INTERNATIONAL MONETARY FUND

Automation and Welfare: The Role of Bequests and Education

Prepared by Manuk Ghazanchyan, Alexei Goumilevski, and Alex Mourmouras

WP/24/11

2024
JAN

WORKING PAPER

WP/24/11

Automation and Welfare: The Role of Bequests and Education*

Prepared by Manuk Ghazanchyan, Alexei Goumilevski, and Alex Mourmouras

Authorized for distribution by Alex Mourmouras
January 2024

ABSTRACT: This paper examines the welfare effects of automation in neoclassical growth models with and without intergenerational transfers. In a standard overlapping generations model without such transfers, improvements in automation technologies that would lower welfare can be mitigated by shifts in labor supply related to demographics or pandemics. With perfect intergenerational transfers based on altruism, automation could raise the well-being of all generations. With imperfect altruism, fiscal transfers (universal basic income) and public policies to expand access to education opportunities can alleviate much of the negative effect of automation.

JEL Classification Numbers:	E13, E62, D64
Keywords:	Automation; Aging; Altruism; Fiscal Policy; Education; Overlapping Generations
Authors' E-Mail Addresses:	mghazanchyan@imf.org; agoumilevski@imf.org; amourmouras@imf.org

☐Manuk Ghazanchyan is an Economist in the Western Hemisphere Department, Alexei Goumilevski is a Senior Scientific Computing Engineer in the Information and Technology Department, and Alex Mourmouras is Division Chief in the Asia Pacific Department.

*Presented at the 28th International Conference on Computing in Economics and Finance in Dallas, Texas, June 17-19th, 2022. We thank conference participants for their useful comments. We are also grateful to Peter Rangazas, Marina Mendes Tavares, and Jeremy Clift for the insightful comments that shaped our paper.

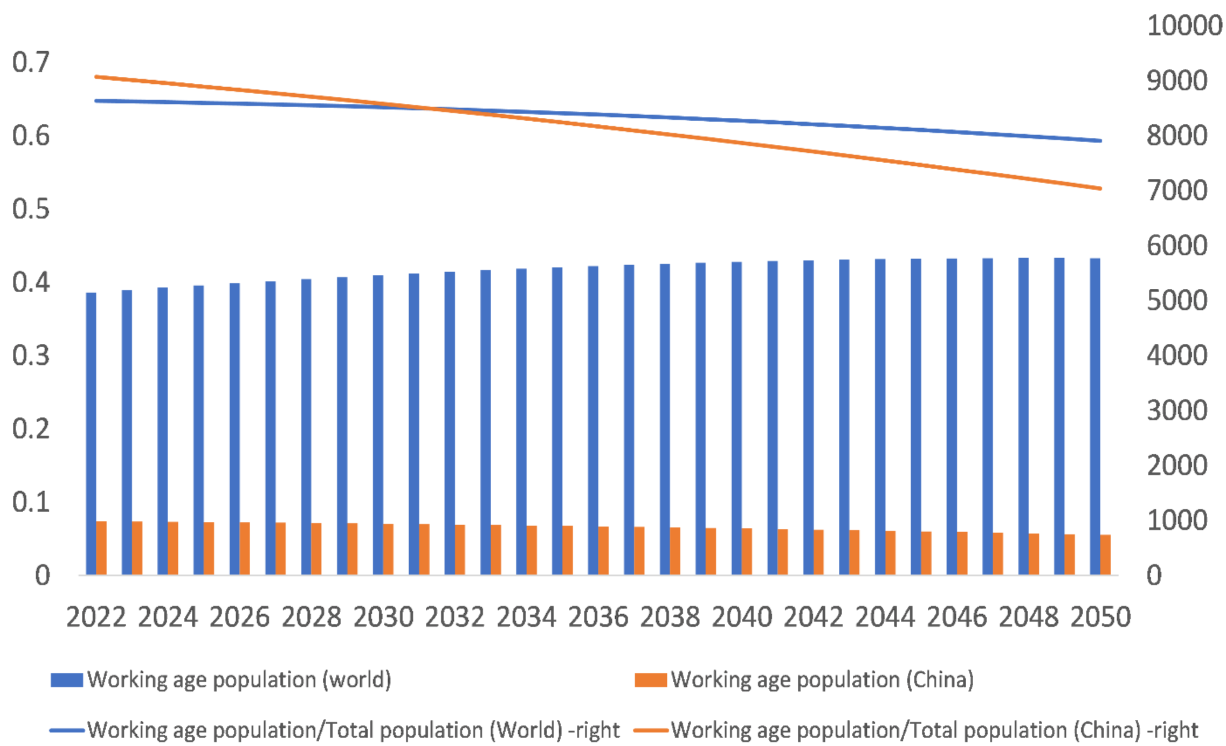
Contents

I. Introduction

Automation has accelerated in recent decades, driven by ongoing improvements in computing and information technologies and associated cost reductions. Machines in a widening range of industries perform increasingly complex tasks, powered by sophisticated, networked software. The acceleration in automation and its economy-wide diffusion in blue- and white-collar occupations alike is creating new employment categories but is also contributing to widening inequality and fueling demand for government policies to reverse long-term income losses of labor. This long-standing promise and concerns are vividly illustrated by the latest breakthrough in Artificial Intelligence involving generative, pretrained transformers.

Looking ahead, while the pace of automation is likely to continue, its effects may be mitigated by offsetting forces. Populations are aging almost everywhere. In the advanced economies, the working-age population has started shrinking for the first time since World War II (Spence, 2022). Globally, the population of working age is expected to continue to grow until about 2040, but the ratio of the working age population to the total is already declining globally (Chart). In the case of China, for example, the working-age population is expected to shrink by a fifth over the next 30 years. As Goodhart and Pradhan stress, our age is one of demographic reversal in which the "long glut of inexpensive labor that had kept prices and wages down for decades, is giving way to an era of worker shortages, and hence higher prices".

Declining trends in working-age population (in millions)



Source: United Nations, World Bank, and staff calculations

Recurring global pandemics also adversely affect labor supply, either by depressing growth in the labor force directly (AIDS pandemic), or indirectly by reducing the participation of older workers and others in contact-intensive occupations (pandemic related to Covid-19). In the absence of mass south-north migration, robots may turn out to be essential in meeting more of the needs of the elderly and reverse declines in aggregate output and welfare[1].

This paper examines the combined welfare effects of automation and lower labor supply using neoclassical growth models with and without intergenerational transfers. It begins by replicating a version of the well-known result of Kotlikoff and Sachs (K&S, 2012) that a one-time improvement in the technical efficiency of machines ends up immiserating all future generations. This striking result relies on two crucial assumptions. First, machines are very good substitutes for unskilled labor throughout the economy, so that improved automation ends up displacing workers and lowering wages across the board. Second, there are no operative intergenerational transfers of any kind, so that the owners of capital end up consuming the entire windfall from the improvements in automation during their lifetime (a generation, roughly thirty years). The positive shock to the efficiency of machines does not raise saving, depresses investment in physical and human capital, and sets in motion a never-ending cycle of declining welfare. Government policy is therefore needed to spread this windfall more equitably across future generations. K&S consider wealth taxes, in particular socializing a portion of the economy's capital stock that allows the government to finance a sustainable income stream (universal basic income) for all future generations. Resorting to compulsion is essential when generations are selfish, precluding any sort of voluntary intergenerational transfers.

In fact, private intergenerational transfers are substantial, with about half of all households planning to leave estates (Laitner and Juster, 2017). The first objective of the paper is to reassess the welfare effects of automation in the presence of intergenerational transfers, both bequests and privately and publicly funded schooling. In a version of the K&S model with operative bequests, we find that intergenerational transfers are positive in equilibrium if the strength of altruism exceeds a certain threshold, mitigating the negative effects of automation. But while it is comfortable to know that the gains from automation may be passed to future generations without the need to nationalize capital, this model of perfect altruism is also extreme: many families in each generation cannot make efficient transfers to their children.

What is needed is a more balanced model, one that features heterogeneity both within and across generations, with some households making efficient bequests and others stuck in a corner solution. We assess whether automation is immiserating in a version of K@S model that incorporates pure altruism, and in Glomm and Ravikumar's model (G&R, 1992) in which parents make investments in the schooling of their children. We study how fiscal and educational policies can best raise welfare, by alleviating financing constraints in the financing of human capital investments and restoring equality of opportunity. Similar results obtain in a version of the model used by Ivanyna, Mourmouras and Rangazas (2018) in with two types of households (the poor, who are bequest-constrained) and the rich (who are unconstrained).

The paper then turns to an analysis of a combined shock involving a jump in automation and a simultaneous reduction in labor supply driven by demographics. As expected, strengthening altruistic bonds raise bequests and human capital investments of the young, providing an additional stimulus to economic growth. In addition, government transfers of tax revenue levied on the rich can improve the

[1] Business leaders have also made a connection between automation and aging recently. One example was the recent article in Fortune by IBM CEO, Arvind Krishna here he points to declining populations to calm fears about A.I. taking jobs. He further added that ultimately "there is going to be job creation" with A.I., as jobs will also be added in areas with more value creation.

welfare of the poor and reduce inequality within and across generations when altruistic links between generations are weak.

II. Relevant Literature

The literature on automation and its economic impact is evolving, with some earlier studies from Gordon (2012), Cowen (2011), Acemoglu and Restrepo (2017, 2018), Sachs and Kotlikoff (2012, 2015), Ford, (2015); Freeman, (2015) amongst pessimists, and Brynjolfsson and McAfee (2014), Autor (2014, 2015) among the optimists. The key issue is whether automation replaces labor share and employment through replacement of routine tasks of ever-increasing scope and complexity or whether, on net, it increases labor participation by creating high-paying jobs in emerging new occupations. Some gloomy scenarios for labor resulting from artificial intelligence and simultaneous automation breakthroughs are described in Bostrom (2014). Graetz et. al. (2018) examined the economic contribution of modern industrial robots in 17 countries for the period 1993-2007.

Contrary to the pessimistic view, these authors found that the increasing use of robots raised the annual growth of GDP and labor productivity by 0.37 and 0.36 percentage points, respectively. Authors conclude that those robots did not significantly reduce total employment, although they did reduce low-skilled workers' employment share. Gaaitzen et al. (2020) studied the effects of adaptation of industrial robots and occupational shifts by task content in the thirty-seven countries for the period from 2005 to 2015. The authors found that increased use of robots is associated with positive changes in the employment share of non-routine analytic jobs and negative changes in the share of routine manual jobs. Of course, enhancing policy including R&D and the regulatory platforms in both private and public sectors to support digital technologies is key to improve productivity. While the 2020-22 pandemic helped to accelerate the digital transformation, many sectors – including the public sector – are lagging, and hence concerns about the effects of automation on employment will persist (Spence, 2022).

Only a few studies examined the effect of automation and population aging on the labor market aside from the classical work by Frey and Osborne (2017) focusing on the probability of automation affecting various jobs and occupations. One of the earliest studies on automation and population aging was by Acemoglu and Restrepo (2017), where the authors examined the relationship between economic growth, population aging, and automation at the country level. Phiromswad et al (2022) is amongst the most recent studies to focus on those effects but also on the interaction effects of automation and population aging on the labor market. Consistent with previous literature including with Graetz and Michaels (2018) the authors found strong evidence that automation negatively affects employment growth holding other factors constant. They also found strong evidence that the disaggregated measures of age-related abilities affect employment growth but not the aggregate measure. As expected and consistent with findings that automation is still evolving in affecting high value jobs, the authors find that with occupations with low score on both the age-appreciated cognitive ability as well age-depreciated physical ability (such as production occupations and food preparation and serving related occupations), the negative effect of automation on employment tends to be strongest. However, for occupations with a high score in both age-appreciated cognitive ability as well as age-depreciated physical ability (such as protective service occupations and healthcare practitioners and technical occupations), the negative effect of automation on employment tends to be weakest.

Aghion-Jones-Jones (2017) study the implications of artificial intelligence for economic growth in light of reconciling evolving automation with the observed stability in the capital share and per capita GDP growth over the past century. The authors create sufficient conditions to generate overall balanced growth with a constant capital share that stays well below 100 percent, even with nearly complete automation. In other words, while Baumol’s cost disease leads to a decline in the share of GDP associated with manufacturing or agriculture (once they are automated), this is balanced by the increasing fraction of the economy that is automated over time. The authors also study the effects of introducing A.I. in the production technology for new ideas and the possibility that A.I. could generate some form of a singularity, where the authors nevertheless claim that the Baumol theme here also remains relevant: even if many tasks are automated, growth may remain limited due to areas that remain essential yet are hard to improve.

Pizzinelli and others (forthcoming) examine the impact of Artificial Intelligence (AI) on labor markets in both Advanced Economies (AEs) and Emerging Markets (EMs). The authors propose an extension to a standard measure of AI exposure, accounting for AI's potential as either a complement or a substitute for labor, where complementarity reflects lower risks of job displacement. Then they analyze worker-level microdata from two AEs (US and UK) and four EMs (Brazil, Colombia, India, and South Africa), revealing substantial variations in unadjusted AI exposure across countries. The authors found that while AI poses risks of labor displacement due to task automation, it also holds promise in its capacity to enhance productivity and complement human labor, especially in occupations that require a high level of cognitive engagement and advanced skills. The authors also find that AEs may expect a more polarized impact of AI on the labor market and are thus poised to face greater risk of labor substitution but also greater benefits for productivity.

