INTERNATIONAL MONETARY FUND

Low for (Very) Long?

A Long-Run Perspective on r* across Advanced Economies

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WP/23/85

2023
APR

WORKING PAPER

 WP/23/85

IMF Working Paper
Research Department

Low for (Very) Long? A Long-Run Perspective on r* across Advanced Economies
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April 2023

ABSTRACT: We provide a long-run perspective on neutral interest rates with new estimates for 16 advanced economies since the 1870s using the Laubach and Williams approach. Our estimates differ substantially from commonly used proxies. We find that, while cross-country heterogeneity was significant in the past, since the 1980s the decline has been common to many countries. Traditional determinants such as population aging and productivity growth are strongly correlated with the changes in neutral rates, while others like the relative price of capital and inequality exhibit weak relationships with r*. We also find that neutral rates co-vary negatively with public debt-to-GDP ratios.

JEL Classification Numbers:	E20, E43
Keywords:	Advanced economies, neutral interest rates, r*, R-Star
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* We thank Giovanni Dell'Arriccia, Rui Mano, Vina Nguyen, Soledad Martinez Peria, Damiano Sandri, Sebastian Weber, and participants to the IMF RES Systemic Issues seminar and the International Macro-finance Research Symposium 2022 at Stanford GSB for comments and suggestions. We thank Chenxu Fu for excellent research assistance and Dennis Quinn for sharing the data on capital account openness.

WORKING PAPERS

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1 Introduction

The evolution of neutral interest rates—the interest rate consistent with a closed output gap and stable inflation—is key for policymakers as it determines the stance of monetary policy. Over the past few years, researchers focused on trying to understand the few-decade-long decline in neutral rates, or r^*. More recently, strong inflationary pressures worldwide led some to argue that neutral interest rates might be on the rise. Yet, r^* is not directly observable. The literature made significant progress in developing methods to identify r^*, with most studies estimating it for a single country or a few countries. The papers examining cross-country dynamics in r^* often rely on observed long-term real interest rates as an approximation, but these do not match the notion of neutral interest rates. Since r^* is hypothesized to be influenced by slow-moving forces (some of which are common to many countries), extending its estimates across time and space is an important task to improve our understanding of r^*.

This paper provides a new dataset of estimates of neutral interest rates for 16 advanced economies spanning almost 150 years from 1878.[1] We rely on the Laubach and Williams (2003) methodology to estimate r^* because, by imposing some economic structure, it returns estimates which are closer to theoretical predictions than simple empirical proxies like observed long-term real rates. For example, time-varying risk premia in the latter can obfuscate movements in neutral rates. Proxies based on observed short-term real rates are usually too volatile and can deviate from the neutral level for prolonged periods (see e.g., Reinhart and Sbrancia, 2015). The approach by Laubach and Williams (2003) is widely used in the empirical literature that focuses on the period after 1960, which facilitates the comparison of our results based on the longer sample period.

Laubach and Williams (2003) propose a semi-structural model to estimate r^*. This relates observables—output, inflation, and short-term interest rate—by postulating standard macroeconomic relationships: an IS curve linking the output gap to a real rate gap, and a Philips curve that relates current inflation to past inflation and expectations of future inflation and a measure of economic slack. The model imposes statistical properties on unobservable state variables, one of which is r^*, and these can be estimated using the Kalman filter. Model coefficients are estimated using maximum likelihood. To keep the model as parsimonious as possible, we do not estimate error-term variances and instead rely on estimates from the literature.[2]

[1] The sample includes Australia, Belgium, Canada, Denmark, Finland, Greece, Germany, Italy, Japan, Netherlands, Norway, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

[2] To check the validity of this simplification, we also have estimated all parameters or only the error-term variances. While in these cases the r^* estimates get more unstable, the estimated error-term variances

Our r^* series reveal considerable cross-country comovements with three distinct phases over the past century and a half. A period with stable to slightly declining neutral rates ranging from the 1870s to World War Two (WWII), an increase in r^* after WWII up until the 1960s, and a steady decline since the 1960s. For the median country, the decline since the 1960s' peak is 4.5 percentage points, bottoming out at 0.5 percent in 2019. This is about 2.5 percentage points below the pre-WWII average of about 3 percent. Looking at the cross-country distribution of r^* over time, there is a clear convergence since the 1980s. This is consistent with greater capital market integration and reminiscent of findings in Del Negro et al. (2019). While comovements are an important feature of r^* estimates, there are notable differences across countries. For example, in the period leading up to World War One (WWI), neutral rates increased only in about half of the countries. Even later on, there are differences in the magnitude of the fluctuations, with Japan standing out as the country with the largest decline in neutral interest rates. Sensitivity analysis shows that the time series patterns are robust, whereas uncertainty about the level of r^* is generally high.

When we correlate our r^* series with common explanatory variables, we generally find results consistent with the theory. Increases in the old-age dependency ratio and life expectancy due to population aging and improvements in healthcare systems tend to depress neutral interest rates. At the same time, variables proxying the evolution of production factors (i.e., population growth, TFP growth, as well as real GDP trend growth) correlate positively with r^*. We also document that the public debt to GDP ratio and r^* are negatively associated. Instead, countries with more open capital accounts, holding the current account constant, tend to have higher neutral interest rates. In contrast to predictions by the theoretical literature, we do not find evidence that inequality or the relative price of capital are significantly related to changes in r^*.[3] However, we do find that the relationships between r^* and its determinants appear to vary over time. Hence, depending on the period covered by the data, results may turn out to be different. Another important finding is that our estimate of r^* shows a surprisingly weak correlation with real interest rates. This might also explain why our results are generally more in line with theory, contrary to studies relying on real interest rates as proxy for r^* (Borio et al., 2017; Lunsford and West, 2019)

We then look at what factors contributed the most to changes in neutral interest rates over different periods. While we are unable to attribute a large part of the changes in r^*

usually do not diverge significantly from the ones we impose.

