productivity of individual job-worker matches—the value of $Ak^{\alpha}$ unit product produced by the match—is denoted by $Apk^{\alpha}$. $p$ is an idiosyncratic part of productivity and stochastically changes over time, reflecting shocks to preference and technology. An idiosyncratic productivity shock refers to a realization of a new value of $p$. Henceforth, $p$ is referred to as idiosyncratic productivity unless confusion arises. New matches are assumed to be most productive, based on an idea that a newly created job-worker match can take advantage of the latest technology and can choose the product that commands the highest value in the market.[5] Formally, idiosyncratic shocks to productivity levels arrive at a Poisson rate $\lambda$. The cumulative distribution function from which a new value of $p$ is drawn in response to an idiosyncratic shock is denoted by $F(\cdot)$. The upper bound and the lower bound of the support of $F$ are denoted by $\overline{p}$ and $\underline{p}$, respectively.

Struck by idiosyncratic productivity shocks, jobs eventually become unproductive, and either the employer or the worker or both sides of a match choose to separate rather than to continue the employment relationship. The employer of a job-worker match dismisses the worker when the present value of expected future profits from the match turns negative. Employed workers, who earn productivity contingent wages, find it optimal to leave the job for an unemployed job search if productivity of a job becomes sufficiently low. Formally, separation happens if, and only if, the value of a match to either of the agents falls below the value obtained by separating. As will become clearer later in the analysis, this separation rule implies that separation happens when productivity falls below a common reservation productivity level, $R$.[6] Hence, the frequency of dismissal is equal to $\lambda F(R)$.

It is assumed that separation is also caused by reasons other than adverse idiosyncratic productivity shocks, such as resignations by workers due to family reasons. This type of match separation may be referred as *exogenous separation* and is assumed to randomly happen at a Poisson rate $\delta$.

Job-worker matches are subject to dismissal restrictions. The dismissal restriction analyzed in this paper is a penalty imposed on the employer for terminating a job-worker match. Essential to the analysis is an assumption that the penalty is a real cost incurred by an employer and is not a severance transfer from an employer to a worker. Indeed, the dismissal penalty does not have to be a monetary penalty. Advance notification requirements with a minimum period and justification requirements, for example, have the same effect as a monetary penalty because they impose a real cost to employers. Henceforth, the dismissal penalty is denoted by $T$.

Since job-worker match productivity $p$ and job-specific investment $k$ determine employers' profits and workers' wage incomes, the values of a job-worker match to both the employer and the worker of a match are contingent on a pair $(p,k)$ of the match. Both values satisfy asset pricing equations.

---

[5] This assumption is used in Mortensen and Pissarides (1994), as well in papers by other authors. The assumption simplifies the analysis because the value of newly created matches is a constant as long as no aggregate uncertainty exists.

[6] This reservation property follows from the fact that the value function of continuing match is increasing in productivity.

The flow value of a job-worker match to a worker is the sum of three parts: a flow wage contingent on productivity and investment level; the option value associated with the arrival of an idiosyncratic productivity shock; and the capital loss due to an exogenous separation. Formally, the value of a *new* match to a worker, $W_o(\bar{p},k)$, satisfies the following pricing equation:

$$rW_o(\bar{p},k)=\omega_o(\bar{p},k)+\lambda\int_{\underline{p}}^{\bar{p}}\max\langle W(x,k),U\rangle dF(x)-\delta[W(p,k)-U]. \tag{3}$$

The first term on the right hand side of the equation is the initial wage that is a function of productivity $p=\bar{p}$ and job-specific investment $k$. This wage rate is negotiated before the formation of the match and is not renegotiated until new productivity arrives. The second term is the option value associated with the arrival of an idiosyncratic productivity shock, and the third term is the capital loss associated with an exogenous separation. The second term takes into account the worker's option to choose between remaining in the job and leaving to become unemployed. Since job-specific investment is made at the onset of the match and remains fixed thereafter, the job-specific investment $k$ is invariant throughout the life of the job. In particular, $k$ remains the same even after an idiosyncratic productivity shock arrives as shown in the expression in the integrand.

The pricing equation for the value of a *continuing* match, $W(p,k)$, can be written in a similar manner to that of equation (3):

$$rW(p,k)=\omega(p,k)+\lambda\int_{\underline{p}}^{\bar{p}}\max\langle W(x,k),U\rangle dF(x)-\delta[W(p,k)-U]. \tag{4}$$

The only difference between equation (4) and equation (3) is the wage rates. The difference of those wages is discussed later.

The flow value of a filled job to an employer is the sum of three parts, just as is the flow value of a job to a worker. The first part is a flow profit contingent on productivity and investment, and the second part is the option value associated with the arrival of an idiosyncratic productivity shock. The third part is the capital loss associated with an exogenous separation. The value of a newly filled job, $J_o(\bar{p},k)$, solves the following equation:

$$rJ_o(\bar{p},k)=A\bar{p}k^{\alpha}-\omega(\bar{p},k)+\lambda\int_{\underline{p}}^{\bar{p}}\max\langle J(x,k),V-T\rangle dF(x)-\delta\left[J_o(\bar{p},k)-V\right]. \tag{5}$$

The first two terms on the right hand side are profits. The second term is the option value associated with the arrival of an idiosyncratic shock, and the third term is the capital loss associated with an exogenous separation. The second term takes into account the dismissal penalty. Should the employer decide to dismiss the worker, the employer incurs the penalty $T$.

The pricing equation for the value of a *continuing* match, $J(p,k)$, can be written in a similar manner to that of equation (5):

$$rJ(p,k) = Apk^{\alpha} - \omega(p,k) + \lambda\int_{\underline{p}}^{\overline{p}} \max\langle J(x,k), V - T\rangle dF(x) - \delta\left[J(p,k) - V\right]. \qquad (6)$$

The only difference between equation (5) and equation (6) is the wage rates.

The value functions that satisfy equations (1)–(6) are the bases for characterizing decision problems faced by employers and workers.

### B. Match Surplus and Wage Rate

Under search and matching frictions, the wage rate of a job reflects not only the productivity of the job but also the value of a worker's outside option and bargaining power. Underlying this property is the fact that job-worker matches have a rent—henceforth referred to as *match surplus*—because both employers and workers of job-worker matches cannot expect the values that are higher than what they are currently enjoying by simply walking away from their employment relationships. The wage rate of a job must somehow divide the match surplus between the employer and the worker. How is the match surplus defined? How is the wage rate determined?

The match surplus for a new job-worker match, $S_o(\overline{p},k)$, is the value of expected profits net of the cost of a job-specific investment, $J_o(\overline{p},k) - t(k)$, plus the value of expected incomes, $W_o(\overline{p},k)$, minus the value of what both parties would get if they chose not to form a match—that is, the value of a vacancy ($V$) and the value of unemployed search ($U$). Hence, the initial match surplus can be written as

$$S_o(\overline{p},k) = J_o(\overline{p},k) - t(k) + W_o(\overline{p},k) - V - U. \qquad (7)$$

The match surplus for a continuing match, denoted by $S(p,k)$, is somewhat different from the one for a new match. On the one hand, the cost of job-specific investment is already sunk. On the other hand, the employer now faces a dismissal penalty $T$. Match surplus, therefore, is the sum of the present value of future employers' profits and workers' incomes plus $T$ minus the values of a vacancy and unemployed search:

$$S(p,k) = J(p,k) + W(p,k) - (V - T) - U. \qquad (8)$$

Rearranging equations (1)-(6) as discussed in the Appendix, one obtains the surplus functions

$$S_o(\overline{p},k) = \frac{A(\overline{p} - R)k^{\alpha}}{r + \lambda + \delta} - (t(k) + T) \qquad (9)$$

and

$$S(p,k) = \frac{A(p - R)k^{\alpha}}{r + \lambda + \delta}. \qquad (10)$$

The surplus for a continuing job-worker match, $S(p,k)$, is the present value of the difference between idiosyncratic productivity $p$ and reservation productivity $R$. $S(p,k)$ is increasing in $p$. The surplus for a new job-worker match, $S_o(\bar{p},k)$, subtracts the cost of job-specific investment and dismissal penalty from the present value of $A(p-R)k^{\alpha}$.

