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Central Banks as Dollar Lenders of Last Resort:

Implications for Regulation and Reserve Holdings

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ABSTRACT: This paper explores how non-U.S. central banks behave when firms in their economies engage in currency mismatch, borrowing more heavily in dollars than justified by their operating exposures. We begin by documenting that, in a panel of 53 countries, central bank holdings of dollar reserves are significantly correlated with the dollar-denominated bank borrowing of their non-financial corporate sectors, controlling for a number of known covariates of reserve accumulation. We then build a model in which the central bank can deal with private-sector mismatch, and the associated risk of a domestic financial crisis, in two ways: (i) by imposing ex ante financial regulations such as bank capital requirements; or (ii) by building a stockpile of dollar reserves that allow it to serve as an ex post dollar lender of last resort. The model highlights a novel externality: individual central banks may tend to over-accumulate dollar reserves, relative to what a global planner would choose. This is because individual central banks do not internalize that their hoarding of reserves exacerbates a global scarcity of dollar-denominated safe assets, which lowers dollar interest rates and encourages firms to increase the currency mismatch of their liabilities. Relative to the decentralized outcome, a global planner may prefer stricter financial regulation (e.g., higher bank capital requirements) and reduced holdings of dollar reserves.

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WORKING PAPERS

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1. Introduction

Central banks around the world hold large balances of foreign currency reserves, with the U.S. dollar accounting for the dominant share of these reserves, at about 59% of the total.[1] In the aggregate, foreign official accounts held \$7.06 trillion of dollar securities in March of 2022, with \$4.06 trillion of this in the form of U.S. Treasury securities.[2] This makes foreign reserve managers among the most important players in the Treasury market, a fact that is often argued to be a key determinant of the level of U.S. interest rates.[3] The implications of these reserve balances for market outcomes were also dramatically highlighted in the COVID-pandemic-induced "dash for cash" in March of 2020, when heavy foreign central bank selling of Treasuries played a major role in the dislocations seen in that market.[4]

What explains the large appetite of global central banks for foreign currency reserves, and for dollar reserves in particular? In this paper, we focus on one potential motive, arising out of the fact that firms in many countries run a significant currency mismatch in their capital structures, borrowing heavily in dollars even when they have largely domestic operating exposures.[5] We argue that in the face of such mismatch, a central bank will have a natural incentive to stockpile dollars, so that it can more effectively serve as a lender of last resort in a state of the world where the local economy and banking system are under stress and need to be bailed out. We then go on to explore the normative implications of this behavior, showing how the reserve-accumulation decisions of individual central banks can be excessive relative to a global optimum to the extent that they do not internalize the general-equilibrium impact of their choices on the aggregate supply of safe dollar assets and on the dollar interest rate.

We begin by presenting some simple empirical relationships to motivate our subsequent theoretical work. We document that in a panel of 53 countries, central bank holdings of dollar reserves (relative to GDP) are significantly correlated with the dollar-denominated bank borrowing

[1] Source: IMF COFER data.

[2] Source: Treasury International Capital data.

[3] Early analyses of the impact of foreign demand for U.S. dollar assets and its effect on interest rates include Bernanke (2005), and Caballero, Farhi and Gourinchas (2008).

[4] See, e.g., Vissing-Jorgensen (2021).

[5] This tendency is documented in Du and Schreger (2022).

of the non-financial corporate sector (again relative to GDP), controlling for a number of known covariates of reserve accumulation. The interpretation we have in mind is that the latter variable is a rough proxy for the currency mismatch of the corporate sector, which in turn drives the currency exposure of the banking system and ultimately the dollar lender-of-last-resort motive in our theory.

Next, building on the framework in Gopinath and Stein (2018), we develop a model of optimal reserve accumulation for a small open-economy central bank that faces a risk of a banking crisis, a risk which is exacerbated when the non-financial corporate sector runs a currency-mismatched capital structure. Importantly, while reserve holdings can help to mitigate the fallout from a banking crisis ex post, they are not the only policy tool available. We consider the possibility that the central bank can also deploy ex ante financial-regulatory tools, such as bank-capital regulation. The optimal mix of these tools depends on a straightforward tradeoff. On the one hand, when ex ante regulation is too stringent, this reduces the profitability of the local banking sector and hence social welfare; this effect tends to favor reserve accumulation. On the other hand, there is a carrying cost associated with reserve holdings, which is roughly given by the difference between the domestic and dollar interest rates. When this carrying cost is larger, the balance tips back towards using more heavy-handed financial regulation, and less reserve accumulation.

While this small open-economy version of the model helps to make sense of the basic cross-sectional empirical patterns we see in the data, a primary contribution of the paper is in fleshing out the model's normative properties. In particular, suppose we have a global economy in which many central banks act as price-takers in the market for safe dollar assets. If each of these central banks sets their regulatory and reserve-holding policies individually, so as to maximize own-country welfare, how does the outcome compare to one in which a global central planner aims to maximize global welfare?

Here the model highlights a novel externality: individual central banks may tend to over-accumulate dollar reserves, relative to what the global planner would choose. This is because individual central banks do not internalize that their hoarding of reserves exacerbates a global scarcity of dollar-denominated safe assets and puts downward pressure on dollar interest rates. This downward pressure on dollar rates in turn exacerbates the tendency of private-sector firms to engage in currency mismatch, increasing their exposure to a dollar appreciation. Relative to the decentralized outcome, a global planner may therefore prefer a different policy mix of policy tools, with stricter financial regulation (i.e., higher bank capital requirements) and reduced holdings of dollar reserves.

The literature on central-bank reserve holdings is well-developed, and has identified a number of potential motives, which can be grouped into two broad categories, sometimes referred to as the "mercantilist" and "precautionary" views.[6] According to the mercantilist view, a central bank that seeks to protect its tradable sector—and hence to prevent its exchange rate from appreciating—will tend to accumulate reserves when it is running a trade surplus.[7]

The precautionary view encompasses several mechanisms that can lead a central bank to stockpile reserves as a buffer against the risk of a future adverse shock. One version focuses on the potential for "sudden stops" in emerging markets—i.e., rapid reversals of *external* capital flows—and the role of reserves in mitigating the damage from such episodes.[8] Another instead emphasizes the possibility that *domestic* depositors might attempt to flee the banking system by converting their local-currency-denominated deposits into foreign currency.[9]

Our starting premise—that central banks hold dollar reserves to deal with the potential consequences arising from a currency mismatch on the part of their corporate sectors—can also be thought of as fitting within the broad precautionary view. In this regard, we are perhaps closest to recent work by Bocola and Lorenzoni (2020), who emphasize the same currency-mismatch motive. We differ from their paper in highlighting the joint roles of financial regulation and reserve holdings in shoring up financial stability, and in showing how the decisions of individual central

[6] This terminology follows Aizenman and Lee (2007).

[7] See Dooley et al (2003), Aizenman and Lee (2010), Benigno and Fornaro (2012), and Korinek and Serven (2016).

[8] See Caballero and Panageas (2008), Alfaro and Kanczuk (2009), Durdu, Mendoza and Terrones (2009), Jeanne and Ranciere (2011), Bianchi, Hatchondo and Martinez (2018), Cespedes and Chang (2020), Arce, Bengui and Bianchi (2022), and Bianchi and Lorenzoni (2021), among others.

[9] This idea is developed in Obstfeld, Shambaugh and Taylor (2010), who use M2/GDP as a proxy for the vulnerability of the domestic banking sector to such an "internal capital drain".

banks lead to too little of the former and too much of the latter, relative to what a global planner would choose.[10]

Our normative analysis also connects to a literature on international coordination in financial regulation. For example, we share the conclusion of Clayton and Schaab (2022) that individual countries acting on their own will tend to impose lower capital requirements on their domestic banks than is globally efficient. This externality motivates the need for international cooperation on the regulatory front, of the sort seen in the so-called Basel Process. However, a key distinction is that in our framework, such regulatory cooperation is not sufficient—there also needs to be a separate mechanism to restrain excessive reserve accumulation by central banks.

The remainder of the paper is organized as follows. In Section 2, we present some motivating evidence on the relationship between central bank holdings of dollar reserves and the dollar-denominated bank borrowing of their corporate sectors. In Section 3, we develop a single-country model in which the central bank can deal with the risks created by currency mismatch in one of two ways: by imposing stricter financial regulation, or by accumulating foreign-exchange reserves. In Section 4, we consider a global economy consisting of many such individual countries and explore the externalities that arise when regulatory policy and reserve holdings are determined at the country level, rather than by a global planner. Section 5 discusses some further extensions of our framework, and Section 6 concludes.

2. Motivating Evidence

Unlike much of the empirical work on foreign-exchange reserve holdings, our focus is on dollar reserves, as opposed to total reserves. This presents something of a data challenge, as the composition of central-bank reserves by currency is not available for all countries. Thus we begin with a panel of 53 non-U.S. and non-Eurozone countries for which we are able to break out the currency composition of reserve holdings, as well as compile a small set of covariates. We exclude the Eurozone countries because, to the extent that they all benefit from either explicit or implicit

[10] Fanelli and Straub (2021) also build a model in which individual countries over-accumulate reserves relative to a global planner's optimum. However the mechanism in their model is quite different, and more "mercantilist" in nature—central bank reserve holdings are driven by a desire to stabilize exchange rate fluctuations.

ECB support, it does not make sense to relate dollar reserve holdings measured at the individual-country level (e.g., dollar reserves on the books of the Bank of Italy) to country level measures of corporate-sector mismatch. The data on dollar reserve holdings is constructed from the union of data in IMF (2020) and Chinn et al (2021); Table A1 in Appendix A lists all of our data sources. Our unbalanced panel of 53 countries has 365 observations, covering the period 2013-2020.[11] Of these 53 countries, 13 are classified by the IMF as advanced economies, 29 are classified as emerging economies, and 11 are classified as developing economies. Table A2 gives a full listing of the countries broken down by these categories.

Our basic objective is to relate a country's dollar reserves to a measure of its private sector's foreign currency mismatch. In thinking about how to best proxy for foreign currency mismatch, we are informed by the following observation: as a general matter, and likely in part as the result of regulation, banks tend not to run large outright currency mismatches on their own books; there is an extremely tight correlation between their dollar-denominated assets and liabilities.[12] Rather, currency mismatch shows up to a greater extent on the balance sheets of the non-financial sector. As discussed in Gopinath and Stein (2021), one way to think about this is that the exogenous variation in the data comes from the fact that the preference on the part of households for dollar-denominated assets is greater in some countries than others. And when a bank finds itself awash in dollar deposits, it seeks to reduce its own currency exposure by cutting the rates on dollar loans, thereby creating an incentive for the non-financial sector to borrow more aggressively in dollars.[13]

With this observation in mind, one simple way to measure mismatch might be to look at the ratio of the dollar-denominated borrowing of the nonfinancial corporate sector to GDP. This

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[11] This number comes after discarding two observations that appear to be data errors—where the ratio of dollar reserves to total reserves is coded as either negative or as exceeding 100%.

[12] For example, in our 365-observation panel, the correlation between the ratio of dollar-denominated bank assets to GDP and the ratio of dollar-denominated bank liabilities to GDP is 0.965. If we instead look at the 53 observations of country-level averages, the correlation is 0.974. The data on total dollar-denominated bank assets and liabilities is from BIS Table A6.1.

[13] Of course, in this case, while the banking sector may appear to be nominally currency hedged, it still bears a significant economic exposure to the dollar—when the dollar appreciates, more of its currency-mismatched borrowers will tend to default on their loans. However, for the purposes of measuring mismatch in the data, this line of argument suggests that we need to look to the balance sheets of the non-financial sector.

would clearly be an imperfect proxy to the extent that it incorporates dollar-denominated borrowing by those firms (e.g. exporters) who may not actually be mismatched. It would in principle be better to capture only dollar-denominated borrowing by purely domestic non-tradable firms; unfortunately we have been unable to create such a measure.

Moreover, even if we look at the aggregate nonfinancial sector, thereby blurring over this distinction, we face a further limitation: for our sample of 53 countries, we have available from the BIS complete data only on those dollar-denominated bank loans to the corporate sector that come from *cross-border banks*, i.e., banks headquartered outside the country in question. To get total dollar-denominated bank lending to the corporate sector in a given country, we need to add loans from *local banks*, but unfortunately, we only are able to obtain this local-lending data by currency for a smaller subsample of 21 countries, 10 of which are advanced economies and 11 of which are emerging economies.[14]

As an admittedly second-best approach, and one that allows us to work with the larger 53-country sample, we use the cross-border lending data in what follows. In doing so, we draw some comfort from the fact that for the 21 countries where we can construct the preferred total (i.e. cross-border plus local) measure of dollar-denominated lending to the nonfinancial corporate sector, it has a correlation of 0.66 with cross-border dollar lending. If we break the data down further into advanced-economy and emerging-economy subsamples, the correlations are higher, at 0.89 and 0.73 respectively. Thus the cross-border lending data may be a tolerably good proxy for total dollar-denominated bank lending.

One reason why this might be the case is that if, say, the U.S.-based subsidiary of a U.K. bank holding company (for example, HSBC Bank USA, which is a subsidiary of U.K.-headquartered HSBC) makes a loan to a U.K. firm, this will be counted in the BIS data as a cross-border loan to the U.K. corporate sector, even though it is in effect a U.K.- headquartered bank holding company lending to a U.K.-domiciled firm. So some of what are categorized by the BIS

[14] The currency composition of banks' local (as opposed to cross-border) claims is in many cases either not available or is treated as confidential by the BIS. Of the 46 countries that report banks' local dollar claims to the BIS, only 8 are non-euro area countries whose data are non-confidential *and* where we also have the currency breakdown of reserves. For the remaining countries, the breakdown of local claims by currency is made available to the IMF from national monetary authorities.

Locational Banking Statistics as cross-border loans may be more closely connected to the domestic banking sector than the label might otherwise suggest. Nevertheless, the imperfect nature of our approach suggests caution in interpreting the results that follow; we cannot be sure that they would continue to hold with a more complete measure of dollar-denominated lending.[15]

With these caveats in mind, Figure 1 presents a first simple univariate visualization of our basic result. For each of the 53 countries in our baseline sample, we plot on the horizontal axis the time-averaged value of cross-border dollar bank loans to GDP, and on the vertical axis the time-averaged value of dollar reserves to GDP. As can be seen, there is a strong positive correlation between these two variables. The R-squared of the regression is 0.53, and the coefficient on the dollar bank loan variable is 5.3, with a t-statistic of 7.6, so that a one-percentage point increase in dollar loans to GDP is associated with a 5.3 percentage point increase in dollar reserves to GDP.

However, as Figure 1 also makes clear, this relationship is driven in important part by one data point—Hong Kong—which is an extreme outlier, with very large values of both dollar bank loans to GDP and dollar reserves to GDP. In Figure 2, we repeat the plot, this time excluding Hong Kong. While there is still a statistically significant relationship, it is considerably attenuated. Now the R-squared of the regression is only 0.083, and the slope coefficient falls to 1.3, with a t-statistic of 2.5. Thus to be conservative in the rest of what follows, we focus on a modified sample that excludes Hong Kong. To be clear, we have no compelling economic reason to do so, and, as the two figures suggest, our results would be stronger with Hong Kong included, but they would be less representative of the general tendencies in the data. Table 1 presents some basic summary statistics for this modified 52-country sample.

