**IMF Working Paper**





WP/99/107

INTERNATIONAL MONETARY FUND

European I Department

**Analysis of the U.S. Business Cycle with a Vector-Markov-Switching Model**

Prepared by Zenon G. Kontolemis[1]

Authorized for distribution by Sharmini Coorey

August 1999

**Abstract**

This paper identifies turning points for the U.S. business cycle using different time series. The model, a multivariate Markov-Swiching model, assumes that each series is characterized by a mixture of two normal distributions (a high and low mean) with switching determined by a common Markov process. The procedure is applied to the series that make up the composite U.S. coincident indicator to obtain business cycle turning points. The business cycle chronology is closer to the NBER reference cycle than the turning points obtained from the individual series using a univariate model. The model is also used to forecast the series, with encouraging results.

JEL Classification Numbers: B41, E32

Keywords: Business Cycle, Regime Switching, NBER Business Cycle Chronology

Author's E-Mail Address: ZKontolemis@imf.org

---

[1] When work on this project began, the author was a research Fellow at the European University Institute in Florence. The work has benefited from comments by Mike Artis, Wenda Zhang, Fabio Canova, and Denise Osborn. Thanks are also due to Hans Jessen for providing the data.

Contents

Page

I. Introduction .......... 3

II. Dating of U.S. 'Business Cycle' .......... 4

III. Forecasting .......... 13

IV. Conclusions .......... 15

References .......... 17

Tables
1. Maximum Likelihood Estimates of Independent Markov Processes .......... 7
2. Turning Points for Individual Coincident Series .......... 9
3. Turning Points for Model Estimated 1961:1-1995:1 .......... 9
4. Vector Estimates .......... 11
5. Alternative Dating of U.S. 'Business" Cycle .......... 12
6. Estimated for Sub-sample 1961:1-1995:1 .......... 12
7. In-Sample Mean Square Forecast Errors .......... 14
8. Post-Sample Mean Square Forecast Errors .......... 14

Figures
1. Probability That Economy Remains in Expansion .......... 18
2. Probability That Economy Remains in Expansion – AR(1) Model .......... 19

## I. INTRODUCTION

In a seminal paper Hamilton(1989) proposed a methodology for analyzing business cycles based on a time series model characterized by discrete changes in regimes. Analysis of the U.S. business cycle showed that this model can make inferences about the state of the economy which are extremely close to the NBER turning points and produce reliable estimates for the growth rates associated with expansions and contractions. According to the estimated equation a typical recession is characterized by a 3 percent drop in the level of GNP. In these type of models the basic task is, having described explicitly the probability law which governs these shifts, to determine when these shifts occur and to estimate the model parameters and the state transition probabilities. The driving force is the probability law which governs shifts from one regime to another. The regime at any given time is presumed to be the outcome of a Markov chain whose realizations are unobserved.

The Markov-Switching model of Hamilton (1989) generalizes in a straightforward way to a vector process (VMS). The latter is useful for analyzing a number of interesting hypotheses but has not been used extensively in the literature. Engle and Hamilton(1990) develop a statistical model of exchange rate dynamics as a sequence of stochastic, segmented time trends and test a number of interesting hypotheses. They report however that extending their analysis, by fitting a number of currencies driven by a single scalar state variable, does not provide them with interesting results. Other applications include Phillips(1991), who applies an augmented-Hamilton model to study international business cycles, and Ravn and Sola(1995) who use a similar to Phillip's model to investigate the correlation between output growth and inflation.

In the context of business cycles an interesting application for a vector type process with regime switching is for dating business cycles. Recent work on dating business cycles mainly discussed issues relating to the sensitivity of turning point dates to different de-trending methods and dating procedures (see Canova, 1995, Boldin, 1994, and references therein) the issue of business versus growth cycle turning points (see Artis, Kontolemis and Osborn, 1997, for example). Most of these recent studies have concentrated on obtaining information from a single series, or, constructing a composite index representing the "overall" activity (see the discussion in Canova, 1995, for example). Stock and Watson (1991) use the dynamic factor model to derive a coincident indicator for the U.S. business cycle as the unobserved variable common to a number of time series. However, their focus is not on the identification of turning points but rather on the derivation of the common unobserved variable which they report as being very similar to the DOC coincident index. Instead, the methodology proposed by Hamilton allows us to focus explicitly on turning points by assuming that the unobserved state is driven by a Markov switching process. A direct generalization of the model of Stock and Watson (1991) into a multivariate Markov switching model is the work by Kim and Yoo (1995).

This paper applies the simple multivariate version of the model used by Engle and Hamilton(1990) to the (four) time series composing the composite coincident indicator in the United States in order to identify the turning points for the U.S. business cycle. According to this model, there are two states, a high mean and low mean period, and transitions from the one state to the other are described by a Markov-chain. In identifying turning points for the "overall" U.S. business cycle a common probability transition mechanism is applied to all series. This is consistent with the practice at the NBER of analyzing several series in order to identify a single "overall" business cycle chronology. Implicitly what is assumed in the NBER methodology is a common underlying cycle. Hence, a regime switching model which encompasses information from these different series and makes inference about the state of the economy is a mechanical interpretation of the NBER methodology. The main difference with Stock and Watson(1991) is the restriction, imposed here, of identical turning points across the different coincident series/sectors of the economy which stems from the common probability law governing jumps from one state to the other. Consequently, by construction this model is a restricted version of the more general specification proposed by Phillips (1991) where each series may be allowed to switch from one state to the other independently or with a lag and where all different combinations of high/low (contraction/expansion) are taken into account.

