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Explaining Foreign Exchange Market Puzzles

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Abstract

The paper develops a flow model of the exchange rate with speculative capital flows integrated in a rigorous manner. The model is consistent with five foreign exchange market puzzles: (1) occasional discontinuous jumps in the exchange rate; (2) periodic short-term regimes of persistent appreciation/depreciation that can develop into a long swing; (3) the forward discount bias; (4) volatility clusters in the foreign exchange market that create conditional heteroskedasticity; and (5) the dual profitability of betting in the short run against any official foreign exchange intervention, and betting with the intervention in the long run.

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I. INTRODUCTON

The world of exchange rate economics is replete with many puzzling facts that include: (1) spot rates that occasionally jump discontinuously from one level to another;¹ (2) periodic short term regimes of persistent appreciation or depreciation that defy fundamentals and that can develop into a long swing;² (3) changes in the spot rate that move in the opposite direction from what is suggested by lagged values for the forward discount, or, equivalently, lagged values for the interest rate differential, i.e., the so-called forward discount bias;³ this puzzle is associated with excess profits for investments in high interest rate countries, thereby suggesting the possibility of market inefficiency;⁴ and (4) exchange rates often experience periods of tranquility that are interrupted by clusters of volatility,⁵ which generates a heteroskedastic frequency distribution for changes in the exchange rate.

A fifth puzzle deals with the more recent and still controversial evidence that perhaps speculators could have made positive profits in the short run if they had bet against the FX market activities of the US Treasury and Fed, but could have earned positive profits in the long run if they had bet in the direction of such FX market intervention activities.⁶ Official FX intervention by the US government has focused on “leaning against the wind”, primarily by attempting to limit or reverse persistent exchange rate movements away from the perceived long run equilibrium value for the dollar. Therefore, puzzle #5 amounts to the fact that it might have been profitable to bet against the dollar’s fundamentals in the short run, but to bet on them in the long run.

There have been alternative theories offered to explain one or two of these puzzles, while the others remain a complete mystery, except for some rather ad hoc explanations. For example, scholars have employed jump-diffusion models in an attempt to replicate discontinuous jumps in the exchange rate, puzzle #1, but no theory exists with regard to what causes this phenomenon.⁷

¹ Cf., for example, Akgiray and Booth (1998) and Jorion (1998).

² The short term drifts or runs appear to occur, at least in part, in response to monetary shocks, thereby generating “delayed overshooting” as in Eichenbaum and Evans (1995), while the long swings relate primarily to the value for the dollar in the 1980s, as documented by Engel and Hamilton (1990).

³ See Froot and Thaler (1990) and Engel (1996) for this literature.

⁴ However, recent evidence suggests that the forward discount bias might exist only for high to medium frequency data, but not for very low frequency data. Cf. Flood and Taylor (1995) and Meredith and Chinn (1998).

⁵ See Mussa (1979).

⁶ See, for example, Leahy (1995) and Neely (1998).

⁷ See Ball and Roma (1993), Malz (1996), and the references in footnote #1 above.

With regard to puzzle #2, Frankel and Froot (1986) show that speculative bubbles can be generated in a model wherein the expectations of portfolio managers are based on a weighted average of fundamentals and the most recent value for the spot rate. However, once they start, such bubbles continue indefinitely, thereby prompting Frankel and Rose (1995) to point out that no one has yet developed a theory that contains a role for fundamentals that eventually ends persistent movements away from the exchange rate dictated by the fundamentals.

Initially, scholars hypothesized that the forward discount bias, puzzle #3, could be explained by the existence of a time varying risk premium, but there has been difficulty generating this with reasonable parameter values within a capital asset pricing model.⁸ Alternatively, this puzzle might arise via exchange rate prediction errors, but the persistence in their signs implied by the forward discount bias seems to imply irrational behavior, unless, perhaps they are associated with peso problems or learning about changes in monetary or fiscal regimes. However, no one has explained how peso problems or learning about regime changes might yield a negative correlation between prediction errors and the interest rate differential (or risk premium on home assets), a condition that must exist if prediction errors are the cause of the forward discount bias.

McCallum (1994), on the other hand, attempts to explain the forward discount bias in a model that yields the appropriate relationship between interest rate differentials and the risk premium. His model contains a central bank reaction function wherein the bank alters the interest rate differential to smooth exchange rate variations that arise via temporary exogenous changes in the risk premium. Meredith, and Chinn (1998) use a more complex version of McCallum's model to generate the forward discount bias in the short run but not the long run. Gourinchas and Tornell (1996) develop a model wherein agents learn gradually about temporary versus persistent changes in interest rate differentials, and where the risk premium increases when exogenous shocks occur. Their model yields the forward discount bias in some but not all cases. Finally, Mark and Wu (1996) obtain the forward discount bias within a foreign exchange market model that contains fundamentalists and noise trades.

The conditional heteroskedasticity generated by volatility clusters, puzzle #4, can be generated via the ARCH model introduced by Engle (1982). Unfortunately, no hypothesis has been offered with regard to the cause of such clusters, except for the feasible but rather unsatisfying observation that this puzzle could be just a statistical phenomenon wherein shock variables exhibit the same patterns of tranquility and volatility. Finally, Neely (1998) and others have suggested that puzzle #5 might arise because some "unknown process" drives the spot rate away from its long run value in the short run, but toward its long run value eventually. Unfortunately, the nature of this process remains "unknown".

The objective here is to develop a model that is consistent with all five puzzles. The theory represents a FX market flow approach to exchange rate theory, with long term speculative activities incorporated in a rigorous manner. This model exhibits many features of asset market models, but it allows for discontinuous regime switching between a relatively high or relatively

⁸ For an excellent review of this literature see Engel (1996).

low value for FX. Each regime switch creates the possibility of endogenous drifts or short term trends in the exchange rate that become more likely to continue as the absolute value of the interest rate differential increases. However, persistent variations in the spot rate in the same direction exert a lagged effect on the trade balance, and this progressively increases the probability that any long swing will end via a regime switch.

The forward discount bias arises because an increase (decrease) in the interest rate differential makes it more likely that the spot rate will jump discontinuously, and/or will drift downward (upward) in the near future. Clusters of volatility can arise if the market remains in the same regime for some time and then jumps to another regime, prompting long term speculators to take profits. Finally, if the interest rate differential is significantly nonzero, but decays substantially over the time horizon of long term speculators, then it can be perfectly rational for short term speculators to bet against the fundamentals, and for long term speculators to bet on the fundamentals.

Section II develops the model and generates puzzle #1, while section III carefully investigates its dynamics, using the results to explain puzzle #2. Section IV shows how the model is consistent with puzzle #3. Then section V generates puzzle #4. Section VI explores the issue of rationality, and offers an explanation for puzzle #5. Finally, section VII summarizes the main conclusions and provides a perspective on the model.

II. THE MODEL AND PUZZLE #1

In the model, home and foreign agents include: (i) traders involved in exports, imports, and non-speculative foreign investment, (ii) long term speculators, and (iii) short term speculators. Long term speculators have a time horizon of “n” periods, which is assumed to be substantial. Short term speculators have a time horizon of one period; they include noise traders of the type found in DeLong, et. al. (1990) and trend chasers or chartists.

Home agents use home money to calculate their wealth and to buy consumer goods, while foreign agents use FX for these purposes. Home and foreign preferences are identical, and nontraded goods do not exist.⁹ The log of the fundamental exchange rate, s^* (in home currency units per FX), is the value for the spot rate that yields a zero fundamental balance of payments (to be defined below). The home minus foreign interest rate differential, $ID = (i - i^*)$, is exogenous, and no distinction is made between nominal and real interest rates.

The fundamental balance of payments consists of the current account plus net capital flows whose motivation is not FX market speculation¹⁰ The fundamental exchange rate, s^* , is the value

⁹ The last two assumptions imply that changes in net foreign assets via a nonzero current account exert no influence on future current accounts, because the resulting change in net international interest earnings is exactly offset by a wealth induced variation in exports and imports. Cf. Dornbusch, Fischer, and Samuelson (1977) and Davis and Miller (1996).