[3] See for example Auclert and Rognlie (2018) and Straub (2019) for inequality, Sajedi and Thwaites (2016) for the relative price of capital.

that took place in the pre-WWI period to any of the explanatory variables considered, for the inter-war period, we find that the decline in neutral rates is largely accounted for by lower trend GDP growth and to a much smaller degree by an increase in the age dependency ratio. After WWII, declining public debt stocks and a liberalization of the capital account contributed to increases in r^*. These contributions, however, are partially offset by a creeping old-age dependency ratio. Coming to the most recent decades, slowing GDP growth, population aging, and debt accumulation were behind the fall in neutral rates during the 70s and the 80s. At the same time, the continued efforts in opening the capital account and other unidentified factors common to all countries played a positive role. Finally, the aging acceleration in the 90s is the main factor behind the fall in r^*. Debt accumulation and the slowing trend GDP growth also contributed to this development.

Literature Several approaches have been developed to estimate neutral rates. Among them, the approach by Laubach and Williams (2003) and Holston, Laubach and Williams (2017) is one of the most widely referenced, with more than 2,100 citations combined at the time of writing. They apply their approach to four advanced economies (United States, United Kingdom, euro area, and Canada) using quarterly data starting in 1960. Several follow-up papers have since used their method to estimate r^* for other countries.[4]

Del Negro et al. (2017) provide two alternative approaches to estimate r^*: a medium-scale DSGE model and a Bayesian common-trend VAR. The latter is extended to a multi-country setting in Del Negro et al. (2019) and applied to a panel of seven advanced economies, on a sample ranging as far back as 1870.[5] Compared to this approach, the one of Laubach and Williams is guided by economic theory and thereby imposes more restrictions that can facilitate the identification of r^*.

Another alternative is presented in Johannsen and Mertens (2016, 2021), who estimate an unobserved components model with stochastic volatility on U.S. data. A feature of their framework is that they explicitly account for the effective lower bound on nominal interest rates. Other approaches include the one of Kiley (2020), which is similar to Laubach and Williams (2003) but uses Bayesian methods and examines the role of several "demand shifters" like asset prices, fiscal policy, and credit conditions; and the one of Bauer and Rudebusch (2020), which obtain r^* estimates from a dynamic term structure model estimated on yield curve data. Brand, Bielecki and Penalver (2018) implement and compare several of these approaches.

[4] Fujiwara et al. (2016) (Japan), Arena et al. (2020) (several euro area members), Armelius, Solberger and Spånberg (2018) (Sweden).

[5] Cesa-Bianchi, Harrison and Sajedi (2022) apply the common-trend VAR of Del Negro et al. (2019) to an unbalanced panel of 31 advanced economies, with their sample starting as early as 1900.

The different approaches and studies generally find a decline in neutral rates over recent decades, although to varying magnitude. Another common finding of all these estimation methodologies is that error bands are large.

Compared to the literature, the contribution of this paper is to use the most widely adopted Laubach and Williams approach and apply it to the largest set of countries over a very long sample period. This has the benefit of yielding estimates that are comparable across countries and that allow to investigate the r^* determinants throughout different macro regimes.

2 r^* estimation

A common approach to constructing a measure for r^* is to take averages of real interest rates. This approach, while simple, has downsides: time series tend to show large volatility, which is at odds with the idea that r^* should be a smooth series. This is especially true if one relies on short-term interest rates and volatile measures of expected inflation. Long-term interest rates tend to be smoother, but a problem can arise due to time-varying risk premia. Prolonged periods of deviation of real interest rates from r^* are conceivable, e.g. due to a deliberate policy. The decades after WWII until the 1970s are by some authors considered to be characterized by financial repression (Reinhart and Sbrancia, 2015), with real rates below equilibrium levels. Lastly, another issue with averaging arises around outlier periods: looking at war periods, real interest rate measures usually swing widely. Averaging over a long time window, and including such outliers, can substantially affect this proxy for r^*.

For these reasons, we go beyond simple averaging and derive r^* with the semi-structural model by Laubach and Williams. This section first describes the Laubach and Williams model. Then, it discusses the estimation procedure. Finally, it reports the sources of the data we rely on.

2.1 The Holston, Laubach and Williams (2017) model

The Laubach and Williams model shares many similarities to standard New Keynesian DSGE models. Dynamics in the output gap and inflation rate are summarized by two equations. The first is an intertemporal IS curve linking the output gap to the real interest rate gap. The second is a Phillips curve linking inflation to the output gap. An important deviating assumption from New Keynesian DSGE is that the neutral rate of interest is a non-stationary process instead of fluctuating along a constant long-run average. That is, the Laubach and Williams model explicitly allows for the possibility that r^* is trending downwards.

The main equations are the following

$$\tilde{y}_t = a_{y,1}\tilde{y}_{t-1} + a_{y,2}\tilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}(r_{t-j} - r^*_{t-j}) + \epsilon_{\tilde{y},t} \tag{1}$$

$$\pi_t = b_\pi \pi_{t-1} + (1-b_\pi)\pi_{t-2,4} + b_y \tilde{y}_{t-1} + \epsilon_{\pi,t} \tag{2}$$

where $\tilde{y}_t$ is the output gap (defined as the log difference of real output and the neutral rate of output), r_t is the real short-term interest rate, r^*_t is the neutral rate of interest, and π_t is the inflation rate. The term $\pi_{t-2,4}$ denotes the average of π_{t-2} to π_{t-4}. The term $\epsilon_{\tilde{y},t}$ is a transitory shock to the output gap and $\epsilon_{\pi,t}$ is a transitory shock to inflation. The coefficients to be estimated are $a_{y,1}$, $a_{y,2}$, a_r, b_π, and b_y.