This paper assumes that the productivity contingent wage rate, denoted by $\omega(p,k)$ (or $\omega_o(p,k)$ in the case of the initial wage), is set to give workers a fixed share $\beta$ of the match surplus. In other words, $\omega(p,k)$ and $\omega_o(p,k)$ support the following rule:

$$W_o - U = \beta S_o \tag{11}$$

and

$$W(p,k) - U = \beta S(p,k). \tag{12}$$

The wage-setting rule, together with the property that $S(p,k)$ is increasing in $p$, implies that match separation takes place if and only if productivity falls below the reservation productivity $R$. One can establish this result as follows. $S(p,k)$ is monotonically increasing in productivity $p$. As already discussed, both $J(p,k) - (V - T)$ and $W(p,k) - U$ are proportional to $S(p,k)$ by virtue of the way the wage rate is determined. Since $V - T$ and $U$ are invariant in $p$, the values $J(p,k)$ and $W(p,k)$ are increasing in $p$. Hence, separation decisions by both employers and workers satisfy the reservation property. Furthermore, the separation decision is unanimous because of the proportionality of $J(p,k) - (V - T)$ and $W(p,k) - U$ to $S(p,k)$, establishing the claim. Also note that $R$ satisfies $J(R,k)-(V-T)=0$ and $W(R,k)-U=0$.

This assumed wage-setting rule is appealing in several aspects. First, the outcome of an axiomatic generalized Nash bargaining yields the assumed wage-setting rule.[7] This assumed wage-setting rule can also be achieved as a perfect equilibrium of a noncooperative strategic bargaining game, providing a micro-foundation. Second, match formation and separation decisions are unanimous under the assumed wage-setting rule. Both parties never disagree when making those decisions. This property supports the supposition that the objective of individual agents is to maximize the expected values of profits and incomes. If decisions were not unanimous, either of the parties who prefers continuing the match would have an incentive to give up part of the surplus for the other party in order to do this.

Wage negotiations for new matches and continuing matches that support the assumed wage-setting rule have different forms. The wage of a continuing match is immediately renegotiated in response to idiosyncratic productivity shocks to guarantee a constant share of the match surplus to the worker and the employer. The wage of a new match (also referred to as *initial wage*) is negotiated to split the surplus $S_o$ *before* a job-specific investment is made and a dismissal penalty becomes effective. As mentioned previously, this paper assumes that the initial wage is not renegotiated until an idiosyncratic productivity shock arrives. The

[7] Details are provided in the Appendix.

assumed timing of wage negotiation reflects an idea that wages are likely to be negotiated before the match is actually formed.[8]

Formally, wage rates can be derived from the outcomes of the generalized Nash bargaining over augmented productivity $Apk^{\alpha}$ given a threat point equal to the flow values of alternatives.[9] The initial wage is

$$\omega_o(\bar{p},k) = rU + \beta\left[A\bar{p}k^{\alpha} - r(V+U) - \lambda T - (r+\delta+\lambda)t(k)\right], \tag{13}$$

while the continuing wage is

$$\omega(p,k) = rU + \beta\left[Apk^{\alpha} - r(V-T) - rU + \delta T\right]. \tag{14}$$

The initial wage $\omega_o(p,k)$ is equal to $rU$ plus the worker's share $\beta$ of the initial flow surplus, $A\bar{p}k^{\alpha} - rV - rU - rt(k)$, less the worker's share $\beta$ of the costs of job-specific investment and dismissal amortized over the period between the formation of the match and the arrival of an idiosyncratic shock, $\lambda T + (\lambda+\delta)t(k)$. Since employers do not incur the cost $T$ when separating due to an exogenous separation (at a rate $\delta$), the amortized cost of $T$ differs from that of $t(k)$. The initial wage in equation (13) implies that workers share the cost of job-specific investment and dismissal penalty. The continuing wage $\omega(p,k)$ is equal to $rU$ plus the worker's share of the surplus, $Apk^{\alpha} - r(V-T) - rU$, plus the adjustment made to take into account the fact that employers do not incur the cost $T$ when workers leave the job due to exogenous events.

The wage-setting rule just discussed illustrates the difference between a pure dismissal restriction and a severance payment. Since pure dismissal penalties ($T$) affect the match surplus $S(p,k)$ and the wage rate $\omega(p,k)$, and since the reservation productivity level ($R$) is defined as $S(R,k) = 0$, dismissal penalties directly affect agents' separation decisions. On the contrary, if $T$ is simply a severance payment, the separation decision is not directly affected by $T$ because the severance payment does not affect the surplus $S(p,k)$. The severance payment, however, still has an adverse effect on the employer's job creation decision because it raises productivity contingent wage rates by lowering the employer's

[8] The assumption that no renegotiation happens until an idiosyncratic productivity shock arrives might appear to be at odds with the fact that the match surplus is $S(\bar{p},k)$ rather than $S_o(\bar{p},k)$ once job-specific investment is made and a dismissal penalty becomes effective. If the wage rate were renegotiated instantaneously after a match is formed, however, the wage bargaining that takes place before the match formation would have no effect on equilibrium. This, however, contradicts the idea of the assumed timing of wage bargaining.

[9] A formal derivation is provided in the Appendix.

threat point of the Nash wage bargaining for continuing matches.[10] While it is straightforward to analyze the effect of severance payments, the following analysis considers only dismissal penalties.

### C. Steady State Equilibrium

This section spells out equilibrium conditions that endogenous variables must jointly satisfy in a steady state equilibrium. Employers and workers make three decisions. The first decision is a job creation decision, the second one is an optimal investment decision, and the third one is a separation decision. The optimality conditions for those three decisions pin down the values of the endogenous variables of the model, that is, market tightness $\theta$, job-specific investment $k$, and reservation productivity $R$.

The first equilibrium condition, the job creation condition, is implied by the profit maximization by potential employers. It is assumed that potential employers are free to post a vacancy. Under this assumption, any expected positive profit induces potential employers to post additional vacancies. As the number of vacancies increases, the expected profit falls. In equilibrium, the competition among potential employers drives down the expected profit from creating a vacancy to zero, and no employer can expect a positive profit by creating a vacancy, i.e., $V=0$. By virtue of the asset pricing equation (2) for $V$, the initial surplus in equation (9), and the wage-setting rule in equation (11) that splits the initial surplus, the job creation condition can be written as

$$c\frac{\theta}{m(\theta)}=(1-\beta)\left\{\frac{A(\overline{p}-R)k^{\alpha}}{r+\lambda+\delta}-(t(k)+T)\right\}. \tag{15}$$

Since $\theta/m(\theta)$ is the average duration of a vacancy, the left hand side is the expected cost of posting and filling a vacancy. The right hand side is the employer's share of the initial match surplus $S_o(\overline{p},k)$.

