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Nonrenewable Resources: A Case for Persistent Fiscal Surpluses

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Abstract

This paper examines whether there is a case for temporary but persistent fiscal surpluses in economies heavily endowed with nonrenewable resources. It finds that there generally is a case. Fiscal surpluses permit replacing nonfinancial wealth with financial assets, the return on which increases public consumption possibilities of future generations for a constant across-generation tax burden. The more biased are a government's preferences toward present generations, the lower will be the initial surpluses; the larger the finite endowment, the larger the initial surpluses. In a more general framework, including public investment, the proposition could be rephrased by replacing surpluses with stronger initial fiscal positions.

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Contents

Page

I. Introduction

The purpose of this paper is to examine, in a stylized theoretical framework, whether the presence of sizable exhaustible resources in an economy affects the path of optimal primary fiscal balances. More specifically, could the existence of nonrenewable resources create a case for transitory but persistent fiscal surpluses? We found that for economies heavily endowed with nonrenewable resources, like Chile, that is generally the case. During the time when fiscal revenues are derived from an exhaustible resource, the government would run fiscal surpluses which would permit the replacement of nonfinancial wealth with financial assets. The return on those financial assets would be used to increase public consumption possibilities of future generations for a constant tax burden per capita over time. Government preferences regarding the welfare of different generations and the size of the finite resource determine the size of the initial surpluses and, for a steady tax burden over time, also the path for government expenditures.

The issue of nonrenewable resources and its optimal depletion has been discussed at length since Hotelling's (1931) seminar paper. The topic of fiscal policy in presence of nonrenewable resources has received recently some attention since it is of great relevance to a group of countries, like Chile, in which the exploitation of exhaustible endowments has a significant impact on their economies and fiscal stances. This literature has focused on general conditions for policy sustainability, on financial and wealth indicators to track the evolution of government solvency, and on intergenerational transfer issues in presence of depletable resources.[2]

The relevance of the issue discussed in this paper is intrinsically linked to the economic size of the nonrenewable resource. In the case of Chile, for example, total copper reserves amount to 163 millions of metric tons or 26 percent of total world reserves,[3] the largest reserve base for any country. The economically usable reserves total 88 millions of metric tons or 28 percent of world reserves. At the current rate of production, Chile has 26 years of economically usable reserves, and almost 50 years of total reserves (which can become usable as technology evolves).

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[2]See for example Chalk (1998) on the issue of sustainability and Liuksila et al. (1994) on wealth indicators for the public sector both in presence of depletable resources. Steigum and Thøgerson (1995) use a computable overlapping generation model to study the effect of oil depletion in Norway, and Auerbach et al. (1993) apply a generational accounting framework with nonrenewable resources to study the extent of intergenerational transfer in Norway.

[3]According to the January 1998 Mineral Commodities Summaries of the U.S. Geological Service. It must be emphasized that estimated reserves tend to be a dynamic concept affected, for example, by technological improvements.

Assuming a decade-average copper price of US$1 per pound and using Chile's 1998 GDP and copper-price discount,[4] the value of total reserves is equivalent to 2.5 times the annual GDP, while economically usable reserves stand to about 1.8 times GDP. Moreover, using an estimated total mining cost of US$0.7 per pound, the net worth of total reserves would amount to 70 percent of 1998 GDP, and economically usable reserves would represent 50 percent of 1998 GDP. In the fiscal accounts, fiscal revenues only from the state copper company CODELCO have averaged 2.2 percent of GDP in the 1990s. It can then be presumed that fiscal policy in countries such as Chile should be subject to the particular considerations that this paper intends to highlight.

The paper is organized as follows: Section II presents a simple framework to analyze fiscal policy in the presence of nonrenewable resources. Section III restates the main findings and policy implications. Appendices I and II develop in detail the model presented in Section II.

II. A Stylized Model of Fiscal Policy with Nonrenewable Resources

This section outlines a simple extension of the nonstochastic overlapping generation (OLG) model which we developed to better understand the economic implications of nonrenewable resources on fiscal policy.[5] Appendix I presents in detail the model discussed in this section. A basic two-generation OLG model is set in the context of a small open economy with free trade in goods and financial assets with the rest of the world. In addition to consumers, there is a government which provides a public good. The government finances its operations with lump-sum taxes levied on consumers and with the proceeds from the exploitation of an exhaustible resource it owns. The country is assumed to have no influence on the international price for the exhaustible resource.

We also assume that there is no production in the economy and consumers are endowed with a constant amount of the consumption good in every period. Consumers make their consumption/saving decisions taking into consideration their intertemporal preferences, the interest rate, the lifetime endowment, and taxes to be paid to the government. Consumers are born with no assets and leave none to their offsprings. In every period consumers derive utility from the consumption of both private and public goods. The public good is consumed in the same amount by all consumers alive in the period that the good is provided.[6]

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[4] The copper price discount considered for Chile's exports is US$0.04 per pound. In addition, the real interest rate used to discount the flow of income is 5 percent.

[5]The OLG model was initially developed by Samuelson (1958) and Diamond (1965).

[6] As discussed later on in Section III, if government expenditure is also investment, the results obtained here could be rephrased as the proposition that the existence of nonrenewable

(continued...)

The government decides on the level of taxation and the provision of the public good taking into account the utility of both current and future generations. We assume that the government pursues only policies that are consistent with public sector solvency.[7] Therefore, the present value of planned primary budget deficits cannot be greater than the government's initial total wealth, including its nonfinancial wealth which in this case comprises only the market value of the reserves of the exhaustible resource. In addition, to abstract from political-economy considerations we assume that the government counts with commitment mechanisms to prevent the appearance of intertemporal inconsistencies in its decision making process.

For the sake of simplicity, we also assume that there are no domestic assets. Both the government and consumers can accumulate foreign assets or liabilities at a given international interest rate.

The distinctive feature of the framework presented here is that the government receives an endowment for a finite number of periods. The present value of the stream of proceeds, derived from such endowments, is considered the government's nonfinancial wealth. Total government wealth at any period of time comprises both nonfinancial and financial wealth, the latter consisting of net financial asset holdings.

For simplicity we assume that the international interest rate is constant over time, the government's endowment is constant during the periods it exists, the world price for the resource is given and constant, and the per capita lump-sum taxes are constant over time and across generations. The latter assumption allows to focus on the evolution in the per capita provision of the public good. That is, for a constant-over-time tax burden per capita, what path of the primary fiscal stance would permit, for example, a constant or increasing provision of the public good?

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[6](...continued)
resources, ceteris paribus, would imply a stronger initial primary balance (larger surplus or lower deficit) than otherwise.

[7]Government solvency is defined as the set of policies satisfying ex ante the government's intertemporal budget constraint (this constraint is always met ex post through the adjustment of the primary fiscal position or the various ways of debt repudiation). On computational problems, see Blejer and Cheasty (1991), and on the study of the government solvency constraint through the analysis of time-series properties of public debt, see Hamilton and Flavin (1986) and Wilcox (1989). A more restricted concept is that of fiscal sustainability which is defined as the set of fiscal policies that the government can, ceteris paribus, continuously pursue without the need of future reversals. Sustainable fiscal policies need to satisfy the solvency constraint, but the converse is not true. The model developed could also be used to analyze policy sustainability. On indicators of sustainability, see Horne (1991), and on debt sustainability, see Zee (1988).