The extent and form of voluntary intergenerational transfers is dictated by the strength of intergenerational altruism and is an important consideration in macroeconomics that is relevant for our paper. Kotlikoff (2001) provides an excellent survey of key works on the role of intergenerational altruism, including empirical findings—for example, the results of Altonji and Hayashi (1994) which are consistent with the pure altruism theory. A closely related area of research concerns the form of human capital investments, specifically the rationale behind education or other bequests in kind. Razin and Rosenthal (1990) show that family taxation as a response to information asymmetry between a parent and a child could reduce the need for government intervention and taxation. Hood and Joyce (2017) provide an excellent update to the empirical relevance of altruism. Our paper is most closely related to Michel, Thibault, and Vidal (2004), who study the effects of altruism and fiscal policies on growth in an overlapping generations model in the tradition of Diamond, and to Glomm and Ravikumar (1992) who study bequests in the form of human capital investments in children. We study privately funded schooling for families with operative bequests and publicly funded education.

We find that government spending on education promote economic growth. These conclusions are supported by a vast volume of research that link individuals’ education attainment to economy-wide prosperity. Fabrizio Carmignani (2016) studied effects of government expenditures on education to economy. Author used the World Bank's World Development Indicator database data on 151 countries for 2000 - 2010 years. He concluded that “increase in education expenditure by 1 point of GDP increases GDP growth by 0.9 percentage points”. Gheraia, Zouheyr et al. (2021) investigated relationship between the cost of education and economic growth in the Kingdom of Saudi Arabia for the period 1990-2017. Authors found that in the long run the rise in educational expenditure by 1% would lead to an increase in economic growth by 0.89%. Similar results were obtained by Yahya, Mohd et al. (2012). Authors analyzed the long-run relationship and causality between the government expenditure in education and economic growth in Malaysia for the period 1970 to 2010. They concluded that economic growth is positively correlated with fixed capital formation, labor force participation and expenditure in education. Regarding Granger causality, the educational expenditure is the short-term Granger cause of economic growth and vice versa. Mehmet Mercana et al. (2014) performed cointegration analysis between the real gross domestic product and total expenses to the education for the case of Turkey for the period 1970-2012. Authors used Autoregressive Distributed Lag model with bounds testing. Authors found that 1% increase in education expenses increases economic growth by 0.3%.

III. Automation and Welfare in Overlapping Generation Models

A. An Analytical Tool: The K&S Model (2012)

We begin with a simple model featuring two period-lived overlapping generations (OLG). Each period $t = 1,2, \dots$ the population consists of young and old households. The young are endowed with one unit of inelastically supplied unskilled labor. They consume part of this income and invest the rest in physical capital (machines, M) and in their own human capital (skilled labor, S). Machines and human capital are perfect substitutes in saving portfolios. In the second period of life, households rent their machines and skills in perfectly competitive markets, consuming all interest and principal. Gross output Q is a constant elasticity of substitution (CES) production function of the economy-wide stocks of M, L, and S:

$$Q = Q(N(uM, L), S) \tag{A.1}$$

M and L combine in a CES production function with elasticity εML to produce an intermediate product N, and N and S combine in a CES production function with elasticity εSN to produce the final output Q. The parameter u is a parameter measuring the technical efficiency of machinery. A rise in u is a pure technical advance. Kotlikoff and Sachs (KS,2012) examine whether a rise in u can reduce economic wellbeing, an outcome they refer to as “im-mesmerizing productivity.”

Competitive firms hire M, L, and S to the point where their marginal products (denoted as Q_i for $i = M, L$, and S) equal their market wages: $Q_i = W_i$. Following K&S the partial derivative of wage with respect to productivity u is,

$$dln(Q_L) \big/ dln(u) = [\varepsilon SN - \theta \varepsilon ML]/\varepsilon ML \tag{A.2}$$

where θ is the share of skilled labor in the economy, equal to $(Q_S * S)/Q$. We see that a rise in machine’s productivity reduces the unskilled wage if $\varepsilon ML > \varepsilon SN/\theta$. Thus, “im-mesmerizing” productivity is more likely if:

- Substitutability of machines and unskilled labor is high (εML large)
- Substitutability of intermediate goods and skilled labor is low (εSN small)
- The share of skilled labor in final output is high (θ high)

Below we present theoretical underpinnings for the K&S conclusions. Note that the income of the young, I_t, is comprised of wages W_t, part of which is invested in machines and human capital. Income when old, I_{t+1}, comes from the ownership of machines and acquired skills. The lifetime budget constraints of generation born at t, who are young in period *t* and old at $t+1$ are:

$$I_t = L_t W_t = C_t + Sv_t \tag{A.3}$$

$$I_{t+1} = \left(\frac{\partial Q}{\partial M}\right)_t M_{t+1} + \left(\frac{\partial Q}{\partial S}\right)_t S_{t+1} = D_{t+1} \tag{A.4}$$

Here C_t is the consumption when young, D_{t+1} is consumption when old, and Sv_t is saving, which is invested in machines and skills:

$$Sv_t = M_{t+1} + S_{t+1} \tag{A.5}$$

This allocation of savings is made under perfect foresight to maximize utility, so that investment in machines and skills are chosen to equalize marginal products of M and S to the gross interest rate in the economy:

$$R_t = \left(\frac{\partial Q}{\partial M}\right)_t = \left(\frac{\partial Q}{\partial S}\right)_t = 1 + r_t \tag{A.6}$$

Here R_t is the gross rate of return and r_t is the interest return on machines. Combining equations (A.3-4), we get:

$$C_t + \frac{D_{t+1}}{R_t} = I_t \tag{A.7}$$

Lifetime utility for a household belonging to generation *t* is:

$$U = \beta log(C_t) + (1-\beta) log(D_{t+1}) \tag{A.8}$$

Parameter β is the time discount factor. It is related to the patience of the agent about consumption - larger β means the agent is predisposed to consume more during youth. The utility maximizing choices of consumption must satisfy the first order condition,

$$\frac{\beta}{C_t} = \frac{1-\beta}{D_{t+1}} R_t \tag{A.9}$$

Turning to the production side of the economy, we follow K&S who postulate a Cobb-Douglas production function for final goods assembled using skilled labor *S* and an intermediate input *N*:

$$Q(N,S) = AN^{\alpha} S^{1-\alpha} \tag{A.10}$$

The intermediate factor N is a linear combination of labor L and machines M, meaning that these two primary factors, machines and labor are perfect substitutes.

$$N = L + u_1\, M \quad \text{(A.11)}$$

Here u is the technical efficiency of machinery. Higher values of u_1 indicate improvements in automation. The marginal products of the factors are,

$$\begin{aligned} Q_L &= A\alpha(N/S)^{\alpha-1} \\ Q_M &= A\alpha u_1(N/S)^{\alpha-1} \\ Q_S &= A(1-\alpha)(N/S)^{\alpha} \end{aligned} \quad \text{(A.12)}$$

Assuming “perfect foresight”, $Q_M = Q_S$, we get,

$$\alpha u S = (1-\alpha)N \quad \text{(A.13)}$$

and

$$\begin{aligned} Q_M = Q_S &= A\alpha u_1 \left(\frac{\alpha u_1}{1-\alpha}\right)^{\alpha-1} \\ W_t &= A\alpha \left(\frac{\alpha u_1}{1-\alpha}\right)^{\alpha-1} \end{aligned} \quad \text{(A.14)}$$

Solving savings equation (A.5), no arbitrage assumption (A.6), first order condition (A.9), and using equation for intermediate production (A.11), we can derive:

$$\begin{aligned} C_t &= \beta L_t W_t \\ D_{t+1} &= (1-\beta) R_t L_t W_t \\ Sv_t &= (1-\beta) L_t W_t \\ M_{t+1} &= \alpha\, Sv_t - (1-\alpha)\, L_t/u_1 \\ S_{t+1} &= (1-\alpha)\, Sv_t + (1-\alpha)\, L_t/u_1 \end{aligned} \quad \text{(A.15)}$$

Machines M_t and skills S_t are proportional to savings and increase/decrease with growth of productivity factor u, respectively.

We are interested in the effects on the economy of different combinations of shocks to productivity and labor, especially in the possibility of immiserating equilibria over time in which inequality within and across generations rises.

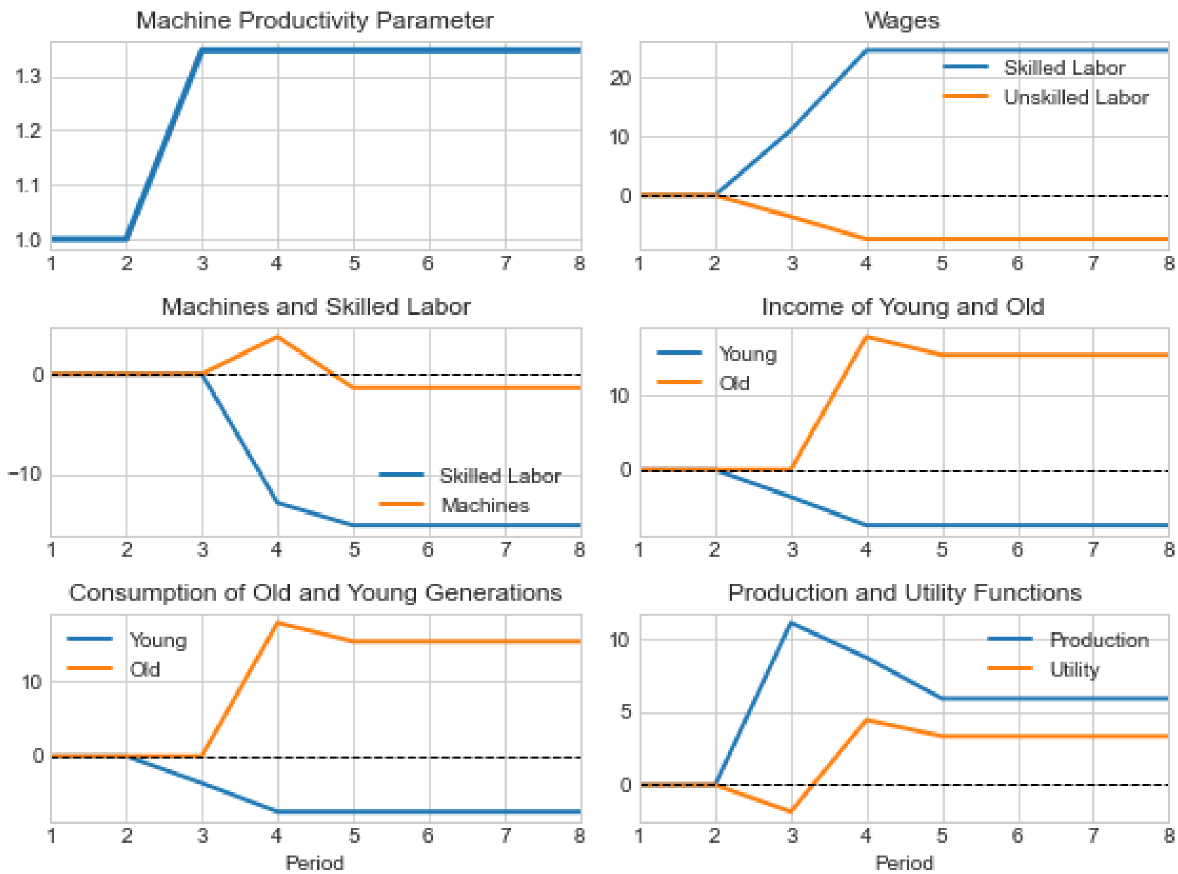
Proposition A1: Improvements in automation benefit the owners of capital (the initial old generation) and lower the welfare of all subsequent generations of workers. A positive shock in the efficiency of machines leads to higher consumption and income of the old. It simultaneously lowers the wage incomes of unskilled workers and lowers their lifetime consumption and utility. Intergenerational inequality rises.

Below we assume that Cobb-Douglas production function has weights $\alpha = 0.74$ on labor and 0.26 on machines and $\beta = 0.5$ on skills. This setup produces the results in Figure 1. In Figure 1 we assume that productivity unexpectedly rises by 35% in period $t = 3$ while labor supply remains the same. In the third period, M and N are unchanged because of the productivity shock, as they were determined by the saving decisions of the young in the first period. The current wage of unskilled workers (i.e., the earnings of the young of generation 3) declines from 2.82 to 2.61 as the result of the rise in u_1. The returns on M and S both rises, and the old generation (that owns both M and S) experiences a boom in income while young generation experiences a bust. Consumption of the old rises while consumption and saving of the young declines. This pushes down the future capital stocks of M and S. By period 5 the economy reaches a new equilibrium characterized by lower wages, lower skills, lower M, and higher total output than in the baseline. The ratio of earnings of skilled workers relative to unskilled workers, W_s/W_l, is permanently raised from 1 to 1.35.

Following K&S, we trace show the implications of improved automation for lifetime utility across generations. The utility of generation 3 is slightly below that of generation 1, as wages have declined while the returns on saving have increased. Yet the utility of generations 4 and later is higher than the baseline utility. Young unskilled workers lose from the rise in machine productivity. The production of generation 3 soars as this generation benefits from high returns to both M and S. The anticipated case produces slightly different but qualitatively similar results.