As in the work of Kim and Yoo (1995) this paper identifies a business cycle chronology which is closer to the NBER reference cycle than the univariate model. In contrast to Kim and Yoo (1995), where an approximate ML method is used to estimate their model, this paper uses the EM algorithm proposed by Hamilton(1990). The VMS model is also used to forecast the level of the (log) of the four coincident series. In general, the VMS model produces more accurate forecasts and these are obtained with considerable ease, compared to the univariate specification.

This paper is organized as follows: Section II describes the model used and the results from the univariate and multivariate estimation. Section III discusses the forecasting performance of the models. Finally, in the last section we present the conclusions.

## II. Dating of U.S. Business Cycle

In order to proceed we must provide the appropriate definition of the business cycle. This definition has been used informally for many years in the United States and has formed the basis for research in the area of business cycles. Burns and Mitchel(1946), henceforth BM, in their pioneering study of business cycles defined business cycles as:

> *"Business Cycles are a type of fluctuation found in the aggregate economic activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle; the sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own."*

The BM definition makes clear that the "business cycle" refers to an aggregate quantity representing all sectors in the economy. Hence the practice at the NBER to use information from several sectors of the economy to decide which dates constitute the turning points of the cycle. Implicit in the BM definition and the NBER methodology is an important assumption: that there exists an unobserved, common to all sectors, state variable which we call the "business cycle". Although some series may lead or lag the other they all share a relationship with a common variable, the business cycle which is unobserved.

This interpretation of the NBER methodology is the basis of the simple application of this paper. The four series which make up the U.S. coincident index, which is regarded as a good approximation of business cycles in the United States, are used to make inferences for the unobserved variable which should, in theory at least, approximate the U.S. "business cycle". The method used assumes that each series is described by a mixture of two normal distributions but the transition to each distribution over time is common to all four variables. This is reasonable for the series used in this study which are all coincident indicators with the business cycle. A richer formulation, with leading indicators, would have required a more general definition perhaps with more dynamics or in the line of the model by Phillips(1991).

There exists, therefore, an unobserved variable $s_t$ that characterizes the state of the process at time $t$. In the more general case there can be $K$ possible regimes although for our purposes $K = 2$ and $s_t$ takes on the value one and two. When $s_t = 1$ the economy is in the upturn and the observed growth of output is presumed to have been drawn from a $N(\mu_1, \sigma_1^2)$ distribution, while when the $s_t = 2$ the economy is in the downturn and this is described by an $N(\mu_2, \sigma_2^2)$ distribution. We begin the analysis with a model which has no autoregressive dynamics but is characterized by varying means and variances across states (a similar model is used in Engel and Hamilton, 1990, for example):

$$\begin{aligned} y_t &\mid s_t \sim N(\mu_{s_t}, \Omega_{s_t}) \\ s_t &= 1, 2 \end{aligned}$$

The conditional distribution of $y_t$ will then be given by

$$p(y_t \mid z_t) = \frac{1}{[2\pi]^{n/2} |\Omega_{s_t}|^{1/2}} \exp\left[\frac{-(y_t - \mu_{s_t})'\Omega_{s_t}^{-1}(y_t - \mu_{s_t})}{2}\right] \tag{1}$$

where $z_t = (s_t, s_{t-1}, ..., s_1, y_{t-1}, y_{t-2}, ...y_1)'$ and $y_t, \mu_{s_t}$ are $m \times 1$ vectors while $\Omega_{s_t}$an $m \times m$ variance-covariance matrix. Within this setup it is fairly straightforward to allow for autoregressive terms. In fact, one can write a more general conditional density which encompasses the above mixture of normals model as a special case[1]. We will briefly comment on the performance of this model later in the paper. The transition between the states follows a first-order Markov chain such that

$$p(s_t = j \mid s_{t-1} = i) = p_{ij} \tag{3}$$

and $\sum_{j=1}^{k} p_{ij} = 1 \; \forall i$. Let $p = (p_{11}, p_{11}, ...p_{KK})'$ be the $(K^2 \times 1)$ vector of Markov transition probabilities. Thus the parameters $\theta = (\mu_{s_t}, \Omega_{s_t}, p)$ for $s_t = 1, ..., K$ describe the probability law for $y_t$ . Since only $y_t$ is observed but not the state $s_t$ the task is to maximize the likelihood function of the observed data $p(y_T, y_{T-1}, ..., y_1; \theta)$ by choosing the parameters $\theta$ and make inferences about the state of the economy. The sample likelihood function is given by

[1]For example the conditional distribution of $y_t$ can be written as:

$$p(y_t \mid z_t) = \frac{1}{[2\pi]^{n/2} |\Omega_{s_t}|^{1/2}} \exp\left[\frac{-(y_t - x_t'\beta_{s_t})'\Omega_{s_t}^{-1}(y_t - x_t'\beta_{s_t})}{2}\right] \tag{2}$$

where $y_t, x_t, \beta_{s_t}$ are $m \times 1$ vectors while $\Omega_{s_t}$an $m \times m$ variance-covariance matrix.