In this set up, we obtained that the path of the per capita provision of the public good and the primary fiscal balance depend on the government's intergenerational preferences. Thus, if the government cares strongly for future generations, it will pursue a policy of increasing per capita expenditures over time which will require larger initial primary fiscal surpluses. In contrast, if the government's preferences are biased toward present and near-future generations, government expenditures per capita will diminish over time which will entail lower initial surpluses. One interesting intermediate case in which the government's preferences determine a constant per capita provision of the public good is also possible.

These three cases are depicted in Figure 1a (in per capita terms) and Figure 1b (in percent of GDP). The fine solid lines show government expenditures which are constant, increasing and decreasing in the top, middle and bottom graphs, respectively. Constant taxes are shown with bold dashed lines, while the income from the nonrenewable resource is depicted with fine dashed lines. The resulting primary fiscal deficit is presented in bold solid lines. The paths of primary deficits show in general an initial primary surplus,[8] i.e., the replacement of nonfinancial wealth with financial assets. The accumulation of financial assets implies higher future interest income and thus the ability to run primary deficits. Case I and ii show a clear pattern of primary surplus when the finite resource exists and primary deficits thereafter. Case iii shows an initial primary surplus followed by a deficit that eventually turns back to a surplus.[9]

Moreover, as illustrated in these figures, the less biased government preferences are toward present generations, the higher the initial surpluses. Conversely, initially weaker primary fiscal stances, ceteris paribus, show implicitly the government's relative disinterest for the future generations, highlighting the intergenerational implications of policies, i.e., lower sustainable primary deficits in the future and a lower provision of the public good.[10]

Furthermore, counter to some casual views that a larger depletable resource would merit less an initial surplus (because it would resemble more closely a permanent resource) we found, as displayed in Figure 2a and 2b, that as long as the resource is finite the larger its size the higher the optimal initial primary surplus in all cases. Also, Figure 3 shows, for the per capita constant expenditure case, that the initial per capita surplus moves equiproportionally with the

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[8] See Appendix II for initial conditions on government financial assets to have initial fiscal surpluses.

[9] Although Figures 1a and 1b do not show the return to a primary surplus, in time as consumption per capita diminishes the constant tax burden will determine a turn around from deficit to surplus in the primary balance.

[10] In this regard, the generational accounting literature highlights the zero-sum nature of fiscal policies from an intergenerational perspective, see Auerbach et al. (1994). On the shortcomings of using conventional budget accounting, see Kotlikoff (1993).

present value of the endowment, i.e., the percentage increase in the primary surplus is the same as that of the endowment.[11]

III. Conclusion and Policy Implications

The aim of this paper has been to analyze fiscal policy in presence of nonrenewable resources that provide a source of temporary revenue for the government. The question posed has been: What would be the optimal path for the primary balance in those circumstances? We constraint the answer to a smaller subset by assuming that lump-sum taxes are constant over time. Thus we ask, for the same tax burden per capita across generations, what path of the primary fiscal stance would permit, for example, a constant or increasing provision of the public good? The answer we found is that in general the government needs to run primary surpluses while the finite resource exists. Depending on the government's intergenerational weights, i.e., how much it discounts future generations' welfare, the path of primary balances will turn into a permanent or transitory deficit thereafter. In terms of a more general class of questions, the existence of nonrenewable resources will constitute a case for temporary but persistent fiscal surpluses.

Two interesting results were also obtained. We found that the larger the resource, the higher the optimal initial primary surplus. Moreover, the per capita primary surplus is shown to move equiproportionally with the size of the finite endowment for the constant expenditure case.

Notwithstanding the robustness of the results obtained, several caveats apply:

- In the model developed, public expenditure is purely a consumption good. In the case where public expenditure is also investment (both in human and physical capital), the analysis becomes more complicated since socially beneficial projects (in presence of, for example, high returns or positive externalities) could determine a path of initially optimal primary deficits because it may be optimal to invest heavily in the initial periods.[12] Nonetheless, ceteris paribus, the existence of a nonrenewable resource would imply a stronger initial primary balance (larger surplus or lower deficit) than otherwise.

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[11]The per capita increasing expenditure case shows a less than proportional percentage increase in the initial surplus, while the expenditure decreasing case exhibits a more than proportional percentage increase.

[12] Notice that this introduces definitional problems in the fiscal accounts since some current expenditures, for example in education and public health, could well be regarded as capital expenditures.

- The model also assumes perfect foresight. However, even if the size of the resource or its future economic value were uncertain,[13] as long as the government is risk-neutral the general result derived above would be maintained.

- The model also ignores optimal depletion issues by assuming a given rate of exploitation of the natural resource. If the government could decide on the rate of exploitation, the problem would become more complex, as new factors would enter into the analysis, but the overall conclusion of the model would still be valid. This is so since it is the net present value of the resource what matters for tax and public expenditure, and any change in the path of the proceeds from the exploitation of the nonrenewable resource will only affect the pattern of the primary balance. The optimal rate of exploitation will depend on production costs and on the intertemporal path of the world price for the resource (on which the country may have an effect if it accounts for a significant share of world output and if the degree of substitutability in consumption is not too high). Given that path of extraction, however, it is still true that the more the government is concerned about future generations, the higher would be the initial surpluses (or the lower the initial deficits).

- With respect to ownership, the fact that we assume that the nonrenewable resource belongs directly to the government should not affect the results. Similar patterns for the path of the primary balance would be expected provided that, if the resource is privately exploited, the government still is able to appropriate over time part of its net present value (through for example levying royalties).[14]

- An important dimension assumed away in the model is that of private offsetting of public savings. The model, for simplicity, rules out explicitly bequest and intergenerational gifts, so that there is no possible offsetting. Conditions for full offsetting in savings are generally very strong, and with partial offsetting the general result for the pattern of primary balances would still be preserved since governments' intergenerational redistribution policies cannot be fully undone by private agents.[15]

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[13] In this regard, there is also the risk of an economic disappearance of the resource, in which a technological breakthrough renders the resource worthless as in the case of nitrates in Chile early this century.

[14] Notice that the government does not have mechanisms to fully receive initially all discounted future fiscal revenues from the nonrenewable resource, which would render the problem irrelevant, because of time-inconsistency problems.

[15] If government's preferences are biased toward present generations more than private agents' preferences but they can only partially offset the intergenerational-redistributing policies by increasing savings, then the government can design a path of fiscal balances which, to accomplish its objective, would have lower initial surpluses than in the case with no possible

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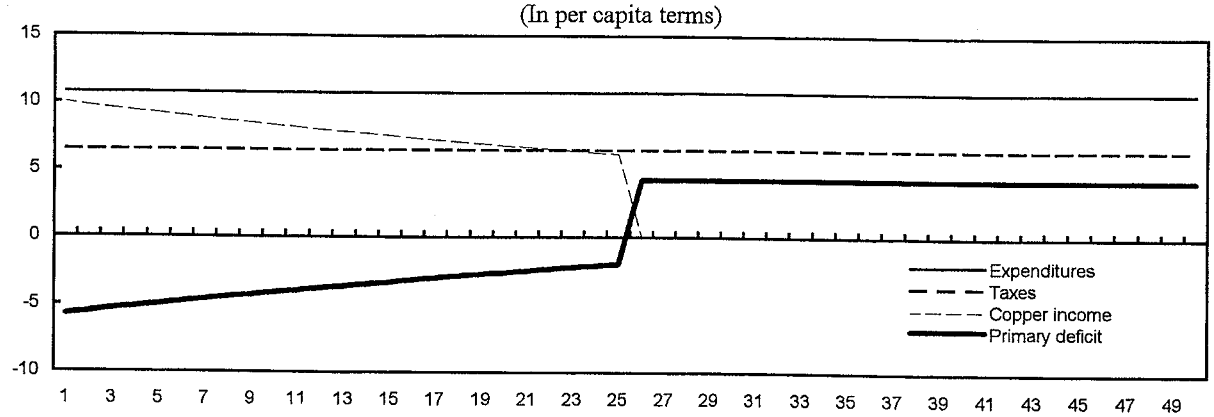
- Lastly, there is an important political-economy consideration not addressed by the model. This is the fact that it may be difficult for governments to resist pressures to spend when fiscal accounts are persistently in surplus. The model developed above ignores this type of issues, but it can still serve well to highlight the intergenerational implications of such pressure-induced policies, given the zero-sum nature of the problem.

