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U.S. Inflation Expectations During the Pandemic

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U.S. Inflation Expectations During the Pandemic Prepared by Euihyun Bae, Andrew Hodge and Anke Weber*

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JEL Classification Numbers:	G20, G23, E52
Keywords:	Inflation Expectations; Learning; Forecasting
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U.S. Inflation Expectations during the Pandemic^{*}

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February 2024

Abstract

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1 Introduction

Inflation surged in the United States in 2021. Originally limited to goods affected by supply disruptions associated with the pandemic, it became much more broad-based (see Figure B.1). This has been accompanied by elevated near-term inflation expectations (see Figure B.2), while medium-term inflation expectations have also moved up, although to a lesser degree.

This paper studies how the inflation expectations of different demographic groups have evolved since the inflation surge of 2021, using micro-level data from consumer surveys. We perform simple tests of the rationality of these expectations, as well as studying whether there is evidence of adaptive learning, in the form of consumers updating their expectations in response to forecast errors. We build a simple, reduced-form model of adaptive learning to determine if it can account for recent movements of inflation expectations, including the differing behaviour of inflation expectations across demographic groups. Additionally, we estimate a micro-founded theoretical model of the U.S. economy that allows for adaptive learning, using a long sample that includes the recent inflation surge.

Our paper contributes to understanding how inflation expectations have evolved during the Covid-19 pandemic. Using micro-level data from the University of Michigan Survey of Consumers, we show that the sample distribution of inflation expectations has widened since 2021, as the mean inflation expectation has increased. The widening of the distribution of expectations has been most pronounced for the bottom quartile of the income distribution. The mean and variance of the sample distribution of inflation expectations have been higher for those with lower education levels (high school or less), as well as for women, although we find little difference across age groups.

We confirm that consumer inflation expectations fail simple tests of rationality using a sample including the inflation surge of 2021. Instead, using a reduced-form empirical model, we find evidence of adaptive learning in forming expectations, showing that consumers revise their inflation expectations in response to past forecast errors that they have made. Within this framework, we find that simpler, autoregressive models of inflation with high levels of persistence perform best in capturing the inflation expectations of women, as well as of those with lower education and lower incomes.

Using a long sample including the 2021 inflation surge, we estimate the micro-founded theoretical model of the U.S. economy with adaptive learning in Slobodyan and Wouters (2012a), where agents form expectations based on small forecasting models. We find that allowing for adaptive learning results in a better in-sample fit than assuming rational expectations. The model with adaptive learning also has superior forecasting performance: cumulative one-period ahead inflation forecast errors, based on forecasts from the time-varying solution of the model with adaptive learning, are below those of the rational expectations variant. The inflation expectations formed by consumers within the model are more consistent with consumer inflation expectations recorded in the University of Michigan Surveys over the sample period, compared with expectations in the rational expectations model which more closely track inflation forecasts from the Survey of Professional Forecasters. The inflation surge of 2021 is interpreted by the theoretical model as a price mark-up shock. These shocks have a more persistent impact on inflation and require a larger and more persistent monetary policy response to return inflation to target in the version of the model with adaptive learning, compared with the rational expectations version.

Our paper relates to several strands of literature. First, our use of micro-data to study demographic differences in inflation expectation is similar to Madeira and Zafar (2015) who used the University of Michigan Survey of Consumers and find demographic differences in inflation expectations, namely larger heterogeneity of expectations and slower updating of expectations for those with less education, as well as for women and ethnic minorities. Our work is also related to a large literature that investigates subjective inflation expectations formed on the basis of agents' life-time experiences (Malmendier and Nagel, 2016).¹ Second, our empirical model of adaptive learning follows the methodologies of Branch and Evans (2006) and Weber (2010), and is in the spirit of the seminal contributions of Evans and Honkapohja (2001) and Marcet and Sargent (1989). Finally, the DSGE model with adaptive learning updates the work of Slobodyan and Wouters (2012a), whose model is based on the DSGE model of the U.S. in Smets and Wouters (2007). Slobodyan and Wouters (2012b) is similar to Slobodyan and Wouters (2012a) but agents form expectations using the same information set as under rational expectations, in which case learning has minimal influence on the dynamics of model variables. Milani (2007) incorporates adaptive learning into a small-scale DSGE model. Elton et al. (2017), Branch and McGough (2009) and Massaro (2013) introduce heterogeneous expectations into DSGE models, where at least a fraction of agents form expectations in a non-rational way. Most recently, Gelain et al. (2019) introduces heterogeneous expectations into the Smets and Wouters (2007) model and finds an improvement in model fit over one with fully rational expectations.

The remainder of this paper is structured as follows. Section 2 examines how consumer inflation expectations in the U.S. have evolved over time and in particular since the emergence of the pandemic, with a focus on heterogeneity among demographic groups. Section 3 describes our simple adaptive learning model and results. Section 4 lays out the DSGE model for the U.S. and compares the model fit under adaptive learning and rational expectations with a discussion of policy implications. Section 5 concludes.

2 Stylized Facts: Inflation and Expectations

2.1 Demographic Differences

In this section, we use data on consumer inflation expectations to analyze how they differ between different demographic groups and over time. We study monthly data from the University of

¹Weber et al. (2022) provide an overview of the recent literature on subjective inflation expectations.

Michigan Surveys of Consumers spanning January 1978 through April 2023. We use responses from two questions posed to survey participants: (i) "By what amount do you expect prices to go up or down, on average, during the next twelve months?'; and (ii) "By what percent per year do you expect prices to go up or down, on average, during the next five to ten years." We interpret the first question as a measure of short-term inflation expectations and the second as a measure of medium-term inflation expectations. Figures B.4 and B.5 show the mean inflation expectation of consumers' at one and five-ten year horizons respectively, disaggregating by education, income, gender and income.

We find that there are substantial demographic differences in inflation expectations and that these differences appear to have become more apparent since the decline of inflation in 2020 and the subsequent surge during 2021-2022. Figure B.4 shows that the mean one-year ahead inflation expectation for those without a high school diploma has been over 5 percentage points higher than the mean inflation expectation of those with college degrees at most times since 2022, a much wider disparity across education levels than recorded pre-pandemic. Inflation expectations of those without high school diplomas have also shown more variance than those of respondents with higher education levels, although again this only becomes apparent during 2022. Similar results are found for inflation expectations five-ten years ahead, although the differences across education levels are smaller.

Differences in inflation expectations have also widened across the income distribution since 2021. The mean and standard deviation of inflation expectations reported by respondents in the bottom quartile of the sample income distribution have been over five percentage points above those of respondents in the top income quartile, at various times during 2022-2023. This implies a higher average level of inflation expectations and a wider dispersion of expectations for those on lower incomes. Again, a pattern also emerges in inflation expectations for the next five-ten years, although the differences across income levels are smaller. These differences by income are illustrated in Tabe B.6, which plots the distribution of sample inflation expectations over time, for the bottom income quartile and top income quartile of survey respondents, at one and five-ten year horizons. Consistent with Figures B.4 and B.5, the sample variance of inflation expectations is revealed to be wider for those on lower incomes, with this variance increasing since 2021.

Differences in inflation expectations by gender are smaller but have been more persistent than differences across education or income levels. Mean inflation expectations for women have been around one percentage point higher than those of men over the past five years, although the discrepancy across genders has widened somewhat since the inflation surge of 2021. The sample variance of female inflation expectations has also increased by more than that for men, although only since 2021. We find no substantial differences in inflation expectations across age groups.

We also use the University of Michigan Surveys of Consumers to study inflation forecast errors. We define forecast errors at one and five-year horizons as the one-year ahead and five-ten year ahead inflation expectations reported one and five years previously less actual headline CPI inflation. We then examine how consumers update their inflation expectations over time. This is possible because the sample includes responses from individuals surveyed twice, with each survey separated by six months. The responses from both surveys can be compared because each participant is given a unique identification number. Specifically, we can compare how the inflation expectations of these consumers changed over the six month interval between surveys. Figure B.7 shows how the mean and standard deviations of inflation forecast errors have evolved over time. After under-estimating inflation around 1980, the mean forecast errors at one and five-year horizons have been mostly positive over the subsequent four decades, suggesting that consumers over-estimate future inflation on average. The exception occurs after 2021, when the mean inflation forecast errors was in trend decline from 1980-2000 and has since been relatively low, with the exception of the post-crisis period around 2010 and following the surge of inflation after 2021.

