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Consumption-Based Interest Rate and the Present-Value Model of the Current Account—Evidence from Nigeria

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Consumption-Based Interest Rate and the Present-Value Model of the Current Account—Evidence from Nigeria

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Abstract

This paper presents a model of current account determination, based upon the permanent-income hypothesis. A present-value relationship among the current account, changes in net output, the exchange rate and the terms of trade is derived and the implications of such a relationship are tested using data for Nigeria during 1960–97.

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I. Introduction

The present-value model of the current account (hereafter PVMCA), a version of the intertemporal model of the current account, has become standard in the theoretical analysis of the current account. This model in its simplest form derives its conclusions from consumption smoothing behavior. It implies that an unanticipated temporary fall in output in a small open economy will produce a deterioration in the current account balance. This approach has its origin in Campbell and Shiller's (1987) seminal work on the relationship between current saving and expected changes in labor income. The extension of this framework to current account determination can be found in Sheffrin and Woo (1990), Milbourne and Otto (1992), and Otto (1992).

Two major findings of the standard[2] PVMCA generated interest in extending it. First, the actual current account has been found in many studies to be more volatile than the "optimal"[3]. Second, the statistical restrictions implied by these models have been rejected for a considerable number of countries. Given the available mixed results, Sheffrin and Woo, Melbourne and Otto, and Otto suggest that for a small open economy, the current account may be more affected by temporary changes in resource prices, indicating it would be useful to relax the assumption of a single good and allow for a distinction between tradable and nontradable goods.

This paper has two main objectives. The first one is to assess whether an intertemporal framework is a valid approach for analyzing the current account in an economy like Nigeria. The second objective is to determine the relative performance of various versions of the PVMCA. To this end, this paper formulates a present-value model of the current account that allows for a variable interest rate as well as changes in the real exchange rate and terms of trade. The theoretical framework presented is along the lines of that developed by Bergin and Sheffrin (2000). Their work explicitly incorporates changes in interest and exchange rates. This paper extends their framework by incorporating expected change in the terms of trade.

In terms of theoretical contribution, this is the first attempt at incorporating the terms of trade in a PVMCA. Apart from this theoretical contribution, the paper constitutes an alternative approach to examining current account determination in Nigeria. Previous studies

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[2] The standard PVMCA has a number of underlying assumptions. First, it assumes that the home country and the trading partners produce goods that are physically identical (so there is no direct role for the terms of trade). Second, it assumes no transport costs, taking all goods to be tradable across countries (i.e., it excludes nontraded goods and as a result fails to take into consideration possible effects of changes in the real exchange rate on the current account).

[3] This is the current account balance obtained from implementing the present-value model of the current account empirically.

on the country emphasized the elasticity, monetary, and absorption approaches[4]. While these studies identified the major determinants of the current account balance, they were static in nature. Analyzing the evolution of the current account requires a model that emphasizes the microeconomic underpinnings of decisions relating to savings and investment[5].

There is evidence to the effect that the PVMCA, an intertemporal approach, constitutes an appropriate framework for analyzing the current account, since despite its simplicity, some of its implications are found to be valid. This position is based on the following results:

- The optimal and actual current account balances are found to be stationary variables;
- The statistical restrictions implied by the PVMCA are not rejected when changes in interest rates, exchange rates and terms of trade are considered;
- The optimal current accounts derived from the PVMCA are able to capture the evolution of the actual current account balance;
- Nonetheless, the actual current account is more volatile than the optimal, indicating that speculative factors are major deriving force behind capital flows.

The paper is organized as follows. Section II outlines a theory of current account determination that incorporates a variable interest rate, changes in the exchange rate and terms of trade and presents the econometric framework. Section III focuses on the data and parameter values. Section IV presents results from the present-value tests. Section V concludes the paper.

II. Theory and Econometric Methods

The presentation here follows the lead of Campbell and Mankiw (1989), Huang and Lin (1993) and Bergin and Sheffrin (2000).

A. Optimal Consumption

Consider a small open economy that can lend and borrow at a variable real world interest rate, r^*. The economy produces both tradable and nontradable goods. The infinitely lived household maximizes the lifetime utility function given by (1) subject to the dynamic budget constraint in (2):

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[4] See, for example, Enunenwosu (1984), Fakiyesi and Umo (1995), Komolafe (1996), Egwhaide (1997). Ghosh and Ostry (1995) made the first attempt at applying the present-value approach to Nigeria.

[5] This conjecture reflects Lucas' (1976) position that policy analysis must be based on the actual forward-looking decision rules of economic agents. The microeconomic foundation of macroeconomic analysis permits the imposition of structure on the macro model.

$$U = E_t\left\{\sum_{t=\tau}^{\infty}\beta^{t-\tau}\left[U(C_{Tt}, C_{Nt})\right]\right\}, \quad 0<\beta<1 \tag{1}$$

$$C_{Tt} + P_t C_{NTt} = Y_t + r_{t+1}{}^* B_t - I_t - G_t = B_{t+1} - B_t = CA_t \tag{2}$$

where: B_t is the net foreign assets; Y_t is the gross domestic product; C_t and G_t capture private and government expenditures, respectively; I_t is private and government investment; and CA_t is the current account balance in period t. C_T is the consumption of traded goods and C_{NT} is the consumption of nontraded goods. P_t is the relative price of home nontraded goods in terms of traded goods. All the variables are measured in terms of traded goods.

The temporal utility function is isoelastic and the allocation of expenditure between tradable and nontradable is in the form of a Cobb-Douglas function:

$$U(C_{Tt}, C_{NTt}) = \frac{1}{1-\sigma}\left(C_{Tt}^{\alpha} C_{NTt}^{1-\alpha}\right)^{1-\sigma} \tag{3}$$

$$\sigma > 0,\ \sigma \neq 1,\ 0 < \alpha < 1$$

The representative agent maximizes (1) subject to the dynamic budget constraint in (2). In carrying out this intertemporal exercise, we must first demonstrate that the dynamic budget constraint in (2) can be written as[6]:

$$P_t{}^* C_t{}^* = Y_t + (1 + r_{t+1}{}^*)B_t - B_{t+1} - I_t - G_t \tag{4}$$

where $C_t{}^* = C_{Tt}^{\alpha} C_{NTt}^{1-\alpha}$, is an index of total consumption. The consumption-based index, P_t*, is the minimum amount of consumption expenditure[7] $C_t = C_{Tt} + P_t C_{NTt}$ such that $C_t{}^* = 1$, for given P_t.

We can now rewrite (3) as:

$$U(C_t{}^*) = \frac{1}{1-\sigma}(C_t{}^*)^{1-\sigma} \tag{5}$$

[6]See Appendix IA for the derivation of (4). The derivation of the optimal consumption is based on the works of Dornbusch (1983) and Obstfeld and Rogoff (1996).

[7]This is the total consumption expenditure from the dynamic budget constraint in (2).