To get a better sense of where the correlation between dollar borrowing and dollar reserve holdings is coming from, in Figure 3 we repeat the graphical exercise for each of three subsamples separately: advanced, emerging and developing economies. As can be seen, the relationships are significant for both the advanced-economy (coefficient estimate of 3.7, t-statistic of 2.2, R-squared of 0.31) and emerging-economy subsamples (coefficient estimate of 2.4, t-statistic of 2.5, R-

[15] Also missing from our measure is dollar-denominated bond-market borrowing, which again we have not been able to assemble for many of the countries in our baseline sample. Though here the theoretical case for including it is arguably murkier, as it is less obvious that the central bank will find itself compelled to bail out bond-market investors as compared to commercial banks.

squared of 0.18).[16] However, in the developing-economy subsample, there is no significant correlation, and the point estimate goes slightly in the wrong direction. Thus our story does not seem to apply to the poorest countries. Of course, in a value-weighted sense, these countries loom less large than they do in our equal-weighted regressions, suggesting that we may nevertheless have something to say about the behavior of the most important holders of dollar reserves.

Table 2 explores these relationships in a series of regressions that exploit the full panel structure of our data, rather than just focusing on country averages. In columns (1)-(4), we run univariate panel regressions of dollar reserves to GDP against the ratio of cross-border dollar loans to nonfinancial firms divided by GDP, for the full sample, and the advanced, emerging and developing-economy subsamples respectively. Consistent with the impressions from Figures 2 and 3, the results for the full sample and the advanced-economy subsample are significant at the 10% level, while that for the emerging-economy subsample is significant at the 5% level.[17] The developing-economy subsample by contrast yields a completely insignificant result.

In columns (5)-(8) we re-run the same regressions, adding several controls familiar from the literature on central-bank reserve holdings: the ratio of M2 to GDP following Obstfeld, Shambaugh and Taylor (2010); a measure of financial openness following Chinn and Ito (2006); bilateral trade with the U.S. scaled by GDP; GDP per capita; and the log of population. Although these controls add substantially to the explanatory power of the regressions, they leave the coefficient estimates close to those in columns (1)-(4).

Finally, in columns (9)-(12), we add country fixed effects (this entails dropping China from the sample, as we only have one observation for China). Because all of the controls in columns (5)-(8) are nearly time-invariant for each country, we omit them in the fixed-effects regressions; however, this makes no meaningful difference to the results.[18] However, because we are now

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[16] It should be noted that the significant result for the advanced-economy subsample disappears if, in addition to removing Hong Kong, we also remove Switzerland from the sample. However, given the relatively small number of countries in this subsample, it is perhaps not surprising that the two most influential observations carry a lot of the explanatory weight.

[17] Standard errors in these regressions are clustered by country.

[18] Another control that is sometimes seen in the literature is an indicator for whether a country anchors its currency to the dollar. However in our sample, this variable has literally no time variation at all, and so is perfectly subsumed by our country fixed effects.

isolating the time-variation in the data, we add a control for the nominal exchange rate, which turns out to be strongly significant in columns (9)-(11). However, even with this added control, the point estimates on our coefficient of interest are again quite similar and are now significant at the 5% level for the full sample as well as both the advanced and emerging-economy subsamples. In all three of these cases the country fixed effects lead to R-squared values that are now in the neighborhood of 0.90, suggesting—not surprisingly—that the lion's share of the variation in the data is between, rather than within countries.

In sum, for both advanced and emerging economies—though not for developing economies—there appears to be a meaningful correlation in the data between dollar-denominated borrowing by their nonfinancial corporate sectors, and dollar reserve holdings by their central banks. Of course, such correlations by themselves do not allow us to say anything about causation. So our empirical results should at most be interpreted as suggestive patterns, which we hope provide some broad-brush motivation for the model that we turn to next; they are certainly not intended as tight tests of any particular causal theory.

3. Optimal Regulation and Reserve Holdings in a Small Open Economy

We begin with a model of central bank regulatory policy and reserve holdings in a single small open economy that takes the dollar interest rate as given. The model, which builds on that in Gopinath and Stein (2018), has three types of agents: households, banks, and the central bank. Importantly, however, the agents we call "banks" should be interpreted as an aggregation of the intermediary sector and the non-financial firms that these intermediaries lend to. And as noted above, when we refer to currency mismatch in the "banking" sector in the model, the real-world counterpart is predominantly mismatch in the capital structure of non-financial firms.

3.1. Households

There are two dates in the model, given by time 0 and time 1. Households have linear utility over consumption at both dates. These households can save in three types of assets at time 0: home-currency-denominated safe assets, D_h, dollar-denominated safe assets, $D_\$$, and home-currency equity K. The representative household consumes only home goods, and has utility given by:

$$U \equiv C_0 + \beta E[C_1] + \underbrace{\theta_d(D_\$ + D_h)}_{\text{Preference for Safe Assets}} + \underbrace{f(D_\$)}_{\text{Extra Preference for the Dollar}} \quad (1)$$

where $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$. The budget constraints are:

$$C_0 = Z - Q_\$ D_\$ - Q_h D_h - Q_K K \quad (2)$$

$$C_1 = \tilde{e} D_\$ + D_h + \pi - X(R_\$) - \Omega(\tau) \quad (3)$$

where Z is the initial household endowment in units of home goods, $Q_\$$ and Q_h are the prices of dollar and home-currency safe assets at time 0 respectively, and Q_K is the time-0 price of a share that delivers an expected payoff of one at time 1. Note that $Q_\$$, Q_h, and Q_K are the reciprocals of one plus the required returns on dollar safe assets, home-currency safe assets, and home-currency equity, respectively. In addition, π is the time-1 profit of the banking sector (net of payments to depositors), $X(R_\$)$ is the net transfer to foreigners on the central bank's reserve position, and $\Omega(\tau)$ are deadweight costs of taxation. In period 0, the nominal exchange rate is given by 1.

The period-1 exchange rate, denoted by $\tilde{e}$, takes on the values $(1 - z)$ and $(1 + z)$, each with probability ½. Our convention is that a higher value of $\tilde{e}$ represents an appreciation of the dollar relative to the home currency. The exchange rate is assumed to be exogenously determined, perhaps as the outcome of financial flows outside the model interacting with limited arbitrage capacity on the part of foreign-exchange traders, as in Gabaix and Maggiori (2015).

We also take households' extra preference for dollar assets, as represented by $f(D_\$)$, to be exogenous, as our interest is in seeing how banks and the central bank respond to the lower interest rate on dollar assets. Given that households consume only home goods, one might rationalize this assumption by arguing that their demand for dollar assets reflects a belief that these assets are "extra safe" and can be counted to pay off in full even in an (unmodelled) severe-disaster state of the world when countries other than the U.S. are unable to bail out their banking sectors.[19]

[19] This approach differs from Gopinath and Stein (2021), who assume that households that purchase more dollar-invoiced imported goods will have a stronger preference for dollar assets, since these serve as a hedge for them against changes in exchange rates.

The first order conditions of the household utility maximization problem yield:

$$Q_K = \beta, \quad Q_h = \beta + \theta_d, \quad Q_\$ = \beta + \theta_d + f'(D_\$) \tag{4}$$

Home-currency assets, D_h and K, are both manufactured locally, as the liabilities of the domestic banking sector. On the other hand, the demand for dollar-denominated assets, $D_\$$, is satisfied both by imported dollar bonds (e.g. U.S. Treasury bonds), $X_\$$, and by domestically-issued dollar-denominated bank liabilities, $B_\$$. A small open economy takes the price of dollar bonds, $Q_\$ > \beta + \theta_d$, as exogenously given. The quantity of dollar savings by households $D_\$$ is then pinned down by the exogenous global price.

3.2. Banks

There is a continuum of banks, with measure equal to one. At time 0, a bank raises funding from households and provides financing for a fixed quantity of projects given by I. To raise funds for these projects, a bank relies on three types of securities: B_h, $B_\$$ and K. Here, B_h denotes deposits denominated in home currency, $B_\$$ denotes deposits denominated in dollars, and K represents outside equity capital. Thus, the bank's balance sheet at time 0 must satisfy:

$$Q_\$ B_\$ + Q_h B_h + Q_K K = I. \tag{5}$$

With probability q, there is a banking crisis at time 1. In a crisis, the revenues of a fraction p of banks fall to zero, while the remainder stay solvent. Those banks whose revenues fall to zero must be bailed out by the government, meaning that the government has to pay off all depositors in full. For the moment, we assume that the probability of a crisis is independent of the exchange rate. We will revisit this assumption below and allow for some correlation between banking crises and exchange rates.

Both in and out of the crisis state, those banks whose revenues don't fall to zero—i.e., banks that are solvent—have sufficient gross revenues from their projects, which we denote by Y, to pay off all depositors, independent of the realization of the exchange rate. However, if a bank is solvent, but the home currency depreciates, which happens with probability $(1 - pq)/2$, the resulting currency mismatch leads to liquidity-constraint costs for the banks and their customers of $\gamma I \left(\frac{B_\$}{I}\right)^2 = \frac{\gamma B_\$^2}{I}$. Concretely, one can think of a currency-mismatched operating firm as having to cut back on positive-NPV investments when its debt-service costs increase due to a depreciation of the home currency relative to the dollar. The logic behind the specific functional form is that such costs scale linearly with project size I but are convex in the degree of capital-structure mismatch—i.e., the proportion of funding coming from dollar deposits, which is given by $\left(\frac{B_\$}{I}\right)$. Thus, the ex-ante expected costs of mismatch (in time-1 units) are given by $(1 - pq)\gamma B_\$^2/2I$.

Given its fixed scale, the bank's problem is simply to minimize the sum of its expected funding and mismatch costs. Note that the bank only pays such costs when it is solvent, which happens with probability $(1 - pq)$. Thus the bank's problem is given by:

$$\min_{B_\$, B_h, K} (1 - pq)\, E\left[\left\{\tilde{e} B_\$ + B_h + K + \gamma B_\$^2/2I\right\}\right] \tag{6}$$

The only constraint faced by banks, unless additional capital requirements are imposed by the central bank, is the time-0 balance sheet condition in (5). Therefore, we have that, with no regulations in place, banks adopt the following capital structure in an interior optimum:

$$B_\$^* = SI/\gamma, \quad B_h^* = \frac{I - Q_\$ B_\$^*}{Q_h}, \quad K^* = 0 \tag{7}$$

with $S \equiv \left(\frac{Q_\$}{Q_h} - 1\right)$ denoting the interest-rate spread between dollar and home-currency deposits. Here and in what follows, a single-asterisk superscript (*) refers to a choice made by an unconstrained bank. Intuitively, dollar deposits are attractive to a bank to the extent that they have a lower interest rate than domestic deposits, with this spread given by S. On the other hand, too much dollar borrowing increases mismatch, and the associated liquidity-constraint costs when the dollar appreciates, with the magnitude of this cost parameterized by γ. And absent financial regulation, there is no motive in our simple model for the bank to finance itself with the more-expensive equity capital.

3.3. Central Bank

To address the risk of having to bail out the banking sector, the central bank can in principle make use of three policy tools: (i) it can accumulate dollar reserves; (ii) it can regulate the equity capital of the banking sector; and (iii) it can regulate the deposit mix of the banking sectors, i.e., the relative proportions of dollar-denominated and home-currency denominated deposits. We discuss each of these tools in turn.

Dollar Reserve Holdings The central bank purchases dollar-denominated reserves, $R_\$$, by issuing government bonds in domestic currency, G_h, at time 0. The value of dollar reserves is therefore equal to the value of government bond issuance i.e. $Q_\$ R_\$ = Q_h G_h$. On average, the central bank earns an expected negative return (in time-1 units) of $X(R_\$) = SR_\$$ on its reserve holdings. This negative return amounts to a net payment to foreigners (e.g. to the U.S. Treasury or other non-domestic issuers of dollar-denominated securities) and so reduces the time-1 consumption of the household sector. We assume that this expected cost associated with the negative carry on reserves does not involve any distortionary taxation.

However, if there is a banking crisis, the central bank has to bail out depositors either by raising taxes on domestic residents, or by using the net profits (or losses) it earns on its reserve holdings. We assume that in the crisis state, fiscal capacity is limited, and the deadweight costs of any incremental taxation are convex and are given by $\psi\tau^2$, where τ is the tax that is raised.

Putting it together, the central bank chooses $R_\$$ to minimize the sum of reserve carrying costs $X(R_\$)$ and deadweight costs of taxation $\Omega(\tau)$:

$$\min_{R_\$} X(R_\$) + \Omega(\tau) = SR_\$ + \psi q/2[(pB_h + (1+z)pB_\$ - zR_\$)^2 + (pB_h + (1-z)pB_\$ + zR_\$)^2] \quad (8)$$

This leads to the following expression for optimal reserve holdings in an interior solution:

$$R_\$^{**} = pB_\$ - \frac{S}{2qz^2\psi} \tag{9}$$

Here and in what follows, a double-asterisk superscript (**) refers to a choice made by the central bank. The expression in (9) holds true for any value of $B_\$$. If there is no financial regulation, then $B_\$$ is given by the bank's choice in (7), and reserves satisfy $R_\$^{**} = pB_\$^* - \frac{S}{2qz^2\psi} = \frac{pSI}{\gamma} - \frac{S}{2qz^2\psi}$. If there is regulation, $B_\$^*$ may or may not be lower, depending on the form of regulation, as we show below. Either way, the logic behind (9) is intuitive: in the limit where taxation is very expensive (as ψ goes to infinity) the central bank holds sufficient reserves $pB_\$$ to fund a bailout entirely via reserves, without resorting to taxation. As deadweight costs of taxation decline, the central bank relies less on reserves and more on taxation, particularly to the extent that the carrying cost S of reserve holdings is significant.

Capital Requirements If the central bank imposes a capital requirement of K^{**}, this will act as a constraint on the *sum* of home-currency and dollar-denominated borrowing but cannot on its own control them individually. Moreover, we can see from the bank's first-order condition in (7) that in an interior optimum, its choice of dollar borrowing $B_\$^* = SI/\gamma$ is independent of the total amount of deposit funding raised. Therefore, it follows that a capital requirement will not change dollar borrowing and can be thought of as equivalent to the regulator simply picking a reduced value of home currency borrowing B_h. It then further follows that a capital requirement will not change the central bank's desired reserve holdings, since from (9) these are only influenced by dollar borrowing and are unrelated to home-currency deposits.

To solve for the central bank's optimal choice of B_h, we need to write down the planner's problem. To do so, note that we can write:

$$C_0 = Z - I - Q_\$(D_\$ - B_\$) - Q_h(D_h - B_h) \tag{10}$$

$$E[C_1] = Y + (D_\$ - B_\$) + (D_h - B_h) - (1 - pq)\gamma B_\$^2/2I - X(R_\$) - \Omega(\tau) \tag{11}$$

Note that (10) follows from (2), combined with the bank's balance-sheet constraint in (5). And (11) reflects the fact that consumption at time 1 is the sum of: (i) the net profits of the banks (gross revenues *Y*, less the repayment of their borrowings, less the liquidity-constraint costs incurred in the event of local-currency depreciation); (ii) the deposit savings that households have accumulated; minus (iii) the carrying costs of central-bank reserve holdings and the deadweight costs of taxation, which are ultimately borne by households.

So overall, social welfare *W* can be written as (neglecting the exogenous terms *Z*, *I* and *Y*):

$$W = -Q_\$(D_\$ - B_\$) - Q_h(D_h - B_h) + \beta\{(D_\$ - B_\$) + (D_h - B_h)\} + \theta_d(D_\$ + D_h) + f(D_\$) - \beta\{(1-pq)\gamma B_\$^2/2I + \big(X(R_\$) + \Omega(\tau)\big)\} \tag{12}$$

This can be simplified to:

$$W = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) + (f(D_\$) - D_\$ f'(D_\$)) - \beta\{(1-pq)\gamma B_\$^2/2I + X(R_\$) + \Omega(\tau)\} \tag{13}$$

The first three terms in (13) have an intuitive interpretation. The first two are the bank's excess profits from borrowing with dollar and home-currency deposits respectively, rather than by issuing equity. The third is related to the utility created for households from their holdings of dollar assets.

