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What are reference rates for?

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Abstract

What is the precise role of reference rates? Why does it matter if LIBOR was manipulated? To address these questions, I analyze the use of reference rates in floating-rate loans and interest-rate derivatives in the context of lending relationships. I develop a simple framework combining maturity transformation with three key frictions which generate meaningful funding risk and a rationale for risk management. Reference rates like LIBOR mitigate contractual incompleteness, facilitating management of funding risk. As bank funding costs move with bank credit risk, it makes sense for the reference rate to have a bank credit risk component. Manipulation can add noise, reducing the usefulness of reference rates for this purpose.

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1 Introduction

Contingent financial contracts allow parties to agree to payments which are determined in the future, depending on how circumstances evolve. In a broad sense contract theory has studied their role in providing incentives and facilitating risk sharing. An important class of contingent contracts, including loans and related derivatives, involves interest rates. These contracts provide for payments that are contingent on reference rates: frequently determined and readily available interest rates. The stated aim for using these contracts is typically risk management. Contingent versions of contracts are also sometimes thought to be cheaper (for instance the interest rate may be perceived to be lower, in expectation, with a variable rate loan compared to a fixed-rate loan). The characteristics of the reference rate determine the ways in which these contracts can be used for risk management.

Many interest-rate-dependent contracts, particularly those denominated in dollars, refer to the London Interbank Offered Rate, LIBOR, which provides a daily measure of interest rates in interbank transactions in major currencies between large banks.[1] Loans and derivatives are large asset classes, and LIBOR is quite dominant: consequently, contracts with a total notional value of as much as $300TN are estimated to refer to LIBOR (FSA 2012). Reference rates have attracted significant attention in recent years due to allegations of their manipulation. In April and May 2008, the Wall Street Journal ran articles suggesting that LIBOR was being manipulated.[2] Global banks have since been fined billions of dollars by regulators from several countries for attempting to manipulate LIBOR. Numerous lawsuits have been filed against banks by US municipalities and GSEs.[3] Belying the importance of well functioning reference rates, an alphabet soup of regulators and official bodies has responded (FSA 2012, BIS 2013, IOSCO 2013, FSOC 2016). LIBOR even merits a dictionary entry providing a timeline of the various allegations of misconduct (Hou & Skeie 2013). Most recently, individual traders have been prosecuted for their role in manipulating LIBOR.[4]

[1] Contracts denominated in Euros typically refer to EURIBOR, which is similar but subtly different; see Section 2 and Footnote 12.

[2] See *The Wall Street Journal, Bankers Cast Doubt On Key Rate Amid Crisis, April 2008, and Study Casts Doubt on Key Rate, May 2008*.

[3] See, for example, *The New York Times, Fannie Mae Sues 9 Banks Over Libor, October 2013*.

[4] One trader convicted in connection with manipulation of LIBOR has been handed a long jail sentence (eight years) and a personal monetary penalty in excess of one million dollars. See *USA Today*, Trader ordered to pay $1.24M for Libor

The small, but growing, academic literature on LIBOR has mainly addressed two issues: detection and quantification of manipulation, and ways LIBOR could be replaced or determined differently.[5] However, relatively little attention has been paid to the purpose that reference rates serve.[6] In light of the new policy relevance of reference rates, this is an important topic to study. In particular, in order to contemplate modifying or replacing LIBOR, it is important to understand how and why it is used. As many have noted, floating interest rates and interest-rate derivatives allow institutions to manage exposure to short-term interest rates. However, it is important to understand which specific risks are being managed and why. For example, as discussed in Section 2, LIBOR has a bank credit risk component. Alternatives that have been proposed, including market rates for Overnight Indexed Swaps (OIS) or repurchase (repo) agreements, do not. Is it important for the reference rate to capture bank credit risk? More generally, how do the properties of reference rates affect how useful linked contracts are for hedging?

In this paper I focus on the role reference rates play in the market they originated from: bank lending to firms. I analyze the use of reference rates in floating-rate loans and in the associated use of interest-rate derivatives using a simple model of maturity transformation. Three key frictions generate a framework in which the use of reference rates makes sense. First, I assume that the lender must use short-term liabilities to fund long-term loans, generating maturity mismatch. This is best thought of as a stronger version of the assumption that short-term funding is cheaper for banks. A long literature argues that deposits provide liquidity and safety (Gorton & Pennacchi 1990, Gorton 2010, Diamond & Dybvig 1983, Stein 2012).

Next, I assume that the bank's own cost of funding is not contractible. This is a natural way to capture the difficulty borrowers have with observing borrowing costs directly and the skepticism they are likely to have about banks' incentives to reveal them truthfully. Reference rates serve the purpose of mitigating this contractual incompleteness. These first two assumptions generate funding risk that, in the absence of a contingent contract, the lender must bear. Finally, I argue that financial frictions

rigging, March 2016.

[5]I discuss this literature further in Section 2.

[6]See Duffie & Stein (2015) for a broad discussion of the roles reference rates can play.

lead to effective risk aversion by making firm profit functions concave, following Froot, Scharfstein & Stein (1993) and Froot & Stein (1998). These three frictions combine to produce a setting in which reference rates facilitate hedging of meaningful funding risk.

This framework permits analysis of how the availability of floating-rate loans and interest-rate derivatives affects credit markets. In their absence, lenders can only offer fixed interest rates and must bear funding risk themselves. When floating rates are available, this risk can be transferred to borrowers, lowering the cost of borrowing. However, whether welfare increases depends on how risk averse firms are. Interest-rate swaps permit this risk to be sold to the market for a price, which depends on how well the market is positioned to bear interest-rate risk. As long as aggregate risk aversion is not too high, it would be better for lenders to use swaps for risk management than to transfer risk to borrowers through floating-rate loans.[7]

In practice, firms typically borrow at floating rates and subsequently manage the risk, at least partially, with swaps.[8] Why might this be better than lenders managing the risk directly? One advantage of this more complicated arrangement is that firms can join the set of institutions bearing interest-rate risk, reducing aggregate risk aversion and the cost of hedging this risk. It may be easier for firms to justify retaining more floating-rate exposure from a floating-rate loan by not hedging it than acquiring such exposure through 'naked' swaps. Alternatively, if derivatives markets are concentrated and lenders provide floating loans and derivatives as a package this arrangement could be a way for lenders to channel volume to high margin activities. As contracts linked to the reference rate serve to insulate the lender against funding risk, it makes sense in this context for the reference rate to be linked to bank funding costs, which are related to bank credit risk. Manipulation which simply makes the reference rate more volatile reduces welfare by diminishing its usefulness for hedging.

The analysis relates to themes considered in several strands of the literature. Brousseau, Chailloux & Durré (2013) propose that reference rates should capture bank funding costs across instruments, in-

[7]This assumes that there are no other costs associated with using interest-rate swaps. Collateral requirements and associated implicit costs can change the nature of the cost of hedging substantially (Rampini & Viswanathan 2010, Rampini, Sufi & Viswanathan 2014, Rampini, Viswanathan & Vuillemey 2015, Ivashina, Scharfstein & Stein 2015, Kirti 2016).

[8]I document that firms only hedge interest-rate risk partially with swaps (Kirti 2016), and argue that collateral requirements limit the use of swaps. Here I explore other potential explanations.

formally arguing that reference rates should be related to bank funding costs. Detragiache (1992) argues that floating-rate loans optimally insure developing countries against fundamental shocks in an incomplete contracting framework given the empirical correlation between LIBOR and these fundamentals. The framework demonstrates conclusions made by Barzel (1982) that good standards are accurate and low cost. I also use the idea that hedging need not be a zero NPV proposition if there is aggregate risk aversion, as suggested by Demsetz (1969). The effects of frictions in financial intermediation have also attracted attention recently. Gabaix & Maggiori (2015) build up effective risk aversion for financial institutions from a commitment problem, studying how this affects exchange rates when intermediaries have to bear exchange rate risk. Shin (2012) constructs a demand curve for risk by modeling risk management as a constraint (either self- or regulator-imposed) on the probability of large losses, arguing that the resulting fluctuations in leverage for global banks transmit financial conditions across borders.

The remainder of this paper is organized as follows. Section 2 provides more background on LIBOR and related literature. Section 3 details the set up and the three key frictions described above. Section 4 analyzes the basic uses of reference rates. Section 5 considers derivatives in more detail, and discusses interpretation and welfare loss from manipulation. Section 6 concludes.

2 Background on LIBOR

Growth in the magnitude of contractual payments affected by LIBOR has primarily been driven by the explosive growth in volumes of interest-rate derivatives over recent decades. These derivatives were introduced as part of the wave of financial innovation which responded to the uncertain environment and restrictions firms faced in the 1970s and 1980s. The breakdown of Bretton Woods and the resulting fluctuations in exchange rates, in addition to high inflation during this period, led firms to try to actively manage risk. Currency swaps, first introduced in 1979, were a more efficient version of agreements that allowed firms to skirt British foreign exchange controls in the 1970s. Interest-rate swaps were a natural extension in which different interest rates in the same currency were exchanged, introduced in London in 1981 and first used in the US in 1982 by Sallie Mae.[9]

One of the drivers of the early growth of the interest-rate swap market was a coincidence of mutual need. In the early 1980s, financial institutions in the US funded long-term, fixed-rate loans such as mortgages with short-term deposits such as CDs. The interest rates on these short-term deposits fluctuated, moving around with T-bill rates or CD rates. Financial institutions were exposed to the risk that their cost of borrowing could substantially rise due to this mismatch between assets and liabilities. At the same time, banks in Europe typically funded themselves through long-term, fixed-rate Eurobonds, but extended variable-rate loans. These opposite problems meant that interest-rate swaps could allow both sets of institutions to manage interest-rate risk. The reason these swaps referenced LIBOR seems to have been that variable-rate loans made to households in Europe referred to LIBOR during this period.[10]

LIBOR is now widely used as a reference rate for interest-rate derivatives as well as for loans to firms and households. The name was first used to label an arrangement created to define payments on a \$80MN loan to the Shah of Iran in 1969 by a syndicate of banks led by Manufacturer's Hanover (now a part of JP Morgan Chase). Shortly before agreed dates at which the interest rate would be updated, large banks in the syndicate reported their funding costs, an average of which became the new interest rate. As the use of LIBOR became more popular, banks' funding costs started to be tied LIBOR, providing an incentive for banks to underreport. A more formal version of LIBOR was introduced in 1986 to address this issue, administered by the British Bankers' Association.[11] LIBOR is determined as a trimmed average of submissions from a panel of large banks: each bank submits one number to a calculating agent, which reports an average after discarding some high and low outliers. Each bank is asked to estimate the rate at which it could borrow on the interbank market 'in reasonable market size' in the morning London time.[12] LIBOR is calculated for several currencies and maturities, though the

[9] See Beckstrom (1986), Campbell & Kracaw (1993) and Marshall & Kapner (1993).