¹⁰ Transactions such as long term bank loans and purchases of foreign stocks or bonds that are uncovered establish nonzero positions in FX, and, hence, can be characterized as “speculative”.

for the nominal spot rate that equates the current account with the optimal net capital flow that falls out of an intertemporal optimization process. Conceptually, s^* is similar to Stein's (1994) NATREX, Clark and MacDonald's (1997) BEER, and the exchange rate determined by the IMF's macroeconomic balance approach as in Isard and Faruquee (1998).

The determinants of s^* are called "the fundamentals". For simplicity these fundamentals are separated into two categories, namely the interest rate differential and all other fundamentals, Z_o .¹¹ The latter are exogenous with normally distributed random shocks that have permanent and temporary components to them. Agents can distinguish between these temporary and permanent shocks, and have static expectations with regard to the permanent components of Z_o . The interest rate differential is also exogenous. Any nonzero value for the interest rate differential is highly persistent, but agents correctly believe that it will decay toward zero in the very long run.

Finally, all agents have the same belief as to the value for the fundamental exchange rate at the end of the time horizon for long term speculators, and this is given by

$$s^*(t+n) = a E[Z_o(t+n)] + b E[ID(t+n)] \quad a > 0, b < 0 \quad (1)$$

Even though the expected value for the interest rate differential approaches zero as "n" approaches infinity, it need not be zero in (1) because "n" is less than infinity. An increase in $E[ID(t+n)]$ reduces $s^*(t+n)$, because a higher home minus foreign interest rate differential will: (i) alter the optimum allocation of world saving toward home assets, (ii) improve the capital account in the fundamental balance of payments, and (iii) necessitate a decrease in s^* in order to deteriorate the home current account sufficiently to keep the fundamental balance of payments zero.

It is assumed that model uncertainty exists, and, consequently, long term speculators do not solve the model to obtain their expectations. Instead, they use a combination of fundamentals and technical analysis in determining their expectations. This is defined as "eclectic expectations", and its use follows the suggestion of Frankel and Froot (1986), Goodhart (1988), and others. In addition eclectic expectations are consistent with the results of surveys of FX dealers such as Allen and Taylor (1990) and Lui and Mole (1998).

In order to capture the essence of eclectic expectations in the simplest possible manner, it is assumed that agents consider the most recent value for the spot rate, and combine this with $s^*(t+n)$ when determining the expected future spot rate at the end of their speculative time horizon. This simplest possible example of eclectic expectations amounts to the often used

However, if the primary motivation of such activities is not to profit from anticipated exchange rate variations, then they are included here in the capital account of the fundamental balance of payments.

¹¹ Surveys consistently find that the interest rate differential is considered to be a very important fundamental by FX dealers. See, for example, Lui and Mole (1998).

regressive expectations assumption which Dornbusch (1976) proved to be Muth-rational under certain conditions.¹²

At the beginning of period t all agents know the value for the spot rate at the end of the previous period, $s(t-1)$. The expected values of all fundamentals for period t as well as the interest rate differential, $ID(t)$, are revealed at the beginning of the period. The value for $ID(t)$ and its anticipated decay are used to determine $E[ID(t+n)]$. This, in turn, is combined with the expected values for the other fundamentals to obtain $s^*(t+n)$ via (1).

Then $s^*(t+n)$ as well as the most recent value for the spot rate are combined to obtain what the simple regressive form of eclectic expectations yields for the value of the spot rate at the end of the time horizon for long term speculators, $s^e(t+n)$. The latter and the current interest rate differential are then used in a manner consistent with risk adjusted uncovered interest parity to determine the FX market activities of long term speculators in period t . More specifically, eclectic expectations for period " $t+n$ ", $s^e(t+n)$, are determined via

$$s^e(t+n) = \psi(t+n)s^*(t+n) + [1-\psi(t+n)]s(t-1) \quad (2a)$$

$$\psi(t+n) = \theta \sum_{k=0}^{n-1} (1-\theta)^k \quad 1 - \psi(t+n) = (1-\theta)^n \quad (2b)$$

$$\lim_{n \rightarrow \infty} \psi(t+n) = 1 \quad (2c)$$

$$s^e(t+n) - s(t-1) = \psi(t+n) [s^*(t+n)] - s(t-1) \quad (2d)$$

With $0 < \theta \leq 1$, the weights given by (2b) are consistent with the results of surveys which indicate that FX market dealers give progressively more weight to fundamentals as the time horizon increases.¹³ The regressive nature of this simple form of eclectic expectations is illustrated by (2d) which is obtained by subtracting $s(t-1)$ from both sides of (2a).

Since home (foreign) agents use home (foreign) money for consumption, they consider foreign (home) bonds to be riskier than home (foreign) bonds. Consequently, no one will buy the

¹² Section VI below shows that the model with regressive expectations yields a rational expectations solution for the exchange rate in the long run equal to the fundamental rate, s^* , under certain reasonable conditions. Also, it is proven that the FX market activities of long term speculators who use the regressive form of eclectic expectations are consistent with rational expectations, RE, provided that their time horizon is of sufficient length. Finally, under reasonable circumstances the sum of eclectic forecast errors (in comparison with RE forecasts) approaches zero in the very long run, implying that the simple form of eclectic expectations used here is unbiased

¹³ See Allen and Taylor (1990) and Lui and Mole (1998).

bonds of the other country unless their expected rate of return exceeds that of local bonds by an amount consistent with the risk-return tradeoff along the relevant Sharpe-Lintner efficiency locus. This means, for example, that FX market speculation by home agents will not occur unless the expected rate of return on foreign bonds exceeds that of home bonds by some threshold magnitude or risk premium, μ . This also applies to the purchase of home bonds by foreign speculators.

These assumptions imply the following activities for long term speculators that are consistent with ex ante uncovered interest parity for risk averse agents.¹⁴

$$\text{Regime 1: Foreign Agents Sell FX if: } s(t) \geq s^e(t+n) + \mu - ID(t) \quad (3a)$$

$$\text{Regime 2: Home Agents Buy FX if: } s(t) \leq s^e(t+n) - \mu - ID(t) \quad (3b)$$

In plain words, long term speculators buy FX when it becomes sufficiently cheap and sell it when it becomes sufficiently dear.

The discontinuous Γ curve in Figure 1A illustrates the excess demand for FX by long term speculators. They supply FX when the spot rate equals s_1 in regime 1, and demand FX when the spot rate is s_2 in regime 2. By assumption the Γ curve becomes perfectly elastic once the threshold values for the spot rate have been reached.¹⁵ From (3a) and (3b) the Γ curve is symmetric around $s^e(t+n)$ if the interest rate differential is zero. The Γ curve shifts downward if: (i) $s^e(t+n)$ declines, or (ii) for any given expected future spot rate, if the interest rate differential increases. Intuitively, if home agents require an expected FX appreciation of $x\%$ in order to buy FX when the interest rate differential is zero, then they will not buy FX unless the expected appreciation is somewhat larger (i.e., s must be lower by the value of ID) if the home interest rate rises above the foreign rate.

Note also from (1) that an increase in the current interest rate differential lowers $s^*(t+n)$, because agents do not anticipate a complete decay in the differential within the next “ n ” periods. Thus, $s^e(t+n)$ falls when the interest rate differential increases, and this means that the Γ curve shifts downward for a second reason. In other words, a monetary shock exerts two influences on s_1 and s_2 : (i) the first influence affects $s^*(t+n)$ and, consequently, the expected future spot rate, $s^e(t+n)$; (ii) the second influence alters the positions of regime 1 and regime 2 with regard to $s^e(t+n)$.

¹⁴ Regime 1 could also include home agents borrowing FX and selling it in order to buy home bonds. Similarly, Regime 2 could include foreign agents borrowing home money and selling it for FX in order to buy foreign bonds.

¹⁵ If long term speculators had heterogeneous exchange rate expectations as found by MacDonald and Marsh (1996) then the discontinuous Γ curve would have a negative slope. The assumption of homogeneous expectations, however, allows the model to generate the stylized facts without undue complexity.

