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Bank Bailouts: Moral Hazard vs. Value Effect[1]

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Abstract

This paper shows that a central bank, by announcing and committing ex-ante to a bailout policy that is contingent on the realization of certain states of nature (for example on the occurrence of an adverse macroeconomic shock), creates a risk-reducing "value effect" that more than outweighs the moral hazard component of such a policy.

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I. INTRODUCTION

Economists (and probably central bankers) disagree on how liberal the access to last-resort support should be. Those who follow Bagehot's gospel think that central banks should only assist temporarily illiquid banks. Others, either stressing the negative externalities associated with bank failures (e.g., Solow (1982), or the difficulties of distinguishing between illiquidity and insolvency (e.g., Goodhart (1988, 1995)), suggest that the access to the lender of last resort (LOLR) should not be *a priori* denied to any bank. In general, however, everyone seems to agree that central banks face a trade-off between being too "tough," and thus increasing the likelihood that the failure of a single bank hampers the confidence in the whole banking system, and being too "soft," thereby creating incentives for banks to engage in excessive risk-taking.[1]

In this paper, we challenge this view, and show that a central bank, by announcing and committing ex-ante to a bailout policy that is contingent on the realization of certain states of nature (for instance on the occurrence of an adverse macroeconomic shock), creates a risk reducing "value effect" that more than offsets the moral hazard component of such a policy.

In our framework, a bank manager faces the following trade-off upon deciding the bank's risk exposure that is not observable, or rather not verifiable, by the central bank: While projects with excessive risk (relative to the one that maximizes expected profits in the absence of limited liability) tend to maximize current bank profits, they also increase the probability that the bank may become insolvent at the end of the period, and thus lose its charter. In this context, the introduction of a bailout scheme has two mutually offsetting effects. On the one hand, it creates moral hazard, as the probability of surviving depends less on the bank's risk choice and more on the central bank's actions. On the other, it increases the bank's probability of survival, thus raising the value at stake and, in turn, the bank's incentives to protect it. Whereas in general the net result of these two influences can move in either direction, we find that a publicly announced policy that commits to bailing out failed institutions only in the event of an adverse macroeconomic shock does indeed yield a lower equilibrium level of risk.

[1] See Bordo (1989) and Giannini (1999) for more exhaustive discussions of LOLR theory and practice.

Among the few formal models of bank bailouts, perhaps the one that more explicitly represents the standard social cost-moral hazard trade-off is Goodhart and Huang (1999). The paper shows that the central bank has incentives to provide LOLR assistance, inasmuch as concerns about bank contagion are weighted more strongly than moral hazard considerations. Moreover, the authors argue that, in order to minimize the moral hazard component, it is optimal for the central bank to use constructive ambiguity in the bailout decisions.

Although Goodhart and Huang's (1999) tougher policies have the advantage of limiting moral hazard, some authors have recently argued that a "soft" policy can be used to induce bank managers to reveal private information about the realization of portfolio returns. Povel (1996), for example, shows that soft policy can induce an entrepreneur protected by limited liability to reveal at an early stage that his firm is in financial distress, thus allowing for a less expensive rescue. A similar idea, for the case of bank bailouts, is proposed by Aghion, et al. (1999).

The link between bank value and risk-taking decisions, namely, the fact that in a dynamic setting the loss of the charter value may act as a disciplinary device against risk taking, is not new to the banking literature. Suarez (1994) shows that the threat of being closed can be seen as "an effective way to induce the bank to be prudent when the present value of its future rents is sufficiently high" (p. 24) from which he infers that the optimal strategy for the central bank is to commit credibly to withdrawing the bank's charter in case of bankruptcy. Along the same lines, a recent paper by Blum (1999) shows that, in a multiperiod model, tough policies such as strict capital requirement, by decreasing the value of the bank, may end up increasing risk-taking incentives. In a related paper, Acharya (1996) shows that regulatory forbearance may be optimal if dead-weight losses associated with the closure of a bank are important. However, he does not model the bank's optimization problem, the effect of forbearance on bank value, and the optimal choice of risk.[2]

Freixas, et al. (1998), develop a model of interbank lending in which banks face liquidity shocks *à la* Diamond and Dybvig (1983), and solvency shocks *à la* Holmström and Tirole (1998). They show that when entrepreneurs have interest in investing in an inferior technology (presence of moral hazard), and it is not possible to distinguish between liquidity and solvency shocks —because of private information— there is excessive liquidation of banks in the absence of central bank intervention. In their model, the role of LOLR assistance is that of mutualizing the solvency shocks by taxing "lucky" banks and subsidizing "unlucky" banks.

The idea of subsidizing "unlucky" banks is also present in our paper, since bank managers are pardoned if the bank fails in bad states of nature when the bank's fate is relatively less dependent on the manager's decisions. This idea is somehow similar to Holmström's

[2] Empirical evidence supporting the link between value and risk is presented in Keeley (1990).

(1979) optimal contract in which a "repairman receives higher pay if it is found that the failure was outside his control than if it is found that a component that he controls failed" (p. 83). While in the same paper the first best contract cannot be written because agents are not risk-neutral,[3] in our model moral hazard cannot be avoided, despite the fact that all agents are risk neutral because of the bankers' limited liability.

The problems of the limited liability constraint in the optimal contract literature was firstly analyzed by Sappington (1983). Closer to our framework is Innes (1990), in which agents choose their actions before observing the state of nature. He shows that the optimal contract is one in which the agent receives maximal reward when the result is good and maximal penalty when the result is bad. Financial contracts of this type are not frequently seen in practice, and the idea of subsidizing banks when profits are high, would take us far away from the bailout issue we are addressing in this paper.[4]

Our results seem to provide a rationale for historical LOLR practice. In particular, they appear to motivate, in a simple way, the tendency to provide subsidized funds to problem institutions in crisis periods. However, there are important differences between our conclusions and what seems to be conventional wisdom concerning bailout policies. First, we assume that the shock on which the policy is contingent is exogenous to the bank manager's decision, a condition that is not necessarily satisfied if bailouts are triggered by systemic banking crises. Although the results still hold in this case, the beneficial effect on risk is weakened, since the moral hazard component becomes relatively more important. Secondly, as opposed to the traditional view, in our framework an explicit commitment to rescue insolvent banks contributes to alleviate the associated moral hazard problem, rather than worsen it. Finally, we show that the "constructive ambiguity" approach often recommended to attenuate moral hazard, in which the terms of the LOLR arrangement are left to the discretion of the central bank, is always dominated by a policy that commits to rescuing failed banks *with certainty*, conditional upon the realization of an adverse aggregate shock.

II. THE MODEL

Bank managers, acting in the interest of risk-neutral shareholders, collect funds from depositors and invest them in a portfolio with stochastic returns. Depositors have the choice of either depositing in the bank or investing their funds in a risk-free asset offering a gross rate of return $r_f \geq 1$. Bank deposits are protected by a comprehensive deposit insurance, so that depositors invest in the bank at any posted deposit rate $r \geq r_f$. For the sake of simplicity,

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3 Harris and Raviv (1979) prove indeed that the problem of moral hazard can be avoided when agents are risk neutral.

4 Also, as Innes recognizes, if enterpreneurs (banks in our framework) have access to short-term borrowing, they would have incentives to inflate their profits to increase the reward.

we assume that the aggregate supply of funds in the economy is fixed, and we normalize it to unity.

Bank managers can affect the risk-return profile of their portfolios. More precisely, we assume that a portfolio X offers a gross rate of return $\tilde{R}$ distributed according to the following two-point distribution:[5]

$$\begin{array}{lll} \tilde{R} = X & \text{with probability} & p(X,\gamma) \\ \tilde{R} = 0 & \text{with probability} & 1 - p(X,\gamma), \end{array}$$

where $\gamma \in [0,1]$ is a stochastic variable distributed over the interval $[0,1]$ with density function $f(\gamma)$, and cumulative distribution function, $F(\gamma)$, $F(1) = 1$, representing a random state of nature. In order to keep our analysis as simple as possible, we assume that $cov(X,\gamma) = 0$, and we impose the following functional form for the probability of success:

$$p(X,\gamma) = \gamma p(X),$$

so that expected returns can be written as:

$$E[\tilde{R}] = \mu p(X) X, \tag{1}$$

with $\mu \equiv \int \gamma f(\gamma) d(\gamma) < 1$. Accordingly, the higher the realization of γ, the higher the probability of success for any given portfolio choice X. Informally, we will refer to high and low values of γ as "good" and "bad" states of nature, respectively.

