

# IMF Working Paper

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## Recursive Utility, Endogenous Growth, and the Welfare Cost of Volatility

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**IMF Working Paper**

IMF Institute

**Recursive Utility, Endogenous Growth, and the Welfare Cost of Volatility<sup>1</sup>**

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Authorized for distribution by Roland Daumont

January 2001

**Abstract**

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This paper proposes a measure of the welfare cost of volatility derived from an endogenous growth model (*AK*) under uncertainty extended to the case of a recursive utility function which disentangles risk aversion from intertemporal elasticity of substitution. It encompasses a direct welfare cost of fluctuations and a welfare cost due to the endogeneity of the consumption. The total welfare cost of volatility increases with both the risk aversion and the intertemporal elasticity of substitution. For plausible values of the agent's preference parameters, the cost of volatility may be greater than measures based on an exogenous process for consumption

JEL Classification Numbers: E32, E60

Keywords: Welfare Cost of Fluctuations, Recursive Utility, Endogenous Growth, Risk

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## I. INTRODUCTION

This paper revisits the following question: would we benefit, and by how much, from reducing all macroeconomic shocks and surprises? Lucas (1987) has provided a well-known response to this question: whenever it could be possible to eliminate fluctuations in consumption, the gain in terms of welfare would be negligible compared with what could be achieved with more growth. Thus economists should look for ways to attain higher growth rates rather than for economic policies to reduce fluctuations in consumption.

As this position is in conflict with the textbook view for which, following Musgrave, stabilization is one of the three goals of political economy, many economists have proposed other measures of the welfare cost of fluctuations. One way of challenging Lucas's conclusion is to relax some of the assumptions under which he performed his computation. Imrohorglu (1989) and Atketson and Phelan (1994) measure the welfare cost of fluctuations for heterogeneous agents in the absence of a perfect insurance market. Preferences of the representative agent have also been reconsidered. Campbell and Cochrane (1995) investigate the case of habit formation while Obstfeld (1994a), Tallarini (1997), and Dolmas (1998) use a recursive utility function. Dolmas (1998) also considers the case of rank-dependent preferences. These studies generally lead to a slightly larger cost of fluctuations but still too small to refute Lucas's conclusion (Dolmas (1998) is an exception). However, even if most of them depart from Lucas's assumption of consumption transitory shocks, they share the hypothesis of an exogenous process for consumption. Otrok (1999) revisits the welfare cost of fluctuations in a complete business cycle model in which consumption is endogenous for various kinds of preferences. His conclusion is that the cost of business cycles is not much larger than Lucas's estimate.

The new point we stress in this paper is that in an endogenous growth model ( $AK$ ), reducing the source of consumption fluctuations modifies the saving/consumption trade-off and thus the growth trend of the entire economy (except in the case where the intertemporal elasticity of substitution is unitary). The issue of the relationship between volatility and growth has recently been treated by Jones, Manuelli, and Stacchetti (1999), and Barlevy (2000). Since we use a model which encompasses a very simple  $AK$  technology while stressing the modeling of preferences using a recursive utility function, we get an analytical resolution which allows us to: 1) explain clearly the relationship between growth and volatility; and 2) split up the total welfare cost between a direct welfare cost (i.e., a cost directly resulting from the fluctuations in consumption), and a cost due to the trend endogeneity. As it will be explained further in the paper, the recursive utility function, which disentangles the risk aversion from the intertemporal elasticity of substitution, allows for cases where the saving/consumption trade-off leads to a rather large welfare cost. Empirical investigation assesses that, for the United States, the likelihood of a high cost of volatility cannot be ignored. Our paper may also be viewed as an application of the idea stressed by Obstfeld (1994b)—who takes into account *both* the reduction of volatility and the induced growth when evaluating the effect of international risk-sharing through capital markets liberalization—to the welfare cost of volatility measurement.

This paper is organized as follows. Section II proposes a straightforward macroeconomic model of endogenous growth under uncertainty extended to the case of a recursive utility function (Smith, 1996). It shows that the growth rate of the economy may decrease with uncertainty. Such a result is also valid in the special case of an additive utility function; the new point is that the size of the effect of volatility on the economy growth rate increases with both the risk aversion and the intertemporal elasticity of substitution. In Section III, a measure of the total welfare cost of volatility as well as a decomposition of this total cost into a direct cost and a cost due to the trend are derived from the theoretical model. Section IV shows that when evaluating this cost, agents' preferences really do matter: a large intertemporal elasticity of substitution (greater than unity) and a not too small risk aversion (greater than two) give rise to a significant cost while small values for the former parameter confirm Lucas's conclusion. Moreover, our figures suggest that, thanks to the consumption endogeneity, the cut in consumption at the time the agent reaches the deterministic economy may be quite different from both the total welfare cost of volatility and the direct cost of volatility evaluated in terms of initial capital. Section V concludes the paper.

## II. VOLATILITY AND GROWTH

### A. The Underlying Macroeconomic Model

#### The recursive utility function

The model is in continuous time since such a modelization allows for a complete analytical resolution. The representative agent maximizes a recursive utility function as defined by Weil (1990), Epstein-Zin (1991) and Svensson (1989):

$$(1-\gamma)U(t) = \left[ c(t)^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-\delta dt} \left( (1-\gamma) E_t U(t+dt) \right)^{\frac{1-\gamma}{\varepsilon}} \right]^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}} \quad \text{with } \gamma \neq 1 \text{ and } \varepsilon > 0, \varepsilon \neq 1 \quad (1)$$

$$U(t) = \left[ c(t)^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-\delta dt} \left( e^{E_t(\ln U(t+dt))} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad \text{with } \gamma = 1 \text{ and } \varepsilon > 0, \varepsilon \neq 1 \quad (2)$$

where  $\gamma$  is the relative risk aversion coefficient,  $\varepsilon$  is the intertemporal elasticity of substitution and  $\delta$  is the time preference rate.  $1/\varepsilon$  may also be understood as a measure of the resistance to intertemporal substitution (sometimes called fluctuations aversion). Note that if  $1/\varepsilon = \gamma$ , the recursive utility function reduces to the familiar time separable intertemporal utility function:

$$U(t) = E_t \left[ \int_t^{\infty} \frac{c(s)^{1-\gamma}}{1-\gamma} e^{-\delta(s-t)} ds \right] \quad \gamma \neq 1 \quad (3)$$

$$U(t) = E_t \left[ \int_t^{\infty} \ln c(s) e^{-\delta(s-t)} ds \right] \quad \gamma = 1 \quad (4)$$

For a utility function as (3), one cannot distinguish between risk aversion and resistance to intertemporal substitution.