Following the methodology in Madeira and Zafar (2015), Table B.1 shows the results of OLS regressions of inflation expectations by participants surveyed twice, as well as of their forecast errors, both in absolute terms and also in terms of the change between the first and second surveys. Each econometric model contains a constant and dummy variables for gender, age, income and education levels. The coefficients in the regressions of absolute forecast errors at the first and second interviews are illustrated in Figure B.8. The estimates suggest that a low-income and / or female consumer will have higher forecast errors, on average, all else equal, with the size of the forecast error being likely to decline between the first and second survey. Age is estimated to have a smaller impact on forecast error, while additional years of schooling are estimated to reduce forecast error on average, all else equal. The results suggest that consumers are updating inflation expectations between surveys based on information received in that time, improving the accuracy of their forecasts. This provides some motivation for the concept of adaptive learning, in the sense that consumers are adjusting expectations as more information becomes available.²

2.2 Rationality Tests

To motivate further our work on adaptive learning, we investigate the rationality of mean monthly household expectations by running the following regression (see also Forsells and Kenny (2004)):

$$\pi_t = \alpha + \beta \pi_t^e + \varepsilon_t \tag{1}$$

where π_t denotes the actual inflation rate in period t and π_t^e denotes the expected inflation rate formed in t - 12 by households. If the joint null hypothesis H_0 : $(\alpha, \beta) = (0, 1)$ cannot be rejected and ε_t exhibits no evidence of autocorrelation, then it follows that expectations

 $^{^{2}}$ Table B.2 shows that there are relatively few households that are classified as high income and low education or low income and with a college degree. As expected, there is a strong correlation between income and education.

are unbiased in a statistical sense. The above rationality tests are conducted by ordinary least squares using covariance matrix corrections suggested by Newey and West (1987). The results (Table B.3) illustrate that the null hypothesis, $H_0: (\alpha, \beta) = (0, 1)$, can be rejected at the 1% and 5% level for overall expectations and also for expectations of different demographic groups. Moreover, the Durbin-Watson statistic shows evidence of autocorrelation. Similar results are obtained when using the simple rationality test proposed in Mankiw et al. (2003) regressing expectation errors on a constant (Table B.4). This suggests that consumer inflation expectations are not rational.

3 A Simple Model of Adaptive Learning

3.1 The Model

In this section, we present a simple, reduced form model of adaptive learning, estimated on a sample including the recent inflation surge. This section follows Branch and Evans (2006) and Weber (2010) and outlines a general state space forecasting model that nests alternative models.

Let π_t denote inflation in period t. It is assumed that the inflation process can be written in its reduced form as

$$\pi_t = \mathbf{b}_t' \mathbf{x}_t + \varepsilon_t \tag{2}$$

where $\mathbf{b}_t = (b_{1t}, b_{2t}, b_{3t}, ..., b_{(n+1)t})'$ and $\mathbf{x}_t = (1, \mathbf{y}_{t-1})'$. Furthermore ε_t is a serially uncorrelated disturbance with mean zero and variance H_t , that is $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = H_t$.

Let \mathbf{y}_t with dimension $n \ge 1$ denote a vector of variables of interest. Thus n is the number of independent variables in our model. These could be lagged values of inflation, output growth or changes in interest rates for example. Economic agents assume that inflation follows the autoregressive process specified in 2. Hence economic agents view inflation in period t as a function of a constant and lagged variables of general interest. Furthermore economic agents are seen as forming their expectations for inflation for the next period using the current values of variables of interest such as inflation and output growth.

Together with the assumption that

$$\mathbf{b}_t = \mathbf{b}_{t-1} + \boldsymbol{\eta}_t \tag{3}$$

where $E(\boldsymbol{\eta}_t) = 0$ and $E(\boldsymbol{\eta}_t \boldsymbol{\eta}_t') = \mathbf{Q}_t$, the above corresponds to a general state space model with \mathbf{b}_t being the state vector.

Conditional forecasts of π_t are given by $\widehat{\pi}_{t:t-1} = \widehat{\mathbf{b}}'_{t-1} \mathbf{x}_t$.

The parameter vector \mathbf{b}_t can be estimated using the Kalman filter.³ The recursion can be written as follows:

 $^{^{3}}$ For an explanation of the basic Kalman filtering procedure, see Hamilton (1994).

$$\widehat{\mathbf{b}}_t = \widehat{\mathbf{b}}_{t-1} + \mathbf{k}_t (\pi_t - \widehat{\mathbf{b}}'_{t-1} \mathbf{x}_t)$$
(4)

where the Kalman gain, \mathbf{k}_t , is given by

$$\mathbf{k}_{t} = \frac{\left(\mathbf{P}_{t-1} + \mathbf{Q}_{t}\right)\mathbf{x}_{t}}{H_{t} + \mathbf{x}_{t}'(\mathbf{P}_{t-1} + \mathbf{Q}_{t})\mathbf{x}_{t}}$$
(5)

and

$$\mathbf{P}_{t} = \mathbf{P}_{t-1} - \frac{(\mathbf{P}_{t-1} + \mathbf{Q}_{t}) \mathbf{x}_{t} \mathbf{x}_{t}' (\mathbf{P}_{t-1} + \mathbf{Q}_{t})}{H_{t} + \mathbf{x}_{t}' (\mathbf{P}_{t-1} + \mathbf{Q}_{t}) \mathbf{x}_{t}} + \mathbf{Q}_{t}$$
(6)

where $\mathbf{P}_t = E(\mathbf{b}_t - \widehat{\mathbf{b}}_t)(\mathbf{b}_t - \widehat{\mathbf{b}}_t)'$.

As shown by Marcet and Sargent (1989) the learning process converges to equilibrium only when the law of motion of the parameters is time invariant.⁴ In other words, convergence requires $\mathbf{Q}_t = 0$. Within the Kalman filter framework it is hence possible to test whether learning is perpetual or whether it converges to equilibrium by examining whether the variance of the state variables is significantly different from zero.

If $\mathbf{Q}_t = 0$ and $H_t = 1$, the Kalman filter recursions, (4)-(6), become equivalent to recursive least squares (RLS) (Sargent (1999)). The system can then be written as

$$\widehat{\mathbf{b}}_t = \widehat{\mathbf{b}}_{t-1} + \gamma_t \mathbf{R}_t^{-1} \mathbf{x}_t (\pi_t - \widehat{\mathbf{b}}_{t-1}' \mathbf{x}_t)$$
(7)

$$\mathbf{R}_{t} = \mathbf{R}_{t-1} + \gamma_{t} (\mathbf{x}_{t} \mathbf{x}_{t}' - \mathbf{R}_{t-1})$$
(8)

where $\gamma_t = t^{-1}$ and \mathbf{R}_t is the matrix of second moments of \mathbf{x}_t . The gain, γ_t , will approach zero as $t \to \infty$. Thus, the above algorithm corresponds to the recursive formulation of ordinary least squares. When economic agents use recursive least squares to update their parameter estimates, these estimates will eventually converge to their rational expectations values (Evans and Honkapohja, 2001).

If $\mathbf{Q}_t = \frac{\gamma}{1-\gamma} \mathbf{P}_{t-1}$ and $H_t = 1-\gamma$, the system becomes equivalent to the constant gain version of recursive least squares (Sargent, 1999), so that $\gamma_t = \gamma$ in equations (7) and (8). Using a constant gain algorithm implies that more weight is placed on recent observations. This algorithm is equivalent to applying weighted least squares where the weights decline geometrically with the distance in time between the observation being weighted and the most recent observation. Thus, the constant gain learning algorithm resembles estimation by ordinary least squares, but with a rolling window of data where the sample size is approximately $\frac{1}{\gamma}$. Past observations are discounted at a geometric rate of $1 - \gamma$. Hence constant gain least squares learning (CGLS) is more robust

⁴The Kalman filter framework allows one to test whether expectations converge towards the rational expectations equilibrium. However, this assumes that agents use the correct model of the economy. If the model used for forecasting is incorrect, expectations may converge towards a so called 'restricted perceptions equilibrium' (Evans and Honkapohja, 2001).

to structural change than recursive least squares learning. Evans and Honkapohja (2001) provide a more detailed explanation of both learning algorithms.

3.2 Simple learning rules

3.2.1 Estimation procedure

We follow Branch and Evans (2006) and divide the sample for inflation and inflation expectations in three parts: a pre-forecasting period in which prior beliefs are formed by estimating (2); an in-sample period in which optimal gain parameters are determined for the case of constant gain least squares, while for recursive least squares learning the gain sequence continues to be updated as t^{-1} ; and finally, an out-of-sample forecasting period.

A fairly long pre-forecasting period, 1961M6-1979M11 is chosen in order to avoid oversensitivity of the initial estimates. The in-sample period is 1979M12-2011M2. The out-of-sample period is hence 2011M3-2022M12. Given the difficulty in predicting the post Covid period, we also calculate mean-square forecast errors up to 2020M2.⁵ Given the monthly frequency of the data, the inflation expectation by households in period t - 12 for period t is hence given by

$$\widehat{\pi}_{t|t-12} = \widehat{\mathbf{b}}_{t-12}' \mathbf{x}_t \tag{9}$$

When economic agents form expectations, the best estimate of the coefficients in period t-12 is used. As new data become available agents update their estimates according to either constant gain least squares learning or recursive least squares learning. The formulae for this updating process under recursive least squares learning are given by equations (7) and (8). Under constant gain least squares learning, γ_t in those recursions is replaced by the constant gain, γ .