The model imposes the following structure on the unobservables r^*_t, y^*_t, g_t

$$r^*_t = g_t + z_t \tag{3}$$

$$g_t = g_{t-1} + \epsilon_{g,t} \tag{4}$$

$$z_t = z_{t-1} + \epsilon_{z,t} \tag{5}$$

$$y^*_t = y^*_{t-1} + g_{t-1} + \epsilon_{y^*,t} \tag{6}$$

where g_t is the growth rate of potential output. The neutral rate r^*_t is the sum of two terms: the growth rate of potential output g_t and a catch-all term z_t. The connection between equilibrium interest rates and the growth rate of productivity is standard in neoclassical growth models, as can be seen in the Euler equation resulting from the household's optimization problem. The term z_t captures all other effects on the neutral rate, like changes in the discount factor, demographic change, and so on. Both g_t and z_t follow a random walk with innovations $\epsilon_{z,t}$ and $\epsilon_{g,t}$, respectively.[6]

Potential output is integrated of order two and $\epsilon_{y^*,t}$ denotes a shock that permanently affects the potential output. The shocks $\epsilon_{g,t}$, $\epsilon_{z,t}$, and $\epsilon_{y^*,t}$ are normally distributed, with variance σ_g, σ_z, and σ_{y^*}, and serially and contemporaneously uncorrelated.[7]

[6] Arena et al. (2020) find that the R^* estimates are sensitive to the random walk assumption. We stick to it, as we find that it makes the estimation results more robust across countries and time.

[7] The neutral rate can be decomposed into a slow-moving, long-run component as well as a short-run component, moving at business-cycle frequency, see Lindé, Platzer and Tietz (2022). Given our interest in long-run trends, the long-run component is the focus of this paper.

2.2 Estimation procedure

The estimation of the Laubach and Williams model is known to be challenging. It imposes many restrictions on the data, it involves unobservable variables, and the number of observations is usually small relative to the number of parameters and restrictions. This often results in non-unique estimates or the estimator not converging.[8] Our goal is to estimate the Laubach and Williams model in a way that makes it easy to replicate and adjust various assumptions. As r^* is generally thought to be driven by slow-moving forces, we work with annual data, which also allow us to cover a much longer sample. The long sample, in turn, is instrumental to analyze r^* dynamics over multiple macroeconomic regimes. This, however, requires us to introduce some simplifications to the estimation procedure.

We estimate the Laubach and Williams model for each country separately. To do so, we define a state-space system based on equations (1)–(6), in which potential output, r^*, and its components are the unobservable variables. The observable variables, measured with our data inputs, are the log-level of real GDP y_t, CPI inflation π_t, and the nominal short-term interest rate i_t. To construct the short-term real interest rate r_t, we calculate inflation expectations as the four-year moving average between t and $t-3$. With the estimated system at hand, we apply one or two-sided filters to derive the time series of the unobserved state variables.

We deviate from Laubach and William's procedure in that we do not estimate the error term variances $(\sigma_{\epsilon_{\tilde{y}}}, \sigma_{\epsilon_\pi}, \sigma_{\epsilon_g}, \sigma_{\epsilon_z}, \sigma_{\epsilon_{y^*}})$ ourselves. Instead, we impose values estimated by previous research whenever available. For example, we use the error term variances estimated by Holston, Laubach and Williams (2017) for the United States, United Kingdom, Canada, and euro area. For Japan, we use the error term variance estimates from Fujiwara et al. (2016). Sensitivity checks confirm the robustness of our main findings to adjusting the imposed parameter values and/or estimating a subset ourselves (unreported). Appendix Table A.1 reports our baseline values, their source, as well as key diagnostic statistics $\lambda_g = \frac{\sigma_{\epsilon,g}}{\sigma_{\epsilon,y^*}}$ and $\lambda_z = \frac{\sigma_{\epsilon,z}}{\sigma_{\epsilon,\tilde{y}}} \times \frac{a_r}{\sqrt{2}}$, where a_r is the initial value of the coefficient on the real rate gap in the IS curve.[9] Arena et al. (2020) tabulate priors and estimates of λ_g and λ_z from the previous literature and note that they differ substantially across countries and studies. Our λ_g values are close to the midpoint of their reported

[8] For example, due to a flat likelihood function, different assumptions about starting values may lead the estimator to converge at different locations and hence different estimates of the parameters and unobservables.

[9] For example, λ_g captures the magnitude of variation in the trend growth rate (one of the two components of r^*) relative to the variance of shocks to the level of potential output. Thus, a higher λ_g means r^* will covary more strongly with trend growth shocks and the z-component will be less important.

range.

We impose the error term variances to make the model as parsimonious as possible. This is necessary as the annual frequency implies a relatively low number of observations per country. We find that this approach makes the estimates more robust. We also provide initial values for the parameters and state variables. For the initial values, we use the closest empirical proxy available. For example, for potential GDP, we use the sample variance of the HP-filtered trend of GDP; for r^*, we use the variance of the HP-filtered trend of the observed real rate.