We assume that a higher X is associated with a higher portfolio risk p, that is, $\partial p/\partial X < 0$. In addition, to avoid corner solutions with infinite risk, we assume that $\partial^2 p/\partial X^2 \leq 0$, so that (1) is strictly concave in the control variable X, and that $X \geq \underline{X} = \frac{r_f}{\mu}$, with $p(\underline{X}) = 1$ and $p'(\underline{X}) > -1/\underline{X}$, so that the risk-free asset is strictly dominated in expected returns by (at least) some risky portfolio. Finally, we assume that the bank's choice of X is not observable (or not verifiable), so that the central bank cannot implement bailout policies that are contingent on the bank's choice of risk.

The time structure of the model is the following: The bank chooses X_t, at the beginning of each period t, $t \in [0,\infty)$. If at the end of the period the portfolio's realized return $\tilde{R}_t$ is equal to zero (and shareholders choose not to recapitalize the bank), all deposit liabilities are transferred to the deposit insurance fund which pays depositors in full. At that point, the central bank decides whether to withdraw the bank licence or not (bailout choice).

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5 The stochastic structure of the model is that of Blum (1999), to which we add a random state of nature limiting the bank's control over risk.

The structure of the model is recursive; in case the charter is not withdrawn, the bank faces the same problem at the beginning of period $t+1$. We assume that there are no bankruptcy costs.

A. Optimal Risk

Before solving for the bank's problem, and the effects of alternative bailout policies on the manager's choice of risk, we derive the (optimal) solution from the point of view of a central planner. The problem that the central planner faces in each time period can be written as:

$$\max_{x} \frac{\mu p(X)X}{1-\delta}.$$

From the necessary and sufficient first order condition we have that the optimal choice of X, henceforth denoted by X^*, is the solution to:

$$p'(X)X + p(X) = 0,$$

an expression that can be rewritten as:

$$\eta(X)_{px} \equiv \frac{p'(X)X}{p(X)} = -1. \tag{2}$$

At the first best solution, the elasticity of the probability of success with respect to the portfolio return in case of success equals minus one. Two points are notable. First, since

$$\frac{\partial \eta(X)_{px}}{\partial X} = \frac{\left[p''(X)X + p'(X)\right]p(X) - \left[p'(X)\right]^2 X}{p(X)^2} < 0, \tag{3}$$

excessive risk (that is, risk levels above the optimum) is associated with $\eta(X)_{px} < -1$. Second,

$$\eta(\underline{X})_{px} = \frac{p'(\underline{X})\underline{X}}{p(\underline{X})} > -1$$

implies that the optimal level of risk is strictly positive, that is, $X^* > \underline{X}$.

B. The Bank's Problem

The bank's problem consists in maximizing the discounted flow of profits V_t, valued at period t, which is given by

$$V_t = \Pi_t + \delta s_t(\cdot)\Pi_{t+1} + \delta^2 s_t s_{t+1}(\cdot)\Pi_{t+2} + \ldots,$$

where $\delta < 1$ is a discount factor, $s_t(X_t, \gamma_t, \cdot)$ is the bank's probability of survival from period t to period $t+1$, and Π_t denotes current expected profits, such that:

$$\Pi_t = \mu p(X)_t \,(X - r)\,. \tag{4}$$

We generically denote the probability of survival by $s\,(\cdot)$, in order to be able to introduce different bailout policies. However, we restrict our attention to time invariant policies, so that the problem that the bank faces is a stationary one, and, dropping time subscripts, the bank's value can be rewritten as:

$$V = \max_X \frac{\Pi}{1 - \delta s(\cdot)}. \tag{5}$$

If the central bank withdraws the bank's charter when the bank does not meet its obligations (no bailout), the bank "survives" with probability

$$s(p, \gamma) = \mu p(X),$$

so that (5) can be rewritten as:

$$V^0 \equiv \max_X \frac{(X - r)p(X)\mu}{1 - \delta p(X)\mu}. \tag{6}$$

The necessary and sufficient first order condition[6] for the maximization in (5) can be written as:

$$\eta(X)_{px} = -\left[1 - \frac{p(X)}{\theta_0}\right]\frac{X}{X - r}, \tag{7}$$

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6 The second order condition is shown to hold in the Appendix.

with $\theta_0 \equiv 1/\delta\mu$. Using (2), (7) and (3), and denoting by X^0 the (unique) solution to equation (7), one can immediately verify that

$$\frac{\mu p(X^*)X^*}{r} \leq \frac{1}{\delta} \Rightarrow X^0 \geq X^*. \tag{8}$$

Accordingly, for a given discount rate, the bank is more prone to engage in excessive risk the lower are the returns of the optimal portfolio relative to those of a risk-free portfolio. Alternatively, for given rates of return, a bank with a stronger preference for the present (a higher discount rate $1/\delta$), tends to invest in more risky portfolios.

Condition (8) suggests that banks may in some cases choose too little risk ($X^0 < X^*$). However, *if at the end of the period, the realized portfolio returns are zero, bank managers still have the option to raise the necessary funds in the capital market, thereby avoiding default.*[7] This "non-default" option will be exercised whenever the value of the charter continues to be positive, that is, when the discounted value of future expected profits exceeds the stock of outstanding deposit liabilities, so that, in equilibrium:

$$r < \delta V = \delta \frac{p(X^*)\mu(X^* - r)}{1 - \delta p(X^*)\mu},$$

a condition that can be rewritten as:

$$\frac{\mu p(X^*)X^*}{r} \geq \frac{1}{\delta}. \tag{9}$$

Then, it follows immediately that if condition (8) is violated, condition (9) holds, and the bank never defaults. But, in that case, the optimal strategy is necessarily that of maximizing expected returns in each time period, and the respective bank's problem coincides with the central planner's. Summarizing:

Lemma 1 *Equilibrium risk is never below the optimum:*

(i) If $\frac{\mu p(X^*)X^*}{r} < \frac{1}{\delta}$, the bank selects a portfolio X^0 strictly riskier than X^*, and defaults when the portfolio yields zero returns.

(ii) If $\frac{\mu p(X^*)X^*}{r} \geq \frac{1}{\delta}$, the bank chooses the optimal portfolio X^*, and never defaults.

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[7] We implicitly assume that existing or prospective shareholders can indeed provide the needed funds, thus abstracting from cases in which liquidity constrained capital markets fail to rescue illiquid but solvent institutions.

Since we are interested in the case in which risk is excessive and bank failures are possible in equilibrium, *in the rest of paper we assume that condition (8) is satisfied.*

III. BAILOUT POLICIES

Assume that the monetary authorities announce (and can credibly commit to) the following policy: In the event that a bank's gross investment returns do not cover outstanding deposit liabilities, with a certain probability β the central bank provides the troubled bank the necessary funds to repay depositors while allowing it to continue its operations.[8] Assume further that this probability is known to be contingent on the realization of the stochastic variable γ, and thus that the bailout rule is a function $\beta(\gamma) : \gamma \longrightarrow [0,1]$. Note that this formulation is quite general and, in particular, encompasses the "constructive ambiguity" approach, according to which the central bank should retain some discretion as to when, and under what circumstances, a distressed bank should be rescued.

We want to study the effects of the implementation of such a rule on the individual bank's choice of risk, and the characteristics that this policy should present in order to minimize its moral hazard component. To simplify the analysis we assume that the bailout policy takes the form of a pure transfer through which the failed bank's (net) liabilities are assumed by the central bank without any requirement to repay the loan, or to contribute additional equity.