Importantly, this third term is exogenous from the perspective of a small-country planner since households' dollar asset holdings are pinned down by the exogenous $Q_\$$ and are thus invariant to any policies that the planner implements. So, in the small-country case, the planner's problem boils down to maximizing local welfare W_L, given by:

$$W_L = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) - \beta\{(1-pq)\gamma B_\$^2/2I + X(R_\$) + \Omega(\tau)\} \tag{14}$$

A local planner who controls only capital requirements effectively picks B_h to maximize this objective function. In this case, the optimal value of B_h in an interior optimum is given by:

$$B_h^{**} = \frac{(Q_h - \beta)}{2\beta\psi q p^2} - B_\$ \,. \tag{15}$$

If the regulator does not control $B_\$$ directly, it continues to be given by the bank's optimum of $B_\$^* = SI/\gamma$. From adding up, this implies that the capital requirement is given by $K^{**} = I - Q_\$ B_\$^* - Q_h B_h^{**}$. And from (9), this implies that reserves are unchanged from the unregulated case and are again given by: $R_\$^{**} = pB_\$^* - \frac{S}{2qz^2\psi} = \frac{pSI}{\gamma} - \frac{S}{2qz^2\psi}$.

Funding Regulation Finally, we consider the case where a planner can also control a bank's funding mix—its proportions of dollar and local-currency deposits—in addition to its capital ratio. We do so for completeness within the logic of the model, while mindful of the fact that this case almost surely overstates the scope of financial regulation in the real world. As we have emphasized, the empirical reality is that the lion's share of currency mismatch occurs on the balance sheets of non-financial firms, where traditional regulatory tools do not reach.[20]

With that caveat in place, this case is equivalent to the planner picking both $B_\$$ and B_h to maximize W_L as given in (14). The first order condition for $B_\$$ in an interior optimum is:

$$B_\$^{**} = \frac{(Q_\$ - \beta) - 2\psi q\beta p^2 B_h^{**} + 2\psi q\beta p z^2 R_\$^{**}}{\left(\frac{\beta(1-pq)\gamma}{I}\right) + 2\psi q\beta p^2(1+z^2)} \tag{16}$$

The first-order conditions for B_h and $R_\$$ continue to be given by equations (15) and (9), respectively. These three equations (i.e., (16), (15), and (9)) can then be solved jointly to yield expressions for the three policy variables as functions of the primitive parameters.

[20] As Acharya, Cecchetti, De Gregorio, Kalemli-Ozcan, Lane, and Panizza (2015) put it: "Policymakers have a challenging task controlling these risks [arising from mismatch on the part of non-financial firms] directly, as it is difficult to intervene to reduce the external foreign-currency borrowing by what are generally unregulated institutions. The concern is less about the direct impact of these firms on the real economy – something that can be managed through traditional stabilization policy – than the impact that non-financial corporate balance sheet stress has on banks and the financial system."

3.4. Banking and Currency Crises are Correlated

Thus far, we have been assuming that the probability of a banking crisis is independent of the exchange rate. This is likely to be too simplistic, as banking crises often coincide with large depreciations of the local currency (Kaminsky and Reinhart 1998). We can easily extend our framework to capture such a correlation. To do so, assume that there is an increased probability $(q + h)$ of a banking crisis when the exchange rate is $(1 + z)$, i.e., when the local currency depreciates against the dollar. And symmetrically, there is a reduced probability $(q - h)$ of a banking crisis when the exchange rate is $(1 - z)$. All else is the same as before. Here the parameter h is a measure of the strength of the correlation between exchange rates and banking crises; note that this setup nests our previous no-correlation case if $h = 0$. With these assumptions in place, we can re-derive our various results. First, we have that an unregulated bank now sets:

$$B_{\$}^{*} = \frac{I((1-qp)S+hpz)}{(1-p(q+h))\gamma} \tag{17}$$

Relative to the previous solution given in equation (7), the primary change is the addition of the hpz term in the numerator of (17). This is a moral hazard effect—since the bank is more likely to default when the dollar has appreciated, it effectively has a call option on the dollar in the crisis state. So, dollar borrowing is increased in this version of the model. A second mechanical effect of the reformulation is that there are fewer states of the world with no crisis and a stronger dollar, so expected mismatch costs are not as important, which also increases dollar borrowing.

The central bank now sets:

$$R_{\$}^{**} = \frac{ph(B_{\$}+B_{h})}{qz} + pB_{\$} - \frac{S}{2qz^{2}\psi} \tag{18}$$

As compared to the no-correlation case in equation (9), central bank reserves are potentially much higher, by an amount $\frac{ph(B_{\$}+B_{h})}{qz}$, and are now influenced by both dollar and home-currency bank deposits, though the effect of the former is still stronger. This is because of a new hedging effect. Now, when there is a banking crisis, we know the dollar is more likely to have strengthened than to have weakened. So, holding dollar reserves is a good way to hedge the possibility of having to bail out both dollar and home-currency deposits. In the zero-correlation case, there was no motive to hedge home-currency deposits with dollar reserves, because the central bank was equally likely to have to bail out these home-currency deposits if the dollar strengthened or weakened.

One implication of this observation is that in the case with correlation between banking crises and exchange rates, any kind of financial regulation that reduces bank deposits of either type will be associated with a decline in reserve holdings. Importantly, this was not true in the zero-correlation case, where capital regulation alone had no impact on reserve holdings.

The local planner's objective function is now given by:

$$W_L = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) - \beta\{\left(1 - p(q+h)\right)\gamma B_\$^2/2I + X(R_\$) + \Omega(\tau)\} \quad (19)$$

where deadweight costs of taxation can now be written as:

$$\Omega(\tau) = \frac{\psi}{2}\left((q+h)(pB_h + (1+z)pB_\$ - zR_\$)^2 + (q-h)(pB_h + (1-z)pB_\$ + zR_\$)^2\right) \quad (20)$$

If the central bank sets just capital requirements, i.e., it just controls B_h, it sets:

$$B_h^{**} = \frac{(Q_h - \beta)}{2\beta\psi q p^2} - \left(1 + \frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$\,, \quad (21)$$

with $B_\$$ and $R_\$$ given by equations (17) and (18) respectively.

If in addition the central bank controls the funding mix, i.e., it also chooses $B_\$$, we have:

$$B_\$^{**} = \frac{(Q_\$ - \beta) - 2(q+zh)\psi\beta p^2 B_h + 2z\left(\frac{h}{q} + z\right)\psi q p \beta R_\$}{\left(\frac{\beta(1-p(q+h))\gamma}{I}\right) + \psi\beta p^2((q+h)(1+z)^2 + (q-h)(1-z)^2)}\,, \quad (22)$$

and in this case the full solution is given by equations (18), (21) and (22).

Numerical Example: Set the parameter values as follows: $I = 100$; $\beta = 0.85$; $\theta_d = 0.1$; $p = 0.5$; $z = 0.75$; $q = 0.05$; $h = 0.025$; $\gamma = 0.06$; $\psi = 0.06$; and $Q_\$ = 0.975$. In this case, the solutions to the model are given in Table 3 below.

Table 3: Numerical Example

	No Regulation	Capital Regulation Only	Capital and Funding Regulation
$B_\$$	59.993	59.993	27.721
B_h	43.716	36.903	69.175
K	0	7.614	8.549
$R_\$$	56.886	54.615	38.479

The parameters in the example are such that absent any regulation (column 1 of the table), banks finance themselves with somewhat more dollar-denominated deposits than local-currency denominated deposits. In this case, the only policy tool the central bank has available is to accumulate dollar reserves, which take on a value of 56.9, relative to private-sector investment (as denoted by the parameter I) of 100. In column 2, we allow the central bank to impose capital regulation, and it sets a capital requirement of approximately 7.6%. With this capital requirement in place—and given that we have assumed a modest correlation between banking crises and exchange rates—dollar reserve holdings fall to 54.6, even though dollar deposits are unchanged, so that the capital requirement only crowds out local-currency deposits. Finally, in column 3, we further allow the central bank to control the volume of dollar deposits directly. When given this power, it keeps the capital requirement roughly the same as in column 2, but significantly cuts back on dollar deposits relative to local-currency deposits. This in turn allows it to further economize on dollar reserve holdings, which decline to 38.5.

To summarize the analysis to this point: we have developed a relatively bare-bones model of how a small open-economy central bank can attempt to mitigate the costs of banking crises that are associated with currency mismatch on the part of the private sector. The central bank can do so either by accumulating dollar reserves, or by imposing various forms of financial regulation.

There is an intuitive tradeoff between these tools: when we give the central bank more scope to deploy regulatory measures, it holds less in the way of reserves.

However, all of this is in a partial equilibrium setting where the small-country central bank takes the interest rate on dollar-denominated assets to be exogenous, and the supply of these assets to be perfectly elastic. We next turn to the question of global externalities, asking whether a planner who internalizes the general-equilibrium effects would strike the balance between regulation and reserve accumulation differently.

4. Global Externalities from Reserve Accumulation

4.1. Basic Setup

We now assume that the global economy consists of a unit measure of identical small countries indexed by $i \in [0,1]$, as well as the United States. We continue to allow for a correlation between banking crises and exchange rates, as in the latter part of the previous section. However, to highlight most starkly the externality of interest, we further assume that: (i) all countries draw the same exchange rate, $\tilde{e}$, against the dollar; and (ii) the occurrence of banking crises is perfectly correlated across countries. These assumptions have the effect of making all risks non-diversifiable, so absent a pecuniary externality with respect to the dollar interest rate, there would be no reason for a global planner to choose a different level of reserve holdings than a local planner. Clearly, if risks were imperfectly correlated across countries, there would be an additional risk-sharing motive for economizing on reserve holdings, but we neutralize this motive for the time being and return to it in the next section.

Finally, in the interest of keeping the analysis concise, and to focus on what we believe to be the more empirically relevant case, we study below only the scenario where regulators are able to impose capital requirements, but not funding-mix requirements.[21] We focus on symmetric outcomes, so now when we refer to any given endogenous variable (e.g., $B_{\$}$), this should be

[21] It turns out that the case where a global planner can control the funding mix leads to a counter-intuitive effect. Because the global planner likes to see a greater supply of dollar-denominated safe assets, all else equal, funding-mix regulation in this case takes the form of the global planner forcing banks to issue *more* dollar deposits than they would themselves choose. This result further leads us to question the empirical relevance of this case.

interpreted as representing the common value of this variable across countries. We can then aggregate global welfare among the mass of small countries as:

$$W_G = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) + \big(f(D_\$) - D_\$ f'(D_\$)\big) - \beta\{\big(1 - p(q+h)\big)\gamma B_\$^2/2I + X(R_\$) + \Omega(\tau)\} \quad (23)$$

In what follows, we set aside the welfare of the U.S. and analyze the global planner's efforts to maximize W_G. The global market clearing conditions are given by:

$$B_\$ + X_\$ = R_\$ + D_\$ \quad (24)$$

$$D_{hi} = B_{hi} + G_{hi}, \quad \forall i \in [0,1] \quad (25)$$

where $X_\$$ represents an exogenous outside supply of safe dollar-denominated assets, such U.S. Treasury bonds. Equation (24) says that the total supply of dollar assets—which comes from either such external sources, or from dollar-denominated deposits in non-U.S. banks—must equal the demand for such assets, which comes from both households and central-bank reserve managers. Equation (25) is an analogous market-clearing condition for local-currency safe assets, stating that household demand for local-currency safe assets can be satisfied either by local banks or by the issuance of local-currency government bonds. Note that this second market-clearing condition has to hold country-by-country, as opposed to globally.

In what follows, we specialize households' utility function from dollar assets, so that it is quadratic in nature. This will allow us to continue writing down all first-order conditions in closed form. In particular, we assume that:

$$f(D_\$) = \theta_{\$1} D_\$ - \frac{1}{2}\theta_{\$2} D_\$^2 \quad (26)$$

Under this specification, we can express the price of safe dollar assets as:

$$Q_\$ = \beta + \theta_d + \theta_{\$1} - \theta_{\$2} D_\$, \quad (27)$$

where we assume that $\theta_{\$1}/\theta_{\$2}$ is large enough such that the dollar interest rate is always lower than the domestic-currency interest rate in equilibrium. It follows that the spread S is given by:

$$S = \frac{\theta_{\$1} - \theta_{\$2}(X_\$ + B_\$ - R_\$)}{\beta + \theta_d} \tag{28}$$

4.2. Global Equilibrium When Reserves and Capital Are Chosen Locally

We begin by solving for the global equilibrium—now with an endogenous value of the interest-rate spread S—that arises when each country sets reserve holdings and capital requirements locally, ignoring their impact on the aggregate supply of dollar claims and hence on S. In Appendix B, we show that in this case, one can solve out the model in terms of primitive parameters to obtain the following expressions for $B_\$^*$ and S^{**}:

$$B_\$^* = a_1 S^{**} + a_2 \tag{29}$$

$$R_\$^{**} = b_1 S^{**} + b_2 \tag{30}$$

$$S^{**} = \frac{\theta_{\$1} - \theta_{\$2}(X_\$ + a_2 - b_2)}{\beta + \theta_d + \theta_{\$2}(a_1 - b_1)} \tag{31}$$

where $a_1 = \frac{I(1-qp)}{\gamma(1-p(q+h))}$, $a_2 = \frac{hpzI}{\gamma(1-p(q+h))}$, $b_1 = \frac{I(1-qp)p}{\gamma(1-p(q+h))} - \frac{q}{2\psi z^2(q^2-h^2)}$, and $b_2 = \frac{hp^2 zI}{\gamma(1-p(q+h))} + \frac{h(Q_h - \beta)}{2\beta\psi pz(q^2-h^2)}$, and with S^{**} denoting the equilibrium interest-rate spread that arises under the decentralized equilibrium. And once we have pinned down $B_\$^*$ and $R_\$^{**}$ the equilibrium value of B_h^{**} follows from equation (21).