The $J()$ function in (4) below indicates the net excess supply of FX from the fundamental balance of payments, as well as from the net FX activities of short term speculators, $(C_s - C_d) + \omega(t)$, and profit taking activities by long term speculators $\Omega(t)$.

$$J(t) = \alpha + \beta s(t) + \beta' s(t-k) + \gamma ID(t) + v(t) + (C_s - C_d) + \omega(t) + \Omega(t) \quad (4)$$

$$\gamma > 0 \quad v \sim N(0, \sigma_v^2) \quad \omega \sim N(0, \sigma_\omega^2)$$

The fundamental balance of payments is determined by the first five terms in (4), i.e., it is a linear function of: (i) the log of the current spot rate, $s(t)$; (ii) a lagged value for the log of the spot rate, $s(t-k)$, and (iii) the interest rate differential. Permanent shocks to the non-interest rate fundamentals, Z_0 , are picked up by changes in the constant term, α .¹⁶ Temporary shocks to the Z_0 fundamentals are represented by $v(t)$. The β and β' coefficients are given by:

$$\beta = \eta + \eta^* - 1 < 0 \quad (5a)$$

$$\beta' = \eta' + \eta'^* - 1 > 0 \quad (5b)$$

Where η and η^* are the home and foreign, respectively, short run price elasticities of demand for the other country's goods, and η' and η'^* are the corresponding long run elasticities. Expression (5a) reflects the assumption that the Marshall-Lerner condition is not satisfied in the short run, thereby giving the J curve a negative slope in Figure 1A. That is, the trade balance and fundamental balance of payments initially improve when home money appreciates, $ds(t) < 0$.

On the other hand, (5b) indicates that the Marshall-Lerner condition is met over a longer time interval, and this is modeled by assuming that the value for the spot rate lagged "k" periods exerts a positive influence on the home trade balance and fundamental balance of payments. Therefore, a home currency appreciation initially improves the trade balance and fundamental balance of payments via a movement down and to the right along the J curve. Then "k" periods later the lagged effect of this appreciation worsens the home trade balance and balance of payments, thereby shifting the J curve to the left. Finally, increases in the home minus foreign interest rate differential improve the home capital account and fundamental balance of payments, thereby shifting the J curve to the right in Figure 1A.

The approach taken here with regard to short term speculators is similar to that of DeLong, et. al. (1990) in that we investigate how their activities affect the dynamics of the exchange rate without presenting a micro-foundation for their behavior. However, later sections provide some insights into such behavior. Therefore, follow DeLong, et. al. by assuming that noise traders are agents who randomly become optimistic or pessimistic with regard to the future

¹⁶ Equation (1) for the fundamental exchange rate, s^* , falls out of the first four terms in (4) if: (i) the fundamental balance of payments is set equal to zero, and (ii) the exchange rate is constant at its fundamental value, $s^* = s(t) = s(t-k)$. This gives

$$s^* = -\alpha/(\beta + \beta') - \gamma ID/(\beta + \beta'), \text{ where } \beta + \beta' > 0 \text{ by assumption.}$$

value for the spot rate, thereby generating normally distributed random net sales, $\omega(t) > 0$, or purchases, $\omega(t) < 0$, in (4). The expected value of the FX activities of noise traders is zero; they simply increase the size of random temporary shifts in the J curve that arise also from random temporary shocks to the Z_0 fundamentals via the $v(t)$ term in (4).

In their survey of 200 chartists in the London FX market, Allen and Taylor (1990) find that chartists' expectations are most accurate when a trend becomes well established. Thus, our model assumes that chartists take a speculative position in FX only after a trend has clearly materialized, and that they hold this position for only one period at a time. They, however, can reenter the market in many consecutive periods. When chartists or trend chasers first enter the FX market, then either the C_d term (for chartists' FX demand) or the C_s term (for chartists' FX supply) is positive while the other term is zero in equation (4). After that both terms are positive because: (i) chartists are wiping out their speculative position from the previous period, and (ii) there are always some chartists who are betting that an established trend will continue.

The value for the exchange rate in each period is determined in Figure 1A where the excess supply of FX as given by the $J(\cdot)$ function in (4) is matched by the excess demand for FX by long term speculators as given by (3a) or (3b). Equivalently, the short run equilibrium exchange rate equates the fundamental balance of payments with minus the net capital flow from the FX activities of all speculators. Figure 1A illustrates that the model generates an unstable equilibrium for the spot rate that is bounded by two stable equilibria. If the J_1 curve is relevant, then the unstable equilibrium occurs at point a' , with exchange rate s' that is not in general equal to the value for the fundamental exchange rate, $s^*(t+n)$, given by (1).¹⁷

With the J_1 curve in Figure 1A, the stable equilibria occur at point 1 in regime 1 and at point 2 in regime 2, respectively. In the simplest case where the activities of noise traders and chartists are zero, and where long term speculators are not taking profits, the J curve represents the fundamental balance of payments. If the fundamental balance of payments is positive, as at point 2 in Figure 1A, then FX market equilibrium requires that long term speculators offset this excess supply of FX with net FX purchases at point 2. On the other hand, at point 1 the net excess demand for FX from a fundamental balance of payments deficit is matched by a supply of FX from long term speculators.

Recall that profit taking activities by long term speculators appear in the J function via the $\Omega(t)$ term. Initially this is zero, but it is assumed that any discontinuous jump in the exchange rate from one regime to the other in the direction in which long term speculators have been betting will elicit FX sales, $\Omega(t) > 0$ in (4), or purchases, $\Omega(t) < 0$, that represent a nontrivial portion, ϕ , of the cumulative FX position of these speculators. The latter is given by the cumulative value for the $J(\cdot)$ function when regime switching occurs. That is, assume that the cumulative value for $J(\cdot)$

¹⁷ This occurs because temporary random shocks to the fundamental balance of payments shift the J curve and change s' , but do not alter s^* . The same is true for the activities of noise traders and chartists. Finally, an increase in the interest rate differential moves s' and s^* in the opposite directions because: (i) it improves the fundamental balance of payments, thereby shifting the J curve to the right and increasing s' , but (ii) it lowers s^* in (1).

was zero in period (t-h) and that R_i ($i = 1,2$) represents the regime in which the market has been operating for the last “h” periods, while $R(t)$ indicates the regime that is relevant in period t. Then

$$\Omega(t) = 0 \quad \text{if } R(t) = R_i \quad (6a)$$

$$\Omega(t) = \phi \sum_{j=1}^h J(t-j) \quad \text{if } R(t) \neq R_i \quad (6b)$$

For example, suppose that the fundamental balance of payments and the value for the $J(\)$ function have been positive and that the market has been in regime 2 for the last “h” periods. This means that long term speculators have been buying FX, thereby accumulating a long position in FX equal to $\sum J(t-j) > 0$ in (6b). If the market jumps up to regime 1, then long term speculators will sell FX in an amount equal to $\phi\%$ of their cumulative position. Such profit taking FX sales by long term speculators shift the J curve to the right in Figure 1A.

This FX market flow approach to exchange rate theory avoids a major criticism of the traditional “elasticities” approach to the exchange rate by allowing for a gross volume of FX transactions that can greatly exceed what arises from exports and imports. In addition, it has several properties exhibited by asset market models. First, the short run dynamics of the exchange rate are dominated by exchange rate expectations and other asset market considerations that shift the Γ curve.

Second, from (3a) and (3b), UIP (adjusted for risk) holds in an ex ante sense for long term speculators at all times. Third, from (1), (2a), and either (3a) or (3b), it follows that a monetary shock that alters the interest rate differential will cause the spot rate to change instantaneously by:

$$ds(t) = ds^e(t+n) - dID(t) = \psi ds^*(t+n) - dID(t) = [\psi bID' - 1] dID(t) \quad (7a)$$

$$\text{Where } ID' = \delta E[ID(t+n)] / \delta ID(t) > 0$$

Thus, an increase in the interest rate differential in favor of home assets will appreciate home money instantaneously by more than the variation in the interest rate differential, as in exchange rate overshooting models.

A fourth similarity with asset market models is that trade flows and the pure flow component of capital flows (that is associated with current period saving and investment) do not directly affect the exchange rate. This is true because variations in the fundamental balance of payments shift the J curve but have no effect on the short run equilibrium value for the spot rate, provided that the FX market remains in the same regime. However, as in traditional asset market models, trade flows can alter the spot rate indirectly via their effect on changes in the expected future spot rate. Permanent changes in the fundamentals which affect the current account and $s^*(t+n)$ will alter the current spot rate via their effect on s_1 and s_2 through the influence of $s^*(t+n)$ on $s^e(t+n)$ in (2a).

$$ds(t) = ds^e(t+n) = \psi ds^*(t+n) = \psi a dZ_o(t) \quad (7b)$$

An important difference between this model and typical asset market models is that the market can jump discontinuously from one stable regime to another, thereby generating puzzle #1 above. Suppose, for example, that the market is initially at point 2 in Figure 1A, where the J curve intersects the regime 2 portion of the Γ curve. If the J curve shifts to the left to J_2 , then an equilibrium no longer exists in regime 2, and the market switches to regime 1 at point 1b via an instantaneous depreciation of home money by the magnitude 2μ . If, in future periods, the J curve ends up in a position such as J_3 , then home money will instantaneously appreciate as the market switches from regime 1 to regime 2.