As a first step, we calculate the constant gain that agents would optimally use to project inflation by using actual inflation data for the US, we refer to this as the optimal in-sample constant gain parameter. This results in hypothetical inflation forecasts, which would prevail if agents used the above model and the optimal constant gain to update their expectations. To compute the optimal gain, the in-sample mean square forecast error

$$MSE_{IN}(\pi) = \frac{1}{T} \sum_{t=t_0}^{T} (\pi_t - \hat{\pi}_{t,t-12})^2$$

is minimised by searching over all $\gamma \in (0,1)$ with $t_0 = 1979M12$ and T = 2011M2. The distances between grids are set at 0.0001. $\hat{\pi}_{t,t-12}$ denotes the hypothetical forecast made in period t - 12 for t, using actual inflation and optimal constant gains. This forecast is generated

⁵This sample period was chosen so that the in- and out-of-sample periods correspond to the period for which household expectations are available. We tried different splits of the data, and the qualitative results remained robust even though the estimation parameters changed somewhat. Results are available from the authors upon request.

by starting the recursions, equations (7) and (8), with $\gamma_t = \gamma$ where the initial values are calculated from the pre-sample period, and then using these recursive equations to calculate $\hat{\mathbf{b}}_t$ and $\hat{\pi}_{t:t-12} = \hat{\mathbf{b}}'_{t-12}\mathbf{x}_t$. When using recursive least squares there is no need to compute an optimal gain parameter as $\gamma = t^{-1}$. The mean square errors can be computed by updating the sequence for $\hat{\mathbf{b}}_t$ with t^{-1} .

Having determined the optimal in-sample values of the constant gain, out of sample MSEs can be computed as

$$MSE_{OUT}(\pi) = \frac{1}{T} \sum_{t=1}^{T} (\pi_t - \hat{\pi}_{t:t-12})^2$$

where t ranges from 2011M3 to 2020M2 or 2022M12.

The second step is to analyze how expectations are actually formed and which constant gain best fits actual mean household inflation expectations, including by demographic group. The best fitting constant gain is computed by minimising the in-sample mean square comparison forecast error

$$MSCE_{IN}(\pi) = \frac{1}{T} \sum_{t=t_0}^{T} (\pi_t^F - \hat{\pi}_{t:t-12})^2$$

by searching over all $\gamma \in (0,1)$ with $t_0 = 1979 \text{M}12$ and T = 2011 M2. π_t^F denotes actual household expectations for period t. The distances between grids are set at 0.0001. Best fitting constant gain parameters are computed to determine whether the best fitting gains that are needed to fit household expectations are equivalent to those needed to fit actual data on inflation in the in-sample period. As before, using the best fitting gains for household expectations, the out-of-sample mean square comparison forecast error is determined. This is given by

$$MSCE_{OUT}(\pi) = \frac{1}{T} \sum_{t=1}^{T} (\pi_t^F - \hat{\pi}_{t:t-12})^2$$

where t ranges from 2011M3 to 2020M2 or 2022M12.

For RLS learning, the in-sample and out-of sample MSCEs are calculated as above. The recursive equations (7) and (8) are updated with t^{-1} .

We consider four different specifications for agents' inflation expectation formation process. Model 1 is a simple AR(1) model where the independent variables are a constant and the lagged value of inflation. Model 2 is a simple AR(2) model with a constant and lagged values of inflation.⁶ Model 3 includes a constant, lagged inflation and lagged output growth, which is approximated by growth in industrial production. Model 4 includes changes in interest rates in

⁶Results for higher order AR models were also computed but it was found that the AR(1) and AR(2) models outperformed higher order models. The AR(1) and AR(2) models led to both smaller out-of-sample MSEs and smaller out-of-sample MSCEs.

addition to the variables in Model 3. Models 1-4 can thus be written as follows:

$$\widehat{\pi}_{t|t-12} = b_{1,t-12} + b_{2,t-12}\pi_{t-12} \tag{Model 1}$$

$$\widehat{\pi}_{t|t-12} = b_{1,t-12} + b_{2,t-12}\pi_{t-12} + b_{3,t-12}\pi_{t-13} \tag{Model 2}$$

$$\widehat{\pi}_{t|t-12} = b_{1,t-12} + b_{2,t-12}\pi_{t-12} + b_{3,t-12}z_{t-12}$$
(Model 3)

$$\widehat{\pi}_{t:t-12} = b_{1,t-12} + b_{2,t-12}\pi_{t-12} + b_{3,t-12}z_{t-12} + b_{4,t-12}w_{t-12} \tag{Model 4}$$

where z_t denotes industrial production growth and w_t denotes changes in interest rates. The interest rate used in the models is the Fed Funds Rate.

3.2.2 Results

In order to assess whether it is possible to fit actual inflation with a learning model, the optimal constant gains by model as well as associated mean-square forecast errors are computed. A higher gain coefficient implies that agents should optimally use fewer years of data to form a prediction of inflation.⁷ Moreover, we also compute mean-square forecast errors for the four models under recursive least squares learning. Table 1 shows out-of-sample mean square forecast errors using both constant gain and recursive least squares learning.

Table 1: Actual Inflation and Fitted Inflation

	CGLS Model 1	Model 2	Model 3	Model 4	RLS Model 1	Model 2	Model 3	Model 4
MSE(pre-Covid)	0.25	0.13	0.16	0.39	1.17	1.20	1.23	1.21
MSE (full sample)	1.85	2.12	4.15	1.51	3.70	3.74	3.21	3.20
Optimal constant gain	0.19	0.14	0.13	0.04	N/A	N/A	N/A	N/A

This table is computes optimal constant gain and out-of-sample mean-squared forecast errors for actual inflation

It can be seen that constant gain dominates recursive least squares learning in terms of forecast accuracy.⁸ No single model fits best for all periods though. The simple model with constant gain learning and lagged inflation and a constant as the independent variables does well if we exclude the Covid period from the out-of-sample MSE. If we include the period following 2020M2, then model 4 does better. Table 1 and Figure B.9 highlight that constant gain least squares learning performs well in fitting actual inflation, especially in the pre-Covid period.

⁷If the gain is denoted by γ , then this gain implies that agents use $(1/\gamma)/f$ years of data, where f denotes the data frequency: f = 1 for yearly data, f = 4 for quarterly data and f = 12 for monthly data.

⁸We performed modified Diebold/Mariano (Diebold and Mariano, 1995) tests with the null of equal forecast accuracy to test whether the differences in MSEs between RLS and CGLS are significant. We test whether the difference between the largest MSE under CGLS and the smallest MSE under RLS is significant. It is found that the null hypothesis of equal forecast accuracy can be rejected at the 5% level of significance. P-values and modified Diebold/Mariano statistics can be provided by the author upon request.

We also analyze which model can best explain data on inflation expectations. Best fitting gains are computed by minimizing the in-sample mean square comparison forecast errors to assess whether there is heterogeneity regarding the best fitting constant gain parameters between models and households with different characteristics.

	CGLS Model 1	Model 2	Model 3	Model 4	RLS Model 1	Model 2	Model 3	Model 4
MSCE (pre-Covid)	4.14	3.82	3.83	1.79	2.22	2.52	2.38	2.04
MSCE (full sample)	5.81	7.60	8.21	3.53	4.11	4.35	3.83	3.99
Best fitting constant gain	0.21	0.15	0.13	0.01	N/A	N/A	N/A	N/A

Table 2: Household Overall Mean Inflation Expectations and Fitted Expectations

This table is computes best-fitting constant gains and out-of-sample mean-squared comparison errors for inflation expectations

From Table 2, it can be seen that best fitting gains to fit mean overall household inflation expectations are broadly in line with optimal gains needed to fit actual inflation. In terms of the lowest forecast errors, model 4 seems to do best for both the pre-Covid and post-Covid period.

We now perform the same analysis but for expectations of different demographic groups since we showed in section 2 that the variance and forecast errors vary significantly across different demographics. Table 3 shows the best fitting model and associated best fitting constant gains mean squared comparison forecast errors.

	Model specification	Best fitting gain	MSCE (pre-Covid)	MSCE (with-Covid)
Low income	1	0.001	3.06	5.04
High income	4	0.01	0.65	1.74
With college degree	4	0.012	0.90	1.80
HS or less	1	0.005	2.70	5.58
Female	1	0.001	2.19	3.74
Male	4	0.011	1.02	2.40

Table 3: Fitting Inflation Expectations by Demographic Characteristics

This table computes best-fitting constant gains for the best fitting model and out-of-sample mean-squared comparison forecast errors for household inflation expectations with different demographics. Low and high-income denote top and bottom tercile.