$$p(y_T, y_{T-1}, ..., y_1; \theta) = \sum_{s_1}^{K} ... \sum_{s_T}^{K} p(y_1, ..., y_T, s_1, ..., s_T; \theta) \quad (4)$$

where the joint distribution of states and observations is given by

$$\begin{aligned} p(y_1, ..., y_T, s_1, ..., s_T; \theta) &= p(y_T \mid s_T; \theta) p(s_T \mid s_{T-1}; \theta) \\ p(y_{T-1} &\mid s_{T-1}; \theta) p(s_{T-1} \mid s_{T-2}; \theta) ... \\ p(y_1 &\mid s_1; \theta) p(s_2 \mid s_1; \theta) p(s_1; \theta) \end{aligned} \quad (5)$$

and $\lambda = (p', \theta', \rho')'$ maximize the likelihood function. Also $p(s_1; \theta) = \widehat{\rho} = \frac{(1-p_{22})}{(1-p_{11})(1-p_{22})}$ is the value of the ergodic probability $p(s_1)$ which simply indicates the unconditional probability of state 1. For the analysis that follows smoothed inferences about the unobserved states are obtained. These are estimates for the probability that $s_t$ took on some value at time $t$ based on observation of $y$ over the entire sample

$$p(s_t \mid y_T, y_{T-1}, ..., y_1; \lambda)$$

The smoothed probabilities can be estimated using the joint probability distribution of observations and states and the probability density of $y_t$ conditional on the states (for details see Hamilton, 1990, for example). Using the smoothed probabilities we can evaluate the sample likelihood and generate estimates for the unknown parameters. The unknown parameters are estimated using the EM algorithm (see Hamilton, 1990, for example). This entails derivation of analytical estimates for the unknown parameters which involve only the smoothed inferences and a simple iterative procedure which provides maximum likelihood estimates. The model is a simple mixture of two normal distributions with Markovian transitions from one state to the other (see Everitt and Hand, 1981, and Titterington et al, 1985, for surveys on mixture of distributions).

The four series used in the construction of the coincident index are the index of industrial production, non-agricultural employment, personal income (less transfer payments) and manufacturing and trade sales. The data is monthly for the period 1948:1-1995:1. Table 1 shows the estimated parameters for the models which assume independent Markov processes — these are the univariate models. The top panel of Table 1 shows the estimated model for the whole period while the lower panel provides estimates for a shorter period which covers 1961:1-1995:1. The numbers in the parentheses are the standard errors and the estimated means denote monthly (log) changes. Notice that although the differences between the estimated means in regime 1 and 2 are rather small, insignificant is some cases, the variances vary considerably across regimes. This finding, also reported in Lahiri and Wang (1994), is consistent with French and Sichel (1993) who report asymmetries in the variance of economic activity. The variance of output is higher after negative shock and it appears to be largest around business cycle troughs. However, the variance differences between the two regimes are less marked if the model is estimated after 1961 (lower panel of Table 1). A plot of these series reveals that for industrial production, sales and employment there are visible differences in the variance before and after 1960-61. Estimating the model for the shorter sample results in smaller differences in the variances across the regimes while it emphasizes more the variation in the means.[2]

[2]Imposing a common variance across the regimes makes the differences between the means more pronounced. However, these models do not provide sensible forecasts and are not discussed in

Table 1: Maximum Likelihood Estimates of Independent Markov Processes

| | | 1948:1-1995:1 | | |
|---|---|---|---|---|
| | IIP | EMP | INC | SAL |
| $\mu_1$ | 0.39(0.04) | 0.25(0.01) | 0.34(0.02) | 0.30(0.05) |
| $\mu_2$ | 0.07(0.14) | 0.02(0.03) | 0.21(0.03) | 0.13(0.26) |
| $p_{11}$ | 0.95(0.02) | 0.96(0.01) | 0.98(0.01) | 0.95(0.02) |
| $p_{22}$ | 0.90(0.04) | 0.94(0.02) | 0.99(0.01) | 0.79(0.07) |
| $\sigma_1$ | 0.37(0.06) | 0.03(0.00) | 0.10(0.01) | 0.85(0.09) |
| $\sigma_2$ | 2.87(0.48) | 0.22(0.02) | 0.42(0.03) | 5.13(1.06) |
| Lik | -239.2 | 471.1 | 79.58 | -373.6 |
| | | 1961:1-1995:1 | | |
| | IIP | EMP | INC | SAL |
| $\mu_1$ | 0.46(0.04) | 0.25(0.01) | 0.35(0.02) | 0.68(0.07) |
| $\mu_2$ | -0.48(0.21) | -0.08(0.04) | 0.15(0.03) | -0.00(0.19) |
| $p_{11}$ | 0.96(0.01) | 0.97(0.00) | 0.98(0.00) | 0.20(0.64) |
| $p_{22}$ | 0.84(0.06) | 0.90(0.03) | 0.99(0.00) | 0.40(0.07) |
| $\sigma_1$ | 0.36(0.03) | 0.02(0.00) | 0.10(0.01) | 1.02(0.21) |
| $\sigma_2$ | 1.38(0.30) | 0.07(0.01) | 0.32(0.03) | 0.95(0.17) |
| Lik | -79.3 | 461.2 | 119.5 | -222.7 |

Notes: IIP - index of industrial production, EMP - non-agricultural employment, INC - personal income less transfers, SAL - manufacturing and trade sales.

Thus industrial production, grows by approximately $\frac{1}{2}$ percent per month during the average expansion while it declines on average by about $\frac{1}{2}$ percent each month during recession (the mean for the recession estimated over the whole sample period implies that industrial production remains more or less flat during downturns). This is comparable, although slightly lower, with the average monthly decline in industrial production during recessions estimated by Artis, Kontolemis and Osborn(1997) to be 0.76 percent. For industrial production the probability that a downturn will continue is 0.84, so this state persists for about 6 months.[3] Table 1 shows the estimates for the other three series. Specification tests indicate that although the equations for personal income and sales are correctly specified with no error autocorrelation (in either of the two states or across regimes) that is not true for industrial production and employment – for details on specification tests for these models Hamilton(1996) provides a good description. We proceed with these models since the alternatives, as we will discuss later on, are not useful for analysis of business cycle monthly data.