As before, for any given bailout rule $\beta(\gamma)$, if in the event of zero portfolio returns no bailout funds are forthcoming, the bank's shareholders will choose to avoid default by raising capital in the capital markets whenever deposit liabilities do not exceed expected future profits. Therefore, we can distinguish two possible scenarios, non-default and default, which we denote with the index d and n, respectively, according to whether or not the non-default condition,

$$r \leq \delta V^n, \tag{10}$$

is satisfied in equilibrium. We proceed to characterize each of these scenarios in turn.

[8] Alternatively, we could assume that the deposit insurance fund fully guarantees the repayment of deposits, while the monetary authority simply allows an insolvent bank to keep its license.

A. Default Scenario

The bank's probability of survival s is now given by the sum of the probability of positive portfolio returns plus the probability that, if returns are zero, the central bank comes to the rescue:

$$s = \int_0^1 [p(X)\gamma + (1 - p(X))\gamma\beta(\gamma)] f(\gamma) d\gamma,$$

or, rearranging,

$$s = p(X)\left(\mu - C\right) + \overline{\beta}, \tag{11}$$

with

$$\overline{\beta} \equiv \int_0^1 \beta\left(\gamma\right) f(\gamma) d\gamma,$$

$$C \equiv \int_0^1 \gamma\beta\left(\gamma\right) f(\gamma) d\gamma,$$

and

$$C < \min\{\mu, \overline{\beta}\}. \tag{12}$$

Inserting (11) into (5), we obtain the following expression for the bank's value:

$$V^d = \max_X \frac{p(X)(X - r)\mu}{1 - \delta\left[p(X)(\mu - C) + \overline{\beta}\right]}. \tag{13}$$

Note that C, which reduces the effect of the bank's portfolio choice on the probability of default, detracts from the marginal cost of risk in terms of lower expected future rents, generating moral hazard. On the other hand $\overline{\beta}$, a pure subsidy component independent of the realization of γ, increases expected future rents and, in turn, the bank's incentives to behave safely.[9]

[9] It is straightforward to verify that $\frac{\partial V^2}{\partial e \partial C} < 0$, and $\frac{\partial V^2}{\partial e \partial B} > 0$.

The necessary and sufficient first order condition[10] for the maximization in (13) can be expressed as:

$$\eta(X)_{px} = -\left[1 - \frac{p(X)}{\theta_d}\right]\frac{X}{X-r}. \tag{14}$$

with

$$\theta_d \equiv \frac{1-\delta\overline{\beta}}{\delta(\mu - C)} > 1.$$

We denote X^d the (unique) solution to (14). It follows directly from (3), (7) and (14) that a bailout policy reduces risk if, and only if, $\theta_n < \theta_0$, a condition that is satisfied whenever

$$\mu\delta\overline{\beta} > C, \tag{15}$$

from which:[11]

Lemma 2 *A necessary condition for a bailout policy $\beta(\gamma)$ to increase monitoring effort (i.e., for $X^d > X^0$) is that*

$$Cov(\beta(\gamma), \gamma) < 0. \tag{16}$$

Proof: $cov(\beta(\gamma),\gamma) = \int_0^1 (\gamma - \mu)\,(\beta(\gamma) - \overline{\beta})f(\gamma)d\gamma = \int_0^1 \gamma\beta\,(\gamma)\,f(\gamma)d\gamma - \int_0^1 \gamma\overline{\beta}f(\gamma)d\gamma +$

$-\int_0^1 \mu\beta\,(\gamma)\,f(\gamma)d\gamma + \mu\overline{\beta} = C - \mu\overline{\beta}$. Finally, $\mu\delta\overline{\beta} > C \Longrightarrow C - \mu\overline{\beta} < 0\,\square$

Condition (16) states that in order to reduce the moral hazard component, the bailout rule has to penalize insolvency to a greater degree under good realizations of the shock. This is not surprising since the probability that insolvency is caused by the bank's imprudent risk behavior is proportional to the value of the shock γ. By withdrawing funds in good states of nature, the central bank indeed indirectly punishes the bank's imprudent practices. Conversely, a bailout policy that does not discriminate between insolvency due to negative exogenous shocks, and to the banker's own decision ($cov(\beta(\gamma), \gamma) = 0$), induces risk-taking.[12] We can thus state that:

Proposition 1 *A state-independent bailout scheme, $\beta(\gamma) = \beta$, for all γ, increases the equilibrium level of risk.*

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10 The second order condition is shown in the Appendix.

11 We thank Giovanni Dell'Ariccia for suggesting this formulation of the necessary condition.

12 This applies, of course, to the particular case of a blanket guarantee such that $\beta(\gamma) = 1$, for all γ.

Let us assume, for the moment, that under all feasible policies the charter value of a bank with realized returns equal to zero is negative, so that in the absence of central bank assistance, shareholders choose to default. In this case, the risk-minimizing bailout policy is characterized in the following Lemma:

Lemma 3 *If under all feasible bailout policies the default condition is satisfied at the equilibrium, bank risk is minimized by a bailout policy*

$$\beta(\gamma) = \begin{cases} 1, & if\ \gamma < \gamma^d; \\ 0, & if\ \gamma > \gamma^d; \end{cases} \qquad (17)$$

with $\gamma^d \equiv \left\{\gamma : \gamma - \frac{1}{\theta(\gamma)} = 0\right\}$. *Furthermore, since* $\gamma^d \in (0, 1)$, *there is always a bailout policy that reduces bank risk.*

Proof: In Appendix.

B. Non-Default Scenario

If the non-default condition holds, the bank covers deposit liabilities whenever returns are zero and bailout funds are not forthcoming, which occurs with probability $1 - s$. Accordingly, the bank's value can be expressed as:

$$V^n = \max_X \frac{p(X)\mu(X - r) - r\left[1 - p(X)(\mu - C) - \overline{\beta}\right]}{1 - \delta}, \qquad (18)$$

where the superscript n denotes equilibrium values under this scenario. In turn, the non-default condition (10) can be rewritten as:

$$r \leq \frac{p(X^n)X^n}{\psi}, \qquad (19)$$

where

$$\psi \equiv \frac{1 + \delta p(X^n)C - \delta\overline{\beta}}{\delta\mu}.$$

For any given policy such that condition (19) is satisfied at the equilibrium, the necessary and sufficient first order condition[13] for the maximization in (18) can be written as:

$$\eta(X)_{px} = -1 + \frac{p'(X)Cr}{\mu p(X)}. \qquad (20)$$

[13] The second order conditions are shown in the Appendix.

We are now in a position to characterize the risk minimizing bailout policy in this scenario:

Lemma 4 *If we restrict our attention to the set of bailout policies that induce the bank to recapitalize, risk is minimized by the policy :*

$$\beta^n(\gamma) \equiv \begin{cases} 1, & if\ \gamma < \gamma^n; \\ 0, & if\ \gamma > \gamma^n; \end{cases} \tag{21}$$

with $\gamma^n = \{\gamma : r = \delta V^n(\gamma)\}$.

Proof: In Appendix.

Comparing Lemmas 3 and 4, we can see that the optimal policy shares the same all-or-nothing nature under both scenarios, namely, that in order to minimize risk the best that the central bank can do, contingent on the realization of a particular value of γ, is either to bail out the bank with probability one or never bail it out.

C. Optimal Bailout Policy

Having derived minimum-risk policies under each scenario, we are ready to characterize the optimal bailout policy, that is, the one that moves banks' choice of risk closer to the first best solution. From Lemmas 3 and 4, we know that candidate risk minimizing bailout functions $\beta(\gamma)$ should belong to the family

$$\dot{\beta}(\gamma) = \begin{cases} 1, & if\ \gamma \leq \dot{\gamma}; \\ 0, & if\ \gamma > \dot{\gamma}; \end{cases}$$

that can be fully described by their threshold level $\dot{\gamma}$. The following Lemma guarantees that the equilibrium level of bank risk is a continuous function of this threshold $\dot{\gamma}$:

Lemma 5 *(i) The optimal choice of risk is continuous in* $\dot{\gamma}$.

(ii) For any bailout policy $\beta(\dot{\gamma})$, *a bank chooses to avoid default if, and only if,* $\dot{\gamma} \geq \gamma^n$.