Another feature of the recursive utility function is that, whereas the standard additive utility function implicitly supposes agents to be indifferent with respect to the timing of the uncertainty resolution, Kreps and Porteus (1978) have shown that it is no more the case with the recursive utility function ((1) and (2)). If  $\gamma > 1/\varepsilon$ , agents dislike risk more than fluctuations and prefer an early resolution of uncertainty; if  $\gamma < 1/\varepsilon$ , agents dislike fluctuations more than risk and prefer a late resolution of uncertainty.

### Technology and volatility

The technology is  $AK$ . Technological shocks continuously perturb the production process. Over the period  $(t, t+dt)$  the flow of output is:

$$F(K)dt = K [A dt + \sigma dz] \quad (5)$$

where  $dz$  is the increment of a standard Wiener process ( $dz = \eta(t)\sqrt{dt}$ ;  $\eta(t) \sim N(0,1)$ ). Equation (5) asserts that the flow of output accumulated over the period  $(t+dt)$  consists into two components: a deterministic component ( $AKdt$ ), and a stochastic component ( $K\sigma dz$ ) reflecting the random influences that impact on the production. The stochastic term  $\sigma dz$  may be referred to as a productivity shock and assumed to be temporally independent, normally distributed with zero mean and variance  $\sigma^2 dt$ . Thus, as far as productivity is concerned, shocks are neither correlated nor persistent. This leads to a stochastic capital accumulation equation:

$$dK(t) = [AK(t) - C(t)]dt + K(t)\sigma dz(t) \quad (6)$$

It is clear from (6) that as soon as consumption does not exactly compensate for productivity shocks, these shocks will have persistent effects on the whole economy through capital accumulation.

### B. The Optimal Growth Rate of Consumption and Capital

Maximizing (1) under (6) gives the optimal stochastic level of consumption:<sup>2</sup>

$$C^* = \left( \varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma \frac{\sigma^2}{2} \right) K \quad (7)$$

and we note  $c(\sigma)$  the propensity to consume the current wealth (capital).

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<sup>2</sup> cf. Appendix.

The common growth rate of consumption, capital, and production is then:

$$\frac{dK}{K} = \frac{dC^*}{C^*} = \left[ \varepsilon(A - \delta) + (1 - \varepsilon)\gamma\sigma^2 / 2 \right] dt + \sigma dz = \mu(\sigma)dt + \sigma dz \quad (8)$$

Thus, the optimal consumption follows a geometric Brownian process, that is, it increases according to a deterministic trend  $\mu(\sigma)$  per unit of time continuously perturbed by shocks. As far as optimal consumption forecasts are concerned, the further the forecast date, the larger the confidence interval. For our purpose, the interesting results in (8) are that (i) the consumption process exhibits a unit root (the exact discretization of (8) is:

$\ln C^*_t - \ln C^*_{t-1} = \left[ \varepsilon(A - \delta) + (1 - \varepsilon)\gamma\sigma^2 / 2 - \sigma^2 / 2 \right] + \sigma\eta_t$ , where  $\eta_t \sim NIID(0, 1)$ ), and (ii) the deterministic trend of the consumption process depends on each structural parameter and also on the size of the uncertainty ( $\sigma$ ). It is straightforward to show that all other macroeconomic variables (capital stock and production) grow at this same rate.

Ignoring the volatility, equation (7) reduces to the relationship between capital and consumption which may be derived from a standard deterministic  $AK$  model: as suggested by intuition, the optimal propensity to consume wealth increases with the time preference rate whereas it decreases with the intertemporal elasticity of substitution, and the risk aversion plays no role.

As far as the effect of volatility is concerned, one may consider the optimal propensity to consume current wealth or equivalently the consumption deterministic trend. The effect of an increase in uncertainty on the latter depends on both the risk-aversion and the intertemporal elasticity of substitution; it is merely:

$$\frac{\partial \mu(\sigma)}{\partial \sigma^2} = -\frac{\partial c(\sigma)}{\partial \sigma^2} = (1 - \varepsilon)\frac{\gamma}{2} \quad (9)$$

When the representative consumer is not too fluctuations-averse ( $\varepsilon > 1$ ), more uncertainty (i.e., a rise of volatility) increases the current marginal propensity to consume current wealth. To escape future uncertainty, she chooses to consume more today and accepts the counterpart of less consumption tomorrow. Following Weil (1990) one may notice that an increase in the volatility reduces the certainty equivalent return on savings ( $A - \gamma\sigma^2 / 2$ ); the way this reduction affects the consumption-saving trade-off depends on the relative strength of the income and substitution effects. Obviously, for a large intertemporal elasticity of substitution, the latter dominates, leading the agent to increase her current consumption. Thus, more uncertainty reduces the deterministic trend in consumption. Whereas the direction of the volatility effect is governed by the intertemporal elasticity of substitution, its size also depends on the risk aversion: the higher the risk aversion, the larger the effect. Of course, when consumers are reluctant to accept intertemporal substitution ( $\varepsilon < 1$ ), more uncertainty urges them to reduce their current consumption.

As a benchmark, it may be useful to recall what one would have got using the standard time-additive utility function. Since the risk aversion is then the inverse of the intertemporal elasticity of substitution, formula (9) reduces to  $\partial\mu/\delta\sigma^2 = (1-\varepsilon)/(2\varepsilon) = (1-\gamma)/2$  and one cannot identify the reason why volatility would increase consumption (small risk aversion or large intertemporal elasticity of substitution?). Furthermore, the higher the risk aversion, the smaller the current consumption, whereas with a recursive utility this is only true when the intertemporal elasticity of substitution is less than unity (see above).