The second equilibrium condition, the optimal investment condition, is implied by the joint surplus maximization by employers and workers. Since employers and workers take a fixed fraction $1-\beta$ and $\beta$ of the surplus, respectively, and since the job-specific investment is made once and for all, the optimal investment level should maximize the initial match surplus $S_0(\overline{p},k)$. By virtue of the expression of $S_0(\overline{p},k)$ derived in equation (9), the optimality leads to the following first order necessary condition that the optimal investment level must satisfy

[10] This result contrasts with a well-known finding by Lazear (1990) that severance payment has no effect on job creation and separation decisions. Underlying Lazear's result is the assumption that before forming a match, workers compete with each other to compensate employers upfront for severance payment obligations. It is possible to consider such upfront payments in the present model. In the equilibrium search framework, such as the one used in this paper, however, it is not sensible to consider such competition for upfront compensation since search frictions most likely prevent such competition.

$$\frac{A\left(-\frac{\partial R}{\partial k}k^{\alpha}+(\overline{p}-R)\alpha k^{\alpha-1}\right)}{r+\lambda+\delta}=\frac{\partial t(k)}{\partial k}. \tag{16}$$

The left hand side is the present value of marginal flow return of the investment while the right hand side is the marginal cost of the investment. The partial derivative $\partial R/\partial k$ is obtained by totally differentiating the other two equilibrium conditions with respect to $k$, as discussed in the Appendix.

The third equilibrium condition, the job destruction condition, follows from the optimal separation decision by employers and workers. Since $S(p,k)$ is increasing in $p$, the reservation productivity level $R$ is defined as idiosyncratic productivity that satisfies $S(R,k) = 0$. Rearranging asset pricing equations (1), (2), (4), and (6), one obtains the asset pricing equation for $S(p,k)$:

$$(r+\lambda+\delta)S(p,k)=Apk^{\alpha}-\frac{\beta c\theta}{1-\beta}+(r+\delta)T+\lambda\int_{R}^{\overline{p}}\frac{A(x-R)k^{\alpha}}{r+\lambda+\delta}dF(x). \tag{17}$$

Hence, the condition is written as

$$\frac{\beta c\theta}{1-\beta}=ARk^{\alpha}+(r+\delta)T+\lambda\int_{R}^{\overline{p}}\frac{A(x-R)k^{\alpha}}{r+\lambda+\delta}dF(x). \tag{18}$$

The left hand side of the condition is a closed form solution for $rU+rV$ under the restriction $V=0$. The condition equates this flow value of separating to an employer and a worker to the flow value of not separating. The first term on the right hand side is the value of output at $p=R$. The second term is the flow cost of dismissal that can be saved by not separating. The third term represents the capital gain associated with the arrival of an idiosyncratic shock. The integrand is the match surplus of a job whose productivity is $x$ as derived in (10).

Now that the conditions that the endogenous variables have to satisfy in a steady state equilibrium are derived, such an equilibrium can be defined.

**Definition**: A steady state equilibrium is a triple $(\theta^*,R^*,k^*)$ that solves three equilibrium conditions (15), (16), and (18).

Given parameter values of the model, equations (15), (16), and (18) form a system of nonlinear equations. A solution of this system $(\theta,R,k)$ that satisfies $0<\theta$, $\underline{p}\le R<\overline{p}$, and $0<k$ is a nontrivial steady state equilibrium.[11]

---

[11] Unlike the case presented in the original model studied by Mortensen and Pissarides (1994), equilibrium is not unique for certain economies in the present model. In the present model, low reservation productivity and high job-specific investment reinforce each other. The higher the job-specific investment is, the lower is the reservation productivity level because both employers and workers expect a greater benefit of waiting for a favorable idiosyncratic productivity shock and postponing separation. However, the lower the reservation productivity level is, the higher is the job-specific investment because both employers and workers expect a

Figure 1 illustrates the equilibrium conditions. Given $k$, the loci of job creation condition (15) and job destruction condition (18) are drawn in the $R$-$\theta$ plane. The job creation condition implies a downward sloping curve while the job destruction condition implies an upward sloping curve. The intersection of the loci, $(\theta^*, R^*)$, and $k^*$ that collectively satisfy (15), (16), and (18) corresponds to an equilibrium. The figure shows the equilibrium of the model calibrated to match the U.S. economy as will be discussed in Section III.

A comparative statics exercise highlights some effects of a dismissal restriction $T$ on labor market outcomes. An increase in $T$ reduces both the reservation productivity level ($R$) (and thus worker turnover rates) and the rate at which an unemployed worker successfully finds a job (referred to as *unemployment hazard*) for a *given* level of job-specific investment. A lower reservation productivity level follows from the fact that separation is more costly with a higher $T$. Unemployment hazard is lower because employers put less effort to recruit a worker expecting that a job to be created will face a costly dismissal penalty in future. The net effect on unemployment is ambiguous.

### D. Labor Market Outcomes

This section defines various measures of labor market outcomes and macroeconomic performance. Those measures can be computed for any steady state equilibrium and are used to compare equilibria of different economies.

The unemployment rate is implied by the steady state condition. When worker flows into unemployment are equal to flows out of unemployment, unemployment is in a steady state. Since flows into unemployment and flows out of unemployment are equal to $(\delta+\lambda F(R))(1-u)$ and $m(\theta)u$, respectively, the steady state unemployment rate reduces to

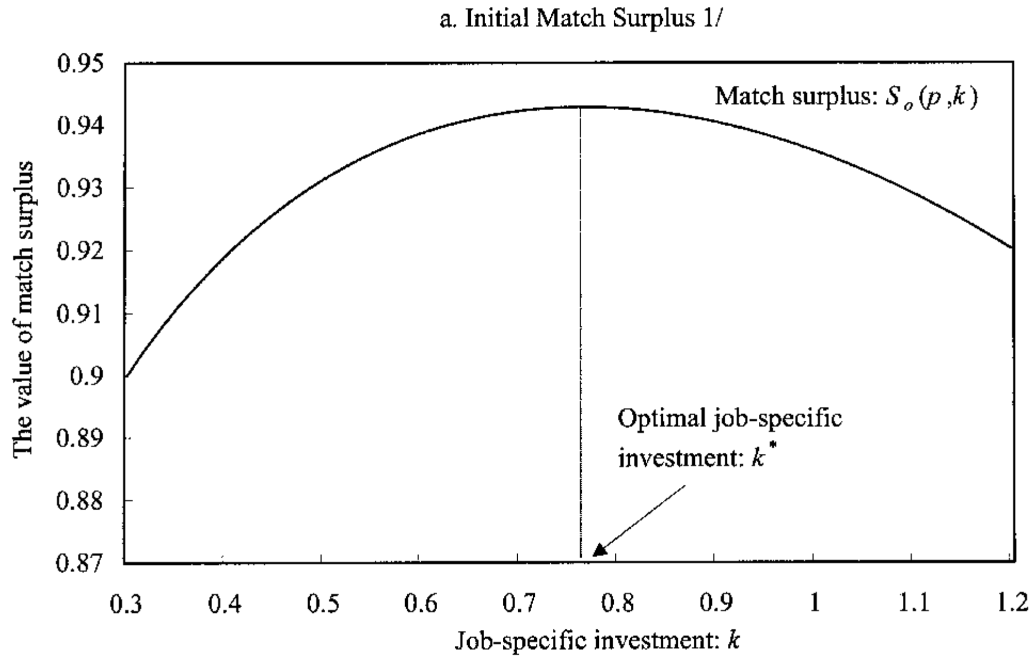
$$u = \frac{\delta + \lambda F(R)}{m(\theta) + \delta + \lambda F(R)}, \tag{19}$$