The problems of choosing traded and nontraded goods intertemporarily in (1) and intratemporarily in (3) have been transformed into one of choosing the composite consumption index C^*. Solving for C^* from (4) and substituting into (1) we have:

$$V(B_t) = E_t\left\{Max\sum_{t=\tau}^{\infty}\beta^{t-\tau}U\left[\frac{Y_t + (1+r^*)B_{t+1} - B_t - I_t - G_t}{P_t^*}\right]\right\} \tag{6}$$

Given the utility function in (5), the first-order condition for this problem is derived in Appendix I. Based on this, the optimal consumption profile is given by (7):

$$E_t\left[\beta(1+r^*_{t+1})\left(\frac{P_t^*}{P_{t+1}^*}\right)\left(\frac{C_t^*}{C_{t+1}^*}\right)^{\sigma}\right] = 1 \tag{7}$$

To ensure empirical implementation, we need to rewrite (7) in terms of consumption expenditure and the relative price of nontraded goods. To achieve this, we reexpress (7) as:

$$E_t\left[\beta(1+r_{t-1}^*)\left(\frac{P_t^* C_t^*}{P_{t+1}C_{t+1}^*}\right)^{\sigma}\left(\frac{P_t^*}{P_{t+1}^*}\right)^{1-\sigma}\right] = 1 \tag{8}$$

We use (3) and (4) to substitute for $\frac{P_t^* C_t^*}{P_{t+1}C_{t+1}^*}$ and $\frac{P_t^*}{P_{t+1}^*}$ respectively:

$$E_t\left[\beta(1+r_{t+1}^*)\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\left(\frac{p_t}{p_{t+1}}\right)^{(1-\sigma)(1-\alpha)}\right] = 1 \tag{9}$$

Assume joint log normality, constant variances and co variances for the gross real world interest rate ($1+r^*_{t+1}$), consumption growth rate ($\Delta c_{t+1} = log\, C_{t+1} - log\, C_t$), and for the percentage change in the relative price of nontraded goods ($\Delta p_{t+1} = log\, P_{t+1} - Log\, P_t$)[8]. Based on the assumption of lognormal distribution, (9) may be rewritten in a log linearized form as[9]:

[8]See Campbell et al (1997): 306–307, for the properties of a random variable that is conditionally lognormal distributed.

[9]Given the condition that the empirical implementation of the model will be based on demeaned variables, the preference parameter, a constant, is dropped from equation (11).

$$E_t \Delta c_{t+1} = \gamma E_t \left[r^*_{t+1} + \frac{1-\gamma}{\gamma}(1-\alpha)\Delta p_{t+1} \right] + \frac{1}{2} \left[\begin{array}{l} \sigma_c^2 + \gamma^2 \sigma_r^2 + (1-\gamma)^2 (1-\alpha)^2 \sigma_p^2 + 2\gamma\sigma_{c,r} \\ +2(1-\gamma)(1-\alpha)\sigma_{c,p} + 2\gamma(1-\gamma)(1-\alpha)\sigma_{r,p} \end{array} \right] \tag{10}$$

where $\gamma = 1/\sigma$, the elasticity of intertemporal substitution.

Equation (10) assumes the approximation, $\log(1+r^*_{t+1}) = r^*_{t+1}$. The first bracket on the right hand side of (10) specifies the consumption based real interest rate, which is different from the world real rate of interest when there is an expected change in the real exchange rate. This is the own rate of interest on the consumption index C_t^*, and is denoted $\hat{r}_t$.

Therefore, the evolution of optimal consumption profile is given by:

$$E_t \Delta c_{t+1} = \gamma E_t \hat{r}_{t+1}; \text{ where } \hat{r}_t = r^* + \frac{1-\gamma}{\gamma}(1-\alpha)\Delta p_{t+1} + constant^{10} \tag{11}$$

The relationship between the evolution of optimal consumption and consumption-based interest rate, $\hat{r}_t$, is given by (11). The consumption-based real interest rate combines both the world interest rate r_t^* and the change in the relative price of nontraded goods. Previous empirical investigation of the intertemporal model of the current account emphasized a consumption profile where the expected change in consumption is zero; household always try to smooth consumption over time by participating in the international financial markets. When the real interest rate is variable, as shown in (11), the representative consumer may decide to alter the path of consumption overtime: increasing or decreasing consumption in some periods due to changes in the terms of borrowing and lending.

A change in the relative price of nontraded good can have similar intertemporal effects as that of a change in the conventional interest rate. For example, an increase in the conventional real interest rate, r_t^*, makes current consumption more expensive in terms of foregone future consumption, and induces substitution toward future consumption with elasticity γ. Relating this scenario to the relative price of nontraded goods, if the price of nontraded goods is temporarily high and expected to fall, then the future repayment of a loan contracted in the current period in traded goods has higher cost in terms of the consumption bundle than in terms of traded goods alone. As a result, the consumption-based interest rate, $\hat{r}$, rises above the conventional interest rate, r^*. This induces a fall in the current total consumption expenditure by elasticity $\gamma(1-\alpha)$. Apart from this intertemporal substitution, a

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[10] The constant term at the end of the expression will drop out of the empirical model when we later demean the consumption based interest rate using (10).

change in the relative price of nontraded goods also induces intratemporal substitution. In the events of a temporarily high relative price of nontraded good, a household will substitute toward traded goods by the intratemporal elasticity, which is a unity under a Cobb-Douglas specification. This increases total current consumption expenditure by elasticity (1-α). This intratemporal effect will dominate the intertemporal effect if the intertemporal elasticity, γ, is less than unity.

B. Intertemporal Budget Constraint

Now, we have to carry out the log-linearization of the intertemporal budget constraint. The dynamic budget constraint in (2) can be rewritten as:

$$\begin{aligned} CA_t &= Y_t - I_t - G_t - \left(C_{Tt} + P_t C_{NTt}\right) + r^*_{t+1} B_t \\ &= Q_t - C_t + r^*_{t+1} B_t \end{aligned} \tag{12}$$

where $Q_t \equiv Y_t - I_t - G_t$, is the net output and $C_t = \left(C_{Tt} + P_t C_{NTt}\right)$

Define $R_{\tau,t}{}^*$ as the market discount factor for date τ consumption, so that

$$R_{\tau,t}{}^* = \frac{1}{\prod_{j=\tau+1}^{t}\left(1 + r^*_j\right)} \qquad (13)^{11}$$

We sum over all periods of the infinite horizon, and impose the *no ponzi game* constraint. Combining this with (13), equation (12) becomes:

$$\sum_{t=0}^{\infty} E_0\left(R^*_t C_t\right) = \sum_{t=0}^{\infty} E_0\left(R^*_t Q_t\right) + B_0 \tag{14}$$

Using the Huang and Lin (1993) procedures (14) is log-linearized. The first step is to log-linearize the present-value of net output. Define:

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[11] This is necessary given the assumption of a variable interest rate. It replaces the usual discount factor $\sum_{t=\tau}^{\infty}\left(\frac{1}{1+r^*}\right)^{t-\tau}$ under the assumption of a constant interest rate. Also, $R_{\tau,\tau} = 1$; $R_{\tau,\tau+1} = \frac{1}{\left(1+r^*_{t+1}\right)\left(1+r^*_{t+2}\right)}$, and so on.

$$\Gamma_0 = \sum_{t=0}^{\infty} E_0(R^*_t\, Q_t) \tag{15}$$

Appendix II shows that (15) implies that:

$$q_0 - \varphi_0 = \sum_{t=1}^{\infty} \rho^t (r_t * - \Delta q_t) + \eta \tag{16}$$[12]

where $q_0 = ln\ Q_0$; $\varphi_0 = ln\ \Gamma_0$ $\Delta q_t = q_t - q_{t-1}$; and η is a constant; ρ is a constant, slightly less than one and can be interpreted as the average value of $1\text{-}Q/\Gamma$.