Results from Table 3 suggest that there is considerable heterogeneity in inflation expectations formation across different types of households. More educated, wealthier, male households seem to use a more complex model to form expectations (Model 4) and their best fitting constant gains match the optimal constant gains quite closely for model 4.⁹ In contrast, low income, less educated, female households tend to use a very simple AR(1) model (Model 1) to form inflation

⁹The time (in months) it takes for the weight given to an observation to fall to 1/2 is given by the following formula: $t_{half} = \frac{\ln 2}{\gamma}$.

expectations and their best fitting constant gains are significantly lower than the optimal constant gain for model $1.^{10}$

These differences could be caused by a greater awareness of the presence of structural breaks by higher income and more educated individuals who rely on a larger set of indicators to project inflation. It could also be the case that those individuals are more willing to incur the costs of updating their information sets than low income, less educated households, which update their information sets less frequently (Carroll, 2003); (Diebold and Mariano, 1995).

Figure B.10, Figure B.11, and Figure B.12 show actual mean household inflation expectations (overall and for the lowest and highest income terciles), and the generated series for expectations of inflation using the optimal model and best fitting constant gain. While the direction of inflation expectations can be predicted well, especially in the pre-Covid period, actual expectations are more volatile than our generated series. This is especially true for lower income households, for which the fit of our predicted series is worse than for high income households. A possible explanation may be that there are certain stochastic shocks and events to which households react and which also influence their expectations. Such events may be more prevalent for lower income households. It could also be the case that households' expectations are very much influenced by prices of goods which form a large share of their consumption. As food and gasoline prices are natural candidates here, we perform the same exercise as in Table 3 but use the combined CPI index for food and energy instead of overall CPI inflation. The results are shown in Table 4. Interestingly, we can fit low-income and less educated household expectations better using food and energy inflation, with a lower MSCE than using the overall CPI, while this is not true for more educated, higher income households.

Table 4:	Fitting	Inflation	Expectations	by	Demographic	Characte	eristics	using	Food	and	Gasol	ine
Inflation	l											

	Model specification	Best fitting gain	MSCE (pre-Covid)	MSCE (with-Covid)
Low income	1	0.004	1.37	3.53
High income	4	0.008	1.00	1.87
With college degree	4	0.005	1.00	3.09
HS or less	1	0.005	1.72	1.58
Female	1	0.005	1.23	1.36
Male	4	0.007	0.84	1.08

This table computes best-fitting constant gains for the best fitting model and out-of-sample mean-squared comparison forecast errors for household inflation expectations with different demographics using a composite CPI for food and gasoline from BLS with fixed weights. Low and high-income denote top and bottom tercile.

¹⁰We also run model 1 for college degree, high income, and male household mean inflation expectations and find that while it results in higher out of sample forecast errors than model 4, the best-fitting constant gain is very close to the optimal constant gain for model 1. Running model 4 for low income, less educated, female household mean expectations results in much lower best fitting gains than would be optimal to fit actual inflation. Results are available from the authors upon request.

4 Adaptive Learning in a Structural DSGE Model

In this section, we assess whether adaptive learning can improve the ability of a larger, microfounded theoretical model to match data on consumer inflation expectations and the evolution of inflation. We estimate a medium-scale, DSGE structural model of the U.S. using quarterly data from 1965Q1 - 2022Q4. ¹¹ The model is estimated under two alternative assumptions about how expectations are formed: (i) rational expectations, so that agents know the full structure of the model, including the distribution of shocks and full history of endogenous variables, when forming expectations at each point in time; (ii) adaptive learning, so that agents form expectations about the future values of certain endogenous variables ("forward" variables), using simple autoregressive forecasting models that they estimate recursively.

The structural model of the U.S. economy is the one used in Slobodyan and Wouters (2012a), which is based on the medium-scale DSGE model in Smets and Wouters (2007). The model contains forward-looking consumers, who make optimal consumption and investment decisions to maximize expected utility over an infinite discrete time horizon. There is an investment-specific production technology, as well as the one for consumption goods. There are a range of frictions in the model that are common in larger DSGE models and help the model mimic certain features of observed data, such as external habits in consumption, investment adjustment costs, as well as sticky prices and wages. Prices and wages are set according to the Calvo mechanism, with partial indexation to past price and wage inflation. Monetary policy is determined by a Taylor-type rule, with inertia, where the Fed Funds Rate also depends on inflation and the output gap.

The model contains 14 endogenous variables, denoted by the vector y_t including seven variables about which agents must form expectations about their future values: vector y_t^f denotes consumption, hours worked, investment, wages, inflation and the price and return on existing capital. The model also contains a vector of seven exogenous variables, denoted by vector w_t . The first five of these exogenous variables are determined by an AR(1) exogenous process, each subject to i.i.d innovations: consumption and investment-specific technological progress, the risk premium, an exogenous demand spending process and monetary policy shocks. The final two exogenous processes are price and wage mark-ups, each of which is determined by a more general ARMA(1,1) process, subject to i.i.d innovations associated with the exogenous processes are denoted ϵ_t . Further information about the model is presented in Appendix A.

4.1 Model Solution under Rational Expectations

Under rational expectations, the solution of the model can be represented in autoregressive form as:

¹¹Data for 2020Q1-Q4 is excluded from the estimation sample, given extreme volatility.

$$z_t = [y_t, w_t]'; z_t = \mu + T z_{t-1} + R \epsilon_t$$
(10)

where T and R are non-linear functions of the parameters of the structural model. The model is solved after being log-linearized, so the fourteen endogenous variables have mean zero. The vector z_t is augmented with seven measurement equations, linking seven observed variables with their counterpart endogenous variables in the model: output growth, consumption growth, investment growth, wage growth, hours worked, inflation and the Fed Funds Rate. The observed variables have non-zero means, which are included as constants in these equations and become additional estimated parameters. The measurement equations are shown in Appendix A, while the data sources for the observable variables used for estimation are listed in Appendix B. The Kalman Filter is used to construct the log-likelihood function of z_t . The structural parameters of the model, as well as the constant terms in the measurement equations, are then estimated using Bayesian techniques.

4.2 Model Solution under Adaptive Learning

The model is also solved relaxing the rational expectations assumption, so that agents do not know the full structural model and can't use it to form expectations of the future values of endogenous variables. Instead, we assume that agents only observe historical data on the seven forward variables y_t^f at each point in time and use simple AR(2) models to forecast each variable, with each model for a forward variable $j \in [1, 2, ...7]$ containing a constant and two lags of the variable to be forecast ¹²:

$$y_{j,t}^{f} = \beta_{0}^{j} + \beta_{1}^{j} y_{j,t-1}^{f} + \beta_{2}^{j} y_{j,t-2}^{f} + u_{j,t};$$
(11)

where u_t is an error term with mean zero, although the possibility of correlation between the error terms in each of the seven forecasting equations is allowed. The system formed by these seven forecasting equations is the 'Perceived Law of Motion (PLM)' that agents use to forecast and form expectations. The system of forecasting equations is assumed to be estimated recursively, with agents using the Kalman Filter to update estimates at each point of the sample. After updating, the estimated PLM is used to form one-quarter ahead expectations (or forecasts) of the seven forward variables. The estimated PLM at each point of the sample is substituted into the equations of the structural model for the seven forward looking variables. The model solution then becomes time-varying, with the matrices T_t and R_t depending on the parameters of the structural model, the intercepts in the measurement equations and on the coefficients of the PLM, which change at each point in the sample. The model's solution can then be written in the following form and is referred to as the Actual Law of Motion (ALM)

 $^{^{12}}$ The AR(2) learning model is found to be a good fit of overall inflation expectations in 3.

$$z_t = [y_t, w_t]'; z_t = \mu + T_t z_{t-1} + R_t \epsilon_t$$
(12)

Using the same observable variables and measurement equations, the Kalman Filter is used to construct the log-likelihood function of the model, which is estimated using Bayesian techniques.

4.3 Comparing Rational Expectations and Adaptive Learning

The prior and posterior distributions of the structural parameters are shown in Tables B.5 and B.6, together with the marginal density of the data in each case. Overall, the results imply that the adaptive learning mechanism captures expectations formation better than the rational expectations model and improves the fit and forecasting performance of the model, at least at short horizons of less than one year.

First, the higher marginal density of the data in the case of adaptive learning suggests that the simple forecasting models recursively estimated, and used to form expectations, improve empirical fit over the case of rational expectations.