### III. THE WELFARE COST OF VOLATILITY

A difficulty arises when evaluating the welfare cost of fluctuations with an endogenous propensity to consume. Following Lucas's seminal monograph, the welfare cost of fluctuations is usually expressed in terms of percentage of consumption the agent is ready to give up *at all dates* to join the deterministic world. But such a measure may be no longer very informative when the agent chooses her propensity to consume. Since the trend in the consumption process may differ as a result of the change in the agent's propensity to consume when reaching the deterministic world, the percentage of loss in consumption varies with respect to time. In fact, for some sets of preference parameters, the agent's consumption may even be higher, after some time spent in the deterministic world, than the one she would have expected at this same time in the stochastic world. Barlevy (2000) chooses to express the welfare cost of fluctuations in terms of initial consumption. This has the advantage of facilitating comparisons with previous measures of the welfare cost of fluctuations. But it may be misleading as well, since an identical cut in consumption may be followed by various consumption trends, and one would wrongly interpret a same initial cut as the fact that the welfare cost is the same. That is the reason why we compute the welfare cost of fluctuations in terms of percentage of the initial capital the agent is ready to give up to join the certain world. Yet for purposes of comparison with previous measures we also provide the formula for the corresponding cut in initial consumption and change in the trend in consumption.

#### A. Deriving a Measure of the Total Welfare Cost of Volatility

Measuring the cost of volatility in this model requires the evaluation of the expected lifetime utility associated with the optimal consumption path. It is straightforward that this lifetime utility may be evaluated as:

$$V^* [K(0); \sigma] = \left[ \varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2 \right]^{\frac{1-\gamma}{1-\varepsilon}} \frac{K(0)^{1-\gamma}}{1-\gamma} \quad (10)$$

*Definition:*<sup>3</sup> *The total welfare cost of volatility is defined as the percentage of capital the representative agent is ready to give up at period zero to be as well off in a certain world as she is in a stochastic one.*

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<sup>3</sup> The definition we used to compute this welfare cost is the compensating variation. One may check that the equivalent variation measure (that is the percentage the agent requires to be as well  
(continued...)



$$V^* [K(0); \sigma] = V^* [(1-k)K(0); \sigma] \quad (11)$$

Using (10) the total cost of volatility may be written:

$$k = \left[ \frac{\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2}{\varepsilon\delta + (1-\varepsilon)A} \right]^{\frac{1}{1-\varepsilon}} \text{ if } \gamma > 0 \text{ and } \lim_{\varepsilon \rightarrow 1} k = 1 - e^{-\gamma\sigma^2/(2\delta)} \quad (12)$$

We note *cic* the corresponding cut in consumption the agent experiences when reaching the deterministic world. It may be expressed using the total welfare cost of volatility  $k$  and the propensity to consume the current stock of capital in each world:

$$(1 - cic) = \frac{c(0)}{c(\sigma)} (1 - k)$$

$$cic = 1 - \left[ \frac{\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2}{\varepsilon\delta + (1-\varepsilon)A} \right]^{\frac{\varepsilon}{1-\varepsilon}}$$

$$\text{and } \lim_{\varepsilon \rightarrow 1} cic = 1 - e^{-\gamma\sigma^2/(2\delta)}$$

The cut in initial consumption depends on the change in the initial capital stock and on the change in the propensity to consume. Thus the cut in initial consumption will be smaller (higher) than the total welfare cost of fluctuations ( $k$ ) when the propensity to consume is higher (smaller) in the deterministic world than in the stochastic one. But one may check that the combination of the two effects will never lead to a negative cut (i.e., a rise) in initial consumption, except in the case of a risk lover agent.

The difference in consumption trend between the deterministic world and the stochastic one is simply (see equation (8)):

$$dg = \mu(0) - \mu(\sigma) = -(1-\varepsilon)\gamma\sigma^2/2$$

off under uncertainty as she is in the deterministic environment, noted *keq* below) is always greater than the compensating variation measure of the welfare cost of volatility:

$$keq = \left[ \frac{\varepsilon\delta + (1-\varepsilon)(A - \gamma\sigma^2/2)}{\varepsilon\delta + (1-\varepsilon)A} \right]^{\frac{1}{\varepsilon-1}} - 1 > k \text{ and } \lim_{\varepsilon \rightarrow 1} keq = e^{\gamma\sigma^2/(2\delta)} - 1 > 1 - e^{-\gamma\sigma^2/(2\delta)}$$

and it is straightforward that the switch to the deterministic world will be followed by a larger (smaller) trend in consumption and by a higher (smaller) growth rate if the intertemporal elasticity of substitution is greater (less) than unity.

### B. Splitting Up the Total Welfare Cost of Volatility

To split up the total welfare cost of fluctuations we will consider three different economies. To the two economies considered above (the stochastic and the deterministic ones), in which the agent is free to choose her saving rate, we add what we call the constrained deterministic economy whereby *constrained* stands for the fact that the agent's propensity to consume is kept equal to the one she had chosen in the stochastic economy. The growth rate of this constrained economy is thus equal to the one expected in the stochastic economy.

Let us now break down the switch from the stochastic economy to the deterministic economy into two successive stages.

The first stage consists in joining the constrained deterministic economy, and we may wonder how much capital the agent is ready to give up to achieve this step. As the two economies only differ in their nature (stochastic/deterministic) and not in their growth trend, one may refer to this cost as to the welfare cost of fluctuations in the narrow sense or as the *direct welfare cost of volatility*. In the following, this cost is noted  $k_F$ .

The second stage consists in switching from the constrained deterministic economy to the optimal one. The task is now to determine how much remaining capital the agent is ready to give up to have the opportunity to choose her propensity to consume. Since the two economies are deterministic, and differ only in their growth trend, one will refer to this cost as to the *trend-related welfare cost of volatility* which is noted  $k_T$ .

These costs are built such that:  $(1-k) = (1-k_F)(1-k_T)$ .