[10] See Campbell & Kracaw (1993). Note that it is typical for swaps to be intermediated, so that these institutions may not have faced each other directly.

[11] This is essentially the present version of LIBOR. See *Business Insider, A Greek Banker on the Early Days of the LIBOR, August 2012* for an account of this first loan. This account argues that demand for these floating loans was driven by a desire for firms and countries to avoid domestic regulation and taxes. As of February 2014, ICE administers LIBOR (the FSA (2012) recommended that the BBA no longer administer LIBOR).

[12] No transactions are required to have taken place at the rates reported, nor are submissions interpreted as quoted rates at

three month dollar LIBOR is the most commonly used rate.

Informally, the interest rate for any transaction can be considered to be the sum of several components, including a risk-free rate, a term premium and a credit risk premium. LIBOR can be thought of as containing the risk free rate, the term premium, and a component pertaining to the credit risk of the panel of banks that submit rates for it. As banks are asked to estimate the cost they would be able to borrow at, some measure of their own credit risk must appear (BIS 2013, Hou & Skeie 2013). Such a rate makes sense in credit markets, where the risk being hedged is that bank funding costs, which have a credit risk component, change.

The interbank lending market is an over-the-counter market. Transactions and interest rates are not publicly announced, and this is why a survey based measure like LIBOR is necessary to create a reference rate tied to the interbank market. At the time LIBOR and interest-rate swaps were first introduced, interbank markets were a liquid market banks used to funded themselves. However, since then large savers have shifted away from bank deposits, forcing banks to fund themselves through CDs or secured financing arrangements like repurchase agreements (Brousseau et al. 2013, BIS 2013). The interbank market, particularly at maturities beyond weeks, is therefore now quite thin. Indeed, even if transactions were collected over a ten day window, only a handful might occur at the three month maturity (Duffie, Skeie & Vickery 2013).

At the same time that the interbank market has thinned, the volume of assets that reference LIBOR has substantially grown, driven by explosive growth in derivatives volumes. Outstanding notional amounts of interest-rate derivatives have grown to hundreds of trillions of dollars.[13] The Wheatley Review conducted by the FSA estimates that $300TN of assets reference LIBOR, including $10TN in syndicated loans.[14] The unfortunate combination of a large volume of transactions related to LIBOR and an opaque and sparse underlying interbank market permitted the recent scandal as banks attempted

which banks are committing themselves to borrow or lend (Gyntelberg & Wooldridge 2008). Note that EURIBOR is subtly different; banks are asked to estimate the rate offered 'by one prime bank to another.'

[13]The BIS publishes statistics on derivative notional amounts semi-annually. Notional amounts have fallen in recent years due to compression.

[14]This estimate may be high as it appears to include derivatives denominated in Euros, which are likely to reference EURIBOR (for example, most euro denominated exchange traded derivatives are linked to EURIBOR). However, the broader point is that the volume of assets which require a reference rate is increasing dramatically.

to manipulate LIBOR. As settlements with regulators make clear, banks' incentives were driven by direct exposure to LIBOR through interest rate derivatives as well as reputational concerns.[15]

Several regulatory bodies have produced reports commenting on LIBOR. A prominent example is the US FSOC, which identifies continued reliance on reference rates as one of its themes for concern about the financial system in several annual reports, including the most recent one (FSOC 2016). The FSA's Wheatley review and the International Organization of Security Commissions (IOSCO) suggest improved regulatory frameworks and audit trails (FSA 2012, IOSCO 2013). The Bank for International Settlements' Economic Consultative Committee emphasizes that network effects and logistics significantly complicate choosing a replacement (BIS 2013). Duffie & Stein (2015) summarize the recommendations put forward by the many groups convened by the Financial Stability Board to address the issue of reference rates. A key recommendation is to reduce reliance on reference rates with a credit risk component.[16]

There is also a growing academic literature on LIBOR. One question of interest has been to identify manipulation using submissions and data on other short-term interest rates. Hou & Skeie (2013) summarize the early papers, arguing that they did not find clear cut evidence of manipulation. More recently, Eisl, Jankowitsch & Subrahmanyam (2013) use the distribution of bids to estimate the maximum amount LIBOR could have been manipulated by a perfectly informed bank. Youle (2015) estimates the magnitude of manipulation by analyzing how bids responded to changes in how much the bank expected to be able to affect LIBOR, estimating that since the crisis LIBOR is about 8 basis points lower than it would be absent portfolio incentives to manipulate. Both of these papers use the fact that as outliers are removed, how much any bank can shift LIBOR by changing its submission depends on the distribution of the remaining bids. Gandhi, Golez, Jackwerth & Plazzi (2015) instead estimate bank exposures to interest rates by regressing stock returns on changes in the reference rate, implicitly

[15] See, for example, Barclays' settlement with the FSA. Barclays was the first bank to settle with regulators. Bank submissions were, until recently publicly disclosed. They are now disseminated after a three month delay, to reduce reputational concerns for banks reporting higher borrowing costs than the rest of the panel. Hou & Skeie (2013) provide a timeline of the various accusations and actions taken in recent years.

[16] As the discussion earlier shows, a US dollar reference rate based on the offshore market for dollar funding is primarily a historical accident. As onshore and offshore markets may not always move in sync, this might lead to more basis risk. Alternatives being put forth now (such as a rate based on general collateral repurchase transactions) are onshore.

assuming that the market can estimate these exposures. They label correlation between submissions and these estimated exposures as manipulation, and estimate that absent this correlation panel banks' market value would have been more than $20 BN lower.

Another question of interest has been whether LIBOR can be calculated differently or replaced. Taking a mechanism design approach, Coulter & Shapiro (2013) propose a generalization of the Moore-Repullo mechanism, and Chen (2013) suggests the AGV mechanism. However, if bank funding costs and exposures to the reference rate are private information, it does not seem possible to construct a mechanism that eliminates manipulation. This does not mean, of course, that mechanisms cannot be designed to reduce manipulability.[17]

The FSA (2012) considers but rejects alternatives such as a random bid or the median bid using historical bid data to assess their impact. Youle (2015) simulates how bids would change if a median

[17] Consider the issues arising from portfolio driven incentives to manipulate. Let X_{it} refer to bank i's exposure to the reference rate at time t, for instance through swaps. As swaps settle based on the level of the reference rate on specific days (based on days the swap payments are to be made, typically at six month frequencies), this exposure can vary significantly at short horizons. Bank settlements with regulators show that traders requested submitters to manipulate on days when exposures were large.

If manipulation is costly (for instance due to ex post punishments if the bank is caught), the bank's payoff might be represented as

$$X_{it}R(r_{it}, r_{-it}) - C(r_i - \bar{r}_{it})$$

where $\bar{r}_{it}$ is the bank's true cost of funding, r_{it} is what it reports, and other banks in the panel report r_{-it}. The first order condition implies that the bank should choose its report to satisfy

$$C'(r_{it} - \bar{r}_{it}) = X_{it}\frac{\partial R(r_{it}, r_{-i})}{\partial r_{it}}$$

As long as the right hand side of this equation is not identically zero, manipulation occurs. Can a mechanism eliminate this problem? Suppose the mechanism outputs $R(r_{it}, r_{-it})$, and assign transfers $T_i(r_{it}, r_{-it})$. The first order condition now becomes

$$C'(r_{it} - \bar{r}_{it}) = X_{it}\frac{\partial R(r_{it}, r_{-it})}{\partial r_{it}} + \frac{\partial T_i(r_{it}, r_{-it})}{\partial r_{it}}$$

It is clearly not possible to set the bank's effect on LIBOR to always be zero. Similarly, as long as X_{it} and the bank's true cost of funding, r_{it} are private information, it is not possible to set transfers to eliminate manipulation. This argument is related to an impossibility result in Hurwicz (1972) driven by private endowments. As noted in the text, this does not mean that mechanisms cannot reduce manipulation.

In contrast, Coulter & Shapiro (2013) assume that each bank's funding costs are known to two other banks, which allows these banks to play the role of a whistleblower if reports are not truthful, in a generalization of the Moore-Repullo multi-stage mechanism. Chen (2013) argues that bank funding costs are tied to LIBOR, and that banks therefore have an incentive to shift LIBOR down. It is assumed that the strength of this incentive relative to the cost of manipulation is the same across banks and constant over time, in which case a version of the AGV mechanism is applicable.

Note that Youle (2015) aims to identify manipulation based on changes in how much the bank expects to affect LIBOR, and not changes in the exposure. I construct a mechanism based on trading which reduces manipulation in Kirti (2013). Duffie & Dworczak (2014) characterize the optimal weights to place on reports from different participants to minimize distortion.

bid was used. Under the distribution of banks' costs of funding which he infers from historical bids, any particular bank would be uncertain whether it would be the median bank, both before and after attempted manipulation, and would therefore find it much less attractive to manipulate. Abrantes-Metz & Evans (2012) propose requiring banks to commit to trade in a 'committed quote' range. Duffie et al. (2013) examine the practicality of using past transactions over extended windows and express concerns that this would lead to a stale reference rate. Brousseau et al. (2013) propose a trade-weighted average across bank wholesale funding operations. If this ideal cannot be achieved, they suggest using rates from overnight indexed swaps (OIS), an alternative that has also been considered by by policy makers (BIS 2013, Duffie & Stein 2015).

In this paper I take a step back and analyze the rationale behind contracts linked to reference rates. What is the precise role of the reference rate? Indeed, why does it matter if LIBOR was manipulated? This analysis should inform how reference rates should be thought about. For example, Brousseau et al. (2013) suggest that the ideal reference rate should more accurately reflect bank borrowing costs across different types of funding operations. Why might this make sense? The framework I construct demonstrates that in the context of bank lending, as the role of the reference rate is to facilitate hedging funding risk, reference rates without such content may be less useful.