For piece-wise constant productivity u_1 these variables are piece-wise constant. Consumption reaches stationary level when $C_t = D_{t+1}$, or when $(1-\beta)R_t = \beta$. This sets condition for total factor productivity, $A = \frac{\beta}{\alpha u_1 (1-\beta)} \left(\frac{\alpha u}{1-\alpha}\right)^{1-\alpha}$. It inversely depends on productivity factor u_1.

Figure 1. Time dependence of variables for Sachs and Kotlikoff settings. At period 3 machines productivity increases by a factor of 1.35. Graphs show percentage difference with respect to the initial steady state. Parameter's settings were: $A = 5,\ \alpha = 0.74,\ \beta = 0.5$.



B. Introducing a One-Time tax

Government action on a large scale is needed to improve on the allocation of resources and welfare in the simplest form of the K&S model, in the form of a permanent intergenerational policy transfer to each young generation. There are several ways to finance this perpetual transfer, and, following K&S, we consider first nationalization of the physical capital stock effected at the time its productivity improves. In subsequent sections we examine the role of bequests and more modest transfers in models with altruism.

Specifically, assume that the government unexpectedly imposes a one-time tax on the machine population M_G at time t=3. Specifically, the government takes an amount $M_G < M$ of the machinery via a wealth levy, while the balance of machinery, $M_P = M - M_G$, is left in the hands of the older generation (private sector). The government choses M_G so that all generations have higher utility than in the baseline. The government income $Q_M M_G$ is split into transfer payment $T = (Q_M - 1)M_G$ which is added to the wage of young generation and the remainder M_G which is reinvested for the next period. Because of that, the government machines M_G are constant after time t. Income of young is comprised of labor wages and transfer T,

$$I_t = L_t W_t + T = C_t + Sv_t \tag{B.1}$$

Savings of the young are:

$$Sv_t = I_t - C_t = (1-\beta) I_t \tag{B.2}$$

Old generation machines and skilled labor are governed by the modified equations (A.5,13):

$$\begin{gathered} M_{P,t+1} + S_{t+1} = Sv_t \\ M_{P,t+1} + M_G + L_t/u = \frac{\alpha S_{t+1}}{1-\alpha} \end{gathered} \tag{B.3}$$

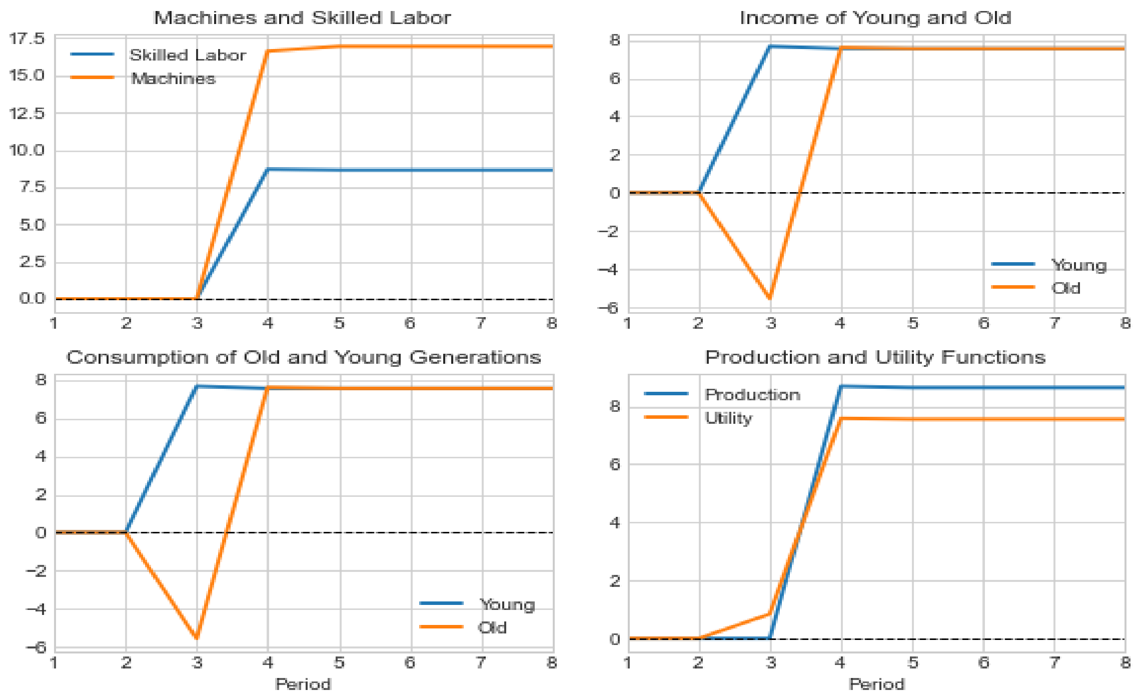
Equations (B.1-3) are employed to predict evolution of machines, skills, savings, and consumption.

Proposition B1: The imposition of a government tax on machines and the redistribution of the windfall from machines from old to young helps increase the welfare of all generations. Future generations are no longer immiserated by automation if the government tax is sufficiently large. Productivity shocks accompanied by taxes on machines result in higher utility for all future generations.

Government intervention via transfer of payments from old to young may improve the well-being of both generations. Below we present results of numerical experiments where we assume that the government imposes a 10% levy on machines owned by the old in period 3 (they are the members of generation 2). The government's income in period 3 is equal to $Q_M M_g$. The government transfers the amount $T = (Q_M - 1)M_g$ to the young of generation 3, whose income is now $LW_L + T$, where T is the transfer payment made by the government. The government also saves and reinvests the sum M_G for the next period. The generation 3 youth make their saving and investment decisions in the knowledge that the government will also be investing in the machinery sector. Total machinery in each period will then be the sum of privately held machinery, M_P, and the government-owned machinery, M_G. In each subsequent period, the government transfers an amount T to the current young generation out of the income that it receives on the income from the government-owned machinery, and it reinvests the sum M_G to maintain a constant level of M_G. This kind of tax-ownership-and-transfer system makes it possible to improve the utility, because of the rise in u. In this example the utility at period 1 is 2.37, and it rises to 2.63 at period 6, leaving generation 3 better off than the baseline despite the capital levy. The income and consumption of old drops at period 3 because of the payment transfer from old to young (see Figure 2). However, in period 4 and subsequent periods consumption and income of old increase. Future generations are even better off because of the government's transfer program.

Figure 2. Time-dependence of variables. The government imposes a tax on the machines owned by the old in period 3. The young receive an interest on machines starting from period three onward.

Graphs show percentage difference between scenarios of imposing and not imposing 10% tax on machines, ceteris paribus.



C. K&S Model with a Bequest

In this section we extend the K&S model by considering the role of bequests. We derive an analytical solution when households' utility depends on their consumption in the two periods of life and on the utility of their children. In this model of pure altruism, a positive automation shock (a rise in u) can improve the welfare of all generations and the role of government intervention can be quite limited. What is more important, the nationalization of capital considered by K&S and in the previous section is not necessary in this environment.

The budget constraints (A.3-4) must be modified to consider the role of bequests. The young household's income I_t is comprised of labor wages W_t and bequest from its parents B_t. When old, the household's income I_{t+1}comes from production of machines and acquired skills:

$$I_t = L_t W_t + B_t = C_t + Sv_t$$

$$I_{t+1} = R_t\, Sv_t = D_{t+1} + (1+n)B_{t+1} \tag{C.1}$$

Here $1+n$ is the number of children per person. We can derive equations for consumption of young and old, savings of young, machines and skills (please see Appendix A):

$$B_{t+1} = \left(1 + \frac{(1-\beta)(1+n)(1-\varphi)}{\varphi}\right)^{-1} \left(\frac{I_{t+1}}{1+n} - m_{t+1}\frac{(1-\beta)(1+n)(1-\varphi)}{\varphi}\right)$$
$$C_t = \beta(1-\varphi)\left[I_t + \frac{1+n}{R_t}m_{t+1}\right]$$
$$D_{t+1} = \frac{(1-\beta)(1-\varphi)}{1-\beta+\beta\varphi}\left[I_{t+1} + (1+n)m_{t+1}\right] \tag{C.2}$$
$$m_t = L_t W_t + \frac{(1+n)m_{t+1}}{R_t}$$

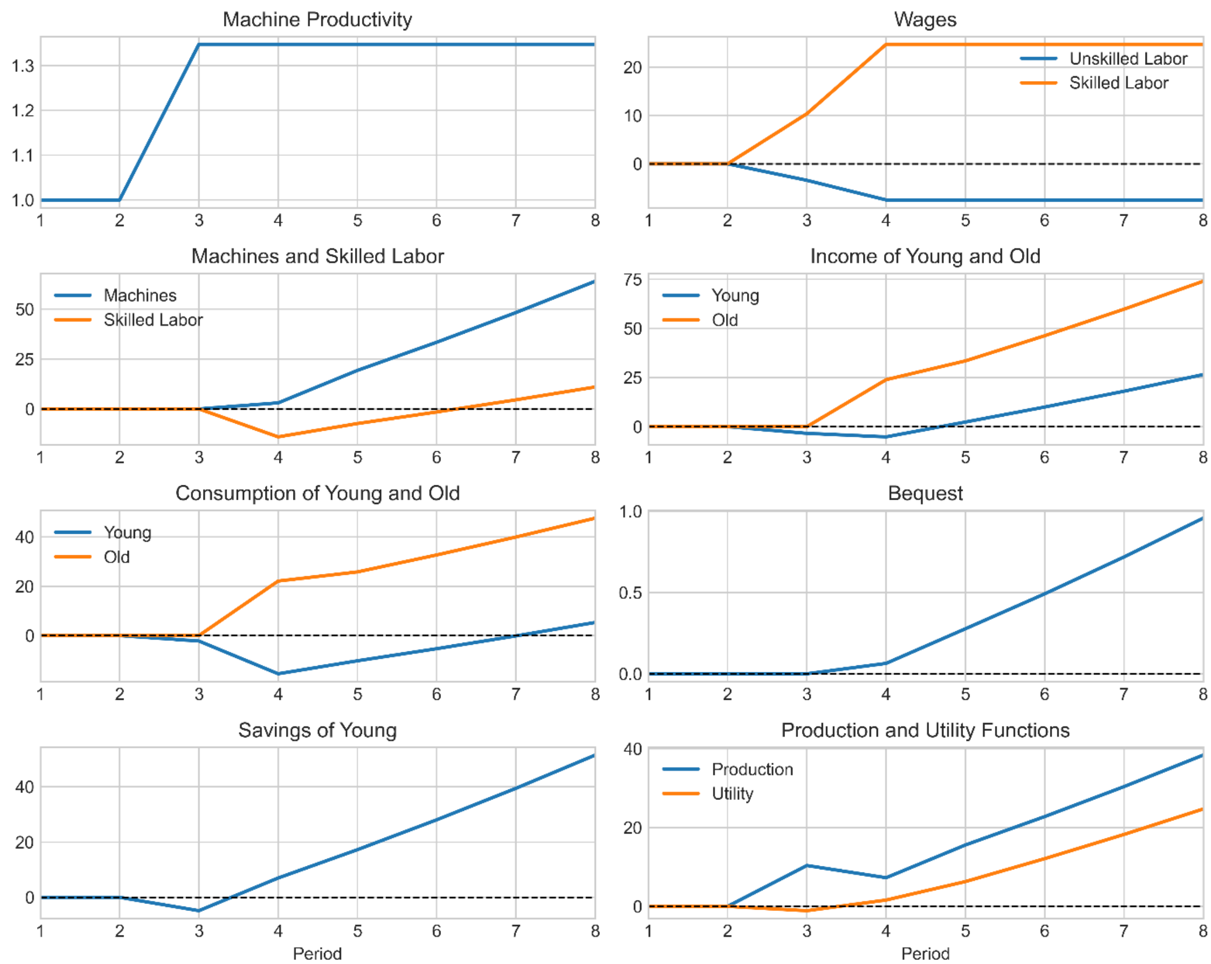
Let's investigate cases of stationary and exponentially growing bequests. Bequest can grow exponentially in time because of positive feed-back loop from old to young. Bequest growth multiplier is, $g = \frac{\varphi \mathrm{R}}{(1+n)(1-\varphi)}$. It is larger than one when, $\varphi > \varphi_{cr} = 1/[1 + R_t/(1+n)]$.

Proposition C1: Bequest from the old to the young in the form of cash or leaving an estate increases welfare of the young and decreases inequality between the old and the young. Future generations are not immiserated if the altruism parameter is above the threshold. In fact, the model sustains exponential growth in consumption and utility driven by accumulation of machines and skills.

Below we present numerical experiments of K&S model with a bequest. We assume that machine productivity grows each year by 1%. For a period of 30 years this results in a jump in the productivity function parameter u_1 by 35%.

We conducted experiments for decreasing selfishness parameter and constant parameters $n = 0, \alpha = 0.74, A = 5, \beta = 0.5$. Below we show results for critical parameter $\varphi = 0.26$. When the selfishness parameter is higher or equal to the critical value, there is no steady state solution to the K&S model with a positive bequest. This is illustrated in Figure 3 for numerical experiment with parameter: $\varphi = 0.3$. In period 3, productivity grows by 35%. Starting from period 4, the economy grows exponentially. As economy improves, so the income and consumption of young. At all times, the wage rate of skilled workers is higher than the wage of unskilled.

Figure 3. Evolution of variables calculated for selfishness parameter greater than critical, $\varphi = 0.3 > \varphi_{cr}$. Graphs show percentage difference with respect to the initial conditions.