#### The direct welfare cost of volatility

The intertemporal lifetime utility of the agent in the constrained deterministic economy ( $\bar{V}$ ) is computed using equation (1) with consumption growing at the same rate as in the stochastic economy. Note that this intertemporal utility is no longer the value of the maximized agent's program since her consumption choice is constrained. One may calculate:

$$\bar{V}[K(0); \sigma] = \left[ \frac{\varepsilon\delta + (1-\varepsilon)A - (1-\varepsilon)\gamma\sigma^2/2}{\left( \varepsilon\delta + (1-\varepsilon)A + \frac{(1-\varepsilon)^2}{\varepsilon}\gamma\sigma^2/2 \right)^{\frac{\varepsilon}{1-\varepsilon}}} \right]^{1-\gamma} \frac{K(0)^{1-\gamma}}{1-\gamma}$$

The percentage of capital the representative agent is ready to give up to join the constrained deterministic economy is then obtained comparing the lifetime utilities  $V^*[K(0); \sigma]$  and  $\bar{V}[(1 - k_F)K(0); 0]$ . The direct welfare cost is then such that:

$$V^*[K(0); \sigma] = \bar{V}[(1 - k_F)K(0); 0] \quad (13)$$

$$\text{that is } k_F = 1 - \left[ \frac{\varepsilon\delta + (1 - \varepsilon)A - (1 - \varepsilon)\gamma\sigma^2 / 2}{\varepsilon\delta + (1 - \varepsilon)A + \frac{(1 - \varepsilon)^2}{\varepsilon}\gamma\sigma^2 / 2} \right]^{\frac{\varepsilon}{1 - \varepsilon}} \quad (14)$$

$$\text{and } \lim_{\varepsilon \rightarrow 1} k_F = 1 - e^{-\gamma\sigma^2 / (2\gamma)} \quad (15)$$

which is always positive. Since consumption grows at the same rate in both economies, this direct welfare cost of volatility is the initial (as well as permanent) cut in the agent's consumption; thus,  $cic_F = k_F$ . This is the measure derived by Obstfeld (1994a).

### The trend-related welfare cost of volatility

Once the agent has reached the constrained deterministic economy, the percentage of capital the representative agent is ready to give up to choose her consumption path is obtained by comparing the corresponding lifetimes utilities  $\bar{V}[K(0); 0]$  and  $V^*[(1 - k_T)K(0); 0]$ :

$$\bar{V}[K(0); 0] = V^*[(1 - k_T)K(0); 0] \quad (16)$$

$$\text{that is } k_T = 1 - \left[ \frac{\varepsilon\delta + (1 - \varepsilon)A - (1 - \varepsilon)\gamma\sigma^2 / 2}{(\varepsilon\delta + (1 - \varepsilon)A)^{\frac{1}{1 - \varepsilon}}} \right] \left( \varepsilon\delta + (1 - \varepsilon)A + \frac{(1 - \varepsilon)^2}{\varepsilon}\gamma\sigma^2 / 2 \right)^{\frac{\varepsilon}{1 - \varepsilon}} \quad (17)$$

$$\text{and } \lim_{\varepsilon \rightarrow 1} k_T = 0 \quad (18)$$

This cost is always positive since the agent switches from a constrained consumption path to an optimal one. In the special case where the intertemporal elasticity of substitution is equal to one, it is null (the constrained path is then the optimal one).

As far as the cut in initial consumption is concerned, one gets:

$$(1 - c/c_T) = \frac{c(0)}{c(\sigma)}(1 - k_T)$$

$$c/c_T = 1 - \left[ \frac{\varepsilon\delta + (1 - \varepsilon)A + \frac{(1 - \varepsilon)^2}{\varepsilon} \gamma \sigma^2 / 2}{\varepsilon\delta + (1 - \varepsilon)A} \right]^{1-\varepsilon} \quad (19)$$

and  $\lim_{\varepsilon \rightarrow 1} c/c_T = 0$

This cut in initial consumption may be positive or negative depending on the value of the intertemporal elasticity of substitution relative to unity. If this elasticity is greater than unity, the agent then has a larger propensity to consume wealth in the constrained deterministic economy than in the optimal one:  $c(0)/c(\sigma) < 1$ ; in such a case, the trend-related initial cut in consumption is always positive and larger than the trend-related welfare cost of volatility since the agent has less capital and wants to consume a smaller part of it. In the opposite case, if  $\varepsilon < 1$ , the optimal propensity to consume is larger and the growth trend smaller in the constrained deterministic economy than in the optimal one. Thus, for the lifetime utilities to be the same in the two economies, the initial consumption has to be smaller in the constrained deterministic economy than in the optimal one (19).

#### IV. IS THE WELFARE COST OF VOLATILITY NEGLIGIBLE ?

##### A. Evaluating the Welfare Cost of Volatility for the United States

The point is now to evaluate the three costs defined above in order first, to appraise whether the total welfare cost of volatility may be significant, and second, to know how much the endogeneity of the saving-consumption trade-off matters when evaluating this total welfare cost. Such evaluations are conducted on U.S. data for a large range of preference parameters. To calibrate the model, we use econometrics performed by Obstfeld (1994) on annual data (1950-90) which gives the volatility and the deterministic trend of the nondurable goods and services consumption:  $\sigma = 0.0112$  and  $\mu = 0.0185$ . For each preference parameters set, we compute

$A = \frac{\mu}{\varepsilon} + \delta + \frac{\varepsilon - 1}{2\varepsilon} \gamma \sigma^2$  such that the theoretical model for the stochastic economy (equation (8)) matches the actual consumption trend and volatility.<sup>4</sup>

The left part of Table 1 gives the values for the total welfare cost of volatility in terms of initial capital (12)) and its decomposition into two parts: the direct welfare cost of volatility (equation (14)) and the trend-related welfare cost (equation (17)).

The total welfare cost of volatility ( $k$ ) for different values of the intertemporal elasticity of substitution and of the risk aversion is presented in the upper left part of Table 1. This cost computed in the special case of the time-additive utility function is in italics: it is then small, since the representative agent accepts to pay about 0.3 percent of her capital to live in a deterministic economy, whatever her risk aversion. Relaxing the constraint imposed by the time additivity, the cost of volatility rises with both the risk aversion and the intertemporal elasticity of substitution. Two effects which apply in the same direction are combined: the effect due to the deterministic trend endogeneity (bottom left part of Table 1), and the direct effect of fluctuations (middle left part of Table 1). The welfare cost of volatility may then be large: for instance, if the representative agent has a high risk aversion (say 20) and a high intertemporal elasticity of substitution (say 5), she accepts to pay more than 30 percent of her initial capital to join the deterministic economy! In this case, the approximate decomposition shows that a non-negligible part ( $k_T = 17.56$  percent) of this cost is trend related. Nevertheless, the direct welfare cost is generally much larger than the trend-related one. In the extreme case in which the agent dislikes fluctuations but is weakly risk averse, ( $\varepsilon = 0.1$  and  $\gamma = 1$ ), the cost of volatility is no longer significant (0.03 percent of the initial capital) as the trend-related cost is zero while the direct cost of volatility is very small.