(iii) Risk is monotonically increasing in $\dot{\gamma}$, *for* $\dot{\gamma} > \gamma^n$.

Proof: In Appendix.

The Lemma clearly illustrates the value effect of a bailout policy: More generous policies are associated with higher bank value which, in turn raises the stakes involved in the bank's risk decision, inducing the bank to invest in safer assets. More important, it tells us that as we increase the threshold $\dot{\gamma}$ (and bank value), and get closer to the point γ^n at which the non-default condition just binds, we may face two cases. If $\gamma^n < \gamma^d$, then policy $\beta^n(\gamma)$ leads to the minimum attainable risk. This is because once bank's value is sufficiently high to ensure its survival, additional central bank aid only makes risky investment decisions less costly, generating moral hazard without any counteracting value effect. If, on the contrary, $\gamma^n < \gamma^d$, then we reach the risk-minimizing policy $\beta^d(\gamma)$ at a point at which bank value is not large enough to prevent default. The extension of the policy to include higher values of γ has a negative impact on risk. By continuity, the same is valid once we eventually hit the non-default constraint.

This intuition is more formally stated in the following Proposition:

Proposition 2 *(i) Bank risk is minimized by the following bailout policy:*

$$\beta^*(\gamma^*) = \begin{cases} 1, & if\ \gamma \leq \gamma^*; \\ 0, & if\ \gamma > \gamma^*; \end{cases} \quad \text{with } \gamma^* = \begin{cases} \gamma^d, & if\ \gamma^d \leq \gamma^n; \\ \gamma^n, & if\ \gamma^d > \gamma^n. \end{cases}$$

Proposition 3 *(ii) The equilibrium level of risk under policy $\beta^*(\gamma^*)$ is always higher than optimal.*

Proof: In Appendix.

The two cases are illustrated in Figure 1 and 2, where the optimal level of risk chosen by the bank is plotted as a function of $\dot{\gamma}$.[14] In Figure 1, we have that $\gamma^n > \gamma^d$, so that the risk minimizing bailout policy is associated with the threshold $\gamma^* = \gamma^d$, and the bank never recapitalizes. In Figure 2, a higher value of δ induces the bank to avoid default at a lower threshold level. As a result, $\gamma^n < \gamma^d$, and the optimal bailout policy lies at $\gamma^* = \gamma^n$.

IV. DISCUSSION

Several important points deserve to be noted in relation to the previous results. First, the minimum risk attainable through a bailout scheme of the sort analyzed in the paper is an increasing function of the deposit rate. A simple inspection of (14) and (20) reveals that, for any given bailout function $\beta(\dot{\gamma})$, higher deposit rates are associated with smaller values of

[14] The figures are plotted for a beta distribution function, with parameters $\nu = 4$, $\omega = 2$, assuming $p(X) = X - .5(1 - X^2)$, and $r_f = 1$. Figures 1 and 2 correspond to $\delta = 0.8$ and $\delta = 0.9$, respectively.

$\eta(X)_{px}$ (alternatively, higher equilibrium risk). This is not surprising since higher financing costs unambiguously detract from the bank's value, and thus increase risk-taking incentives. Thus, when deposit rates are low enough, $r < \frac{p(X^n(\gamma^c))X^n(\gamma^c)}{\psi(\gamma^c)}$, the optimal bailout policy not only achieves a lower equilibrium risk level (thus reducing the frequency with which the central bank assumes bank losses) but also, by eliminating defaults altogether, transfers to the private bank the financial losses incurred in those states of nature for which the bailout is not activated. The same can be said of the discount rate $\frac{1}{\delta}$, since the present value of future rents, through which the value effect operates, is higher the lower the rate at which they are discounted.[15] In both cases, the impact on the optimal policy differs according to whether γ^d is below or above γ^n. Whereas in the first case, the threshold increases with δ,[16] in the second an increase in δ has the opposite effect, since banks recapitalize under less generous policies, and the central bank can afford to limit its support while obtaining a lower level of risk at the same time.[17]

Second, the optimal bailout policy requires that in those states of nature for which the bailout scheme is activated ($\gamma \leq \gamma^*$), funds should be made available with certainty ($\beta = 1$). Thus in our framework a "constructive ambiguity" approach that leaves the possibility of financial support in a given state of nature at the discretion of the central bank yields a higher equilibrium risk level than (at least some) non-discretionary policies that bail out failed banks in some states, and provide no support in the remaining ones. This is a logical corollary of the model, in which the relevant trade-off weights the incentives to reduce risk associated with the "value" effect against the moral hazard effect of ensuring the continuance of the bank. This finding contrasts with the context in which the "constructive ambiguity" approach is usually developed, where the moral hazard effect, the only channel through which the central bank is assumed to (negatively) influence risk-taking decisions, is weighted against potential losses arising from disruptions in the financial system.

In this analysis, we deliberately assumed away these losses, as well as any bankruptcy costs, although it can immediately be seen that they would only contribute to the positive impact of a bailout scheme. A number of more important explicit and implicit simplifying assumptions made in the paper deserve some further analysis. We address some of them in turn.

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[15] The negative link between financing costs and discount rates, on the one side, and the choice of risk, on the other, is a natural implication of our basic model and does not depend on the existence of a bailout policy. In particular, it suggests that countries with higher costs of capital are more prone to engaging in excessive risk.

[16] It is easy to verify this by applying the implicit function theorem to (28), from which we have that

$$\frac{\partial \gamma^d}{\partial \delta} = -\frac{\partial G}{\partial \delta} / \frac{\partial G}{\partial \gamma} = \frac{\int_{\gamma^d}^{1} \gamma f(\gamma) d\gamma - \gamma^* \int_0^{\gamma^*} f(\gamma) d\gamma}{1 - \delta \int_0^{\gamma^*} f(\gamma) d\gamma} > 0.$$

[17] A lower threshold implies a lower value of C which, under the non-default scenario, leads to lower equilibrium risk.

A. Partial Deposit Insurance

The assumption of full deposit insurance, which simplified the derivation of the result by making the deposit rate r an exogenous variable, may be relaxed without altering the main findings of the paper. In the absence of insurance, risk-neutral depositors would demand a return on bank deposits equal to:[18]

$$r = \frac{r^f}{E(s)},$$

where r^f is the return on a risk-free asset, and $E(s)$ denotes depositors' expectation of the probability that the deposit contract is honored.

In this environment, a bailout policy would reduce the deposit rate both through its beneficial effect on portfolio risk (higher $p(X)$), as uninformed but rational depositors anticipate the impact of the policy on individual banks' decisions, and through an increase in the probability of repayment $s = p(X)(\mu - C) + \overline{\beta}$, which is higher under a bailout scheme for any level of portfolio risk. In turn, as discussed above, lower deposit rates would result in a second order positive effect on risk from the increase in value due to the widening of intermediation margins, amplifying the impact of the policy.

B. Alternative Central Bank Objectives

In the previous section, we implicitly assumed that the central bank's sole concern was minimizing risk and we abstracted from the impact of central bank outlays. However, it may be the case that, due to efficiency costs associated with these outlays, the central bank wants to minimize the combined quasi-fiscal cost under different bailout schemes.[19]

Regarding this point, the analysis should distinguish between two possible cases, according to whether or not unassisted banks with zero returns choose to avoid default in equilibrium. In the first case, in which $\gamma^n < \gamma^d$ (Figure 2), it is easy to verify that the risk-minimizing bailout policy $\gamma^* = \gamma^n$ also minimizes disbursements. Any bailout policy $\dot{\gamma} < \gamma^d$ would not only be associated with a higher level of risk, but it would also induce insolvent banks to default for $\gamma > \dot{\gamma}$, increasing the central bank's bill. On the other hand, a policy $\dot{\gamma} > \gamma^d$ would increase the banks' risk-taking incentives while committing the central bank to reimbursing depositors in situations in which private banks would otherwise voluntarily produce the funds.

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18 It is straightforward to show that the following argument holds when deposits are partially insured.