3 The basic framework

In order to understand the role of reference rates, I develop a simple model of maturity transformation incorporating uncertainty regarding future short-term financing costs. As discussed below, these costs are not contractible. Therefore, in the absence of reference rates, lenders must bear the associated funding risk. Reference rates are estimates of current short-term funding costs. Their existence permits contracts to refer to their levels in future, mitigating contractual incompleteness. In Section 3.1 I set up the model. Section 3.2 discusses key frictions in the presence of which managing funding risk is desirable. I analyze the use of floating-rate loans and derivatives to do this in Sections 4 and 5.

What is funding risk? The financial crisis provided one prominent example. Figure 1 shows the outstanding volume of C&I loans (for US commercial banks) and LIBOR. Of particular interest is the month (shaded) when the crisis was in its acute phase. Suppose that loans had to have fixed interest rates, and that banks borrowed short term to make the loans. In this case, banks would have to bear the risk that their cost of funding might rise, as it did in this period. In contrast, with floating rates, this risk is transferred to borrowers. I ask what incidence of risk makes sense and why institutions might care about this risk.[18]

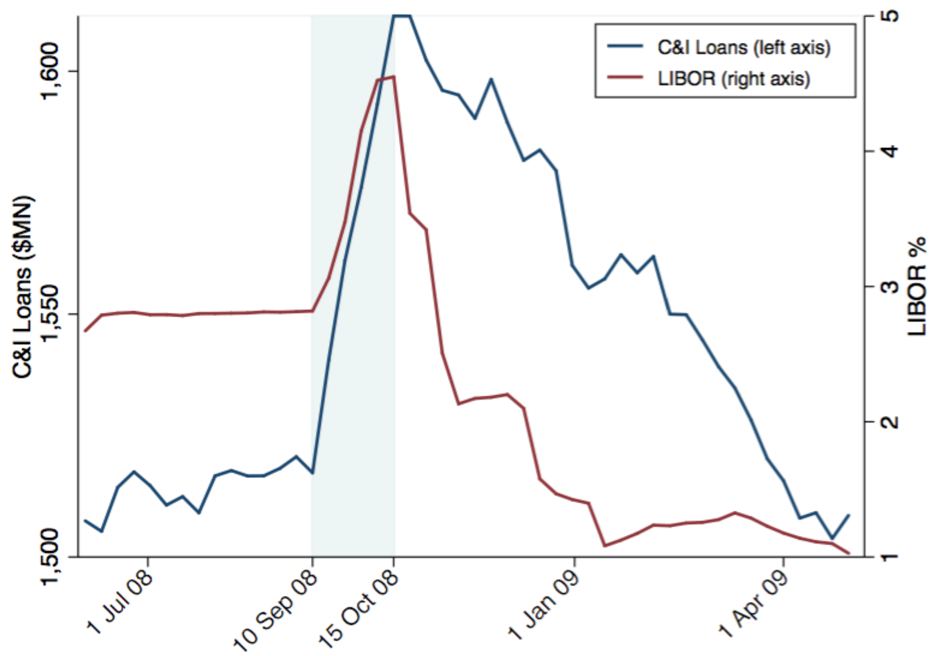

Figure 1: *Interest rate risk in the crisis (Sources: FRED and Bloomberg)*

3.1 Set up

In general banks take on several forms of risk when they provide loans at a different interest rate and maturity than the liabilities that fund them. I focus on funding, or interest-rate, risk, which arises from uncertainty regarding the cost of short-term financing. Term premium, which is related to the opportunity cost of locking up a given amount of resources and being unable to invest them in better

[18]Note that the reported jump in loan volume occurred because firms drew down on commitments (Ivashina & Scharfstein 2010). In the analysis that follows, I focus on loans for simplicity.

opportunities that may arise subsequently, is perhaps on a basic level the risk that banks are compensated for bearing. Credit risk due to uncertainty about how much the borrower will be able to or willing to repay is often an important consideration. I abstract away from the latter two sources of risk.

I analyze a model with three periods: t_0, t_1, t_2. I will consider a transaction between a single firm and a single bank – the analysis applies more generally. The bank lends L to a firm at t_0 for a project with cash flows $L(1+P)$ at t_2 (and no intermediate cash flows). In return, the firm promises to pay $L(1+\mu)$ at t_2. Thus μ is the (fixed) interest rate. The quantity of credit, L, and the interest rate μ, will be determined in equilibrium.

The bank finances itself with a sequence of two short-term loans from investors. It borrows L from investors at t_0, and must pay $L(1+S_0)$ back at t_1 (I make the normalization $S_0 = 0$ to simplify notation). It finances its repayment at t_1 by borrowing L again at t_1, promising to pay $L(1+S_1)$ at t_2. Figure 2 illustrates the timeline. Vertical arrows indicate payments and their directions, in the periods that they occur.

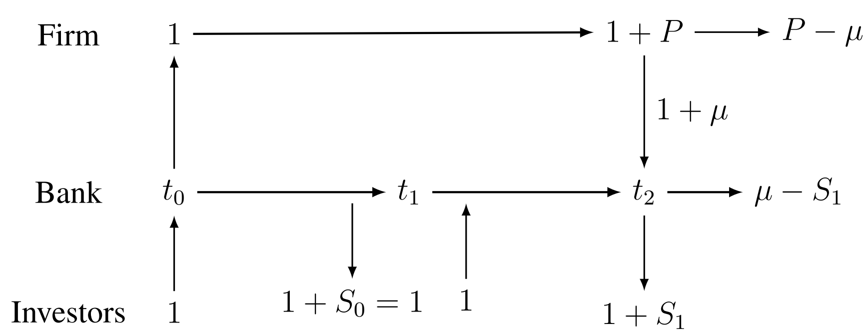

Figure 2: *Timeline (per dollar of lending)*

Project outcomes, P, and short-term funding costs, S_1, are assumed to be Normally distributed (for a generic random variable X, I use the notation $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$, where μ_X and σ_X^2 are the mean and variance). I assume $\mu_P > \mu_S$ so that in expectation the project is worth funding.

The publicly determined reference rate, denoted R_1, will also be Normally distributed. To simplify notation, I assume that the reference rate has mean zero ($\mu_R = 0$). I denote the covariances with short-term funding costs and project outcomes $\text{Cov}(S_1, R_1) = \rho > 0$ and $\text{Cov}(P, R_1) = \pi$ respectively. Clearly the reference rate should be positively correlated with funding costs: this is its basic role. I note assumptions regarding these covariances as used. A baseline case to keep in mind is $\pi = 0$, i.e. that project outcomes are not correlated with funding costs.

3.2 Frictions

Three key frictions generate meaningful funding risk and a desire to reduce it. First, short-term funding must be used. This assumption reflects the fact that maturity transformation is central to bank intermediation. While in practice some longer term financing is used, banks do not fully match the maturities of their assets and liabilities. A substantial majority of commercial bank liabilities continue to be in the form of interest-bearing deposits. In their seminal paper, Diamond & Dybvig (1983) model banks as transformers of maturity: investing in long-term illiquid assets, while providing liquidity insurance through short-term or demand deposits. More recently, Stein (2012) argues that a preference for the safety associated with short-term assets makes it cheaper for banks to fund themselves with shorter term liabilities.[19]

Second, S_1 is not contractible. A contract that referred directly to an individual bank's funding cost would likely be viewed with suspicion. For example, some retail mortgages in the UK are pegged to discretionary rates. During the crisis banks lowered these rates slowly while policy rates were drastically lowered. Since the crisis banks have been raising their rates even as policy rates have stayed low.[20] One interpretation is that large corporate borrowers have sufficient bargaining power to insist on a formal index. As discussed in Section 2, LIBOR began as a syndicate specific survey for a specific loan. The current formal version was introduced due to concerns with such an arrangement. Reference rates can thus be viewed as the standardized, publicly available and hence contractible version of bank funding costs.

Finally, S_1 is not known until t_1, and financial frictions prevent the bank from being risk neutral. In practice, banks in particular and firms in general do pay attention to hedging risks. However, while

[19]The literature on banking, e.g. (Diamond & Rajan 2001), has also argued that short-term financing provides financial intermediaries the necessary incentives to monitor.

[20]These rates are known as 'Standard Variable Rates'. See, for example, *The Telegraph*, Mortgage borrowers locked into high rates, January 2013.

short-term financing costs are volatile and unpredictable uncertainty regarding profits per se does not invalidate the standard view that firms should maximize expected profits (Modigliani & Miller 1958). As I argue that contracts linked to reference rates exist in order to facilitate this hedging, it is important to understand why hedging is necessary in the first place.

The Modigliani-Miller view is that shareholders can replicate any hedging strategy the firm may follow, where the firm's actions are treated as independent of financial transactions. In contrast, Campbell & Kracaw (1993) argue that the prospect of losses might prevent firms from executing their strategy. Froot et al. (1993) formalize a similar argument. I follow Froot et al. (1993) and Froot & Stein (1998), and assume financial frictions, in the presence of which banks and firms are effectively risk averse.[21] The crux of the argument is that if today's risky payoffs affect how much can be invested in the following period when investment opportunities are concave, the profit function inherits this concavity. This concavity creates effective risk aversion. To understand the intuition, suppose that following the three periods that are modeled above, the bank has concave investment opportunities $F(I)$. Denote the internal funds that depend on risky payoffs by w. If the firm can raise no external funds, the profit function inherits this concavity: $P(w) = F(w)$. A firm maximizing expected profits when the profit function is concave is effectively risk averse.

Froot et al. (1993) point out that even if external financing is available, the profit function is still concave if external financing is costly. Denote the difference between investment and internal funds by $e = I - w$. If this external financing is associated with a convex cost $C(e)$, the profit function continues to be concave.

$$P_{ww} = F_{II} \frac{dI^*(w)}{dw} \propto -F_{II} C_{ee} \tag{1}$$

What financial frictions generate a convex cost of external finance? I follow Froot et al. (1993) and use a version of the costly state verification model of Townsend (1979). More precisely, the output of production denoted by $F(I)$ cannot be used as collateral to borrow (for instance because the project has no liquidation value). Instead all borrowing must be collateralized by risky cash flows y, distributed

[21] As discussed in Section 1, there other approaches to understanding why risk management is relevant. Gabaix & Maggiori (2015) build up effective risk aversion for financial institutions from a commitment problem. Shin (2012) constructs a demand curve for risk by assuming a constraint on the probability of large losses (a VaR constraint).