Second, the estimated persistence of the price mark-up shock is lower under adaptive learning than rational expectations - i.e. the estimated AR(1) parameter and MA(1) parameter in the exogenous ARMA(1,1) process for the price mark-up shock is lower. At the same time, the estimated degree of indexation of prices to past inflation is higher under adaptive learning. This suggests that the adaptive learning mechanism allows the model to account endogenously for the gradual propagation of price mark-up shocks, without having to interpret the shocks as being highly persistent. By contrast, the wage mark-up shock is estimated to be highly persistent under both rational expectations and adaptive learning. This finding is somewhat different to that of Slobodyan and Wouters (2012a), who estimate this model over a much shorter sample, ending in 2008, finding that the persistence of both the wage and price mark-up shocks declines when the model is estimated under adaptive learning. Another novel finding is that the parameter determining the elasticity of labor supply is now estimated to be much larger under adaptive learning than rational expectations, whereas Slobodyan and Wouters (2012a) found little difference between estimated structural parameters under rational expectations and adaptive learning, other than for wage indexation.

Third, of particular interest in this paper, the model with adaptive learning has better forecasting performance than that with rational expectations. Figure B.13 shows cumulative onequarter ahead forecast errors of the PLM and ALM of the model with adaptive learning, compared with the model with rational expectations. Forecast errors are defined as the inflation forecast less actual inflation. The rational expectations model persistently over-estimates inflation during the 1990's and early 2010's, whereas the PLM and ALM of the model with adaptive learning are more accurate, suggesting that the adaptive learning mechanism helps the model capture the impact of the Great Moderation.¹³

Finally, the PLM of the model with adaptive learning captures broad trends in actual data on consumer survey expectations, in contrast to the rational expectations model, which produces forecasts closer to those in surveys of professional forecasters. In order to make this comparison, forecasts from the estimated DSGE model under rational expectations and adaptive learning are now produced for the next year ahead, in line with those in surveys. Figure B.14 compares forecast errors from the PLM with those from the University of Michigan Survey of Consumer Expectations. The forecast errors of the autoregressive PLM are larger at one-year horizons, compared with the one-quarter ahead errors shown in Figure B.13, particularly in the 1990s's and 2000's. Since the Global Financial Crisis though, the University of Michigan survey forecasts over-estimated inflation significantly, with cumulative forecast errors trending upwards, similar to those from the PLM. Figure B.15 compares the cumulative one-year ahead forecast errors from the rational expectations model and ALM of the adaptive learning model, with forecasts in the Survey of Professional forecasters. The forecast errors from the rational expectations model track those from the Survey of Professional Forecasters much more closely than those from the adaptive learning ALM, which are much lower, suggesting that professional forecasts reflect similar considerations to those in a structural DSGE model, making them less accurate than those in the adaptive learning model. It is important to note that the rational expectations and adaptive learning model significantly underestimated inflation since 2021, as did consumers and forecasters in the University of Michigan and Survey of Professional Forecasters.¹⁴

Slobodyan and Wouters (2012b) also find that the adaptive learning model improves upon the rational expectations model, despite the model being estimated on an earlier sample period (1966-2008), with improved marginal density of the data, better forecasting performance and better ability to match movements in survey measures of inflation expectations.

4.4 Changing Inflation Dynamics over Time

The recent surge of inflation since 2021 appears to have been estimated to be a large and shortlived price mark-up shock in both the rational expectations and adaptive learning versions of the model, even though price mark-up shocks in the adaptive learning model are not estimated to be inherently persistent, as shown above when discussing the estimated coefficients of the

¹³This is in line with Reis (2022) who shows that in normal times household survey inflation expectations data lags professional surveys and is less accurate, but during large changes in inflation, household data were more informative. Especially when taking into account shifts in the distribution of household inflation expectations, household expectations are shown to have informational value beyond what is contained in professionals' expectations (see also Brandao-Marques et al. (2023)).

¹⁴Forecast errors are used to assess forecasting performance across models and surveys, to abstract from complications caused by estimating the models using latest historical data, which reflects ex post data revisions. Specifically, the forecast errors from the models are computed by reference to latest historical data, on which the models are estimated, while the forecast errors from surveys of consumers and professional forecasters are constructed by reference to real time data, since this was available to the survey participants at the time of forecasting.

relevant ARMA(1,1) process. Figure B.19 and Figure B.20 show the estimated innovations to the exogenous processes w_t over the sample. It is perhaps unsurprising that the surge of inflation is interpreted as a large price mark-up shock, given the comparative stability of inflation over the preceding three decades. Nonetheless, the short-lived shock has a prolonged impact in the adaptive learning model, given the propagation mechanism that learning introduces.

One way to illustrate this is to examine the estimated persistence of inflation, in the PLM used by consumers in the model to make forecasts (form expectations). Figure B.17 shows how the sum of the estimated coefficients of the first and second lags of inflation have changed in the model over the sample, as the model is estimated recursively. The main observation is that inflation has been estimated to have been persistent over the whole sample, with a coefficient of around 0.9. Nonetheless, the estimated PLM showed substantial volatility in the late 1970's and 1980's, which dissipated over the subsequent three decades but has since re-emerged during the surge of inflation during 2022. The estimated intercept term in the inflation forecasting equation has been close to zero over the sample (see Figure B.18), suggesting little drift in inflation over the sample. It is important to note that Slobodyan and Wouters (2012a) find that the estimated autoregressive coefficients in the inflation forecasting equation declined more sharply during the Great Moderation. This was in a version of the structural model where the price and wage mark-up shocks were modeled as white noise processes, rather than ARMA(1,1). Slobodyan and Wouters (2012a) simplified their model with white noise price and wage mark-up shocks, after estimating the model with ARMA(1,1) processes and finding the estimated coefficients of the ARMA(1,1) parameters (AR(1) and MA(1) coefficients) to be consistent with white noise for both prices and wages. We estimate Slobodyan and Wouters (2012a) over a significantly longer sample, including recent data and find that wage mark-up shocks still follow a persistent ARMA(1,1), even though price mark-up shocks are closer to white noise under adaptive learning.

Another way to look at the prolonged impact of price mark-up shocks in the model with adaptive learning is to study the Impulse Response Functions (IRFs) of endogenous variables to a price mark-up shock. Comparing these IRFs to those from the model with rational expectations illustrates the impact of adaptive learning on the propagation of these mark-up shocks. Given that the ALM and model solution is time-varying in the model with adaptive learning, IRFs change over the sample. Figure B.16 shows the IRFs from the model with adaptive learning in (i) 1981, before the Great Moderation; (ii) in 2019, after the Great Moderation and before the pandemic; and (iii) 2022, after the inflation surge of 2021. It also shows the IRFs from the model with rational expectations. Given that the estimated PLM coefficients have been stable over the sample, as discussed above, implying that inflation is perceived to be persistent, the IRFs in the model with adaptive learning change little over time.

The main difference is between the IRFs in the adaptive learning model with those under rational expectations. A one standard deviation price mark-up shock (equal to around 0.1 percentage points) has a similar initial impact on inflation in the models with adaptive learning and rational expectations, of around 0.3 percentage points, but the impact is substantially more persistent under adaptive learning, still being around 0.1 percentage points above steady state after 12 quarters. This is consistent with the reduction in output in the adaptive learning model being only around one third as deep as in the rational expectations model, although more persistent. The increase in the Fed Funds Rate required to return inflation to the two percent objective is more than 50 percent larger in the model with adaptive learning than in the model with rational expectations. It is also significantly more persistent, with the Fed Funds Rate remaining around 0.1 percentage points above steady state after 20 quarters. Wage mark-up shocks also have a more persistent impact in the model with adaptive learning, compared with that of rational expectations.

5 Conclusion

The widening of the distribution of inflation expectations since 2021 is striking, as is the sharp rise in short-term inflation expectations. Micro data shows that lower income and less educated consumers, as well as women, appear to have higher and more heterogeneous expectations during this period.

This paper has presented empirical and theoretical evidence showing that an adaptive learning framework is consistent with these facts. We have shown survey evidence that lower income and female consumers make larger inflation forecast errors, producing a higher mean and variance of inflation expectations within these groups. A reduced form model of adaptive learning is consistent with their behavior, if these types of consumers place less weight on more recent data and have more persistent inflation expectations. A theoretical DSGE model estimated on a US sample including the recent inflation surge suggests adaptive learning better matches how consumer inflation expectations are formed, producing consumer expectations more in line with consumer survey measures. The model with adaptive learning has better overall forecasting performance for inflation than one with rational expectations. The model shows that adaptive learning makes the inflation caused by cost push shocks more persistent (interpreting the recent surge as a cost push shock). The model implies that higher interest rates, maintained for longer, are required to return inflation to target in response to these shocks.

A precise, micro-founded explanation for the large differences in inflation expectations between demographic groups (e.g. income and education), within an adaptive learning framework, is beyond the scope of this paper. Further research is needed into whether different information sets, different consumption baskets, or different ways of forming and updating expectations, are responsible for the disparity across these groups.

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A Technical Appendix

The following log-linearized equations define the equilibrium of the DSGE model of Slobodyan and Wouters (2012a), which is similar to that of Smets and Wouters (2003) and Smets and Wouters (2007), and is used in Section 4. A variable x which appears as \hat{x} is log-linearized.