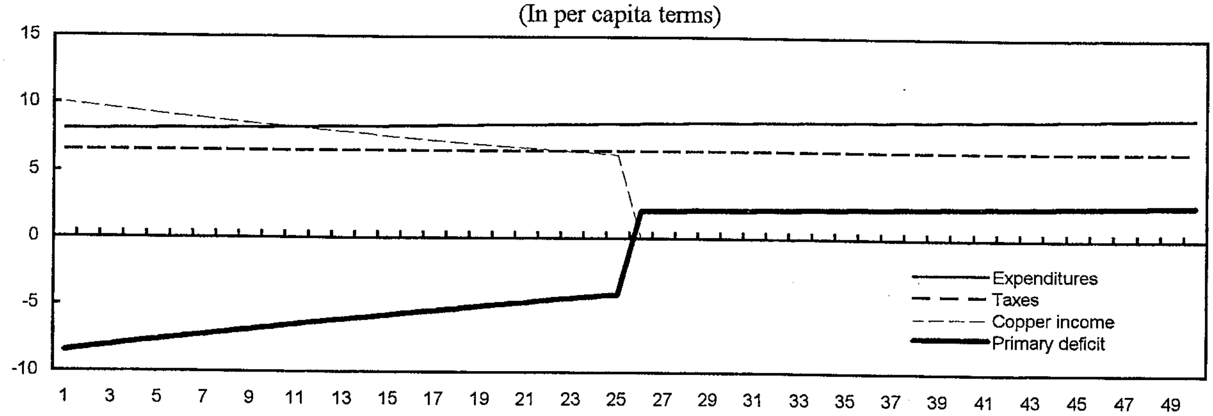
[15](...continued)
offsetting. Conversely, if the government's preferences care for future generations more than private agents' preferences, then the government's intergenerational transfers (embedded in the initial fiscal surpluses) cannot be fully undone since current generations cannot borrow against the income of future ones.

Figure 1a. Paths of Primary Deficits

Constant Government Expenditure Case
(In per capita terms)



Increasing Government Expenditure Case
(In per capita terms)



Decreasing Government Expenditure Case
(In per capita terms)

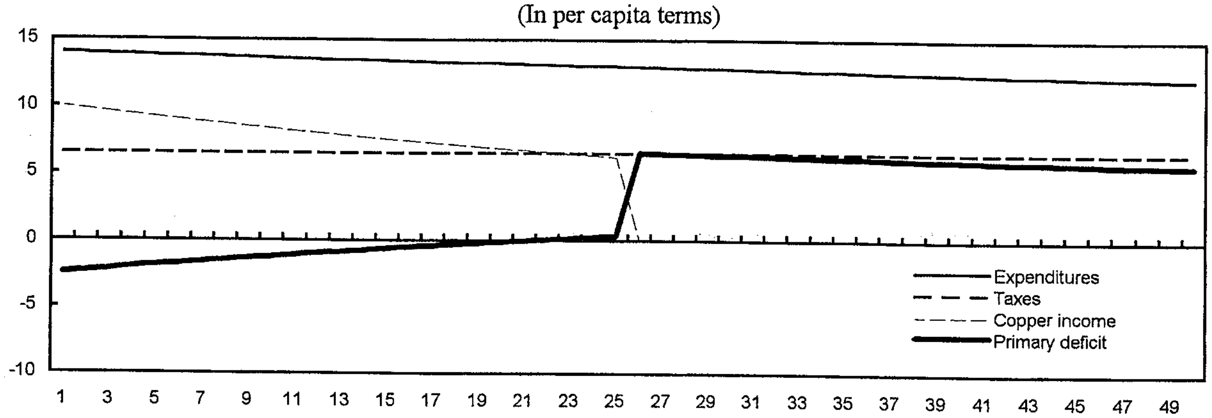
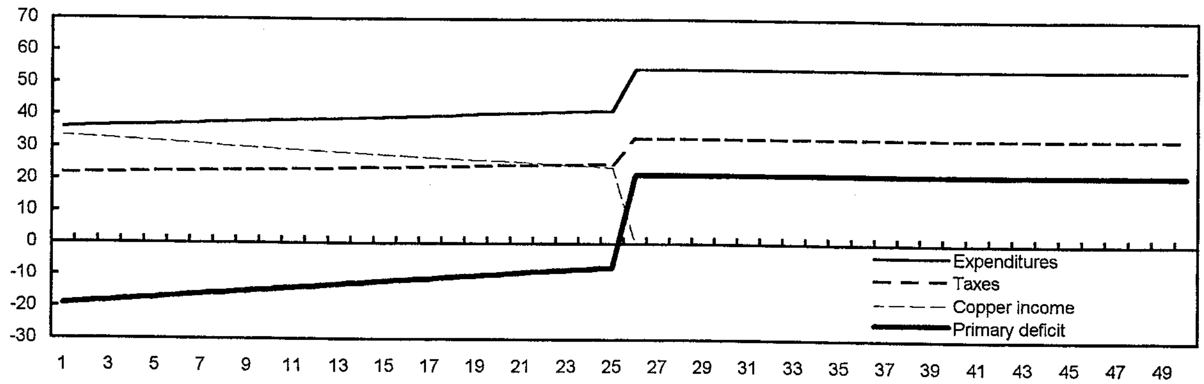
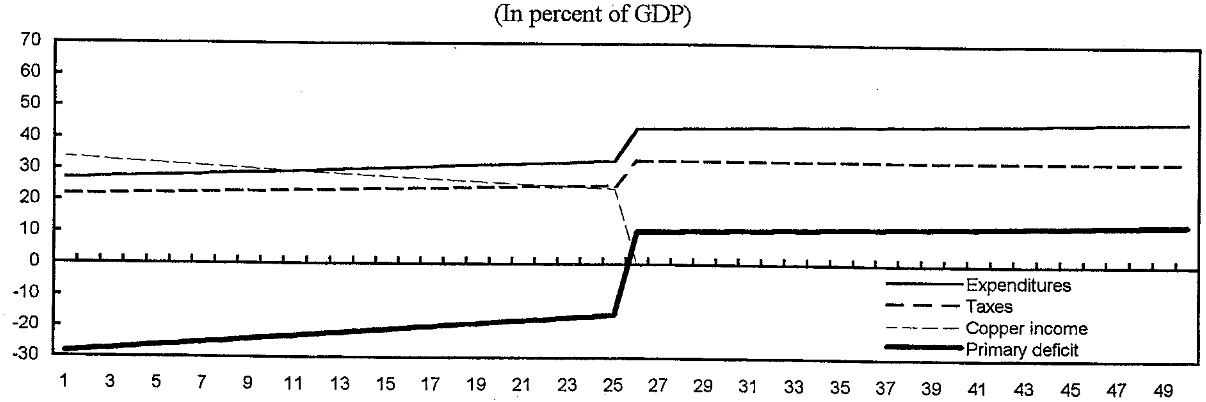