The second step involves log-linearization of the present-value of current and future consumption:

$$X_0 = \sum_{t=0}^{\infty} E_0(R_t * C_t) \tag{17}$$

Using the procedures in Appendix II, (18) implies that:

$$c_0 - x_0 = \sum_{t=1}^{\infty} \rho^t (r_t * - \Delta c_t) + \eta \tag{18}$$

where $c_0 = ln\ C_0$; $x_0 = ln\ X_0$; $\Delta c_t = c_t - c_{t-1}$; η is a constant; ρ is constant, slightly less than one and can be interpreted as the average value of $1\text{-}C/X$

Based on the same procedures in Appendix II, the intertemporal budget constraint $\Gamma_0 = X_0 + B_0$, implies that:

$$\varphi_0 - x_0 = \left(1 - \frac{1}{\Omega}\right)[b_0 - x_0] + k \tag{19}$$

where $b_0 = ln\ B_0$; Ω is constant, slightly less than one, and can be interpreted as the average value of $1\text{-}B/X$; and k is a constant.

Substituting for φ_0 and x_0 respectively from (16) and (18) into (19), produces the following log-linearized intertemporal budget constraint for the representative agent for $t \geq 0$.

$$-\sum_{t=1}^{\infty} \rho^t (\Delta q_t)\left[-\frac{1}{\Omega}\Delta c_t - \frac{\Omega - 1}{\Omega} r_t *\right] = q_0 - \frac{1}{\Omega} c_0 + \frac{\Omega - 1}{\Omega} b_0 \tag{20}$$

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[12] Appendix II provides the detailed derivation of (16). The same procedure is used in transforming (17) and the entire intertemporal budget constraint.

Taking the expectation of (20), and using the optimal consumption profile in (11), equation (20) may be written as:

$$-E_t\sum_{i=1}^{\infty}\beta^i\left(\Delta q_{t+i}\right)-\frac{\gamma}{\Omega}\hat{r}_{t+1}-\frac{\Omega-1}{\Omega}r_t*=q_t-\frac{1}{\Omega}c_t+\frac{\Omega-1}{\Omega}b_t \tag{21}$$

The right hand side of (21) is similar to the definition of the current account in (12), except that its components are in log terms. We call the transformed representation of the current account as $CA**$. This is the optimal current account balance when the interest rate is variable and there is an expectation of a possible change in the real exchange rate.

We follow the convention of choosing the steady state around which we linearize to be the one in which net foreign assets are zero. In this case, Ω=1 and (21) may be reexpressed as:

$$CA_t**=-E_\tau\sum_{t=\tau+1}^{\infty}\beta^{t-\tau}\left(\Delta q_t-\gamma\hat{r_t}\right) \tag{22}$$

This condition stipulates that expected fall in net output would produce a current account surplus in the current period as the representative agent smoothes consumption. Apart from the shocks emanating from the three components of the net output, (22) also implies that a rise in the consumption-based interest rate will lead to an improvement in the current account position by inducing the representative household to lower consumption below its smoothed level.

The restrictions implied by (22) are tested using the procedure applied by Sheffrin and Woo (1990), but augmented with an additional variable, $\hat{r}_t$. This produces:

$$\begin{bmatrix}\Delta Q_t\\ CA_t\\ \hat{r}_t\end{bmatrix}=\begin{bmatrix}\psi_{11} & \psi_{12} & \psi_{13}\\ \psi_{21} & \psi_{22} & \psi_{23}\\ \psi_{31} & \psi_{32} & \psi_{33}\end{bmatrix}\begin{bmatrix}\Delta Q_{t-1}\\ CA_{t-1}\\ \hat{r}_{t-1}\end{bmatrix}+\begin{bmatrix}V_{1t}\\ V_{2t}\\ V_{3t}\end{bmatrix} \tag{23}$$

We need to determine the expected values of ΔQ_{t-i} and $\hat{r}_{t+i}$. To arrive at (24), we use the conditions that $E_t[X_{t+j}]=\Psi^j X$; that $E(V_{1t})=E(V_{2t})=E(V_{3t})=0$.; and that $\Psi = matrix\ [\psi_{ij}]$. Therefore,

$$hz_t=-\sum_{t=\tau+1}^{\infty}\beta^{t-\tau}\left(g_1-\gamma g_2\right)\Psi^{t-\tau}z_t \tag{24}$$

where z_t, $=(\Delta Q_t\ CA_t\ \hat{r}_t)$, $g_1=[1\ 0\ 0]$, $g_2=[0\ 0\ 1]$, and $h=[0\ 1\ 0]$. For a given z_t, the right hand side of the last equation can be reexpressed as:

$$CA_t ** = kz \tag{25}$$

$$k = -(g_1 - \gamma g_2)\beta\Psi(I - \beta\Psi)^{-1}$$

Equation (25) provides the model's prediction for the current account consistent, with the VAR and the restrictions of the intertemporal theory.

To evaluate the model, we have to test the hypothesis that $k = [0\ 1\ 0]$ in (25) so that $CA**_t = CA_t$. This implies that the model may be tested statistically by using the delta method to calculate a χ^2 statistic for the hypothesis that $k = [0\ 1\ 0]$. Let k^* be the difference between the actual k and the hypothesized value. Then $\boldsymbol{k}^{*\prime}\left((\partial k/\partial\Psi)V(\partial k/\partial\Psi)'\right)^{-1}\boldsymbol{k}^*$ will be distributed chi-squared with 3 degrees of freedom, where V is the variance-covariance matrix of the underlying parameters in the VAR, and $(\partial k/\partial\Psi)$ is the matrix of derivatives of the k vector with respect to these underlying parameters.

Once the optimal current account series has been calculated, a number of other tests may be performed. Under the null, another implication of the PVMCA is the equality of the variances of the actual current account and the optimal current account. If this is found to be valid, in the empirical section the Nigerian economy can be considered as receiving sufficient capital flows to ensure consumption smoothing. If the actual current account is less volatile than the optimal, this implies that capital inflows into Nigeria are not sufficient for consumption smoothing. On the other hand, if the actual current account is more volatile than the optimal, this represents excessive capital flows and therefore there are speculative motives behind capital flows.

The last implication is the stationarity of both the actual and optimal current account balances. The optimal current account is an I (0) process,[13] given the null of equality between the actual and optimal current accounts; the actual current account is expected to be an I (0) process as well.

To summarize, the implications of the present-value model when the interest rate is not constant are as follows:

- there is equality between the optimal and actual current account;
- there is equality between the variance of the optimal and actual current accounts and finally;
- the actual current account is a stationary variable based on the stationarity of the optimal current account balance.

[13] The stationarity of the optimal current account is due to the stationarity of changes in net output adjusted for the consumption based interest rate.