$g(y)$, generated by existing assets. The only action external financiers can take to force repayment is to liquidate the assets generating these risky cash flows, at a cost. As Appendix A shows, under general conditions this set up generates concavity of the production function.[22]

I use a simplified version of the framework employed by Froot & Stein (1998) to generate a tractable form for the risk aversion arising from the concavity of the profit function. Suppose the bank has earlier chosen some level of capital K, and can currently add θ units of a Normally distributed payoff X to its balance sheet. The funds available to be invested next period are then $w = w_0 + \theta X + K(1 - \tau)$. τ is a deadweight cost of cash held on the firm's balance sheet so that the unmodeled choice of capital structure is meaningful.[23]

Now the firm's optimal allocation to the payoff X can be chosen to maximize expected profits. The first order condition provides

$$\begin{aligned}\frac{dEP(w)}{d\theta} &= E\left(P_w \frac{dw}{d\theta}\right) = \operatorname{Cov}\left(P_w, \frac{dw}{d\theta}\right) + EP_w E\frac{dw}{d\theta} \\ &= EP_{ww} \operatorname{Cov}(w, X) + EP_w EX \\ &= \theta EP_{ww}\sigma_X^2 + EP_w \mu_X = 0\end{aligned}$$

where the third equality uses the fact that for x, y normally distributed, $\operatorname{Cov}(f(x), y) = E(f_x) \operatorname{Cov}(x, y)$ and that $\frac{dw}{d\theta} = X$. Solving for the optimal allocation

$$\theta^* = \frac{\mu_X}{A\sigma_X^2} \implies A = -\frac{EP_{ww}}{EP_w} \tag{2}$$

the allocation is the same as what would arise from a CARA utility function. The endogenous coefficient A is similar to the standard coefficient of risk aversion, except that the level of curvature of the profit function depends on the earlier level of capital chosen by the bank.[24] I make the argument with reference to banks, but it evidently applies to firms more generally.

[22] It is also necessary that g has sufficient weight on the right tail. See Appendix A.

[23] Froot & Stein (1998) motivate this as the tax disadvantage of not issuing debt.

[24] Financial frictions generate a positive coefficient of risk aversion as the profit function is concave.

In the remainder of this paper I directly view the bank and firm as maximizing CARA utility functions, with exogenous coefficients of risk aversion A_B and A_F respectively. The usual first order conditions provide

$$\arg\max_{\theta} -E\exp(-A(W_0 + \theta X)) = \frac{\mu_X}{A\sigma_X^2} \tag{3}$$

The correspondence with Equation 2 is clear. I refer to $A\sigma_X^2$ as the utility cost of risk. This optimal allocation trades off expected return against the utility cost of risk. If there are N participants with CARA preferences, their aggregate behavior is equivalent to that of a single agent with risk aversion $\frac{\bar{A}}{N}$, where $\bar{A}$ is the harmonic mean of individual risk aversion. In this sense the analysis is not specific to the case of a single bank dealing with a single firm.

4 Basic uses of reference rates

In the absence of a reference rate, a fixed-rate loan is the only contract available. As illustrated in Figure 2, per unit borrowed, the firm's payoff is $P - \mu$. Similarly, the bank's payoff is $\mu - S_1$. The equilibrium interest rate, μ^*, equates the quantity of credit demanded by the firm and supplied by the bank based on the optimal allocation in Equation 3.

$$D(\mu^*) = \frac{\mu_P - \mu^*}{A_F\sigma_P^2} = \frac{\mu^* - \mu_S}{A_B\sigma_S^2} = L(\mu^*) \tag{4}$$

For example, the bank might lend \$10 MM to the firm at an interest rate of 500 basis points, or 5%. I refer to the sum of the utility costs of risk, $\Phi = A_F\sigma_P^2 + A_B\sigma_S^2$, as the total utility cost of risk.

When reference rates are available, there are two ways the lender can reallocate funding risk: floating interest rates and interest-rate derivatives. Both types of contracts are derivatives in the sense that the payments they require are not known in advance, but are determined based on the realized reference rate (R_1). Facilitating the existence of these contracts is the basic purpose of reference rates. Floating rates shift the incidence of funding risk from the bank to the firm. Interest-rate derivatives shift them to the broader market, at an explicit cost. Sections 4.1 and 4.2 show that if these contracts must be used independently, derivatives are better in welfare terms.

4.1 Floating rates

I denote the interest rate on floating-rate loans by $R_1 + \nu$. This corresponds to an interest rate of, for example, LIBOR + 300 bps. R_1 is the reference rate, and ν is the fixed premium, agreed in advance, charged over the realized reference rate. Recall that I assume the reference rate has zero mean, though the numerical example reflects the more realistic case of a positive mean. I assume that when a reference rate is available, there is no direct cost to writing a floating-rate contract, reflecting the public availability of the reference rate. Figure 3 shows the modified timeline.

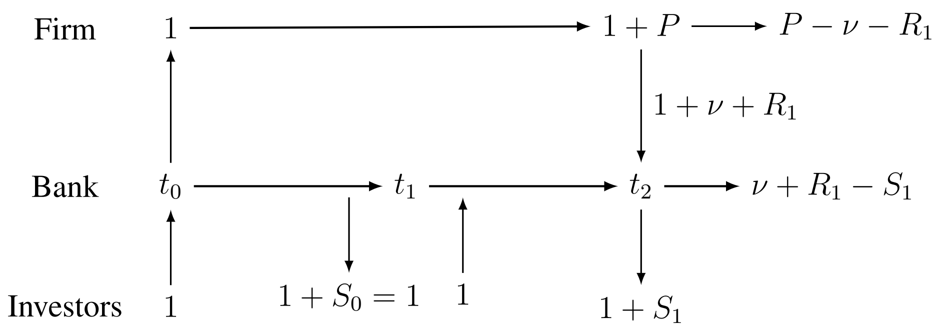

Figure 3: *Timeline for floating rate*

As the reference rate has positive covariance with funding costs, the floating-rate loan shifts some funding risk from the bank to the firm. However both the bank and the firm now bear risk due to volatility of the reference rate. As long as the reference rate is not too volatile, and $\rho > \frac{\sigma_R^2}{2}$, the bank reduces its risk from $\mathrm{Var}(S_1)$ to $\mathrm{Var}(R_1 - S_1) = \sigma_S^2 + \sigma_R^2 - 2\rho$. This is achieved by increasing risk for the firm from $\mathrm{Var}(P)$ to $\mathrm{Var}(P - R_1) = \sigma_P^2 + \sigma_R^2 - 2\pi$. I retain π in the notation here, but the baseline case is $\pi = 0$. The new equilibrium interest rate is

$$D'(\nu^*) = \frac{\mu_P - \nu^*}{A_F(\sigma_P^2 + \sigma_R^2 - 2\pi)} = \frac{\nu^* - \mu_S}{A_B(\sigma_S^2 + \sigma_R^2 - 2\rho)} = L'(\nu^*) \tag{5}$$

Relative to Equation 4, the credit supply and demand curves effectively rotate downwards. I refer to $\Phi(R_1) = A_F(\sigma_P^2 + \sigma_R^2 - 2\pi) + A_B(\sigma_S^2 + \sigma_R^2 - 2\rho)$ as the total utility cost of risk when floating rates are used.[25]

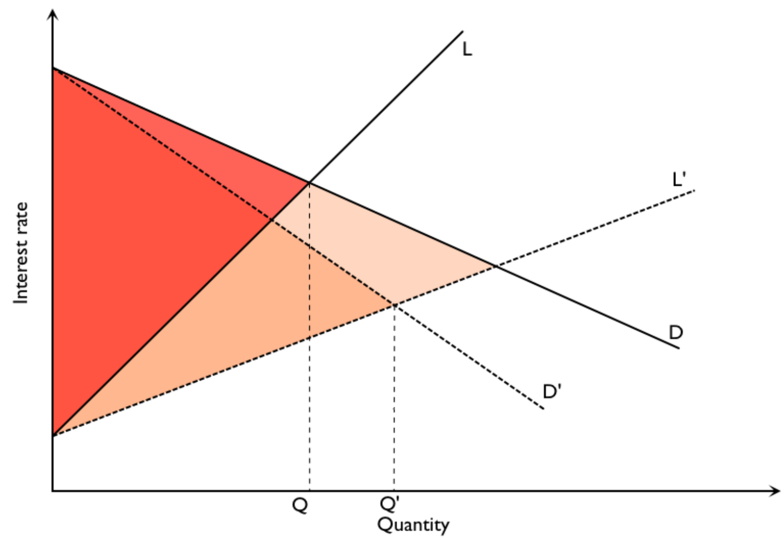

Figure 4: *Fixed and floating rates*

Figure 4 illustrates the comparison. D and L are the baseline credit demand and supply curves. D' and L' are the demand and supply curves when a reference rate is introduced. Interest rates ($L' = D'$) fall relative to the baseline case ($L = D$) both because the bank bears less risk and the firm bears more risk. In this framework, welfare is proportional to quantity transacted.[26] Floating rates are therefore welfare improving as long as borrowers are not too risk averse and not too much risk is transferred to them.

However, if the risk transfer is one-for-one, floating rates only improve welfare compared to fixed rates if firms are less risk averse than banks. Floating rates involve the transfer of risk from the bank to the firm, with the constraint that the reference rate *as a percentage of the loan amount* is transferred. While standard risk sharing arguments imply that a small amount of risk transfer from the bank to the firm should be optimal, floating rates do not necessarily provide the necessary flexibility. As discussed in Section 4.2, derivatives do provide this flexibility. Proposition 1 formalizes these observations.

[25]The floating-rate loan can reduce the overall risk faced by the firm if reference rates are sufficiently correlated with firm firm outcomes, i.e. $\pi > \frac{\sigma_R^2}{2}$. However, as discussed in Section 3.1, firms frequently borrow at floating rates but then swap away the floating-rate exposure. This suggests that the level of exposure to floating rates provided by a floating loan is too high, and the relevant case is the one discussed in the text here. I discuss how firms can use swaps in Section 5.2.

[26]Unit surplus $\mu_P - \mu_S$, and quantity of credit are the welfare triangle's base and height respectively.