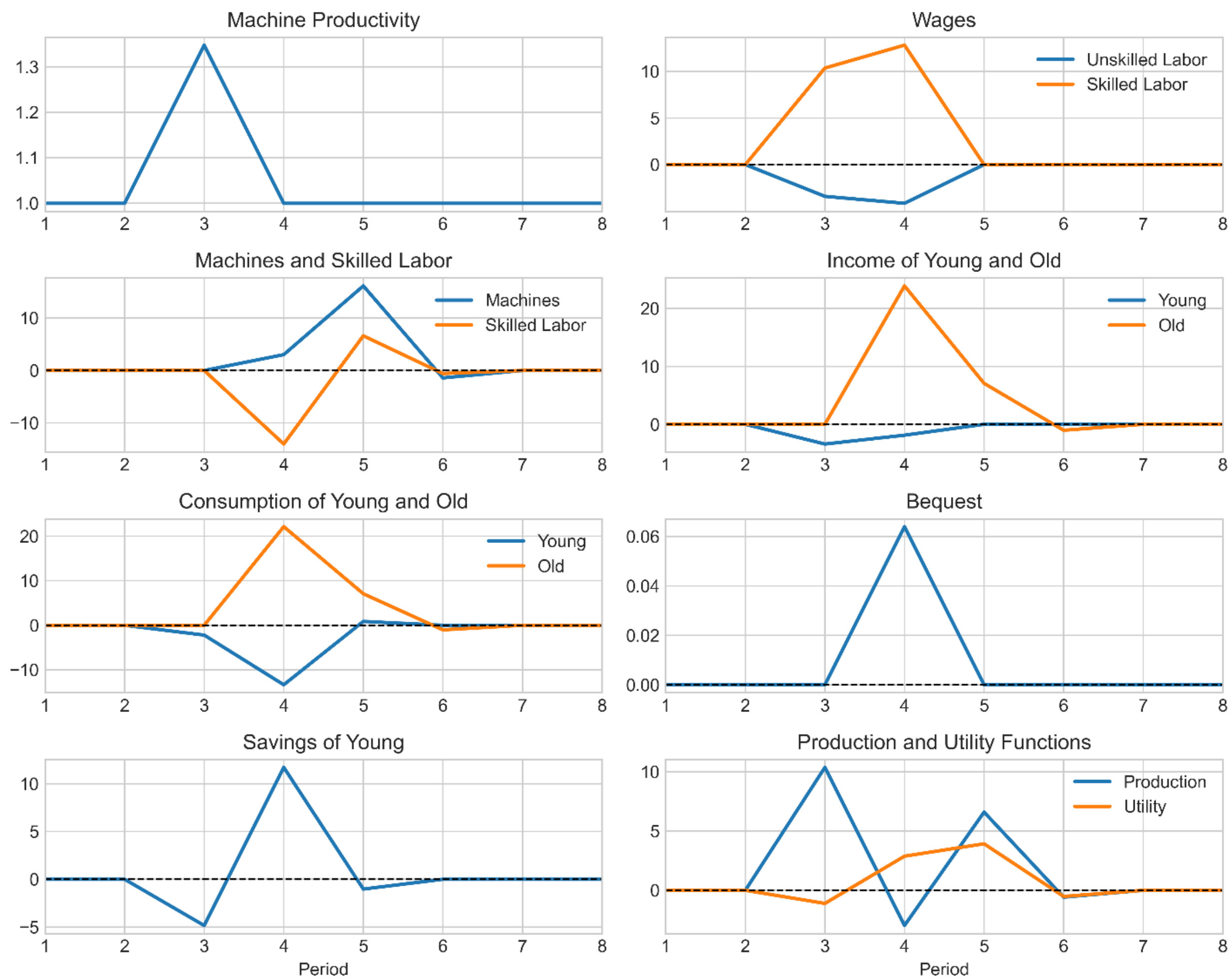


Note: We have performed calculations for a range of impatience parameters β. While critical values of φ_{cr} were different, these results were qualitatively like the case of $\beta = 0.5$.

When the altruism parameter is less than critical, i.e., $\varphi < \varphi_{cr}$, this model predicts a new steady state with lower income and consumption of young.

The impact of the temporary shock of machine productivity on economic variables is shown below. This increase in productivity causes positive bequests in period 4. At period 6, economy reaches a steady state.

Figure 4. Time evolution of variables calculated for altruism parameter $\varphi = 0.3$ greater than critical, $\varphi_{cr} = 0.28$. Temporary shock to machine productivity occurs at period 3.



D. K&S Model with Bequest and Education

Expanded access to education not only improves economic opportunity for young generation, but also strengthens the overall economy. We generalize the K&S model to account for old generation's bequest and expenses E_t that they spend on their children's education. We repeat derivations of equations presented earlier in section C.

Proposition D1: Individuals with private schooling poses higher income and consumption than those with public schooling. Bequest-constrained poor households may no longer afford rising educational cost. This problem could be alleviated in part by transfer of tax revenue imposed on machines of rich households with private schooling. However, for selfishness parameter less than critical (when old invest more into their young), this transfer of funds is no longer necessary. Targeted transfer to poor households with public education increases level of income and consumption of these households for selfishness parameter larger than critical; however, this transfer is not beneficial when it is less. Public schooling provides equal opportunities for individuals and decreases population inequality.

1. Private Education

In the private education regime, the old choose educational expenses based on individual decisions. The income of young generation is comprised of their wage earnings, bequest from old generation to young, and expenses that old spend on their children education. Savings of young Sv_t is invested into machines M_t and skills S_t. The income of old generation is the return on investment in machines and skills. The budget equations of young and old are:

$$I_t = L_t W_t + B_t + E_t = C_t + Sv_t$$
$$I_{t+1} = R_t Sv_{t+1} = D_{t+1} + (1+n)B_{t+1} + (1+n)E_{t+1} \tag{D.1}$$

Value function is now augmented by educational expenses term,

$$V(B_t, E_t) = \beta\ log(C_t) + (1-\beta)\ log(D_{t+1}) + \varphi\ V(B_{t+1}, E_{t+1}) \tag{D.2}$$

Individuals chose their consumption that maximize lifetime value function,

$$\max_{C_t, D_{t+1,} E_{t+1}} V, \quad \text{s.t. } B_{t+1} \geq 0, E_{t+1} \geq 0 \tag{D.3}$$

We seek solution of this Bellman equation in the form,

$$V(B_t, E_t) = a + b\ log(B_t + E_t + m_t) \tag{D.4}$$

Taking partial derivatives of (D.3) and using (D.1,4), we obtain expressions for optimal values:

$$\frac{\beta}{C_t} = \frac{2\varphi b}{(1+n)(B_{t+1}+E_{t+1}+m_{t+1})} R_t$$
$$\frac{1-\beta}{D_{t+1}} = \frac{2\varphi b}{(1+n)(B_{t+1}+E_{t+1}+m_{t+1})} \tag{D.5}$$

Substituting expressions (D.5) into (D.2), and re-arranging terms we get,

$$b\ log(B_t + E_t + m_t) = (1+\varphi b)\log(B_{t+1} + E_{t+1} + m_{t+1}) + const \tag{D.6}$$

Let's find a relationship between the bequest and education expenses of old and young generations. By substituting optimal consumptions (D.5) into budget constraint equations (D.1) yields:

$$b \log\left(B_t + E_t + m_t\right) = \left(1+\varphi b\right) log\left(B_t + E_t + L_t W_t + \frac{1+n}{R_t}\, m_{t+1}\right) + const \qquad \text{(D.7)}$$

Applying the method of undetermined coefficients to equations (D.6,7) results in formulas identical to (C.16-17):

$$b = \frac{1}{1-\varphi}$$

(D.8)

$$m_t = L_t\, W_t + \frac{(1+n)m_{t+1}}{R_t}$$

Like Glomm and Ravikumar model, private agents spend half of their saving on bequests to their children and the half on their educational expenses. Solving equations (D.1,5), we get:

$$C_t = \beta\, \frac{1-\varphi}{1+\varphi}\left[I_t + \frac{1+n}{R_t} m_{t+1}\right]$$

$$D_{t+1} = \frac{1-\beta}{\beta} R_t C_t \qquad \text{D.9)}$$

$$B_{t+1} = E_{t+1} = \left[2 + \frac{(1+n)(1-\beta)(1-\varphi)}{\varphi}\right]^{-1} \left[I_{t+1} + (1+n)\, m_{t+1}\right] - \frac{m_{t+1}}{2}$$

To account for human capital, we modify the Cobb-Douglas production function as following:

$$\mathrm{Q}(N_t, H_t) = A N_t^{\alpha} H_t^{1-\alpha} \qquad \text{(D.10)}$$

Here α is the elasticity of intermediate product N_t, and $H_t = S_t + E_{t-1}$ is the human capital. It has two sources: skills level and education attainment.

The perfect foresight equations for machines and human capital are:

$$\frac{L}{u_1} + M_{t+1} = \frac{\alpha}{1-\alpha} H_{t+1}$$

(D.11)

$$M_{t+1} + S_{t+1} = Sv_t$$

Equations (D.9,11) describe evolution of variables of extended K&S model with individuals' private education.

2. Public Education

Under public education regime, the amount of educational expenses is subsidized by average tax revenue:

$$\overline{E}_t = \tau\, \overline{\mathrm{Sv}_t} = \tau\left(\overline{\mathrm{M}}_{t+1} + \overline{S}_{t+1}\right) \tag{D.12}$$

This education expense is determined by an average savings and tax rate τ. This tax base is similar to the human capital tax base of Glomm and Ravikumar model. We assume that tax rate τ is flat across the population. State authorities socialize portion τ of machines and skills to pay for public school education expenses. Income of old now comes from production of smaller number of machines and skills (to prevent double counting). The budget constraint equations become:

$$\begin{aligned} I_t &= W_t L + B_t + \overline{E}_t = C_t + Sv_t \\ I_{t+1} &= R_t(1-\tau)\, Sv_t = D_{t+1} + (1+n)B_{t+1} + (1+n)\overline{E}_{t+1} \end{aligned} \tag{D.13}$$

Individuals maximize lifetime utility by choosing consumption levels C_t, D_{t+1}

$$\max_{C_t, D_{t+1}} V, \qquad \text{s.t. } B_{t+1} \geq 0 \tag{D.14}$$

$$V(B_t) = \beta\, log(C_t) + (1-\beta) log(D_{t+1}) + \varphi\left[a + b\, log\left(B_{t+1} + \overline{E}_{t+1} + m_{t+1}\right)\right]$$

Taking partial derivatives of value function and equating it to zero, we get:

$$\begin{aligned} \frac{\beta}{C_t} &= \frac{\varphi b(1-\tau)}{(1+n)\left(B_{t+1} + \overline{E}_{t+1} + m_{t+1}\right)} R_t \\ \frac{1-\beta}{D_{t+1}} &= \frac{\varphi b}{(1+n)\left(B_{t+1} + \overline{E}_{t+1} + m_{t+1}\right)} \end{aligned} \tag{D.15}$$

Solving equations (D.13,15) yields:

$$\begin{aligned} C_t &= \beta(1-\varphi)\left[I_t + \frac{(1+n)}{(1-\tau)R_t} m_{t+1}\right] \\ D_{t+1} &= \frac{(1-\beta)(1-\tau)}{\beta} R_t\, C_t \\ B_{t+1} &= \frac{\varphi}{1-\beta+\beta\varphi}\left(\frac{I_{t+1}}{1+n} + m_{t+1}\right) - \overline{E}_{t+1} - m_{t+1} \\ M_{t+1} &= \alpha\, Sv_t - (1-\alpha)\, L_t/u_1 + \alpha\, \overline{E}_t \\ S_{t+1} &= (1-\alpha)\, Sv_t + (1-\alpha)\, L_t/u_1 - \alpha \overline{E}_t \end{aligned} \tag{D.16}$$

The evolution of skills and the machines is governed by perfect foresight conditions (A.6). These equations are similar to equations (A.15) of K&S model. Machines increase with growth of savings, productivity factor u and education expenses $\bar{E}_t$. Human capital $H_t = S_t + \bar{E}_{t-1}$ is proportional to savings and decreases with productivity factor increase.

