IMF Working Paper

WP/99/174

INTERNATIONAL MONETARY FUND

Fiscal Affairs Department

Risk, Resources, and Education—Public Versus Private Financing of Higher Education

Prepared by Berthold U. Wigger and Robert K. von Weizsäcker [1]

Authorized for distribution by Vito Tanzi

December 1999

Abstract

This paper develops a public education scheme that takes uncertainty aspects of private educational investments explicitly into account. In the author's framework, the social merits of public education schemes are related to the lack of markets in which students can insure against educational risks. A case is made for tuition fees that depend on the expected returns of investments in education. The consideration of uncertainty provides a neglected link between educational choice, resource endowment, and productivity growth, which may serve to redefine the public role of education financing.

JEL Classification Numbers: H52, I22, D81

Keywords: Public education, tuition fees, choice under uncertainty

Author's E-Mail Address: Weiz@econ.uni-mannheim.de

[1] This paper was written while Professor von Weizsäcker was a visiting scholar in the Fiscal Affairs Department. Professor von Weizsäcker is Professor of Economics and Finance at the University of Mannheim and Dr. Wigger is Assistant Professor at the University of Mannheim.

Contents

I. Introduction

In virtually all developed countries the government is engaged in higher education. One of the arguments commonly used is that higher education contributes to economic growth. In fact, recent developments in the theory of economic growth, initiated by the seminal papers by Romer (1986) and Lucas (1988), have identified investment in education, especially in higher education, as a key element of economic growth.[1]

The view of higher education as being an essential contribution to economic growth does not, however, as such provide a case for a public provision. First of all, it should be asked whether and why investments in higher education regulated by private markets do not provide an efficient level of higher education.

The argument of a *public good* can hardly be employed. In general, higher education is rival and excludable. Sometimes, however, it is argued that higher education produces *positive externalities.* The classical argument that a stable democracy can only be based upon appropriately educated citizens may be applied to elementary schooling rather than to higher education. Undoubtedly, university graduates provide the society with valuable services. On the other hand, they earn comparatively high incomes which may have internalized the economic surplus of university graduates. But even if some work by university graduates causes external effects beyond the respective salary differential, a Pigou-internalization by subsidizing *all* university students, which is common practice, will hardly be successful.[2]

A second frequently stated argument attributes educational subsidies to *imperfect capital markets.* Human capital acquired by education is non-tradable, it cannot be lent ahead and thus cannot be used as a security for student loans. This can lead to an inefficiently small demand for higher education by those who are

---

[1] Most contributions to the theory of endogenous growth link the rate of growth to the level of education. See, e.g., models with rising product qualities [Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991a), or Aghion and Howitt (1992)], models with rising product varieties [Romer (1990), or Grossman and Helpman (1991b)], or models with convex technologies [Jones and Manuelli (1990) or Rebelo (1991)]. For a recent survey see Barro and Sala-i-Martin (1995).

[2] See, e.g., Rosen (1995, chap. 6) and Lüdecke (1996) for a discussion on positive externalities of higher education.

not able to finance a cost-intensive education due to liquidity constraints.[3] Capital market imperfections, however, do not necessarily suggest educational subsidies. Publicly provided student loans should suffice to redress a too small demand for higher education due to capital market imperfections.[4]

One should bear in mind, though, that individual returns to education are generally *uncertain*. The individual can neither be sure about finishing his education successfully nor about his future returns after a successful examination. In fact, educational returns display a very high variance as many students do not graduate, income differences between graduates are large and even the risk to become unemployed exists.[5] Publicly provided student loans which have to be paid back irrespective of educational success generally do not change the nature of individual educational risk. Yet, the risk an individual faces with an investment in education can be expected to constitute a significant disincentive to invest in education as individuals have hardly a chance to insure against these risks on private markets. Therefore, it can be assumed that risk-averse individuals do not adjust expected marginal returns on educational investments to marginal costs. Instead, they *underinvest* in education.

What matters for society as a whole are average returns of all university graduates. Society should invest in higher education until average marginal returns of educational investments equal their marginal costs. A publicly provided educa-

---

[3] Most of the literature concerned with the relationship between income distribution and growth assumes some kind of capital market imperfection. See, e.g., Galor and Zeira (1993), Perotti (1993), and Barham, Boadway, Marchand and Pestieau (1995), as well as the survey by Perotti (1994).

[4] See Lott (1987) for a more in-depth survey of the normative justifications for subsidizing higher education. Trostel (1993, 1996) provides a further efficiency argument for educational subsidies relying on the notion that income taxation distorts the investment decision on education which, in turn, can be corrected by an education subsidy. In contrast to the arguments above, however, this constitutes a second best argument for subsidizing education. It should be noted that there is an alternative strand of the economics of education literature which emphasizes a public choice perspective of public education financing rather than stressing efficiency arguments. This literature includes Peltzman (1973), Johnson (1984), and Fernandez and Rogerson (1995).

[5] Already Becker (1964, pp. 104) has pointed out that educational returns are characterized by a very high coefficient of variation. This observation is supported by more recent studies as, for instance, Miller and Volker (1993). For a theoretical approach to educational risk see Levhari and Weiss (1974).

tional program with a *success-dependent* cost participation of university students may contribute to an individual realization of this rule.