The consumption Euler equation takes the following form:

$$\hat{c}_{t} = (\gamma/(\gamma + \eta))E_{t}[\hat{c}_{t+1}] + (\eta/(\gamma + \eta))\hat{c}_{t-1} + (\gamma/(\gamma + \eta))(\sigma_{c} - 1)(wL/C\sigma_{c})(\hat{L}_{t} - E_{t}[\hat{L}_{t+1}]) - (\gamma/\sigma_{c}(\gamma + \eta))((\gamma - \eta)/\gamma)(\hat{R}_{t} - E_{t}[\pi_{t+1}] + \epsilon_{t}^{\hat{b}})$$
(13)

where ϵ_t^b is the AR(1) exogenous process for the risk premium, η is the external habits parameter, γ is the steady-state growth rate of the economy and σ_c is the inverse of the intertemporal elasticity of consumption. The Euler equation for private investment is:

$$\hat{i}_{t} = (1/(1+\beta\gamma^{2-\sigma_{c}}))\hat{i}_{t-1} + ((\beta\gamma^{2-\sigma_{c}}))\hat{i}_{t+1} + (1/(1+\beta\gamma^{2-\sigma_{c}})\gamma^{2}\varphi)\hat{Q}_{t}^{k} + \hat{\epsilon}_{t}^{q}$$
(14)

where ϵ_t^q is the AR(1) process for the production technology specific to investment goods, φ is the elasticity of the capital adjustment cost and \hat{Q}_t^k is the value of the capital stock. It satisfies the following equation:

$$\hat{Q}_{t}^{k} = -(\hat{R}_{t} - E_{t}[\pi_{t+1}] + \hat{\epsilon}_{t}^{b}) + (r^{k*}/(r^{k*} + (1-\delta)))E_{t}[r_{t+1}^{k}] + ((1-\delta)/(r^{k*} + (1-\delta)))E_{t}[Q_{t+1}^{k}]$$
(15)

Aggregate demand and aggregate supply are expressed as follows and must be equal in equilibrium:

$$\hat{y}_t = (c_*/y_*)\hat{c}_t + (i_*/y_*)\hat{i}_t + \hat{\epsilon}_t^g + (r_{k*}k_*/y_*)\hat{u}_t$$
(16)

$$\hat{y}_t = \Phi_p(\alpha \hat{k}_t + (1 - \alpha)\hat{L}_t + \hat{\epsilon}_t^a)$$
(17)

where aggregate demand is based on expenditure and aggregate supply on the production technology of final goods, with Φ_p representing the price mark-up in steady state. The AR(1) exogenous processes associated with aggregate demand and total factor productivity are $\hat{\epsilon}_t^g$ and $\hat{\epsilon}_t^a$ respectively. The equation determining price setting is:

$$\hat{\pi_t} - \iota_p \pi_{t-1} = \beta \gamma^{2 - \sigma_c} (E_t[\pi_{t+1}] - \iota_p \hat{\pi_t}) - z_p \mu_t^p + \epsilon_t^p$$
(18)

where $z_p = (1 - \beta \gamma^{2-\sigma_c} \xi_p)(1-\xi_p)/[\xi_p)(1+(\Phi_p-1)\epsilon_p]$ and $\hat{\mu}_t^p$ is the inverse of real marginal cost $\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha \hat{r}_t^k - \hat{A}_t$. The parameter ξ_p is the probability of a price change in the Calvo model, while ι_p is the parameter determining indexation in the Calvo model. The parameter ϵ_p determines the curvature of the aggregator function. The equation determining wage setting is:

$$\hat{\pi_t^w} - \iota_w \hat{\pi_{t-1}} = \beta \gamma^{2-\sigma_c} (E_t[\pi_{t+1}^{\hat{w}}] - \iota_w \hat{\pi_t}) - z_w \hat{\mu_t^w} + \epsilon_t^w$$
(19)

where $z_w = (1 - \beta \gamma^{2 - \sigma_c} \xi_w) (1 - \xi_w) / [\xi_w) (1 + (\phi_w - 1)\epsilon_w]$ and the wage markup is $\hat{\mu}_t^w = \hat{w}_t - (\gamma / (\gamma + \eta))\hat{c}_t + (\eta / (\gamma + \eta))\hat{c}_{t-1} - \sigma l\hat{L}_t$. Capital accumulation evolves according to:

$$\hat{\bar{k}_t} = (1 - (i_*/\bar{k}_*))\hat{k_{t-1}} + (i_*/\bar{k}_*)\hat{i}_t + (i_*/\bar{k}_*)(1 + \beta\gamma^{2-\sigma_c})\gamma^2 S''\hat{\epsilon}_t^{\hat{q}}$$
(20)

Capital services used in production are then given by:

$$\hat{k_t} = \hat{\mu_t} + k_{t-1}^{-1} \tag{21}$$

and optimal capital utilization is given by:

$$\hat{\mu_t} = (1 - \psi)/\psi r_t^k \tag{22}$$

with ψ being the elasticity of capital utilization. The condition determining the optimal combination of capital and labor inputs is given by:

$$\hat{k_t} = \hat{w_t} - \hat{r_t^k} + \hat{L_t} \tag{23}$$

Finally, the monetary policy rule is given by:

$$\hat{R}_{t} = \rho_{R} \hat{R}_{t-1} + (1 - \rho_{R}) (r_{\pi} \hat{\pi}_{t} + r_{y} (y g \hat{a} p_{t}) + r_{y\Delta} \Delta (y g \hat{a} p_{t})) + \hat{\epsilon}_{t}^{\hat{r}}$$
(24)

where the output gap is given by

$$ygap_t = \hat{y}_t - \Phi_p \hat{\epsilon}_t^a$$

B Data Sources

In Section 4, the data sources for the seven observable variables used in estimation of the structural model are as follows: (i) the quarter-over-quarter growth rate of real personal consumption expenditure is based on nominal personal consumption expenditure from the Bureau of Economic Analysis (BEA), divided by the GDP deflator, also from the BEA, all divided by the non-institutional civilian population (over age 16), as sourced from the Bureau of Labor Statistics (BLS); (ii) the quarter-over-quarter growth rate of private fixed investment, divided by the GDP deflator (both from BEA), all divided by the non-institutional civilian population (over age 16) (BLS); (iii) the quarter-over-quarter growth rate of real gross domestic product (BEA) divided by the civilian non-institutional population (BLS); (iv) non-farm business average weekly hours multiplied by total non-civilian employment (over age 16) (both BLS), all divided by the civilian non-institutional population (over age 16) (BLS), expressed as an index; (v) the quarter-over-quarter growth rate of the GDP deflator (BEA); (vi) the quarter-over-quarter growth rate of non-farm business compensation per hour (BLS), divided by the GDP deflator (BEA); and (vii) the annual effective Federal Funds Rate, from the Federal Reserve Board, expressed as a quarterly rate by dividing by four.



Figure B.1: Headline and Core CPI Inflation

Figure B.2: Short-Term Measures of Inflation Expectations



Sources: University of Michigan and Federal Reserve Bank of Philadelphia. Notes: The chart shows the inflation expected over the next year by consumers in the University of Michigan Survey and by professional forecasters in the Survey of Professional Forecasters.

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Figure B.3: Medium-Term Measures of Inflation Expectations

Sources: University of Michigan, Federal Reserve Bank of Philadelphia and Haver. Notes: The chart shows the inflation expected over the next five years by consumers in the University of Michigan Survey and by professional forecasters in the Survey of Professional Forecasters. It also shows the inflation expectations implied by the difference between yields on 5-year U.S. Treasuries and the inflation-protected 5-year TIPS securities.