We do the following sensitivity tests to see how our r^* estimates depend on our assumptions:

S1) increase error term variances by 50 percent

S2) decrease error term variances by 50 percent

S3) force error term variances fall by 50 percent after 1990

S4) force the Phillips curve to flatten by 50 percent after 1980

S5) change initial value of r^* -50 percent

S6) change initial value of r^* +50 percent

S7) estimate only from 1950 onwards

We implement the changes in the error term variances (S1–S3) in both the IS curve and the Phillips curve. Sensitivity check S3 gets to the notion that macroeconomic stability increased during the Great Moderation. S4 captures the result in earlier studies that the relationship between inflation and unemployment weakened substantially since the 1970s and 1980s (Roberts, 2004; Mishkin, 2007). Of course, the ideal approach would be to let the data determine the precise timing of these events as well as the magnitude by which the structural relationships change. In unreported results, we experimented with this and the results were not stable for multiple countries. Therefore we opted to impose the regime changes exogenously. S5 and S6 aim to account for the large impact initial guesses can have on the estimates from the Kalman filter. S7 aims to show how our estimates change if we exclude the volatile period before the end of WWII.

2.3 Data

Our main data source is the Macrohistory Database of Jordà, Schularick and Taylor (2017) (henceforth JST), which provides macroeconomic and financial time series starting in

1870. To analyze some key facts about the estimated r^*, we augment the JST dataset with other data sources. First, demographic data combines three sources: the World Bank's World Development Indicators, which we extend with data from the Human Mortality Database and the International Historical Statistics (Mitchell 2007). Second, we add data on income inequality from the World Inequality Database (www.wid.world). Third, we include data on capital openness from Quinn (2003). And finally, all remaining data comes from the IMF's World Economic Outlook database.[10]

Our sample consists of annual data on 16 advanced economies: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, Netherlands, Norway, Spain, Sweden, Switzerland, United Kingdom, and the United States. The sample is unbalanced and starts as early as 1878 and ends in 2019.[11] To avoid outliers driving our results, we exclude from the sample the years around the two World Wars, specifically 1913 to 1921 and 1939 to 1947.[12]

3 r^* estimates

Figure 1 plots the time series of average and median r^* estimates across countries alongside the interquartile range. The plot shows three different phases for r^*: a relatively stable, or slightly declining, series at the beginning of the sample up until WWII, an increase in r^* after WWII until the 1970s, and a decline to all-time lows thereafter. Two characteristics stand out. First, the average r^* exhibits the same downward trend starting around 1960 as the previous literature documented. Second, the range of r^* across countries is substantially narrower in the latter half of the sample, which could be the result ever-increasing integration of the global economy.[13] A finding common with the related literature is the relatively wide standard errors, as large as 1.9% percentage points. Figure A.2 in the Appendix shows the average r^* estimate with a one-standard-error range.

Another key output of the estimation procedure is the estimate of the output gap. Appendix Figure A.3 plots the time series of the average output gap across countries, as well as the output gap of the United States along with the NBER recession phases. Based on the assumption imposed in the estimation, the output gap is stationary around zero. Its biggest drop, on average, is observed around the Great Depression to around 5 percent of potential GDP, even though individual countries observed output gaps greater than 10 percent of potential GDP. We observe other large drops in output gaps around the oil

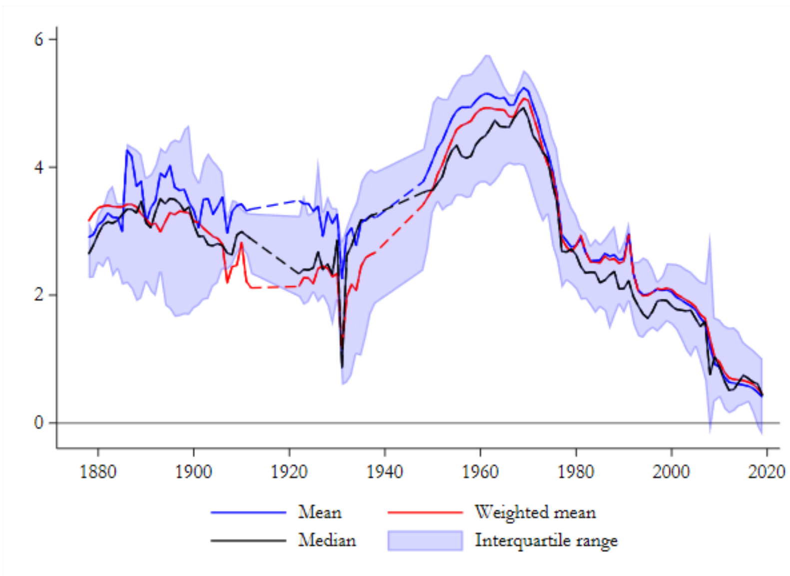
[10] Appendix Table A.2 reports the summary statistics of all variables.

[11] All series start at least in the 1880s, except for Canada (1951), Finland (1922), and Switzerland (1923).

[12] For Spain, we also exclude the period of the Spanish Civil War 1936–1939.

[13] Figure A.1 in the Appendix shows the individual country r^* estimates.

Figure 1: r^* estimates
(Percent)



Notes: The Figure shows the mean, PPP GDP-weighted mean, median, and interquartile range of the baseline r^* estimates across countries. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*.

crisis in 1979 and the Global Financial Crisis.

3.1 Sensitivity checks

Figure 2 shows the results of our sensitivity checks. They suggest that the baseline r^* estimates are not particularly sensitive to the changes in assumptions, at least in terms of the time series of cross-country averages. For individual countries, the differences between the baseline and variants can be larger (see Appendix Figure A.4). Overall, we find that it is mostly the level of the r^* estimates that is sensitive to assumed initial values and error term variances. Instead, the time series patterns appear robust. The behavior of r^* over time is the same across sensitivity tests and the time series only differ by a constant.