Individuals possess different educational risks depending on their talent, determination, and flexibility with respect to unexpected events. Educational policymakers have to take this into account. However, they hardly have information of that kind. An efficient educational program should therefore induce individuals to voluntarily reveal their educational characteristics and to choose an education which fits best their educational risks.

In this paper we develop a public education program which explicitly takes uncertainty of private education into account. The policy objective is characterized by a maximization of a tax dividend of public education financing. First, we derive a public scheme on the assumption that individual educational risks are observable. Next, we suppose that individual educational risks constitute private information. Optimal education financing may then be shown to include individual income-dependent contributions. In this way, our model provides a case for *success-dependent tuition fees* as they already exist in some countries.[6]

The consideration of educational risk gives an interesting insight into the interrelation between individual educational choice, initial resource endowment, and productivity growth. We show that the extent of private educational financing as well as the social surplus of a public funding of education depend on both individual resource endowments and productivity growth. The necessity for a public scheme becomes less compelling when better endowed individuals are increasingly willing to accept educational risks. Productivity growth may also lead to an increase in private educational investments. This effect, however, tends to be much weaker than the impact of an increase in initial endowments.

II. The Basic Framework

We consider two types of individuals, denoted by $i = 1, 2$, facing some amount of risk when deciding on the level of educational investment. Such an investment has two possible outcomes: the education undertaken may either be successful or fail,

[6] An example is the *Higher Education Contribution Scheme* which was introduced in Australia in 1989. See Chapman (1997) for an economic analysis of this scheme.

where success and failure are measured in terms of disposable income.[7] Let y_i^S and y_i^F be disposable income in case of success and in case of failure, and let π_i denote the probability that the educational investment of an individual of type i fails. This probability can be understood as a measure of the lack of individual talent and purposefulness. We assume that each individual knows his own probability to study without success.[8] Expected utility of an individual of type i may be written as:

$$Eu_i = \pi_i \, u(y_i^F) + (1 - \pi_i) \, u(y_i^S), \qquad i = 1, 2. \tag{1}$$

The von Neumann-Morgenstern function $u : \mathbf{R}_+ \to \mathbf{R}$ is assumed to be twice continuously differentiable. It is assumed that the individuals are risk-averse and anxious to realize positive consumption in any state. This implies $u' > 0$, $u'' < 0$, and $u'(x) \to \infty$, if $x \to 0$. In what follows it is assumed that $\pi_1 > \pi_2$, i.e. individuals of type 1 have a lower chance of success than individuals of type 2.

If i's educational investment is not successful, his disposable income is given by:

$$y_i^F = x - e_i, \qquad i = 1, 2, \tag{2}$$

where e_i marks the educational investment of i, and x is the labor income of an unskilled individual. Hence, an individual failing in education has less income at its disposal than an individual which has not undertaken an education at all. If education is successful, disposable income is given by:

$$y_i^S = z(e_i) - e_i, \qquad i = 1, 2, \tag{3}$$

where the function $z : \mathbf{R}_+ \to \mathbf{R}_+$ indicates the return on an educational investment e_i. It is assumed that $z(0) = x$, $z' > 0$ and $z'' < 0$, i.e. marginal returns on education are positive but decreasing.

---

[7] The model could easily be extended by considering more than two outcomes of educational investments. This would not, however, affect the main results.

[8] Alternatively, we could have assumed that individuals learn their educational type when they are studying. However, as will be seen later on, what matters in our context is that individuals are better informed about their talent than other people. This, in turn, is introduced most straightforwardly by assuming that individuals perfectly know their type.

III. Education Financing

III.1 Private Education Financing

If education is financed privately an individual of type i will choose an educational investment e_i such that expected utility takes on a maximum:

$$\bar{e}_i = \operatorname{argmax}\{\pi_i\, u(x - e_i) + (1 - \pi_i)\, u(z(e_i) - e_i) : e_i \geq 0\}, \qquad i = 1, 2. \tag{4}$$

The ex ante optimal investment $\bar{e}_i$ fulfills the following first-order condition:

$$-\pi_i\, u_i'(\bar{y}_i^F) + (1 - \pi_i)(z'(\bar{e}_i) - 1)\, u_i'(\bar{y}_i^S) \leq 0, \qquad i = 1, 2, \tag{5}$$

where $\bar{y}_i^F$ and $\bar{y}_i^S$ denote disposable income in case of failure and success, respectively, if an educational investment $\bar{e}_i$ was undertaken. Assuming an interior solution[9], the first-order condition may be expressed as:

$$(1 - \pi_i) z'(\bar{e}_i) = 1 + \pi_i \frac{u_i'(\bar{y}_i^F) - u_i'(\bar{y}_i^S)}{u_i'(\bar{y}_i^S)} > 1, \qquad i = 1, 2. \tag{6}$$

Hence, the individuals do not equate expected marginal returns on education ($= (1 - \pi_i) z'(e_i)$) and marginal costs (=1), but due to risk considerations invest a lower amount in education. Straightforward manipulation of equation (6) yields:

$$\frac{1}{z'(\bar{e}_i) - 1} = \frac{(1 - \pi_i)\, u'(\bar{y}_i^S)}{\pi_i\, u'(\bar{y}_i^F)}, \qquad i = 1, 2. \tag{7}$$

The right hand side gives the marginal rate of substitution between income in case of success and failure of an individual of type i. It indicates how much income in case of failure an individual is willing to give away to receive an additional income unit in case of success. The expression on the left hand side is the marginal rate of transformation between income in case of failure and success of an individual of type i.[10] It indicates how much income an individual has to give up in case of failure to receive an additional income unit in case of success. Hence, equation (7) states that if education is financed privately, the individual marginal evaluation

---

[9] With the additional assumption $z'(e) \to \infty$ for $e \to 0$, for instance, the solution of (4) is always an interior one.