Tables 2 and 3 show the NBER turning points for the U.S. business cycle and those for the individual components of the composite index. Table 3 is based on the estimated model over 1961:1-1995:1. The rule for dating the business cycle is based on whether the economy is more likely than not to stay in one of the two phases. For example, $s_t = 1$ if $P(s_t = 1) > a$ and $y_t$ is classified as having come from the upturn. We set $a = 0.5$ initially, though we discuss the sensitivity of the results to this value later. This value has also been used by Hamilton(1989), for example. For the purpose of this exercise the conclusions are not changed if other values of this parameter are used. In addition, we impose a minimum duration (md) requirement which is essential in eliminating spurious cycles in the monthly series we are using. We require that each phase has a minimum duration of 6 months, which corresponds to the widely used definition of a cycle with a minimum duration of two quarters. Different definitions are used later in the paper for comparison purposes.

The four series exhibit different cycles at different times. First note that personal income and sales display few cycles compared with industrial production and employment. Looking at this information it is hard to decide on an overall classification of the business cycle. Very few turning points are common across all the series and there appear to be a number of idiosyncratic cycles experienced by one or, at most, two component series. Additionally, the procedure comes up with "spurious cycles" in a number of cases and fails to identify important cyclical fluctuations. The case of industrial production provides one example. The classification in Table 2 shows a peak in 1956M3 to be followed by a trough in 1961M6. The latter however takes a lower value than the former, both are therefore excluded from the list of "tentative" turning points, by definition. This is of course due to the failure of the model for industrial production to classify the decline from the late 1957 until early 1958 as a downturn. Table 3 shows that the ability to identify turning points improves somewhat when inference is based on the shorter sample. In previous studies Markov-Switching models have been used to date business cycles (see Hamilton, 1989, Goodwin, 1993, for example). These papers use quarterly data and the identification of turning points is therefore more straightforward.

---

this paper.

[3]This is equal to $1/(1 - p_{11}) = 6.2$.

Table 2: Turning Points for Individual Coincident Series

| | NBER | IIP | EMP | INC | SAL |
|---|---|---|---|---|---|
| P | | 48M6 | | | |
| T | | 50M9 | | | |
| P | | 52M2 | | | |
| T | | 52M10 | | | |
| P | 53M7 | 53M5 | | | |
| T | 54M5 | 54M1 | 54M1 | | |
| P | | 54M9 | | | |
| T | | 55M4 | | | |
| P | | 56M3 | 56M5 | | |
| T | | | | | |
| P | 57M8 | | | | |
| T | 58M4 | | | | |
| P | 60M4 | | | | |
| T | 61M2 | 61M6 | 61M3 | 60M12 | |
| P | | | | | |
| T | | | | | |
| P | 69M12 | | 69M11 | | |
| T | 70M11 | | 70M1 | | |
| P | 73M11 | | | 72M3 | |
| T | | | | | |
| P | | 74M8 | 74M6 | | 74M7 |
| T | 75M3 | 75M3 | 75M5 | 75M3 | 75M3 |
| P | 80M1 | | | 79M11 | |
| T | 80M7 | | | | |
| P | 81M7 | 81M7 | 81M5 | | |
| T | 82M11 | 82M12 | 83M8 | | |
| P | | | | | |
| T | | | | | |
| P | 90M7 | | 90M5 | | |
| T | 91M3 | | 91M12 | | |

Notes: $P(S_1 = 1) < 0.5$ determines the dates, minimum duration of regime 6 months.

Table 3: Turning Points for Model Estimated over 1961:1-1995:1

| | NBER | IIP | EMP | INC | SAL |
|---|---|---|---|---|---|
| P | 69M12 | 69M9 | 69M11 | | 66M9 |
| T | 70M11 | 70M11 | 71M1 | | |
| P | 73M11 | 74M5 | 74M5 | 72M3 | |
| T | 75M3 | 75M2 | 75M5 | 75M3 | |
| P | 80M1 | | | 79M10 | |
| T | 80M7 | | | | |
| P | 81M7 | 81M7 | 81M2 | | |
| T | 82M11 | 82M12 | 83M8 | | |
| P | | | | | |
| T | | | | | |
| P | 90M7 | 90M8 | 90M4 | | |
| T | 91M3 | 91M2 | 92M2 | | |

Notes: $P(S_1 = 1) < 0.5$ determines the dates, minimum duration of regime 6 months.

Instead of assuming four independent Markov processes we estimate a VMS model. The estimated parameters are shown in Table 4 with the lower panel showing the estimates over the shorter sample as elsewhere. Both industrial production and sales display the largest variance over both the high mean and low mean distributions. They also seem to be correlated more than the other two series[4]. The estimated means and variances are sensible and extremely close to the ones shown in Table 1 although the standard errors are somewhat higher. As it is the case for the estimated univariate models, here too, the estimates based on the restricted sample imply smaller differences between the variances across regimes and larger differences between the means. Although the expected duration for expansions is 25 months, based on full sample estimates, it is estimated to be 33 months based on the restricted sample estimates which is close to the estimate by Hamilton(1989) of 32 months. These are more realistic than the estimated probabilities shown in Table 1.

Figure 1 shows the probability that the economy remains in expansion, obtained from the VMS model of Table 4 (estimated over the whole sample). The shaded areas represent the NBER downturns. With the exception of the recession in the beginning of the 1970s (1969:12-70:11) the estimated probabilities track relatively well the NBER downturns. Table 5 shows alternative business cycle turning points obtained for different values of $\alpha$ and the minimum duration of regimes (md) based on the estimates over the whole sample. We comment on two sets of results, those for $(md = 6, \alpha = 0.5)$ and $(md = 6, \alpha = 0.9)$. Both identify turning points which are close to the NBER cycles. For $\alpha = 0.5$ the model does not identify a downturn from 1957:6-1958:4 or the subsequent expansion until 1960:4 considered by the NBER as a cycle. Nor does it consider the short cycle 1980:1-1980:7 as a business cycle. Instead, it identifies a longer recession from 1981:8-1983:8 to have taken place at the beginning of the 1980s. The later finding is consistent with other findings which have found the second of the two dips in the early 1980s to be the more significant (see Artis, Kontolemis and Osborn, 1997, for example). Similar cycles are identified when we require $\alpha = 0.9$, though the resulting turning points seem to be closer to the NBER ones. A notable difference is the trough of the 1981-82 cycle which is identified more accurately when $\alpha = 0.9$. Table 6 shows the turning points obtained when the model is estimated over the shorter sample size. The turning points obtained are remarkably close to the NBER reference cycle in particular those obtained when $(md = 3, \alpha = 0.9)$. In this case the model classifies 1980:2-1980:6 as a short recession consistent with the NBER reference cycle.