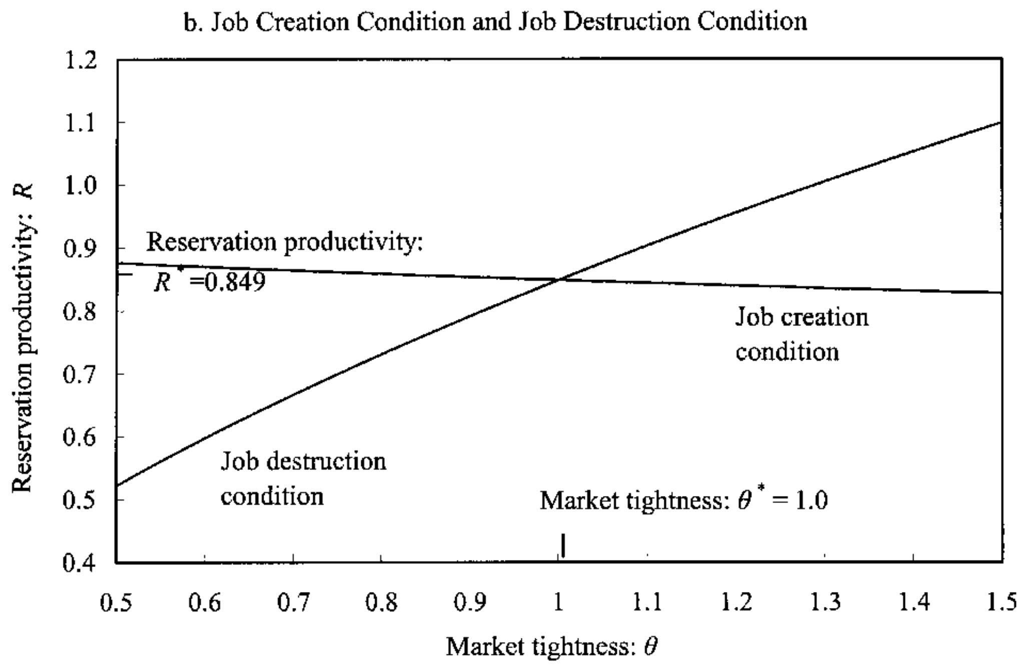
which is a function of the endogenous variables $\theta$ and $R$.

Aggregate output is defined as average productivity of job-worker matches weighted by the distribution of employment over productivity. By letting $n(\cdot)$ denote the employment density over productivity, the aggregate output can be written as

higher benefit of making job-specific investments. This feedback process is the potential source of multiple equilibria. It is possible to find a set of parameter values that leads to two equilibria—a corner solution (high investment equilibrium) and an interior solution (low investment equilibrium). Furthermore, equilibria can be ranked in the Pareto sense. Multiple equilibria is a secondary issue in this paper and is not discussed below.

Figure 1. Equilibrium of the Model: the United States




1/ For each value of job-specific investment $k$, the value of $S_o(p,k)$ is evaluated by solving for a pair of market tightness and reservation productivity $(\theta, R)$ from the steady state equilibrium conditions (15) and (18) given $k$.

$$Ak^{\alpha}\int_{R}^{\overline{p}} x\cdot n(x)dx. \tag{20}$$

Labor productivity of the economy is computed as the aggregate output minus the aggregate cost of posting vacancies and the cost of job-specific investments, divided by the size of the employed:

$$\frac{Ak^{\alpha}\int_{R}^{\overline{p}} x\cdot n(x)dx - c\cdot v - \lambda F(R)(1-u)T}{1-u}. \tag{21}$$

$c\cdot v$ denotes the aggregate cost of vacancy posting. Since the number of matches that separate at any point in time due to unfavorable idiosyncratic shocks is $\lambda F(R)(1\text{-}u)$, $\lambda F(R)(1\text{-}u)T$ is the aggregate cost of dismissal.

Consumption per participating worker is the comprehensive measure of the welfare of the model economy. Since all agents are risk neutral and no assets are accumulated in the model economy, one can properly measure the welfare of an economy by measuring how much a participating worker can consume. Consumption per participating worker is equal to the aggregate output plus the value of aggregate leisure minus the aggregate cost of posting vacancies minus the aggregate cost of job-specific investment minus the aggregate dismissal cost.[12] Since the number of matches formed at any point in time is $m(\theta)u$, the consumption per participating worker can be written as

$$Ak^{\alpha}\int_{R}^{\overline{p}} x\cdot n(x)dx + b\cdot u - c\cdot v - t(k)\cdot m(\theta)u - \lambda F(R)(1-u)T, \tag{22}$$

where $b\cdot u$ denotes the aggregate value of leisure enjoyed by unemployed workers.

Since all newly created jobs start with productivity $\overline{p}$, and since the probability that jobs with productivity $\overline{p}$ change their productivity at any point in time is less than one, the employment density $n$ has a mass at $p=\overline{p}$.

## III. The Macroeconomic Effect of Dismissal Restrictions

This section calibrates the model to match the stylized data for Portugal and a country chosen for a comparison and tries to answer two questions. One question is how much welfare loss the Portuguese suffer due to dismissal restrictions, and the other question is to what extent job-specific investment offsets the welfare loss resulting from the dismissal restrictions. The

---

[12] An alternative definition results from not netting out the aggregate dismissal penalties. Such a definition implicitly assumes that the penalty is in the form of a dismissal tax and that all tax revenues are transferred to consumers on a lump sum basis. Treating the dismissal penalty $T$ as a dismissal tax changes none of the decision problems and none of the equilibrium conditions.

estimated welfare loss is substantial, and the offsetting effect due to job-specific investment is marginal.

## A. Calibration

The calibration exercise retrieves parameter values of the model by comparing equilibrium outcomes of two model economies with the stylized facts of two actual economies. As previously mentioned, the Portuguese economy is the focus of the analysis. Since the United States is ranked by the OECD as the least restrictive economy in employment protection legislations, the United States is used as a country of comparison. It is assumed that the Portuguese economy and the U.S. economy are identical except for the workers' bargaining power ($\beta$), the dismissal penalty ($T$), and the common technology level ($A$). Consequently, any difference in equilibrium outcomes across the two economies in the model reflects differences in $\beta$, $T$, and $A$. Because of this simplified assumption, the finding of the analysis should not be regarded as a definite conclusion.

The calibration retrieves five parameters: (i) the workers' bargaining power $\beta$ and (ii) the dismissal penalty $T$ for Portugal; (iii) the difference in common technology level $A$ between Portugal and the United States; (iv) the elasticity of investment return $\alpha$; and (v) the value of leisure $b$. The calibration iterates the following two-step procedure. In the first step, $\alpha$ and $b$ are retrieved by matching the model to the U.S. economy. Given particular values for $\beta$, $T$, and $A$ that are set a priori for the U.S. economy, the values for $\alpha$ and $b$ are obtained by matching the unemployment rate and the average unemployment duration implied by the model to those actually observed in the United States. The values for $\alpha$ and $b$ also characterize the Portuguese economy because the two economies are assumed to be the same except for $\beta$, $T$, and $A$. In the second step, $\beta$ and $T$ for Portugal are retrieved. Given $\alpha$ and $b$ that are just retrieved, as well as $A$ that is set a priori for the Portuguese economy, the values for $\beta$ and $T$ in Portugal can be obtained by matching the unemployment rate and the average unemployment duration implied by the model to those actually observed in Portugal. Once parameters are retrieved, labor productivity implied by the model as in the formula (21) can be computed for each economy. If the difference in labor productivity between the two economies implied by the model does not match the difference actually observed in the data, the two-step procedure is performed again with a new value for $A$ for the U.S. economy. The iteration of the two-step procedure continues until the difference in labor productivity implied by the model matches the difference actually observed.