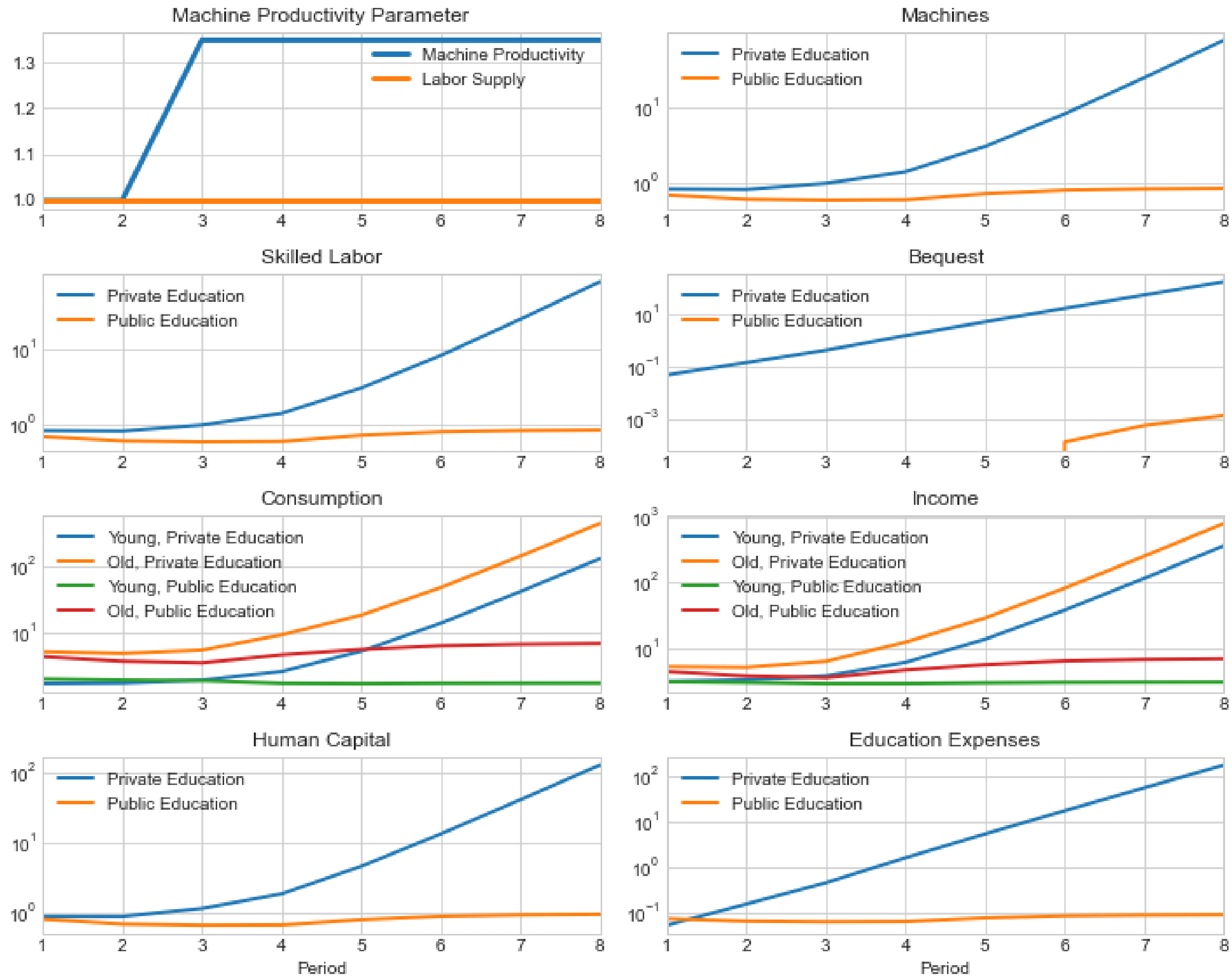
Proposition D2: Inequality within generations is reduced when public education is financed by income taxes levied on skilled workers' households. Inequality across generations declines. In both models, there is no need to resort to socializing physical capital. In the pure altruism model, private inter-generational transfers do the job of spreading the wealth windfall from the technological improvement across generations. Government investments in education help to equalize opportunities.

We present numerical experiments of K&S model with a bequest and an education. We conducted experiments for heterogeneous agents which differ by the degree of selfishness. Initial values of human capital, income, and consumption were the same. The selfishness parameter distribution was a truncated normal with lower and upper bounds of $(0,1)$. The number of draws was 1,000. The variance $\sigma = 0.022,\ 0.012$ of altruism parameter φ was chosen such that the cumulative distribution function of truncated normal was 0.5 at the critical value of selfishness parameter of $\varphi_{cr} = 0.14$ and 0.08 for public and private education regimes, respectively. In other words, only half of households can afford to leave a bequest to their children.

Consumption and income of young and old grow exponentially for private schooling regime. For public education the old generation income up to six periods is not sufficient to leave a bequest and these variables reach a new low steady state.

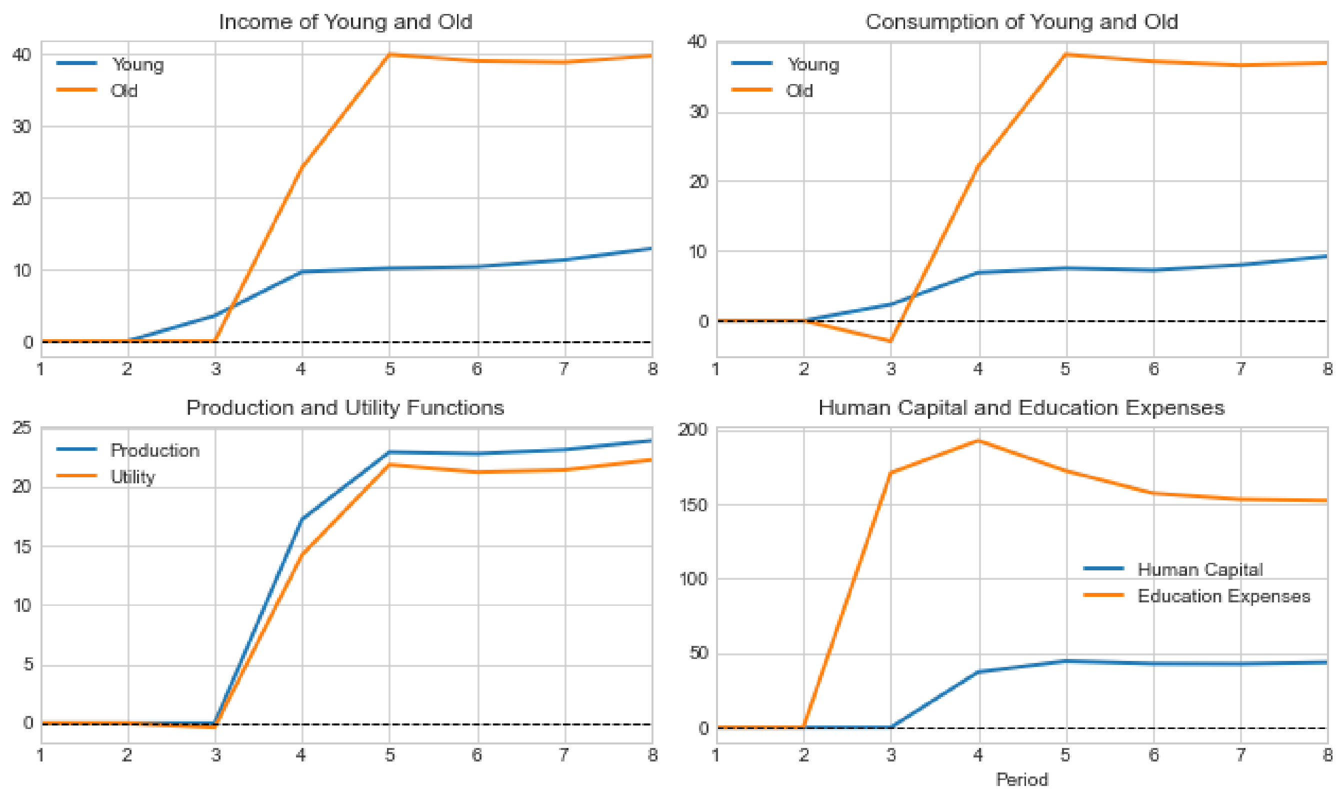
Poor households may no longer afford rising educational costs. Under private regime individuals on average have higher income, consumption, and can afford higher bequest than the same individuals under public regime. We observe that the K&S model with a bequest and education predicts higher values of bequest, utility function, income, savings, and consumption compared with the K&S model with a bequest alone.

Figure 5. Time evolution of mean variables for private and public education regimes. For the public regime bequest is positive and is growing in magnitude starting from period 6 onward. The mean altruism parameter was $\bar{\varphi} = 0.15$ and the tax rate $\tau = 5\%$.



Raising revenue through a tax levy of poor households may not be sufficient to cover educational expenses of their children. This shortage of educational cost can be covered in part by taxing rich households. This tax plays a role of an additional income to poor households. Below we assume that public educational expenses are subsidized in part by tax revenue on machines of private education households. We assume that the government unexpectedly imposes a one-time tax on the 5% of the machine population at time t=3. Similar to the section B, government transfer a constant payment of $T = 0.05 * (Q_M - 1) * M$ to the poor public households starting from period 3 onward. This transfer increases the human capital of poor households and provides means to pay for rising educational costs.

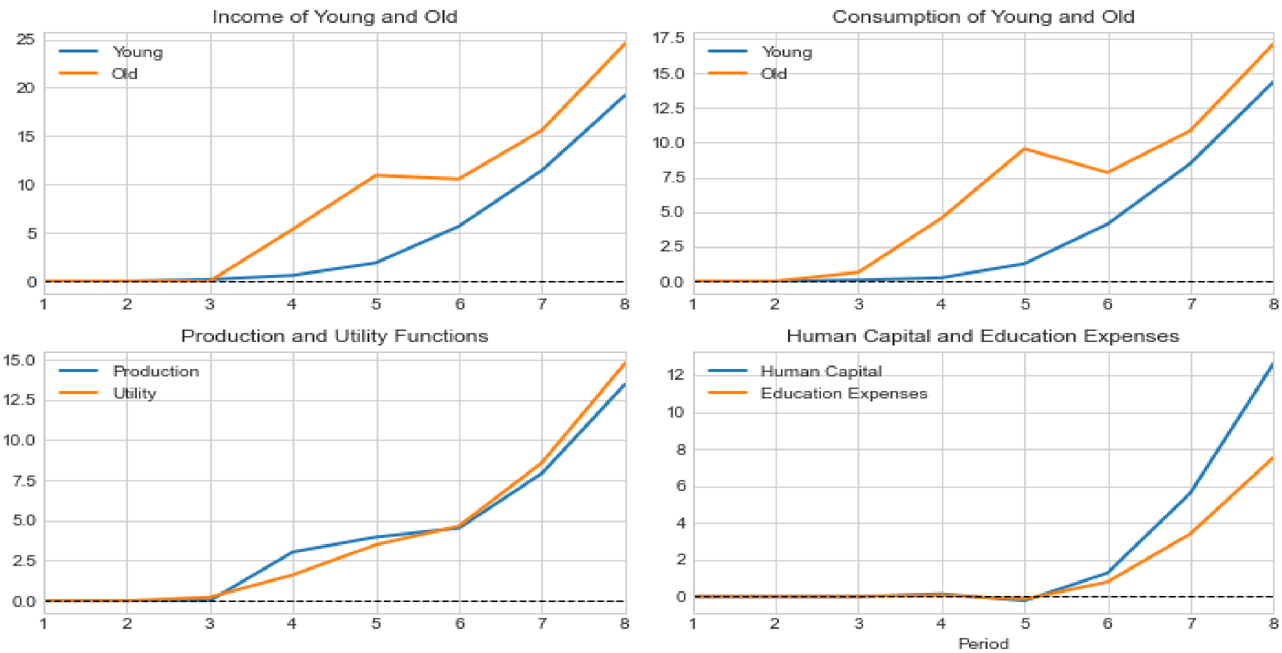
Figure 6. The educational cost of poor households with public education is covered in part by a 5% tax revenue levy imposed on rich households' machines. The rich and poor households are classified by their ability to leave a bequest to their children. Graphs show percentage difference of variables in public education regime.



Note: For impatience parameters $\beta < 0.5$ poor households can invest more into the machines and education compared to their consumption and afford higher educational costs.

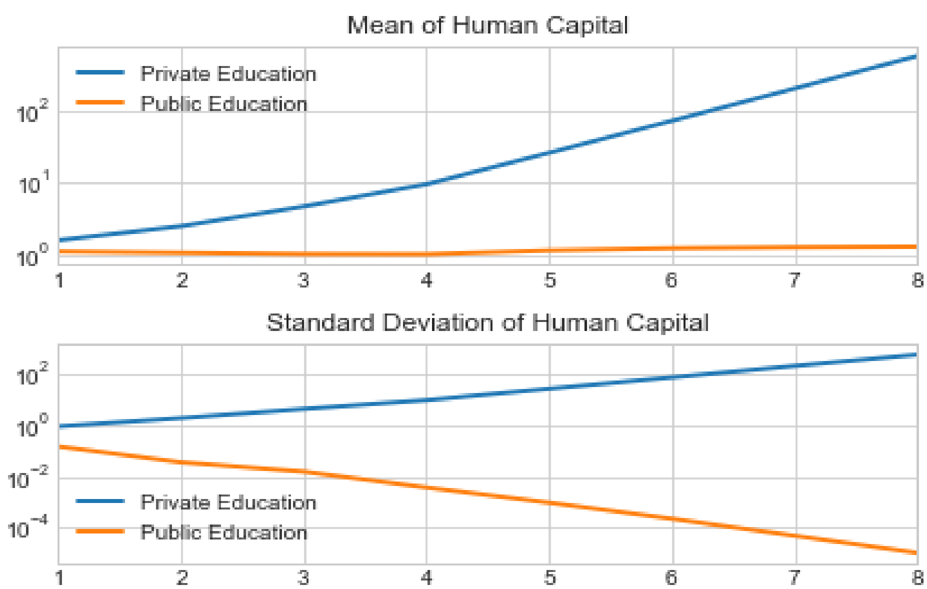
We investigated effects of targeted transfers to poor households with public schooling. We ran two scenarios: in scenario 1 payments were transferred to households with $\varphi < \overline{\varphi}$ (group 1), and scenario 2 – to households with $\varphi > \overline{\varphi}$ (group 2). Targeted transfers to bequest-constrained poor households improve income and consumption levels of young and old generations. Figure 7 shows mean of relative difference between scenario 1 and 2. Poor and wealthy households receive the same amount of transfer. Income of poor households is smaller compared to the wealthy households, but the relative change due to transfer is larger. It means that poor households are much better off than the wealthy ones.