As explained in the earlier section this model can be generalized to a vector autoregressive switching model. Figure 2 shows the probability that the economy is in the expansionary phase obtained from the model with one autoregressive term. The model with the autoregressive terms fails to track the NBER reference cycle during the entire sample period. Similar results are obtained if one includes more than one autoregressive parameter[5]. Although in a slightly different setting, Lahiri and Wang(1994) also find, in their evaluation of the Commerce Department's Composite Index of Leading Economic Indicators as a predictor of business cycle turning points, that by including autoregressive terms the performance of the Hamilton filter deteriorates quite significantly. In contrast they also report that the simple model without any autoregressive

[4]This may be due to accounting practises in the contruction of these two series.
[5]These results are available upon request from the author.

Table 4: Vector Estimates

| | IIP | EMP | INC | SAL |
|---|---|---|---|---|
| | 1948:1-1995:1 | | | |
| $\mu_1$ | 0.43 (0.04) | 0.24(0.01) | 0.32(0.02) | 0.38(0.05) |
| $\mu_2$ | -0.00(0.34) | 0.01(0.09) | 0.10(0.11) | 0.04(0.21) |
| $p_{11}$ | | 0.96(0.01) | | |
| $p_{22}$ | | 0.91(0.02) | | |
| | | $\Omega_1$ | | |
| IIP | 0.42(0.04) | 0.04(0.01) | 0.08(0.02) | 0.26(0.04) |
| EMP | - | 0.03(0.00) | 0.02(0.00) | 0.05(0.01) |
| INC | - | - | 0.18(0.01) | 0.09(0.02) |
| SAL | - | - | - | 0.91(0.08) |
| | | $\Omega_2$ | | |
| IIP | 2.70(0.30) | 0.63(0.08) | 0.71(0.11) | 1.81(0.26) |
| EMP | - | 0.25(0.03) | 0.22(0.03) | 0.44(0.08) |
| INC | - | - | 0.56(0.06) | 0.43(0.11) |
| SAL | - | - | - | 3.12(0.36) |
| Lik | | -253.18 | | |
| | 1961:1-1995:1 | | | |
| | IIP | EMP | INC | SAL |
| $\mu_1$ | 0.44 (0.03) | 0.26(0.00) | 0.33(0.02) | 0.42(0.05) |
| $\mu_2$ | -0.22(0.13) | -0.06(0.03) | -0.07(0.05) | -0.17(0.12) |
| $p_{11}$ | | 0.97(0.01) | | |
| $p_{22}$ | | 0.90(0.02) | | |
| | | $\Omega_1$ | | |
| IIP | 0.36(0.03) | 0.03(0.00) | 0.07(0.01) | 0.22(0.03) |
| EMP | - | 0.02(0.00) | 0.02(0.00) | 0.04(0.00) |
| INC | - | - | 0.18(0.01) | 0.10(0.02) |
| SAL | - | - | - | 0.97(0.07) |
| | | $\Omega_2$ | | |
| IIP | 1.39(0.21) | 0.22(0.04) | 0.27(0.07) | 0.85(0.17) |
| EMP | - | 0.07(0.01) | 0.05(0.01) | 0.13(0.03) |
| INC | - | - | 0.25(0.03) | 0.06(0.06) |
| SAL | - | - | - | 1.27(0.20) |
| Lik | | 457.0 | | |

Table 5: Alternative Dating of US 'Business' Cycle

| | NBER | $\alpha = 0.5$ md=3 | $\alpha = 0.5$ md=6 | $\alpha = 0.9$ md=3 | $\alpha = 0.9$ md=6 |
|---|---|---|---|---|---|
| T | | | | 48M5 | |
| P | | | | 48M8 | |
| T | | 51M8 | | 51M7 | |
| P | | 51M11 | | 51M11 | |
| T | | 52M10 | 52M10 | 52M10 | 52M10 |
| P | 53M7 | 53M5 | 53M5 | 53M6 | 53M6 |
| T | 54M5 | 54M7 | 54M7 | 54M5 | 54M5 |
| P | | 56M5 | | 56M5 | |
| T | | 56M8 | | 56M8 | |
| P | 57M8 | 57M6 | 57M6 | 57M8 | 57M8 |
| T | 58M4 | | | 58M12 | |
| P | 60M4 | | | 59M5 | |
| T | 61M2 | 61M3 | 61M3 | 61M3 | 61M3 |
| P | | 64M8 | | | |
| T | | 64M11 | | | |
| P | 69M12 | 70M2 | 70M2 | 70M3 | 70M3 |
| T | 70M11 | 70M11 | 70M11 | 70M11 | 70M11 |
| P | 73M11 | 73M10 | | 73M11 | |
| T | | 74M2 | | 74M2 | |
| P | | 74M9 | 74M9 | 74M9 | 74M9 |
| T | 75M3 | 75M3 | 75M3 | 75M3 | 75M3 |
| P | 80M1 | 80M1 | | 80M2 | |
| T | 80M7 | 80M6 | | 80M6 | |
| P | 81M7 | 81M8 | 81M8 | 81M9 | 81M9 |
| T | 82M11 | 82M12 | | 82M9 | 82M9 |
| P | | 83M5 | | | |
| T | | 83M8 | 83M8 | | |
| P | 90M7 | 90M5 | 90M5 | 90M8 | 90M8 |
| T | 91M3 | 91M3 | 91M3 | 91M3 | 91M3 |