19 The argument applies irrespective of whether the bailed out bank's liabilities are assumed by the deposit insurance fund or are paid for by bailout money provided directly to the bank by the central bank.

More interesting is the case in which $\gamma^n > \gamma^d$, where the risk-minimizing policy is such that $\gamma^* = \gamma^n$, and default is possible. As it is clear from a simple inspection of Figure 1, by implementing the alternative policy γ^n, the central bank can transfer part of the expected financial loss to troubled banks that are now willing to recapitalize in good states of nature, at the cost of a higher risk level. Which of the two policies minimizes expected quasi-fiscal outlays is not straightforward. The relevant trade-off is illustrated in Figure 3. Lines AF and AD represent expected disbursements, $(r\,(1 - p(X)\mu))$, as a function of γ, under bailout policies γ^d or γ^n, respectively. Note that, since the bank takes on more risk when the policy γ^n is implemented, the line AF is flatter than AD. By implementing a bailout policy γ^n, the central bank induces the bank to recapitalize when $\gamma > \gamma^n$, and thus saves an amount equal to the area of the trapezoid $EBDG$. At the same time, for all $\gamma < \gamma^n$, the expected disbursement increase by the area of the triangle ABC. Note that, while the relative size of these areas depends on parameter values, as γ^d gets closer to γ^n, the central bank may find it optimal to go beyond the risk-minimizing policy if its priority is to reduce its quasi-fiscal outlays.

C. Observability of the Aggregate Shock

Critical to our results are the assumptions regarding the information structure, namely that while the portfolio choice is the banks' private information, the aggregate shock γ is observed by the central bank. The first one is based on our understanding that, even if the regulator may have information about the composition of the investment portfolio of the troubled bank at the time emergency funds are requested, in practice this information tends to be rather vague and, in any case, difficult to assess objectively in order to be used as a condition for the provision of open assistance.

On the other hand, we interpret the aggregate shock as a proxy for overall macroeconomic conditions beyond the influence of individual bank managers. It is crucial to note that if banks' behavior has any impact on the value of the variable γ, as it would be the case, for example, if the bailout scheme were activated whenever individual bank problems threaten to become systemic, then the policy would generate additional moral hazard on the part of large institutions that anticipate that their eventual failure may act as an automatic trigger for the provision of emergency funds.

Whereas an exact measure of the relevant aggregate factors influencing investment returns may be difficult to devise, one can think of a number of directly quantifiable variables affecting the cost of (short-term) financing or the repayment capacity of bank debtors, that can be used as signals of a sudden deterioration of the financial environment.[20] However, in

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[20] Examples include negative real shocks depressing the level of economic activity, sudden changes in country risk that push up domestic interest rates, or sharp devaluations that increases the risk of default on foreign currency loans to producers of non-tradables.

reality one should not interpret our bailout scheme as a strict contract that specifies a unique value of any of these variables beyond which assistance is denied, but rather as a guide for central bank intervention that calls for some unavoidable degree of discretion at the time of judging the current state of nature.

V. FINAL REMARKS

The results of this study seem to be consistent with the existing evidence from past bailouts, in that troubled banks have more often received financial aid from the central bank in times of financial crises than at other times. However, the motivations we provide are of a very different nature than those usually invoked to justify the standard LOLR practice. We argue that a well-designed and properly announced bailout policy can indeed reduce rather than worsen moral hazard in banking by increasing the value at stake at the time of risk-taking decisions. In particular, as regards the LOLR function, we found that: (i) Optimal policies should unambiguously commit to making funds available during periods of *adverse macroeconomic conditions*, as opposed to crises that are in itself intimately related with banks' past portfolio decisions; (ii) the central bank should intervene systematically (i.e., as a rule) contingent on those conditions, either providing or withholding emergency funds, to *all* requesting banks.

This paper may be extended in several interesting ways.[21] For example, the analysis suggests that charging a penalty rate on central bank aid may limit the incentive effect of the policy as the burden of the debt reduces expected future rents. On the other hand, provided there are efficiency costs in the use of public money to ensure the continuance of insolvent banks, even if this insolvency was due solely to bad luck, the central bank may find it optimal to charge an "access fee" to the bailout facility, in order to limit the associated quasi-fiscal cost. At any rate, there is an obvious constraint on the size of this fee, as shareholders of financial institutions in distress would accept to participate in a rescue plan only if expected future rents exceed the required contribution. If the bank's share of the burden is too costly, the institution may choose to follow an alternative strategy: forced to make a killing or die, it may choose to gamble for resurrection, with the undesired effects of increasing risk-taking incentives and, by placing central bank money at risk, possibly increasing central bank losses. Thus the threat of a punishment carries no weight if it is associated with a scenario in which the one who is being punished has nothing else to lose and penalties are ex-post unenforceable.

A similar argument applies if we consider the case, deliberately ignored in the previous discussion, of a central bank that cannot commit the necessary resources for a complete bailout, as is typical in economies where financial intermediation is largely conducted in a foreign currency, of which the central bank has only limited holdings. If bank managers

[21] These extensions will be discussed in detail in a future companion paper.

anticipate incomplete bailouts (that is, bailouts that demand a proportional contribution from the troubled bank's shareholders) in the event of a sudden worsening of macroeconomic conditions, the beneficial effect emphasized in the paper may be reduced or simply eliminated. The accumulation of international reserves can certainly limit this possibility, but this solution does not come without costs. This suggest one way in which the present model can be extended to an international context. Inasmuch as aggregate shocks are not perfectly correlated across countries, an international agency could provide similar "insurance services" at a much lower cost in terms of the required reserves. More in general, our results suggest that an international lender of last resort that ensures that liquidity constrained central banks implement the optimal bailout policy in full could contribute to reduce bank risk in the long run.

Second order condition of the bank's maximization problem in the different scenarios

No bailout

Problem:

$$\max_X \frac{(X-r)p(X)\mu}{1-\delta p(X)\mu}.$$

FOC:

$$FOC \equiv \frac{\partial\left(\frac{(X-r)p(X)\mu}{1-\delta p(X)\mu}\right)}{\partial X} = \frac{(X-r)p' + p\,(1-\delta p\mu)}{[1-\delta p\mu]^2} = 0. \tag{22}$$

SOC:

$$\frac{\partial^2\left(\frac{(X-r)p(X)\mu}{1-\delta p(X)\mu}\right)}{\partial X^2} = \frac{(1-\delta p\mu)\left((X-r)p'' + 2p' - 2\delta\mu p p'\right)}{[1-\delta p\mu]^3} + \frac{2\delta p' FOC}{1-\delta p\mu}$$

From (22), $1 > \delta p\mu \Longrightarrow \frac{\partial^2\left(\frac{(X-r)p(X)\mu}{1-\delta p(X)\mu}\right)}{\partial X^2} < 0.$

Default Scenario

Problem:

$$\max_X \frac{p(X)(X-r)\mu}{1-\delta\left[p(X)(\mu-C)+\overline{\beta}\right]}.$$

FOC:

$$FOC_d \equiv \frac{\partial\left(\frac{p(X)(X-r)\mu}{1-\delta\left[p(X)(\mu-C)+\overline{\beta}\right]}\right)}{\partial X} = \frac{(X-r)p' + p\,(1-\delta p\mu) - \delta p^2(\mu-C)}{\left[1-\delta(p(\mu-C)+\overline{\beta})\right]^2} = 0. \tag{23}$$

SOC:

$$\frac{\partial^2\left(\frac{p(X)(X-r)\mu}{1-\delta\left[p(X)(\mu-C)+\overline{\beta}\right]}\right)}{\partial X^2} =$$

$$\frac{\left[1-\delta(p(\mu-C)+\overline{\beta})\right]^2(1-\delta\overline{\beta})(p''(X-r)-2p') - 2\delta p p'(\mu-C)}{\left[1-\delta(p(\mu-C)+\overline{\beta})\right]^3} + \frac{2\delta p'(\mu-C)FOC_d}{1-\delta(p(\mu-C)+\overline{\beta})}$$