C. Consumption Based Interest Rate and the Terms of Trade

As noted by Bergin and Sheffrin (2000), a small open economy may be strongly affected by external shocks. The external shock considered important for current account determination is changes in the terms of trade. A change in the terms of trade impacts on the current account through a number of channels. The first channel is the income effect. An unanticipated deterioration in the terms of trade reduces current income below its permanent level, with a constant consumption level; this tends to produce a decline in the current account position. The increase in the current price of importables relative to the future price may induce agents to tilt consumption towards the future, thereby reducing current consumption. Moreover, a drop in the relative price of exports affects current account through changes in the exchange rate. The deterioration in the terms of trade induces domestic economic agents to demand for nontradables, leading to an increase in its relative price (exchange rate appreciation). The increase in the current price of nontradables relative to the future price may further reduce the current consumption of nontradables relative to future consumption. To accommodate possible impacts of changes in the terms of trade and exchange rate on the current account, an interest rate that captures expected changes in both terms of trade and exchange rate is used in this paper (consumption-based interest rate).

In order to incorporate changes in the terms of trade, the traded goods are disaggregated into exportables and importables. It is assumed that the exportable is not consumed at home. In Nigeria shifts in the world terms of trade play an important role in the relative domestic demand between imported and nontraded goods.

The consumption and world discount factors will differ from each other whenever the terms of trade (the relative price of importables) or the real exchange rate (the reciprocal of the relative price of nontaxable) is not expected to remain constant through time. Allowing for the consumption of imported and nontraded goods permits the incorporation of the terms of trade changes, where the terms of trade are now the relative price of imports in terms of exports and the real exchange rate is the relative price of nontradables in terms of the price of exports. This formulation is along the lines of Ghosh and Reinhart (1992) and Ostry (1988).

D. Derivation of Optimal Consumption

Consider a small open economy that can lend and borrow at a varying real world interest rate, r^*, and consumes both importables and nontradable goods. The infinitely lived household maximizes the expected value of the lifetime utility function in (26) subject to the dynamic budget constraint in (27):

$$U = E_t\left\{\sum_{t=\tau}^{\infty}\beta^{t-\tau}\left[U(C_{mt}, C_{Nt})\right]\right\},\ U'(C_t) > 0\ ;\ U''(C_t) < 0;\ 0 < \beta < 1 \tag{26}$$

$$\frac{P_t^m}{P_t^e}C_{mt} + \frac{P_t^n}{P_t^e}C_{NTt} = \frac{Y_t + (1+r^*)B_t - B_{t+1} - I_t - G_t}{P_t^e}$$

$$P_t^m C_{mt} + P_t^n C_{NTt} = Y_t + (1+r^*)B_t - B_{t+1} - I_t - G_t$$

$$P^*_t C^*_t = Y_t + (1+r^*)B_t - B_{t+1} - I_t - G_t \tag{27}$$[14]

The variables are as defined in Section B except that they are now measured in terms of exportable goods. C_{mt} is the consumption of imported goods and C_{NTt} is the consumption of nontraded goods. The relative price of imports in terms of export goods is given by $P^t_m/P_e = P^t_m$ (this is the case because exportables are the numeraire and their price is set to 1). The relative price of nontradable goods in terms of export goods is captured by $(P^t_n/p_e - Pn)$.

The intratemporal utility function is isoelastic and the allocation of expenditure between tradable and importable is in the form of a Cobb-Douglas function:

$$U(C_{mt}, C_{NTt}) = \frac{1}{1-\sigma}\left(C_{mt}^{\alpha} C_{NTt}^{1-\alpha}\right)^{1-\sigma} \tag{28}$$

Appendix III derives the first-order conditions for this problem and uses them to derive the following optimal consumption profile:

$$E_t\left[\beta(1+r_{t+1})\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\left(\frac{p_{nt}}{p_{nt+1}}\right)^{(1-\sigma)(1-\alpha)}\left(\frac{p_{mt}}{p_{mt+1}}\right)^{\alpha(1-\sigma)}\right] = 1 \tag{29}$$

$$E_t\left[\beta^{\gamma}(1+r_{t+1})^{\gamma}\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{p_{nt}}{p_{nt+1}}\right)^{(\gamma-1)(1-\alpha)}\left(\frac{p_{mt}}{p_{mt+1}}\right)^{(\gamma-1)(\alpha)}\right] = 1 \tag{30}$$

Based on the assumption of lognormal distribution, (30) may be rewritten in a log-linearized form as:

$$E_t\Delta c_{t+1} = \gamma E_t\left[r^*_{t+1} + \frac{1-\gamma}{\gamma}(1-\alpha)\Delta p_{nt+1} + \frac{1-\gamma}{\gamma}\alpha\Delta p_{mt+1}\right] \tag{31}$$

Therefore, the evolution of optimal consumption profile is given by:

$$E_t\Delta c_{t+1} = \gamma E_t\hat{\hat{r}}_{t+1}\text{; where } \hat{\hat{r}}_t = r^* + \frac{1-\gamma}{\gamma}(1-\alpha)\Delta p_{nt+1} + \frac{1-\gamma}{\gamma}\alpha\Delta p_{mt+1} \tag{32}$$

The difference between the evolution of consumption in (32) and that in (10) is the inclusion of changes in the terms of trade. All the restrictions derived in Section B are equally applicable here, but with the difference that the consumption based interest rate, $\hat{\hat{r}}$, is now defined to include changes in the terms of trade.

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[14] We allude to the derivation in Appendix I. Further details are provided in Appendix III.

The optimal current account is given by:

$$CA_t *** = -E_\tau \sum_{t=\tau+1}^{\infty} \beta^{t-\tau} \left(\Delta q_t - \gamma \hat{\hat{r}}_t \right) \tag{33}$$

Following the same procedures used to derive (25), equation (33) can be rewritten as:

$$CA_t *** = kz_t \tag{34}$$

$$k = -(g_1 - \gamma g_2)\beta\Psi(I - \beta\Psi)^{-1}$$

where $z_t = (\Delta Q_t \; CA_t \; \hat{\hat{r}}_t)$, $g_1 = [1\ 0\ 0]$, $g_2 = [0\ 0\ 1]$, and $h = [0\ 1\ 0]$.

E. Assumptions of the Intertemporal Model of the Current Account and the Applicability of the Model to the Nigerian Economy

One of the assumptions of the intertemporal model of the current account is that current consumption depends on permanent income. Since consumption depends on permanent income, a current tax cut that will be offset with future tax increases will have no impact on the current consumption, based on the Ricardian equivalence hypothesis[15]. The PVMCA also assumes unrestricted access to the international financial market, so that any negative shocks to current output, with permanent income unchanged, will not affect current consumption but result in the accumulation of external liabilities. Furthermore, the assumption of unrestricted access to the international financial market suggests separation of investment decision from consumption decisions. Any productive investment that generates returns that is greater than the world interest rate should be financed through the international capital market.