Model parameters that are not retrieved by the calibration need to be set to certain values. $\beta$ and $T$ for the U.S. economy are one set of such parameters. While noncooperative bargaining theory implies a 50 percent workers' share, a higher value of 0.65 is assumed for the following exercise.[13] Since the OECD (1999) ranks the U.S. economy as the least restrictive

[13] In part, the choice of $\beta$ is limited by the model's ability to replicate the difference in labor productivity across the two countries. For a range of values for $\beta$, the model fails to replicate the observed difference in labor productivity.

economy in terms of employment protection legislations, the dismissal penalty $T$ is assumed to be zero. The remaining parameters to be set are the exogenous separation rate $\delta$, the idiosyncratic productivity shock arrival rate $\lambda$, the interest rate $r$, and the vacancy posting cost $c$. For those parameters, this paper uses the same values as those used in existing studies. Based on survey data, Millard and Mortensen (1997) use $c = 0.33$ a quarter for the United States and the United Kingdom—the same value used in this paper. Following Millard and Mortensen (1997), the exogenous separation rate $\delta$ and the idiosyncratic shock arrival rate $\lambda$ are set to 1.5 percent per quarter and 10 percent per quarter, respectively. The quarterly interest rate $r$ is set to 1 percent, which is the value frequently used when calibrating neoclassical growth models. The common technology level ($A$) for Portugal is normalized to 1.

In order to compute the model, one needs to specify the matching function $m(\theta)$, the cost function $t(k)$, and the distribution $F$. In line with existing studies, a functional form $m(\theta) = \theta^{\eta}$ is assumed for the matching function. Underlying this functional form is an assumption that the matching function is constant returns to scale. Estimates of the matching function elasticity in empirical studies vary between 0.4 and 0.6.[14] This paper uses a middle point of $\eta = 0.5$. The cost function is parameterized as a quadratic function $t(k) = k^2$. Idiosyncratic shocks to productivity of existing jobs are assumed to be uniformly distributed between 0 and 1.

The unemployment rate and the average unemployment duration observed in the data form the basis of the calibration. Average unemployment rates in both the United States and Portugal during a period from 1986–2000 were 6.0 percent, according to the annual labor force survey compiled by the OECD. However, basing the calibration on the actual unemployment rate is inaccurate. The class of the models similar to the one in this paper does not allow worker flows between participation and nonparticipation. This simplification leads to an unemployment rate higher than the one that would follow if flows into and out of nonparticipation were allowed. In order to adjust for this bias, Blanchard and Portugal (2001) reconstructed the unemployment rates in Portugal and the United States as the product of the average unemployment duration and total flows out of employment.[15] The resulting unemployment rates were 9.0 percent for both countries. Therefore, this paper uses 9.0 percent as the basis of the calibration. As regards average unemployment duration, both publications by Millard and Mortensen (1997) and Blanchard and Portugal (2001) use one-quarter average unemployment duration for the U.S. economy as the basis of their calibration exercises. Blanchard and Portugal (2001) report and use three-quarters average unemployment duration for the Portuguese economy. Therefore, this paper uses one-quarter average unemployment duration for the United States and three-quarters average unemployment duration for Portugal.

[14] See Blanchard and Diamond (1989), Pissarides (1986), and Van Ours (1991).

[15] In the model, average unemployment duration is $1/m(\theta)$, and total flows from employment is $\left[\delta + \lambda F(R)\right](1-u)$. By virtue of (5), the product is equal to the unemployment rate.

Table 1. Parameter Values

| | |
|---|---|
| Vacancy posting cost: $c$ | 0.33 |
| Idiosyncratic productivity shock arrival rate: $\lambda$ | 0.10 |
| Exogenous separation rate: $\delta$ | 0.14 |
| Interest rate: $r$ | 0.01 |
| Workers' bargaining power (U.S.): $\beta$ | 0.65 |
| Dismissal penalty (U.S.): $T$ | 0.0 |
| Matching elasticity: $\eta$ | 0.50 |
| Common technology level (Portugal): $A$ | 1.00 |

Another fact used in the calibration exercise is the difference in labor productivity between two countries. The PPP GDP per capita in the United States has been roughly twice as large as that in Portugal over the decade ending in 2001. This difference in PPP GDP per capita is used as a basis for retrieving the parameter $A$ for the U.S. economy.

Parameter values backed out of the model are reported in Table 2. Measured by the output of the most productive job, $A\bar{p}k^{*\alpha}$, the value of leisure is 46.1 percent of the value of the most valued product in Portugal. For the U.S. economy, the value of the leisure is 35.0 percent of the most valued product in the United States. The implied workers' bargaining power in Portugal is greater than the bargaining power assumed for the United States. The dismissal penalty per incidence in Portugal (5.2) amounts to the value of the most valued product produced over the duration of 5.1 quarters (i.e., $A\bar{p}k^{*\alpha}$ times 5.1).

Table 2. Estimated Parameter Values

| | |
|---|---|
| Value of leisure: $b$ | 0.465 |
| Elasticity of investment return: $\alpha$ | 0.103 |
| Workers' bargaining power (Portugal): $\beta$ | 0.751 |
| Dismissal penalty (Portugal): $T$ | 5.175 |
| Common technology level (U.S.): A | 1.291 |

Table 3 reports equilibrium outcomes for the two economies as well as the results of the experiments. Table 4 reports selected equilibrium outcomes that are normalized by the respective values obtained for the calibrated Portuguese economy. Figure 2 compares the equilibrium of the U.S. economy and that of the Portuguese economy. Since the employment distribution over productivity in Portugal has a lower mean, labor productivity is also lower in Portugal. Labor productivity and consumption per participating worker in Portugal are just 50.0 percent and 51.9 percent of those in the United States, respectively.

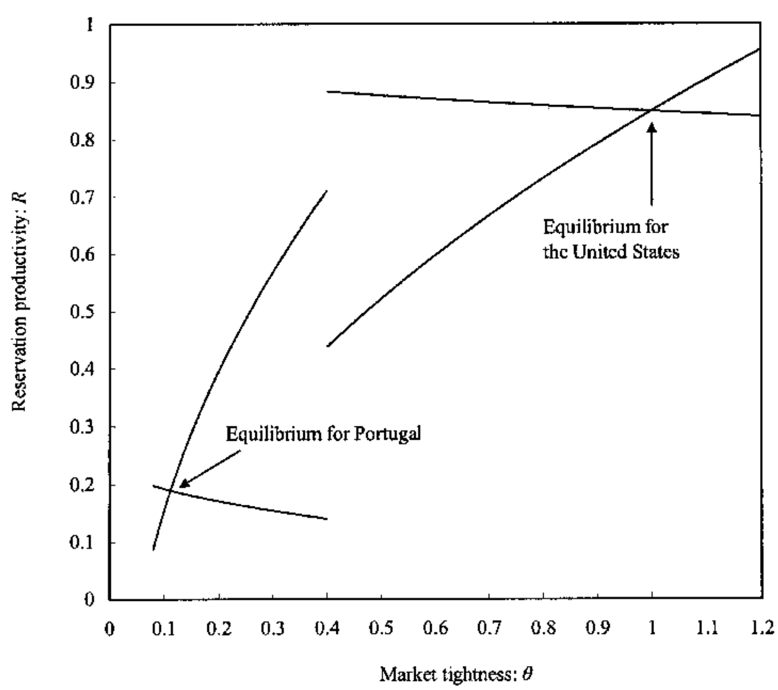
Table 3. Equilibrium Outcomes: the United States and Portugal

| | United States | Portugal | Portugal ($T = 0$) | Portugal ($k$ fixed) |
|---|---|---|---|---|
| Job-specific investment: $k^*$ | 0.766 | 0.934 | 0.672 | 0.672 |
| Reservation productivity: $R^*$ | 0.849 | 0.190 | 0.840 | - |
| Unemployment rate: $u$ | 0.090 | 0.090 | 0.142 | - |
| Average unemployment duration: $1/m(\theta^*)$ | 1.0 | 3.0 | 1.684 | - |
| Labor productivity: | 1.210 | 0.605 | 0.930 | 0.582 |
| Consumption per participating worker: | 1.091 | 0.566 | 0.826 | 0.558 |
| Aggregate job creation cost: $c{\cdot}v$ | 0.030 | 0.003 | 0.017 | 0.003 |
| Aggregate investment cost: $t(k){\cdot}m(\theta)u$ | 0.053 | 0.026 | 0.038 | 0.014 |
| Aggregate dismissal cost: $\lambda F(R)(1\text{-}u)T$ | 0.0 | 0.089 | 0.0 | 0.089 |

Table 4. Normalized Labor Productivity and Welfare
(Portugal=100.0)

| | United States | Portugal | Portugal ($T$=0) | Portugal ($k$ fixed) |
|---|---|---|---|---|
| Labor productivity: | 200.0 | 100.0 | 153.7 | 96.1 |
| Consumption per participating worker: | 192.5 | 100.0 | 145.9 | 98.5 |

Figure 2. Equilibrium of the Model: a Comparison Between Portugal and the United States



As noted previously, the difference between the equilibrium outcomes for the United States and Portugal reflects differences in workers' bargaining power ($\beta$), dismissal penalty ($T$), and the level of common technology level ($A$). Measuring the effect caused purely by dismissal penalty requires re-computing the equilibrium by changing certain parameters while holding other parameters constant.