Proposition 1. *Floating rates transfer funding risk from the bank to the firm. If* $\rho > \frac{\sigma_R^2}{2}$ *and* $\pi < \frac{\sigma_R^2}{2}$*:*

- *Interest rates fall*

$$\mu^* > \nu^* \iff \frac{2\rho - \sigma_R^2}{\sigma_S^2} + \frac{\sigma_R^2 - 2\pi}{\sigma_P^2} > 0 \tag{6}$$

- *However, welfare increases only if the utility benefit of reduced funding risk for banks outweighs the utility cost of increased risk for firms*

$$\Omega(R_1) > \Omega \iff A_B(2\rho - \sigma_R^2) > A_F(\sigma_R^2 - 2\pi) \tag{7}$$

Proof. See Appendix B.1. □

4.2 Interest-rate derivatives

Interest-rate swaps provide a natural alternative strategy for the bank to reallocate funding risk. There are three important differences between swaps and floating rates. First, the risk no longer needs to be shifted to the firm; it can be transferred on the market for swaps to any party willing to take the other side. I model swaps, denoted $\mathcal{R}_1$, as a contract agreed to at t_0, which (per dollar of notional value) obligates the buyer to pay a fixed rate λ in return for R_1, where all payments are made at t_2.[27,28] This highlights the remaining differences. The bank can choose a hedging ratio in this case (the notional value chosen need not be the loan amount) and there is a direct cost associated with using swaps, to the extent that they are not zero NPV transactions. I begin by taking the unit cost of hedging, λ, as exogenous and defer endogenizing it until Section 5.

How does the use of swaps compare with floating interest rates? For now I consider a return to fixed interest rates, and consider the bank's joint choice of credit supply and hedging with swaps. Denote the fixed rate charged here with κ. The bank's problem is

$$\max_{\theta,\alpha} EU[W_0 + \theta(\kappa - S_1 + \alpha\mathcal{R}_1)] \tag{8}$$

[27]As the exchange only occurs once this is really a Forward Rate Agreement.

[28]I do not consider indirect costs of hedging here. In particular, I assume collateral costs of hedging away here.

where α is the hedge ratio: the notional value of swaps purchased per unit of lending. With the use of swaps, the bank can construct a contract with payments according to any linear function of the reference rate R_1.[29] Figure 5 shows the modified timeline.

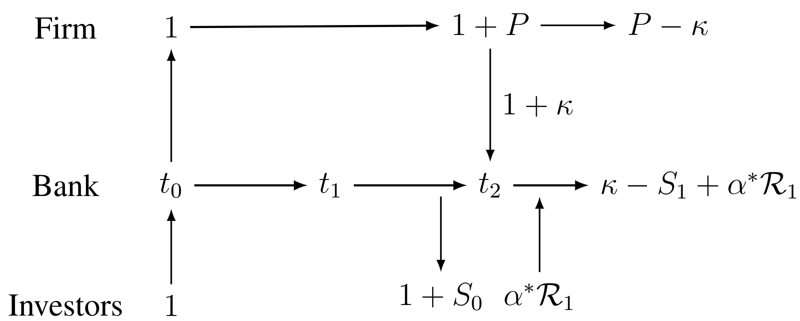

Figure 5: *Timeline when swaps are available*

First order conditions provide

$$L''(\kappa) = \theta^* = \frac{\kappa - \mu_S - \lambda\alpha^*}{A_B\left(\sigma_S^2 - \frac{\rho^2}{\sigma_R^2}\right)} \tag{9}$$

$$\alpha^*(\kappa, \lambda) = \underbrace{\frac{\rho}{\sigma_R^2}}_{\alpha^*} - \frac{\lambda}{A_B\theta^*\sigma_R^2} \tag{10}$$

If the swap is zero NPV ($\lambda = 0$), the resulting optimal hedging ratio is $\alpha^* = \alpha^*(\kappa, 0) = \frac{\rho}{\sigma_R^2}$. This optimal hedging ratio is the beta of S_1 with respect to R_1. The bank is willing to supply more because it faces less funding risk: $\text{Var}(-S_1 + \alpha^*\mathcal{R}_1) = \sigma_S^2 - \frac{\rho^2}{\sigma_R^2}$ is the residual funding risk after optimal hedging. The optimal level of risk reduction is therefore the beta of funding costs with the reference rate multiplied by their covariance.

When the derivative is costly, the bank reduces its position size in the derivative: $\alpha^*(\kappa, \lambda) \leq \alpha^*$. However, its demand for loans is the same as when the optimal hedge portfolio is purchased (see Appendix C.1). Equivalently, starting from its optimal hedging position, the bank is willing to bear some of the risk itself in exchange for the expected return of λ. Regardless of how volatile the reference rate is, the principle of participation means that the bank is willing to keep some of the risk.

[29]If non-linear derivatives on the reference rate were available, it might be optimal to use them. See Appendix C.2 for a discussion of the limited set of situations in which the optimal contract is likely to be linear.

The equilibrium condition $L''(\kappa) = D(\kappa)$ now allows κ to be determined. Figure 6 illustrates what happens. As before, D and L are the baseline credit demand and supply curves, and D' and L' are the demand and supply curves when a reference rate is introduced. When derivatives are used for optimal hedging, interest rates ($L'' = D$) are lower than the baseline case ($L = D$) but potentially higher than with reference rates ($L' = D'$).

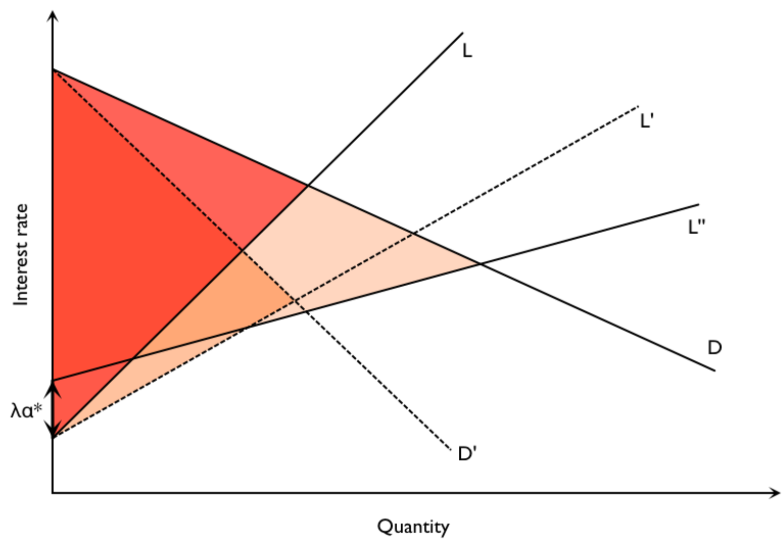

Figure 6: *The effect of reference rates*

As long as hedging with derivatives is not too expensive, welfare is improved relative to both fixed rates and floating rates. The total cost of hedging, $\lambda\alpha^*$, is best thought of as a fraction of the initial surplus, $\mu_P - \mu_S$ (the interest-rate-intercepts of the original demand and supply curves, D and L in Figure 6). Relative to a fixed rate, as long as this cost is not to high, the reduction in funding risk for the bank lowers interest rates and increases welfare. The effect on interest rates relative to floating rates is ambiguous: optimal hedging means that the risk is lower than with floating rates (strictly, unless $\alpha^* = 1$), pushing rates lower. On the other hand, the firm no longer has to bear risk, which pushes rates higher. Both of these effects raise welfare relative to floating rates. Proposition 2 formalizes these observations.

Proposition 2. *If interest rate swaps are zero NPV transactions ($\lambda = 0$)*

- *The effect on interest rates relative to floating rates is ambiguous*

$$\nu^* > \kappa^* \iff \frac{\sigma_R^2(\alpha^* - 1)^2}{\sigma_S^2 - \frac{\rho^2}{\sigma_R^2}} > \frac{\sigma_R^2 - 2\pi}{\sigma_P^2} \tag{11}$$

- *Welfare is improved relative to floating rates*

$$\Omega(\mathcal{R}_1, \lambda = 0) > \Omega(R_1) \iff A_B\sigma_R^2(\alpha^* - 1)^2 + A_F(\sigma_R^2 - 2\pi) > 0 \tag{12}$$

If interest rate swaps are costly, welfare is still increased as long as

$$\lambda\alpha^* \leq \bar{C} = (\mu_P - \mu_S)\left(1 - \sqrt{\frac{\Phi(\mathcal{R}_1)}{\Phi(R_1)}}\right) \tag{13}$$

where $\Phi(\cdot)$ is the total utility cost of risk.

Proof. See Appendix B.2. □

5 Derivatives, interpretation and welfare

The analysis in Section 4 shows that swaps are a more effective method than floating rates for banks to reduce funding risk, as long as they are not too costly. However, floating rates are widely used, particularly in the syndicated lending market.[30] Moreover, borrowing firms often hedge the interest-rate risk they acquire in this manner with swap positions of their own. To understand this behavior it is important to consider how firms would like to use derivatives. I begin by connecting the cost of hedging to aggregate risk tolerance for funding risk in Section 5.1. Section 5.2 then shows that the combined use of floating rates and firm swaps positions broadens the set of market participants bearing funding risk, lowering the cost of hedging. Section 5.3 cautions that this analysis requires derivatives markets to

[30] See Kirti (2016).

be sufficiently competitive. Section 5.4 shows that manipulation which makes the reference rate more volatiletile reduces welfare when swaps are not too costly.

5.1 Cost of hedging

If hedging with swaps is a zero NPV transaction, there is no reason for floating rates to be used, since they transfer funding risk to effectively risk averse firms. But why should swaps be costly in the first place? Standard models of swap pricing begin with the assumption that initial contractual terms are set in a way such that they are zero NPV transactions. However, if the market in aggregate is risk averse about funding risk, insuring it should not be a zero NPV proposition, a point made for example by Demsetz (1969). Indeed, all that is required is that both parties should be indifferent between taking either side of the trade, but if both are risk averse, hedging should be costly.

Suppose, then, that the other side of swap transactions the bank makes is taken up by a set of dealers with aggregate risk aversion A_D.[31] These dealers provide a supply function for swaps according to Equation 3. The equilibrium cost of hedging, λ^*, is how much these dealers need to be paid to satisfy the bank's hedging demand (hedging demand per unit of credit extended is shown in Equation 10). This condition requires

$$L''(\kappa)\alpha^* - \frac{\lambda^*}{A_B\sigma_R^2} = \frac{\lambda^*}{A_D\sigma_R^2} \tag{14}$$

Recall that $L''(\kappa)$ is the amount of credit supplied, and α^* is the optimal hedging ratio.

As discussed in Section 4.2, the bank does not try to shift as much risk to the swap market as implied by the optimal hedging ratio when $\lambda > 0$. Instead, it keeps some of the risk, taking into account that transferring it is costly. Effectively, 'market' risk tolerance combines the risk tolerance of swap dealers and the bank. Define this tolerance, as $T_{D,B}$:

$$T_{D,B} = \frac{1}{A_D} + \frac{1}{A_B} \tag{15}$$

[31] Dealers should be thought of as an example of the class of entities willing to bear interest-rate risk.