In addition to the sensitivity of the level of r^* to various modeling assumptions, all estimates exhibit sizeable standard errors, meaning that estimation uncertainty is high (this is illustrated in Figure A.2 and discussed further below).

A further sensitivity check is to obtain the state variable estimates with a one-sided, as opposed to a two-sided, filter. Figure A.5 in the Appendix plots the results. As is usually the case, the one-sided estimates are more volatile than the two-sided ones. However, overall, the time series patterns are similar.

The uncertainty around the level of r^* is unsatisfactory for some purposes, e.g., to gauge the appropriate level of real interest rates and the implied monetary stance. However, regressions analyses that control for the level of r^* (e.g. through country fixed effects) to study its determinants will yield robust results.

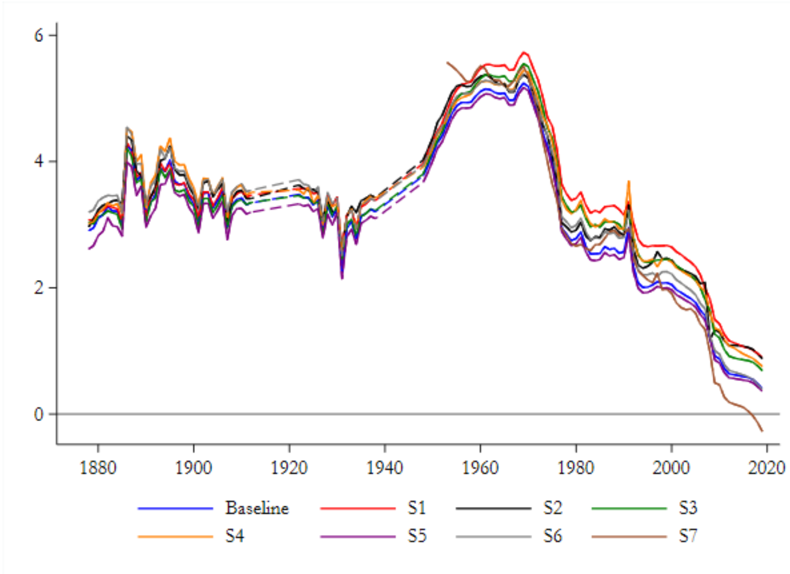
3.2 Comparison to other r^* measures

Figure 3 plots our r^* estimates alongside the original estimates of Holston, Laubach and Williams (2017) and the real interest rate for Canada, Germany, the United Kingdom, and the United States.[14] On the common sample period, our r^* tracks closely the shape of the estimates provided by Holston, Laubach and Williams (2017), except a few periods. For example, for the United Kingdom our estimate shows a more pronounced drop since 2000.[15] There is a difference in level for both United States and Canada. However, as

[14] We construct the real interest rate by taking the nominal yield on a long-term government bond and subtracting a ten-year trailing moving average of inflation. We also experimented with an inflation expectations measure based on leading data, following the approach in Hamilton et al. (2016) and Borio et al. (2017), but found the measure based on trailing data to exhibit more intuitive properties.

[15] There is also a deviation for the estimate of Germany, especially for the period of 1990 to about 2010. However, we compare our estimate, which uses data for Germany, to the HLW estimate for the euro zone. HLW do not provide a separate estimate for Germany.

Figure 2: Sensitivity checks for r^* estimates
(Percent)



Notes: The Figure shows the mean of the baseline r^* estimates along with the results of sensitivity checks S1 to S7. S1: increase error term variances by 50 percent; S2: decrease error term variances by 50 percent; S3: force error term variances to fall by 50 percent after 1990; S4: force Phillips curve to flatten by 50 percent; S5: change initial value of r^* -50 percent; S6: change initial value of r^* +50 percent; S7: estimate only from 1950 onwards. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*.

discussed with the sensitivity checks, the level of r^* is not precisely identified for our estimates. Our estimates are also smoother than the ones of HLW, which is likely due to the different data frequency (annual vs quarterly) and not to the error term variances, as shown in the results of our sensitivity checks.

Figure 4 compares our mean and median r^* to the advanced economy r^* estimate in Rachel and Summers (2019). They estimate a single r^* series for OECD countries using the approach of Holston, Laubach and Williams (2017). This is like assuming OECD members were a single country, where the underlying idea is that OECD countries are sufficiently integrated. The input for the estimation are aggregated series for the full OECD bloc. The estimate is available since 1971 and shows a steady decline of r^* from about 3.5 percent at the beginning of the sample to about 0.4 percent by 2017. Our r^* estimate shows a very similar decline. The figure also shows the mean of advanced economy real rates over the sample horizon.

As shown in the left panel of Figure 5, the observed real interest rate is much more volatile than the structural r^* estimates[16]. Another period of marked difference is after WWII: while the real rate is very low in this period, and below the prewar average way into the 1960s, the estimated r^* is above the pre-war average. A relatively high and increasing r^* is consistent with theory in as much as growth was high in this period. The low real interest rate might be explained by financial repression (Reinhart and Sbrancia, 2015). The right panel of Figure 5 shows a scatter plot of the real interest rate and r^*, which suggests that the relationship is weak: the correlation coefficient between the two measures is only 7 percent. Even after excluding extreme values for both metrics (i.e., outside of the $[-5, 5]$), the correlation remains low at 18 percent.

4 Some stylized facts about determinants of r^*

After estimating neutral interest rates, we move on to study their association with some of their key determinants. We start by presenting some stylized facts based on bilateral relationships, then we move to a multivariate regression framework, and finally we attempt to attribute changes in r^* observed over the past century to the variables mentioned above.