Finally, the model generates hysteresis because it will remain in an existing regime even if the events that induced an earlier switch to that regime are completely reversed. For example, in Figure 1A, if the market switches from regime 2 to regime 1 via a shift in the J curve from J_1 to J_2 , then the market will remain in regime 1 even if the J curve returns to the position given by J_1 . Consequently, the model yields at least two values for the spot rate for any given set of values for the fundamentals. This means that empirical reduced form exchange rate equations are likely to have low explanatory power and to experience unstable coefficients.

III. DYNAMICS AND PUZZLE #2

This section first summarizes the dynamics of the model. Then it explores how these dynamics can generate an endogenous movement in the exchange rate away from its fundamental value, and what determines: (i) the maximum size of such movements, and (ii) the probability that such movements will develop into a long swing. Finally, it is shown that long swings sow the seeds of their own demise. In summary form, the dynamics of the model are as follows:

1. Assume that an exogenous monetary shock prompts the FX market to jump discontinuously from regime 1 to regime 2.
2. Calculate the value for $s_2 = s^e(t+n) - \mu - ID(t)$, as given by (3b), in each period.
3. Use the value for s_2 and the values for all exogenous variables to determine the value for $J()$ in each period via equation (4).
 - a. If $J() \geq 0$, then the market remains in regime 2.
 - b. If $J() < 0$, then the market switches to regime 1.
4. If the market switches to regime 1, then use the value for s_1 in each period to calculate $J()$ in each period, and proceed as in (3a) and (3b).

In more detail, assume initially that at time $(t-1)$ the market is in regime 1, with an exchange rate of $s_1(t-1)$ at point 1 in Figure 1B. Also, assume a zero interest rate differential, and a zero value for the expected future differential, $E[ID(t+n)] = 0$. Finally, the initial fundamental exchange rate, $s^*(t+n)$, and the initial value for $s^e(t+n)$ both lie between $s_1(t-1)$ and $s_2(t-1)$, but they are not necessarily equal to each other. Then let an exogenous increase in the

interest rate differential reduce s^* by $bE[ID(t+n)]$, where $E[ID(t+n)]$ is a fraction of $ID(t)$. Hence, $s^e(t+n)$ decreases by $\psi bE[ID(t+n)]$. The positive interest rate differential also: (i) makes the Γ curve unsymmetric around $s^e(t+n)$, and (ii) shifts the J curve to the right via (4), as from J1 to J2 in Figure 1B.

Therefore, the upper and lower values for the spot rate at which long term speculators enter the market are reduced by $\psi bE[ID(t+n)] - ID(t)$, thereby shifting the Γ curve downward by this amount, say from Γ to Γ' in Figure 1B. Assume that this shift and the rightward shift in the J curve are sufficient to induce a switch from regime 1 to regime 2 in Figure 1B, as the market moves from point 1 to point 2. This yields an instantaneous decrease in the spot rate of

$$ds_0 = -2\mu + \psi bE[ID(t+n)] - ID(t) \quad (8a)$$

The instantaneous appreciation in the spot rate in period t by ds_0 reduces $s^e(t+n)$ in the next period by $(1-\psi)ds_0$. This shifts the Γ curve downward again to Γ'' in the next period and (if the market remains in regime 2) decreases the exchange rate in period $t+1$ by $(1-\psi)ds_0$, as is illustrated in moving from point 2 to point 3 in Figure 1B. If the market continues to remain in regime 2 with s^* constant and (for analytical simplicity) the interest rate differential constant at $ID(t)$, then the: (i) change in the spot rate in period $(t+k)$, which equals any change in $s^e(t+n)$ that occurs in period $(t+k)$, denoted here by $ds^e[(t+n) | (t+k)]$, as given by (2a); (ii) the value for the spot rate in period $(t+k)$; and (iii) the limit to the movement away from s^* are¹⁸

$$ds(t+k) = ds^e[(t+n) | (t+k)] = ds_0(1-\psi)^k \quad (8b)$$

$$\begin{aligned} s(t+k) &= s^e[(t+n) | (t+k)] - \mu - ID(t) \\ &= \left\{ \psi s_0^* + \psi bE[ID(t+n)] + (1-\psi)s(t-1) + ds_0 \sum_{n=1}^k (1-\psi)^n \right\} - \mu - ID(t) \end{aligned} \quad (8c)$$

$$\lim_{k \rightarrow \infty} s(t+k) = s(t) + ds_0(1-\psi)/\psi \quad (8d)$$

The intuition here is important. From (2a) and (3a) we know that in this situation: (a) the expected future spot rate generated by (2a) exceeds the actual spot rate, and (b) long term speculators buy FX. If the current spot rate, s_2 , lies below the rate determined by fundamentals, $s^*(t+n)$, then such expectations and actions appear to be stabilizing. However, in this case they are destabilizing, because they generate progressive decreases in the value for the spot rate at which stabilizing FX purchases begin. Therefore, continuous movements of the spot rate away

¹⁸ Note that $s^e(t+n)$ is the spot rate that long term speculators think will exist “n” periods later than any given time period. From (2a) it follows that with s^* constant, $s^e(t+n)$ varies in response to the actual change in the spot rate in the previous period. Thus, $ds^e[(t+n) | (t+k)]$ indicates how $s^e(t+n)$ changes in period $(t+k)$ in response to any $ds(t+k-1)$. Also, note that for notational simplicity these expressions use ψ instead of $\psi(t+n)$.

from its fundamental value do not require speculators to sell FX when the exchange rate is below its fundamental value. All that is needed is for a previous fall in the spot rate to decrease the value of the exchange rate at which stabilizing speculation begins.

If the shock terms are relatively large, then long swings in the exchange rate will be unlikely (as in reality) because any existing drift or short term trend is apt to end abruptly via a switch from regime 2 to regime 1. This can occur at any time if shocks to the J curve shift it sufficiently to the left (as from J2 to J3 in Figure 1B) so that an equilibrium in regime 2 no longer exists. This has the interesting implication that an increase in the variance of noise trader speculative activity, σ^2_{ω} , can at times reduce volatility in the FX market by making it more likely that drifts or runs in the spot rate end via a regime switch.¹⁹

Even if a long swing in the exchange rate is unlikely, the model suggests that an increase in the interest rate differential raises the conditional probability of the market remaining in regime 2, $\Pr[R2 | R2]$, thereby tending to prolong any existing downward drift in the spot rate.

$$\delta \Pr[R2 | R2] / \delta ID(t) = F' \left\{ \gamma + \beta[-1 + \psi bID'] + (\psi bID' - 1) \sum_{n=1}^k (1-\psi)^n \right\} > 0 \quad (9a)$$

where $F[]$ represents the cumulative frequency distribution for the normally distributed sum of the shock terms $[v(t) + \omega(t)]$ in (4), and, hence, $F' > 0$.

A higher interest rate differential affects the probability of remaining in regime 2 in two general ways, each of which is tied up with a larger value for the J function, namely: (i) it directly improves the capital account and fundamental balance of payments via the γ term in (9a) and (4), and (ii) it indirectly improves the fundamental balance of payments by generating a lower value for the spot rate in period $s(t+k)$ via the $+\beta[-1 + \psi bID'] + (\psi bID' - 1) \sum (1-\psi)^n$ term in (9a).

These influences are illustrated in Figure 2A, which assumes that the market has been drifting downward recently, and would be at point d in period $t+k$ if the interest rate differential were zero. In such a case, it would take a shift to the left in the J curve that is larger than the distance cd in order to elicit a switch to regime 1. The direct effect of an increase in the interest rate differential improves the fundamental balance of payments and shifts the J curve to the right as to J' in Figure 2A. Now it takes a negative shock to the J curve exceeding the distance ce to prompt a regime switch.