The important question is: how do these assumptions apply to a developing economy such as Nigeria? Surveys by Gersovitz (1988) and Deaton (1989) provide several explanations as to why household consumption behavior in developing countries may be quite different from that of the developed countries. First, the demographic structure of the household in developing countries appears to be different from that of the developed countries. The size of the household in developing countries is larger than in developed countries. Also, the members of the household in developing countries engage in resource

[15] There are many reasons why Ricardian equivalence does not generally hold in practice, including liquidity constraints, rule of thumb consumption behavior, and the presence of distortionary taxes. As Milesi-Ferretti and Razin (1996) suggest, strong links between the current account and fiscal balances should be observed in underdeveloped financial system (i.e., where liquidity constraints are likely to be more binding). However, there is one argument that may work towards the validity of Ricardian equivalence in the context of the Nigerian economy: the public debt levels are high, and the possibility of budget deficits leading to imminent tax increases is conceivable, thereby enhancing the link between public and private sector savings behavior.

sharing. Resource sharing implies that there is no need to save for the retirement years. The older generations support the younger ones by lowering their consumption. When the younger generation becomes older, they depend on the younger members of the household for their consumption expenditure. Thus, the life-cycle model of consumption with the assumption of "hump" saving cannot explain consumption behavior of the household in most developing countries. Also, the fact that the household lives forever through resource transfers from one generation to another, so that the permanent income hypothesis may capture the actual experience in developing countries.

The second factor distinguishing the household consumption behavior of developed economies from that of the developed countries is the high volatility of income. This reflects the agrarian base of most developing countries and terms of trade changes. In the face of uncertainty in income, there is higher impetus to save for precautionary reasons as this sort of uncertainty cannot be diversified through aggregate resource pooling (Deaton, 1989, Grimard, 1997). In addition, a typical household in developing countries is often liquidity constrained. This has the implication that when income is less than its permanent level, the household may not be able to borrow in order to maintain the same level of consumption. In this situation, Ricardian equivalence may fail to hold.

For Nigeria, the idea of a dynasty of consumers supports the assumption that the consumers live infinitely, which implies the permanent income hypothesis. Another fact that favors the use of permanent income hypothesis is that a high number of households in developing countries operate at near-subsistence income levels. This may strengthen the need to smooth consumption over time, so that a sudden increase in income, which is considered to be temporary, is likely to be saved. The fact that the households may not be able to smooth consumption in the face of negative income shocks constitutes a limitation of the application of the rational-expectation permanent income consumption for Nigeria.

There are other features of the Nigerian economy that lend support to the application of PVMCA. The country experienced a civil war over the 1967–70 period. This was period of increasing real government expenditures due to the civil war. If they were viewed as temporary phenomenon, one would expect deficits in the current account in the course of this war. Thus, the Nigerian economy constitutes a natural framework for examining the validity of one of the implications of the intertemporal model of the current account, that is, an unexpected temporary increase in government expenditure generates a current account deficit.

As noted by Mwau and Handa (1994), a major point of contention in the economic literature on the developing economies is the extent to which the economic behavior of the government and the private sector can be adequately studied through the use of neoclassical models based on the assumption of rational optimizing agents. There are two choices available to a researcher working on a developing economy like Nigeria. The first choice is to work with a neoclassical framework and adjust it for special features of developing economies. An alternative choice is to make use of a descriptive or *ad hoc* theoretical framework that is specific to the developing country being studied. This carries the implication that the results obtained from such analysis is only country-specific and cannot be used as a basis for policy formulation in other developing economies. The former approach is followed in this paper; the intertemporal model of the current account is applied to the Nigerian economy. However, variables such as the terms of trade and the real exchange rate are taken into consideration.

III. The Empirical Analysis

A. Data and Parameter Values

Equations (25) and (34) are tested using annual data over the 1960–97 period. Data on private consumption, government consumption, investment, Gross Domestic Product (GDP), and Gross National Product (GNP) in billions of local currency (Naira) are collected. All variables are cast in *real per capita terms* by dividing the nominal variables by the GDP deflator (1995=100) and the level of total population. The National Income Accounting (NIC) data used for the analysis originate from the World Bank, Social Indicators of Development data base. This is in conformity with the previous empirical investigation of the intertemporal model of the current account for a number of developing countries by Ghosh and Ostry (1995).

The actual current account series, ca_t, was constructed by subtracting the log of per capita real private consumption expenditure from the log of net output, (q_t)[16]. The net output series; q_t is computed by subtracting government and investment expenditures from the GDP (all in log forms). The change in net output, $cneo_t$, was arrived at by taking the log and first difference of q_t.

Following the method of Barro and Sala-i-Martin (1990), Bergin and Sheffrin (2000), a proxy for the world real interest rate, r_t* is computed. This paper assumes that the world interest rate is given by the weighted average of the money market rates in the following countries: France, Japan and the USA[17]. The consumer price index for each country is used to derive estimates of inflation for that country, and expected inflation is based on one-year auto regression. An *ex-ante* real interest rate for each country is the nominal rate adjusted for expected inflation. The time varying weight, the share of each country in the total GDP is used to compute the weighted average real interest rate. The data used for the computation of the real interest rate are from *International Financial Statistics.*

The available time series data on the real effective exchange rate[18] for Nigeria covers 1980–97 period. In order to arrive at estimates of real effective exchange rate during

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[16] Bergin and Sheffrin (2000) used the same approach to arrive at the current account balance.

[17] Data availability during the period covered by this study influenced the choice of the three countries.

[18] Real exchange rates are used as proxies for the relative prices of nontradables in terms of either tradables or exportables. The fact that the relative prices of nontradables expressed in

(continued…)

1960-97, first the bilateral exchange rate between Nigeria and a trading partner is computed. This is summed across six major trading partners: France, Germany, Japan, Netherlands, U.K., and the USA. The weight assigned to a trading partner reflects the extent of trade flows between that partner and Nigeria. The Wholesale Price Index (WPI) is used in computing the real effective exchange rate, as this is more likely to be a good proxy for the price of tradable. The terms of trade is defined as the relative price of imports in terms of exports. An *ex-ante* expected change in the exchange rate and terms of trade are computed using a one-year auto regression. The first set of consumption based interest rate, $\hat{r}$, is given by the world interest rate, r^* adjusted for expected change in the exchange rate and the second set of consumption based interest rates, $\hat{\hat{r}}$, adjusts for both changes in the exchange rate and terms of trade. Given the interest in dynamic implications of the intertemporal model, the series $cneo_t$, ca_t, $\hat{r}$, and $\hat{\hat{r}}$ are expressed as deviations from their means in the VAR processes.

There are three other parameters that are important to estimating equations (25) and (34): the elasticity of intertemporal substitution, γ, the share of tradable/importable goods in the total consumption expenditure, α, and the preference parameter, β. Outside studies are used in order to come up with estimates for these parameters. Arguments abound in the literature as to the size of the intertemporal elasticity parameter. This paper tends to support the position of Ostry and Reinhart (1992) that the intertemporal elasticity of substitution is significantly different from zero. This emanates from the fact that their study and this paper allow for nontradable goods. In their study, the intertemporal elasticity of substitution ranged between 0.38 and 0.50. The preference parameter fell in the range of 0.96–0.99[19]. The share of tradable good was assigned the value of 0.85. Their estimates are used in conducting the preset value tests.