With this notation, Equation 14 can be rearranged to show that the equilibrium cost of hedging is

$$\lambda^* \alpha^* = \frac{1}{T_{D,B}} L''(\kappa) \frac{\rho^2}{\sigma_R^2} \tag{16}$$

Intuitively, the cost of hedging depends on how much risk tolerance there is, how big underlying credit markets are, and how much swaps reduce risk with optimal hedging. Thus Proposition 2 implies that swaps are more effective than floating rates when $T_{D,B}$ is high enough ($\lambda\alpha^* < \bar{C}$).

5.2 Complementarity between floating rates and swaps

The argument leading to Equation 15 makes clear that aggregate risk tolerance, and therefore the cost of hedging, depends on how broadly funding risk is borne. If firms were to enter swap markets as well, they would have hedging demand of their own, similar to the bank demand shown in Equation 10.

$$\beta^*(\kappa, \lambda) = \underbrace{-\frac{\pi}{\sigma_R^2}}_{\beta^*} - \frac{\lambda}{A_F D(\kappa) \sigma_R^2} \tag{17}$$

Note that π enters in the opposite direction given that the firm's problem is a mirror image. This would lead to more risk tolerance for funding risk. However, firms directly holding swaps positions would likely be labeled speculators.

In contrast, if firms have floating-rate obligations, it is a more legitimate transaction for them to (partially) hedge the risk this comes with. This is a potential advantage of floating interest rates: they can draw firms into the market for interest-rate risk. Per unit of credit, the floating rate transfers one unit of exposure to the reference rate from the bank to the firm. This increases the firm's desired hedging ratio by one, and correspondingly decreases the bank's desired hedging ratio by one. Figure 7 shows the modified timeline.

Recall that the baseline case is that $\pi = 0$: that the firm has no initial desire to use reference rates to reduce cash flow risk. In this case $\beta^* = 0$. With floating rates, then, the firm's desired hedging ratio is one, and the bank would like $\alpha^* - 1$. Effectively the firm moves back to the situation with a fixed

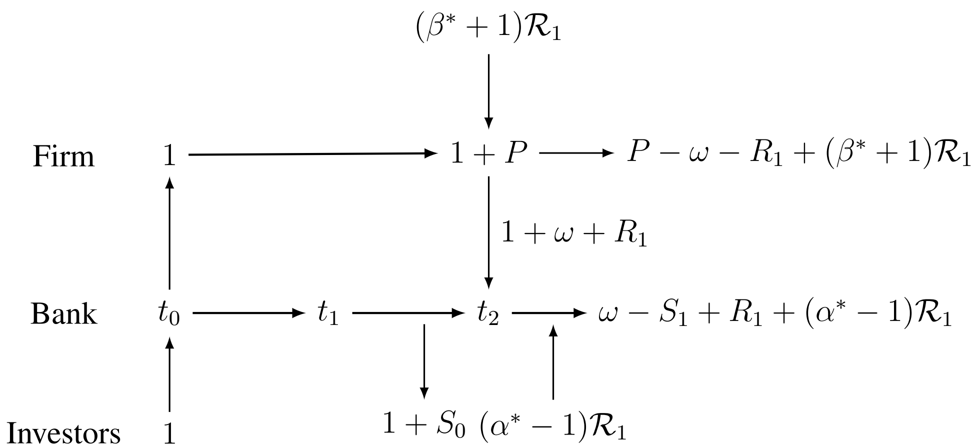

Figure 7: *Timeline with floating rates when both parties can use swaps*

interest rate: it has no exposure to the reference rate. In this case the total demand for swaps would be the same as in Equation 14 except that the firm is also willing to bear some risk. The cost of hedging is now determined by

$$L''(\omega)[1 + (\alpha^* - 1)] - \frac{\lambda^*}{A_B \sigma_R^2} - \frac{\lambda^*}{A_F \sigma_R^2} = \frac{\lambda^*}{A_D \sigma_R^2} \tag{18}$$

As the firm absorbs some risk, market risk tolerance is now larger

$$T_{D,B,F} = \frac{1}{A_D} + \frac{1}{A_B} + \frac{1}{A_F} > T_{D,B} \tag{19}$$

This means hedging is cheaper and derivatives are more effective

$$\lambda^* \alpha^* = \frac{1}{T_{D,B,F}} L''(\kappa) \frac{\rho^2}{\sigma_R^2} \tag{20}$$

Note that if reference rates are positively correlated with project outcomes, the exposure the firm obtains through floating rates insures it against cash-flow risks. The firm would then elect to keep even more of the exposure, further reducing the cost of hedging. Proposition 3 formalizes these observations.[32]

[32] I abstract away from basis risk between swaps and floating-rate borrowing arrangements here. The presence of basis risk would reduce demand for hedging. One prominent example of basis risk relates to variable rate demand notes. These are short-term borrowing arrangements, regularly rolled over if the borrower can obtain a letter of credit from a bank. Many municipalities borrow in this manner, and enter swaps to hedge interest-rate risk. However, as was the case in the crisis, this arrangement can expose the borrower to basis risk. If a letter of credit is not forthcoming, the borrower may have to refinance at a fixed rate, while still being party to a pay-fixed swap.

Proposition 3. *If $T_D = \infty$ ($\implies \lambda = 0$) and $\pi = 0$:*

- *Fixed rates with bank hedging and floating rates with hedging on both sides are equivalent*
- *Compared to fixed rates without hedging, both lower interest rates in proportion to the reduction in risk $\left(\frac{\rho^2}{\sigma_R^2}\right)$ and increase welfare in proportion to the utility benefit of this reduction in risk $\left(A_B \frac{\rho^2}{\sigma_R^2}\right)$*

If $T_D < \infty$:

- *Welfare is higher with floating rates than with fixed rates as firms can bear some of the risk*
- *If $\pi \in (0, \rho)$, as firms' optimal hedging ratio is $1 - \frac{\pi}{\sigma_R^2} < 1$, the cost of hedging is decreasing in π and welfare is increasing in π*

Proof. See Appendix B.3. □

5.3 Competition in derivatives markets

I argue so far that floating rates, when firms optimally hedge the risk this transfers to them, are a better contractual arrangement than swaps alone. This draws firms into the market for swaps written on the reference rate, broadening the set of players that bear funding risk, and lowering the cost of hedging. However, this argument relies on the competitiveness of derivatives markets. If, as is the case, a small set of dealers dominates the market for swaps, hedging may be sufficiently costly that a floating rate or even a fixed rate with no hedging would be better. This is important to consider as derivatives markets are very concentrated. US bank regulatory data shows that the top six bank holding companies account for about 95% of the total for swaps and for derivatives.[33]

[33] The OCC releases quarterly reports on bank trading and derivatives activities. The text refers to the total notional value held by Citigroup, JP Morgan, Goldman Sachs, Bank of America, Morgan Stanley and Wells Fargo as a fraction of the total held by the top 25 banks in Q2 2016 (which is approximately the total held by US bank holding companies; the 25th, Principal Financial, accounts for 0.002%). Swaps here include currency swaps. However, the majority of derivatives volume is tied to interest rates. Moreover, currency swaps can also exchange variable-rate payments (in different currencies). While gross notionals do not necessarily capture net exposure for these dealers, they do provide a sense for market concentration.

Suppose that there are N oligopolistic dealers in the swap market, each symmetrically contributing risk tolerance $\frac{T_D}{N}$. In a symmetric Cournot equilibrium, the cost of of hedging is

$$\lambda' \alpha^* = \frac{1 + \frac{T_D}{N T_{B,F}}}{T_{D,B,F} + \frac{T_D}{N}} L''(\omega) \frac{\rho^2}{\sigma_R^2} > \lambda^* \alpha^* \tag{21}$$

In comparison with the competitive case, shown in Equation 20, the cost of hedging is higher (see Appendix C.3 for details). For instance, if these six large dealers account for even 80% of the total risk tolerance in the market for swaps, Equation 21 implies a cost of hedging 50% higher than the competitive situation described in Equation 20. If the market for swaps is sufficiently concentrated, even with the addition of firms to the set of players that bear funding risk, the cost of hedging may be above the threshold at which they are useful, discussed in Proposition 2.

Why might floating rates and swaps still be used? It is possible that the effect of firms being drawn in to swaps markets is useful for a different reason than wider risk sharing. If universal banks both lend to large clients and act as dealers writing derivatives contracts with them, this arrangement may be a way for banks to generate business with derivatives. If lending is a much more competitive business than derivatives, banks may be willing to lose money while lending to achieve this.

5.4 Welfare and the cost of manipulation

Contracts linked to reference rates facilitate hedging. Intuitively, their usefulness for this purpose is increased if the covariance between the reference rate and the underlying risk, $\text{Cov}(R_1, S_1) = \rho$, increases. Similarly, their use requires institutions to bear risk as the reference rate itself is volatile. As discussed in Section 2 and Footnote 17, manipulation of LIBOR was driven by both portfolio incentives and reputational concerns. Portfolio incentives fluctuated significantly over time, as they were particularly strong on days when banks had significant amounts of swaps contracts settling. These were the days submitters to the process were asked to modify their submissions. Manipulation due to this can be thought of as adding pure noise to the reference rate, which intuitively makes associated hedging less useful. Reputational concerns could lead to manipulation which just changes the level of the rate.

However, if the intensity of reputational concerns also changes over time, such manipulation can also add noise.

In this framework, the hedging properties of contracts of reference rates are determined by their covariance with risks and with their volatility. A change in their level does not affect welfare. Of course, if LIBOR were to be shifted down by 200 bps and remain otherwise unchanged this might not be welfare neutral as reference rates are used for purposes other than risk sharing. In particular, reference rates are used as a gauge for the health of the financial sector, and an artificially low rate may have played a role in delaying recognition of the crisis. Reference rates are also used to discount streams of payments, and changes in the level might distort investment decisions.