Figure 1b. Paths of Primary Deficits

Per Capita-Constant Government Expenditure Case
(In percent of GDP)



Per Capita-Increasing Government Expenditure Case
(In percent of GDP)



Per Capita-Decreasing Government Expenditure Case
(In percent of GDP)

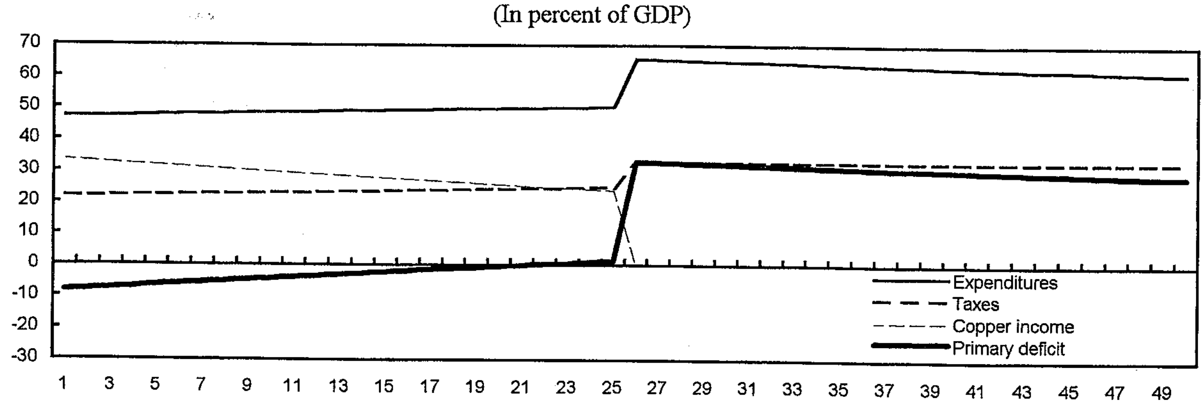
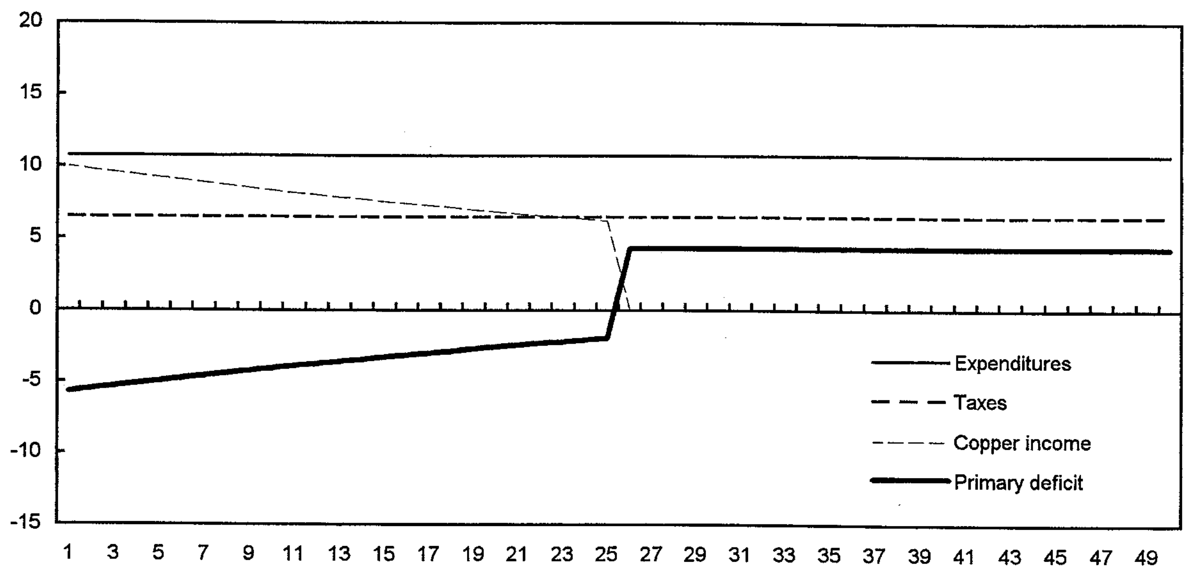


Figure 2a. Price or Volume Effect

Constant Government Expenditure Case
(In per capita terms)



Double Endowment Size Case
(In per capita terms)

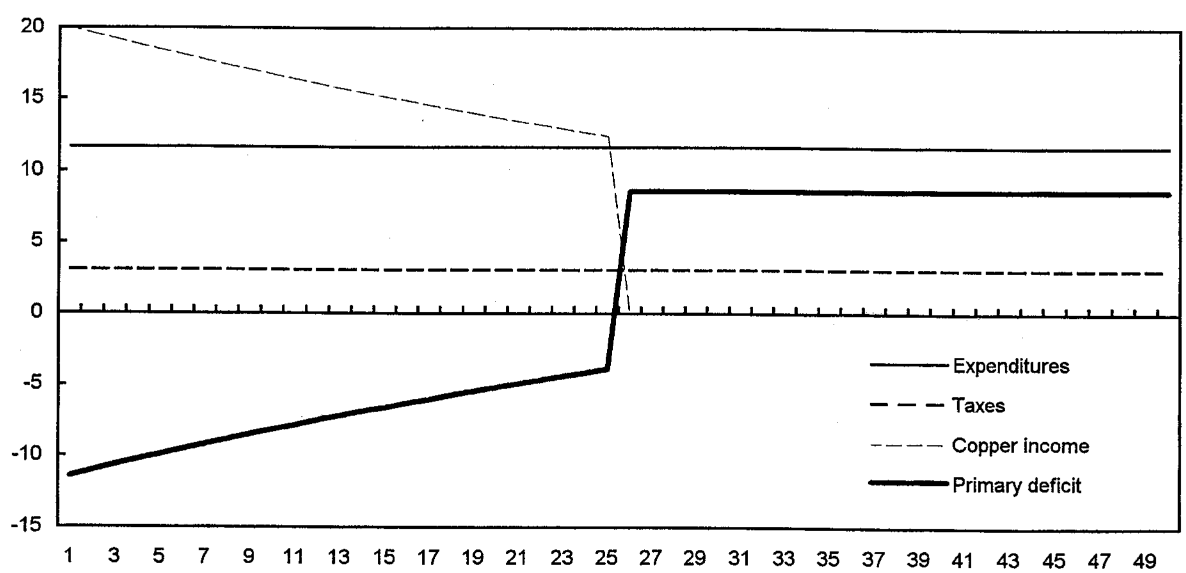
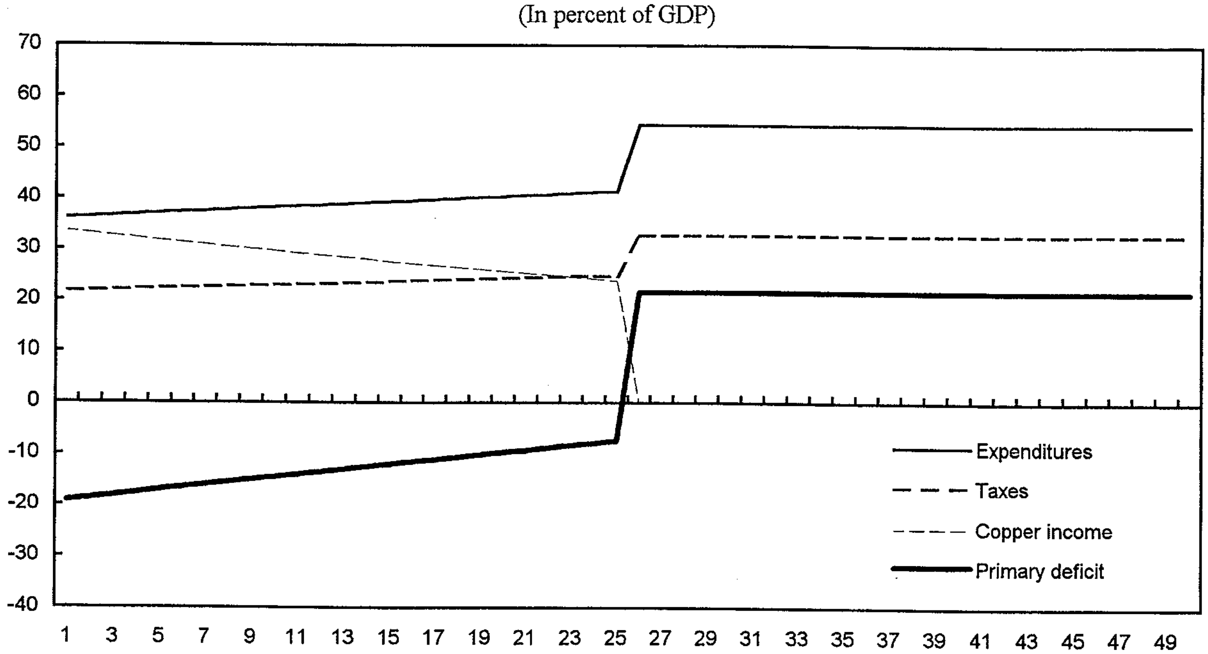