¹⁹ Note that with s^* and the interest rate differential constant, a switch from regime 2 to regime 1 is a necessary but not sufficient condition for a negative drift in the spot rate within regime 2 to end. If the spot rate has been drifting downward in regime 2 for “k-1” periods, and then it switches up to regime 1 in period $t+k$, the change in the spot rate that occurs is given by $ds(t+k) = 2\mu + (1-\psi)^k ds_0$, where $ds_0 < 0$ is the value for the initial jump down to regime 2. Thus, a downward drift in the spot rate can conceivably continue in spite of a switch from regime 2 to regime 1. This becomes progressively less likely as the age of the short term trend increases.

The indirect effect of a larger interest rate differential is associated with the fact that the latter yields (at any point in time after a switch to regime 2) a Γ curve that lies farther down in Figure 2A. For example, if the Γ curve were relevant in period $t+k$ with a zero interest rate differential, then a positive differential might yield the Γ' curve. In this case, it takes a negative shock to the J curve in excess of the distance $c'e'$ in order to induce a switch to regime 1.

This conclusion suggests that a positive interest rate differential makes it less risky for trend chasers to bet that a downward drift in the spot rate will continue in the short run even though the spot rate lies below its long run fundamental value. Such speculation would involve selling FX spot, thereby giving $C_s - C_d > 0$ in (4), and shifting the J curve to the right. This raises the conditional probability of remaining in regime 2 by

$$\delta \Pr[R2 | R2] / \delta(C_s - C_d) = F' > 0 \quad (9b)$$

Therefore, a large positive interest rate differential in favor of the home country creates a situation where the probability of persistent home appreciations increases, provided that the positive interest rate differential persists.²⁰ This might provide an insight into the dazzling ascent of the dollar in the first half of the 1980s, when US interest rates were significantly above foreign rates.

If a positive shock to the interest rate differential makes persistent appreciations more likely, then the probability that this drift will abruptly end via a jump up to regime 1 gradually increases as the positive interest rate differential decays. Consequently, a substantial increase in the interest rate differential that is followed by its slow decay creates the possibility that the exchange rate will trend downward for some time, and then switch from regime 2 to regime 1 when the differential has decayed sufficiently. Thus, the model is consistent with the “delayed overshooting” found empirically by Eichenbaum and Evans (1995).

An obvious question is “Does anything guarantee that a long swing will not continue indefinitely?” The answer is an emphatic “Yes”, because persistent movements in the exchange rate exert a lagged influence on the trade balance in a direction that progressively raises the probability of a regime switch. For example, if the spot rate has been appreciating continuously in regime 2, then the lagged value for the spot rate in (4) becomes progressively smaller, and the probability of remaining in regime 2 changes by

$$\delta \Pr[R2 | R2] / \delta s(t-k) = F'\beta' > 0 \quad (9c)$$

A negative value for $\delta s(t-k)$ reduces the probability of remaining in regime 2, because persistent decreases in the lagged value for the exchange rate worsen the trade balance and fundamental balance of payments, thereby shifting the J curve to the left. Thus, as a long swing downward in

²⁰ It is easily proven that if the market is initially in regime 1, then an increase in the interest rate differential raises the probability that the market will jump discontinuously to regime 2, thereby initiating a downward drift in the spot rate.

regime 2 continues, it becomes progressively more likely that the market will jump up to regime 1, thereby guaranteeing that the long swing ends eventually. Fundamentals win in the long run!

Expression (9c) holds everything (especially the current change in the spot rate) constant but the lagged value for the change in the spot rate. In any given period $(t+k)$ after a negative drift in the spot rate has begun, a leftward shift in the J curve (via the lagged effect of home appreciation on the fundamental balance of payments) tends to reduce the value for the J function in (4). However, if the downward drift in the spot rate continues in this period, then this appreciation tends to increase the value for the J function. The net effect will be negative if

$$\eta' + \eta^{*'} > 1 + (1-\psi)^k \quad (9d)$$

where η' and $\eta^{*'}$ are the long run price elasticities of the fundamental balance of payments. Since the Marshall-Lerner condition is assumed to be satisfied in the long run, condition (9d) is certain to be met eventually. However, this yields the empirically testable conclusion that the average duration of endogenous drifts or short term trends in the spot rate will be longer for countries whose long run trade elasticities are lower.²¹

In summary, this section has shown that the model is consistent with puzzle #2 in that it can endogenously generate regimes of persistent appreciation or depreciation that can conceivably develop into a long swing. The probability that a drift or run in the spot rate will continue is a positive function of the absolute value of the interest rate differential. If a shock to the interest rate differential gradually decays, then this can enhance the possibility of “delayed overshooting”. Finally, even with a constant nonzero interest rate differential, the lagged effects of persistent appreciations or depreciations will alter the trade balance in a manner that ensures that any long swing will eventually end via a regime switch.

IV. THE FORWARD DISCOUNT BIAS: PUZZLE #3

This section shows how the model is consistent with the forward discount bias. In the process it also provides an insight as to why the forward discount bias might exist only for high frequency data. What we seek is a negative relationship between the interest rate differential in any period $(t+k)$ and the change in the spot rate in the next period.

The existence of stochastic terms within the model means that anything can happen to the spot rate at any time. Thus, the analysis considers the rational expectations, RE, value for the future change in the spot rate. This is defined as the weighted average of all possible values for $ds(t+k+1)$ generated by the model in any given situation, with the weights given by the conditional probability of each possible outcome, as determined by the model. If the market has been drifting downward in regime 2 for k periods following a shock to the spot rate in period t of $ds_0 < 0$, the RE value for the future change in the spot rate is given by

²¹ Since developing countries typically export primary products whose price elasticities are relatively low, this suggests that such countries are likely to experience longer lasting endogenous drifts or runs in their exchange rates.

$$E[ds(t+k+1)] = +2\mu \{1 - \Pr[R2 | R2]\} + (1-\psi)^{k+1} ds_0 \quad (10a)$$

If the interest rate differential increases in period $t+k$ by $dID(t+k)$, this exerts a negative influence on the spot rate in that period of

$$\Psi b \, dE[ID(t+n)] - 1 \, dID(t+k) = [\psi b ID' - 1] \, dID(t+k) < 0 \quad (10b)$$

The shock to the spot rate given by (10b) reduces the expected future spot rate, which, in turn, begins another potentially infinite series of persistent movements in the exchange rate.

In period $t+k+1$, the influence of this second series is given by the expression in (10b) multiplied by $(1-\psi)$. Adding this to (10a), substituting $ID(t+k) - ID(t+k-1)$ for $dID(t+k)$, and differentiating the resulting expression with respect to the interest rate differential yields

$$\delta E[ds(t+k+1)]/\delta ID(t+k) = (1-\psi)[\psi b ID' - 1] - 2\mu \delta \Pr[R2 | R2]/\delta ID(t+k) < 0 \quad (11)$$

where $\delta \Pr[R2 | R2]/\delta ID(t+k)$ is positive from (9a). Thus, the model is consistent with the forward discount bias.

The intuition is as follows. First, an increase in the interest rate differential in period $(t+k)$ instantaneously generates a negative shock to the spot rate in period $(t+k)$. From (2a) this affects $s^e(t+n)$ in a negative manner in the next period, thereby reducing the algebraic value for $ds(t+k+1)$, regardless of its sign. Second, an increase in the interest rate differential raises the probability that the market will end up in regime 2, thereby giving a greater weight to the value for the spot rate in regime 2 when calculating $E[ds(t+k+1)]$. Thus, the expected value for the future change in the spot rate decreases as the interest rate differential rises, provided, of course, that ID is not expected to fall substantially in the next period.²²

In sum, the model suggests that the forward discount bias is likely to occur from one time period to another if a monetary shock creates a nonzero interest rate differential that remains essentially intact over two consecutive periods. On the other hand, it is less likely that the forward discount bias will exist between any two consecutive periods if a nonzero interest rate differential dissipates substantially from the first period to the second. This suggests a hypothesis as to why the forward discount bias appears to be a high to medium frequency phenomenon in reality. If interest rate differentials decay slowly, then any current period differential will be essentially unchanged in the next period if the time interval is brief, as is assumed in (11). On the other hand, if the duration of one time period is quite long, say several years, it is more likely that the interest rate differential will decay substantially from one period to the next. Consequently, the forward discount bias will be less likely with low frequency data.