From (23) $\frac{1-\delta\overline{\beta}}{\delta(\mu-C)} = \theta > p \Longrightarrow \frac{\partial^2\left(\frac{p(X)(X-r)\mu}{1-\delta[p(X)(\mu-C)+\overline{\beta}]}\right)}{\partial X^2} < 0.$

Non-default scenario

Problem:

$$\max_X \frac{Xp\mu X - r(1+pC-\overline{\beta})}{1-\delta}.$$

FOC:

$$\frac{\partial\left(\frac{Xp\mu X - r(1+pC-\overline{\beta})}{1-\delta}\right)}{\partial X} = \frac{Xp'\mu + p\mu - p'Cr}{[1-\delta]} = 0, \tag{24}$$

SOC:

$$\frac{\partial^2\left(\frac{Xp\mu X - pCr + r\overline{\beta} - r}{1-\delta}\right)}{\partial X^2} = \frac{Xp''\mu + 2p'\mu - p''Cr}{[1-\delta]}.$$

Finally, $\mu > C \Longrightarrow \frac{\partial^2\left(\frac{Xp\mu - pCr + r\overline{\beta} - r}{1-\delta}\right)}{\partial X^2} < 0.$

Proof of Lemma 3:

(i) First note that to find the bailout policy that minimizes the solution to (14), it suffices to find the bailout function $\beta(\gamma)$ that minimizes θ_d.

By definition of the Stieltjes integral, we have that

$$\overline{\beta} = \int_0^1 \beta(\gamma)f(\gamma)d\gamma = \lim_{||\Delta||\to 0}\sum_{k=1}^{n}\beta(\xi_k)[f(x_k) - f(x_{k-1})],$$

and

$$C = \int_0^1 \gamma\beta(\gamma)f(\gamma)d\gamma = \lim_{||\Delta||\to 0}\sum_{k=1}^{n}\gamma_{\xi_k}\beta(\xi_k)[f(x_k) - f(x_{k-1})],$$

with

$$0 = x_0 < x_1 < ... < x_n = 1,$$

$$||\Delta|| = \max(x_1 - x_0, x_2 - x_1, ...x_n - x_{n-1}),$$

and

$$x_{k-1} \leq \xi_k \leq x_k.$$

Let us define:

$$\dot{\theta} \equiv \frac{1 - \delta \sum_{k=1}^{n} \beta(\xi_k)[f(x_k) - f(x_{k-1})]}{\delta(\mu - \sum_{k=1}^{n} \gamma_{\xi_k} \beta(\xi_k)[f(x_k) - f(x_{k-1})]},$$

noting that

$$\theta_d = \lim_{||\Delta|| \to 0} \dot{\theta}_d.$$

It is easy to check the following:

$$\frac{\partial \dot{\theta}}{\partial \beta(\xi_k)} < 0 \Leftrightarrow \gamma_{\xi_k} < \frac{\delta(\mu - \sum_{k=1}^{n} \gamma_{\xi_k} \beta(\xi_k)[f(x_k) - f(x_{k-1})]}{1 - \delta(\sum_{k=1}^{n} \beta(\xi_k)[f(x_k) - f(x_{k-1})]} = \frac{1}{\dot{\theta}_d}.$$

Since this is true for any arbitrary γ_{ξ_k} in any arbitrary partition, defining $\widehat{\gamma} \equiv \lim_{||\Delta|| \to 0} \gamma_{\xi_k}$, and $\gamma^c \equiv \lim_{||\Delta|| \to 0} \frac{1}{\dot{\theta}}$ we have that

$$\frac{\partial \theta}{\partial \beta(\widehat{\gamma})} < 0 \Leftrightarrow \widehat{\gamma} < \gamma^d. \tag{25}$$

This in turn implies that the bailout policy that minimizes θ, and in turn, maximizes the equilibrium effort can be characterized as:

$$\beta^d(\gamma) \equiv \begin{cases} 1 & if\ \gamma < \gamma^d, \\ 0 & if\ \gamma > \gamma^d \end{cases}. \tag{26}$$

If the optimal bailout policy $\beta^d(\gamma)$ is implemented, we have that

$$\overline{\beta}(\beta^d(\gamma)) \equiv \int_0^{\gamma^d} f(\gamma) d\gamma,$$

$$C(\beta^d(\gamma)) \equiv \int_0^{\gamma^d} \gamma f(\gamma) d\gamma,$$

and

$$\theta(\beta^d(\gamma)) = \frac{1 - \delta \int_0^{\gamma^d} f(\gamma) d\gamma}{\delta(\mu - \int_0^{\gamma^d} \gamma f(\gamma) d\gamma)} = \frac{1 - \delta \int_0^{\gamma^d} f(\gamma) d\gamma}{\delta \int_{\gamma^d}^{1} \gamma f(\gamma) d\gamma}. \tag{27}$$

It remains to be proven that there is a unique $\gamma^d \in (0, 1)$. From (27), γ^d must satisfy

$$\gamma^d = \frac{\delta \int_{\gamma^d}^{1} \gamma f(\gamma) d\gamma}{1 - \delta \int_0^{\gamma^d} f(\gamma) d\gamma},$$

or

$$G(\gamma^d) \equiv \gamma^d - \delta \int_{\gamma^d}^{1} \gamma f(\gamma) d\gamma - \delta \gamma^d \int_0^{\gamma^d} f(\gamma) d\gamma = 0. \tag{28}$$

Since $G(\gamma)$ is continuous in γ, $G(0) = -\delta\mu < 0$, and $G(1) = 1 - \delta > 0$, there is at least one $\gamma^d \in (0,1)$ such that $G(\gamma^d) = 0$. Finally, the uniqueness of γ^d is ensured by the fact that $\frac{\partial G(\gamma)}{\partial \gamma^d} = 1 - \delta \int_0^{\gamma^d} f(\gamma) d\gamma > 0$. □

Proof of Lemma 4:

We proceed in steps.

(i) First, we show that the risk minimizing bailout policy belongs to the family of bailout policies

$$\dot{\beta}(\gamma) = \begin{cases} 1 & if\ \gamma \leq \dot{\gamma}, \\ 0 & if\ \gamma > \dot{\gamma}. \end{cases}$$

Notice, since from (20) risk is increasing in C, a necessary condition for a bailout policy to be optimal is that no other feasible policy exists (i.e., such that $r \leq \delta V^n = \frac{Xp\mu X - r(r\overline{\beta} + pCr + r)}{1-\delta}$) leading to a lower level of risk. In particular, since $\partial V^n / \partial C < 0$, if $\hat{\beta}(\gamma)$ is an optimal bailout policy, no other policy $\tilde{\beta}(\gamma)$ should exist such that $\overline{\beta}(\tilde{\beta}(\gamma)) = \overline{\beta}(\hat{\beta}(\gamma))$, and $C(\tilde{\beta}(\gamma)) < C(\hat{\beta}(\gamma))$. Assume that a bailout policy $\hat{\beta}(\gamma)$ with $\hat{\beta}(\gamma) \in (0,1)$ for $\gamma \in [\widehat{\gamma}^-, \widehat{\gamma}^+]$ be a risk minimizing bailout policy. Consider now a bailout policy $\tilde{\beta}(\gamma)$ with $\tilde{\beta}(\gamma) = \hat{\beta}(\gamma)$ for $\gamma \notin [\widehat{\gamma}^-, \widehat{\gamma}^+]$, $\tilde{\beta}(\gamma) = 1$ for $\gamma \in [\widehat{\gamma}^-, \gamma^T]$, and $\tilde{\beta}(\gamma) = 0$ for $\gamma \in [\gamma^T, \widehat{\gamma}^+]$, with $\gamma^T = \{\gamma : \overline{\beta}(\tilde{\beta}(\gamma)) = \overline{\beta}(\hat{\beta}(\gamma))$. Since $C(\tilde{\beta}(\gamma)) - C(\hat{\beta}(\gamma)) = cov(\tilde{\beta}(\gamma), \gamma) - cov(\hat{\beta}(\gamma), \gamma)$, and, by construction of $\tilde{\beta}(\gamma)$, $cov(\tilde{\beta}(\gamma), \gamma) < cov(\hat{\beta}(\gamma), \gamma)$, we have that $C(\tilde{\beta}(\gamma)) < C(\hat{\beta}(\gamma))$, a contradiction. A similar argument proves that no bailout policy such that $\beta(\gamma) = 0$ for $\gamma < \dot{\gamma}$, and $\beta(\gamma) = 0$ for $\gamma > \dot{\gamma}$ can be a risk minimizing bailout policy.