Notes: $P(S_1 = 1) < \alpha$ determines the dates, md refers to minimum duration rule for each phase

Table 6: Estimated for Sub-sample 1961:1-1995:1

| | NBER | $\alpha = 0.5$ md=6 | $\alpha = 0.9$ md=6 | $\alpha = 0.5$ md=3 | $\alpha = 0.9$ md=3 |
|---|---|---|---|---|---|
| P | 69M12 | 69M9 | 69M11 | 69M9 | 69M11 |
| T | 70M11 | 71M1 | 70M11 | 71M1 | 70M11 |
| P | 73M11 | 73M10 | 73M11 | 73M10 | 73M11 |
| T | 75M3 | 75M5 | 75M3 | 75M5 | 75M3 |
| P | 80M1 | | | 80M2 | 80M2 |
| T | 80M7 | | | 80M6 | 80M6 |
| P | 81M7 | 81M3 | 81M6 | 81M3 | 81M6 |
| T | 82M11 | 83M1 | 82M12 | 83M1 | 82M12 |
| P | | | | 83M5 | |
| T | | | | 83M8 | |
| P | 90M7 | 90M4 | 90M5 | 90M4 | 90M5 |
| T | 91M3 | 92M1 | 91M12 | 92M1 | 90M12 |

Notes: $P(S_1 = 1) < \alpha$ determines the dates, md refers to minimum duration rule for each phase structure gives "*...very sharp signals*".

## III. Forecasting

Although the emphasis of the paper is on the use of the VMS model to identify the business cycle turning points, in this section the forecasting performance of the VMS is also examined. Does the additional information provided by the common unobserved component help us in predicting economic variables compared with using information from one series or sector alone? We calculate in-sample and post-sample forecast errors for the univariate and VMS models. The aim is to compare the forecasting performance of the univariate relative to the VMS model. Although an interesting exercise by itself, a comparison of the forecasting performance of these models with other time series models (ARIMA, VAR models) is not the aim of this paper. For the latter, few studies have undertaken useful comparisons with mixed results (see Phillips, 1991, and Goodwin, 1993, for example).

The in-sample forecasts for time $t$ are obtained after taking the MLE estimate of $\widehat{\lambda}$ for $y_1, y_2, ..., y_T$ , that is making inferences based on the full sample. Following Engel and Hamilton(1990) we can write the forecast made on the basis of observations of $y$ through date $t$ and based on knowledge of $\widehat{\lambda}$ as:

$$\widehat{y}_{t+j|t} = E\left[y_{t+j} \mid y_t, y_{t-1}, ..., y_1; \widehat{\lambda}\right] = \widehat{\mu}_2 + \left\{\widehat{\rho} + (-1 + \widehat{p}_{11} + \widehat{p}_{22})^j \left[p(s_t = 1 \mid y_1, ..., y_t; \widehat{\lambda}) - \widehat{\rho}\right]\right\} \cdot (\widehat{\mu}_1 - \widehat{\mu}_2) \quad (6)$$

and the k-period ahead forecast of the level of the log of the coincident series:

$$\widehat{x}_{t+k|t} = x_t + \widehat{y}_{t+1|t} + \widehat{y}_{t+2|t} + ... + \widehat{y}_{t+k|t}$$

The mean squared forecast error (MSE) for 1, 3, 6 and 12-months given by:

$$\sum_{t=1}^{T-k} (\widehat{x}_{t+k|t} - x_{t+k})^2 / (T-k)$$

In a similar fashion we calculate post-sample forecasts and MSEs by estimating the population parameters using a restricted sample which excludes the last 48 months of the period under study. The forecasts for the multivariate model are given by:

$$\widehat{\mathbf{y}}_{t+j|t} = \mathbf{a}'\mathbf{P}^m \boldsymbol{\mu}$$

where $\mathbf{a}$ is a $(1 \times k)$ vector of inferences for the $k$ states at time $t$, $\mathbf{P}$ is the $(k \times k)$ transition probability matrix and $\boldsymbol{\mu}$ the estimated means for each state.

Tables 7 and 8 show the in-sample and post-sample MSEs for horizons of 1, 3, 6 and 12 months for the univariate models (denoted in the table as "Univ.") which assume independent Markov Switching processes (as shown in Table 1) and the vector-switching model (denoted in the table as "Multi.") which imposes a common "unobserved" cycle for all four series (estimated over 1961:1-1995:1). The forecasts are calculated for a period of 48 months which is used as a post-sample forecast period. The row entitled as "Relative MSE" shows the mean squared error

Table 7: In-Sample Mean Square Forecast Errors (model estimated 1961:1-1995:1)

| | Model | Horizon(months) 1 | 3 | 6 | 12 |
|---|---|---|---|---|---|
| IIP | Univ. | 1.07 | 5.54 | 10.35 | 14.07 |
| | Multi. | 0.93 | 4.80 | 9.29 | 13.91 |
| | **Relative MSE** | **0.87** | **0.87** | **0.90** | **0.99** |
| EMP | Univ. | 0.11 | 0.78 | 2.04 | 4.11 |
| | Multi. | 0.10 | 0.70 | 1.81 | 3.62 |
| | **Relative MSE** | **0.91** | **0.90** | **0.89** | **0.88** |
| INC | Univ. | 0.21 | 1.24 | 2.73 | 4.19 |
| | Multi. | 0.26 | 1.64 | 3.62 | 5.12 |
| | **Relative MSE** | **1.26** | **1.32** | **1.33** | **1.22** |
| SAL | Univ. | 1.96 | 6.41 | 13.45 | 26.53 |
| | Multi. | 1.79 | 5.74 | 10.67 | 15.34 |
| | **Relative MSE** | **0.92** | **0.90** | **0.79** | **0.58** |