Figure 2b. Price or Volume Effect

Per Capita-Constant Government Expenditure Case
(In percent of GDP)



Double Endowment Size Case
(In percent of GDP)

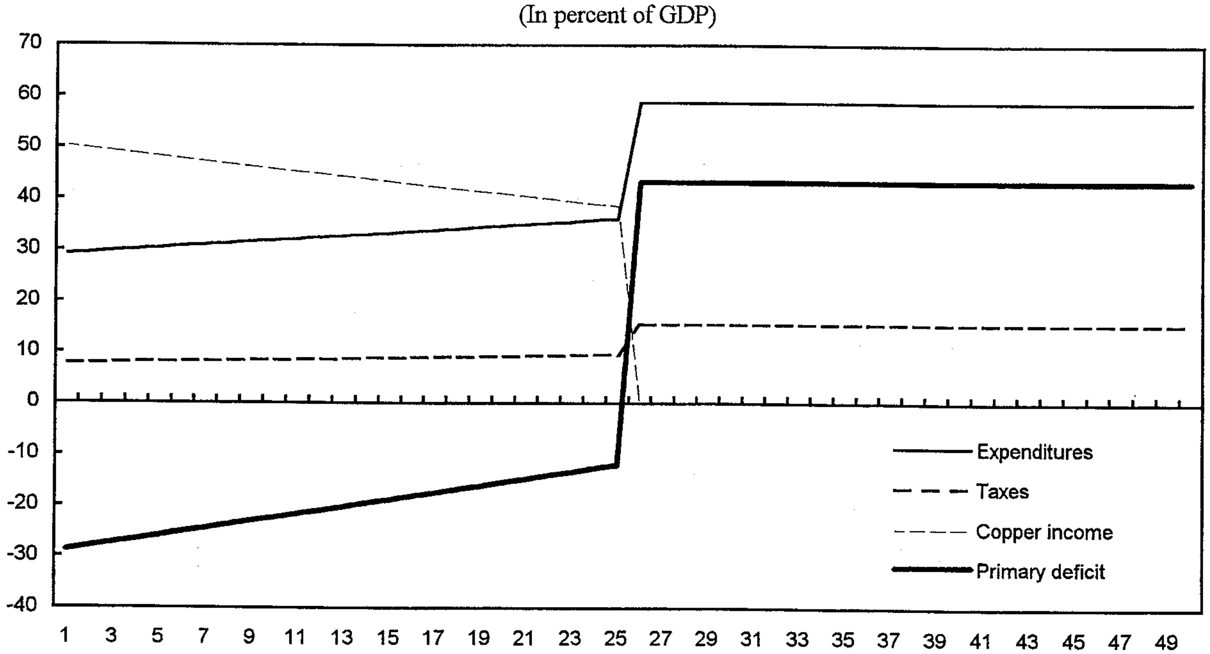
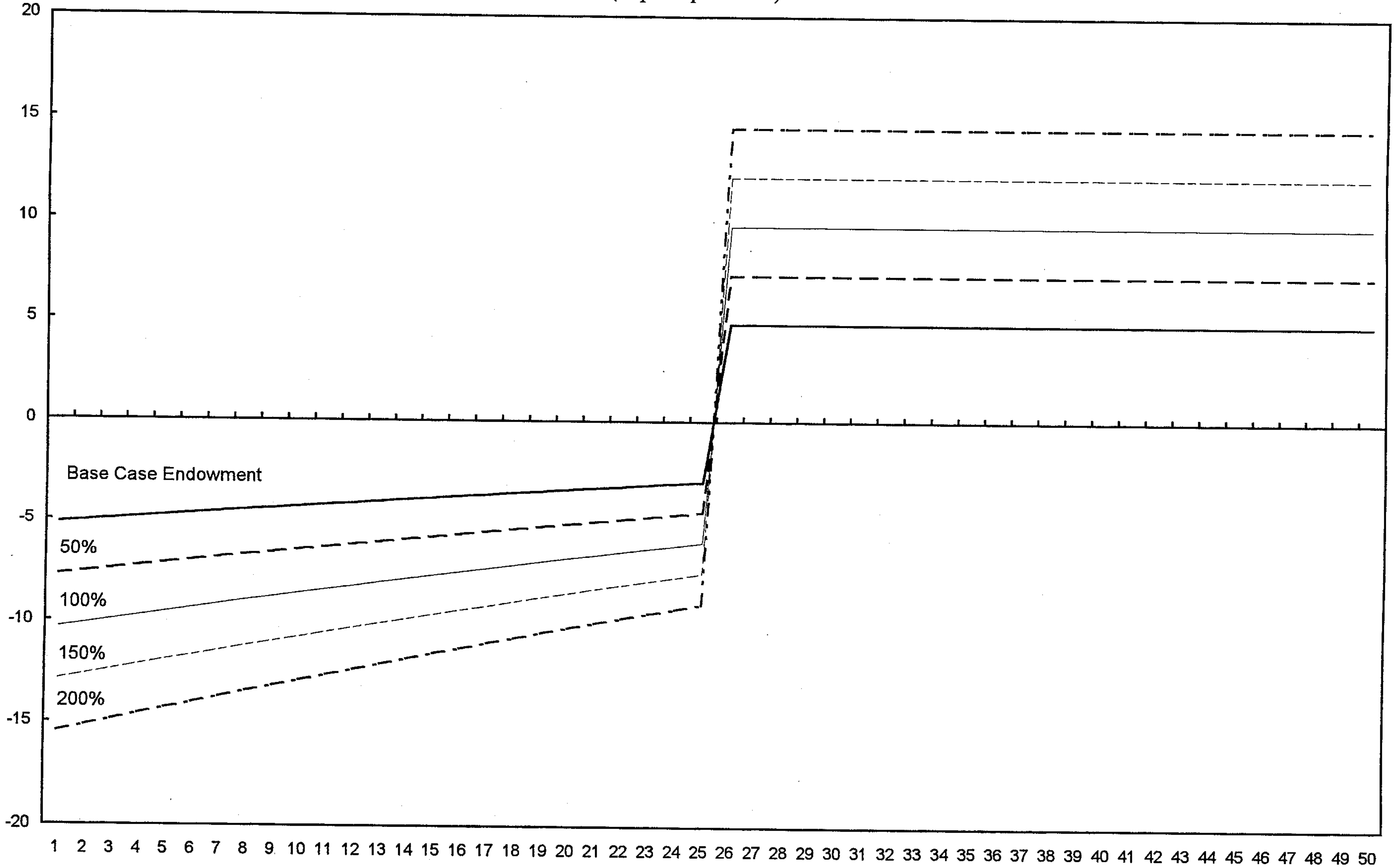