(ii) Second, we show that the equilibrium level of risk $X^n(\dot{\gamma})$ increases with the bailout threshold $\dot{\gamma}$, and is always greater than X^*. In fact, that since $\frac{\partial C(\dot{\gamma})}{\partial \dot{\gamma}} = \gamma f(\gamma) > 0$, from (3), and (20), we have that $\frac{\partial X^n(\dot{\gamma})}{\partial \dot{\gamma}} > 0$.

From (i) and (ii) it then follows that, in the non-default scenario, the optimal bailout policy is given by $\beta^n(\gamma)$. □

Proof of Lemma 5:

We proceed by steps. First we show that the value of the bank increases in $\dot{\gamma}$, for all $\dot{\gamma} \in [0,1]$. For $\dot{\gamma} < \gamma^n$ (default scenario), differentiating (13) with respect to $\dot{\gamma}$ (for any given X, and thus a fortiori when we allow the bank to adjust its choice of risk) yields:

$$\frac{\partial V^d}{\partial \dot{\gamma}} = \frac{\delta p(X)(X-r)\mu}{[1-\delta\left(p(X)(\mu - C) + \overline{\beta}\right)]^2}\left[f(\gamma) - p(X)\gamma f(\gamma)\right] > 0.$$

Similarly, for $\dot{\gamma} > \gamma^n$ (non-default scenario), differentiating (18) with respect to $\dot{\gamma}$:

$$\frac{\partial V^n}{\partial \dot{\gamma}} = \frac{r}{1-\delta}[f(\gamma) - p(X)\gamma f(\gamma)] > 0.$$

By definition of γ^n, we have that $r = \frac{p(X(\gamma^n))X^n(\gamma^n)}{\psi(\gamma^n)}$. Substituting this value into (20) and (14), and rearranging:

$$\lim_{\dot{\gamma}\to\gamma^n} \eta(X)_{px} = \lim_{\dot{\gamma}\to\gamma^n} \eta(X)_{px} = -1 - \frac{\delta C(\gamma^n) p\left(X(\gamma^n)\right)}{1-\delta\bar{\beta}(\gamma^n)\mu p\left(X(\gamma^n)\right)}.$$

which implies that $V^n = V^d$ at $\dot{\gamma} = \gamma^n$, and thus that the optimal choice of risk is continuous in $\dot{\gamma}$. Then it follows that $\delta V^n \gtrless r$ iff $\dot{\gamma} \gtrless \gamma^n$. The fact that risk is monotonically increasing in $\dot{\gamma}$, for $\dot{\gamma} > \gamma^n$ follows from the fact that the risk choice is continuous in $\dot{\gamma}$, and from the proof of Lemma 4.□

Proof of Proposition 2:

(i) By Lemma 4, among all policies $\dot{\beta}(\gamma)$ with $\dot{\gamma} > \gamma^n$, the policy associated with the minimum risk is $\beta^n(\gamma)$. Then, for $\gamma^d > \gamma^n$, $X^n(\gamma^d) > X^n(\gamma^n)$. On the other hand, if $\gamma^d \leq \gamma^n$, we know that $X^d(\gamma^d) \leq X^d(\dot{\gamma})$, for all $\dot{\gamma} \geq \gamma^d$and, in particular, by Lemma 5, $X^d(\gamma^d) \leq X^d(\gamma^n) = X^n(\gamma^n)$.

(ii) If $\gamma^d > \gamma^n$, it follows directly from (2), (3), and (20) that $-1 + \frac{p'(X)C(\gamma^n)r}{\mu p(X)} < -1 \Rightarrow X^n \geq X^*$. If $\gamma^d \leq \gamma^n$, we know that $\psi(\gamma^d) = \theta(\gamma^d) - \frac{C(\gamma^d)(\theta(\gamma^d) - p(X^d))}{\mu} > \theta(\gamma^d)$, from which $r \geq \frac{p(X^d)X^d}{\psi} > \frac{p(X^d)X^d}{\theta_n}$, and substituting into (14), $\eta(X^d)_{px} < -1$. □

Figure 1: Bank's Optimal Risk Choice (δ=0.8)

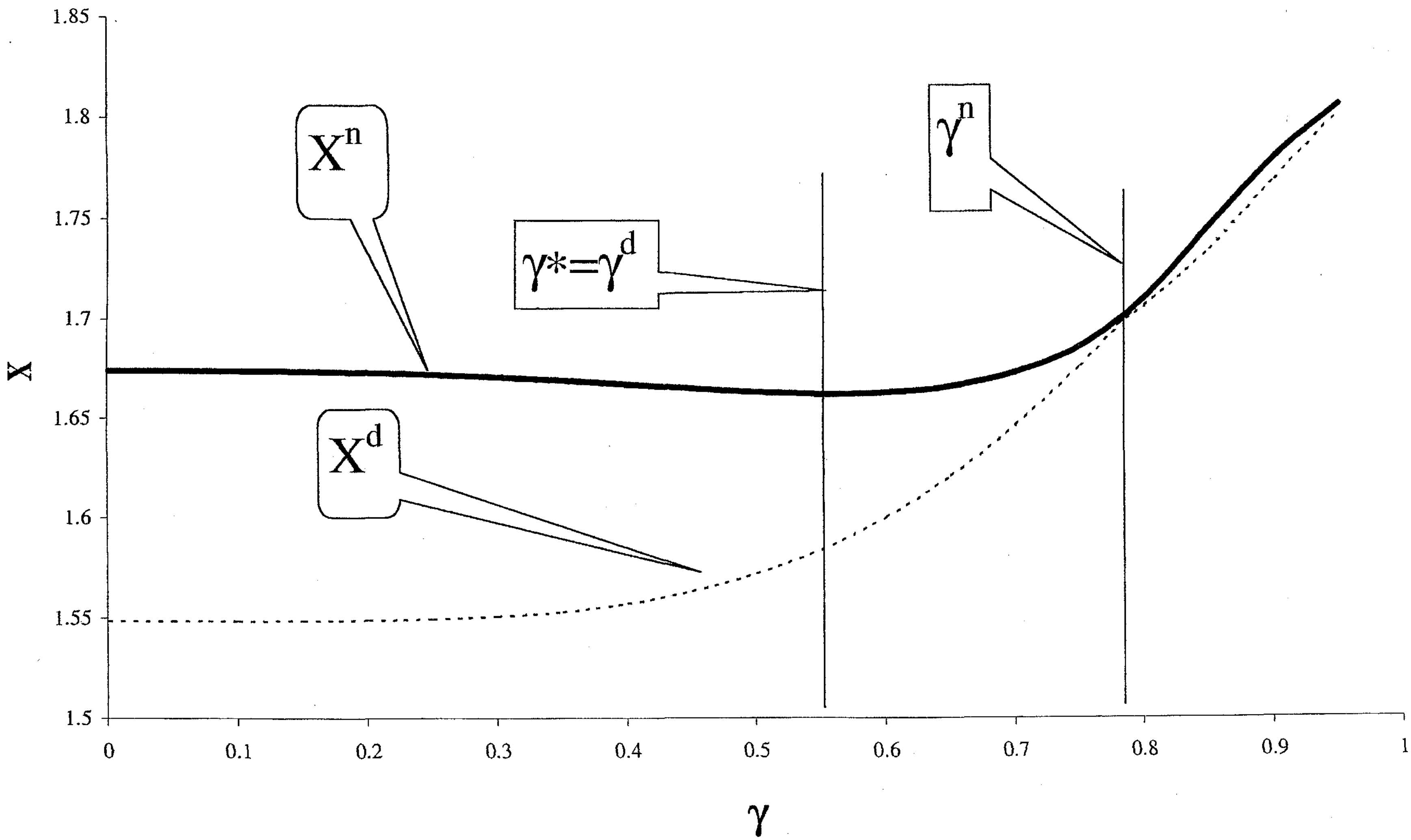


Figure 2: Bank's Optimal Risk Choice (δ=0.9)