Sources: University of Michigan and authors' calculations



Figure B.5: Five to Ten-Year Ahead Consumer Inflation Expectations

Sources: University of Michigan and authors' calculations



Figure B.6: Distribution of Inflation Expectations by Income

Sources: University of Michigan and authors' calculations

Figure B.7: Forecast Errors of Consumers' Inflation Expectations



Sources: University of Michigan and authors' calculations

	1-year infla	tion forecast		Absolu	te error^2
Variable	1st survey	2nd survey	Abs revision of point forecasts ¹	1st survey	2nd survey
	(1)	(2)	(3)	(4)	(5)
Female	1 110***	0.90/***	0 306***	0.966***	0 50/***
1 cillate	(0.0407)	(0.0365)	(0.0311)	(0.0349)	(0.0317)
Young ³	0.422^{***}	0.168***	0.084*	0.256^{***}	0.024
	(0.0631)	(0.0566)	(0.0481)	(0.0541)	(0.0491)
Mid-age	(0.344^{***})	0.259***	0.032	0.076*	0.040
	(0.0495)	(0.0444)	(0.0379)	(0.0424)	(0.0387)
Lowest income tercile	0.897***	0.858***	0.423***	0.850***	0.504***
	(0.0560)	(0.0502)	(0.0429)	(0.0480)	(0.0438)
Middle income tercile	0.297***	0.278***	0.119***	0.180***	0.172***
	((0.0471))	(0.0424)	(0.0357)	(0.0404)	(0.0365)
Education	-0.203***	-0.168***	-0.077***	-0.223***	-0.112***
	(0.0091)	(0.0082)	(0.0070)	(0.0078)	(0.0072)
Inflation in survey month	-0.177***	-0.027	× ,	· · ·	· · · ·
	(0.0309)	(0.0233)			
Realized 1-year ahead inflation	0.255***	0.339***			
	(0.0320)	(0.0231)			
Absolute error in first survey			0.643^{***}		
			(0.0032)		
Actual chg. inflation bet. surveys			0.038**		
			(0.0159)		
Constant	6.390^{***}	4.961^{***}	1.831^{***}	5.973^{***}	3.320^{***}
	(0.2198)	(0.1727)	(0.1104)	(0.1207)	(0.1126)
Observations	82,865	$82,\!858$	68,977	82,865	68,213
R^2	0.084	0.070	0.419	0.084	0.172

Table B.1: Intercept Coefficients: Consumers' Inflation Expectations and Forecast Errors

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

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¹ Defined as |1-year ahead inflation expectation reported in second survey - 1-year ahead inflation expectation reported in first survey|.

 2 Defined as |respondent's 1-year ahead inflation expectation - Actual realized 1-year ahead inflation|.

 3 Young is defined as age < 31; mid-age is defined as age > 30 & age < 61.

Sources: University of Michigan and authors' calculations



Figure B.8: Intercept Coefficients: Consumers' Inflation Expectations and Forecast Errors

Sources: University of Michigan and authors' calculations

Table B.2: Education & Income Quartile Observations

		Income Q	uartile		
Education	Bottom 25%	26-50%	51-75%	Top 25%	Total
HS or Less	$33,\!520$	$24,\!449$	$24,\!899$	15,785	98,653
Some College	$13,\!925$	$17,\!869$	$20,\!285$	$17,\!109$	69,188
College or Above	8,694	$17,\!581$	$31,\!277$	$47,\!974$	105,526
Total	56,139	$59,\!899$	76,461	80,868	273,367

Note: The data covers the period from January 1978 to December 2023.

Sources: University of Michigan

VARIABLES	α	β	χ^2	DW	Observations
Bottom 33%	-0.475	0.718***	39.35	0.1014519	516
	(0.7586)	(0.1604)	[0.0000]		
Middle 33%	-0.105	0.767***	13.96	0.1024677	516
	(0.5937)	(0.1510)	[0.0000]		
Top 33%	0.241	0.800***	3.54	0.1048172	516
	(0.4076)	(0.1161)	[0.0297]		
Male	-0.944	1.137***	3.78	0.1015178	537
	(0.5856)	(0.1715)	[0.0235]		
Female	-1.579**	1.025***	24.90	0.0918531	537
	(0.7315)	(0.1621)	[0.0000]		
HS or Less	-1.740**	1.055^{***}	27.69	0.1008745	537
	(0.8419)	(0.1892)	[0.0000]		
Some College	-1.176*	1.050^{***}	12.51	0.0907454	537
	(0.7115)	(0.1842)	[0.0000]		
College	-0.719	1.064***	3.48	0.0967579	537
	(0.5552)	(0.1593)	[0.0314]		
18-34	-0.934	0.978***	12.04	0.1020401	537
	(0.6379)	(0.1554)	[0.0000]		
35-54	-0.918*	0.984^{***}	11.73	0.0898236	537
	(0.6213)	(0.1557)	[0.0000]		
55 +	-1.868**	1.255***	11.49	0.1211024	537
	(0.8201)	(0.2152)	[0.0000]		
All	-1.333**	1.085^{***}	14.34	0.2075865	539
	(0.5956)	(0.1492)	[0.0000]		

Table B.3: Rationality Tests for Household Expectations

 $\overline{\text{Standard errors in parentheses, P-values in brackets. *** p<0.01, ** p<0.05, * p<0.1. DW denotes Durbin Waton statistic.}} \chi^2 \text{ tests for the joint null of } H_0: (\alpha, \beta) = (0, 1),$

Sources: University of Michigan and authors' calculations .

Variable	α	Observations
Bottom 33%	1.885^{***}	516
	(0.0766)	
Middle 33%	1.086***	516
	(0.0737)	
Top 33%	0.480***	516
	(0.0704)	
Male	0.410***	537
	(0.0796)	
Female	1.454***	537
	(0.0797)	
HS or Less	1.466***	537
	(0.0797)	
Some College	0.951***	537
	(0.0828)	
College	0.465^{***}	537
	0.0780	
18-34	1.031***	537
	(0.0791)	
35-54	0.991^{***}	537
	(0.0794)	
55 +	0.777***	537
	(0.0848)	
All	0.952***	539
	(0.0778)	

 Table B.4: Mean Forecast Error on Constant

Standard errors in parentheses, P-values in brackets. *** p<0.01, ** p<0.05, * p<0.1.

Sources: University of Michigan and authors' calculations .





Sources: University of Michigan, BLS, and IMF Staff Calculations .





Sources: University of Michigan, BLS, and IMF Staff Calculations





Sources: University of Michigan, BLS, and IMF Staff Calculations





Sources: University of Michigan, BLS, and IMF Staff Calculations. Notes: high and low income denote top and bottom tertiles of income distribution

		Prior D	istributi	on
Estimated 1	Parameters	Type	Mean	Std. Dev.
Structural I	Parameters			
α	Cobb-Douglas Intermediate Pdn Fn Parameter	Normal	0.3	0.1
ψ	Fn of elasticity of capital util. adj. cost function	Beta	0.5	0.2
$100(\beta^{-1}-1)$	Fn of discount factor of households	Gamma	0.3	0.2
$100(\gamma - 1)$	Gross deterministic growth rate of economy	Normal	0.4	0.1
ϕ	s.s. elasticity of the capital adjustment cost fn	Normal	4.0	1.5
η	External habit formation of consumers	Beta	0.7	0.1
$\sigma^{ m c}$	Inverse inter-temporal elasticity of substitution	Normal	1.5	0.4
ϕ_p	Price mark-up in s.s.	Normal	1.3	0.1
ι_p	Degree of indexation to past price inflation	Beta	0.5	0.2
$\hat{\xi_p}$	Degree of price stickiness	Beta	0.5	0.1
ι_w	Degree of indexation of wages to past price inflation	Beta	0.5	0.2
ξ_w	Degree of wage stickiness	Beta	0.5	0.1
σ^{ι}	Labor supply parameter	Normal	2.0	0.5
$ ho_R$	Inertia parameter in monetary policy rule	Beta	0.8	0.1
$ au_{\pi}$	Inflation coefficient in monetary policy rule	Normal	1.5	0.3
$ au_{y}$	Output coefficient in monetary policy rule	Normal	0.1	0.1
$ au_{\Delta y}$	Change in output coefficient in monetary policy rule	Normal	0.1	0.1
$\overline{\pi}$	s.s. inflation in measurement equation	Gamma	0.6	0.1
\overline{l}	s.s. hours worked in measurement equation	Normal	0.0	2.0
Persistence	of exogenous processes			
ρ_{a}	Total factor productivity	Beta	0.5	0.2
ρ_b	Risk premium	Beta	0.5	0.2
ρ_a	Exogenous spending	Beta	0.5	0.2
ρ_a	Investment-specific technology	Beta	0.5	0.2
ρ_r	Monetary policy	Beta	0.5	0.2
$a q^{\rm b}$	Coefficient on TFP innovation in invspecific tech. process	Normal	0.5	0.3
ρ_n	Price mark-up AR(1) parameter	Beta	0.5	0.2
ρ_{uv}	Wage mark-up AR(1) parameter	Beta	0.5	0.2
θ_n	Price mark-up MA(1) parameter	Beta	0.5	0.2
θ_w^p	Wage mark-up $MA(1)$ parameter	Beta	0.5	0.2
Standard de	eviation of innovation to the exogenous processes			
σ_a	Total factor productivity	Inverse Gamma	0.1	2.0
σ_{h}	Risk premium	Inverse Gamma	0.1	2.0
σ_{σ}	Exogenous spending	Inverse Gamma	0.1	2.0
σ_{a}	Investment-specific technology	Inverse Gamma	0.1	2.0
σ_{q}	Monetary policy	Inverse Gamma	0.1	2.0
σ_r	Price mark-up	Inverse Gamma	0.1	2.0
σ_p	Wage mark-up	Inverse Gamma	0.1	2.0
$\circ w$	trage mark up	inverse Gamma	0.1	2.0
$\frac{Fixed \ param}{s}$	<u>neter</u>		0.0	
0	Depreciation rate		0.0	
ϕ_w	s.s. labor market mark-up		1.6	
cg			0.2	
ϵ_p	Curvature of the Kimball goods market aggregator		10	
ϵ_w	Curvature of the Kimball labor market aggregator		10	

Table B.5: Prior Distributions of Estimated Parameters in Structural DSGE Mo
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Sources: authors' calculations and Slobodyan and Wouters (2012a). Notes: The choice of priors follows Slobodyan and Wouters (2012a).