Fig. 3: Sensitivity of Constant-Expenditure Primary Deficits to Endowment Size
(In per capita terms)



Chile—A Stylized Model of Fiscal Policy with Nonrenewable Resources

This appendix spells out in detail the framework used in Section II. We develop a simple overlapping generation (OLG) model to present the case of sustained fiscal surpluses with depletable resources, and to highlight some key underlying elements. We use a two-generation nonstochastic OLG model in which the main feature is the existence of a finite (nonrenewable) resource owned by the government which generates a temporary source of fiscal revenue. Thus, we assume an endowment economy in which each consumer receives a fix endowment and the government has a temporary endowment.

There are two consumption goods: a private good (same as the private endowment), and a public good (public expenditure). Both endowments can be internationally traded at given prices, and the government's endowment can only be exported (not consumed domestically). Both, private consumers and the government, can buy and sell foreign assets at a given international interest rate. We assume here that the government pursues only solvent fiscal policies (no Ponzi games), and that it counts with commitment mechanisms to avoid dynamic inconsistencies.

The population grows at a constant rate n, and the model is developed in per young terms, i.e., all variables are divided by the young population in every period.[1] Capital letters denote total values in consumption good terms, small letters denote per young values, and overlined letters denote individual-agent values.

The demographics can then be portrayed as follow:

$\eta = (1+n)$	= population growth factor.
N_t^Y	= total number of young agents in period t.
N_t^O	= total number of old agents in period t.
$N_t = N_t^Y + N_t^O$	= total population in period t.

[1]The model is developed in per young terms for algebraic simplicity. To express the results of the model in per capita terms, the per young results need to be multiplied by the scalar $\eta/(1+\eta)$, where $\eta=(1+n)$.

Which implies:

$$N_t = \eta N_{t-1}$$

$$N_t^O = \eta N_{t-1}^O = N_{t-1}^Y$$

$$N_t^Y = \eta N_{t-1}^Y$$

$$\frac{N_t^Y}{N_t^O} = \eta \qquad \frac{N_t^Y}{N_t} = \frac{\eta}{1+\eta} \qquad \frac{N_t^O}{N_t} = \frac{1}{1+\eta}$$

Consumers

Let:

C_t^x	= consumption of agents type x (x=y -young-, o -old-, p -private-) in period t.
E_t^x	= endowment of agents type x (x=y,o,p) in period t.
β	= intertemporal discount factor.
A_t^p	= private sector net financial assets holdings at the end of period t-1.
S_t^x	= savings by agents type x (x=y,o,p) in period t.

Period budget constraint:

$$\overline{e_t^y} - \overline{c_t^y} = \tau_t^y + \overline{a_{t+1}^p} \qquad \text{young agent.}$$

$$\overline{e_t^o} - \overline{c_t^o} = \tau_t^o - R_t \overline{a_t^p} \qquad \text{old agent.}$$

Intertemporal budget constraint (life-time):

$$(\overline{e_t^y} - \tau_t^y) + \frac{(\overline{e_{t+1}^o} - \tau_{t+1}^o)}{R_{t+1}} = \overline{c_t^y} + \frac{\overline{c_{t+1}^o}}{R_{t+1}}$$

Net wealth definition (beginning at period t):

$$\overline{nw_t^y} = (\overline{e_t^y} - \tau_t^y) + \frac{(\overline{e_{t+1}^o} - \tau_{t+1}^o)}{R_{t+1}}$$ young agents (NPV of disposable income).

$$\overline{nw_t^o} = (\overline{e_y^o} - \tau_t^o) + R_t\ \overline{a_t^p}$$ old agents.

We assume that:

a1. There is no bequest motive nor intergenerational gifts.

a2. Private agents take fiscal policy as given (i.e., no political economy considerations).

a3. Utility function is additively separable over time.

a4. Period utility function separable and logarithmic on both goods in every period.

Then the period utility function is depicted as:

$$u(\overline{c_t^x}, g_t) = \ln(\overline{c_t^x}) + \ln(g_t) \qquad x = y, o$$

This form of utility function implies that the path of private consumption is independent of the path of government expenditure.

Then the lifetime utility function becomes:

$$U = u(\overline{c_t^y}, g_t) + \beta\ u(\overline{c_{t+1}^o}, g_{t+1})$$

The old agents' problem can be stated as follows:

$$Max\ u(\overline{c_t^o}, g_t)$$

$$s.t.\ \overline{c_t^o} = \overline{nw_t^o}$$

$$given\ g_t$$

The solution to this problem is:

$$\overline{c_t^o} = \overline{nw_t^o}$$

$$\overline{s_t^o} = -\overline{a_t^p} = (R_t - 1)\ \overline{a_t^p} + (\overline{e_t^o} - \tau_t^o) - \overline{c_t^o}$$

The young agents' problem is:

$$Max\ u(\overline{c_t^y}, g_t) + \beta\ u(\overline{c_{t+1}^o}, g_{t+1})$$

$$s.t.\ \overline{c_t^y} + \frac{\overline{c_{t+1}^o}}{R_{t+1}} = \overline{nw_t^y}$$

$$given\ g_t,\ g_{t+1}$$

The solution to this problem is:

$$\overline{c_{t+1}^o} = \beta\ R_{t+1}\ \overline{c_t^y}$$

$$\overline{c_t^y} = \frac{1}{1+\beta}\ \overline{nw_t^y}$$

$$\overline{c_{t+1}^o} = R_{t+1}\ \frac{\beta}{1+\beta}\ \overline{nw_t^y} = \overline{nw_{t+1}^o}$$

$$\overline{s_t^y} = \overline{a_{t+1}^p} = (\overline{e_t^y} - \tau_t^y) - \overline{c_t^y}$$

Aggregating (in per young terms) yields the following functions:

Consumption:

$$c_t = \overline{c_t^y} + \frac{\overline{c_t^o}}{\eta} = \frac{1}{1+\beta}\,\overline{nw_t^y} + \frac{1}{\eta}\,\overline{nw_t^o}$$

Saving:

$$s_t^p = \overline{s_t^y} + \frac{\overline{s_t^o}}{\eta} = \overline{a_{t+1}^p} - \frac{a_t^p}{\eta} = \frac{\beta}{1+\beta}\,\overline{nw_t^y} - \frac{1}{\eta}\,\overline{a_t^p}$$

The government

Let:

G_t = government expenditure (public good) in period t.

E_t^g = government's endowment in period t.

A_t^g = government's net financial assets position at the end of period t-1.

T_t = total tax revenue in period t.

τ_t^Y = lump-sum tax levied on each young agents in period t.

τ_T^O = lump-sum tax levied on each old agents in period t.

p_t = international price of the government's endowment.

T = number of periods during which the government receives the endowment.