Here I focus on understanding the effect of manipulation which adds pure noise to the reference rate. I parametrize the results of such manipulation by considering a post-manipulation reference rate

$$\tilde{R}_1 = R_1 + \sqrt{K-1}Z \tag{22}$$

where Z is a normally distributed variable independent of R_1 as well as risks including S and P, with $\sigma_Z^2 = \sigma_R^2$. K is a parameter which captures the extent of manipulation. Now $\mathrm{Cov}(\tilde{R}_1, S_1) = \mathrm{Cov}(R_1, S_1)$ and $\mathrm{Cov}(\tilde{R}_1, P) = \mathrm{Cov}(R_1, P)$, while $\mathrm{Var}(\tilde{R}_1) = K\sigma_R^2$. I consider the effect of adding a small amount of manipulation, i.e. slightly increasing K from 1.

Intuitively, if hedging is a zero NPV activity, such manipulation must reduce welfare as institutions now have to bear more risk as the reference rate moves. However, it is less clear what happens when hedging is costly, as institutions will choose to bear more of the underlying risk themselves as contracts linked to reference rates become less useful for hedging. This, in turn, can lower the cost of hedging. Proposition 4 formalizes the idea that this effect is not important when the market, in aggregate, is not too risk averse about risk related to the reference rate.

Proposition 4. *For aggregate risk tolerance T high enough, welfare is decreasing in added noise, i.e.*

$$\lim_{T\to\infty} \frac{\partial \Omega(K)}{\partial K} = -\frac{(\mu_P - \mu_S)^2 \Phi'(K)}{2\Phi^2(K)} < 0 \tag{23}$$

where $\Phi'(K)$ is the added utility cost of risk participants bear as the reference rate becomes more volatile.

Proof. See Appendix B.4. □

6 Conclusion

The growing literature on LIBOR has not focused on what purpose reference rates serve. This is an important question to consider as markets and regulators consider how LIBOR might be modified or replaced. In this paper, I construct a simple model of credit markets with three key frictions, in the presence of which reference rates can be understood to be useful to hedge funding risk. Constraining lenders to using short-term funding generates maturity mismatch. As bank funding costs are not contractible, lenders, in their absence, bear funding risk. Due to financial frictions, profit functions become concave, implying that bearing this risk is costly. Reference rates mitigate contractual incompleteness and facilitate risk management.

Floating-rate loans transfer funding risk from lenders to borrowers. While this transfer may lower the cost of borrowing, whether it improves welfare depends on how costly it is for borrowers to bear funding risk. Indeed, if the cost of hedging through swaps is sufficiently low, it would be better for lenders to manage risk through swaps. In practice, firms borrow at floating rates and swap some of the exposure away. If firms find it easier to take exposure to interest rates when they have taken floating-rate loans, this more complicated arrangement may be a way to include firms in the set of institutions bearing interest rate risk, lowering the cost of hedging it. In concentrated derivatives markets, it may also be a way for banks to generate profitable activity.

I focus on understanding the uses of reference rates in credit markets and associated derivatives markets; as discussed in Section 2, this is the market reference rates emerged from. While understanding the broader use of reference rates is important, this analysis clarifies the purpose of reference rates and should facilitate a clearer discussion of issues with manipulation and potential replacements. As the risk being managed in credit markets is uncertainty regarding the bank's cost of funding, reference rates that better capture bank funding costs across instruments should be given more consideration (Brousseau et al. 2013). A good reference rate for use in credit markets is highly correlated with bank funding costs and stable, corresponding to the old idea that standards should be accurate and low cost (Barzel 1982). Manipulation driven by varying exposure of banks to interest rate derivatives can add noise, making reference rates less useful for hedging. However, in credit markets, it makes sense for reference rates to capture bank credit risk. Alternatives being considered to LIBOR may be less manipulable, but also less useful for hedging risks in credit markets.

As Duffie & Stein (2015) note, the various groups convened by the FSB have recommended that markets be steered away from reference rates incorporate credit risk. To the extent that reference rates with credit risk are needed, the suggestion is that the two separate reference rates should be used. This paper suggests that there is demand from lenders relying on short-term funding to use pay-fixed interest-rate swaps to reduce funding risk. This raises a broader question about the extent to which demand and supply would be naturally balanced if interest-rate derivative markets are split into two separate markets (one set with reference rates incorporating credit risk, the other with reference rates not incorporating credit risk).

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A Effective institutional risk aversion

Why should banks or firms care about risk management? In this Appendix, I follow Froot et al. (1993) and relate institutional risk aversion to financial frictions. Suppose a bank has risky internal funds w. In the following period it has concave productive opportunities $f(I)$. As internal funds may not be sufficient, the bank would sometimes like to borrow in order to invest more. However, investment and output are not observable to external financiers and cannot be used to collateralize borrowing. Instead, all borrowing must be collateralized by risky cash flows y, distributed $g(y)$, generated by existing assets. This cash flow is observable to external financiers at a cost of c. Financiers are risk neutral and the competitive rate of return they require is normalized to 0.

The bank wants to maximize the value of output and the portion of y it retains, subject to the lender's IR constraint:

$$P(w) = f(I) + \int_D^\infty (y - D)g(y)dy + \lambda \left(\int_{-\infty}^D (y - c)g(y)dy + \int_D^\infty Dg(y)dy - (I - w) \right) \quad (24)$$

The first order conditions are

$$\frac{\partial P}{\partial D} = (\lambda - 1)(1 - G(D)) - \lambda c g(D) = 0 \quad (25)$$

$$\frac{\partial P}{\partial I} = f_I - \lambda = 0 \quad (26)$$

Bankruptcy costs generate underinvestment

$$f_I = \frac{1 - G(D)}{1 - G(D) - cg(D)} > 1 \quad (27)$$

assuming an interior solution (i.e. $1 - G(D) - cg(D) > 0$) and positive monitoring costs.

Note that Equation 26 implies that $\lambda = f_I$, while the first order condition with respect to w implies that $P_w = \lambda$. Therefore the concavity of the profit function is determined by

$$P_{ww} = f_{II}\frac{dI^*}{dw} \tag{28}$$

This is the same expression as Equation 1. In order for the profit function to be concave, then, investment must respond positively to the level of internal funds so that $P_{ww} < 0$. This can be guaranteed when g has an increasing hazard rate.

Begin by combining Equations 25 and 26 and totally differentiating with respect to w:

$$\begin{aligned} &(f_I(I^*(w)) - 1)(1 - G(D(w))) = f_I(I^*(w))cg(D(w)) \\ \implies\ & f_{II}\frac{dI^*}{dw}(1 - G(D)) - G'(D)D'(w)(f_I - 1) = f_{II}\frac{dI^*}{dw}cg(D) + f_I cg'(D)D'(w) \end{aligned} \tag{29}$$

Next, I solve for $D'(w)$, by totally differentiating the lender's IR constraint, which determines how D responds to changes in w:

$$\begin{aligned} &\int_{-\infty}^{D(w)} (y - c)g(y)dy + \int_{D(w)}^{\infty} D(w)g(y)dy = I^*(w) - w \\ \implies\ & D'(w) = \frac{\frac{dI^*}{dw} - 1}{1 - G(D) - cg(D)} \end{aligned} \tag{30}$$

Substituting Equations 30 and 27 into Equation 29 provides an expression for $\frac{dI^*}{dw}$ in terms of f'', c, and g:

$$\begin{aligned} \frac{dI^*}{dw} &= \frac{1}{1 - f_{II}\Gamma} \\ \text{where } \Gamma &= \frac{1}{c}\frac{(1 - G(D) - cg(D))^3}{g(D)G'(D) + g'(D)(1 - G(D))} \end{aligned}$$

As f is concave and I have already assumed $1 - G(D) - cg(D) > 0$, $\frac{dI^*}{dw}$ has the same sign as

$$g(D)G'(D) + g'(D)(1 - G(D)) \propto \frac{d}{dD}\frac{g(D)}{1 - G(D)}$$

For concavity of the profit function it is therefore sufficient for the hazard rate of g to be strictly increasing. It can also be shown that the same condition generates a convex cost function (where the cost is the additional deadweight cost arising from external finance). This condition is satisfied for the Normal and Uniform distributions. In the case of the uniform distribution it is possible to explicitly calculate the face value of debt and deadweight cost of external finance. If y is distributed uniform on $[0, U]$:

$$D(e) = U - c - \sqrt{(U-c)^2 - 2 \times 30 \times e}$$
$$C(e) = \frac{c}{U} D(e)$$

The requirement that the hazard rate of g be increasing is not always satisfied - for instance, for the exponential distribution the hazard rate is 1. Indeed, it can be shown that in this case the deadweight cost of external finance is linear and not convex, which does not generate the concavity of the profit function.

B Proofs

B.1 Proof of Proposition 1

Let $X_D = A_F \sigma_P^2$ and $X_S = A_B \sigma_S^2$ be the utility costs of risk on the demand and supply sides of the market when a fixed rate is used. Then the equilibrium price is determined as

$$\frac{\mu_P - \mu^*}{X_D} = \frac{\mu^* - \mu_S}{X_S} \tag{31}$$

$$\implies \mu^* = \frac{\mu_S X_D + \mu_P X_S}{X_D + X_S} \tag{32}$$

Denote welfare when a fixed rate is used by Ω. Welfare depends on the surplus from lending, $\mu_P - \mu_S$:

$$\Omega = \frac{1}{2} \frac{(\mu_P - \mu_S)^2}{X_D + X_S} \tag{33}$$

The introduction of a floating rate changes the utility costs of risk to

$$Y_D = A_F \left(\sigma_P^2 + \sigma_R^2 - 2\pi\right)$$

$$Y_S = A_B \left(\sigma_S^2 + \sigma_R^2 - 2\rho\right)$$

The expressions for ν^* and $\Omega(R_1)$ (welfare when a floating rate is used) are similar to Equations 32 and 33. Rearrangements show that the change in interest rates is

$$\mu^* - \nu^* = \frac{(\mu_P - \mu_S)(Y_D X_S - X_D Y_S)}{(X_D + X_S)(Y_D + Y_S)} \tag{34}$$

This can be written as

$$\text{Sgn}(\mu^* - \nu^*) = \text{Sgn}\left(\frac{X_S - Y_S}{X_S} - \frac{X_D - Y_D}{X_D}\right) \tag{35}$$

Similarly the sign of the welfare difference is

$$\text{Sgn}\left(\Omega(R_1) - \Omega\right) = \text{Sgn}((X_D - Y_D) + (X_S - Y_S)) \tag{36}$$

This proves Proposition 1.