### B. Results of the Experiments

This section discusses the effect of dismissal restrictions and job-specific investments on equilibrium outcomes. Since the model has a micro-foundation, it is possible to perform experiments based on hypothetical parameter values. Here, the experiment is to set the dismissal penalty $T$ to zero but hold other parameters the same as those of the calibrated Portuguese economy. A new equilibrium can then be obtained from the three equilibrium conditions (15), (16), and (18). A comparison between the calibrated Portuguese economy and the hypothetical Portuguese economy with $T=0$ reveals the effect of the dismissal penalty.

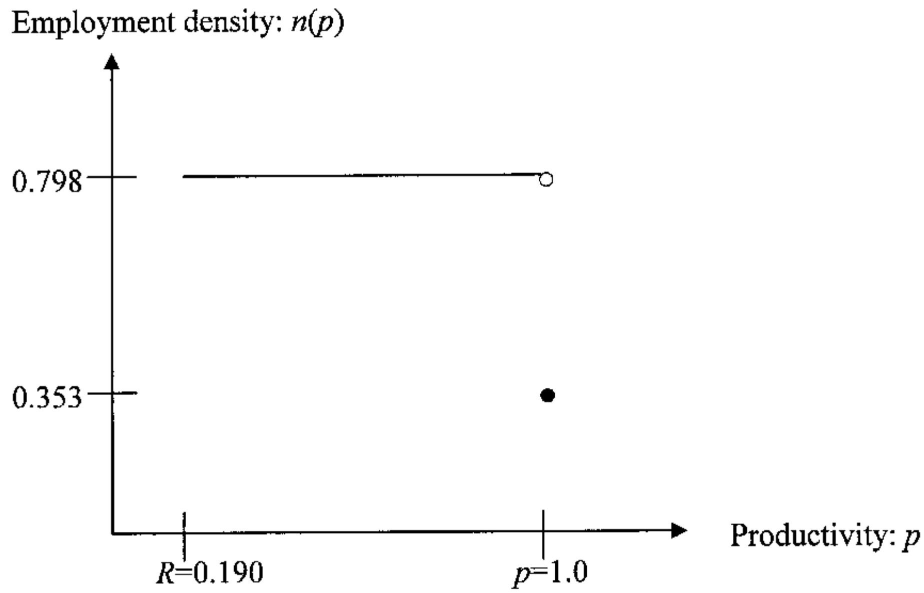
The experiment shows that the dismissal penalty causes a large welfare loss, despite the offsetting effect of the higher job-specific investment. With a dismissal penalty, the labor productivity and consumption per participating worker are 34.9 percent and 31.4 percent lower, respectively, than they would be otherwise.[16]

By comparing equilibrium outcomes for Portugal, the United States, and the hypothetical Portuguese economy with $T=0$, one can quantify how much of the observed difference in labor productivity between the two countries is due to dismissal restrictions. The results reported in Table 4 show that dismissal restrictions account for 53.7 percent of the difference in labor productivity between Portugal and the United States and 45.9 percent of the difference in consumption per participating worker between the two countries. The dismissal penalty lowers reservation productivity, resulting in a greater number of unproductive jobs. Figure 3 shows the employment densities for the two economies to illustrate this point. Those unproductive jobs are the cause of the economy's lower labor productivity and the lower consumption per participating worker.

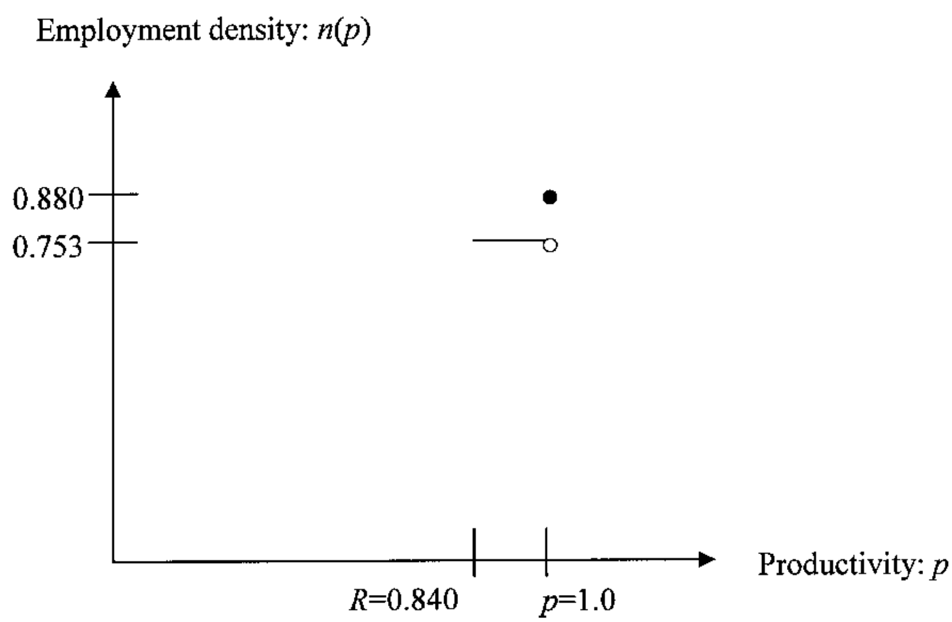
[16] The quantified welfare loss is substantially higher than the 14 percent loss estimated by Blanchard and Portugal (2001). The difference is due to differences in the models and in the assumptions used in the calibration exercises. Blanchard and Portugal assume that the baseline economy corresponds to one with minimal dismissal restrictions ($T=1$), while the baseline in this paper assumes no dismissal restrictions ($T=0$). Their assumption is likely to result in a smaller estimate of the welfare loss. Blanchard and Portugal also assume higher values for the shock arrival rate $\lambda$ and interest rate $r$. Furthermore, they assume a log normal distribution for productivity of job-worker matches while this paper uses a uniform distribution. Unlike the difference in the value of the firing penalty $T$, those differences have ambiguous effects on the welfare loss.

Figure 3. Employment Densities Implied by the Model: the Calibrated Portuguese Economy and the Hypothetical Portuguese Economy with $T$=0

a. Employment Density for the Calibrated Portuguese Economy




b. Employment Density for the Hypothetical Portuguese Economy with $T$=0



A comparison between the calibrated Portuguese economy and the hypothetical Portuguese economy with $T$=0 also illustrates that the unemployment rate is not an appropriate measure of welfare. Despite the welfare loss involved, the unemployment rate under dismissal penalty is lower than the one under no dismissal penalty.