4.2. Equilibrium With a Global Planner

We now turn to the case where a global planner sets both reserve holdings and capital requirements. The crucial difference in this case is that a global planner recognizes that the choice of $R_\$$ impacts the dollar interest rate, and hence the interest rate spread S, as can be seen in equation (28). To see this explicitly, we can take the global planner's first order condition for $R_\$$, which is given by:

$$\frac{\mathrm{d}W_G}{\mathrm{d}R_\$} = \frac{\mathrm{d}}{\mathrm{dR_\$}}\big(\mathrm{B_\$}(\mathrm{Q_\$} - \beta)\big) - \beta\frac{\mathrm{d}}{\mathrm{dR_\$}}\Big(\big(1 - p(q+h)\big)\gamma B_\$^2/2I\Big) + \frac{\mathrm{d}}{\mathrm{dR_\$}}\big(f(D_\$) - f'(D_\$)\,\big) - \beta\frac{\mathrm{d}}{\mathrm{dR_\$}}X(R_\$) - \beta\frac{\mathrm{d}}{\mathrm{dR_\$}}\Omega(\tau) = 0, \tag{32}$$

where the five individual components of (32) can be expressed as:

$$\frac{\mathrm{d}}{\mathrm{dR_\$}}\big(\mathrm{B_\$}(\mathrm{Q_\$} - \beta)\big) = \phi(\mathrm{Q_\$} - \beta) + B_\$\big(\theta_{\$2}(1 - \phi)\big) \tag{33}$$

$$\frac{\mathrm{d}}{\mathrm{dR_\$}}\Big(\big(1 - p(q+h)\big)\gamma B_\$^2/2I\Big) = \phi\big(1 - p(q+h)\big)\gamma B_\$/I \tag{34}$$

$$\frac{\mathrm{d}}{\mathrm{dR_\$}}\Big(f(D_\$) - D_\$ f'(D_\$)\,\Big) = -(1 - \phi)\theta_{\$2}(B_\$ + X_\$ - R_\$) \tag{35}$$

$$\frac{\mathrm{d}}{\mathrm{dR_\$}}X(R_\$) = S + R_\$(\theta_{\$2}(1 - \phi)/\mathrm{Q_h}) \tag{36}$$

$$\frac{\mathrm{d}}{\mathrm{dR_\$}}\Omega(\tau) = \big(2\psi\phi \mathrm{q}\mathrm{p}^2 - 2\psi zhp(1 - p\phi)\big)(\mathrm{B_h} + \mathrm{B_\$}) + \big(2\psi\phi zp\mathrm{h} - 2\psi q z^2(1 - p\phi)\big)(\mathrm{pB_\$} - \mathrm{R_\$}), \tag{37}$$

and where:

$$\phi \equiv \frac{dB_\$}{dR_\$} = \left(\frac{\theta_{\$2}\mathrm{I}(1-\mathrm{qp})}{\gamma}\right) / \left((1 - \mathrm{p}(\mathrm{q} + \mathrm{h}))(\beta + \theta_{\mathrm{d}}) + \frac{\theta_{\$2}\mathrm{I}(1-\mathrm{qp})}{\gamma}\right) \tag{38}$$

To begin to understand the intuition for why the global planner's solution differs from that with a local planner, note that the two coincide when $\theta_{\$2} = 0$, i.e., when the dollar interest rate is insensitive to the supply of dollar assets and hence to central-bank reserve-holding decisions. Hence, we can see where the wedges between the two solutions occur by looking at the terms that are influenced by $\theta_{\$2}$. The first three terms in (32), given by (33)-(35), drop out completely when $\theta_{\$2} = 0$, so they are each a source of difference between the global and local-planner outcomes.

The first term, in (33), captures the effect of reserve holdings on bank profits. This term actually pushes in the direction of the global planner wanting to hold *more* reserves than the local planner, because the global planner internalizes the fact that this lowers dollar interest rates and thereby increases the profitability of the banking sector. All the other terms, however, go in the opposite direction. The second term, in (34), says that the global planner would like to hold fewer reserves than the local planner, because increasing the dollar interest rate leads banks to engage in less mismatch, and hence reduces the liquidity costs associated with mismatch. The third term, in (35), again tends to reduce the global planner's desired holdings of reserves, in this case because a higher level of central-bank reserves takes away from the dollar claims available to households, and hence reduces household utility.

Finally, the last two terms in (32), given by (36) and (37), also differ from their analogs in the local-planner scenario, and once again tilt the global planner away from holding reserves. In the former case, this is because fewer reserves means a higher dollar interest rate, which reduces the carrying cost associated with reserve holdings; this is the global planner behaving like a large buyer who internalizes the impact of their own demand on the market price. In the latter case, the logic is that the deadweight costs of taxation are lower when a higher dollar interest rate results in a lesser degree of currency mismatch in the banking sector.

As far as capital regulation goes, recall that capital regulation is equivalent to picking a value of domestic-currency deposits B_h. It is straightforward to show that the global planner's first-order condition for this variable is identical to that of an individual central bank as given in (21). The intuition is simple: domestic-currency deposits do not affect the externalities we are focused on, since these externalities involve only the quantity and price of dollar-denominated assets. So, holding all else fixed, there is no motive for a global planner's behavior to diverge from that of a local planner. Similarly, given that we are studying the case where there is only capital regulation, and no funding-mix regulation, the value of dollar-denominated deposits $B_\$$ continues to be chosen by the banks themselves, and so the relevant first-order condition is again given by (17). Of course, while the partial-equilibrium first-order conditions are the same, the ultimate general-equilibrium values of B_h and $B_\$$ will differ with a global planner, because they depend on the planner's choice of $R_\$$ and the resulting value of *S*.

Putting it all together, the solutions for the global-planner case, which we denote with triple-asterisk superscripts (***), are obtained by combining (17), (21), and (32), along with the formula for the spread *S* given in (28) and expressing everything in terms of primitive parameters. In Appendix B, we derive:

$$B_\$^{*} = a_1 S^{***} + a_2 \tag{39}$$

$$R_\$^{***} = b_3 S^{***} + b_4 \tag{40}$$

$$S^{***} = \frac{\theta_{\$1} - \theta_{\$2}(X_\$ + a_2 - b_4)}{\beta + \theta_d + \theta_{\$2}(a_1 - b_3)} \tag{41}$$

where a_1 and a_2 are the same as in equation (30), and b_3 and b_4 are given by $b_3 = \frac{(2\beta\psi q z^2 p(1-p\phi) - 2\beta\psi z^2 p h^2(1-p\phi)/q - \beta\phi(1-p(q+h))\gamma/I)\left(\frac{I(1-qp)}{(1-p(q+h))\gamma}\right) + \phi Q_h - \beta}{(2\beta\psi q z^2(1-p\phi) + (1-\phi)\theta_{\$2}(Q_K/Q_h - 1) - 2\beta h^2\psi z^2(1-p\phi)/q)}$, and by $b_4 = \frac{-\theta_{\$2}(1-\phi)X_\$ + hz(Q_h - \beta)(1/p - \phi)/q + hpzI\,(2\beta\psi q z^2 p(1-p\phi) - 2\beta\psi z^2 p h^2(1-p\phi)/q - \beta\phi(1-p(q+h))\gamma/I)/(\gamma(1-p(q+h)))}{(2\beta\psi q z^2(1-p\phi) + (1-\phi)\theta_{\$2}(Q_K/Q_h - 1) - 2\beta h^2\psi z^2(1-p\phi)/q)}$.

Based on this solution, Appendix B establishes the following proposition:

Proposition 1: There is a threshold value $\bar{\beta}$ such that $R_\$^{***} < R_\$^{**}$ holds for all $\beta \geq \bar{\beta}$.

Simply put, the proposition states that the global planner's solution involves lower reserve holdings than the decentralized solution as long as β is not too low, or equivalently, as long as the cost of equity capital is not too high. The intuition is straightforward. As we have noted, there is only one "wrong-way" effect pushing the global planner to hold more reserves, while there are four effects going in the other direction. Moreover, this wrong-way effect comes from the impact of reserve holdings on bank profitability, and as can be seen from (33), is closely related to $(Q_\$ - \beta)$, the spread between the bank's cost of equity capital and the rate it pays on dollar deposits.

In order for the wrong-way effect to overwhelm all the others, this spread—and thus its impact on bank profitability—needs to be large. Assuming a zero lower bound on all interest rates, which bounds $Q_\$$ from above at one, this can only happen if β is sufficiently low, i.e. if equity capital is very expensive.

Just how small is the threshold value $\bar{\beta}$? In Appendix B, we show that:

$$\bar{\beta} < \tilde{\beta} = \frac{\theta_d\left(R_\$^g[S^{**}] - X_\$\right)\gamma/I + \theta_d S^{**}}{hpz + \gamma X_\$/I}, \tag{42}$$

where S^{**} denotes the equilibrium interest-rate spread when local regulators choose $R_\$$, and where $R_\$^g[S^{**}]$ is the level of reserves a global planner would choose at that spread. As long as we impose the natural condition that reserve holdings in any candidate equilibrium cannot exceed the supply of Treasury securities and other safe dollar bonds, the first term in the numerator of (42) is negative, which taken alone would mean that there is *no positive value of* β for which the wrong-way bank-profit effect dominates. More generally, we have done extensive numerical experimentation with the model, and in no case have we been able to find an interior solution where $\bar{\beta}$ exceeds 0.5. In other words, the only way we have been able to reverse the conclusion that the global solution involves lower reserve holdings than the decentralized solution is by assuming an absurdly high cost of equity capital, in excess of 100%. Thus we conclude that the wrong-way effect associated with bank profitability is a second-order consideration in our framework, and that the other effects we have isolated dominate the overall outcome.

Numerical Example (continued): Set the parameter values as follows: $I = 100$; $\beta = 0.85$; $\theta_d = 0.1$; $p = 0.5$; $z = 0.75$; $q = 0.05$; $h = 0.025$; $\gamma = 0.06$; and $\psi = 0.06$. Unlike in the partial-equilibrium case, $Q_\$$ is no longer exogenous. Rather, we have to specify three further parameters that now serve to pin it down. To do so, we set: $\theta_{\$1} = 0.1$; $\theta_{\$2} = 0.001$; and $X_\$ = 70$. These values are chosen so that, in the case where local central banks choose reserve holdings and capital requirements, $Q_\$$ endogenously turns out to be 0.975, just as it was in the partial-equilibrium version of the example.

Table 4 summarizes this example, showing how all the endogenous variables are affected as we consider different policy regimes. Column 1 displays the outcomes for a completely unregulated economy, in which there are no reserve holdings or capital requirements. Column 2 examines the case where policies are set by local central banks, which control both reserve holdings and capital requirements (note that column 2 of Table 4 is identical to column 2 of Table 3 from the partial-equilibrium case). And column 3 asks what happens when instead the global planner chooses reserve holdings.

Table 4: Numerical Example

	No Regulation or Reserve Holdings	Local Planners Set Reserves and Capital	Global Planner Sets Reserves
$Q_\$$	0.955	0.975	0.965
S	0.005	0.026	0.016
$B_\$$	25.043	59.993	42.975
B_h	80.089	36.903	40.361
K	0	7.614	23.767
$R_\$$	0	54.615	27.994

Perhaps the most policy-relevant comparison in the table is between columns 2 and 3 of Table 4, which contrasts local central-bank determination of reserves and capital requirements with the globally coordinated solution. One can see that in the latter case, reserve holdings decline sharply, from 54.62 to 27.99. At the same time, the capital requirement becomes much stricter, with K rising from 7.61 to 23.77. This comparison highlights our central point: a global planner prefers tougher capital regulation and less reserve accumulation than does a local central bank.[22]

[22] Interestingly, in our model, once the global planner has set reserve holdings, the choice of the capital requirement can be decentralized back to the local central banks. Of course, in a richer and more realistic setting, there are good reasons why international coordination in setting capital requirements may have additional value. See, e.g., Clayton and Schaab (2022).

And as a result of the reduced reserve holdings, the interest rate spread S is lower—i.e. the dollar interest rate is higher—in the global-planner regime. The magnitude of this effect is substantial in our example, on the order of one percentage point. One important consequence of this drop in S is that even though regulation cannot control banks' dollar borrowing $B_{\$}$ directly, dollar borrowing is nevertheless meaningfully reduced, from 59.99 to 42.98, in the global-planner case. This is because the incentive for banks to borrow in dollars declines when the dollar interest rate goes up.

Table 5 presents a detailed welfare decomposition, showing how each component of aggregate social welfare varies across the policy regimes. The values are normalized so that total welfare in the case without regulation or reserve holdings is equal to 100.

Table 5: Welfare Decomposition

	No Regulation or Reserve Holdings	Local Planners Set Reserves and Capital	Global Planner Sets Reserves
Total Welfare	100	113.417	120.350
Bank Profits	164.458	172.642	138.799
Household Dollar Deposit Utility	69.828	43.921	55.802
Carry Cost of Reserves	0	-18.602	-5.823
Deadweight Cost of Taxation	-131.876	-70.710	-61.335
Liquidity Cost	-2.411	-13.834	-7.093

Comparing columns 2 and 3 of the table, we can see how the five different effects summarized in equations (33)-(37) play out as we move from the local-planner to the global-planner solution. Bank profits decline significantly, but this is more than offset by the cumulative impact of an increase in household utility from holding dollar deposits, and reductions in the carrying cost of reserves, the deadweight costs of taxation and the liquidity costs associated with bank mismatch.

5. Further Implications: Global Risk Sharing

In the previous section, we assumed that banking crises were perfectly correlated across countries. This assumption helps to most cleanly isolate the externalities in reserve accumulation that are our primary interest. However, one can also ask how things change if crises are imperfectly correlated, so that there is scope for global risk-sharing in reserve holdings. Crucially, however, such risk-sharing is only possible if countries can agree to a mechanism that allows them to re-distribute reserves ex post to those who are experiencing a crisis. For example, a supra-national institution like the IMF might hold the reserves, and then allocate them to countries on an as-needed basis. This approach clearly raises a set of challenging moral hazard and monitoring issues that don't arise when simply capping the reserve holdings of individual central banks. Will countries now take the proper ex ante precautions to avert crises? Will it be clear ex post how severe a given crisis is, and thus how large is the required draw on the common pool of reserves?

For the moment, we set aside these important considerations, and just assume that there is a frictionless mechanism to implement the ex-post allocation of reserves. To see the forces at play in the simplest possible way, we revert back to the more tractable case where $h = 0$, so that there is no correlation between exchange rates and banking crises. We also set the spread S to be a fixed constant, which is tantamount to saying that the demand for dollar safe assets is linear, i.e. that $\theta_{\$2} = 0$. However, we now assume that, instead of there being a probability q that all countries experience a banking crisis simultaneously at time 1, it is a certainty that a fractional mass q of countries will experience a crisis; this is equivalent to thinking of crises as completely independent and uncorrelated occurrences in our continuum of countries.

In the case of $h = 0$, with correlated banking crises, the global planner's objective function was given by (13), which we reproduce here for convenience:

$$W_G = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) + (f(D_\$) - D_\$ f'(D_\$)) - \beta\{(1 - pq)\gamma B_\$^2/2I + X(R_\$) + \Omega_c(\tau)\} \quad (43)$$

where the deadweight costs of taxation $\Omega_c(\tau)$ (with the subscript "c" denoting the correlated-crises case) could be written as:

$$\Omega_c(\tau) = \psi q/2[(pB_h + (1+z)pB_{\$} - zR_{\$})^2 + (pB_h + (1-z)pB_{\$} + zR_{\$})^2] \tag{44}$$

By contrast, when banking crises are uncorrelated, the only modification to the global planner's objective function is in this cost-of-taxation term, which we denote by $\Omega_u(\tau)$ and which now takes the form:

$$\Omega_u(\tau) = \psi q/2[\left(pB_h + (1+z)pB_{\$} - \frac{z}{q}R_{\$}\right)^2 + \left(pB_h + (1-z)pB_{\$} + \frac{z}{q}R_{\$}\right)^2] \tag{45}$$

The only change from (44) to (45) is that the terms involving $R_{\$}$ are multiplied by $\frac{z}{q}$, rather than by z. This reflects the fact that an individual country in crisis now has access to a $1/q > 1$ share of the total pool of reserves, rather than just a pro-rata share. Or said differently, with uncorrelated crises, a dollar of reserves held by the supra-national institution goes further than a dollar of reserves held at the individual-country level, because it can be reallocated to those countries who need it.

With $h = 0$ and S fixed, we have already solved for the optimal level of reserve holdings in the correlated-crises case; it is given by equation (9), reproduced below:

$$R_{\$c}^{***} = pB_{\$} - \frac{S}{2qz^2\psi} \tag{46}$$

If we recompute the first-order condition for optimal reserves using the new expression in (45) for the deadweight costs of taxation in the uncorrelated-crises case, we get:

$$R_{\$u}^{***} = pqB_{\$} - \frac{Sq}{2z^2\psi} \tag{47}$$

Comparing equations (46) and (47), we can see that there are two competing effects. On the one hand, the first term in (47) is reduced by a factor of q relative to that in (46). This cuts in the direction of reserves being lower in the uncorrelated-crises case. The intuition here is that it only takes reserves of $pqB_{\$}$, as opposed to $pB_{\$}$, to fully cover all possible needs, on account of the risk-sharing effect.