[10] After replacing e_i in (3) considering (2) one has: $y_i^S = z(x - y_i^F) - x + y_i^F$, and differentiation yields: $-dy_i^F / dy_i^S = 1/(z' - 1)$.

of income will equal individual marginal cost. As Figure 1 illustrates, individuals of type 2, i.e. individuals with a lower risk of educational failure, will undertake higher educational investments; for any (y^S, y^F) combination their marginal rate of substitution of income in case of failure for income in case of success is higher than that of type 1 individuals.[11] Hence, the indifference curves of type 2 individuals are steeper in the y^S-y^F-diagram than the indifference curves of type 1 individuals. This property is known as the single crossing property. It will prove to be very helpful in determining the public program of education financing.

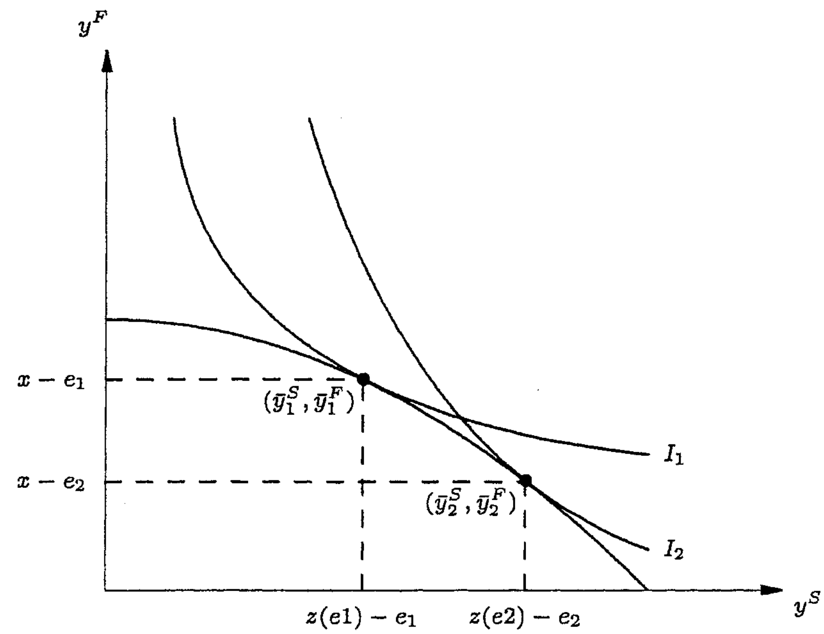

Figure 1: Private Education Financing

Before we can proceed to public education funding, we need a measure for the value individuals assign to privately financed education. It is convenient to employ the concept of the certainty equivalent. The certainty equivalent of an individual of type i is that amount of income $\hat{y}_i$ that, if received for certain, the individual regards as just as good as the expected income when it undertakes an educational investment $\bar{e}_i$. Thus, $\hat{y}_i$ is implicitly defined by:

$$u(\hat{y}_i) = \pi_i \, u(\bar{y}_i^F) + (1 - \pi_i) \, u(\bar{y}_i^S), \qquad i = 1, 2. \tag{8}$$

[11] The shape of the transformation curve in Figure 1 implies that $z'(e_i) > 1$ for all $e_i \in [0, x]$. This, however, is not essential for the results derived in this paper.

Obviously, a public education scheme has to offer an individual of type i a utility level of at least $u(\hat{y}_i)$. Otherwise, the individual will finance the education privately.

III.2 Public Education Financing

Suppose that the government encourages individuals by means of a public program to invest more in their education than they would do otherwise. This increases the average return on educational investments if there is a sufficiently large number of individuals who aim at an higher education and whose educational risks are distributed independently. It is sometimes argued that a public program of higher education financing will also be advantageous for that part of the population which does not participate in higher education, if the additional tax revenues exceed the costs of the scheme. The social (tax) return on higher education is called the *tax dividend* of a public education scheme.[12]

Following the notion of the tax dividend we assume that the objective of the policymaker is to maximize the net social return on educational investments. The net social return on educational investments is defined as the sum of individual returns on educational investments minus the cost of education and minus the disposable incomes of the individuals having undertaken an education. The public program comprises two educational packages – one for each type. An educational package is a triplet consisting of an educational investment e_i, a disposable income in case of failure y_i^F, and a disposable income in case of success y_i^S. Hence, a public program is a pair of educational packages $\{e_i, y_i^F, y_i^S\}_{i=1,2}$.