Job-specific investment mitigates the welfare loss due to the dismissal penalty, but the size of the effect is small relative to the loss associated with lower labor mobility due to the dismissal penalty. To measure the mitigating effect of the higher job-specific investment due to the dismissal penalty, the following exercise is performed. Labor productivity and consumption per participating worker are re-computed for the Portuguese economy holding the job-specific investment at the level obtained for the hypothetical Portuguese economy with $T$=0. The values for other parameters and endogenous variables are held the same as those for the calibrated Portuguese economy. The mitigating effect of job-specific investments can then be measured by looking at how much additional losses of welfare the economy would incur if job-specific investment were held constant. Normalized measures of welfare (Table 4) show that the additional loss would be 6.7 percent of the potential loss of labor productivity (i.e., (100-96.1)/(153.7-96.1) ) and 3.3 percent of the potential loss of consumption per participating worker (i.e., (100-98.5)/(145.9-98.5) ). Therefore, the welfare saving due to job-specific investments is marginal when compared with the size of the welfare loss due to dismissal restrictions.

## IV. Conclusion

This paper found a substantial welfare loss due to dismissal restrictions, even after taking into account the mitigating effect due to job-specific investment. What is the policy implication of this finding? If higher job-specific investment, induced by dismissal restrictions, had largely offset the effect of lower reservation productivity, no major reform of dismissal restrictions would be seen necessary. The finding of this paper, however, points to the opposite, namely, that reducing dismissal restrictions would substantially improve economic welfare.

One caveat to keep in mind is that the estimated welfare loss is likely to be an upper bound of the true loss. If agents are risk averse—and not risk neutral, as assumed in this paper—dismissal restrictions would provide an insurance to workers by lowering the frequency of unemployment. The welfare loss due to dismissal restrictions would be smaller in that case.

The quantitative results could also be affected if the model included on-the-job search. If workers are allowed to search for a higher productivity job while employed and move to a new job without experiencing an unemployment incidence, some match separations would no longer be affected by dismissal restrictions. The job-to-job transition likely smoothes the reallocation of workers from low productivity jobs to high productivity jobs, and the welfare loss might not be as large as the one estimated in this paper.

## The Derivation of Key Analytical Results

### A. Nash Bargaining

This appendix derives the outcome of Nash bargaining problems that underlies the wage-setting rules (11) and (12).

Nash bargaining problem for continuing matches can be written as

$$\begin{aligned} &\omega(p,k) \in \arg\max_{\omega} (W(p,k)-U)^{\beta}(S(p,k)-(W(p,k)-U))^{1-\beta} \\ &s.t.\ S(p,k) = J(p,k)+W(p,k)-U+T. \end{aligned} \tag{23}$$

One can transform the first order necessary condition of this problem to

$$\beta\left[J(p,k)+T\right]\frac{\partial W(p,k)}{\partial \omega}+(1-\beta)\left[W(p,k)-U\right]\frac{\partial J(p,k)}{\partial \omega}=0$$

where

$$\frac{\partial J(p,k)}{\partial \omega}=-\frac{1}{r+\delta+\lambda} \text{ and } \frac{\partial W(p,k)}{\partial \omega}=\frac{1}{r+\delta+\lambda}$$

by virtue of the asset pricing equations (4) and (6). The transformed condition is the bargaining outcome given in equation (12).

The outcome of Nash bargaining for new matches can be derived in a way similar to that for continuing matches. The Nash bargaining problem for new matches is

$$\begin{aligned} &\omega_o \in \arg\max_{\omega_o} (W_o-U)^{\beta}(S_o-(W_o-U))^{1-\beta} \\ &s.t.\ S_o = J_o+W_o-t(k)-U. \end{aligned} \tag{24}$$

It is straightforward to derive the outcome in (11) from the first order condition of this problem, and thus the derivation is omitted.

### B. Match Surplus and Wage Rates

This appendix derives match surpluses in equations (9) and (10) as well as wage rates in (13) and (14). The derivation builds on the outcome of bargaining problems derived in the previous appendix.

The surplus for a continuing match is derived first. Adding both sides of equation (6) and equation (4) produces

$$\begin{aligned} & r\left[J(p,k)+W(p,k)\right] \\ = \quad & Apk^{\alpha}+\lambda\int_{\underline{p}}^{\overline{p}}\max\left\langle J(x,k),-T\right\rangle dF(x)+\lambda\int_{\underline{p}}^{\overline{p}}\left\langle W(x,k),U\right\rangle dF(x) \\ & -\lambda\left[J(p,k)+W(p,k)\right]-\delta\left[J(p,k)+W(p,k)-U+T\right]+\delta T. \end{aligned}$$

Subtracting $rU$ from and adding $rT$ to both sides of the equation and rearranging terms, one obtains:

$$\begin{aligned} & r\left[J(p,k)+W(p,k)-U+T\right] \\ = \quad & Apk^{\alpha}-rU+rT+\lambda\int_{\underline{p}}^{\overline{p}}\max\left\langle J(x,k)+W(x,k)-U+T,0\right\rangle dF(x) \\ & -\lambda\left[W(p,k)+J(p,k)-U+T\right]-\delta\left[W(p,k)+J(p,k)-U+T\right]+\delta T. \end{aligned}$$

Noting $S(p,k)=J(p,k)+W(p,k)-U+T$ under $V=0$ and collecting terms with $S(p,k)$ to the left hand side of the equation, this equation can be transformed to the following form:

$$(r+\lambda+\delta)S(p,k)=Apk^{\alpha}-rU+(r+\delta)T+\lambda\int_{\underline{p}}^{\overline{p}}\left\langle S(x,k),0\right\rangle dF(x).$$

The right hand side of this equation is a contraction that maps a function $S(p,k)$ increasing in $p$ into itself given $k$. Indeed, $S(p,k)$ is linear by virtue of $\partial S(p,k)/\partial p=k^{\alpha}/(r+\lambda+\delta)$. Since reservation productivity R is defined to satisfy $S(R,k,)=0$, the surplus $S(p,k)$ is written as

$$S(p,k)=\frac{A(p-R)k^{\alpha}}{r+\lambda+\delta}.$$

The surplus for a new match is derived here. Adding both sides of equation (5) and equation (3) and subtracting $rU$ and $rt(k)$ from the resulting equation lead to

$$\begin{aligned} & r\left[J_o+W_o-U-t(k)\right] \\ = \quad & A\overline{p}k^{\alpha}-rU-rt(k)+\lambda\int_{\underline{p}}^{\overline{p}}\max\left\langle J(x,k)+W(x,k)-U+T,0\right\rangle dF(x)-\lambda T \\ & -\lambda\left[J_o+W_o-U-t(k)\right]-\delta\left[J_o+W_o-U-t(k)\right]-\lambda t(k)-\delta t(k). \end{aligned}$$

Noting $S_o=J_o+W_o-U-t(k)$ under $V=0$ and collecting terms with $S_o$ to the left hand side of the equation, one obtains

$$(r+\lambda+\delta)S_o=A\overline{p}k^{\alpha}-rU-(r+\lambda+\delta)t(k)-\lambda T+\lambda\int_{R}^{\overline{p}}S(x,k)dF(x).$$

Note that the lower limit of the integration is $R$ because $S(x,k)$ is negative for any $x$ below $R$. By virtue of the job destruction condition in (18),

$$\lambda\int_{R}^{\overline{p}}S(x,k)dF(x)-rU=-ARk^{\alpha}-(r+\delta)T.$$

Substituting this expression into the functional equation for $S_o$ and transforming the resulting equation results in

$$S_o = \frac{A(\overline{p}-R)k^{\alpha}}{r+\lambda+\delta} - (t(k)-T).$$

The wage rate for continuing matches in (14) is derived as follows. Multiplying (1-$\beta$) to and subtracting $(r+\lambda)(1-\beta)U$ from both sides of the asset pricing equation (4) generates the following equation:

$$\begin{aligned}
&(r+\lambda)(1-\beta)\left[W(p,k)-U\right] \\
&= (1-\beta)\omega(p,k)+\lambda(1-\beta)\int_{\underline{p}}^{\overline{p}} \max\left\langle W(x,k)-U,0\right\rangle dF(x) \\
&\qquad -\delta(1-\beta)\left[W(p,k)-U\right]-r(1-\beta)U.
\end{aligned} \tag{25}$$

Similarly, multiplying β and adding $(r+\lambda)\beta T$ to both sides of the asset pricing equation (6) generates the following equation:

$$\begin{aligned}
&(r+\lambda)\beta\left[J(p,k)+T\right] \\
&= \beta Apy^{\alpha}-\beta\omega(p,k)+\beta\lambda\int_{\underline{p}}^{\overline{p}} \max\left\langle J(x,k)+T,0\right\rangle dF(x) \\
&\qquad -\delta\beta\left[J(p,k)+T\right]+\delta\beta T+r\beta T.
\end{aligned} \tag{26}$$

Subtracting both sides of equation (26) from respective sides of equation (25) leads to the wage rate in (14).