B.2 Proof of Proposition 2

Equations 11 and 12 follow from the same argument used to prove Proposition 1. This leaves Equation 13. Denote welfare when swaps are used and their exogenous cost is λ by $\Omega(\mathcal{R}_1, \lambda\alpha^*)$. I have already established that $\Omega(\mathcal{R}_1, 0) > \Omega(R_1)$. Let Z_D and Z_S be the utility costs of risk when swaps are used. The threshold cost of optimal hedging can be found by equating these levels of welfare

$$\Omega(\mathcal{R}_1, \bar{C}) = \Omega(R_1)$$
$$\frac{1}{2}\frac{(\mu_P - \mu_S)^2}{Y_D + Y_S} = \frac{1}{2}\frac{(\mu_P - \mu_S - \bar{C})^2}{Z_D + Z_S}$$

From this quadratic equation I select the smaller root

$$\bar{C} = (\mu_P - \mu_S)\left(1 - \sqrt{\frac{Z_D + Z_S}{Y_D + Y_S}}\right)$$

as the maximal cost must be smaller than the surplus $\mu_P - \mu_S$. The total utility costs of risk are $\Phi(\mathcal{R}_1) = Z_D + Z_S$ and $\Phi(R_1) = Y_D + Y_S$.

This proves Proposition 2.

B.3 Proof of Proposition 3

The discussion prior to Proposition 3 explains why when hedging is a zero NPV transaction, fixed rates with the bank hedging with swaps and floating rates with both the firm and the bank hedging with swaps are equivalent. For this part I assume that $\pi = 0$. The same reasoning used in the proof of Proposition 1 explains the interest rate and welfare differences relative to fixed rates.

Similarly, the discussion before Proposition 3 establishes that the cost of hedging is lower with floating rates when firms also hedge, as risk tolerance is higher. Welfare, as a function of risk tolerance in the market for swaps, is

$$\Omega(T) = \frac{1}{2}\frac{(\mu_P - \mu_S - \lambda(T)\alpha^*)^2}{\Phi(\rho)} \tag{37}$$

where $\Phi(\rho)$ takes into account that the bank behaves as if it has reduced its risk by $\rho\alpha^*$ (continue to assume $\pi = 0$ for now). Substituting for the cost of hedging from Equation 16, I find

$$\begin{aligned} \lambda(T)\alpha^* &= \frac{\mu_P - \mu_S}{T\Phi(\rho)}\rho\alpha^* \\ \implies \Omega(T) &= \frac{1}{2}\frac{(\mu_P - \mu_S)^2}{\Phi(\rho)}\left(1 - \frac{1}{T\Phi(\rho)}\frac{\rho^2}{\sigma_R^2}\right)^2 \end{aligned} \tag{38}$$

When $\pi \neq 0$, this expression becomes[34]

$$\Omega(T) = \frac{1}{2}\frac{(\mu_P - \mu_S)^2}{\Phi(\rho, \pi)}\left(1 - \frac{1}{T\Phi(\rho, \pi)}\frac{(\rho - \pi)^2}{\sigma_R^2}\right)^2 \tag{39}$$

For $\pi \in (0, \rho)$, the cost of hedging is decreasing in π and welfare is increasing in π.

This proves Proposition 3.

B.4 Proof of Proposition 4

Welfare as a function of the level of manipulation K is

$$\Omega(K) = \frac{1}{2}\frac{(\mu_P - \mu_S)^2}{\Phi(K)}\left(1 - \frac{1}{T\Phi(K)}\frac{(\rho - \pi)^2}{K\sigma_R^2}\right)^2 \tag{40}$$

The effect of increasing K is

$$\frac{\partial\Omega(K)}{\partial K} = \left(1 - \frac{1}{T\Phi(K)}\frac{(\rho - \pi)^2}{K\sigma_R^2}\right)\left(\frac{1}{T}\frac{(\mu_P - \mu_S)^2(\rho - \pi)^2}{2\Phi^2(K)K\sigma_R^2}\underbrace{\left(\frac{2}{K} + \frac{3\Phi'(K)}{\Phi(K)}\right)}_{>0} - \underbrace{\frac{(\mu_P - \mu_S)^2\Phi'(K)}{2\Phi^2(K)}}_{>0}\right) \tag{41}$$

The two marked terms are positive because $\Phi'(K) > 0$: both lenders and borrowers bear more risk as

[34] Note that the corresponding version of Equation 16 then provides an expression for $\lambda(\alpha^* + \beta^*)$.

K increases. For example, the optimal level of risk reduction for the bank is $\frac{\rho^2}{K\sigma_R^2}$. The effect of added risk when hedging is zero NPV is a clear negative effect when T is sufficiently large

$$\lim_{T\to\infty} \frac{\partial\Omega(K)}{\partial K} = -\frac{(\mu_P - \mu_S)^2\Phi'(K)}{2\Phi^2(K)} < 0 \tag{42}$$

This proves Proposition 4.

C Details

C.1 Optimal hedging with costly derivatives

Begin by simplifying the notation: let $H = \kappa - \mu_S$, $\sigma_R^2 = 1$ and $\sigma_S^2 = \sigma^2$. I want to show

$$\begin{aligned} & \frac{\kappa - \mu_S - \lambda\alpha^*(\kappa,\lambda)}{A_B \operatorname{Var}(-S_1 + \alpha^*(\kappa,\lambda)\mathcal{R}_1)} = \frac{\kappa - \mu_S - \lambda\alpha^*}{A_B \operatorname{Var}(-S_1 + \alpha^*\mathcal{R}_1)} \\ \iff & \frac{H - \lambda\left(\rho - \frac{\lambda}{A\theta}\right)}{A(\sigma^2 + \left(\rho - \frac{\lambda}{A\theta}\right)^2 - 2\left(\rho - \frac{\lambda}{A\theta}\right)\rho)} = \frac{H - \lambda\rho}{A(\sigma^2 - \rho^2)} \end{aligned}$$

The equality can be verified from

$$\begin{aligned} \frac{H - \lambda\left(\rho - \frac{\lambda}{A\theta}\right)}{A(\sigma^2 + \left(\rho - \frac{\lambda}{A\theta}\right)^2 - 2\left(\rho - \frac{\lambda}{A\theta}\right)\rho)} &= \frac{A\theta(H-\lambda\rho) - \lambda^2}{A^2\theta(\sigma^2 - \rho^2) + \frac{\lambda^2}{\theta}} \\ &= \frac{A\theta(H-\lambda\rho)}{A^2\theta(\sigma^2-\rho^2)} \underbrace{\frac{\overbrace{A^2\theta(\sigma^2-\rho^2)}^{=X}}{A^2\theta(\sigma^2-\rho^2) + \frac{\lambda^2}{\theta}}}_{=Y} + \underbrace{\frac{\lambda^2}{A^2\theta(\sigma^2-\rho^2) + \frac{\lambda^2}{\theta}}}_{=Y} = \theta \end{aligned}$$

where the final equality follows from $\theta(Y - X) = \lambda^2$.

C.2 Optimal contracting

In general the optimal contract need not be linear. Consider a simplified problem: suppose P, S are functions of a random variable R. In the more general problem I analyze in this paper, P, S_1 and R_1 are jointly distributed, but this simple version provides useful intuition.[35] Consider a standard optimal risk sharing problem. What is the optimal function for the payment from the firm to the bank $f(R)$ subject to a participation constraint for the bank? Suppose R has pdf $g(R)$. The Lagrangian for this problem is

$$\mathcal{L} = \max_{f(R)} \int U^F[P(R) - f(R)]g(R)dR + \lambda\left(\bar{U} - \int U^B[f(R) - S(R)]g(R)dR\right) \tag{43}$$

[35] A similar exercise is performed by Froot et al. (1993).

Pointwise maximization provides the Borsch risk sharing rule, where I write the marginal utilities as functions of the resulting wealth.

$$-\frac{U_W^F(P(R)-f(R))}{U_W^B(f(R)-S(R))}=\lambda \tag{44}$$

Implicit differentiation and rearrangement provides

$$\left[\underbrace{-\frac{U_{WW}^F}{U_W^F}}_{A_F}\left(\frac{df^*(R)}{dR}-\frac{dP}{dR}\right)\right]+\left[\underbrace{-\frac{U_{WW}^B}{U_W^B}}_{A_B}\left(\frac{df^*(R)}{dR}-\frac{dS}{dR}\right)\right]=0 \tag{45}$$

and therefore

$$\frac{df^*(R)}{dR}=\frac{A_F\frac{dP}{dR}+A_B\frac{dS}{dR}}{A_F+A_B} \tag{46}$$

Thus f should be linear only if P and S are linear functions of R.

C.3 Competition in derivatives markets

Equation 18 can be rearranged to find an inverse demand function from the perspective of dealers, as a function of Q, the total demand for swaps that all N dealers choose.

$$\lambda(Q)=(L''(\omega)\alpha^*-Q)T_{B,F}\sigma_R^2 \tag{47}$$

$T_{B,F}$ is the combined risk tolerance of banks and firms. An individual dealer's problem is then to maximize production, q_i, holding $r=\sum_{j\neq i}q_j$ fixed. This objective can be written as

$$\max_{q_i}\ q_i\lambda(q_i+r)-\frac{1}{2}A_iq_i^2\sigma_R^2 \tag{48}$$

The dealer takes into account how its demand decision will affect its return for accepting risk as well as the utility cost of bearing this risk. Recall that I have assumed $A_i=\frac{N}{T_D}$. The first order condition provides

$$q_i = \frac{T_{B,F}(L''(\omega)\alpha^* - r)}{2T_{B,F} + A_i} \tag{49}$$

In a symmetric equilibrium it must be the case that $r = (N-1)q$. Substituting this in, each dealer demands

$$q^* = \frac{T_{B,F}L''(\omega)\alpha^*}{(N+1)T_{B,F} + A_i} \tag{50}$$

The equilibrium cost of hedging, as in Equation 21, is

$$\lambda(Nq^*)\alpha^* = \frac{1 + \frac{T_D}{NT_{B,F}}}{T_{D,B,F} + \frac{T_D}{N}} L''(\omega)\frac{\rho^2}{\sigma_R^2} \tag{51}$$

As N goes to infinity, this approaches the competitive cost of hedging, $\lambda^*\alpha^*$, shown in Equation 20. To see that it is always greater, note that

$$\frac{1 + \frac{T_D}{NT_{B,F}}}{T_{D,B,F} + \frac{T_D}{N}} = \frac{1}{T_{D,B,F}} \left(1 + \underbrace{\frac{T_D^2}{T_{B,F}(T_D + NT_{D,B,F})}}_{>0} \right) \tag{52}$$