4.1 Bivariate relationships

The factors usually associated with movements in the neutral interest rates include demographic trends, factors of production, public debt, inequality, as well as restrictions to capital flows.[17] We focus on variables for which we could obtain data for all 16 advanced economies in the sample and over a relatively long period (i.e., at least since 1970s): the old-age dependency ratio, life expectancy, population growth, the relative price of capital, TFP growth, the trend in real GDP growth, the ratio of public debt to GDP, inequality, and the index of capital account openness of Quinn (2003).

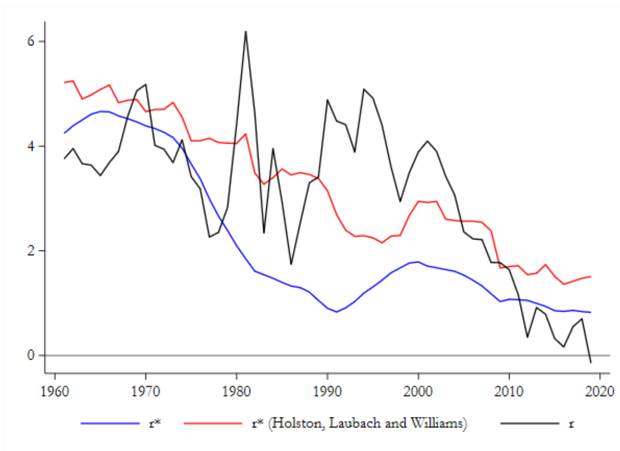
Figure 6 reports the median neutral interest rate of the sample along with the median value of each determinant. To provide a sense of the cross-country dispersion in the variables, we also plot the interquartile ranges. Owing to population aging in advanced economies and significant improvements in health care over decades, the old-age dependency ratio and life expectancy display a steady upward trend over time, which negatively correlates with the decline in neutral rates since the 1970s. Before that, how-

[16]Figure A.6 in the Appendix compares our r^* estimate to a short-term real interest rate.

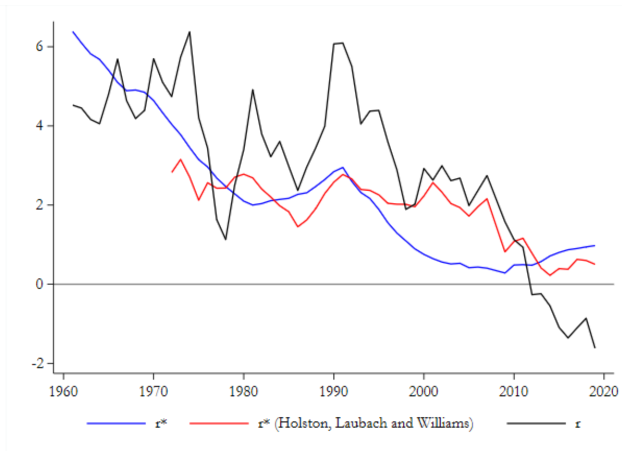
[17]While there could be many other factors playing a role in the dynamics of r^*, we center our attention here on these variables because of their direct theoretical relationship with neutral interest rates or because they are prominent in recent empirical work.

Figure 3: Comparison to Holston, Laubach and Williams (2017)