²² Expression (11) simplifies by assuming that ID is not expected to change between periods $(t+k)$ and $(t+k+1)$.

V. VOLATILITY CLUSTERS AND HETEROSKEDASTICITY: PUZZLE #4

This section addresses the fact that volatility clusters occasionally interrupt prolonged periods of exchange rate tranquility, and, hence, generate a time series for exchange rate variations that exhibits conditional heteroskedasticity. Section I pointed out that this could arise simply because random shocks experience this same time series pattern. More interestingly, our model suggests that this can arise from profit taking when regime switching occurs.

Suppose that speculators had a zero net FX position in period t , and that the market has remained in regime 2 for “ h ” periods, during which time the spot rate has been relatively tranquil. Since long term speculators buy FX when the market is in regime 2, it follows that they will have accumulated a net positive position in FX equal to the cumulative sum of the value for the J function during this tranquil time interval. Such a situation is represented in Figure 1B at point 4, where J_1 intersects the Γ' curve.

Assume next that the relative positions of the J curve and Γ curve change in period $t+h$ such that an equilibrium no longer exists in regime 2, as is true for J_3 and the Γ' curve. Such an event induces a jump up to regime 1, with an instantaneous home depreciation and a new equilibrium at point 5 in Figure 1B. The depreciation of home money when the market moves in period $t+h$ from point 4 to point 5 means that the exchange rate moves in the direction in which long term speculators have been betting.

From (6b) this means that long term speculators will take profits equal to $\phi\%$ of their cumulative positive FX position. Such profit taking sales of FX temporarily shift the J curve to the right in Figure 1B. If the previous period of tranquility had been relatively brief, then the $\Omega(t)$ term in (6b) and (4) might not be large enough to shift the J curve very far to the right in Figure 1B, as from J_3 to J_1 . In this case, the market will remain in regime 1 and a volatility cluster does not arise.

On the other hand, as the duration of the previous period of tranquility increases, the volume of profit taking given by (6b) rises. This means that the J curve will shift farther to the right in period $t+h+1$, thereby creating the possibility that the market can immediately switch back to regime 2. For example, if J_3 shifts to J_2 in Figure 1B, then the market will end up in period $t+h+1$ in a position such as point 2. Thus, one regime switch can be followed by a reversal of this switch in the next period.

This need not be the end of the story. Suppose that the market ends up at point 2, in period $t+h+1$, with the volume of profit taking by long term speculators equal to the horizontal distance between the J_3 and J_2 curves. Since the market has returned to regime 2, profit taking sales of FX by long term speculators cease. Consequently, if the fundamentals do not change between period $t+h$ and period $t+h+2$, then the J curve will return to J_3 in period $t+h+2$. This induces another regime switch and an instantaneous depreciation as the market jumps up to regime 1. Consequently, the volatility cluster persists, and it will end when a regime switch does not elicit enough profit taking to induce another regime switch. In sum, the model can generate clusters of volatility, and it suggests that a cluster is more likely and will be more pronounced as

the consecutive time spent in a given regime increases. This, of course, will generate a time series for changes in the spot rate that exhibits conditional heteroskedasticity.

VI. RATIONALITY AND PUZZLE #5

This section explores several aspects of rationality, namely the conditions (if any) under which: (i) the model is consistent with the apparently random expectations and FX activities of noise traders of the type used here and in DeLong, et. al.; (ii) the model generates a long run RE solution for the exchange rate that equals s^* ; (iii) the expectations of short term trend chasers are Muth-rational, i.e., it is rational to bet against the fundamentals in the short run; (iv) regressive expectations (as a simple example of eclectic expectations) are equivalent to rational expectations; (v) the speculative activities of long term speculators (who use eclectic expectations) are Muth-rational because such expectations are in the right direction; and (vi) it is rational to simultaneously bet against the fundamentals in the short run, and to bet on them in the long run?

The first issue can be restated as, “Does the model offer any rationale for why some agents might suddenly become optimistic or pessimistic with regard to the current value for the exchange rate?” Clearly, this might occur if these agents believe that they can predict the timing of regime switches. However, if the anticipated regime switch does not occur, then the hunches of noise traders would appear to be unfounded, i.e., random.

The second question is whether the model generates a long run equilibrium RE solution for the exchange rate equal to s^* . In such a case, the expected value for the exchange rate in the long run is constant (for any given values for the fundamentals) and equals $s^*(t+n)$ as “n” approaches infinity.

$$E[s(t+n)] = E[s(t+n+1)] = s^*(t+n) \quad (13a)$$

If (13a) holds when we replace $E[s(t+n+1)]$ by the regressive form of eclectic expectations, $s^{\circ}(t+n+1)$, then the answer to the question is “Yes”! Thus, we seek the conditions under which this is true.

$E[s(t+n)]$ is found by taking a weighted average of the values for: (i) the expected value for the spot rate in regime 1 in period $(t+n)$, and (ii) the expected value for the spot rate in regime 2 in period $(t+n)$, with the weights determined by the true probabilities of the market being in each regime. Assume that the market has been operating in regime 2 and represent the conditional probability that the market will be in regime 2 in period $t+n$ by $Pr(t+n)$. Consequently,

$$E[s(t+n)] = E[s_1(t+n)][1-Pr(t+n)] + E[s_2(t+n)] Pr(t+n). \quad (13b)$$

Long term speculators will buy FX in period $t+n$ if the spot rate equals $s^{\circ}(t+n+1)$ minus the sum of the interest rate differential and risk premium in period $t+n$. Therefore, the expected value of $s_2(t+n)$ is given by $s^{\circ}(t+n+1)$ minus the sum of the expected value for the interest rate differential

in period $t+n$ and the risk premium, μ , as in (14a) below. Inserting (14a) into (13b) and noting that $E[s_1(t+n)] = E[s_2(t+n)] + 2\mu$, gives (14b).

$$E[s_2(t+n)] = s^e(t+n+1) - E[ID(t+n)] - \mu \quad (14a)$$

$$E[s(t+n)] = s^e(t+n+1) - E[ID(t+n)] + \{1 - 2Pr(t+n)\}\mu \quad (14b)$$

Consequently, the model yields a Muth-rational solution of s^* in the very long run if: (a) $s^e(t+n+1)$ equals the fundamental value for the exchange rate, and (b) the last two terms on the rhs of (14b) sum to zero. From (2a) through (2c) it follows that condition (a) holds as the time horizon approaches infinity. Condition (b) will be satisfied if: (i) the expected interest rate differential is zero, and the conditional probability of being in regime 2 in period $t+n$ is 0.50, or (ii) if a nonzero expected interest rate differential is just offset by the $\{1 - 2Pr(t+n)\}\mu$ term. Since the probability of the latter is zero, it can be ignored.

The model clearly satisfies the first half of condition (bi) because the interest rate differential moves toward zero in the long run by assumption. Furthermore, if the expected value for the net speculative activity included in the J function in (4) is zero in the very long run, then the J function and the J curve represent simply the fundamental balance of payments in the very long run. Under these conditions, a zero interest rate differential means that s' in Figure 2B, (the value for the exchange rate at the unstable equilibrium point) will equal s^* , the exchange rate that yields a zero fundamental balance of payments.

Moreover, with a zero interest rate differential, the Γ curve is symmetric around the expected future spot rate, as in Figure 2B, and (since $s' = s^*$) the distance ab equals the distance cd. Thus, over an infinite number of periods, it is equally likely that the market will be in regime 1 and regime 2. Consequently, $Pr(t+n) = 0.50$ in (14b), and the second part of condition (bi) is satisfied. In sum, the model yields a RE solution of s^* in the very long run under these seemingly reasonable conditions.

Next turn to issue (iii) that deals with the rationality of the speculative activities of short run trend chasers. Rewrite (13b) and (14a) for $n = 1$ to obtain an expression similar to (14b) in the short run; use (2a) to substitute for $s^e(t+2)$, and then subtract $s(t-1)$ from both sides.

$$E[s(t+1)] - s(t-1) = \psi(t+1)[s^*(t+n) - s(t-1)] - ID(t) + \{1 - 2Pr(t+1)\}\mu \quad (14c)$$

Where $\psi(t+1)$ is the weight from (2b) that eclectics give to $s^*(t+n)$ in period $(t+1)$.²³ If the fundamental exchange rate exceeds the most recent value for the spot rate, then the first term on the rhs of (14c) is positive. That is, the fundamentals suggest that home money will depreciate in the short run. However, in regime 2 the $\{1 - 2Pr(t+1)\}\mu$ term is negative, because the conditional probability of remaining in regime 2 exceeds 0.50.