Table 8: Post-Sample Mean Square Forecast Errors

| | Model | Horizon(months) 1 | 3 | 6 | 12 |
|---|---|---|---|---|---|
| IIP | Univ. | 1.11 | 5.80 | 10.96 | 14.72 |
| | Multi. | 0.97 | 5.11 | 10.01 | 14.67 |
| | **Relative MSE** | **0.88** | **0.88** | **0.91** | **1.00** |
| EMP | Univ. | 0.12 | 0.79 | 2.04 | 3.99 |
| | Multi. | 0.11 | 0.70 | 1.77 | 3.47 |
| | **Relative MSE** | **0.90** | **0.88** | **0.87** | **0.87** |
| INC | Univ. | 0.21 | 1.25 | 2.76 | 4.21 |
| | Multi. | 0.27 | 1.69 | 3.72 | 5.18 |
| | **Relative MSE** | **1.29** | **1.35** | **1.35** | **1.23** |
| SAL | Univ. | 1.94 | 6.30 | 13.05 | 24.89 |
| | Multi. | 1.81 | 5.90 | 10.92 | 14.90 |
| | **Relative MSE** | **0.94** | **0.94** | **0.84** | **0.60** |

Notes: Relative MSE is that of the multivariate relative to univariate model of the multivariate relative to the univariate model. With the exception of one series, sales, and for both in-sample and post-sample forecasts the VMS model produces more accurate forecasts than the univariate specification[6]. These conclusions remain essentially unchanged if other sample and forecast periods are used. In all cases there is an improvement in forecasting three of the four series and usually some deterioration in the forecasting performance of one of the series[7].

It thus appears, that the information from a set four coincident time series, as opposed to information obtained from one series, is important and enables us to improve our forecasts. Although the evidence in favor of the multivariate model is not overwhelming, it is strong. A more in-depth analysis could assess the forecasting performance of a VMS model relative to the univariate specification and examine under what circumstances does such an improvement occur. There are various interesting reasons why one would expect the VMS model to produce superior forecasts as compared with the univariate specifications. The interactions between these economic variables are rich and should provide useful information for prediction purposes. Idiosyncratic movements in some of these series do not necessarily reflect the overall business climate and may therefore be reversed if an overall downturn is not realized. Consequently, the VMS model provides a smoother cyclical component which appears to help in predicting these times series. One way to extend this model would be to include other series in the system which have leading indicator information. This would require modification of the model used in this exercise perhaps by allowing richer dynamics in the line of Phillips(1991).

## IV. CONCLUSIONS

This paper identifies turning points for the U.S. "business cycle" using information from four coincident series. This is consistent with the practice at the NBER which analyses a number of series to identify a single business cycle chronology. Implicitly what is assumed in the NBER methodology is a common underlying cycle. Hence, a regime switching model which encompasses information from these different series and makes inference about the state of the economy is a mechanical interpretation of the NBER methodology.

The model used assumes that each series is represented by a mixture of two normal distributions (a high and low mean) with the switching from one to the other determined by a common Markov process. The procedure is applied to the series composing the composite

[6]Although the vector-switching model does not seem to improve overall the forecasting ability for the coincident cycles it significantly reduces the cost of forecasting in terms of computer time. For the post-sample forecasts the univariate-Markov switching model required 106,42,86 and 125 replications for convergence for IIP, Employment, Income and Sales. When the vector-switching model is used convergence is achieved after only 38 replications. Thus, estimating the parameters in this way leads to a quicker estimation and some improvement in the forecasting ability for two out of the four series.

[7]We have also experimented with a model that restricts the variance to be common across the two regimes. Although the model produces turning points very close to the NBER reference cycle it fails when it comes to forecast the series. The deterioration in the forecasting performance in this case in dramatic.

coincident indicator in the U.S. to obtain business cycle turning points. The business cycle chronology is closer to the NBER reference cycle than the turning points obtained the from individual series.

Finally, it compares the forecasting performance of the univariate and VMS models. The VMS model produces more accurate forecasts relative to a simple univariate specification. Thus, using the extra information and imposing a common cycle does enable us to improve the forecasting performance of the basic Hamilton model. Furthermore, both in-sample and post-sample forecasts are obtained at a considerably lower cost, in terms of computer time, compared with the standard univariate switching models.

## REFERENCES


Artis, M.J., Kontolemis, Z.G., and Osborn, D.R. (1997), "Business Cycles for G7 and European Countries", *Journal of Business*, April, vol.70, no.2, 249-279.

Boldin, M.D. (1994), "Dating Turning Points in the Business Cycle", *Journal of Business*, vol.67, 97-131.

Canova, F. (1995), "Does Detrending Matter for the Determination of the Reference Cycle and the Selection of Turning Points", Universitat Pompeu Fabra, Economics Working Paper 113.

Engel, C. and Hamilton, J.D. (1990), "Long Swings in the Dollar: Are They in the Data and Do Markets Know it?", *American Economic Review*, vol.80, no.4, 689-713.

Everitt, B.S. and Hand, D.J. (1981), *Finite Mixture Distributions*, Chapman and Hall.

French, M.W. and Sichel D. F. (1993),"Cyclical Patterns in the Variance of Economic Activity", *Journal of Business and Economic Statistics*, 11:113-119.