B. Unit Root Tests

The variables entering the VAR $cneo_t$, ca_t, r^*, $\hat{r}$ and $\hat{\hat{r}}$ must be stationary if (25) and (34) are to be valid. In order to test for the presence of unit roots, and hence the degrees of integration of these variables, two unit root tests are used: the Augmented Dickey-Fuller (ADF) approach and Phillips and Perron (1988) procedure. These approaches have low power against plausible trend-stationary alternatives. This weakness should be kept in mind when interpreting the results obtained from their application. The results from applying the ADF and PP tests, using one lag, presented in Table 1 does not include a constant and a time trend as variables are expressed in deviations from their means. The current account, ca_t,

terms of either tradables or exportables may not necessarily move with real exchange rates in reality is recognized. See, for example, Kakkar and Ogaki (1999) for a detailed discussion on the relationship between the relative price of nontradables and exchange rates.

[19] In the current context, $\beta = \dfrac{1}{1+\bar{r}^*} = 0.97$. $\bar{r}^*$ is the average of the computed world interest rate.

change in net output, $cneo_t$, world interest rate, r^*, consumption based interest rates, $\hat{r}$ and $\hat{\hat{r}}$, are found to be stationary variables at significance levels of 5 percent.

Moreover, the intertemporal models of the current account as indicated in equation (25) and (34) imply that optimal current accounts are stationary variables. If the intertemporal model is valid, that is, the actual current account equals the optimal current account, and then the actual current account must also be a stationary variable. Using the Augmented Dickey Fuller and Phillips-Perron Unit root tests, this implication is found to be valid, as the actual current accounts appear to be stationary variables. This constitutes evidence in favor of the present-value model of the current account.

Table 1: Unit Root Tests

Variable	*ADF* *Level*	*PP* *Level*
cneo	*-5.23*	*-5.88*
ca	*-4.27*	*-4.28*
r^*	*-3.91*	*-3.28*
$r^\wedge$; $\gamma=0.45;\ \alpha=0.85$	*-4.04*	*-3.82*
$r^{\wedge\wedge}$ $\gamma=0.45;\ \alpha=0.85$	*-4.62*	*-5.56*

r^*_t is the world interest rate; $r^\wedge_t$ is the consumption-based interest rate that includes expected change in the exchange rate; $r^{\wedge\wedge}_t$ is the consumption based interest rate that includes both changes in the terms of trade and exchange rate.

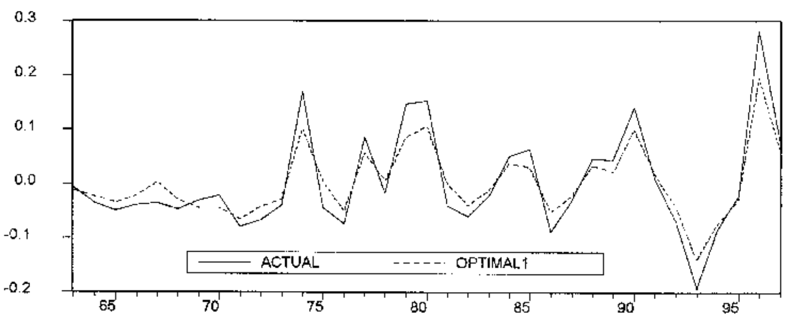
C. Formal and Informal Tests of the Model

Formal tests of the restrictions implied by the present-value relationship, are based upon the estimates reported in Table 2. The first column shows the results obtained from estimating the benchmark model that excludes changes in the interest rate, exchange rate and terms of trade. The second column reports the results from adjusting the benchmark model for changes in the interest rate. The third column reports the results from estimating a model that accommodates changes in both the interest rate and the exchange rate. Column four contains the empirical results from adjusting the benchmark model for changes in the interest rate, exchange rate and terms of trade. Each column contains the estimated k-vector and the accompanied $\chi 2$ statistic and its p-value. The volatility of the optimal current account is reported as a percentage of the actual current account. Informal tests are captured by the graphical representation of the actual and optimal current accounts.

1. The Standard Present-Value Model of the Current Account

The standard PVMCA excludes changes in the interest rate, exchange rate and terms of trade A one-lag VAR for the variables of interest, change in net output ($cneo_t$) and actual current account, (ca_t) was chosen by the Akaike Information Criterion (AIC) and Schwarz Criterion (SC). In Figure 1 are shown the actual current account data derived from national income account statistics and the optimal current account, covering the period 1963–97.

Figure 1. Nigeria: Actual and Optimal Current Accounts
(Excluding Interest Rate, Exchange Rate and Terms of Trade)



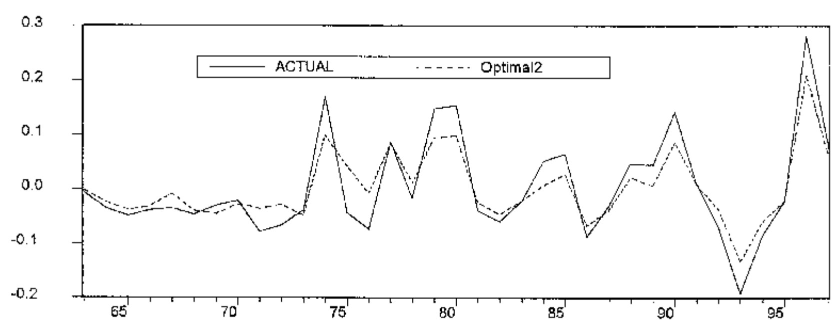
As shown in Figure 1, the standard model is able to predict the general direction of current account movements, especially the current account deficits in the 1960s, the 1974 and 1979–80 current account surpluses arising from global increases in oil price, the 1981–83 current account deficits and the improvements in current account position in the last two years covered by this study. Despite this performance, the statistical test rejects the model. With one lag and two variables, the k-vector based on the PVMCA is expected to be [0 1]. However, the estimated values are [-0.14 0.74]. While the coefficient on the current account at date t is statistically different from zero, it is well different from 1 (see column1, Table 2,). The change in net output, $cneo_t$ however, is found not to be significantly different from its theoretical value of zero. The model's current account is only 47 percent as volatile as the actual current account. Overall, the $\chi2$ test strongly rejects the model, with a p-value of zero. As noted by Bergin and Sheffrin (2000), this result is typical of most previous empirical investigation in this area: while a simple graphical analysis appears to suggest that the standard PVMCA can explain a significant portion of the movements in the actual current account, the model rarely satisfies statistical tests. The failure of the model in terms of statistical tests can be attributed to exclusion of channels through which external shocks can impact upon the current account.

2. The Standard Model Adjusted for Changes in the World Interest Rate

There is no change in the coefficient on the current account at date t as a result of introducing the real interest rate into the standard PVMCA. However, the coefficient on the change in net output, $cneo_t$, marginally moves toward its theoretical value of zero, increasing from -0.14 to -0.05. Nonetheless, instead of the coefficient on the varying interest rate being zero, it has a negative value of 0.8. The equality between each coefficient and its theoretical value is tested. It is somehow difficult to interpret the observed result. Despite the fact that point estimates are distant from their theoretical values, the existence of large standard errors prevents a rejection of the null. The introduction of a varying interest rate does not necessarily lead to much increase in the volatility of the optimal current account, as the optimal current account is only 47.5 percent of the actual current account. Focusing on the overall performance of the intertemporal model, the $\chi 2$ test rejects the model, with a zero p-value.