Figure 7. The educational cost of poor households with public education is covered in part by a 5% tax revenue levy imposed on rich households' machines. In scenario 1 transfer payment was received by households with $\varphi < \overline{\varphi}$ and in scenario 2 – by households with $\varphi > \overline{\varphi}$. Mean altruism parameter was $\overline{\varphi} = 0.15$. Graphs show mean of relative difference between scenario 1 and 2 in percentage points.



Finally, we looked at effects of public and private schooling on inequality. We assumed that individuals initially have different skills levels but the same level of selfishness parameter. We run numerical simulations for this ensemble of heterogenous agents. The number of draws was 1,000. Public schooling provides an equal opportunity to individuals. Because of that the distribution of human capital, consumption and income tends to become homogenous. On the contrary, variance of human capital of private schooling households grows after the increase in the productivity.

Figure 8. Effects of initial distribution of skills on human capital. Initial distribution of skills was a truncated normal with standard deviation of $\sigma = 1$. Selfishness parameter was $\varphi = 0.15$ and tax rate $\tau = 10\%$.



Overall, education provides additional means to improve the well-being of younger and older generations and the economy.

IV. Artificial Intelligence

A. K&S Model

Proposition D1: While automation may lead to immiseration of young, the adoption of Artificial Intelligence improves wellbeing of all generations.

Now we turn our attention to the effects of Artificial Intelligence (AI) on wellbeing of young and old generations. While automation mostly affects blue-color unskilled workers by replacing routine jobs by machines, AI affects white-color skilled workers jobs by AI technology displacement. Following K&S model we introduce a second intermediate input in the production function to replace skills S,

$$Q = Q\big(N_1(u_1 M_1, L), N_2(u_2 M_2, S)\big) \tag{A.1}$$

Machines M_1 replace unskilled labor, while M_2 is the intelligence of machines or software that replaces skilled labor. While unskilled labor and machines M_1 are perfect substitutes for each other, machines M_2 cannot substitute jobs that require expert human judgements. We assume Cobb-Douglas production function form for final goods and CES functional form for intermediate goods N_2,

$$\begin{aligned} Q &= A N_1^{\alpha} N_2^{1-\alpha} \\ N_1 &= L + u_1 M_1 \\ N_2 &= \left(S^{\gamma} + u_2 M_2^{\gamma}\right)^{1/\gamma} \end{aligned} \tag{A.2}$$

Taking partial derivatives of production Q,

$$\begin{aligned} Q_L &= A\alpha (N_1/N_2)^{\alpha-1} \\ Q_S &= A(1-\alpha)(N_1/N_2)^{\alpha} N_2^{(1-\gamma)/\gamma}\, S^{\gamma-1} \\ Q_{M_1} &= A\alpha u_1 (N_1/N_2)^{\alpha-1} \\ Q_{M_2} &= A(1-\alpha) u_2 (N_1/N_2)^{\alpha} N_2^{(1-\gamma)/\gamma} M_2^{\gamma-1} \end{aligned} \tag{A.3}$$

and imposing "perfect foresight" condition, $R_t = Q_{M_1} = Q_{M_2} = Q_S$, we get equations for the next period of machines and skills,

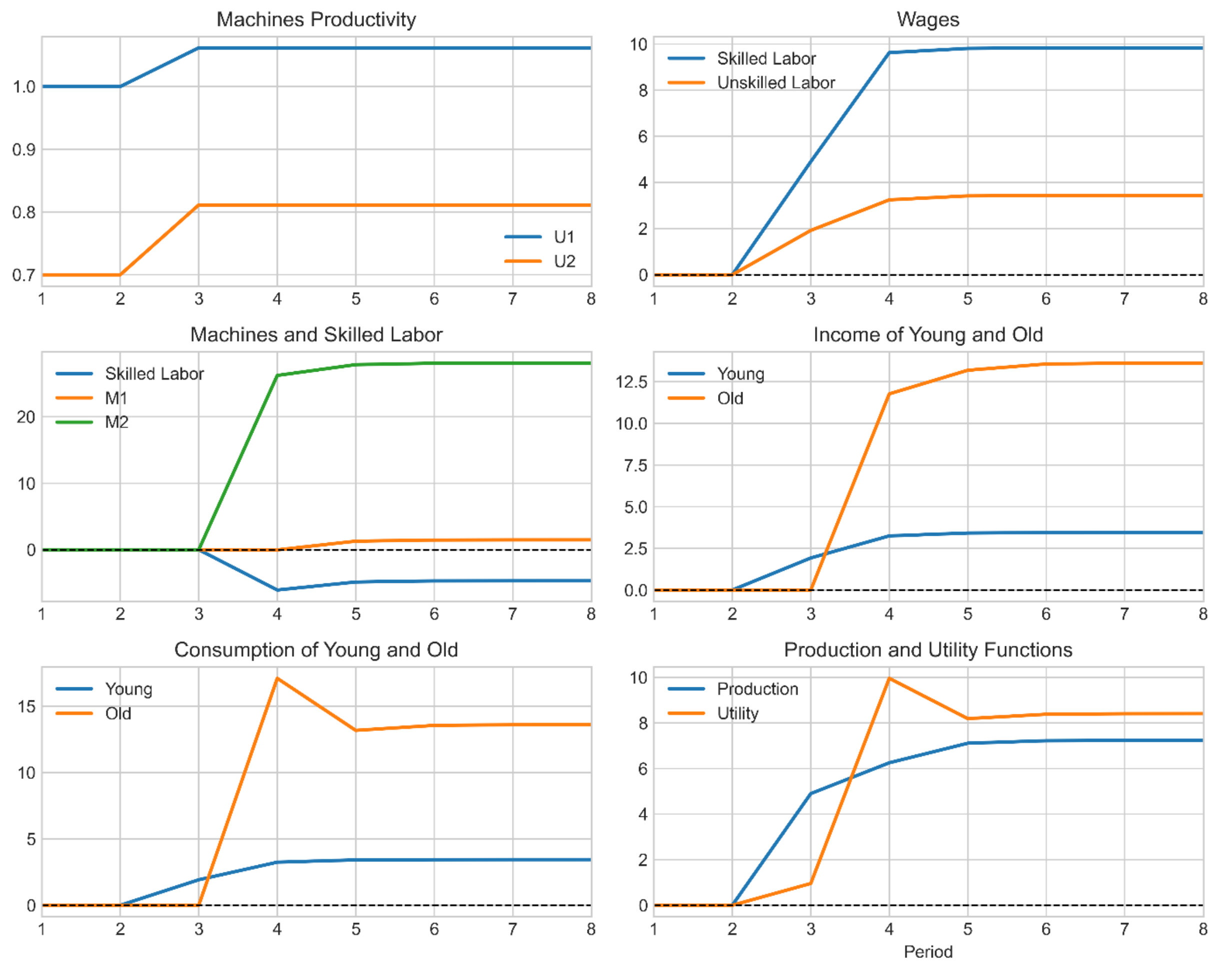
$$M_{1,t+1} + M_{2,t+1} + S_{t+1} = Sv_t$$
$$u_2 M_{2,t+1}^{\gamma-1} = S_{t+1}^{\gamma-1} \tag{A.4}$$
$$(1-\alpha) u_2 N_{1,t+1} N_{2,t+1}^{1/\gamma-2}\, M_{2,t+1}^{\gamma-1} = \alpha u_1$$

These equations coupled with standard equations of KS model describe dynamics of consumption, savings, skills, and machines:

$$C_t = \beta L_t Q_L$$
$$D_{t+1} = (1-\beta) R_t L_t Q_L \tag{A.5}$$
$$Sv_t = (1-\beta) L_t Q_L$$

We ran numerical experiments were parameter u_2 was increased at period 3 from 0.7 to $1.02^{30} - 1 = 0.81$, while parameter u_1 from 1 to $1.002^{30} = 1.06$. The underlying assumption was that AI adoption increases at 2% per year. Higher rate of AI technology penetration compared to automation means that software innovations and adoption occurs at a higher rate than machinery/robots' penetration in assembly lines. Parameter $\gamma = (\sigma - 1)/\sigma$, which describes elasticity of substitution σ, was equal 0.5 ($\sigma = 2$). At period 3 wage of skilled and unskilled workers suddenly increased. At period 5 these wages reached the new steady state which is above their initial values. This is to be expected: the increase in the productivity of machines squeezes the marginal product of S used to produce N_2. These old workers in period 3 face two opposing effects: the return of their skills SQ_S and machines which replace unskilled labor $M_1 Q_{M_1}$ has unexpectedly gone down, but the return of the intelligent machines $M_2 Q_{M_2}$ they also own has gone up (see bottom left and right plot of Figure 9). The net effect is positive and consumption and income of young and old increase.

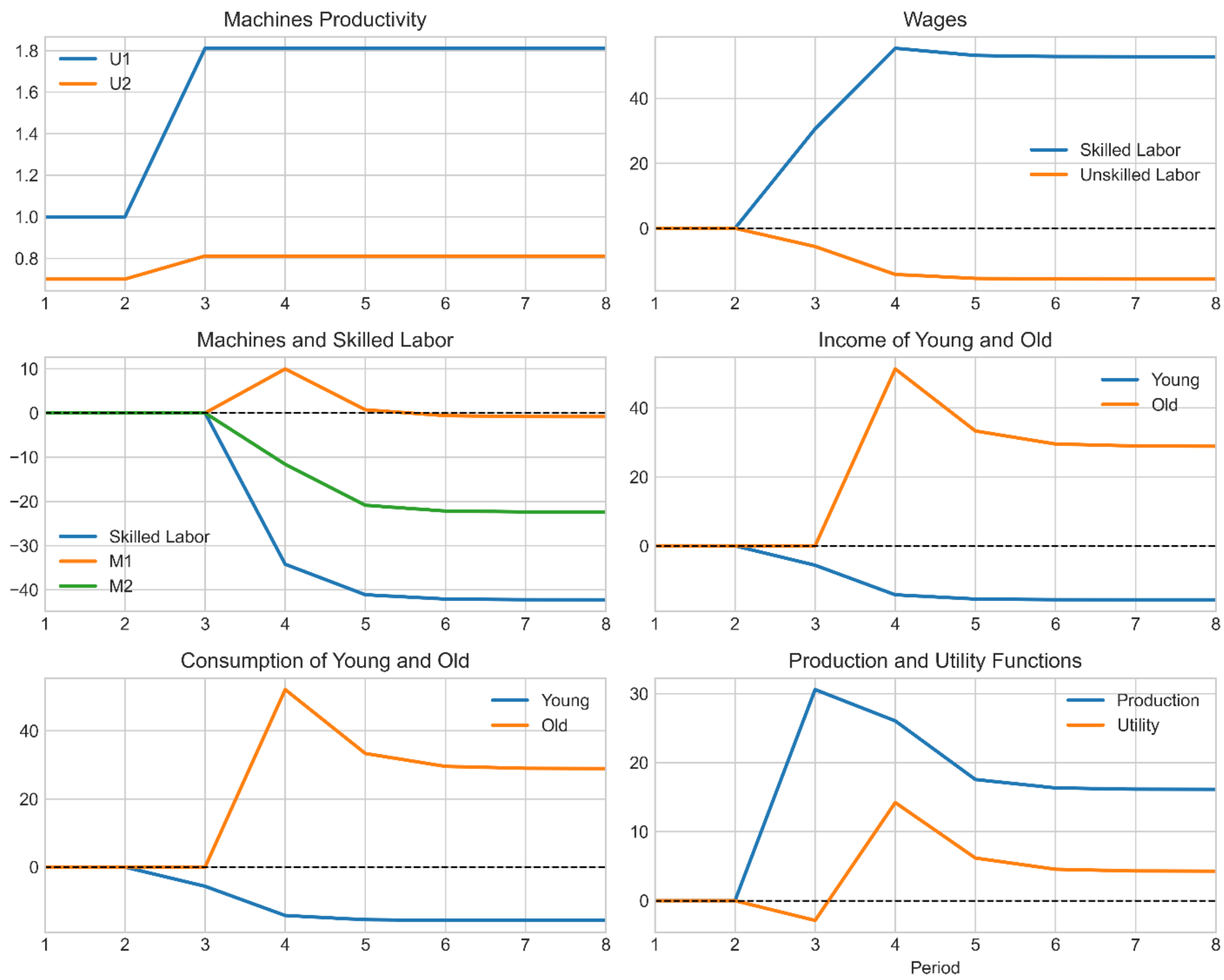
Figure 9. Time dependence of variables for Sachs and Kotlikoff settings. At period 3 machines factor productivities u_1, u_2 increased by 16 and 6 percents, respectively. Parameter's settings were: $A = 5, \alpha = 0.6, \beta = 0.5, \gamma = 0.5\ (\sigma = 2)$. Graphs show percentage difference with respect to the initial conditions.