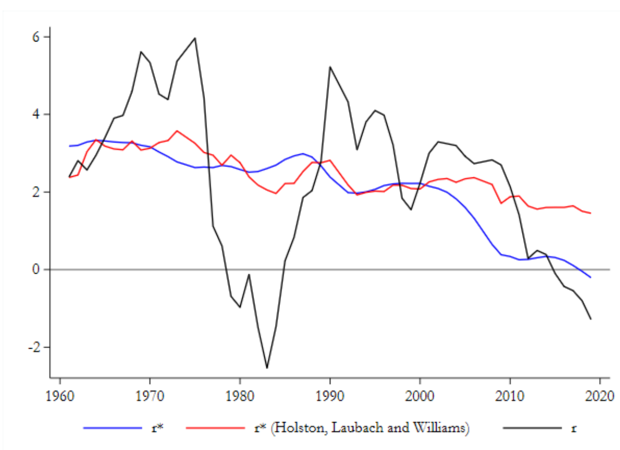
(a) Canada



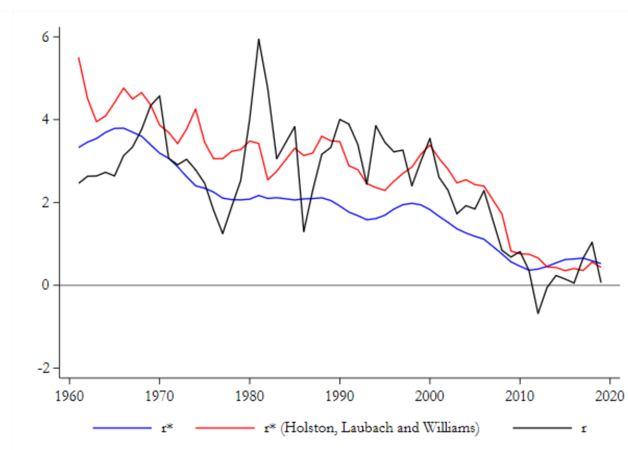
(b) Germany



(c) United Kingdom

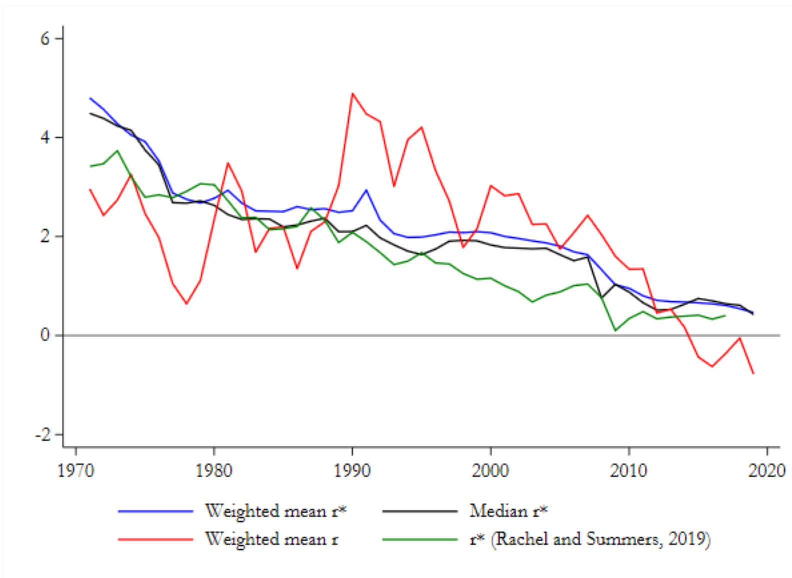


(d) United States



Notes: The real interest rate (r) is constructed as the nominal yield on long-term government bonds (10y when available) minus 10-year trailing moving average of inflation. For Germany, the r^* from Holston, Laubach and Williams (2017) corresponds to the euro area estimate. Observations corresponding to the war periods (1913–1921 and 1939–1947) are excluded from the estimation of r^*.

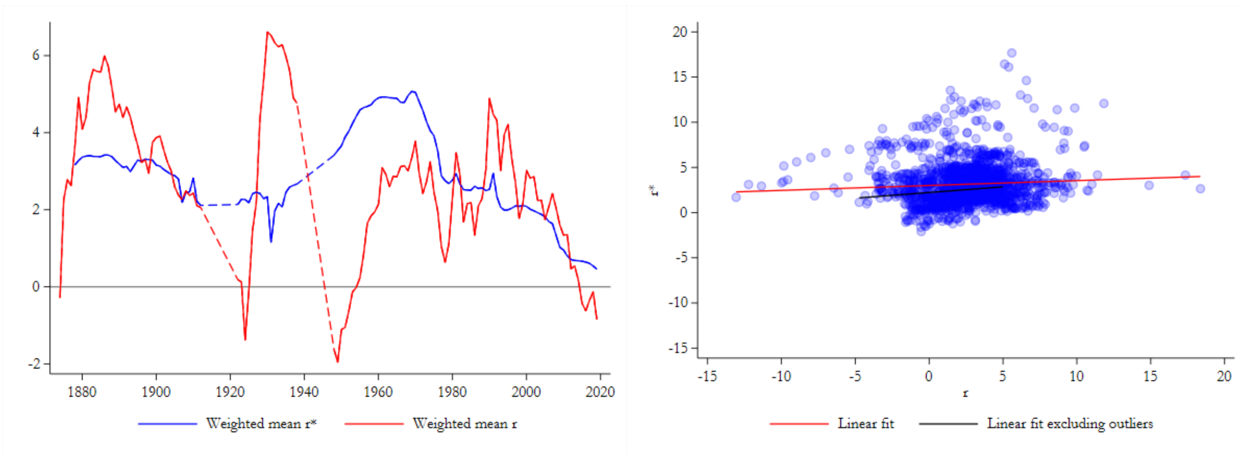
Figure 4: Comparison to Rachel and Summers (2019)
(Percent)



Notes: The real interest rate (r) is constructed as the nominal yield on long-term government bonds (10y when available) minus the 10-year trailing moving average of inflation. Observations corresponding to the war periods (1913–1921 and 1939–1947) are excluded from the estimation of r^*. Weighted means are constructed using PPP GDP weights.

Figure 5: r^* vs r
(Percent)

(a) Time series (b) Correlation



Notes: The real interest rate (r) is constructed as the nominal yield on long-term government bonds (10y when available) minus the 10-year trailing moving average of inflation. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*. Weighted means are constructed using PPP GDP weights. The correlation coefficient between the two measures is 6.8 percent; excluding outliers (i.e., removing observations outside the [-5,5] interval for r and r^*) brings the correlation to 18.3 percent.

ever, the relationship is not apparent. The growth rate of the population and of trend real GDP, on the other hand, shows a tight relationship with the neutral rates since at least the beginning of the 20th century. Among the variables that are available for at least a hundred years, there is the ratio of public debt to GDP, which shows a positive correlation until WWI and a clearly negative correlation thereafter.

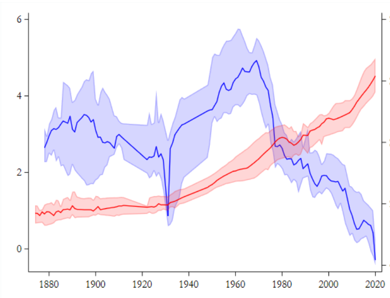
We now turn to variables with shorter time series. The well-known decline in the relative price of capital (Lian et al., 2020) as well as the decline in TFP growth coincide with the decline in neutral rates. Neutral rates, however, started their descent well before inequality started to increase in the set of countries of our sample. Finally, capital account openness displays a negative correlation with neutral rates for most of the sample.

Some of the time series in Figure 6 display an evident time trend, which can potentially deliver a spurious estimate of the relationship between r^* and these variables. To mitigate this concern, in Figure 7 we plot binned scatter plots based on the same sample in which variables are residualized with respect to year fixed effects, which capture any form of time trend that is common across countries (as well as country fixed effects).