Let the number of individuals being able to participate in higher education be given and normalized to 1, and let q be the proportion of type 1 individuals. The net social return on educational investments is then given by:

$$\begin{aligned} T(e_1, e_2, y_1^F, y_2^F, y_1^S, y_2^S) = & q\,[\pi_1 x + (1-\pi_1) z(e_1)] \\ & + (1-q)\,[\pi_2 x + (1-\pi_2) z(e_2)] \\ & - q\,[e_1 + \pi_1\, y_1^F + (1-\pi_1)\, y_1^S] \\ & - (1-q)\,[e_2 + \pi_2\, y_2^F + (1-\pi_2)\, y_2^S]. \end{aligned} \tag{9}$$

[12] See, e.g., Barr (1993, chap. 13).

In what follows, we first assume that the policymaker can observe the individual types. Afterwards, we consider the more realistic case that the types constitute private information.

III.2.a Public Education Financing when Types are Observable

Even if types are observable, the policymaker does not have complete freedom in his choice. Since the individuals can always undertake a privately financed education, the public program must offer individuals at least as much utility as the private alternative. An individual of type i will only participate in the public education scheme if:

$$\pi_i \, u(y_i^F) + (1 - \pi_i) \, u(y_i^S) \geq u(\hat{y}_i), \qquad i = 1, 2, \tag{10}$$

i.e., if the program offers at least the utility of the certainty equivalent of a privately financed education. These constraints ensure voluntary participation in the public program. When types are observable, these are the only constraints the policymaker has to consider. The optimal public program, denoted by PP I, can then be formulated as:

$$\begin{aligned}
&\text{PP I}: \\
&\max_{\{e_i, y_i^F, y_i^S\}_{i=1,2}} \quad T(e_1, e_2, y_1^F, y_2^F, y_1^S, y_2^S), \\
&\qquad\qquad \text{subject to}: \\
&\qquad\qquad\qquad \pi_1 \, u(y_1^F) + (1 - \pi_1) \, u(y_1^S) \geq u(\hat{y}_1), \\
&\qquad\qquad\qquad \pi_2 \, u(y_2^F) + (1 - \pi_2) \, u(y_2^S) \geq u(\hat{y}_2).
\end{aligned}$$

The first-order conditions are given by:

$$(1-\pi_1)\, z'(e_1) - 1 = 0, \tag{11}$$

$$q - \lambda\, u'(y_1^F) = 0, \tag{12}$$

$$q - \lambda\, u'(y_1^S) = 0, \tag{13}$$

$$(1-\pi_2)\, z'(e_2) - 1 = 0, \tag{14}$$

$$(1-q) - \mu\, u'(y_2^F) = 0, \tag{15}$$

$$(1-q) - \mu\, u'(y_2^S) = 0, \tag{16}$$

where λ and μ denote non-negative Lagrange multipliers associated with the voluntary participation constraints. Straightforward manipulation yields:

$$(1-\pi_1)\, z'(e_1) = 1, \tag{17}$$

$$y_1^F = y_1^S = \hat{y}_1, \tag{18}$$

$$(1-\pi_2)\, z'(e_2) = 1, \tag{19}$$

$$y_2^F = y_2^S = \hat{y}_2. \tag{20}$$

The optimal program of education financing when types are observable is characterized by the condition that the marginal expected return of each educational investment e_i equals its marginal costs, and that each individual receives a disposable income amounting to his certainty equivalent of the privately financed alternative, irrespective of whether the educational investment is successful or not. Figure 2 illustrates the i-th educational package of PP I. An individual of type i participating at PP I attains the same utility level as if he had chosen a privately financed educational investment, indicated by the indifference curve I_i. Since the individual receives a disposable income of $\hat{y}_i$ in any case, his marginal rate of substitution is given by: $(-dy_i^F/dy_i^S)_{y_i^F=y_i^S=\hat{y}_i} = (1-\pi_i)/\pi_i$. In Figure 2, this is illustrated by the slope of the dashed line tangent to the indifference curve. The marginal rate of transformation, illustrated by the slope of the dashed line tangent to the transformation curve at point P is also given by: $-dy_i^F/dy_i^S = (1-\pi_i)/\pi_i$.[13] The distance between the two dashed lines measures the net return of the i-th educational package – horizontally in units of income in case of success and vertically in units of income in case of failure. The distance is maximized when individuals receive their certainty equivalent of privately financed education irrespective of the

[13] This follows from (17) and (19), respectively, and footnote 10.

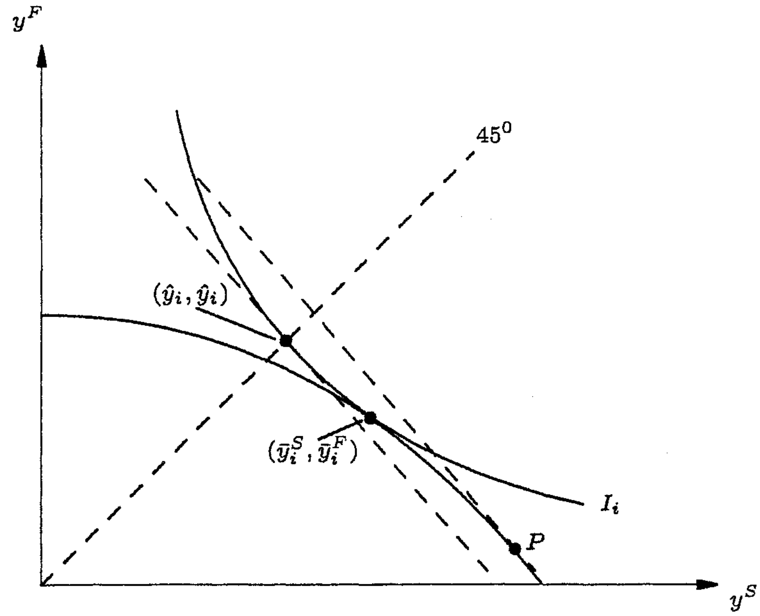

Figure 2: Optimal Public Program when Types are Observable

educational result.