To sum up, the total welfare cost of volatility is significant and the trend-related welfare cost of volatility matters when both the intertemporal elasticity of substitution and the risk aversion parameters are greater than two.

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<sup>4</sup> Another way for evaluating the welfare cost of volatility would be to keep the same value for  $A$  (the productivity parameter in the production function) whatever the set of preference parameters. One drawback of this alternative solution is that, for most sets of preference parameters, the resulting consumption path in the stochastic economy (the only one for which data are available) is completely different from what is observed. That is why we choose to recalibrate the model (calculate a new  $A$ ) for each set of parameters, using the actual trend and volatility in consumption as the benchmark. Meanwhile, the figures one would have obtained using this alternative possibility to evaluate the welfare cost of fluctuations would not have been dramatically different from the one reported in Table 1.

**Table 1: Welfare Cost of Volatility and Initial Cut in Consumption**

		Total welfare cost $k$					Initial cut in consumption $cic$				
$\gamma \rightarrow$	$\varepsilon \downarrow$	1	2	5	10	20	1	2	5	10	20
0.1		0.03	0.07	0.17	0.35	0.71	0.00	0.01	0.02	0.03	0.07
0.5		0.16	0.33	0.81	1.64	3.28	0.08	0.16	0.41	0.82	1.66
1		0.31	0.63	1.56	3.09	6.08	0.31	0.62	1.56	3.09	6.08
2		0.58	1.16	2.85	5.67	11.0	1.16	2.31	5.67	11.0	20.8
5		1.22	2.46	5.72	13.5	33.9	5.94	11.7	27.9	51.6	87.4

		Direct welfare cost $k_F$					Initial cut in consumption (direct) $cic_F$				
$\gamma \rightarrow$	$\varepsilon \downarrow$	1	2	5	10	20	1	2	5	10	20
0.1		0.03	0.07	0.17	0.34	0.69	0.03	0.07	0.17	0.34	0.69
0.5		0.16	0.33	0.81	1.63	3.26	0.16	0.33	0.81	1.63	3.26
1		0.31	0.63	1.56	3.09	6.08	0.31	0.63	1.56	3.09	6.08
2		0.58	1.16	2.85	5.59	10.7	0.58	1.16	2.85	5.59	10.7
5		1.19	2.36	5.72	10.9	19.8	1.19	2.36	5.72	10.9	19.8

		Trend-related welfare cost $k_T$					Initial cut in consumption (trend related) $cic_T$				
$\gamma \rightarrow$	$\varepsilon \downarrow$	1	2	5	10	20	1	2	5	10	20
0.1		0.00	0.00	0.00	0.00	0.02	-0.03	-0.06	-0.15	-0.31	-0.62
0.5		0.00	0.00	0.00	0.00	0.03	-0.08	-0.16	-0.41	-0.82	-1.66
1		0	0	0	0	0	0	0	0	0	0
2		0.00	0.00	0.02	0.09	0.34	0.58	1.16	2.90	5.75	11.3
5		0.02	0.1	0.65	2.94	17.6	4.80	9.55	23.5	45.6	84.2

The three welfare costs are such that:  $(1-k)=(1-k_F)(1-k_T)$ . The feasibility condition is met for the parameters. Time preference rate:  $\delta=2$  percent. Reading indications: For a risk aversion equal to 5 and an intertemporal elasticity of substitution equal to 2, the representative agent accepts to give up 2.88 percent of her initial capital (upper left part of Table 1) which corresponds to a 5.67 percent cut in initial consumption (upper right part of Table 1) to meet the deterministic economy. Had the trend remained constant after the switch into the deterministic economy, the cost would have been 2.85 percent of her initial capital (middle left part of Table 1) which corresponds as well to a 2.85 percent cut initial in consumption (middle right part of Table 1). Having the opportunity to leave the constrained deterministic economy to optimally choose her level of savings, the agent accepts to give up 0.02 percent size of her initial capital (bottom left part of Table 1) which corresponds to a 2.90 percent cut in initial consumption (bottom right part of Table 1).

As far as the cuts in initial consumption are concerned, the upper right part of Table 1 shows that the initial cut in consumption due to the switch from the stochastic to the deterministic economy may be much larger than the total welfare cost of volatility. It is the case when the intertemporal elasticity of substitution is greater than unity. For example, let us consider the case where both the intertemporal elasticity of substitution and the risk aversion are equal to two. The total cut in initial consumption is as large as 2.31 percent whereas the agent only gives up 1.16 percent of her capital, because of the change in her propensity to consume as she join the deterministic economy. One can break down this initial cut in consumption into two successive cuts (see also Figure 1): there is a first 1.16 percent cut due to the switch to the constrained deterministic economy, and the remaining consumption is then cut further by 1.16 percent as the agent joins the

unconstrained deterministic economy. Nevertheless, after this large cut in her initial consumption, she will experience a higher growth rate (+0.01 points per year, see Table 2) in her consumption path, as shown by Figure 1. In this example, the two successive cuts in consumption are of the same magnitude, but for a larger intertemporal elasticity of substitution (say 5), the second cut in consumption is about four times larger than the first one whatever the risk aversion parameter.

**Table 2: Consumption Dynamics**

		Difference in consumption determinist trend				
$\gamma \rightarrow$ $\varepsilon \downarrow$	1	2	5	10	20	
0.1	-0.01	-0.01	-0.03	-0.06	-0.11	
0.5	0.00	-0.01	-0.02	-0.03	-0.06	
1	0.00	0.00	0.00	0.00	0.00	
2	0.01	0.01	0.03	0.06	0.13	
5	0.03	0.05	0.13	0.25	0.50	

The feasibility condition is met for the parameters. Time preference:  $\delta = 2$  percent. Readings indications: For a risk aversion equal to 5 and an intertemporal elasticity of substitution equal to 2, the annual growth rate of the consumption is 0.03 points higher.