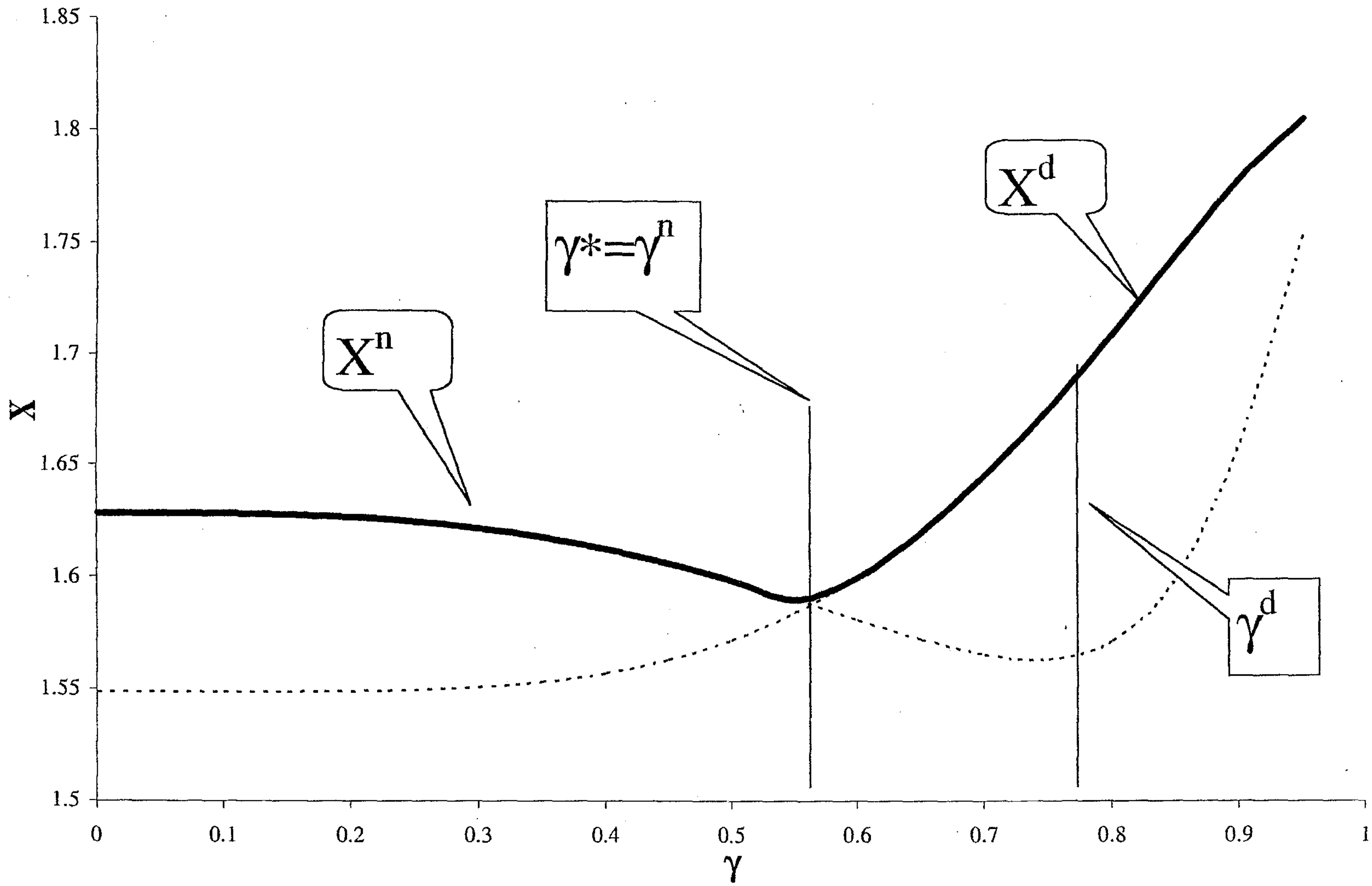
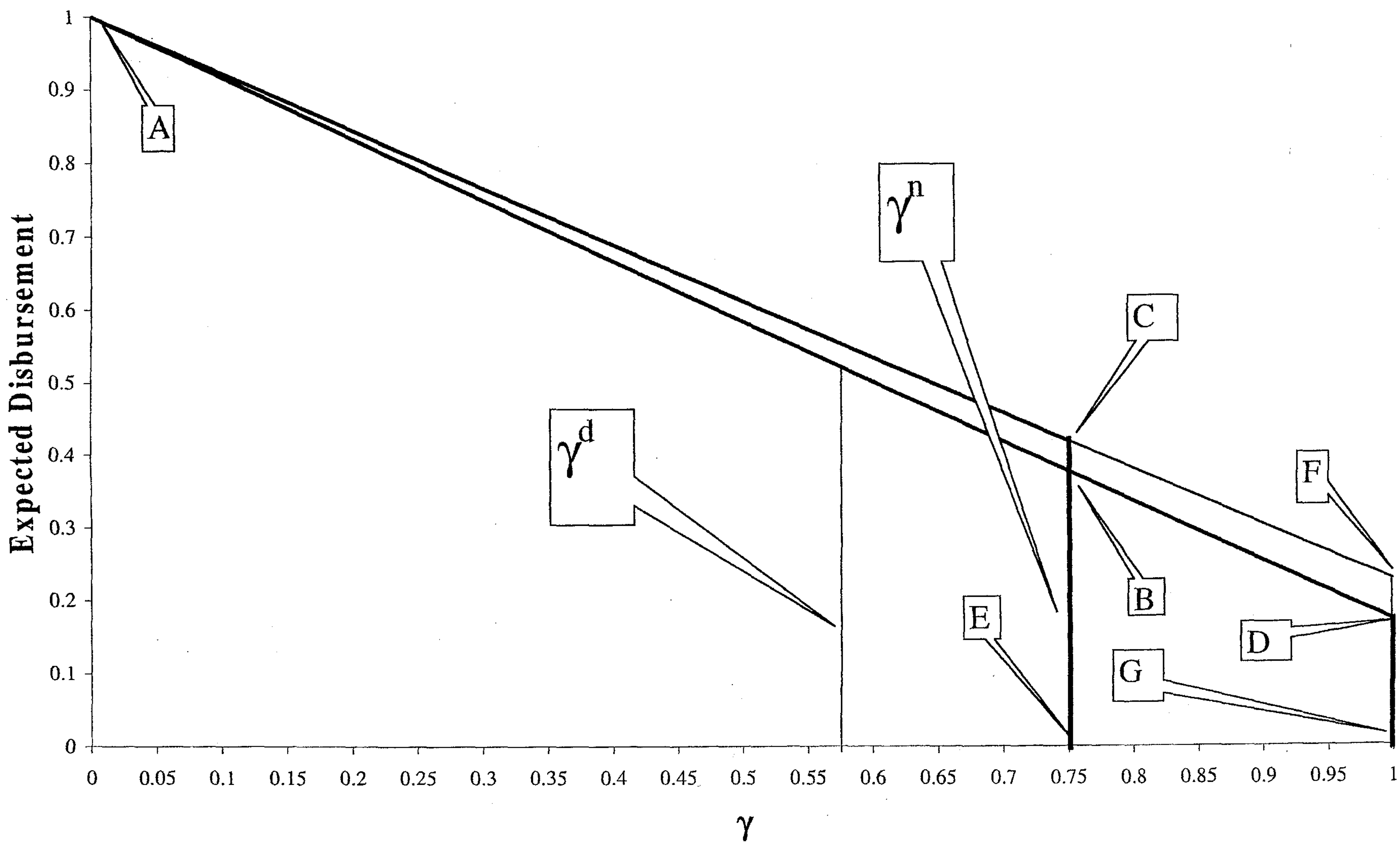


Figure 3: Quasi-Fiscal Outlays (δ=0.8)



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