The temporariness of the government's endowment can be depicted as:

$$E_i^g > 0 \qquad \forall\ i \leq t{+}\mathrm{T}-1$$

$$E_i^g = 0 \qquad \forall\ i > t{+}\mathrm{T}-1$$

Total tax revenue in period t can be expressed by:

$$T_t = N_t^Y\ \tau_t^Y + N_t^O\ \tau_t^O$$

$$t_t = \tau_t^Y + \frac{\tau_t^O}{\eta} \qquad \text{in per young terms}$$

The government's nonfinancial wealth at the end of period t-1 is defined by:

$$\sum_{i=t}^{\infty} Q_i^{t-1}\ E_i^g\ p_i = \sum_{i=t}^{t-1+\mathrm{T}} Q_i^{t-1}\ E_i^g\ p_i = NFW_i^g$$

$$\sum_{i=t}^{\infty} Q_i^{t-1}\ \eta^{i-(t-1)}\ e_i^g\ p_i = \sum_{i=t}^{t-1+\mathrm{T}} Q_i^{t-1}\ \eta^{i-(t-1)}\ e_i^g\ p_i = nfw_i^g \qquad \text{in per young terms}$$

$$\eta^{1-t} \sum_{i=t}^{\infty} Q_i^{t-1}\ \eta^i\ e_i^g\ p_i = \eta^{1-t} \sum_{i=t}^{t-1+\mathrm{T}} Q_i^{t-1}\ \eta^i\ e_i^g\ p_i = nfw_i^g$$

Where:

$R_t = (1+r_t)$ = international interest factor on one-period bonds maturing in period t.

$Q_f^t = 1 \qquad$ *for* $f = t$.

$Q_f^t = \dfrac{1}{\prod_{v=t+1}^{f} R_v} \qquad$ *for* $f \geq t$ = present value in period t of a unit of good delivered in period f.

The government's intertemporal budget constraint, assuming away Ponzi games and beginning at period t, is defined by:

$$R_t\ [A_t^g + NFW_t^g]\ +\ \sum_{i=t}^{\infty} Q_i^t T_i\ =\ \sum_{i=t}^{\infty} Q_i^t G_i$$

$$R_t\ [a_t^g + nfw_t^g]\ +\ \sum_{i=t}^{\infty} Q_i^t \eta^{i-t} t_i\ =\ \sum_{i=t}^{\infty} Q_i^t \eta^{i-t} g_i \qquad \text{in per young terms}$$

$$R_t\ [a_t^g + nfw_t^g]\ =\ \eta^{-t} \sum_{i=t}^{\infty} Q_i^t \eta^i (g_i - t_i)$$

These equations, in absolute and per young terms, state the well-known condition that the present value of primary budget deficits must equal the initial total wealth of the government, including its nonfinancial wealth which in this case comprises only the government's temporary endowment.

In presence of finite resources, one could think of alternative definitions for the primary budget deficit. A conventional fiscal accounting definition would include the receipts from the nonrenewable resource as income; while an economic definition should exclude the proceeds of the resource, regarding them as a below-the-line asset management decision. These can then be expressed as:

$$(G_t - T_t) \qquad \text{economic definition.}$$

$$(G_t - T_t)\ -\ E_t^g\ p_t \qquad \text{fiscal account definition.}$$

The implication of distinguishing between such definitions is clear: conventional fiscal accounts understate the economic version of the primary budget deficit during those periods in which the government receives the proceeds from the endowment.

Similarly, government savings can be defined in an analogous fashion:

$$(R_t - 1)\ [A_t^g + NFW_t^g]\ +\ (T_t - G_t) \qquad \text{economic definition.}$$

$$(R_t - 1)\ A_t^g\ +\ E_t^g\ p_t\ +\ (T_t - G_t) \qquad \text{fiscal accounts definition.}$$

These two definitions highlight an important difference. While the economic definition treats as income only the interest-equivalent component of the nonfinancial wealth, the fiscal account definition considers as revenue the full amount of proceeds from the nonrenewable resource. This portrays clearly that conventional accounting regards as income a share of proceeds that would otherwise be considered as a below-the-line operation. In this vein, government savings are overstated in the conventional fiscal accounts with respect to the economic accounting since:

$$E_t^g\, p_t > (R_t - 1)\, NFW_t^g$$

In other words, the inequality above shows that these two definitions would coincide only if the government's endowment were a perpetual annuity (assuming constant endowment prices), which is precisely what was ruled out when considering a finite resource.

Welfare function

We assume that:

a5. Welfare weights decrease over time but are equal across contemporaneous generations.

a6. The government chooses a path of taxes and public expenditures to maximize welfare.

Then the government's problem is:

$$Max \sum_{i=t}^{\infty} \phi_i \, [N_i^y u(\overline{c_i^y}, g_i) + N_i^o u(\overline{c_i^o}, g_i)]$$

$$= \eta^{-(t+1)} N_t^y \sum_{i=t}^{\infty} \phi_i \, \eta^i \, [\eta \ln(\overline{c_i^y}) + \ln(\overline{c_i^o}) + (1+\eta) \ln(g_i)]$$

$$s.t. \quad R_t \, [a_t^g + nfw_t^g] = \eta^{-t} \sum_{i=t}^{\infty} Q_i^t n^i (g_i - t_i)$$

$$t_t = \tau_t^y + \frac{\tau_t^o}{\eta}$$

$$\overline{c_t^y} = \frac{1}{1+\beta} [(e_t^y - \tau_t^y) + \frac{(e_{t+1}^o - \tau_{t+1}^o)}{R_{t+1}}]$$

$$\overline{c_t^o} = \frac{\beta}{1+\beta} [R_t \, (e_{t-1}^y - \tau_{t-1}^y) + (e_t^o - \tau_t^o)]$$

A stylized application:

For the sake of exposition and to reduce the dimensionality of the problem some simplifying assumptions are used, namely:

a7. Interest rates are constant over time.

a8. The government's temporary endowment is constant over the period it exists.

a9. The price of the government's endowment is constant over time.

a10. Levied lump-sum taxes are constant over time and across generations in every period.

Assumption a7 implies:

$$Q_i^t = R^{t-i}$$

Assumptions a7, a8 and a9 imply:

$$NFW_t^g = \sum_{i=t}^{t-1+T} Q_i^{t-1} E_i p_i = R \frac{1-R^{-T}}{R-1} E p$$

$$nfw_t^g = R \frac{1-R^{-T}}{R-1} \frac{E p}{N_t^Y} \qquad \text{in per young terms}$$

Assumption a10 implies:

$$t_t = \tau \frac{1+\eta}{\eta}$$

The assumption that lump-sum taxes are constant over time allows to emphasize the issue of sustainability in the per capita provision of the public good. That is, for the same tax burden per capita, what path of the primary fiscal stance would permit, for example, a constant or increasing provision of the public good?