The higher productivity of S affects the equilibrium utility of the young unskilled workers usage of machines M_2 in intermediate N_2. The comparative advantage of N_2 is analogous to the trade theory of Stolper-Samuelson and Rybczynsky theorems that relate the production and prices of the affected factors. In the end, more M_2 is used in making N_2 and less M_1 in making N_1. This helps to raise the wage of unskilled workers: see equation (A3), where marginal product of labor depends on the ratio$(N_2/N_1)^{1-\alpha}$, meaning it rises as N_2 is getting higher following the increase in u_2. Young workers born in period 3 face a decision of optimal portfolio allocation between skills and machines. Recalling that S and M_2 are perfect substitutes in saving, we conclude that young workers are indifferent in selection. Young will end up investing more in M_2 and less in S to accommodate the increase in u_2 in production. Interestingly, the increase in machine M_2 productivity is not enough to compensate the fall in wages, consumption, and income of young caused by the effects of automation. As rate of substitution between machines M_2 and skills S becomes larger, the ratio of intermediate products N_2/N_1becomes smaller and the relative difference of consumption and income of old and young decreases. However, these larger values of σ result in smaller values of consumption and income of both generations.

Next, we performed numerical experiments when both machine productivities u_1 and u_2 growth rate is 2% per year. Increased productivity of machines M_1 squeezes unskilled labor out. This results in decrease of unskilled labor wage at period 3. Positive effects of increased productivity of intelligent machines M_2 are not sufficient to overcome negative effects of automation and consumption and income of young decrease.

Figure 10. Time dependence of variables for Sachs and Kotlikoff model. At period 3 machines productivities u_1 and u_2 increased by 16%. Parameter's settings were: $\gamma = 0.5$ $(\sigma = 2)$.



Overall, AI improves wellbeing of all generations. Inequality between old and young generations decreases as elasticity of substitution σ becomes larger.

IV. Conclusions

In this research, we applied an overlapping generation model (K&S) to study the effects of bequests by older generations to the young generation, of government taxation on machines, and of public and private schooling. While this model is quite simple in its nature, it can nevertheless provide insights into the positive effects of increased machine productivity, the benefits of bequests from older to younger generations, and schooling of younger generations on the wealth of all generations.

Robotization, increased machine productivity, and widespread mechanization of labor can cause hardship for young workers with low skills levels, according to the K&S model. Fiscal policies can be implemented to improve the wellbeing of the younger generation. We show that automation in altruistic models does not have these conclusions. In both general and the K&S growth models with altruism we find critical values of model parameters and regimes when all generations benefit from the automation process.

Private bequests, whether in the form of inter-generational transfers or private education to younger generations, can improve the outcomes for both older and younger generations and for global output. Even in cases when poor households are bequest- constrained, government financing of public education can help avoid the recourse to increased public debt. As a complement to government fiscal taxation of machines and socialization of future "machine-based" capital, utility of all generations can be improved through expanded access to private education based on encouraged altruism to improve the skills pool. This conclusion supports the Keynesian notion that deficit spending can boost overall economic performance.

Lastly, we investigated effects of Artificial Intelligence on wellbeing of generations. We found that the adoption of AI improves welfare of all generations. This effect is more pronounced for lower values of elasticity of substitution of the intermediate production function N_2.

Appendices

Appendix A. K&S Model with Bequest

The young household's income I_t is comprised of labor wages W_t and bequest from its parents B_t. When old, the household's income I_{t+1}comes from production of machines and acquired skills:

$$I_t = L_t W_t + B_t = C_t + Sv_t$$
$$I_{t+1} = R_t\, Sv_t = D_{t+1} + (1+n)B_{t+1} \tag{1}$$

Here $1+n$ is the number of children per person.

Combining equations (A.5) and (1), we get:

$$C_t + \frac{D_{t+1} + (1+n)B_{t+1}}{R_t} = I_t \tag{2}$$

The value function of a household belonging to generation t is the of utility from its consumption during its own life cycle and the value function of generation $t+1$:

$$V_t = U_t + \varphi\, V_{t+1};\ \ 0 \leq \varphi \leq 1 \tag{3}$$

Here φ is the degree of intergenerational altruism. Parameter φ governs the fraction of wealth that the old generation bequest to their young. This value function can be expressed as an infinite sum of current and future generations utilities in the form of Bellman equation,

$$V_t = \sum_{j=t}^{\infty} \varphi^{j-t} U_j \tag{4}$$

providing the transversality condition is satisfied, $\lim_{t\to\infty} \varphi^t\, U_t = 0$. Utility from consumption during the life cycle is:

$$U = \beta\ log(C_t) + (1-\beta)\ log(D_{t+1});\ 0 \leq \beta \leq 1 \tag{5}$$

Here C_t is consumption when young, D_{t+1} is consumption when old, and β is the consumption preference parameter: the larger β means more consumption today versus tomorrow. Individuals chose the consumption and bequest that maximizes life-time value function,

$$V(B_t) = \max_{C_t, D_{t+1}} \{U(C_t, D_{t+1}) + \varphi\, V(B_{t+1})\}, \text{ s.t. } B_{t+1} \geq 0 \tag{6}$$

The Lagrangian associated with this constrained optimization problem is,

$$L = U(C_t, D_{t+1}) + \varphi\, V(B_{t+1}) + \mu B_{t+1}. \tag{7}$$

The Karush–Kuhn–Tucker necessary conditions for maximizing L are:

$$\begin{aligned} &\nabla V(C_t^*, D_{t+1}^*) - \mu \nabla B_{t+1} = 0 \\ &\mu \geq 0 \\ &\mu B_{t+1} = 0 \end{aligned} \tag{8}$$

The first equation is the first-order condition for value function, and the last two equations are dual feasibility and complementary slackness conditions. If bequest is positive, the constant μ is zero, while if bequest is zero, this constant is non-negative. Values $C_t^*,\ D_{t+1}^*$ are the optimal values of consumption when young and old. In what follows, we drop variables asterisk notation for brevity.

Taking partial derivatives of (6) and using (2), we get:

$$\begin{aligned} &\frac{\beta}{C_t} - \varphi \frac{\partial V}{\partial B_{t+1}} \frac{R_t}{1+n} + \mu \Upsilon \frac{R_t}{1+n} = 0 \\ &\frac{1-\beta}{D_{t+1}} - \varphi \frac{\partial V}{\partial B_{t+1}} \frac{1}{1+n} + \mu \Upsilon \frac{1}{1+n} = 0 \end{aligned} \tag{9}$$

Here $1+n$ is the number of children per person, and Υ is the indicator variable. This indicator $\Upsilon = 0$ for positive bequest, $B_{t+1} > 0$ and $\Upsilon = 1$ for zero bequest, $B_{t+1} = 0$.

Following Michel et al. (2004), we seek a solution to Bellman equation (3) of the form,

$$V(B_t) = a + b\, log(B_t + m_t). \tag{10}$$

Equations (8-9) assume that consumption is,

$$\begin{aligned} &C_t = (1+n)\, \beta\, \frac{B_{t+1} + m_{t+1}}{b\varphi R_t}; \text{ if } B_{t+1} > 0 \\ &C_t = \frac{(1+n)\beta}{R_t\left(\frac{b\varphi}{m_{t+1}} - \mu\right)}; \text{ if } B_{t+1} = 0 \end{aligned} \tag{11}$$

$D_{t+1} = \frac{1-\beta}{\beta} R_t C_t$

Firstly, we consider the case of zero bequest, $B_{t+1} = 0$. Since consumption is positive, constant μ is upper bounded, i.e., $0 \leq \mu < \frac{b\varphi}{m_{t+1}}$, and economy reaches a steady state as factors R_t and m_t become constants.

Secondly, we consider case of positive bequest, $B_{t+1} > 0$. Then $\mu = 0$, and by substituting (11) into (6) we get:

$$a + b \ log(B_t + m_t) = log\left(\frac{(1+n)(B_{t+1} + m_{t+1})}{b \ \varphi R_t}\right) + \beta \ log\left(\frac{\beta \ (1+n)(B_{t+1} + m_{t+1})}{b \ \varphi}\right)$$
$$+\varphi \ a + \ \varphi \ b \ log(B_{t+1} + m_{t+1}) \qquad (12)$$

By re-arranging terms in this equation:

$$b \ log(B_t + \mathrm{m_t}) = [1 + \varphi \ b] \ log(B_{t+1} + m_{t+1}) + \ log\left(\frac{(1-\beta) \ (1+n)}{b\varphi}\right) + (\varphi - 1)a \qquad (13)$$

We need to find a relationship between the bequest given by the older generation B_{t+1} and the bequest received by the younger generation B_t. By substituting optimal consumptions (9) into budget constraint equations (2) we get:

$$I_t = L_t \ W_t + B_t = \frac{1+n}{R_t}\left(B_{t+1} + \ \frac{B_{t+1} + m_{t+1}}{b\varphi}\right) \qquad (14)$$

By re-arranging terms again:

$$B_{t+1} + m_{t+1} = \frac{b\varphi R_t}{(1+n)(1+b\varphi)}\left[L_t W_t + B_t + \ \frac{(1+n)\mathrm{m_{t+1}}}{R_t}\right] \qquad (15)$$

Applying method of undetermined coefficients to equations (13,15), we get:

$$b = \frac{1}{1-\varphi}$$

$$m_t = L_t \ W_t + \frac{(1+n)m_{t+1}}{R_t} \qquad (16)$$

$$a = log\left(\frac{\varphi R_t}{1+n}\right) + \ \frac{1}{(1-\varphi)} log\left(\frac{(1-\beta) \ (1-\varphi)(1+n)}{\varphi}\right)$$

Parameter m depends on factor u, total factor productivity A, elasticity of substitution α and is piece-wise constant:

$$m_t = \frac{L_t W_t R_t}{R_t - 1 - n} \tag{17}$$

Rearranging (15), we get:

$$B_{t+1} = \frac{\varphi R_t}{(1+n)(1-\varphi)}(L_t W_t + B_t) - \frac{m_{t+1}}{1-\varphi} \text{ , s.t. } B_{t+1} \geq 0 \tag{18}$$

For small values of φ or low value of labor W_t, bequest B_{t+1} is zero and old generation consumes all their income. By induction we can obtain solution to (18):

$$B_{t+k} \cong g^k B_t + \frac{g^k - 1}{g-1} \frac{m}{1-\varphi} \left(\frac{\varphi R}{1+n} - 1 \right) \text{ , s.t. } B_{t+k} \geq 0 \tag{19}$$

Here g = $\frac{\varphi \mathrm{R}}{(1+n)(1-\varphi)}$ is the growth rate of bequest.

We can compute the optimal value of bequest:

$$B_{t+1} = \left(1 + \frac{(1-\beta)(1+n)(1-\varphi)}{\varphi}\right)^{-1} \left(\frac{I_{t+1}}{1+n} - m_{t+1} \frac{(1-\beta)(1+n)(1-\varphi)}{\varphi}\right) \tag{20}$$

and consumption of young and old:

$$C_t = \beta(1-\varphi)\left[I_t + \frac{1+n}{R_t} m_{t+1}\right]$$
$$D_{t+1} = \frac{(1-\beta)(1-\varphi)}{1-\beta+\beta\varphi} \left[I_{t+1} + (1+n) m_{t+1}\right] \tag{21}$$

Equations (20,21) describe the dynamics of macro variables of K&S model with automation and bequest from old to young.

For Cobb-Douglass production interest factor R and factor m are functions of productivity:

$$R = A\alpha u_1 \left(\frac{\alpha u_1}{1-\alpha}\right)^{\alpha - 1}$$

$$m = \frac{Q_L\, Q_M}{Q_M - 1 - n} = \frac{A^2 \alpha^2 u_1 \left(\frac{\alpha u - 1}{1-\alpha}\right)^{2\alpha - 2}}{A \alpha u_1 \left(\frac{\alpha u_1}{1-\alpha}\right)^{\alpha - 1} - 1 - n} \tag{22}$$

Appendix B. Replicating Kotlikoff and Sachs (2012)

Below we present results of a run of K&S model with parameters:
$L = 1,\ A = 10,\ \alpha = 0.6,\ \beta = 0.5.$

T	u	M	S	Wl	Ws	Sv	Q
1	1	1.1	1.4	5.10	5.10	2.55	18.12
2	1	1.1	1.4	5.10	5.10	2.55	18.12
3	10	1.1	1.4	2.53	14.6	2.04	51.88
4	10	0.7	0.6	2.03	20.3	1.79	27.72
5	10	0.6	0.5	2.03	20.3	1.79	22.66
6	10	0.6	0.5	2.03	20.3	1.79	22.66
7	10	0.6	0.5	2.03	20.3	1.79	22.66
8	10	0.6	0.5	2.03	20.3	1.79	22.66
9	10	0.6	0.5	2.03	20.3	1.79	22.66

Table A1. Replication of Sachs and Kotlikoff results.

Here T is the time, u is the productivity parameter, M is the machines, S is the skills, Wl is the earnings of young, Ws is the earnings of old, Sv is the savings of young, Q is the Cobb-Douglas productivity function.

These results reproduce Table 1 (unanticipated case) from Sachs and Kotlikoff paper except those authors claim that they obtained their results for slightly different set of parameters: $L = 1, \alpha = 0.5, A = 10, \beta = 0.5$

Appendix C. Robustness Checks

Following Michel (2004), we further conducted a robustness check of steady state of the neo-classical growth model with a bequest.