²³ The value of $\psi(t+1)$ is an interpolated one, because long term speculators calculate only $\psi(t+n)$.

The intuition for this last statement is as follows. In regime 2 the value for the J function is positive, and, thus, the market will switch to regime 1 only if a sufficiently large negative shock to the J curve occurs. However, since all shock terms are normally distributed, it follows that: (i) the probability that the shocks will be net positive equals 0.50, and (ii) the probability that the shocks are net negative, but not large enough in absolute value to induce a switch in regimes, exceeds zero. Consequently, the probability of remaining in regime 2 exceeds 0.50.

A negative $\{1 - 2Pr(t+1)\}\mu$ term in (14c) works against the positive influence of the fundamentals on the RE change in the spot rate, so that the net effect is uncertain. However, the RE value for the short run change in the spot rate is more likely to be negative in (14c) if the interest rate differential is positive, because: (i) a larger ID(t) directly reduces the rhs of (14c); and (ii) a higher value of ID(t) increases the expected value of the interest differential in period (t+1); this, in turn, raises the conditional probability of remaining in regime 2; thus, the negative $\{1 - 2Pr(t+1)\}\mu$ term increases in absolute value as the current period's interest rate differential grows. Therefore, it can conceivably be Muth-rational to expect the spot rate to move away from its fundamental value in the short run if the market is in regime 2 (regime 1) and the current interest rate differential is strongly positive (negative).

Next turn to issue (iv), i.e., Under what conditions does the simple form of eclectic expectations used here approach rational expectations?" To explore this, assume that the market is in regime 2. Then rewrite (14c) for period t+n in order to obtain an expression for the RE value for the change in the spot rate over the time horizon of long term speculators.

$$E[s(t+n)] - s(t-1) = \psi(t+n)[s^*(t+n) - s(t-1)] - E[ID(t+n-1)] + [1 - 2Pr(t+1)]\mu \quad (15a)$$

Thus, the RE value for the change in the exchange rate over the time horizon of long term speculators has two components to it. The first component, $\psi(t+n)[s^*(t+n) - s(t-1)]$, is the eclectic forecast from (2d). The second component, $-E[ID(t+n-1)] + \{1 - 2Pr(t+1)\}\mu$, represents the difference between the RE forecast and the eclectic forecast, defined here as the "expected eclectic forecast error, ε ". The latter will vary with the state of the world, "j", at the time when eclectic expectations are formulated. That is,

$$\begin{aligned} \varepsilon_j &= E[s(t+n) - s(t-1)] - \psi(t+n)[s^*(t+n) - s(t-1)]_j \\ &= \{-E[ID(t+n-1)] + [1 - 2Pr(t+1)]\mu\}_j \end{aligned} \quad (15b)$$

where there are an infinite number of possible states of the world, $j = 1 \dots \infty$.

From the discussion of issue (ii) above, it is clear that the regressive form of eclectic expectations will not in general equal the RE forecast. However, the eclectic forecast will be unbiased over long periods of time if the expected forecast errors sum to zero.

$$\lim_{n \rightarrow \infty} \sum_{j=1}^n \varepsilon_j = 0 \quad (15c)$$

From (15b), condition (15c) will be satisfied if the mean value for the interest rate differential is zero in the very long run, and, thus, the probability of being in each regime is 0.50. In sum, the regressive expectations version of an eclectic forecast will not in general equal the rational expectations forecast, but over the very long run the eclectic forecast is unbiased under seemingly reasonable conditions.

Next, issue (v) relates to the weaker condition as to whether the sign of the regressive expectations version of an eclectic forecast, $\psi(t+n)[s^*(t+n) - s(t-1)]$, is the same as the sign of the RE forecast, $E[s(t+n) - s(t-1)]$, in (15a). If so, then the speculative activities of long term speculators will be rational. As before, if $s^*(t+n)$ exceeds the most recent value for the exchange rate, then the eclectic forecast exerts a positive influence on the RE long run change in the exchange rate in (15a). However, from (2b) and (2c) this positive influence grows progressively larger as the time horizon of long term speculators increases, because (2b) indicates that the size of $\psi(t+n)$ in (15a) increases with "n". Thus, the sign of the eclectic forecast is more likely to be the same as the sign of the RE forecast as the time horizon of long term speculators grows.

In addition, since any initially nonzero interest rate differential decays over time, $E[ID(t+n)]$ in (15a) will be smaller than any positive differential in period t . This directly increases the value for the RE change in the spot rate in (15a), and it indirectly increases the value for the RE change because it reduces $Pr(t+n)$ below $Pr(t+1)$. Consequently, the speculative activities of long term speculators who use eclectic expectations will be Muth-rational, provided that their time horizon is long enough for the expected value of the interest rate differential to decay sufficiently.

Finally, the answer to rationality issue (vi) follows directly from the analysis of issues (iii) and (v). If the interest rate differential is currently significantly nonzero, but if it decays substantially over the time horizon of long term speculators, then it can be perfectly rational to bet against the fundamentals in the short run, and to bet on them in the long run. In such a case, the model is consistent with the fact that positive profits could have been earned by betting against US intervention in the FX market in the short run, and by betting with the intervention in the long run, provided that the intervention generally leaned against any movement in the spot rate away from its fundamental value.

In summary, this section has shown that with the apparently reasonable condition that the expected interest rate differential is zero in the very long run, the model yields a long run RE solution for the exchange rate that equals the fundamental rate.²⁴ Furthermore, if the time horizon for long term speculators is long enough for any current period nonzero interest rate differential to decay substantially, then the regressive form of eclectic expectations will be in the correct direction, thereby making the speculative activities of eclectic speculators rational.

In addition, even though the long run forecast given by the regressive form of eclectic expectations will not in general equal the RE forecast, the eclectic forecast will be unbiased in the long run if the mean value for the interest rate differential is zero. Thus, in the long run the

²⁴ This was also a necessary condition for the regressive expectations in Dornbusch (1976) to be rational.

regressive form of eclectic expectations appears to be an excellent pragmatic approach in a world where model uncertainty exists. Finally, the model is consistent with puzzle #5 (i.e., it can be rational to bet against the fundamentals in the short run, and to bet on them in the long run) if the current interest rate differential is significantly nonzero, but if it decays substantially over the time horizon of long term speculators.

VII. CONCLUSIONS AND A PERSPECTIVE

1. The model is consistent with puzzles #1 and #2 because it suggests that the FX market can jump discontinuously from one stable regime to another, and each regime switch creates the possibility of a short term endogenous drift or run. Furthermore, the market is more likely to experience persistent movements away from the fundamental exchange rate as the interest rate differential increases in absolute value. However, any long swing in the exchange rate will eventually be reversed because lagged movements in the spot rate in the same direction alter the trade balance and fundamental balance of payments in a direction that progressively increases the probability of a regime switch. This implies that countries with relatively low long run trade elasticities are apt to experience longer swings.

2. The model is consistent with the forward discount bias, puzzle #3, because an increase in the interest rate differential makes it more likely that a downward drift in the spot rate will begin and/or will continue. Delayed overshooting can occur if: (a) a positive shock to the interest rate differential initiates a downward drift in the spot rate; and (b) the differential decays slowly until this decay eventually induces a switch in regimes. Finally, the forward discount bias might not exist for extremely low frequency data if the interest rate differential decays substantially from one long time interval to another.

3. The model can generate volatility clusters, puzzle #4, if a regime switch induces a substantial volume of profit taking. This becomes more likely as the consecutive time spent in any given regime increases. Finally, if the interest rate differential is large in absolute value, but it decays substantially over the time horizon of long term speculators, then it can be Muth-rational to bet that the spot rate will move away from s^* in the short run, and simultaneously to bet that the spot rate will move toward s^* in the long run. This is consistent with puzzle #5 if official US intervention in the FX market has generally attempted to limit or reverse movements in the spot rate away from its fundamental value.