Things happen differently when the representative agent has a low intertemporal elasticity of substitution. Initial cuts in consumption are then much lower than those for the total welfare cost. Let us consider the case where the intertemporal elasticity of substitution is 0.5 and the risk aversion is equal to 2. Note that for such parameter values the recursive utility function then reduces to the standard additive utility function. The total cut in initial consumption is small and equal to 0.16 percent, half the corresponding reduction in the capital stock (i.e., the total welfare cost of volatility). Nevertheless, this small cut in initial consumption is composed of a larger cut in consumption (0.33 percent) as the agent joins the constrained deterministic economy and of an increase in initial consumption (+0.16 percent, so the cut is negative) as she reaches the optimal economy (see Figure 2). Once in the optimal deterministic economy, the representative agent experiences a flatter consumption path (-0.01 points per year, see Table 2) as shown by Figure 2.

### B. Defining a Plausible Range for the Total Welfare Cost of Volatility

The evaluations performed above lead to a wide range for the total welfare cost of volatility. To restrict this range one may look at evaluations available in the empirical literature for the two relevant preference parameters. There are three potential sources to evaluate preference parameters. The first one is rather crude and incomplete: it relies on cross-country estimations of the link between the annual average growth rate of per capita production and the standard deviation of this growth rate. In our model, this link is negative when the intertemporal elasticity of substitution is less than unity and otherwise positive. Ramey and Ramey (1995) obtain a significantly negative link on a 92-country panel over the period studied (1962-85). Their conclusion is that countries with a higher volatility are also those that have lower growth rates. As far as our model is concerned, these results suggest the intertemporal elasticity of substitution to be greater than unity. More precisely, in our model the annual growth rate of the economy is:  $\varepsilon(A-\delta) - (1-\varepsilon)\gamma\sigma^2/2 - \sigma^2/2 - \sigma\eta_t$ , where  $\eta_t$  are iid shocks; the mean growth rate of the economy is thus  $\varepsilon(A-\delta) + (1-\varepsilon)\gamma\sigma^2/2 - \sigma^2/2$ . Using the Ramey and

Ramey's 92-country panel, one may perform a rough econometric estimation for the slope of the variance/mean growth rate relationship. This gives a point estimation of -0.842 with a standard deviation of 0.07. To match this point estimate, an intertemporal elasticity of substitution equal to 1.34 is required for a risk aversion equal to 2. For a risk aversion equal to 5, the required intertemporal elasticity of substitution is 1.14.

The second source to evaluate preference parameters lies in the literature on equity risk premium in the case of recursive utility. Three different methods have been developed to bring the portfolio choice model to the data. Their results are summarized in the upper part of Table 3. Epstein and Zin (1991) propose an econometric estimation of the Euler equations derived from the model. Kocherlakota (1996) calibrates these same equations. Solving further the model, Weil (1989) calibrates the expressions for the risk-free rate and for the risk premium. Unanimously, they retain a high value for the risk aversion parameter (which may reach 20). As far as the intertemporal elasticity of substitution is concerned, it seems to be less than unity. However, since the portfolio choice model with recursive utility fails to solve the equity risk premium puzzle and the risk free puzzle, these evaluations should be taken cautiously.

**Table 3: Some Evaluations for the Preference Parameters**

Authors	$\varepsilon$	$\gamma$	Approach
Weil (1989)	0.1	45	Calibration on equity risk premium
Epstein and Zin (1991)	0.2 → 0.4	0.4 → 1.4	Econometrics on portfolio choice and saving behavior
Giovanini and Jorion (1993)	0.1 → 0.3	5.4 → 11.9	
Normandin and Saint-Amour (1996)	0.3 → 0.8	0.4 → 3.6	
Kocherlakota (1996)	0.2 → 1.8	7 → ∞	Calibration on portfolio choice and saving behavior
Koskiewicz (1999)	3.2	0.1	Econometrics on consumption/leisure trade-off

$\varepsilon$ : Intertemporal elasticity of substitution;  $\gamma$ : Relative risk aversion.

The third source for preference parameters values is provided by econometrics on the consumption-leisure trade-off. Koskiewicz (1999) finds an intertemporal elasticity of substitution greater than unity (3.2) and a low risk aversion (0.1).

Thus, according to these evaluations, the intertemporal elasticity of substitution seems to be between 0 and 3.3. As far as the welfare cost of fluctuations is concerned, only values less than or equal to 2 for the intertemporal elasticity of substitution will be later considered. For the risk aversion parameter we restrict the range to [0,10]. This leads to a maximum total welfare cost of volatility equal to 5.67 percent of the initial capital.

### C. Comparison with Previously Proposed Measures

In order to compare our evaluation with previously proposed measures, we have to turn to the variation in initial consumption corresponding to the initial capital the agent accepts to give up to reach the deterministic economy. We have already argued (see Section III) that this amount of initial consumption is not that much informative because the trend in consumption is affected by



the change to the deterministic economy. Since we got a 5.67 percent of initial capital maximum total welfare cost of volatility, the corresponding initial cut in consumption may reach 11.2 percent (see Table 1). Compared with other measures, this result appears much larger than some of the previously proposed evaluations:

**Table 4: Maximum Welfare in Fluctuations—Cut in Initial Consumption**

Exogenous consumption	
Time-additive utility function	Other utility functions
Lucas (1987) - $kmax=0.5\%C(0)=0.5\%\bar{C}$	Obtsfeld (1994a) – Recursive Utility - $kmax=1.5\%C(0)=1.5\%\bar{C}$
	Dolmas (1998) – First-order Risk Aversion - $kmax=1.5\%C(0)=1.5\%\bar{C}$
Endogenous consumption	
Time-additive utility function	Other utility functions
Barlevy (2000) – $kmax=7.2\%C(0)$	Otrok (1999) – Habit Formation and Durability – $kmax = 0.004\%Cst$ This Paper – Recursive Utility – $kmax = 11.2\%C(0)$

$kmax$  is the larger welfare cost accepted in each paper.  $\bar{C}$  stands for permanent consumption, while  $C(0)$  corresponds to initial consumption and  $Cst$  represents steady state consumption.