III.2.b Public Education Financing when Types are Unobservable

In this section we assume that individual types are unobservable. The policy-maker knows that a proportion q of that part of the population which is capable of participating in higher education is of type 1, but he cannot observe the type of each individual separately. As a consequence, PP I is not feasible – see Figure 3. Individuals of type 1 would simply masquerade as individuals of type 2 and choose the educational package intended for individuals of type 2 to achieve the utility level $I_1' \,(> I_1)$. Thus, PP I is not incentive compatible. When types are unobservable, feasibility of a public financing program requires that no individual finds it worthwhile to choose the educational package designed for individuals of a different type. This requires:

$$\pi_i\, u(y_i^F) + (1 - \pi_i)\, u(y_i^S) \geq \pi_i\, u(y_j^F) + (1 - \pi_i)\, u(y_j^S), \quad i = 1, 2,\ j \neq i. \quad (21)$$

Figure 3 indicates that the public scheme is constrained by the requirement that individuals of type 1 do reveal their true educational characteristics. In fact,

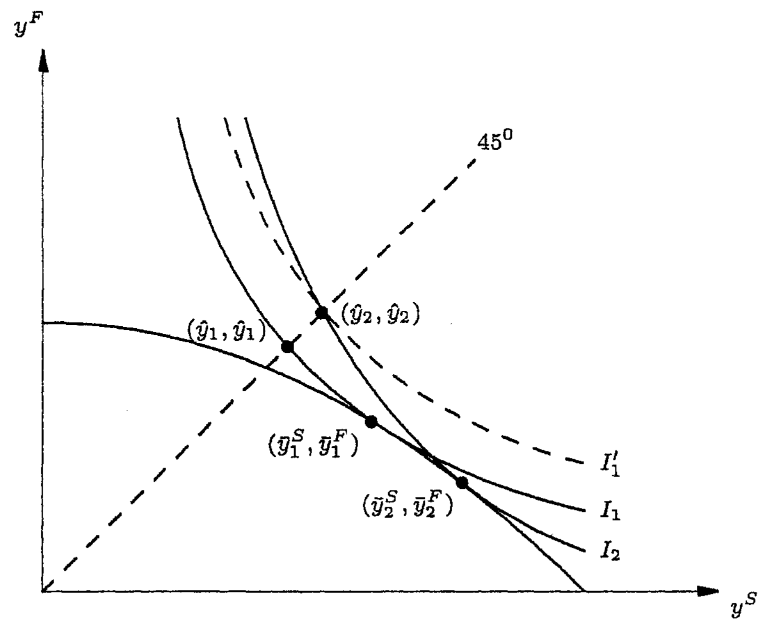

Figure 3: Infeasibility of PP I when Types are Unobservable

standard arguments show[14] that in an optimal setting only the incentive compatibility constraint in which individuals of type 1 prefer to claim the educational package intended for individuals of type 2 is binding. Thus, an optimal program implies:

$$\pi_1 u(y_1^F) + (1-\pi_1) u(y_1^S) = \pi_1 u(y_2^F) + (1-\pi_1) u(y_2^S), \tag{22}$$

$$\pi_2 u(y_2^F) + (1-\pi_2) u(y_2^S) > \pi_2 u(y_1^F) + (1-\pi_2) u(y_1^S). \tag{23}$$

On the other hand, whether one or both of the voluntary participation constraints bind is not clear a priori, since individuals of different types would undertake different private educational investments. As Figure 4 illustrates, this implies that either both constraints bind or only the one in which individuals of type 2 prefer to undertake private educational investments. Figure 4 displays two different incentive compatible public funding programs. One implies a bundle of disposable incomes given by $[(y_1^S, y_1^F),\ (y_2^S, y_2^F)]$ and the other one a bundle given by $[(y_1^{S'}, y_1^{F'}), (y_2^{S'}, y_2^{F'})]$. In the first case both voluntary participation constraints are binding, while in the second only the voluntary participation constraint of

[14] See, e.g., Kreps (1990, chap. 18).