The wage rate for new matches in (13) can be derived in a way similar to that for continuing matches, and thus the derivation is omitted.

## C. The Effect of Job-Specific Investment on R and θ

This appendix drives the partial derivative of reservation productivity $R$ and market tightness $\theta$ with respect to job-specific investment $k$ given the assumed functional form for $F$.

Given the functional form for $F$, job destruction condition reads as

$$\int_R^1 \frac{A(x-R)k^{\alpha}}{r+\lambda+\delta}dF(x) = \frac{Ak^{\alpha}(R-1)^2}{2(r+\lambda+\delta)}.$$

Totally differentiating this condition with respect to $k$ leads to the following equation:

$$0 = A\frac{\partial R}{\partial k}k^{\alpha}+AR\alpha k^{\alpha-1}-\frac{\beta c}{(1-\beta)}\frac{\partial\theta}{\partial k}+\frac{\lambda A}{2(r+\lambda+\delta)}\left\{\alpha k^{\alpha-1}(R-1)^2+2k^{\alpha}(R-1)\frac{\partial R}{\partial k}\right\}.$$

By rearranging terms, one obtains:

$$\frac{\partial R}{\partial k}\left\{1+\frac{\lambda(R-1)}{r+\lambda+\delta}\right\}Ak^{\alpha}-\frac{\partial\theta}{\partial k}\frac{\beta c}{1-\beta} = -AR\frac{\partial y}{\partial k}-\frac{\lambda A}{2(r+\lambda+\delta)}\alpha k^{\alpha}(R-1)^2. \tag{27}$$

Totally differentiating job creation condition and rearranging the resultant equation leads to

$$\frac{\partial\theta}{\partial k}\frac{c}{m(\theta)}(1-\eta)+\frac{\partial R}{\partial k}\frac{(1-\beta)Ak^{\alpha}}{r+\lambda+\delta}=\frac{(1-\beta)(1-R)}{r+\lambda+\delta}Ak^{\alpha}-(1-\beta)\frac{\partial t(k)}{\partial k}. \tag{28}$$

Two equations (27) and (28) form the following system of linear equations for $\partial R/\partial k$ and $\partial\theta/\partial k$:

$$\begin{pmatrix} Ak^{\alpha}\left(1+\frac{\lambda(R-1)}{r+\lambda+\delta}\right) & -\frac{\beta c}{1-\beta} \\ \frac{(1-\beta)Ak^{\alpha}}{r+\lambda+\delta} & \frac{c}{m}(1-\eta) \end{pmatrix}\begin{pmatrix} \frac{\partial R}{\partial k} \\ \frac{\partial\theta}{\partial k} \end{pmatrix}=\begin{pmatrix} -AR\alpha k^{\alpha-1}-\frac{A\lambda\alpha k^{\alpha-1}(R-1)^2}{2(r+\lambda+\delta)} \\ \frac{(1-\beta)(1-R)}{r+\lambda+\delta}A\alpha k^{\alpha-1}-(1-\beta)\frac{\partial t(k)}{\partial k} \end{pmatrix}.$$

The solution of this system of linear equations is

$$\begin{pmatrix} \frac{\partial R}{\partial k} \\ \frac{\partial\theta}{\partial k} \end{pmatrix}=\frac{1}{C}\begin{pmatrix} \frac{c}{m}(1-\eta) & \frac{\beta c}{1-\beta} \\ -\frac{A(1-\beta)k^{\alpha}}{r+\lambda+\delta} & Ak^{\alpha}\left(1+\frac{\lambda(R-1)}{r+\lambda+\delta}\right) \end{pmatrix}\begin{pmatrix} -AR\alpha k^{\alpha-1}-\frac{A\lambda\alpha k^{\alpha-1}(R-1)^2}{2(r+\lambda+\delta)} \\ \frac{(1-\beta)(1-R)}{r+\lambda+\delta}A\alpha k^{\alpha-1}-(1-\beta)\frac{\partial t(k)}{\partial k} \end{pmatrix}$$

where

$$C=Ak^{\alpha}\left(1+\frac{\lambda(R-1)}{r+\lambda+\delta}\right)\frac{c}{m(\theta)}(1-\eta)+\frac{A\beta ck^{\alpha}}{r+\lambda+\delta}>0.$$

**References**

Belot, M., J. Boone, and J. Van Ours, 2001, *Welfare Effects of Employment Protection*, (unpublished; the Netherlands: Tilburg University).

Bertola G., T. Boeri, and S. Cazes, 1999, "Employment Protection and Labour Market Adjustment in OECD Countries: Evolving Institutions and Variable Enforcement," *Employment and Training Papers*, No. 48, Employment and Training Department, (Geneva, Switzerland: ILO).

Blanchard, O. and P. Diamond, 1989. "The Beveridge Curve," *Brookings Papers on Economic Activity*, Vol.1, pp. 1–60.

Blanchard, O., and P. Portugal, 2001, "What Hides Behind an Unemployment Rate: Comparing Portuguese and U.S. Labor Markets," *American Economic Review*, Vol. 91, pp. 187–207.

Bover, O., P. García-Perea, and P. Portugal, 2000, "Labour Market Outliers: Lessons from Portugal and Spain," *Economic Policy*, Vol. 31, pp. 379–428.

Hogan S., and C. Ragan, 1998, "Job Security with Equilibrium Unemployment," *Labour Economics*, Vol. 5, pp. 185–204.

Lazear, E. P., 1990, "Job Security Provisions and Unemployment," *Quarterly Journal of Economics*, Vol. 55, pp. 699–726.

Millard S. P. and D. T. Mortensen, 1997, "The Unemployment and Welfare Effects of Labour Market Policy: A Comparison of the USA and the UK," in *Unemployment Policy: Government Options for the Labour Market*, ed. by D. J. Snower and G. de la Dehesa (Cambridge, England: Cambridge University Press).

Mortensen, D. T., and C. A. Pissarides, 1994, "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies*, Vol. 66, pp. 397–415.

Mortensen, D. T., and C. A. Pissarides, 1999, "Job Reallocation, Employment Fluctuations and Unemployment," in *Handbook of Macroeconomics*, Vol.1, ed. by J. B. Taylor and M. Woodford (Amsterdam, the Netherlands: Elsevier Science B.V.).

Organization for Economic Cooperation and Development, 1999, *Employment Outlook* (OECD, Paris).

Pissarides, C.A., 1986, "Unemployment and Vacancies in Britain," *Economic Policy*, Vol. 3, pp. 499–559.

Van Ours, J. C., 1991, "The Efficiency of the Dutch Labour Market in Matching Unemployment and Vacancies," *De Economist*, Vol. 139.