Let us first compare the cut in initial consumption we calculate with the one obtained by Lucas (1987) who considers that the log of consumption is generated by a trend stationary process ( $\ln C_t = \ln \bar{C} + \mu t - \frac{1}{2}\sigma_u^2 + u_t$  where  $u_t \sim NIID(0, \sigma_u^2)$ ) and uses a standard additive CRRA utility function. Computing the welfare cost of volatility in terms of equivalent variation on this basis leads to a maximum (for  $\gamma = 10$ ) cost of 0.5 percent of the permanent consumption  $\bar{C}$ . As mentioned by Obstfeld (1994a), in Lucas's framework, shocks on consumption are transitory. If we turn to a martingale for the consumption process ( $\ln C_t = \ln C_{t-1} + \mu - \frac{1}{2}\sigma_z^2 + z_t$  where  $z_t \sim NIID(0, \sigma_z^2)$ ) where shocks on consumption are persistent, the cut in initial consumption is of the same magnitude as Lucas's result. One may check that restricting to the switch to the constrained deterministic economy when the standard CRRA additive utility function holds, with  $\gamma = 10$  (and  $\varepsilon = 0.1$ ),  $cic_F$  is equal to 0.3 percent of the consumption (see middle right part of Table 1). The endogeneity of the trend lowers this figure, and there is thus an even smaller cut in consumption at the time of the switch from the stochastic economy to the deterministic one (see upper right part of Table 1).

Let us now compare our results to measures for the welfare cost in terms of consumption when utility is no longer time-additive and the exogenous consumption process allows for persistent shocks. Using a recursive utility function as proposed by Obstfeld (1994a) increases

the welfare cost of volatility to a maximum of 5.78 percent of permanent consumption (for  $\gamma = 10$ ,  $\rho = 2$  and  $\delta = 2\%$ ).<sup>5</sup> Dolmas (1998) considers a class of preferences displaying ‘‘first-order’’ risk aversion. Assuming that shocks in the consumption growth are unit root, he calculates that the welfare cost of fluctuations may reach 20 percent of initial consumption. Nevertheless, the welfare cost of fluctuations becomes less than 1 percent as soon as disturbances wanders from a unit root (for instance with an autocorrelation coefficient equal to 0.98).

Turning to measures which take the endogeneity of the consumption trend into account, Otrok (1999) uses a utility function with habit formation and durability to appraise the welfare cost of business cycles. He estimates the preference parameters consistent with the observed business cycle fluctuations in an exogenous growth model, and then deduces a welfare cost of volatility almost of the same magnitude as the one calculated by Lucas. Note that since he uses an exogenous growth model, shocks do not lead to a unit root in the consumption process. In a model of endogenous growth with diminishing returns to innovation and encompassing a standard utility function, Barlevy (2000) shows that eliminating shocks may lead to an increase in growth from 2.0 percent to 2.36 percent and conclude that this figure delivers costs of business cycles that are far larger than reported in previous work. It stresses that when evaluating the welfare cost of volatility one has to take the dependence of growth with respect to volatility into account and reminds that Lucas (1987) argues that a one-point increase in growth would result in a 20 percent welfare gain in terms of consumption (with  $\gamma = 1$ ). Finally, Jones, Manuelli, and Stacchetti (1999) conclude that less volatility would lead to less growth except when the intertemporal elasticity of substitution is greater than one, a configuration they do not consider very relevant since they use an additive utility function with a risk aversion parameter greater than unity (nevertheless, they investigate the case where the risk aversion is 0.9). As the recursive utility function allows for both the risk aversion and the intertemporal elasticity of substitution to be greater than unity, our model shows that more volatility may be related with less growth even when the relative risk aversion is greater than one.

To sum up, all these evaluations suggest that the welfare cost of volatility depends on four major determinants: the persistence of shocks on consumption (or even unit root), the endogeneity of the consumption process, the endogeneity of growth, and the parameters of the utility function (especially the intertemporal elasticity of substitution). In our endogenous growth model which leads to a unit root in the consumption process, the recursive utility function may generate high levels for the welfare cost of volatility as well as small ones. Thus, we join Jones, Manuelli, and Stacchetti (1999) when asking for a sharp estimation of the curvature parameter(s) of the utility function.

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<sup>5</sup> The small discrepancy with the corresponding figure in middle right part of Table 1 derives from the equivalent variation measure proposed by Obstfeld as opposed to the compensating variation measure we use.

## V. CONCLUSION

In this paper, we have proposed a measure of the total welfare cost of volatility. This total cost is derived from a stochastic endogenous growth model, and it is computed as the percentage of capital the representative agent is ready to give up to join a deterministic economy. Since we consider a whole economy, and not solely the consumption process, the total welfare cost of volatility we obtain has two components: a direct welfare cost of volatility and a trend-related one. The former is close to the welfare cost of fluctuations first put forward by Obstfeld (1994a) when shocks the consumption process exhibits a unit root; in our whole economy framework it is computed as the percentage of capital the representative agent would accept to give up to join the deterministic economy while being constrained to keep the saving rate she had chosen in the stochastic world. The trend-related welfare cost of volatility is then computed as the percentage of her remaining capital stock the representative agent is ready to give up to get the opportunity to choose her saving rate. This trend-related cost is linked to the *cost of reducing growth* considered by Lucas (1987). The recursive utility function we used to model the representative agent's preferences allows us to show that the total cost of volatility increases with both the risk aversion and the intertemporal elasticity of substitution.

Calibrating our stochastic endogenous growth model to match the actual consumption process for the United States, we show that when the intertemporal elasticity of substitution is greater than 2, the trend-related welfare cost of volatility may be significant and should not be neglected when evaluating the welfare cost of volatility. Yet, for an intertemporal elasticity of substitution close to unity, the endogeneity of the consumption process does not matter when computing the welfare cost of volatility. Meanwhile, our results suggest that, thanks to the endogeneity of consumption, the trend-related welfare cost may be quite different from both the total welfare cost of volatility and the direct cost of volatility. As far as policy-making is concerned, our calibrations suggest that since the reduction of volatility may induce growth, economic policies which smooth fluctuations may be good for growth as well. However, the design of such policies are beyond the scope of this paper.