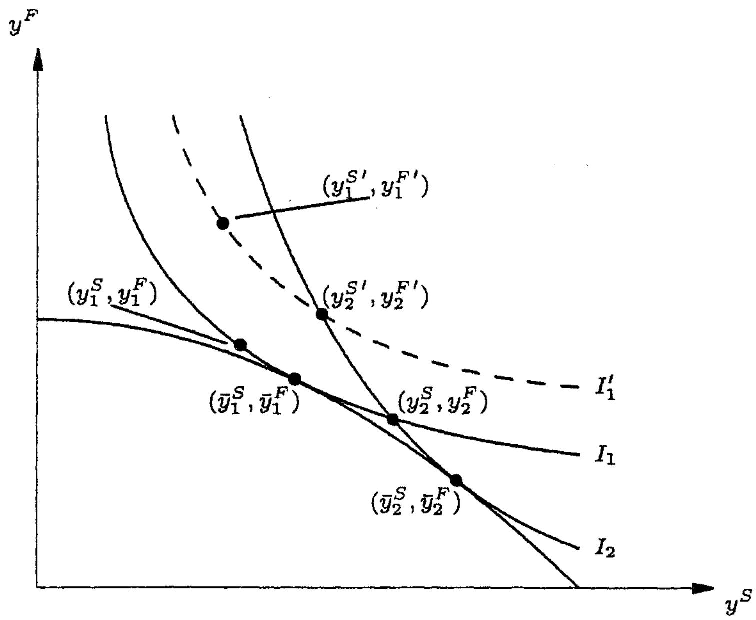

Figure 4: Binding Voluntary Participation Constraints

type 2 individuals is binding.[15]

Hence, the optimal incentive compatible public scheme, denoted by PP II, may be written as:

$$\text{PP II}:$$

$$\max_{\{e_i, y_i^F, y_i^S\}_{i=1,2}} \quad T(e_1, e_2, y_1^F, y_2^F, y_1^S, y_2^S),$$

subject to :

$$\pi_1\, u(y_1^F) + (1-\pi_1)\, u(y_1^S) \geq u(\hat{y}_1),$$
$$\pi_2\, u(y_2^F) + (1-\pi_2)\, u(y_2^S) \geq u(\hat{y}_2),$$
$$\pi_1\, u(y_1^F) + (1-\pi_1)\, u(y_1^S) = \pi_1\, u(y_2^F) + (1-\pi_1)\, u(y_2^S).$$

15 Obviously, only the voluntary participation constraint of type 2 individuals would bind if individuals of both types had only one identical private alternative.

The first-order conditions are given by:

$$(1-\pi_1)\, z'(e_1) - 1 = 0, \tag{24}$$

$$q - (\lambda + \nu)\, u'(y_1^F) = 0, \tag{25}$$

$$q - (\lambda + \nu)\, u'(y_1^S) = 0, \tag{26}$$

$$(1-\pi_2)\, z'(e_2) - 1 = 0, \tag{27}$$

$$(1-q) - \left(\mu - \nu \frac{\pi_1}{\pi_2}\right) u'(y_2^F) = 0, \tag{28}$$

$$(1-q) - \left(\mu - \nu \frac{1-\pi_1}{1-\pi_2}\right) u'(y_2^S) = 0, \tag{29}$$

where λ and μ are defined as before, and ν is a non-negative Lagrange multiplier associated with the binding incentive compatibility constraint. It follows that:

$$(1-\pi_1)\, z'(e_1) = 1, \tag{30}$$

$$y_1^F = y_1^S \geq \hat{y}_1, \tag{31}$$

$$(1-\pi_2)\, z'(e_2) = 1, \tag{32}$$

$$y_2^F < \hat{y}_2 < y_2^S. \tag{33}$$

Again, the optimal public program is characterized by an equalization of marginal returns and marginal costs of educational investments for both types. The information of the policymaker, whether full or incomplete, does not affect the optimal level of public educational investments. Furthermore, as in the case of full information, the public program should take over the whole educational risk of individuals of type 1. Whether they receive only their certainty equivalent or more depends on whether their participation constraint is binding or not. Individuals of type 2, on the other hand, should still face some of their educational risk. The intuition behind this result is as follows. Participation in educational risk is less attractive for type 1 than for type 2 individuals. If type 2 individuals face some educational risk this restrains type 1 individuals from masquerading and from choosing the educational package intended for type 2 individuals. The net social return on educational investments of PP II is less than that of PP I, even if both participation constraints are binding.[16] If type 2 individuals are given a share in their educational risk, their average disposable income has to exceed their certainty

[16] Note, however, that the net social return is non-negative. A public program of the form $\{\bar{e}_i, \bar{y}_i^F, \bar{y}_i^S\}_{i=1,2}$, i.e. a public program which simulates private educational financing, is incentive compatible and implies a net social return equal to zero.

equivalent. Otherwise, due to risk aversion, they would not participate in the public program.

To summarize, the optimal public financing scheme is characterized by a simple condition: marginal returns on educational investments should equal marginal costs. This condition, however, is *not* attained by means of private financing. *The government has to take over a part of the individual educational risk* to meet this condition. The design of the public program depends on the information the government has about individual educational risk. *If the government can observe individual educational risk, it should take all of it. If the government cannot observe individual educational risk, more expensive education should include higher individual risk participation.*

It should be noted, however, that a public takeover of educational risks may reduce the incentive for individuals to make an effort to be successful. Faced with this kind of *moral hazard*, a public program of education financing should not only ensure that there remains sufficient individual risk leading to correct individual education choices, but also that there is still an incentive to study towards success.

IV. Initial Endowments and Productivity Growth

Hitherto, we have not considered the financial background of the individuals and its potential impact on private educational investments. It is conceivable that a privately undertaken educational investment depends on whether it has to be financed solely out of success-dependent income or whether the individual can fall back on additional financial resources. Furthermore, the absolute amount of disposable income may also affect private educational investments. This is important since, due to productivity growth, incomes of skilled and unskilled individuals have risen continuously during recent decades.