On the other hand, the second term in (47) is also reduced relative to its counterpart in (46), this time by a factor of q^2. This cuts in the other direction. The idea here is that when one spends an amount S to add a unit of reserves, it now buys more effective coverage than before, since the reserves can be deployed more efficiently.

Putting it together, it is apparent that for relatively small values of S, the first effect will dominate, and the ability of countries to share risk will lead to a lower equilibrium value of reserve holdings when crises are uncorrelated. However, it is possible for this conclusion to be reversed if S is sufficiently high. To see why, set S high enough so that reserves $R_{\$c}^{***}$ are exactly equal to zero in the correlated-crises case, i.e., so that $pB_{\$} = \frac{S}{2qz^2\psi}$. From (47), we can see that reserves in the uncorrelated-crises case $R_{\$u}^{***}$ will still be positive and given by $R_{\$u}^{***} = pqB_{\$}(1-q)$.

To the extent that the small-S configuration is the more empirically relevant one, this analysis would further underscore the general message of the paper, namely that there may be considerable efficiencies to be obtained from international coordination in the management of dollar reserves. We now have seen two distinct mechanisms which can push in this direction: the first being the internalization of the impact of reserve accumulation on the overall scarcity of dollar assets, and the second being a risk-sharing motive that arises when banking crises are imperfectly correlated across countries. Again, however, it should be underscored that taking advantage of the latter risk-sharing benefit is likely to entail significantly greater institutional challenges.[23]

[23] One way to see this point is to note that, taken literally, our risk-sharing solution with uncorrelated banking crises requires that *all* dollar reserves be housed in a supra-national institution like the IMF. Or alternatively, it requires giving such an institution the right to take reserves away from countries that are not currently experiencing a crisis and to re-distribute them to those who are. More generally, in an in-between world where banking crises are neither perfectly correlated across countries nor perfectly independent, a hybrid solution—with some reserves held by local central banks, and some held by a supra-national institution—may be more appropriate. Nevertheless, even in such a case, similar governance challenges will arise for the supra-national institution.

6. Conclusions

Central banks around the world hold large volumes of dollar-denominated reserves. Our empirical work suggests that one important motive for these reserve holdings is a concern on the part of central banks with currency mismatch in the composition of private-sector liabilities in their countries, with many firms financing themselves heavily with relatively cheap dollar borrowing. Ironically, however, the collective reserve-accumulation decisions of individual price-taking central banks can exacerbate this mismatch problem, because they drive down dollar interest rates and thereby further increase the incentive for the private sector to over-borrow in dollars.

Given this externality, we have shown that a global regulator would prefer to see individual central banks holding fewer dollar reserves, and instead using their existing regulatory tools—such as bank capital requirements—more aggressively in an effort to shore up financial stability. However, unlike with capital regulation, where the importance of international cooperation in standard-setting is well-understood, and is enshrined in the Basel process, the potential benefits of coordinating reserve-holding behavior across countries are less fully appreciated. This paper can be thought of as an initial attempt to highlight these benefits, and perhaps to help kick-start a conversation over what such a coordination process might look like.

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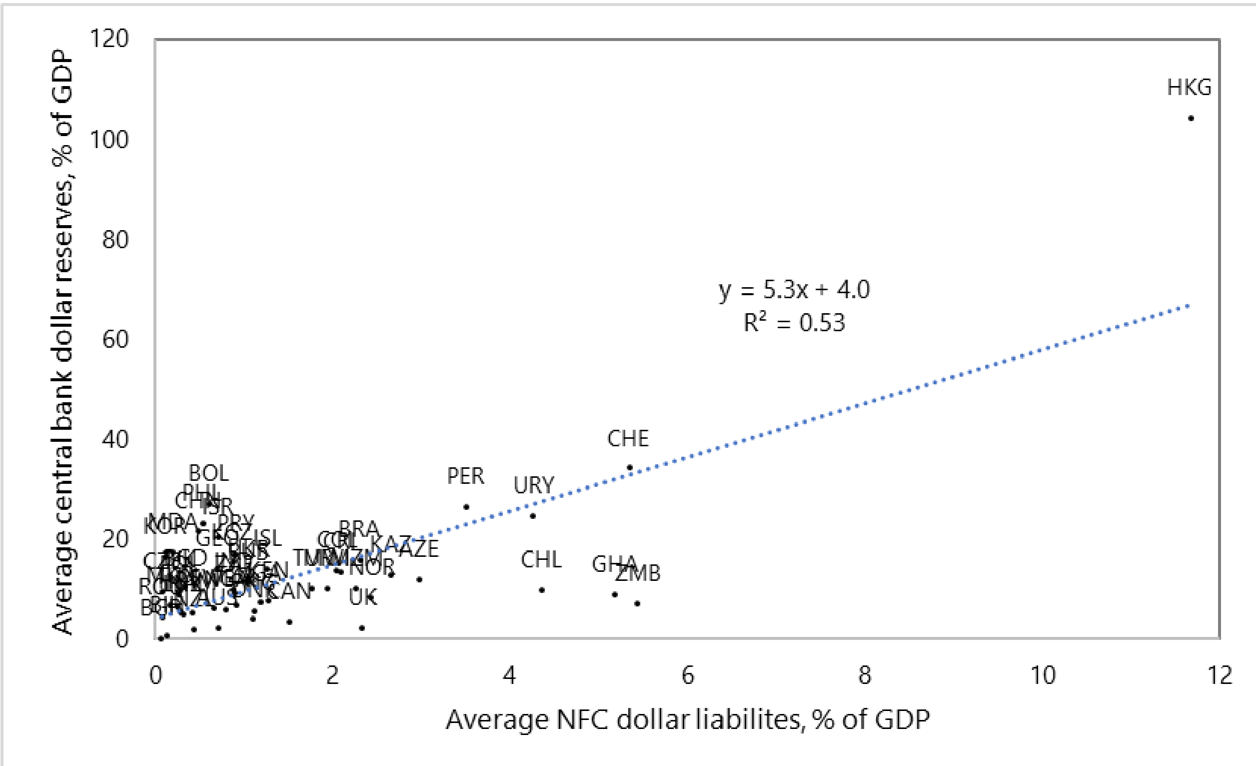
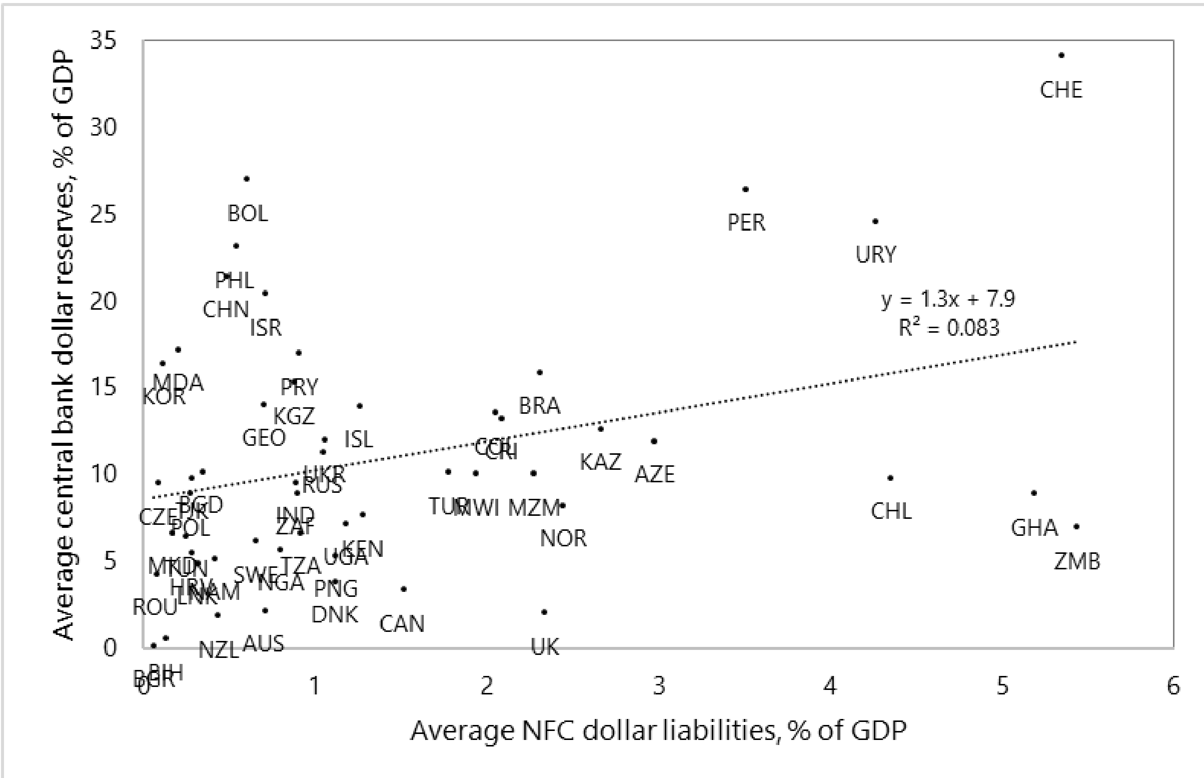
Figure 1
Nonfinancial company dollar loans and central bank dollar reserves: country averages (2013-20, 53 countries)



The horizontal axis in Fig. 1 shows average NFC dollar loans from cross-border banks, scaled by GDP. The vertical axis shows central bank dollar reserves, also scaled by GDP. Average loans and average reserves are calculated over different years across different countries, but the same years within a country (ranging from 1-8 years).

Sources: BIS, Data.imf.org, IMF (2020), Chinn, Ito and Macauley (2021).

Figure 2
Nonfinancial company dollar loans and central bank dollar reserves: country averages excluding Hong Kong (2013-20, 52 countries)



The horizontal axis in Fig. 1 shows average NFC dollar loans from cross-border banks, scaled by GDP. The vertical axis shows central bank dollar reserves, also scaled by GDP. Average loans and average reserves are calculated over different years across different countries, but the same years within a country (ranging from 1-8 years). Relative to Figure 1, this figure drops Hong Kong SAR (HKG).

Sources: BIS, Data.imf.org, IMF (2020), Chinn, Ito and Macauley (2021).

Figure 3
Nonfinancial company dollar loans and central bank dollar reserves: disaggregation across advanced, emerging and developing countries

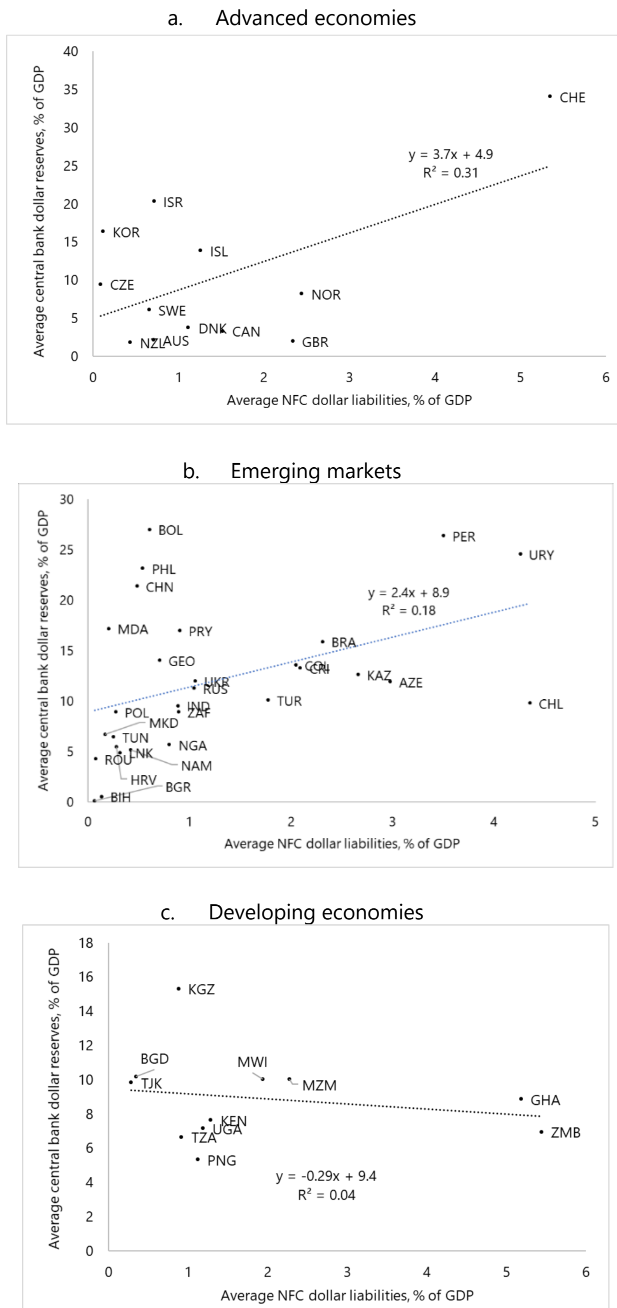


Figure 3 shows the same data as in Figure 2, disaggregated into advanced, emerging and developing economies. The identities of the countries in each group are given in Appendix A Table A2.

Sources: BIS, Data.imf.org, IMF (2020), Chinn, Ito and Macauley (2021).

Table 1
Summary statistics: Central bank dollar reserves and nonfinancial company dollar loans, % of GDP

		N	Countries	Mean	Median	Std Dev	Min	Max
Foreign reserves denominated in USD	Total	357	52	10.5	8.7	7.6	0	49.2
	AE	93	12	10.1	6.5	9.9	0	49.2
	EM	184	29	11.4	10.7	7.4	0.01	36.9
	DE	80	11	7.9	8.8	3.6	1.5	19.3
NFC dollar loans	Total	357	52	1.4	0.8	1.6	0	8.1
	AE	93	12	1.4	0.9	1.5	0.06	6.8
	EM	184	29	1.2	0.6	1.4	0	5.3
	DE	80	11	1.9	1.2	2.0	0	8.1

Notes. Summary statistics are provided for all country-years for which data on central bank reserve currency composition and NFC dollar loans from cross-border banks are available, with the exception of Hong Kong SAR which is dropped due to significant outliers (see Figure1). We also drop two country-year observations where dollar reserve shares are either greater than 100% or less than 0%.

Sources: IMF (2020), Chinn et al. (2021), Data.imf.org, BIS.