Finally, we have to acknowledge that the wish to obtain analytical formula for the total welfare cost of volatility and its decomposition prevented us from incorporating some important features: the technology is  $AK$  (there is no labor), the shocks are uncorrelated and agents' heterogeneity is ignored. Introducing labor/leisure in both the recursive utility and the production functions prevents obtaining a closed-form solution for the model and would require a numerical resolution (as in Greenwood and others, 1988, for example). Introducing a recursive utility function in the model recently proposed by Jones, Manuelli, and Siu (2000), which cleverly studies the business cycle properties of a stochastic endogenous growth model with human and capital formation, as well as leisure in the utility function, might be a good starting point to go further in the exploration of the welfare cost of volatility in an endogenous growth model.

APPENDIX

Identifying the optimal consumption path<sup>6</sup>

The Bellman function associated with the program is:

$$(1-\gamma)V(t) = \underset{\{c(t)\}}{\text{Max}} \left[ c(t)^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-\delta dt} \left( (1-\gamma) E_t V(t+dt) \right)^{\frac{1-\varepsilon}{\varepsilon}} \right]^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}} \quad (20)$$

By analogy with the standard time-additive deterministic program, one can guess that the value function is:

$$V(t) = B^{\frac{1-\gamma}{1-\varepsilon}} \frac{K(t)^{1-\gamma}}{1-\gamma}$$

where  $B$  is a constant to be calculated, and that the optimal consumption at  $t$  is a linear function of the current wealth; that is  $C(t) = DK(t)$  where  $D$  is a constant to be calculated.

Calculating  $E_t [V(K(t+dt))]$ :

$$E_t [V(K(t+dt))] - E_t [V(K(t))] = E_t [dV] = B^{\frac{1-\gamma}{1-\varepsilon}} \frac{E_t [K(t+dt)^{1-\gamma}]}{1-\gamma} - B^{\frac{1-\gamma}{1-\varepsilon}} \frac{E_t [K(t)^{1-\gamma}]}{1-\gamma} \quad (21)$$

Moreover, applying Itô's lemma, one calculates:

$$E_t [dV] = \frac{\partial V}{\partial K} E [dK] + \frac{1}{2} \frac{\partial^2 V}{\partial K^2} E [dK]^2 \quad (22)$$

where  $E [dK] = (A - D)K(t)dt$  and  $E [dK]^2 = \sigma^2 K(t)^2 dt$  when neglecting power of  $dt$  superior to one.

From (21) and (22) one calculates:

$$E_t [V(K(t+dt))] = B^{\frac{1-\gamma}{1-\varepsilon}} \left[ \left( A - D - \frac{1}{2} \sigma^2 \right) dt + \frac{1}{1-\gamma} \right] K(t)^{1-\gamma} \quad (23)$$

<sup>6</sup> Cf. Smith (1996).

Substituting (23) into (20), the Bellman equation can be rewritten:

$$B^{\frac{1-\gamma}{1-\varepsilon}} K(t)^{1-\gamma} = \underset{\{D\}}{\text{Max}} \left[ D^{\frac{\varepsilon-1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} dt + e^{-\delta dt} \left( (1-\gamma)^{\frac{\varepsilon-1}{(1-\gamma)\varepsilon}} B^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} \left[ (A-D-\gamma\sigma^2/2) dt + \frac{1}{1-\gamma} \right]^{\frac{\varepsilon-1}{(1-\gamma)\varepsilon}} \right)^{\frac{(1-\gamma)\varepsilon}{\varepsilon-1}} \right] \quad (24)$$

using the fact that  $\lim_{x \rightarrow 0} (1+x)^y = 1+xy$  and  $\lim_{x \rightarrow 0} e^x = (1+x)$ , leads to:

$$B^{\frac{1-\gamma}{1-\varepsilon}} K(t)^{1-\gamma} = \underset{\{D\}}{\text{Max}} \left[ D^{\frac{\varepsilon-1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} dt + B^{\frac{1}{\varepsilon}} K^{\frac{\varepsilon-1}{\varepsilon}} \left( \frac{\varepsilon-1}{\varepsilon} (A-D-\gamma\sigma^2/2) dt - \delta dt + 1 \right) \right] \quad (25)$$

The optimization shows that  $D$ , the optimal propensity to consume current wealth is equal to  $B$ .  $B$  is then identified by replacing,  $D$  in (24):

$$B = D = \varepsilon \left( \delta - \frac{\varepsilon-1}{\varepsilon} (A - \gamma\sigma^2/2) \right)$$

The value function is then:

$$V(K(t)) = \left[ \varepsilon \left( \delta - \frac{\varepsilon-1}{\varepsilon} (A - \gamma\sigma^2/2) \right) \right]^{\frac{1-\gamma}{1-\varepsilon}} \frac{K(t)^{1-\gamma}}{1-\gamma}$$

The feasibility condition requires  $C^* > 0$ , which may be rewritten:

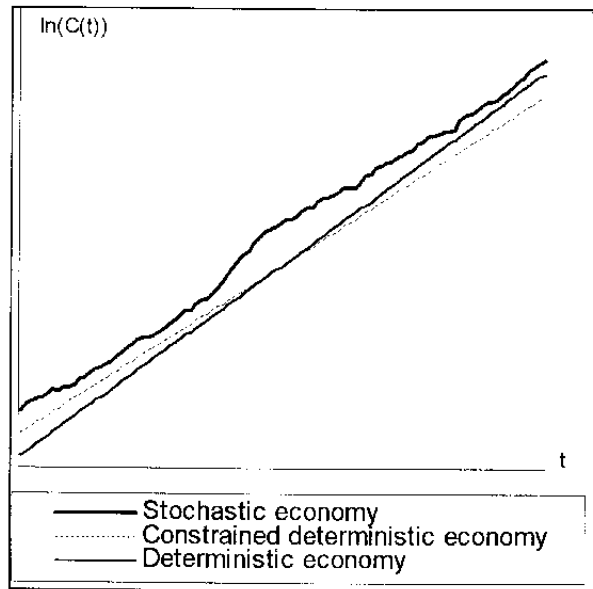
$$\varepsilon \left( \delta - \frac{\varepsilon-1}{\varepsilon} (A - \gamma\sigma^2/2) \right) > 0 \Leftrightarrow \delta > \frac{\varepsilon-1}{\varepsilon} (A - \gamma\sigma^2/2)$$

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**Figure 1 : Consumption paths ,  $\varepsilon > 1$**



**Figure 2 : Consumption paths ,  $\varepsilon < 1$**