In this section we consider the impact of both initial resource endowments and productivity growth on private educational investment. We start by considering the effect of increasing initial resources. For simplicity we assume that there is only one type of individual, characterized by a probability of educational failure π. Let a be an initial endowment. The state dependent budget constraints are then given by:

$$y^F = a + x - e, \tag{34}$$

$$y^S = a + z(e) - e; \tag{35}$$

where y^F and y^S now denote disposable resources rather than incomes in case of failure and success, respectively. The privately undertaken amount of educational investments is determined by:

$$\bar{e} = \operatorname{argmax}\{\pi\, u(a + x - e) + (1 - \pi)\, u(a + z(e) - e) : e \geq 0\}. \tag{36}$$

Assuming an interior solution, the first-order condition is given by:

$$\Omega(\bar{e}, a) = -\pi\, u'(a + x - \bar{e}) + (1 - \pi)(z'(\bar{e}) - 1)\, u'(a + z(\bar{e}) - \bar{e}) = 0. \tag{37}$$

This condition implicitly defines a function $\bar{e} = \bar{e}(a)$, which describes the relationship between the optimal amount of privately financed educational investments and the initial endowment. Employing the implicit function theorem, it follows:

$$\frac{d\bar{e}}{da} = -\frac{-\pi u''(a + x - \bar{e}) + (1 - \pi)(z'(\bar{e}) - 1)u''(a + z(\bar{e}) - \bar{e})}{\partial\Omega/\partial\bar{e}}. \tag{38}$$

Elimination of $z'(\bar{e}) - 1$ by making use of (37) yields:

$$\frac{d\bar{e}}{da} = [A(a + x - \bar{e}) - A(a + z(\bar{e}) - \bar{e})]\, \underbrace{\frac{\pi u'(a + x - \bar{e})}{-\partial\Omega/\partial\bar{e}}}_{>0}, \tag{39}$$

with

$$A(y) = -\frac{u''(y)}{u'(y)},$$

where $A(y)$ is the Arrow-Pratt measure of *absolute* risk aversion. Since $\bar{e}$ is the optimal amount of private educational investments, $\partial\Omega/\partial\bar{e}$ has to be negative. Hence, the second factor in (39) is positive, so that:

$$\frac{d\bar{e}}{da} \gtreqless 0 \quad - \quad A(a + x - \bar{e}) \gtreqless A(a + z(\bar{e}) - \bar{e}). \tag{40}$$

Since $x < z(e)$ for all $e > 0$, the amount of private educational investments is increasing in initial endowment if absolute risk aversion is decreasing. Considering the discussion of the previous section, decreasing absolute risk aversion implies that public education financing is the more important the less individuals can fall back on sufficient initial resources. In view of the standard argument that educational policies should facilitate access to higher education for the poor, this result gains in significance. In fact, it implies that *the amount of educational investments* *undertaken by the poor differs most markedly from the socially optimal level.* In this way the result provides a link between the distribution of initial resources and the net social return of public education financing.

When an increase in initial resource endowments leads to higher private educational investments, one may ask, whether a rise in disposable incomes due to productivity growth affects private investments in a similar way. If a in equations (34) and (35) is interpreted as an income component depending on productivity, one could conclude that private educational investments go up due to productivity growth, and a public program of education financing would cease to be necessary in course of time. However, such an interpretation would, among other things, imply that relative educational costs decrease over time. If one observes that educational costs *increase* over time, such an analogy is less obvious. Suppose that educational costs are governed by the same trend as incomes.[17] The state dependent budget constraints may then be written as:

$$y^F = \tau \cdot (x - e), \tag{41}$$

$$y^S = \tau \cdot (z(e) - e), \tag{42}$$

where the parameter τ measures productivity at a certain point in time. The optimal amount of privately financed educational investments takes the form:

$$\bar{e} = \mathrm{argmax}\{\pi\, u(\tau(x - e)) + (1 - \pi)\, u(\tau(z(e) - e)) : e \geq 0\}, \tag{43}$$

and the first-order condition is given by:

$$\Omega(\bar{e}, \tau) = -\pi\, u'(\tau(x - \bar{e})) + (1 - \pi)(z'(\bar{e}) - 1)\, u'(\tau(z(\bar{e}) - \bar{e})) = 0. \tag{44}$$

Proceeding in the same way as before we have:

$$\frac{d\bar{e}}{d\tau} \gtreqless 0 \quad - \quad R(\tau(x - \bar{e})) \gtreqless R(\tau(z(\bar{e}) - \bar{e})), \tag{45}$$

where

$$R(y) = -\frac{u''(y)}{u'(y)} y$$

is the Arrow-Pratt measure of *relative* risk aversion. An increase in productivity

[17] This may be the case, for instance, if educational costs are linked to the employment of teachers.

leads to higher privately financed educational investments if relative risk aversion is decreasing. The condition for productivity growth leading to higher private educational investments is thus much more restrictive than the condition for increasing resource endowments causing the same effect. Taking the results of the previous section into account, it has to be doubted that productivity growth may lead to privately financed educational investments at the socially optimal level.