Table 2

Regressions of central bank dollar reserves vs. nonfinancial company dollar loans

Dependent variable: central bank dollar reserves as % of GDP

	No fixed effects				No fixed effects				Fixed effects			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	All	AE	EM	DE	All	AE	EM	DE	All	AE	EM	DE
NFC dollar liabilities	1.645*	3.825*	2.330**	-0.104	1.731*	3.274*	1.906**	0.0176	1.213**	3.566**	0.946**	0.466
	(0.918)	(1.756)	(0.885)	(0.177)	(0.954)	(1.794)	(0.737)	(0.258)	(0.594)	(1.445)	(0.433)	(0.392)
M2					0.0209	0.248***	-0.0624	0.102				
					(0.0492)	(0.0653)	(0.0494)	(0.108)				
Financial openness					-1.772	16.73	5.075	-1.781				
					(2.977)	(10.67)	(3.291)	(1.382)				
Bilateral trade w/US					0.141	0.130	0.153	0.345				
					(0.268)	(0.153)	(0.263)	(0.753)				
GDP per capita					-0.0230	-0.434	-0.442**	-0.0251				
					(0.0748)	(0.281)	(0.167)	(0.765)				
Ln Population					-0.325	-8.067***	0.970	-1.356				
					(0.627)	(2.268)	(0.743)	(1.284)				
Nominal dollar ER									2.074***	15.41**	2.884***	1.320
									(0.667)	(6.353)	(0.366)	(0.875)
Observations	357	93	184	80	345	89	184	72	356	93	183	80
# of Countries	52	12	29	11	52	12	29	11	51	12	28	11
Adj r-sq	0.117	0.352	0.178	0.003	0.138	0.625	0.385	0.195	0.862	0.934	0.861	0.437

Notes. NFC dollar liabilities are dollar liabilities to cross-border banks. AE, EM and DE are as per the IMF classification (Appendix Table 3). Standard errors are clustered by country. Central bank dollar reserves, NFC dollar liabilities, bilateral trade with the US, and M2 are in % of GDP. Nominal dollar ER is the nominal exchange rate vis-à-vis the U.S dollar. Columns (9)-(12) drop China for which we have only one year's data. *** p<0.01, ** p<0.05, * p<0.1.

Appendix Table A1
Data sources

Variable	Source	Notes
NFC cross-border USD liabilities to banks, loans and deposits	BIS, Locational Banking Statistics Table A6.1	Loans and Deposits liabilities in USD only
NFC local USD liabilities to banks, loans and deposits	BIS, Restricted Locational Banking Statistics; central banks and authorities; IMF	Loans and Deposits liabilities in USD only
Foreign reserves	Data.imf.org	International reserves, billons of USD
Nominal GDP	Data.imf.org	In billions of USD
Currency composition of reserves	IMF (2020) + Chinn et al (2021)	Share of reserves denominated in USD, EUR, JPY, GBP.
M2	Data.imf.org ,Data.worldbank.org, Haver for EMU	In millions of USD
Bilateral trade with the U.S.	Data.imf.org	Sum of exports and imports; in billions of USD
Financial Openness (index)	Chinn and Ito (2006), updated	Last available observation (2019) is maintained until 2020
Population	Data.worldbank.org	In millions
PPP GDP per capita	Data.worldbank.org	Current dollars
Nominal dollar ER	International Financial Statistics	Nominal local currency per U.S. dollar end of period exchange rate

Appendix Table A2
Countries in advanced, emerging and developing sub-samples

ISO country code	Full sample	Advanced	Emerging	Developing
AUS	Australia	Australia	Azerbaijan	Bangladesh
AZE	Azerbaijan	Canada	Bolivia	Ghana
BGD	Bangladesh	Czech Republic	Bosnia and Herzegovina	Kenya
BOL	Bolivia	Denmark	Brazil	Kyrgyz Republic
BIH	Bosnia and Herzegovina	Iceland	Bulgaria	Malawi
BRA	Brazil	Israel	Chile	Mozambique
BUL	Bulgaria	Korea	China	Papua New Guinea
CAN	Canada	New Zealand	Colombia	Tajikistan
CHL	Chile	Norway	Costa Rica	Tanzania
CHN	China	Sweden	Croatia	Uganda
COL	Colombia	Switzerland	Georgia	Zambia
CRI	Costa Rica	United Kingdom	India	
HRV	Croatia		Kazakhstan	
CZE	Czech Republic		Moldova	
DEN	Denmark		Namibia	
GEO	Georgia		Nigeria	
GHN	Ghana		North Macedonia	
ISL	Iceland		Paraguay	
IND	India		Peru	
ISR	Israel		Philippines	
KAZ	Kazakhstan		Poland	
KEN	Kenya		Romania	
KOR	Korea		Russia	
KGZ	Kyrgyz Republic		South Africa	
MWI	Malawi		Sri Lanka	
MDA	Moldova		Tunisia	
MZM	Mozambique		Turkey	
NAM	Namibia		Ukraine	
NZL	New Zealand		Uruguay	
NGA	Nigeria			
MKD	North Macedonia			
NOR	Norway			
PNG	Papua New Guinea			
PRY	Paraguay			
PER	Peru			
PHL	Philippines			
POL	Poland			
ROM	Romania			
RUS	Russia			
ZAF	South Africa			
ESP	Spain			
LNK	Sri Lanka			
SWE	Sweden			
CHE	Switzerland			
TJK	Tajikistan			
TAN	Tanzania			
TUN	Tunisia			
TUR	Turkey			
UGN	Uganda			
UKR	Ukraine			
GBR	United Kingdom			
URY	Uruguay			
ZAM	Zambia			

Appendix B: Proofs

B.1. Derivation of equations (29) and (30)

Take the case of the local central bank, which takes the dollar spread S as given, allowing banking and currency crises to be correlated. The local planner's objective function is:

$$W_L = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) - \beta\{\left(1 - p(q+h)\right)\gamma B_\$^2/2I + X(R_\$) + \Omega(\tau)\}$$

where the deadweight cost of taxation is:

$$\Omega(\tau) = \frac{\psi}{2}\left((q+h)(pB_h + (1+z)pB_\$ - zR_\$)^2 + (q-h)(pB_h + (1-z)pB_\$ + zR_\$)^2\right)$$

We are interested in the case where the planners choose the level of dollar reserves ($R_\$$) and capital requirements (B_h). In this case, $B_\$ = \frac{I((1-qp)S+hpz)}{(1-p(q+h))\gamma}$ is set by the unregulated bank. Take the first-order condition of W_L with respect to B_h and we recover:

$$(Q_h - \beta) - \beta\frac{d\Omega(\tau)}{dB_h} = 0$$

where $\frac{dB_\$}{dB_h} = 0$ and $\frac{dR_\$}{dB_h} = 0$. Plugging in for $\frac{d\Omega(\tau)}{dB_h}$ and solving for B_h^{**} :

$$(Q_h - \beta) - \beta\psi p[(q+h)(pB_h + (1+z)pB_\$ - zR_\$) + (q-h)(pB_h + (1-z)pB_\$ + zR_\$)] = 0$$

$$(Q_h - \beta) - \beta\psi p[2qpB_h + 2(qp + zhp)B_\$ - 2hzR_\$] = 0$$

$$B_h^{**} = \frac{(Q_h - \beta)}{2\beta\psi qp^2} - \left(1 + \frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$$$

Next, we take the first-order condition of W_L with respect to $R_\$$. Note that in the case of the local planner, they do not internalize the effect of $R_\$$ on the dollar spread S.

$$-\beta\frac{dX(R_\$)}{dR_\$} - \beta\frac{d\Omega(\tau)}{dR_\$} = 0$$

$$-S - z\psi[-(q+h)(pB_h + (1+z)pB_\$ - zR_\$) + (q-h)(pB_h + (1-z)pB_\$ + zR_\$)] = 0$$

$$[-2hpB_h - 2p(qz+h)B_\$ + 2qzR_\$] = -\frac{S}{\psi z}$$

$$R_\$^{**} = \frac{hp}{qz}[B_h + B_\$] + pB_\$ - \frac{S}{2\psi qz^2}.$$

We can rewrite $R_\$^{**}$ as

$$R_\$^{**} = \frac{2\beta\psi zhp\left(B_\$+B_h\right)+2\beta\psi qz^2pB_\$-\beta S}{2\beta\psi qz^2}$$

Now, we can write the first order conditions for the small open economy as:

$$B_\$^{**} = \frac{I\left((1-qp)S+hpz\right)}{\left(1-p(q+h)\right)\gamma} \equiv a_1S + a_2$$

$$B_h^{**} = \frac{(Q_h-\beta)}{2\beta\psi qp^2} - \left(1+\frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$$$

$$R_\$^{**} = \frac{hp}{qz}\left[B_h + B_\$\right] + pB_\$ - \frac{S}{2\psi qz^2}$$

Note that $B_\$$ is a linear function of S where $a_1 \equiv \frac{I(1-qp)}{\gamma\left(1-p(q+h)\right)}$ and $a_2 \equiv \frac{hpzI}{\gamma\left(1-p(q+h)\right)}$. Using the expression for B_h^{**}, we can write the term $\frac{h}{qz}\left(B_h^{**}+B_\$^{**}\right)+B_\$^{**}$, which appears in the simplified version of $R_\$^{**}$, as:

$$\frac{h}{qz}\left(B_h^{**}+B_\$^{**}\right)+B_\$^{**} = \frac{h(Q_h-\beta)}{2\beta\psi q^2p^2z} - \frac{h^2}{q^2}B_\$ + \frac{h^2}{pq^2}R_\$ + B_\$.$$

Plug this into the expression for $R_\$^{**}$,

$$R_\$^{**} = p\left(\frac{hp}{qz}\left[B_h^{**}+B_\$^{**}\right]+B_\$^{**}\right) - \frac{S}{2\psi qz^2}$$

$$R_\$^{**} = p\left(\frac{h(Q_h-\beta)}{2\beta\psi q^2p^2z} - \frac{h^2}{q^2}B_\$ + \frac{h^2}{pq^2}R_\$ + B_\$\right) - \frac{S}{2\psi qz^2}$$

$$\left(1-\frac{h^2}{q^2}\right)R_\$^{**} = p\left(\frac{h(Q_h-\beta)}{2\beta\psi q^2p^2z} + \left(1-\frac{h^2}{q^2}\right)B_\$\right) - \frac{S}{2\psi qz^2}$$

$$R_\$^{**} = pB_\$ + \frac{h(Q_h-\beta)}{2\beta\psi pz(q^2-h^2)} - \frac{Sq}{2\psi z^2(q^2-h^2)}$$

Plug in for $B_\$^{**}$ to solve explicitly for the optimal level of dollar reserves as a function of the dollar spread, S:

$$R_\$^{**} = p\frac{I\left((1-qp)S+hpz\right)}{\left(1-p(q+h)\right)\gamma} + \frac{h(Q_h-\beta)}{2\beta\psi pz(q^2-h^2)} - \frac{Sq}{2\psi z^2(q^2-h^2)}$$

$$R_\$^{**} \equiv b_1S + b_2$$

where:

$$b_1 = \frac{I(1-qp)p}{\gamma(1-p(q+h))} - \frac{q}{2\psi z^2(q^2-h^2)}, b_2 = \frac{hp^2zI}{\gamma(1-p(q+h))} + \frac{h(Q_h-\beta)}{2\beta\psi pz(q^2-h^2)}.$$

We want to solve for the equilibrium dollar spread. Note that:

$$B_\$^{**} - R_\$^{**} = a_1S + a_2 - (\, b_1S + b_2)$$

$$B_\$^{**} - R_\$^{**} = (a_1 - b_1)S + (a_2 - b_2)$$

To solve for the equilibrium spread in the local planner case, we use the equilibrium spread condition given by equation (28). Since we assume a unit mass of identical local planners, we plug in for the local planner's optimal decision (found above) and solve for the equilibrium spread. We have from equation (28):

$$S = \frac{\theta_{\$1} - \theta_{\$2}(B_\$ + X_\$ - R_\$)}{\beta + \theta_d} = \frac{\theta_{\$1} - \theta_{\$2}(X_\$ + (a_1 - b_1)S + (a_2 - b_2))}{\beta + \theta_d}$$

Hence, we can pin down the explicit equilibrium solution as follows:

$$S = \frac{\theta_{\$1} - \theta_{\$2}(X_\$ + a_2 - b_2)}{\beta + \theta_d + \theta_{\$2}(a_1 - b_1)}$$

$$B_\$^{**} = \frac{I((1-qp)S + hpz)}{(1-p(q+h))\gamma}$$

$$R_\$^{**} = pB_\$ + \frac{h(Q_h-\beta)}{2\beta\psi pz(q^2-h^2)} - \frac{Sq}{2\psi z^2(q^2-h^2)}$$

$$B_h^{**} = \frac{(Q_h-\beta)}{2\beta\psi qp^2} - \left(1 + \frac{zh}{q}\right) B_\$ + \frac{zh}{pq} R_\$.$$

B.2. Derivation of equations (39) and (40)

In this section, we solve for the explicit general equilibrium solution for the global planner problem when the planner chooses the amount of dollar reserves, $R_\$$, and capital requirements, B_h, allowing for correlated banking and currency crises. So, the global planner equivalent of Appendix B.1. In this case, $B_\$$ is chosen by the banking sector and given by:

$$B_\$^* = \frac{I((1-qp)S + hpz)}{(1-p(q+h))\gamma}.$$

Note that the equilibrium dollar spread will solve:

$$S = \frac{\theta_{\$1} - \theta_{\$2}(X_\$ + B_\$ - R_\$)}{\beta + \theta_d}$$

where $B_\$$ is that given above and $R_\$$ will come from the optimization problem of the global planner.

The welfare function for the global planner is given by equation (23) in the text:

$$W_G = B_\$(Q_\$ - \beta) + B_h(Q_h - \beta) + \big(f(D_\$) - D_\$ f'(D_\$)\big) - \beta\{\big(1 - p(q+h)\big)\gamma B_\$^2/2I + X(R_\$) + \Omega(\tau)\}$$

Consider first the first-order condition with respect to B_h. We can see from the welfare function above that the first-order condition for B_h will take the same form as that for the local planner in Appendix B.1. Hence, we have:

$$B_h^{***} = \frac{(Q_h - \beta)}{2\beta\psi q p^2} - \left(1 + \frac{zh}{q}\right) B_\$ + \frac{zh}{pq} R_\$.$$

Next, we need to determine the equilibrium dollar reserve policy for the global planner. In the global planner case, we must now take into account that the global planner internalizes the impact $R_\$$ has on the dollar spread, S. The global planner's first-order condition with respect to $R_\$$ is given by:

$$\frac{dW_G}{dR_\$} = \frac{d}{dR_\$}\big(B_\$(Q_\$ - \beta)\big) - \beta \frac{d}{dR_\$}\Big(\big(1 - p(q+h)\big)\gamma B_\$^2/2I\Big) + \frac{d}{dR_\$}\big(f(D_\$) - f'(D_\$)\big) - \beta \frac{d}{dR_\$} X(R_\$) - \beta \frac{d}{dR_\$}\Omega(\tau) = 0.$$

Note that $B_\$$ is a linear function of S where $a_1 \equiv \frac{I(1-qp)}{\gamma(1-p(q+h))}$ and $a_2 \equiv \frac{hpzI}{\gamma(1-p(q+h))}$.