Despite the fact that the standard errors are quite big, we can still make the following conclusions. Introducing a time varying interest rate into the PVMCA is not sufficient to improve the fit of the model. The overall model is rejected and there is not much improvement in the volatility of the optimal current account. The optimal current account associated with introducing a varying world interest rate into the standard PVMCA and the actual current accounts are presented in Figure 2. As the case with the standard PVMCA, this model as well is able to mimic the actual behavior of the current account.

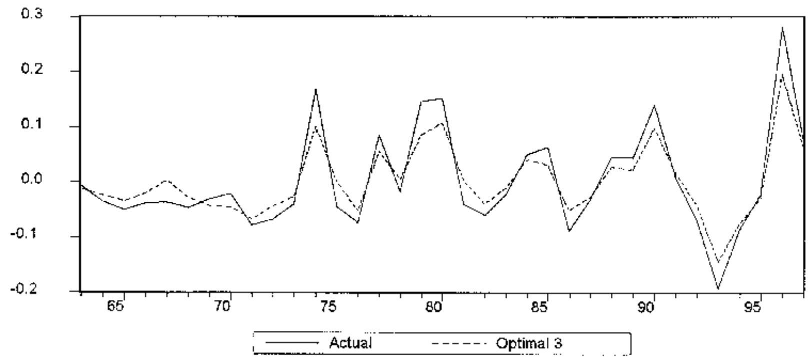
Figure 2. Nigeria: Actual and Optimal Current Accounts
(Excluding Exchange Rate and Terms of Trade)



3. The Standard Model Adjusted for Changes in the World Interest Rate and Real Exchange Rate

By introducing expected change in the real exchange rate, the fit of the PVMCA improves. Despite the fact that there is not much change in the coefficient on current account at date t. and net output, $cneo_t$, relative to a model that incorporates varying interest rate alone or the standard model, the coefficient on the consumption based interest rate is close to its theoretical value of zero (see column 3, Table 2). Again, the problem of large standard errors prevents the rejection of the difference between theoretical and estimated values. Also, while not much, the volatility of the optimal current account increased to 48 percent of that of the actual. However, focusing on the overall performance of the intertemporal model, the $\chi 2$ test does not reject the model, with a p-value of 0.9501. The optimal current account when the consumption based interest rate is introduced better matches the actual current account balance graphically (Figure 3).

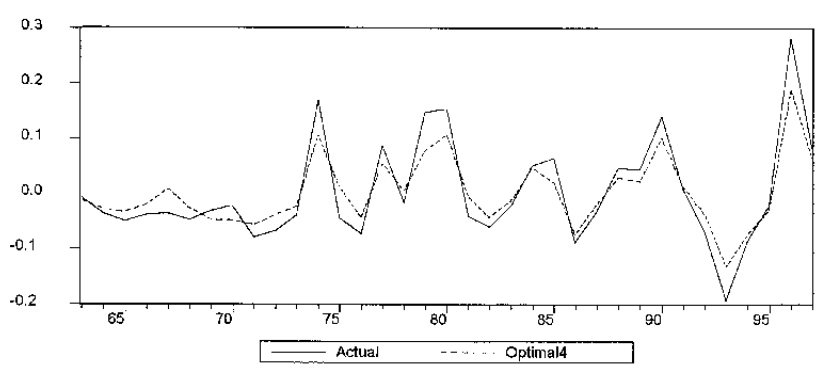
Figure 3. Nigeria: The Standard Model Adjusted for Changes in the World Interest Rate and Real Exchange Rate



4. The Standard Model Adjusted for Changes in the World Interest Rate, Real Exchange Rate and Terms of Trade

By introducing the terms of trade into the present-value model of the current account, additional key insights emerged. First, the coefficient on current account at date t was 0.7042 instead of the theoretical value of 1. This is less than 0.7425, when changes in the interest rate and exchange rate are considered. The coefficient on the change in net output, $cneo_t$, is 0.1591, which is different from its theoretical value of zero and is even farther than the value of -0.1446, when the terms of trade was not taken into consideration. The value on the consumption based interest rate when the terms of trade was taken into consideration was -0.0257, whereas, it was 0.0693 when only the expected change in the exchange rate was taken into consideration. Secondly, the volatility of the optimal current account is 46 percent of that of the actual current account. Lastly, the $\chi 2$ test does not reject the model, with a p-value that is close to one. The incorporation of the terms of trade does not appear to improve the fit of the PVCMA (when changes in the interest rate and exchange rate have already been taken into consideration). However, it outperforms a model that does not incorporate the transmission mechanisms through which an external source can impact on the current account. As the results of this analysis tend to suggest, it seems what is important for current account determination in an intertemporal analysis is not the source of the shocks but the transmission mechanisms though which the shock affects the current account.

Figure 4. Nigeria: The Standard Model Adjusted for Changes in the World Interest Rate, Real Exchange Rate and Terms of Trade



IV. Conclusion

This paper has examined the impact of introducing a time varying interest rate into the present-value model of the current account. The fact that a small open economy may be vulnerable to external shocks necessitated the need to incorporate transmission mechanisms through which external shocks can affect the domestic economy. Allowing for a distinction between importable and nontradable goods allowed changes in both the terms of trade and real exchange rate to affect the current account balance.

Overall, after taking into consideration the channels through which changes in external shocks can affect the current, one can conclude that PVMCA is a valid framework for analyzing the Nigerian current account. Also, the optimal current accounts derived from the PVMCA are able to capture the evolution of the actual current account balance. Nonetheless, the actual current account is more volatile than the optimal, suggesting that speculative factors are a major driving force behind capital flows. The empirical results clearly show that in analyzing the current account from an intertemporal perspective, the channels through which external shocks impact on the current account must be taken into consideration and not necessarily the source of the shocks.

Finally, the result obtained from the empirical implementation of the present-value model of the current account is also important for understanding the behavior of households in responding to external shocks, such as a terms of trade deterioration. This is crucial for a government that is interested in using policy instruments at its disposal to respond to external shocks. As an illustration, if a government is interested in achieving a given current account balance, an unintended terms of trade shock may be perceived by the government as damaging its ability to achieve the target, and consequently the need to use, fiscal, exchange and trade policy measures to achieve the target. However, such a reaction may be counterproductive and generate distortions. Knowing that intratemporal and intertemporal effects will accompany unanticipated changes in the terms of trade and that the overall impact on the current account will be less than what would be expected when these effects are not taken into consideration would determine the magnitude of the policy response. This clearly shows the importance of analyzing the current account balance from an intertemporal perspective.