Goodwin, T.H. (1993), "Business-Cycle Analysis with a Markov-Switching Model", *Journal of Business and Economic Statistics*, vol.11, no.3, 331-339.

Hamilton, J.D. (1989), "A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle", *Econometrica* , vol.57, no.2, 357-384.

Hamilton, J.D. (1990), "Analysis of Time Series subject to changes in Regime", *Journal of Econometrics*, vol.45, 39-70.

Hamilton, J.D. (1996), "Specification testing in Markov-switching time-series models", *Journal of Econometrics*, vol.70, 127-157.

Kim, M-J and Yoo, J-S (1995), "New Index of coincident indicators: A multivariate Markov Switching Factor Model approach", *Journal of Monetary Economics*, 36, 607-630.

Lahiri, K. and Wang, A.G. (1994), "Predicting Cyclical Turning Points with Leading Index in a Markov Switching Model", *Journal of Forecasting*, vol.13, 245-263.

Phillips, K.L. (1991), "A two-country model of stochastic output with changes in regime", *Journal of International Economics*, 31, 121-142.

Ravn, M.O. and Sola, M. (1995), "Stylized facts and regime changes: Are prices procyclical?", *Journal of Monetary Economics*, 36, 497-526.

Stock, J. H. and Watson, M.W. (1991), "A Probability Model of the coincident economic indicators", in K.Lahiri and G.H.Moore, eds, *Leading economic indicators: New approaches and forecasting records*, (Cambridge University Press, New York, NY) 63-85.

Titterington, D.M, Smith, A.F.M and Makov, U.E. (1985), *Statistical Analysis of Finite Mixture Distributions*, New York, John Wiley.

**Figure 1. USA: Probability that Economy Remains in Expansion**

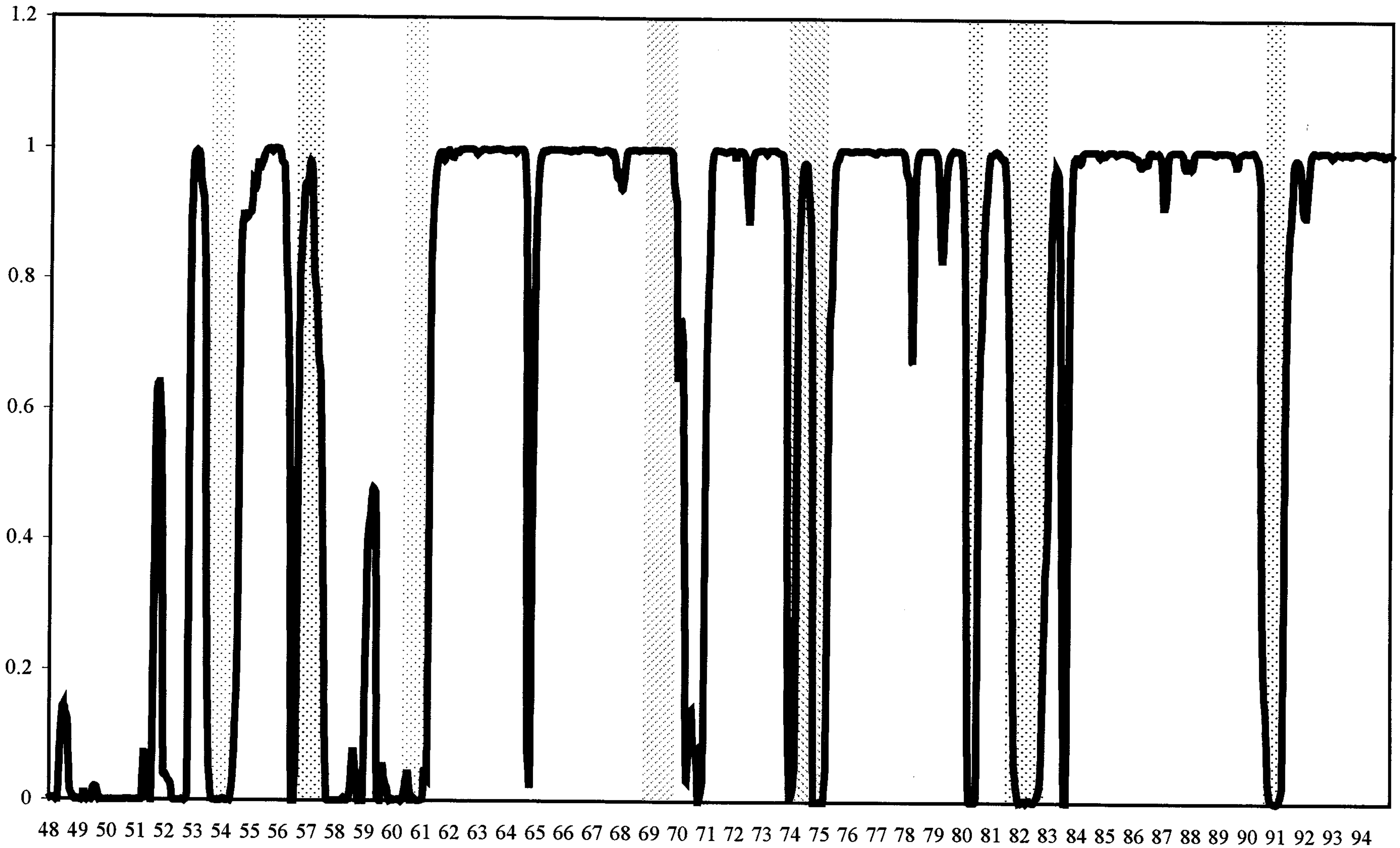

**Figure 2. USA: Probability that Economy Remains in Expansion - AR(1) Model**