Moving forward, we have:

$$B_\$ = \frac{I\Big((1-qp)\big(\big(\beta + \theta_d + \theta_{\$1} - \theta_{\$2}(B_\$ + X_\$ - R_\$)\big)/Q_h - 1\big) + hpz\Big)}{\big(1 - p(q+h)\big)\gamma}$$

which leads to:

$$\frac{dB_\$}{dR_\$} = \frac{I(1-qp)\theta_{\$2}}{\big(\big(1-p(q+h)\big)\gamma Q_h + I(1-qp)\theta_{\$2}\big)} \equiv \phi$$

$$\frac{dD_\$}{dR_\$} = \frac{dB_\$}{dR_\$} - 1 = \phi - 1$$

$$\frac{dQ_\$}{dR_\$} = \theta_{\$2}(1 - \phi)$$

Using these expressions, we have that the derivatives of each term in W_G with respect to $R_\$$ are below (and given by equations (33)-(38) in the text):

$$\frac{d}{dR_\$}\big(B_\$(Q_\$ - \beta)\big) = \phi(Q_\$ - \beta) + B_\$\big(\theta_{\$2}(1 - \phi)\big)$$

$$\frac{d}{dR_\$}\Big(\big(1 - p(q + h)\big)\gamma B_\$^2/2I\Big) = \phi\big(1 - p(q + h)\big)\gamma B_\$/I$$

$$\frac{d}{dR_\$}(f(D_\$) - D_\$ f'(D_\$)) = -(1 - \phi)\theta_{\$2}(B_\$ + X_\$ - R_\$)$$

$$\frac{d}{dR_\$}X(R_\$) = S + R_\$(\theta_{\$2}(1 - \phi)/Q_h)$$

$$\frac{d}{dR_\$}\Omega(\tau) = \big(2\psi\phi qp^2 - 2\psi zhp(1 - p\phi)\big)(B_h + B_\$) +$$

$$\big(2\psi\phi zph - 2\psi qz^2(1 - p\phi)\big)(pB_\$ - R_\$),$$

and where:

$$\phi \equiv \frac{dB_\$}{dR_\$} = \left(\frac{\theta_{\$2}I(1-qp)}{\gamma}\right) \Big/ \left((1 - p(q + h))(\beta + \theta_d) + \frac{\theta_{\$2}I(1-qp)}{\gamma}\right)$$

Plug these into the first-order condition:

$$\phi(Q_\$ - \beta) + B_\$\big(\theta_{\$2}(1 - \phi)\big) - \beta\phi\big(1 - p(q + h)\big)\gamma B_\$/I - (1 - \phi)\theta_{\$2}(B_\$ + X_\$ - R_\$) - \beta S -$$

$$\beta R_\$(\theta_{\$2}(1 - \phi)/Q_h) - \beta\big(2\psi\phi qp^2 - 2\psi zhp(1 - p\phi)\big)(B_h + B_\$) - \beta\big(2\psi\phi zph -$$

$$2\psi qz^2(1 - p\phi)\big)(pB_\$ - R_\$) = 0.$$

Isolate the $R_\$$ terms to get:

$$\left[-2\beta\psi\phi zph + 2\beta\psi qz^2(1-p\phi) + (1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right)\right] R_\$ = \phi(Q_\$-\beta) - (1-\phi)\theta_{\$2}X_\$ - \beta S + (2\beta\psi zhp(1-p\phi) - 2\beta\psi\phi qp^2)(B_h + B_\$) - \beta\left(2\psi\phi zp^2h - 2\psi qz^2(1-p\phi)p + \frac{\phi(1-p(q+h))\gamma}{I}\right) B_\$$$

Plug in that $B_h^{***} = \frac{(Q_h-\beta)}{2\beta\psi qp^2} - \left(1+\frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$$:

$$\left[-2\beta\psi\phi zph + 2\beta\psi qz^2(1-p\phi) + (1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right)\right] R_\$ = \phi(Q_\$-\beta) - (1-\phi)\theta_{\$2}X_\$ - \beta S + (2\beta\psi zhp(1-p\phi) - 2\beta\psi\phi qp^2)\left(\frac{(Q_h-\beta)}{2\beta\psi qp^2} - \left(1+\frac{zh}{q}\right)B_\$ + \frac{zh}{pq}R_\$\right) - \beta\left(2\psi\phi zp^2h - 2\psi qz^2(1-p\phi)p + \frac{\phi(1-p(q+h))\gamma}{I}\right) B_\$$$

Isolate the $R_\$$ terms and combine the $B_\$$ terms to get:

$$\left[-\frac{2\beta\psi z^2h^2(1-p\phi)}{q} + 2\beta\psi\phi pzh - 2\beta\psi\phi zph + 2\beta\psi qz^2(1-p\phi) + (1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right)\right] R_\$ = \phi(Q_\$-\beta) - (1-\phi)\theta_{\$2}X_\$ - \beta S + (2\beta\psi zhp(1-p\phi) - 2\beta\psi\phi qp^2)\left(\frac{(Q_h-\beta)}{2\beta\psi qp^2}\right) - \left(\left(\frac{zh}{q}\right)(2\beta\psi zhp(1-p\phi) - 2\beta\psi\phi qp^2) + 2\beta\psi\phi zp^2h - 2\beta\psi qz^2(1-p\phi)p + \frac{\beta\phi(1-p(q+h))\gamma}{I}\right) B_\$$$

Rearranging and combining terms results in:

$$\left[-\frac{2\beta\psi z^2h^2(1-p\phi)}{q} + 2\beta\psi qz^2(1-p\phi) + (1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right)\right] R_\$ = \left(2\beta\psi qz^2(1-p\phi)p - \frac{2\beta\psi z^2h^2p(1-p\phi)}{q} - \frac{\beta\phi(1-p(q+h))\gamma}{I}\right) B_\$ + \phi(Q_\$-\beta) - \phi(Q_h-\beta) - (1-\phi)\theta_{\$2}X_\$ - \beta S + \left(\frac{zh(1-p\phi)(Q_h-\beta)}{qp}\right) \quad \text{(A1)}$$

Substituting $B_\$ = \frac{I\left((1-qp)S+hpz\right)}{\left(1-p(q+h)\right)\gamma}$ and $Q_\$ = Q_h(S+1)S$ into the equation,

$$\left(2\beta\psi qz^2 + (1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right) - 2\beta p\psi\phi qz^2 - \frac{2\beta\psi z^2h^2(1-p\phi)}{q}\right)R_\$ =$$

$$\left(2\beta\psi qz^2p(1-p\phi) - \frac{2\beta\psi z^2h^2p(1-p\phi)}{q} - \frac{\beta\phi\left(1-p(q+h)\right)\gamma}{I}\right)\frac{I\left((1-qp)S+hpz\right)}{\left(1-p(q+h)\right)\gamma} - \beta S -$$

$$\theta_{\$2}(1-\phi)X_\$ + \phi Q_h S + \left(\frac{zh(1-p\phi)(Q_h-\beta)}{qp}\right)$$

We can explicitly solve for $R_\$$ and express it as a linear function of S:

$R_\$ = b_3 S + b_4$

where

$$b_3 \equiv \frac{\left(2\beta\psi qz^2p(1-p\phi) - \frac{2\beta\psi z^2h^2p(1-p\phi)}{q} - \frac{\beta\phi\left(1-p(q+h)\right)\gamma}{I}\right)\left(\frac{I(1-qp)}{\left(1-p(q+h)\right)\gamma}\right) + \phi Q_h - \beta}{\left(2\beta\psi qz^2(1-p\phi) + (1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right) - \frac{2\beta\psi z^2h^2(1-p\phi)}{q}\right)}.$$

$$b_4 \equiv \frac{-\theta_{\$2}(1-\phi)X_\$ + \frac{zh(1-p\phi)(Q_h-\beta)}{qp} + \left(2\beta\psi qz^2p(1-p\phi) - \frac{2\beta\psi z^2h^2p(1-p\phi)}{q} - \frac{\beta\phi\left(1-p(q+h)\right)\gamma}{I}\right)\frac{hpzI}{\left(1-p(q+h)\right)\gamma}}{\left(2\beta\psi qz^2(1-p\phi) + (1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right) - \frac{2\beta\psi z^2h^2(1-p\phi)}{q}\right)}.$$

Finally, recall that

$S = \frac{\theta_{\$1} - \theta_{\$2}\left(X_\$ + a_1S + a_2 - b_3S - b_4\right)}{\beta + \theta_d}$

which is solved as in Appendix B.1.

The explicit solution in the global planner case is therefore described by the following equations:

$S = \frac{\theta_{\$1} - \theta_{\$2}\left(X_\$ + a_2 - b_4\right)}{\beta + \theta_d + \theta_{\$2}(a_1 - b_3)}$

$B_\$^* = \frac{I((1-qp)S+hpz)}{\left(1-p(q+h)\right)\gamma}$

$R_\$^{***} = b_3 S + b_4$

$$B_h^{***} = \frac{(Q_h - \beta)}{2\beta\psi q p^2} - \left(1 + \frac{zh}{q}\right) B_\$^{**} + \frac{zh}{pq} R_\$^{***}.$$

B.3. Proof of Proposition 1

We want to show that: There is a threshold value $\bar{\beta}$ such that $R_\$^{***} < R_\$^{**}$ holds for all $\beta \geq \bar{\beta}$. First, we take equation (A1) from Appendix B.2. and isolate $R_\$$. This will give an expression for the optimal dollar reserve holdings of the global planner given a dollar spread S, $R_\$^{***}[S]$.

$$\left[\frac{1}{q}\left(2\beta\psi q^2 z^2(1 - p\phi) - 2\beta\psi z^2 h^2(1 - p\phi)\right) + (1 - \phi)\theta_{\$2}\left(\frac{Q_K}{Q_h} - 1\right)\right] R_\$ =$$

$$\left(\frac{p}{q}\left(2\beta\psi q^2 z^2(1 - p\phi) - 2\beta\psi z^2 h^2(1 - p\phi)\right) - \frac{\beta\phi\left(1 - p(q+h)\right)\gamma}{I}\right) B_\$ + \phi(Q_\$ - Q_h) -$$

$$(1 - \phi)\theta_{\$2} X_\$ - \beta S + \left(\frac{zh(1 - p\phi)(Q_h - \beta)}{qp}\right)$$

where we slightly re-expressed the coefficient term on $R_\$$, simplified the right-hand side, and re-expressed the coefficient term on $B_\$$ from (A1). Further re-expressing the coefficients and re-arranging:

$$\left[\frac{1}{q} 2\beta\psi z^2(1 - p\phi)(q^2 - h^2)\right] R_\$ = \left(\frac{p}{q} 2\beta\psi z^2(1 - p\phi)(q^2 - h^2)\right) B_\$ + \phi(Q_\$ - Q_h) -$$

$$(1 - \phi)\theta_{\$2} X_\$ - \beta S + \left(\frac{zh(1 - p\phi)(Q_h - \beta)}{qp}\right) - (1 - \phi)\theta_{\$2}\left(\frac{Q_K}{Q_h} - 1\right) R_\$ - \frac{\beta\phi\left(1 - p(q+h)\right)\gamma}{I} B_\$$$

Divide by the left-hand side coefficient on $R_\$$ to get:

$$R_\$ = pB_\$ + \left(\frac{h(Q_h - \beta)}{2\beta\psi z p(q^2 - h^2)}\right) - \frac{qS}{2\psi z^2(1 - p\phi)(q^2 - h^2)} +$$

$$\frac{q\left(\phi(Q_\$ - Q_h) - (1 - \phi)\theta_{\$2} X_\$ - (1 - \phi)\theta_{\$2}\left(\frac{Q_K}{Q_h} - 1\right) R_\$ - \frac{\beta\phi\left(1 - p(q+h)\right)\gamma}{I} B_\$\right)}{2\beta\psi z^2(1 - p\phi)(q^2 - h^2)}$$

Use that $Q_\$ = Q_h(S + 1)$, which implies that $Q_\$ - Q_h = Q_h S$. In addition, add and subtract $\frac{qS(p\phi)}{2\psi z^2(1 - p\phi)(q^2 - h^2)}$ to get:

$$R_\$ = pB_\$ + \left(\frac{h(Q_h-\beta)}{2\beta\psi zp(q^2-h^2)}\right) - \frac{qS}{2\psi z^2(1-p\phi)(q^2-h^2)} + \frac{qS(p\phi)}{2\psi z^2(1-p\phi)(q^2-h^2)} -$$

$$\frac{q\beta S(p\phi)}{2\psi\beta z^2(1-p\phi)(q^2-h^2)} + \frac{q\left(\phi Q_h S-(1-\phi)\theta_{\$2}X_\$-(1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right)R_\$-\frac{\beta\phi(1-p(q+h))\gamma}{I}B_\$\right)}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}$$

$$R_\$ = pB_\$ + \left(\frac{h(Q_h-\beta)}{2\beta\psi zp(q^2-h^2)}\right) - \frac{(1-p\phi)qS}{2\psi z^2(1-p\phi)(q^2-h^2)} + \frac{q\left(\phi Q_h S-\beta p\phi S-\frac{\beta\phi(1-p(q+h))\gamma}{I}B_\$\right)}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} -$$

$$\frac{q(1-\phi)\theta_{\$2}X_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} - \frac{q(1-\phi)\theta_{\$2}\left(\frac{Q_K}{Q_h}-1\right)R_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}$$

Plug in for $B_\$$, and use that $Q_h = \beta + \theta_d = Q_K + \theta_d$:

$$R_\$ = pB_\$ + \left(\frac{h(Q_h-\beta)}{2\beta\psi zp(q^2-h^2)}\right) - \frac{(1-p\phi)qS}{2\psi z^2(1-p\phi)(q^2-h^2)} + \frac{q(\phi Q_h S-\beta p\phi S-\beta\phi((1-qp)S+hpz))}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} -$$

$$\frac{q(1-\phi)\theta_{\$2}X_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} + \frac{q(1-\phi)\theta_{\$2}\theta_d R_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}$$

Use that $\phi = \frac{I(1-qp)\theta_{\$2}}{\left((1-p(q+h))\gamma Q_h+I(1-qp)\theta_{\$2}\right)}$ to write everything in terms of $Q_h = \beta + \theta_d$:

$$R_\$ = pB_\$ + \left(\frac{h(Q_h-\beta)}{2\beta\psi zp(q^2-h^2)}\right) - \frac{(1-p\phi)qS}{2\psi z^2(1-p\phi)(q^2-h^2)} + \frac{q\left(\phi Q_h S-\beta p\phi S-\beta\phi((1-qp)S+hpz)\right)}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} +$$

$$q\left(\frac{(1-p(q+h))\gamma(\beta+\theta_d)}{(1-p(q+h))\gamma(\beta+\theta_d)+I(1-qp)\theta_{\$2}}\right)\left(\frac{\theta_{\$2}\theta_d R_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} - \frac{\theta_{\$2}X_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}\right).$$

Rearranging and simplifying gives:

$$R_\$^{\text{global}}[S] = pB_\$ + \left(\frac{h(Q_h-\beta)}{2\beta\psi zp(q^2-h^2)}\right) - \frac{qS}{2\psi z^2(q^2-h^2)} + \frac{q\phi([\theta_d-\beta p(1-q)]S-\beta hpz)}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} +$$

$$q\left(\frac{(1-p(q+h))\gamma(\beta+\theta_d)}{(1-p(q+h))\gamma(\beta+\theta_d)+I(1-qp)\theta_{\$2}}\right)\left(\frac{\theta_{\$2}\theta_d R_\$^{global}[S]}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} - \frac{\theta_{\$2}X_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}\right).$$

Recall that $R_\$^{\text{local}}[S] = pB_\$ + \frac{h(Q_h-\beta)}{2\beta\psi pz(q^2-h^2)} - \frac{qS}{2\psi z^2(q^2-h^2)}$ when $h > 0$. Thus, when β is sufficiently high (hence $\theta_d \to 0$ due to $\beta + \theta_d \leq 1$) it follows that:

$$R_\$^{global}[S] \to \; pB_\$ + \left(\frac{h(Q_h-\beta)}{2\beta\psi zp(q^2-h^2)}\right) - \frac{qS}{2\psi z^2(q^2-h^2)} - \frac{q\phi(\beta p(1-q)S+\beta hpz)}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} -$$

$$q\left(\frac{(1-p(q+h))\gamma\beta}{(1-p(q+h))\gamma\beta+I(1-qp)\theta_{\$2}}\right)\left(\frac{\theta_{\$2}X_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}\right) = R_\$^{local}[S] - \frac{q\phi(\beta p(1-q)S+\beta hpz)}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} -$$

$$q\left(\frac{(1-p(q+h))\gamma\beta}{(1-p(q+h))\gamma\beta+I(1-qp)\theta_{\$2}}\right)\left(\frac{\theta_{\$2}X_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}\right)$$

Since the last 2 terms are negative (for $q^2 > h^2$), we have that $R_\$^{global}[S] < R_\$^{local}[S]$ for all S when β is sufficiently high.