Table 2: Present Value Tests

[1] Benchmark model	*[2] Benchmark Model with r**	*[3] Benchmark Model with r^ $\gamma=0.45; \alpha=0.85$*	*[4] Benchmark Model with r^^ $\gamma=0.45; \alpha=0.85$*
-0.1400	-0.0550	-0.1446	-0.1591
[0.149]	[0.75752]	[0.36866]	[0.34887]
0.7400	0.7437	0.7425	0.7042
[0.016]	[0.62531]	[0.26677]	[0.73249]
	-0.8319		
	[1.49596]		
		0.0693	
		[0.1750]	
			-0.0257
			[0.91468]
265.6550	156.3914	0.3513	0.3990
0.0000	0.0000	0.9501	0.9404
47	47.5	48	46

Derivation of the Dynamic Budget Constraint with an Index of Total Consumption
(Excluding Changes in the Terms of Trade)

This Appendix demonstrates the equality of (2) and (4). To show this, we first need to derive an expression for P_t*. To achieve this, we use the period utility function in (3) and the constraint that $C_t = C_{Tt} + P_t C_{NTt}$ to solve for the optimal intratemporal allocation of expenditure between the tradable and non-tradable goods. Using the first order condition that the marginal rate of substitution between tradable and non-tradable goods should equal their relative price, we have:

$$C_{NTt} = (1-\alpha)\frac{C_t}{P_t}, C_{Mt} = \alpha C_t \tag{35}$$

The optimal values of tradable and non-tradable goods are substituted into the definition for C*, which yields:

$$C_t^* = (\alpha C_t)^a \left[(1-\alpha)\frac{C_t}{P_t}\right]^{1-\alpha} \tag{36}$$

Now use the condition that P* is defined such that C*–1. This produces:

$$C_t^* = \left(\alpha P_t^*\right)^a \left[(1-\alpha)\frac{P_t^*}{P_t}\right]^{1-a} = 1 \tag{37}$$

Solving (37) for the consumption based price index, P_t*, yields:

$$P_t^* = P_t^{(1-a)}\left[\frac{1}{\alpha^a (1-\alpha)^{(1-a)}}\right] \tag{38}$$

Note that C_t/P_t* is the ratio of spending, measured in units of tradables, to the minimum price, in tradables, of a single unit of the consumption index. Therefore, C_t/P_t* is the index of the total real consumption of an optimizing consumer.

Let:

$$C_t * = \frac{C_t}{P_t *}$$

Given that:

$$C_t * = \frac{C_T + p_t C_{NTt}}{P_t *}$$

$$P_t * C_t * = C_t + P_t C_{NTt} \tag{39}$$

This provides the justification for writing the dynamic budget constraint in (2) as (4).

Derivation of the Log-Linearization of the Intertemporal Budget Constraint

Equation (16) was:

$$\Gamma_0 = \sum_{t=0}^{\infty} E_0(R^*_t\, Q_t)$$

Equation (16) implies the law of motion for Γ_t:

$$\Gamma_{t+1} = (1+r_t^*)(\Gamma_t - Q_t), \text{ for } t \geq 0. \tag{40}$$

Dividing (40) by Γ_t, and taking logarithms on both sides, yields:

$$\varphi_{t+1} - \varphi_t = ln(1+r^*_t) + ln\left(1 - \frac{Q_t}{\Gamma_t}\right) \tag{41}$$

$$\approx r^* + ln(1 - exp(q_t - \varphi_t))$$

where $\varphi = log\, \Gamma$ and we made use of the approximation that $log\,(1+r_t^*) = r_t^*$

Next, take a first-order Taylor expansion of ln $(1\text{-}exp(q_t\text{-}\varphi_t))$ in (41) around a normal level of $(q_t\text{-}\varphi_t)$. This yields:

$$ln(1 - exp(q_t - \psi_t)) \approx k + \left(1 - \frac{1}{\rho}\right)(q_t - \varphi_t) \tag{42}$$

Therefore, we can write equation (42) as:

$$\varphi_{t+1} - \varphi_t \approx r_t^* + k + \left(1 - \frac{1}{\rho}\right)(q_t - \varphi_t) \tag{43}$$

where

$\rho = 1 - exp(q_t - \varphi_t) = 1 - \frac{Q_t}{\Gamma_t}$, a number slightly less than one, and

$$k = ln(\rho) - \left(1 - \frac{1}{\rho}\right) ln(1-\rho)$$

[20] This presentation follows the work of Huang and Lin (1983) closely. While their presentation focuses on fiscal policy, it is equally applicable in analyzing the current account balance.

Note that:

$$\varphi_{t+1} - \varphi_t \approx \Delta q_{t+1} + (q_t - \varphi_t) - (q_{t+1} - \varphi_{t+1})r_t * + k + \left(1 - \frac{1}{\rho}\right)(q_t - \varphi_t) \tag{44}$$

Substituting (43) into (44) yields:

$$-(q_{t+1} - \varphi_{t-1}) + \left(\frac{1}{\rho}\right)(q_t - \varphi_t) \approx -(\Delta q_{t+1} + r_t * + k) \tag{45}$$

Solving the above difference equation forward produces:

$$q_0 - \varphi_0 = \sum_{t=1}^{\infty} \rho^t (r * - qc_t) + \eta \tag{46}$$

where $q_0 = \ln Q_0$; $\varphi_0 = \ln \Gamma_0$; $\Delta q_t = q_t - q_{t-1}$; and η is a constant.

Derivation of the Dynamic Budget Constraint with an Index of Total Consumption

The derivation here is quite similar to that in Appendix I. Our intention is to show that the dynamic budget constraint in (27) is valid.

Using the first-order condition that the marginal rate of substitution between importable and nontradable goods should equal their relative price, we have:

$$C_{NTt} = \frac{(1-\alpha)}{p_{nt}} C_t \,;\, C_{mt} = \frac{\alpha}{P_{mt}} C_t \tag{47}$$

Once we derive the optimal choice of both importable and non-tradable goods, they are substituted into the definition for C^*, which in turn yields:

$$C_t^* = \left(\frac{\alpha}{P_{mt}} C_t\right)^{\alpha} \left[(1-\alpha)\frac{C_t}{P_{nt}}\right]^{1-\alpha} \tag{48}$$

Now we use the condition that P^* is defined such that $C^*=1$. This gives:

$$C_t^* = \left(\frac{\alpha}{p_{mt}} P_t^*\right)^{\alpha} \left[(1-\alpha)\frac{P_t^*}{P_{n_t}}\right]^{1-\alpha} = 1 \tag{49}$$

Solving (49) for the consumption based price index, P_t^*, produces:

$$P_t^* = p_{m_t}^{\alpha} P_{n_t}^{(1-\alpha)} \left[\frac{1}{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}\right] \tag{50}$$

Note that C_t/P_t^* is the ratio of spending, measured in units of exportables, to the minimum price, in exportables, of a single unit of the consumption index. Thus C_t/P_t^* equals the level of the total real consumption index C_t^* that an optimizing consumer enjoys.

Therefore:

$$C_t^* = \frac{C_t}{P_t^*}; \quad C_t = P_{mt} C_{mt} + P_{nt} C_{NTt}$$

$$C_t^* = \frac{p_m C_{mt} + p_{n_t} C_{NTt}}{P_t^*}$$

$$P_t^* C_t^* = p_{mt} C_{mt} + p_{n_t} C_{NTt} \tag{51}$$

This provides the justification for writing the dynamic budget constraint as (27).

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