V. Summary and Conclusion

This paper has dealt with private educational investments under risk. It has derived an economic role for the state concerning education financing. In our framework the social merits of public education schemes are related to a non-existence of markets in which students can insure against educational risks. Hence, a public program of education financing should contain elements of risk insurance. This could be realized, for instance, by success-dependent tuition fees. A complete public takeover of all educational risks cannot, however, be recommended. An individual participation in educational risk should rather be oriented at educational costs: Cost-intensive courses should require more individual risk participation. This would ensure that only individuals with adequate educational abilities undertake costly education.

The role of public education financing also depends on the disposability of individual incomes and wealth. If absolute risk aversion is a decreasing function of income, the role of a public scheme becomes less important when educational risks are taken anyhow to a socially desirable extent due to a sufficient financial background. In this respect, the frequently used argument that public educational programs serve to facilitate access to higher education for members of low-income groups gains in importance. As long as absolute risk aversion decreases, it has to be expected that members of low-income groups will invest less in their education than members of groups with higher incomes, even if they have the same educational abilities.

A general rise in incomes caused by productivity growth may also reduce the public role of education financing, but this would require a distinctly higher income effect on risk aversion. Insofar, the view that economic growth could lead to a stimulation of private demand for education has to be met with caution.

References

Aghion, P. and **Howitt, P.** (1992): "A Model of Growth Through Creative Destruction"; *Econometrica*, 60, 323-351.

Barham, V., **Boadway, R.**, **Marchand, M.** and **Pestieau, P.** (1995): "Education and the Poverty Trap"; *European Economic Review*, 39, 1257-1275.

Barr, N. (1993): *The Economics of the Welfare State*; second edition, Oxford: Oxford University Press.

Barro, R.E. and **Sala-i-Martin, X.** (1995): *Economic Growth*; New York: McGraw-Hill.

Becker, G. (1964): *Human Capital – A Theoretical and Empirical Analysis, with Special Reference to Education*; New York: Columbia University Press.

Chapman, B. (1997): "Conceptual Issues and the Australian Experience with Income Contingent Charges for Higher Education"; *Economic Journal*, 107, 738-751.

Fernandez, R. and **Rogerson, R.** (1995): "On the Political Economy of Education Subsidies"; *Review of Economic Studies*, 62, 249-262.

Galor, O. and **Zeira, J.** (1993): "Income Distribution and Macroeconomics"; *Review of Economic Studies*, 60, 35-52.

Grossman, G.M. and **Helpman, E.** (1991a): "Quality Ladders in the Theory of Growth"; *Review of Economic Studies*, 58, 43-61.

Grossman, G.M. and **Helpman, E.** (1991b): "Trade, Knowledge Spillovers, and Growth"; *European Economic Review*, 35, 517-526.

Johnson, G.E. (1984): "Subsidies for Higher Education"; *Journal of Labor Economics*, 2, 303-318.

Jones, L.E. and **Manuelli, R.** (1990): "A Convex Model of Equilibrium Growth: Theory and Policy Implications"; *Journal of Political Economy*, 98, 1008-1038.

Kreps, D.M. (1990): *A Course in Microeconomic Theory*; New York: Harvester Wheatsheaf.

Levhari, D. and **Weiss, Y.** (1974): "The Effect of Risk on the Investment in Human Capital"; *American Economic Review*, 64, 950-963.

Lott, J.R.Jr. (1987): "Why is Education Publicly Provided? A Critical Survey"; *Cato Journal*, 7, 475-501.

Lucas, R.E. (1988): "On the Mechanics of Economic Development"; *Journal of Monetary Economics*, 22, 3-42.

Lüdeke, R. (1996): "Gesellschaftliche Erträge als Rechtfertigung der heutigen Bildungs- und Hochschulfinanzierung? Die Zerstörung einer Illusion"; Contribution to the Second Hochschulforum, Kiel/Kronshagen, May 8, 1996.

Miller, P. and **Volker, P.** (1993): "Youth Wages, Risk, and Tertiary Finance Arrangements"; *Economic Record*, 69, 20-33.

Peltzman, S. (1973): "The Effect of Government Subsides-in-Kind on Private Expenditures: The Case of Higher Education"; *Journal of Political Economy*, 81, 1-27.

Perotti, R. (1993): "Political Equilibrium, Income Distribution, and Growth"; *Review of Economic Studies*, 60, 755-776.

Perotti, R. (1994): "Income Distribution and Investment"; *European Economic Review*, 38, 827-835.

Rebelo, S. (1991): "Long-Run Policy Analysis and Long Run Growth"; *Journal of Political Economy*, 99, 500-521.

Romer, P.M. (1986): "Increasing Returns and Long-Run Growth"; *Journal of Political Economy*, 94, 1002-1037.

Romer, P.M. (1990): "Endogenous Technological Change"; *Journal of Political Economy*, 94, 71-102.

Rosen, H.S. (1995): *Public Finance*; 4. Auflage, Homewood, Ill.: Irwin.

Segerstrom, P.S., **Anant, T.C.A.** and **Dinopoulos, E.** (1990): "A Schumpeterian Model of the Product Life Cycle"; *American Economic Review*, 80, 1077-1091.

Trostel, P.A. (1993): "The Effect of Taxation on Human Capital"; *Journal of Political Economy*, 101, 327-350.

Trostel, P.A. (1996): "Should Education be Subsidized?"; *Public Finance Quarterly*, 24, 3-24.