Let $\Delta[S] = \frac{q\phi(\beta p(1-q)S+\beta hpz)}{2\beta\psi z^2(1-p\phi)(q^2-h^2)} + \; q\left(\frac{(1-p(q+h))\gamma\beta}{(1-p(q+h))\gamma\beta+I(1-qp)\theta_{\$2}}\right)\left(\frac{\theta_{\$2}X_\$}{2\beta\psi z^2(1-p\phi)(q^2-h^2)}\right) > 0$ so that we can write $R_\$^{global}[S] = R_\$^{local}[S] - \Delta[S]$.

Now, consider the equilibrium interest rate, S, which is determined by:

$$S = \frac{\theta_{\$1}-\theta_{\$2}\left(X_\$+B_\$[S]-R_\$[S]\right)}{\beta+\theta_d} \tag{A2}$$

Define:

$$f^{global}[S] = \frac{\theta_{\$1}-\theta_{\$2}\left(X_\$+B_\$[S]-R_\$^{global}[S]\right)}{\beta+\theta_d}$$

$$f^{local}[S] = \frac{\theta_{\$1}-\theta_{\$2}\left(X_\$+B_\$[S]-R_\$^{local}[S]\right)}{\beta+\theta_d}$$

We consider 3 cases:

Case 1: $\frac{df^{global}[S]}{dS} < 0$

Assume that $S^{**} < S^{***}$. Recall that for a given S, $R_\$^{global}[S] < R_\$^{local}[S]$. This implies that, for a given S, $f^{global}[S] < \; f^{local}[S]$. Hence, we have $S^{**} = f^{local}[S^{**}] > f^{global}[S^{**}] > f^{global}[S^{***}] = S^{***}$ where the first and last equalities come from equation (A2) holding with equality for the local and global equilibrium S values respectively. The second inequality, $f^{global}[S^{**}] > f^{global}[S^{***}]$, uses the assumption that $S^{**} < S^{***}$ and that $\frac{df^{global}[S]}{dS} < 0$.

However, we have a contradiction: $S^{**} > S^{***}$. Hence, when : $\frac{df^{global}[S]}{dS} < 0$, we must have that : $S^{***} < S^{**}$. This then implies that $B_{\$}[S^{***}] < B_{\$}[S^{**}]$. We also have that if $S^{***} < S^{**}$:

$$\frac{\theta_{\$1} - \theta_{\$2}\left(X_{\$} + B_{\$}[S^{***}] - R_{\$}^{global}[S^{***}]\right)}{\beta + \theta_d} < \frac{\theta_{\$1} - \theta_{\$2}\left(X_{\$} + B_{\$}[S^{**}] - R_{\$}^{local}[S^{**}]\right)}{\beta + \theta_d}$$

$$B_{\$}[S^{***}] - R_{\$}^{global}[S^{***}] > B_{\$}[S^{**}] - R_{\$}^{local}[S^{**}]$$

$$R_{\$}^{local}[S^{**}] - R_{\$}^{global}[S^{***}] > B_{\$}[S^{**}] - B_{\$}[S^{***}] > 0$$

$$R_{\$}^{local}[S^{**}] > R_{\$}^{global}[S^{***}].$$

In equilibrium, when $\frac{df^{global}[S]}{dS} < 0$, we have $S^{***} < S^{**}$, $B_{\$}[S^{***}] < B_{\$}[S^{**}]$, and $R_{\$}^{global}[S^{***}] < R_{\$}^{local}[S^{**}]$.

<u>Case 2</u>: $0 < \frac{df^{global}[S]}{dS} < 1$

Assume that $S^{**} < S^{***}$. Recall that for a given S, $R_{\$}^{global}[S] < R_{\$}^{local}[S]$. This implies that, for a given S, $f^{global}[S] < f^{local}[S]$. Hence, we have:

$$S^{**} = f^{local}[S^{**}] > f^{global}[S^{**}] \tag{A3}$$

Consider the equation for the equilibrium spread, S, in the global planner case which uses equation (A2):

$$S^{***} = f^{global}[S^{***}]$$

Since $0 < \frac{df^{global}[S]}{dS} < 1$, if we decrease S^{***} to some value $S^{***} - \Delta$, for $\Delta > 0$, then the left-hand side of the above equality will decrease faster than the right-hand side of the above equality. Hence,

$$S^{***} - \Delta < f^{global}[S^{***} - \Delta]$$

Since by assumption, $S^{**} < S^{***}$, set Δ such that $S^{**} = S^{***} - \Delta$. Then we have

$S^{**} < f^{global}[S^{**}]$.

We have a contradiction since here, $S^{**} < f^{global}[S^{**}]$, while we have $S^{**} > f^{global}[S^{**}]$ from equation (A3). Hence, when $0 < \frac{df^{global}[S]}{dS} < 1$, in equilibrium we have that $S^{***} < S^{**}$. This then implies that $B_\$[S^{***}] < B_\$[S^{**}]$. As in case 1, $R_\$^{global}[S^{***}] < R_\$^{local}[S^{**}]$.

<u>Case 3</u>: $\frac{df^{global}[S]}{dS} > 1$

In this case, since $f^{global}[S]$ is linear in S, if the intercept of $f^{global}[S]$ is positive, we will never have a global equilibrium. This is because $f^{global}[S] > S$ for all S > 0. We exclude this case. In addition, we can exclude this case by invoking equilibrium stability. Consider that $S^{***} = f^{global}[S^{***}]$. This is an unstable equilibrium. Consider a small deviation, such as increasing S^{***} to $S^{***} + \epsilon,\ \epsilon > 0$. Then, $\frac{-\theta_{\$2}(B_\$[S]-R_\$^{global}[S]}{\beta+\theta_d}$ will increase by more than ϵ since $\frac{df^{global}[S]}{dS} > 1$. This will lead to a higher value of S, to a higher value of $\frac{-\theta_{\$2}(B_\$[S]-R_\$^{global}[S]}{\beta+\theta_d}$ and so on. So, there is a divergence if we perturb the equilibrium value of S. We therefore exclude this case.

Finally, in the cases we do not exclude, cases 1 and 2, the equilibrium is such that $R_\$^{global}[S^{***}] = R_\$^{***} < R_\$^{**} = R_\$^{local}[S^{**}]$ for all $\beta \geq \bar{\beta}$ (β sufficiently large). Note that we used the $\beta \geq \bar{\beta}$ (β sufficiently large) condition to relate $R_\$^{global}[S] < R_\$^{local}[S]$.

B.4. Derive the threshold $\bar{\beta}$

The proof in Appendix B.3. uses the β sufficiently large condition for $R_\$^{global}[S] < R_\$^{local}[S]$ to hold for a given S. Recall from Appendix B.3. we have, before taking the limit as $\theta_d \to 0$,

$$\mathrm{R}_{\$}^{\mathrm{global}}[\mathrm{S}] = \mathrm{pB}_{\$} + \left(\frac{\mathrm{h}(\mathrm{Q_h}-\beta)}{2\beta\psi \mathrm{zp}(\mathrm{q}^2-\mathrm{h}^2)}\right) - \frac{\mathrm{qS}}{2\psi \mathrm{z}^2(\mathrm{q}^2-\mathrm{h}^2)} + \frac{\mathrm{q}\phi([\theta_{\mathrm{d}}-\beta\mathrm{p}(1-\mathrm{q})]\mathrm{S}-\beta\mathrm{hpz})}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)} +$$

$$\mathrm{q}\left(\frac{(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})}{(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})+\mathrm{I}(1-\mathrm{qp})\theta_{\$2}}\right)\left(\frac{\theta_{\$2}\theta_{\mathrm{d}}\mathrm{R}_{\$}^{global}[S]}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)} - \frac{\theta_{\$2}\mathrm{X}_{\$}}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)}\right)$$

Since $\mathrm{R}_{\$}^{\mathrm{local}}[\mathrm{S}] = \mathrm{pB}_{\$} + \frac{\mathrm{h}(\mathrm{Q_h}-\beta)}{2\beta\psi\mathrm{pz}(\mathrm{q}^2-\mathrm{h}^2)} - \frac{\mathrm{qS}}{2\psi\mathrm{z}^2(\mathrm{q}^2-\mathrm{h}^2)}$ when $h > 0$,

$$\mathrm{R}_{\$}^{\mathrm{global}}[\mathrm{S}] = \mathrm{R}_{\$}^{\mathrm{local}}[\mathrm{S}] + \frac{\mathrm{q}\phi([\theta_{\mathrm{d}}-\beta\mathrm{p}(1-\mathrm{q})]\mathrm{S}-\beta\mathrm{hpz})}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)} +$$

$$\mathrm{q}\left(\frac{(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})}{(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})+\mathrm{I}(1-\mathrm{qp})\theta_{\$2}}\right)\left(\frac{\theta_{\$2}\theta_{\mathrm{d}}\mathrm{R}_{\$}^{global}[S]}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)} - \frac{\theta_{\$2}\mathrm{X}_{\$}}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)}\right)$$

Rearranging and specifying that the expressions have similar denominators gives (for a given S):

$$\left[(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})+\mathrm{I}(1-\mathrm{qp})\theta_{\$2}\right]\left(\mathrm{R}_{\$}^{\mathrm{global}}[\mathrm{S}]-\mathrm{R}_{\$}^{\mathrm{local}}[\mathrm{S}]\right) =$$

$$\frac{\mathrm{q}\phi([\theta_{\mathrm{d}}-\beta\mathrm{p}(1-\mathrm{q})]\mathrm{S}-\beta\mathrm{hpz})}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)}\left[(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})+\mathrm{I}(1-\mathrm{qp})\theta_{\$2}\right] +$$

$$\frac{\mathrm{q}\theta_{\$2}(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})}{2\beta\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)}(\theta_{\mathrm{d}}\mathrm{R}_{\$}^{global}[S]-X_{\$})$$

The bracketed coefficient on the left-hand side is positive and we want to find the $\widetilde{\beta}$ such that for $\beta \geq \tilde{\beta}, R_{\$}^{global}[S] < R_{\$}^{local}[S]$. If this condition held, then the left-hand side of the equality above would be negative. To sign the right-hand side, we can ignore the denominators, since they are positive. So, just fix β in the denominator as $\bar{\beta}$, which will not matter for determining the threshold.

$$\left[(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})+\mathrm{I}(1-\mathrm{qp})\theta_{\$2}\right]\left(\mathrm{R}_{\$}^{\mathrm{global}}[\mathrm{S}]-\mathrm{R}_{\$}^{\mathrm{local}}[\mathrm{S}]\right) =$$

$$\frac{\mathrm{q}\phi([\theta_{\mathrm{d}}-\beta\mathrm{p}(1-\mathrm{q})]\mathrm{S}-\beta\mathrm{hpz})}{2\bar{\beta}\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)}\left[(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})+\mathrm{I}(1-\mathrm{qp})\theta_{\$2}\right] +$$

$$\frac{\mathrm{q}\theta_{\$2}(1-\mathrm{p}(\mathrm{q}+\mathrm{h}))\gamma(\beta+\theta_{\mathrm{d}})}{2\bar{\beta}\psi\mathrm{z}^2(1-\mathrm{p}\phi)(\mathrm{q}^2-\mathrm{h}^2)}(\theta_{\mathrm{d}}\mathrm{R}_{\$}^{global}[S]-X_{\$}).$$

We want a condition on β such that $R_{\$}^{global}[S] - R_{\$}^{local}[S] < 0$. This implies that

$$\left[\left(1-p(q+h)\right)\gamma(\beta+\ \theta_d)+I(1-qp)\theta_{\$2}\right]\left(R_{\$}^{global}[S]-R_{\$}^{local}[S]\right)<0$$

$$\frac{q\phi([\theta_d-\beta p(1-q)]S-\beta hpz)}{2\bar{\beta}\psi z^2(1-p\phi)(q^2-h^2)}\left[\left(1-p(q+h)\right)\gamma(\beta+\ \theta_d)+I(1-qp)\theta_{\$2}\right]+$$

$$\frac{q\theta_{\$2}\left(1-p(q+h)\right)\gamma(\beta+\theta_d)}{2\bar{\beta}\psi z^2(1-p\phi)(q^2-h^2)}\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)<0$$

$$q\phi([\theta_d-\beta p(1-q)]S-\beta hpz)\left[\left(1-p(q+h)\right)\gamma(\beta+\ \theta_d)+I(1-qp)\theta_{\$2}\right]+$$

$$q\theta_{\$2}\left(1-p(q+h)\right)\gamma(\beta+\ \theta_d)\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)<0$$

$$([\theta_d-\beta p(1-q)]S-\beta hpz)I(1-qp)\theta_{\$2}+\ \theta_{\$2}\left(1-p(q+h)\right)\gamma(\beta+$$

$$\theta_d)\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)<0,$$

where the last line uses the definition of $\phi = \frac{I(1-qp)\theta_{\$2}}{\left(\left(1-p(q+h)\right)\gamma Q_h+I(1-qp)\theta_{\$2}\right)}$ to simplify the first term. Simplify and rearrange to get:

$$([\theta_d-\beta p(1-q)]S-\beta hpz)I(1-qp)+\ \left(1-p(q+h)\right)\gamma(\beta+\ \theta_d)\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)<0$$

$$\theta_d SI(1-qp)-\beta I(1-qp)[p(1-q)S-hpz]+\left(1-p(q+h)\right)\gamma\beta\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)+$$

$$\left(1-p(q+h)\right)\gamma\theta_d\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)<0$$

$$\theta_d\left[S+\frac{\left(1-p(q+h)\right)}{(1-qp)}\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)\right]<\beta\left[[p(1-q)S-hpz]-\right.$$

$$\left.\frac{\left(1-p(q+h)\right)}{(1-qp)}\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)\right]$$

If the bracketed term on the right-hand side of the inequality above is positive, that is

$$\left[[p(1-q)S-hpz]-\frac{\left(1-p(q+h)\right)}{(1-qp)}\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S]-X_{\$}\right)\right]>0,$$

dividing through gives:

$$\tilde{\beta} = \frac{\theta_d\left[S + \frac{(1-p(q+h))\gamma}{(1-qp)}\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S] - X_{\$}\right)\right]}{[p(1-q)S - hpz] - \frac{(1-p(q+h))\gamma}{(1-qp)}\frac{}{I}\left(\theta_d R_{\$}^{global}[S] - X_{\$}\right)} < \beta$$

where we define $\tilde{\beta}$ to equal the left-hand side of the inequality. Hence, we have found the threshold.

If we take $hpz \to 0$ and $\theta_d S \to 0$,

$$\tilde{\beta} \approx -\theta_d \left[\frac{\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S] - X_{\$}\right)}{\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S] - X_{\$}\right) - \frac{p(1-q)(1-qp)S}{(1-p(q+h))}}\right]$$

Then in this special case, if $X_{\$} > \theta_d R_{\$}^{global}[S]$, the numerator in the bracket, $\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S] - X_{\$}\right) < 0$ and both terms in the denominator of the bracket term are negative: $\frac{\gamma}{I}\left(\theta_d R_{\$}^{global}[S] - X_{\$}\right) - \frac{p(1-q)(1-qp)S}{(1-p(q+h))} < 0$. Hence the bracketed term is positive and $\tilde{\beta} < 0$ (approximately) for $\theta_d > 0$. But, then for any $\beta > 0$, we will have $\tilde{\beta} \leq \beta$ in this special case.

PUBLICATIONS

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