			F	Posterior	Distribution			
		Rational	Expect	tions	Adaptivo	Adaptiva Learning		
		national Expectations		Adaptive Learning				
		D	C.1.		Destate	0.1		
		Posterio	5%	95%	Posterio	5%	95%	
		Mean			Mean			
Estimated Parmeters								
Structural Parameters								
α	Cobb-Douglas Intermediate Pdn Fn Parameter	0.2	0.1	0.3	0.2	0.1	0.2	
ψ	Fn of elasticity of capital util. adj. cost function	0.8	0.7	0.9	0.7	0.6	0.9	
$100(\beta^{-1}-1)$	Fn of discount factor of households	0.3	0.1	0.4	0.2	0.1	0.3	
$100(\gamma - 1)$	Gross deterministic growth rate of economy	0.4	0.4	0.4	0.4	0.4	0.4	
ϕ	s.s. elasticity of the capital adjustment cost fn	6.1	4.3	8.2	7.4	5.5	9.0	
η	External habit formation of consumers	0.7	0.5	0.8	0.9	0.8	0.9	
σ^{c}	Inverse inter-temporal elasticity of substitution	1.5	1.1	1.8	1.4	1.2	1.6	
$\phi_{\mathcal{P}}$	Price mark-up in s.s.	1.5	1.4	1.7	1.5	1.4	1.6	
Lp	Degree of indexation to past price inflation	0.3	0.1	0.4	0.5	0.3	0.7	
ξ_n^r	Degree of price sticckiness	0.9	0.8	0.9	0.9	0.8	0.9	
	Degree of indexation of wages to past price inflation	0.5	0.3	0.8	0.3	0.1	0.4	
E _w	Degree of wage stickiness	0.8	0.7	0.9	0.8	0.7	0.8	
σ^{ν}	Labor supply parameter	1.3	0.5	2.2	1.9	1.1	2.6	
0 P	Inertia parameter in monetary policy rule	0.9	0.8	0.9	0.9	0.9	0.9	
τ_{π}	Inflation coefficient in monetary policy rule	1.7	1.4	1.9	1.4	1.1	1.6	
т., Т.,	Output coefficient in monetary policy rule	0.1	0.1	0.1	0.1	0.1	0.1	
T_{y}	Change in output coefficient in monetary policy rule	0.1	0.1	0.2	0.1	0.1	0.1	
$\frac{T\Delta y}{\pi}$	s s inflation in measurement equation	0.9	0.7	1.0	0.7	0.6	0.9	
7	a a hours worked in measurement equation	0.1	26	1.0	0.1	1.0	1.9	
L	s.s. nours worked in measurement equation	-0.1	-2.0	1.4	0.1	-1.9	1.0	
Persistence of erogenous nr	2008808							
1 ersistence of exogenous pro	Total factor productivity	0.98	0.95	0.00	0.96	0.94	0.08	
Pa	Bisk promium	0.65	0.30	0.55	0.33	0.23	0.45	
<i>P</i> _b	Exogenous spending	0.05	0.00	0.01	0.05	0.25	1.00	
p_g	Investment specific technology	0.38	0.50	0.95	0.53	0.37	0.64	
ρ_q	Manatam a lim	0.78	0.71	0.85	0.55	0.43	0.04	
ρ_r	Drive mark up AD(1) recorded	0.09	0.03	0.17	0.14	0.04	0.23	
ρ_p	Price mark-up AR(1) parameter	0.85	0.11	0.93	0.35	0.08	0.01	
ρ_w	Wage mark-up AR(1) parameter	0.97	0.96	0.99	0.95	0.91	0.98	
θ_p	Price mark-up MA(1) parameter	0.66	0.5.	0.84	0.46	0.26	0.65	
θ_w	Wage mark-up MA(1) parameter	0.95	0.92	0.98	0.87	0.80	0.95	
a_g^{D}	Coefficient on TFP innovation in invspecific tech. process	0.32	0.22	0.42	0.36	0.26	0.45	
a								
Standard deviation of innov	ation to the exogenous processes	0.0	0.5	0 -	0.0	0 5	0.0	
σ_a	Total factor productivity	0.6	0.5	0.7	0.6	0.5	0.6	
σ_b	Risk premium	0.1	0.1	0.2	0.1	0.1	0.1	
σ_g	Exogenous spending	0.5	0.5	0.6	0.5	0.5	0.5	
σ_q	Investment-specific technology	0.4	0.3	0.4	0.3	0.3	0.4	
σ_r	Monetary policy	0.2	0.2	0.2	0.2	0.2	0.2	
σ_p	Price mark-up	0.1	0.1	0.1	0.1	0.1	0.2	
σ_w	Wage mark-up	0.3	0.3	0.4	0.3	0.2	0.2	
Fixed parameter		0.0			0.0			
0	Depreciation rate	0.0			0.0			
ϕ_w	s.s. labor market mark-up	1.6			1.6			
cg		0.2			0.2			
ϵ_p	Curvature of the Kimball goods market aggregator	10			10			
ϵ_w	Curvature of the Kimball labor market aggregator	10			10			
Marginal Density of Data								
marginal Density of Data		-1386.8			-1356 1			

Table B.6: Posterior Distributions of Estimated Parameters in Structural DSGE Model

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Sources: authors' calculations and Slobodyan and Wouters (2012a). Notes: The choice of priors follows Slobodyan and Wouters (2012a).





Notes: The chart shows the cumulative sum of one-quarter ahead forecast errors for inflation from the AR(2) Perceived Law of Motion (PLM) model used by agents in the model to forecast inflation, as well as cumulative forecast errors from the structural model itself under rational expectations (RE) and adaptive learning (AL).



Figure B.14: Cumulative Inflation Forecast Errors: A Comparison of PLM and Surveys

Sources: University of Michigan and authors' calculations. Notes: The chart shows the cumulative sum of forecast errors for inflation over the next year from the AR(2) Perceived Law of Motion (PLM) model used by agents in the structural model to forecast inflation, compared with one-year ahead inflation forecasts from the University of Michigan Survey of Consumers (UMich).

Figure B.15: Cumulative Inflation Forecast Errors: A Comparison of Structural Models and Surveys



Sources: Federal Reserve Bank of Philadelphia and authors' calculations. Notes: The chart shows the cumulative sum of forecast errors for inflation over the next year from the structural DSGE model, under rational expectations (RE) and adaptive learning (AL), compared with one-year ahead inflation forecasts from the Survey of Professional Forecasters (SPF).



Figure B.16: Impulse Response Functions: Structural Model under Rational Expectations (RE) and Adaprtive Learning (AL)

Sources: authors' calculations. Notes: The solution of the structural DSGE model under adaptive learning (AL) is timevarying, since agents form expectations using a recursively estimate forecasting model (PLM), so that impulse response functions can differ across the estimation sample.

Figure B.17: Estimated Autoregressive Coefficients of the Perceived Law of Motion (PLM) for Inflation



Sources: authors' calculations. Notes: The chart shows the sum of the estimated first and second lags of the dependent variable in the AR(2) PLM model used to forecast inflation. Estimated coefficients are time-varying since agents estimate the PLM recursively.





Sources: authors' calculations. Notes: The chart shows the estimated intercept term of the AR(2) PLM model used to forecast inflation. Estimated coefficients are time-varying since agents estimate the PLM recursively.



Figure B.19: Estimated Shocks in the Structural Model with Rational Expectations

Sources: authors' calculations. Notes: The chart shows the estimated innovations over the sample to the exogenous processes determining total factor productivity (ea), the risk premium (eb), investment-specific technology (eq), spending (eg), monetary policy (em), price mark-ups (epinf) and wage mark-ups (ew). The integers on the horizontal axes correspond to the number of quarters in the sample: 228 quarters from 1965Q1-2022Q4, excluding 2020Q1-Q4.



Figure B.20: Estimated Shocks in the Structural Model with Adaptive Learning

Sources: authors' calculations. Notes: The chart shows the estimated innovations over the sample to the exogenous processes determining total factor productivity (ea), the risk premium (eb), investment-specific technology (eq), spending (eg), monetary policy (em), price mark-ups (epinf) and wage mark-ups (ew). The integers on the horizontal axes correspond to the number of quarters in the sample: 228 quarters from 1965Q1-2022Q4, excluding 2020Q1-Q4.



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