The young generation income I_t is comprised of labor wages W_t and bequest from its parents:

$$I_t = W_t + B_t = C_t + Sv_t \tag{1}$$

Savings of young generation are invested in capital, $Sv_t = (1+n)K_{t+1}$. Note that the income of older generation comes from the proceeds of savings of young,

$$I_{t+1} = R_t Sv_t = D_{t+1} + (1+n)B_{t+1} \tag{2}$$

The derivation of equations in this case is almost identical to the K&S model with bequest. Following Mitchel et al. we introduce shadow prices of bequest, p_t. The value function is augmented with the change of these shadow prices:

$$V = log(C_t) + \hat{\beta} log(D_{t+1}) + \phi p_{t+1} B_{t+1} - p_t B_t \tag{3}$$

By setting derivatives of this utility function to zero, we get:

$$C_t = \frac{1+n}{\phi p_{t+1} R_t} \tag{4}$$

$$D_{t+1} = \frac{(1+n)\hat{\beta}}{\phi p_{t+1}}$$

Maximizing the Lagrange function (3) with respect to B_t gives:

$$p_{t+1} = \frac{(1+n)}{\phi R_t} p_t \text{ , if } \quad B_t > 0 \tag{5}$$

$$p_{t+1} < \frac{(1+n)}{\phi R_t} p_t \text{ , if } \quad B_t = 0$$

Let us consider the classical Cobb-Douglas production function:

$$Q(K, L) = A(K/L)^\alpha L \tag{6}$$

We assume that young supply one unit of labor, $L = 1$. The wage rate and interest factor are described by standard equations:

$$W_t = (1-\alpha)AK_t^\alpha$$

$$R_t = \alpha A K_t^{\alpha-1} \tag{7}$$

Equations (1,2,4,5,7) govern the dynamics of this system. We repeated experiments by using Michel's et al. (2004) analytical solution shown below:

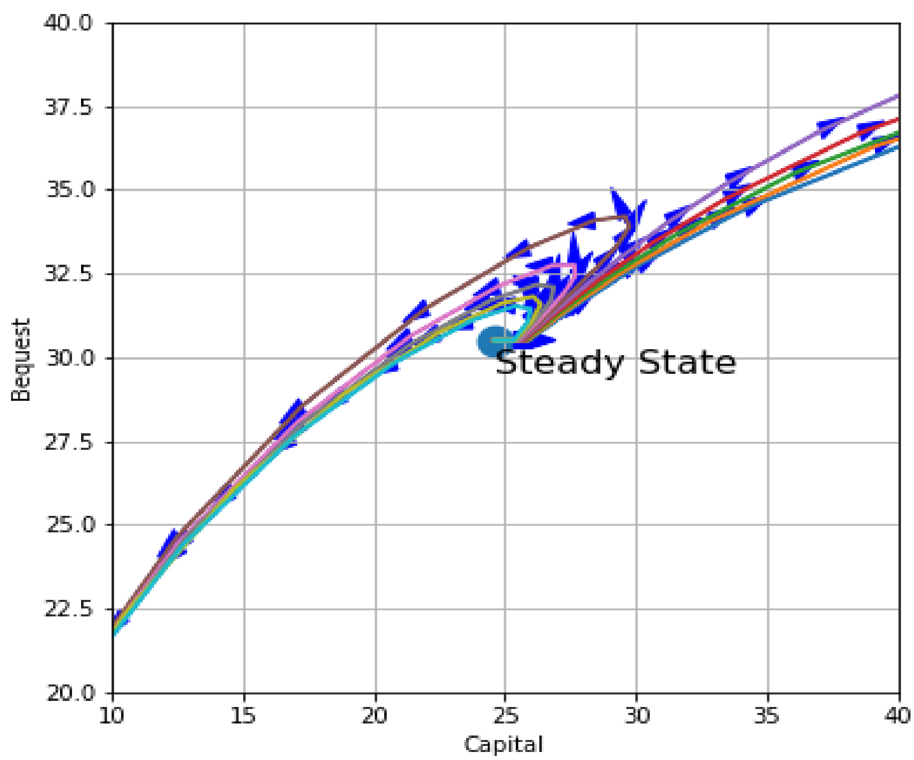
$$v_t = A\, p_t\, K_t^{\alpha}$$
$$v_{t+1} = \frac{1}{\alpha\hat{\varphi}}\left(v_t - 1 - \frac{\hat{\beta}}{\hat{\varphi}}\right)$$
$$B_t = \left(\alpha - \frac{\hat{\beta}(1-\alpha\hat{\varphi})}{\hat{\varphi}+\hat{\beta}}\right) A\, K_t^{\alpha} \quad (8)$$
$$p_t B_t = \alpha\, v_t - \frac{\hat{\beta}}{\hat{\varphi}}$$

These equations have a steady state solution for a specific set of initial conditions,

$$v_0 = v_t = \frac{1}{1-\alpha\hat{\varphi}}\left(1 + \frac{\hat{\beta}}{\hat{\varphi}}\right) \quad (9)$$

However, this solution is unstable and grows exponentially in time. This exponential growth is caused by variable v_t multiplier $\frac{1}{\alpha\hat{\varphi}} > 1$ (since $\alpha < 1$, $\hat{\varphi} \leq 1$). Figure 1 shows phase diagram of capital and bequest trajectories computed according to equations (8) with initial conditions in the range $v \in (0.99v_0, 1.01v_0)$. This example illustrates instability of this steady state solution with positive bequest.

Figure 1. Phase diagram of Michel et al. (2004) analytic solution.



A. References

Acemoğlu, Daron and Restrepo, Pascual, 2017. “Robots and Jobs: Evidence from US Labor Market”, Working Paper 23285, NBER, https://www.nber.org/papers/w23285.

Acemoğlu, Daron and Restrepo, Pascual, 2018. “Modeling Automation”, AEA Papers and Proceedings, Volume 108, pp. 48-53.

Acemoğlu, Daron and Restrepo, Pascual, 2019. “Automation and New Tasks: How Technology Displaces and Reinstates Labor”, Journal of Economic Perspectives, Volu.33, Number 2, pp. 3–30.

Acemoğlu, Daron and Restrepo, Pascual, 2021. “Tasks Automation, and the Rise in US Wage Inequality”, Working Paper 28920, National Bureau of Economic Research.

Aghion Philippe, Jones Benjamin F. and Jones Charles I., 2017, “Artificial Intelligence and Economic Growth “, *NBER Working Paper Series*, Working Paper 29283.

Altonji, Joseph G., Fumio Hayashi, and Laurence J. Kotlikoff, 1997. "Parental Altruism and Inter Vivo Transfers: Theory and Evidence," Journal of Political Economy, Volume 105, pp. 1121-1166.

Andreoni James, 1989. “Giving with Impure Altruism: Application of Charity and Ricardian Equivalence”, The Journal of Political Economy, Volume 97, Issue 6, pp. 1447-1458.

Andreoni, James, 1990. “Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving”, The Economic Journal, Volume 100, Issue 401, pp.464-477.

Artige, Lionel, 2010. “Endogenous Growth and Parental Funding of Education in an OLG Model with a Fixed Factor”, *ResearchGate*.

Autor, David, 2014, “Skills education and the rise of earnings in equality among the “other 99 percent”, Science, Volume 344, pp. 843-851.

Autor, David, 2015. “Why Are There Still So Many Jobs? The History and Future of Workplace Automation”, Journal of Economic Perspectives, Vol. 29, Number 3, pp 3–30.

Benzell Seth G., Kotlikoff Laurence J., LaGarda Guillermo, and Sachs Jeffrey D., 2015. “Robots Are Us: Some Economics of Human Replacement”, NBER Working Paper 20941, http://www.nber.org/papers/w20941.

Bolton, Gary and Ockenfels, Axel, 2000. “Inequality Aversion, Efficiency, and Maximin Preferences in Simple Distribution Experiments: Comment”, *The American Economic Review* Volume 96, No. 5, pp. 1906-1911.

Brynjolfsson, Erik and McAfee, Andrew, 2014. “Race Against the Machine. How the Digital Revolution Is Accelerating Innovation, Driving Productivity, and Irreversibly Transforming Employment and the Economy”, W. Norton & Company, Inc. 500 Fifth Avenue, New York, N.Y. 10110.

Carmignani Fabrizio (2016). “Does government spending on education promote economic growth”, The Conversation, Griffith University, https://theconversation.com/does-government-spending-on-education-promote-economic-growth-60229.

Cowen Tyler, 2011. “The Great Stagnation: How America Ate All the Low-Hanging Fruit of Modern History, Got Sick, and Will (Eventually) Feel Better: A Penguin Special from Dutton, Kindle Edition.

Chernoff, Alex and Warman, Casey, 2020. “COVID-19 and Implications for Automation”, Working Paper 27249, National Bureau of Economic Research.

Chowdhury, Subhasish and Jeon, Joo Young, 2014. “Impure Altruism or Inequality Aversion? An experimental investigation based on income effects”, Journal of Public Economics, Volume 118, pp. 143-150.

Diamond, Peter, 1965. "National debt in a neoclassical growth model". American Economic Review. Vol. 55, No. 5, pp. 1126–1150.

Elizabeth N. Appiah, (2017). “The Effect of Education Expenditure on Per Capita GDP in Developing Countries”, International Journal of Economics and Finance; Vol. 9, No. 10, pp. 136-144.

Fehr, Ernst and Schmidt, Klaus M., 1999. “A Theory of Fairness, Competition and Cooperation”, Working Paper Series ISSN 1424-0459, The Quarterly Journal of Economics’, August 1999, 817-868.

Ford, Martin, 2015. “Rise of the Robots Technology and the Threat of a Jobless Future”, Basic Books. A Member of the Perseus Books Group New York.

Freeman, Richard, 2015. “Who owns the robots rules the world. Workers can benefit from technology that substitutes robots or other machines for their work by owning part of the capital that replaces them”, IZA World of Labor.

Gaaitzen J. de Vries, Elisabetta Gentile, Sébastien Miroudot, Konstantin M. Wacker, 2020. “The Rise of Robots and the Fall of Routine Jobs”, *Labor Economics*, V. 66, 101885, https://doi.org/10.1016/j.labeco.2020.101885.

Georg, Graetz and Guy Michaels, 2018. “Robots at Work”, Review of Economics and Statistics. ISSN 0034-6535, http://eprints.lse.ac.uk/87218.

Glomm, Gerhard and Ravikumar B., 1992. “Public Versus Private Investment in Human Capital: Endogenous Growth and Income Inequality”, *The Journal of Political Economy*, Vol. 100, No. 4, pp. 818-834, The University of Chicago Press.

Goodhart, C. and Pradhan, M., 2020. “The Great Demographic Reversal: Ageing Societies, Waning Inequality, and an Inflation Revival”, 1st ed., Palgrave Macmillan.

Gordon, Robert, 2012. “Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds”, NBER Working Paper, N 18315, https://www.nber.org/papers/w18315.

Gheraia, Zouheyr, Mohammed, Benmeriem, Abdelli, Hanane and Saadaoui, Sawssan, (2021). “The Effect of Education Expenditure on Economic Growth: The Case of the Kingdom of Saudi Arabia”, Humanities and Social Sciences Letters, Vol 9, pp. 14-23.

Ivanyna M., Mourmouras A., and P., Rangazas, 2018. “The Macroeconomics of Corruption: Governance and Growth, Springer Texts in Business and Economics.

Kotlikoff, Laurence J., Razin Assaf, and Rosenthal Robert W., 1990. “A Strategic Altruism Model in which Ricardian Equivalence Does Not Hold”, The Economic Journal, Vol.100, No. 403, pp. 1261-1268.

Laitner, John and F. Juster, Thomas, 1996. “New Evidence on Altruism: A Study of TIAA-CREF Retirees”, *American Economic Association*, Vol. 86, No. 4, pp. 893-908.

Lutz, Hendricks, 2020. “Overlapping Generations Model Bequests and Altruism”, Econ 720.https://lhendricks.org/econ720/olg/OLG_Bequest_SL.pdf.

Mehmet Mercana, Sevgi Sezer, 2014. “The effect of education expenditure on economic growth: The case of Turkey”, Procedia - Social and Behavioral Sciences, Vol. 109, pp. 925 – 930.

Müller, Vincent C. and Bostrom, Nick, 2014. “Future Progress in Artificial Intelligence: A Survey of Expert Opinion”, Fundamental Issues of Artificial Intelligence (Synthese Library; Berlin: Springer).

Phillipe Michel, Emmanuel Thibault and Jean-Pierre Vidal, 2004. “Intergenerational Altruism and Neoclassical Growth Models”, *Working Paper Series*, No. 386, European Central Bank.

Carlo Pizzinelli, Augustus Panton, Marina M. Tavares, Mauro Cazzaniga,
Longji Li (forthcoming). “Labor Market Exposure to AI: Cross-country Differences and Distributional Implications”, IMF Working Paper.

Sachs, Jeffrey D. and Kotlikoff, Laurence J., 2012. “Smart Machines and Long-Term Misery”, *NBER Working Paper Series*, Working Paper 18629.

Tahsin, Saadi Sedik and Jiae, Yoo, 2021. “Pandemics and Automation: Will the Lost Jobs Come Back?”, IMF Working Paper.

Yahya, Mohd, Muhammad, Fidlizan, Abu-Hussin, Mohd Fauzi, Razak Azila,
(2012). “Education Expenditure and Economic Growth: A Causal Analysis for Malaysia”, Journal of Economics and Sustainable Development, Vol. 3, No. 7, pp. 71-81.

PUBLICATIONS

Automation, Labor and Welfare: The Role of Bequests and Education

Working Paper No. WP/2024/011