Thus, the welfare maximization problem becomes:

$$Max\ \eta^{-(t+1)} N_t^y \sum_{i=t}^{\infty} \phi_i \eta^i [\eta \ln(\overline{c_i^y}) + \ln(\overline{c_i^o}) + (1+\eta)\ln(g_i)]$$

$$s.t.\quad R\ (a_t^g + (R\frac{1-R^{-T}}{R-1})\frac{E\,p}{N_t^Y}) = \sum_{i=t}^{\infty}(\frac{\eta}{R})^{i-t}(g_i - (\frac{1+\eta}{\eta})\tau)$$

$$\overline{c_i^y} = \frac{1}{1+\beta}[(\frac{R+1}{R})(e-\tau)]$$

$$\overline{c_i^o} = \frac{\beta}{1+\beta}[(R+1)(e-\tau)]$$

$$\overline{c_t^o} = \frac{\beta}{1+\beta}[(R+1)e - (R\tau_{t-1}+\tau)]$$

$$given\quad \tau_{t-1},\ a_t^g,\ N_t^Y$$

Solving for g_i and τ yields the following close-form solution:

$$g_{i+1} = R\frac{\phi_{i+1}}{\phi_i}g_i$$

$$\tau = \frac{1}{2}[e - k_0\frac{\eta}{(1+\eta)}[\sum_{i=t}^{\infty}(\frac{\eta}{R})^{i-t}]^{-1}]$$

$$k_0 = R_t\, a_t^g + (R^2\frac{1-R^{-T}}{R-1})\frac{E\,p}{N_t^Y}$$

$$g_i = R^{i-t}\frac{\phi_i}{\phi_t}\frac{1}{2}[e\frac{(1+\eta)}{\eta}\sum_{i=t}^{\infty}(\frac{\eta}{R})^{i-t} + k_0][\sum_{i=t}^{\infty}\eta^{i-t}\frac{\phi_i}{\phi_t}]^{-1}$$

$$d_i^{FD} = R^{i-t}\frac{\phi_i}{\phi_t}\frac{1}{2}[e\frac{(1+\eta)}{\eta}\sum_{i=t}^{\infty}(\frac{\eta}{R})^{i-t} + k_0][\sum_{i=t}^{\infty}\eta^{i-t}\frac{\phi_i}{\phi_t}]^{-1}$$

$$-\frac{1}{2}\frac{(1+\eta)}{\eta}[e-k_0\frac{\eta}{(1+\eta)}[\sum_{i=t}^{\infty}(\frac{\eta}{R})^{i-t}]^{-1}] - \frac{E^g\,p}{\eta^{i-t}N_t^Y}\qquad iff\ i\leq T+t$$

Primary fiscal deficit (fiscal accounting)

The solution above presents three different cases depending on the term $R\phi_{i+1}/\phi_i$, which reflects the government's relative time-preference of people's welfare adjusted by the opportunity cost of spending an extra unit today:

i. If $R\phi_{i+1}/\phi_i > 1$, then government expenditure per young (and per capita) increases over time, reflecting its relative care for future generations.

ii. If $R\phi_{i+1}/\phi_i = 1$, i.e., if the government seeks intergenerational equity, then its expenditure per young (and per capita) is constant over time.

iii. If $R\phi_{i+1}/\phi_i < 1$, then government expenditure per young (and per capita) diminishes over time because the adjusted generational preference is biased toward present and near-future generations.

If government intertemporal preferences evolve smoothly, so will government expenditure in all of the cases above. As mentioned in Section II, these three cases are depicted in Figure 1a (in per capita terms) and Figure 1b (in percent of GDP). The paths of primary fiscal deficits, in the conventional fiscal accounting sense, show in general an initial primary surplus,[2] i.e., the replacement of nonfinancial wealth with financial assets. The accumulation of financial assets implies higher future interest income and thus the ability to run primary deficits. Case i and ii show a clear pattern of primary surplus when the finite resource exists and primary deficits thereafter. Case iii shows an inverted u-shape path for the primary deficit.

Moreover, as illustrated in these figures, the less biased government preferences are toward present generations the higher the initial surpluses. Conversely, initially weaker primary fiscal stances, ceteris paribus, show implicitly the government's relative disinterest for the future generations, highlighting the intergenerational implications of policies, i.e., lower primary deficits in the future and a lower provision of the public good.

Furthermore, counter to some casual views that a larger depletable resource would merit less an initial surpluses (because it would resemble more closely a permanent resource); we found, as displayed in Figure 2a and 2b, that as long as the resource is finite the larger its size the higher the optimal initial primary surplus in all cases. Also, Figure 3 shows, for the per capita constant expenditure case, that the initial per capita surplus moves equiproportionally with the present value of the endowment, i.e., the percentage increase in the primary surplus is the same as that of the endowment.[3]

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[2]See Appendix II for initial conditions.

[3]The per capita increasing expenditure case shows a less than proportional percentage increase in the initial surplus, while the expenditure decreasing case exhibits a more than proportional percentage increase.

Chile—Initial Conditions for Initial Primary Surpluses

Assume $R > \eta$

Let $\Phi_i = \phi^{i-t}$ *with* $0<\phi<1$

$$d_t^{FD}<0 \ \ iff \ \ a_t^g < \frac{E^g p}{N_t^Y}\frac{2}{\theta_2}\frac{1}{R} - \frac{1}{\theta_2}\frac{1}{R}e\frac{(1+\eta)}{\eta}\theta_1 - R\frac{(1-R^{-T})}{(R-1)}\frac{E^g p}{N_t^Y}$$

Where:

$$\theta_1 = \frac{R}{(R-\eta)}(1-\eta\phi) - 1$$

$$\theta_2 = \frac{(R-\eta)}{R} + (1-\eta\phi)$$

<u>*Case ii*</u>: $R\frac{\Phi_{i+1}}{\Phi_i} = R\phi = 1$

$$\theta_1 = 0$$

$$\theta_2 = 2\frac{(R-\eta)}{R}$$

$$d_t^{FD}<0 \ \ iff \ \ a_t^g < \frac{E^g p}{N_t^Y}\frac{1}{(R-\eta)} - R\frac{(1-R^{-T})}{(R-1)}\frac{E^g p}{N_t^Y} = \frac{E^g p}{N_t^Y}\frac{1}{(R-\eta)} - \frac{1}{R}NFW_t = \theta_3$$

Since $\frac{E^g p}{N_t^Y} > (R-1)\,NFW_t$ *then* $\theta_3>0$

Thus, in the per capita constant expenditure case, as long as the initial stock of financial assets is not too large the initial fiscal primary balance will be in surplus.

<u>*Case i*</u>: $R\frac{\Phi_{i+1}}{\Phi_i} = R\phi > 1$

$\theta_1 < 0$

$0 < \theta_2 < 2\,\frac{(R-\eta)}{R}$

$$d_t^{FD} < 0 \quad iff \quad a_t^g < \frac{E^g p}{N_t^Y}\,\frac{2}{\theta_2}\,\frac{1}{R} - \frac{1}{\theta_2}\,\frac{1}{R}\,e\,\frac{(1+\eta)}{\eta}\,\theta_1 - R\,\frac{(1-R^{-T})}{(R-1)}\,\frac{E^g p}{N_t^Y} = \theta_3^*$$

Where:

$\theta_3^* > \theta_3 > 0$

Then the per capita increasing expenditure case will have initial fiscal surpluses even under a larger stock of initial financial assets than in the constant expenditure case.

<u>*Case iii*</u>: $R\frac{\Phi_{i+1}}{\Phi_i} = R\phi < 1$

$\theta_1 > 0$

$\theta_2 > 2\,\frac{(R-\eta)}{R} > 0$

$$d_t^{FD} < 0 \quad iff \quad a_t^g < \frac{E^g p}{N_t^Y}\,\frac{2}{\theta_2}\,\frac{1}{R} - \frac{1}{\theta_2}\,\frac{1}{R}\,e\,\frac{(1+\eta)}{\eta}\,\theta_1 - R\,\frac{(1-R^{-T})}{(R-1)}\,\frac{E^g p}{N_t^Y} = \theta_3^{**}$$

Where:

$\theta_3^{**} < \theta_3$

The per capita decreasing expenditure case supports initial primary surpluses under more restricted conditions than the constant or decreasing expenditure cases, i.e., smaller initial stocks of financial assets could call for initial deficits, since the future reduction in per capita expenditures will determine future surpluses permitting early transitory deficits.

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