4. If model uncertainty exists, then the regressive expectations form of eclectic expectations appears to be quite reasonable. However, it is useful to ascertain the extent to which the conclusions reached here are driven by this assumption concerning exchange rate expectations.

a. Puzzle #1 (discontinuous jumps in the spot rate) arises because of the assumptions that the Marshall-Lerner condition is not satisfied in the short run and that stabilizing speculation occurs only if the expected rate of return is sufficiently large.

b. The first part of puzzle #2 (endogenous drifts in the spot rate away from its fundamental value) will occur if long term speculators utilize any expectations wherein

the most recent value for the spot rate exerts a positive influence on the expected value for the spot rate at the end of the time horizon for long term speculators.

c. The second part of puzzle #2 (the potential for long swings) as well as puzzle #3 (the forward discount bias) are also independent of the specific form of the expectations function, provided that the latter satisfies the condition given in (b) above. The model is consistent with these puzzles because an increase in the interest rate differential lowers the expected future spot rate, and (for any given expectations) it also alters the values for the spot rate at which stabilizing speculation begins. Both effects tend to make it more likely that the spot rate will drift downward, or, at least, drift upward more slowly.

d. Puzzle #4 (volatility clusters and conditional heteroskedasticity) is independent of the expectations function. Such clusters arises because of: (i) the assumptions given in (a) above yield an unstable equilibrium that is bounded by two stable regimes, and (ii) the assumption that a substantial amount of profit taking occurs when the spot rate jumps in the direction in which long term speculators have been betting.

e. Finally, puzzle #5 is driven by the assumptions given in (b) and (c) above.

Figure 1A : The Model

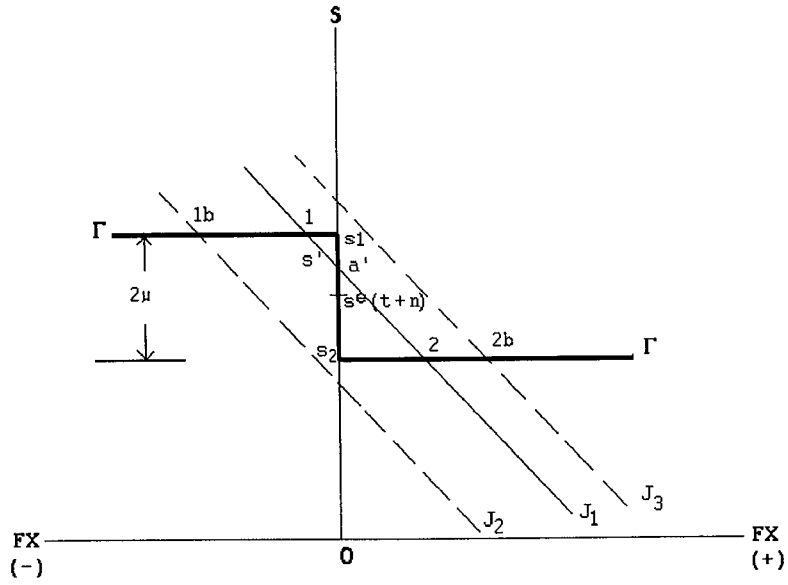


Figure 1B : Endogenous Drifts

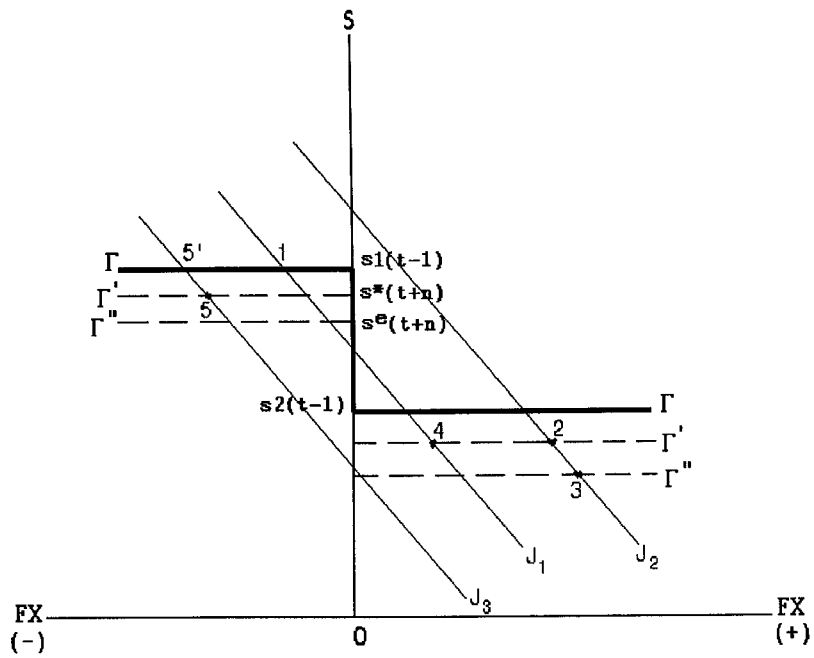


Figure 2A : Conditional Probabilities

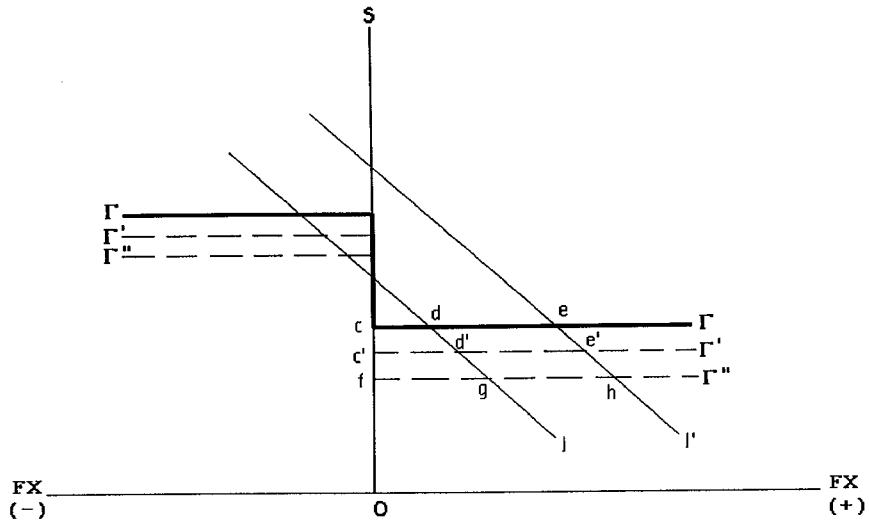
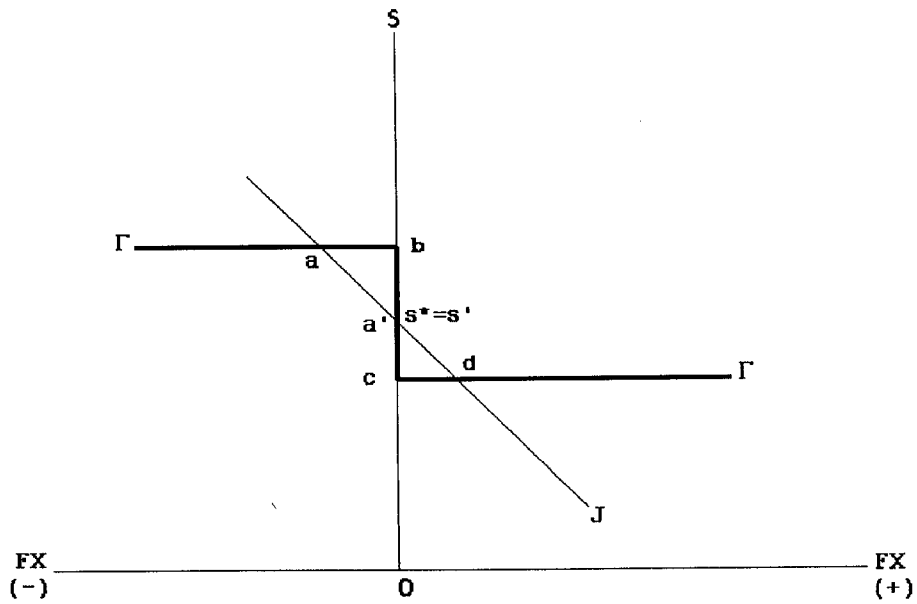


Figure 2B : Rationality



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