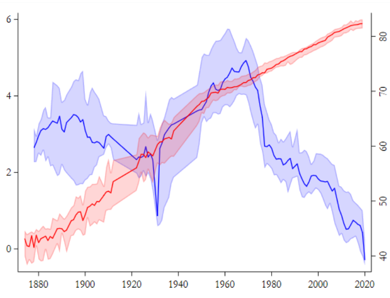
An older population should reduce the number of workers in the economy and therefore deliver lower output. As the economy's productive capacity declines, the neutral

Figure 6: r^* and its determinants over the long run

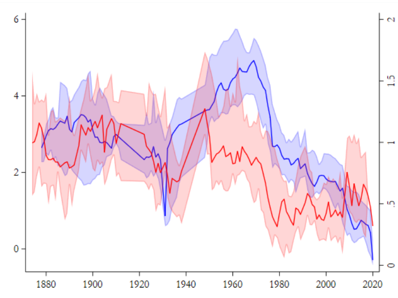
(a) Old-age dependency ratio (Percent)



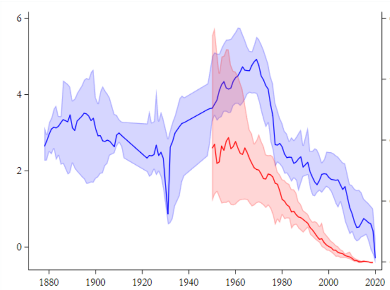
(b) Life expectancy at birth (Years)



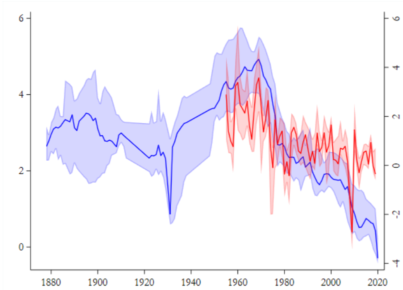
(c) Population growth (Percent)



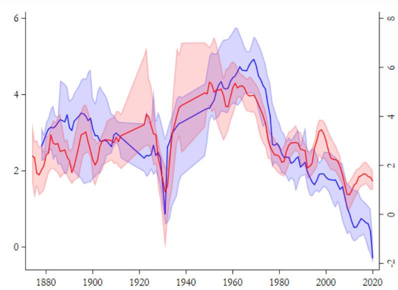
(d) Relative price of capital (CPI deflated)



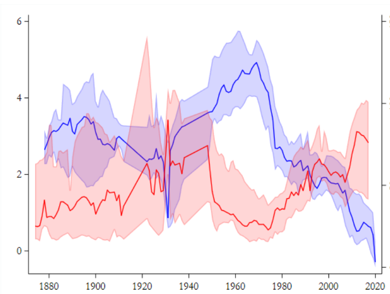
(e) TFP growth (Percent)



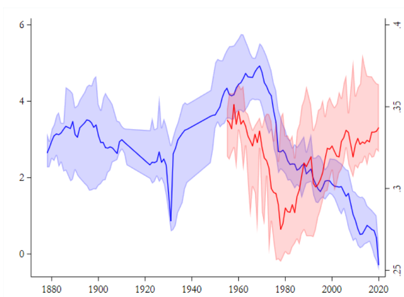
(f) Real GDP trend growth (Percent)



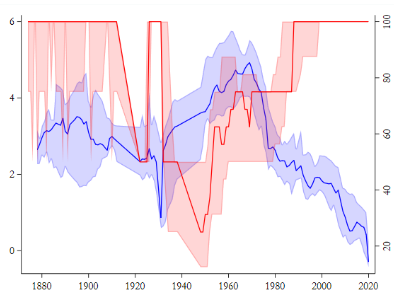
(g) Public debt to GDP (Percent)



(h) Inequality (Income share of top 10 percent)



(i) Capital account openness (Index)



Notes: The blue (red) solid line denotes the cross-country median of the neutral interest rate (variable in the panel title), shaded areas denote the interquartile ranges. Neutral interest rate determinants are measured on the right axis.

interest rate also falls. The life-cycle hypothesis, however, predicts that an increase in the share of older people in the population should be associated with lower saving rates (Attanasio and Weber, 2010). Yet, when life expectancy increases, people will need to save for a longer horizon, especially if they feel that safety nets may fail (Carvalho et al., 2017). Consistent with that, we find that both the old-age dependency ratio and life expectancy are negatively related to the dynamics of r^*, even though the coefficient on life expectancy is not statistically significant once year effects are considered. Faster population growth is positively and statistically significantly associated with neutral interest rates.

The higher price of capital goods relative to consumption goods and faster TFP growth are associated with higher r^*, at least partially due to their positive effect on investment and thus savings demand. Theory, in particular the Euler Equation, implies that there is another force at work: higher per-capita output growth, therefore higher TFP growth as well, shifts household savings supply inwards due to consumption smoothing, and consequently leads to a higher r^*. Once we residualize with respect to time fixed effects, we find that only TFP growth turns out to be statistically significant. In a similar way, the variable tracking trend growth in real GDP displays a tight positive relationship with r^*, which is also statistically significant. It is important to stress that this finding does not result mechanically from our estimation procedure. It is correct that, in the LW model, one of the two components of r^* is the (model-implied) trend growth rate g_t. However, no data on observed trend GDP growth is used in the estimation and the statistical properties imposed on g_t during estimation (that it follows an I(1) process) do not force r^* to be closely related to the observed trend growth rate. The finding of a positive correlation between growth and r^* also stands in contrast with Rogoff, Rossi and Schmelzing (2022) who find that, over the period from 1311 to 2021, growth and real rates exhibit a negative correlation. The different results can be explained by their focus on time trends while we focus on cross-country differences. We also show below that once we replace our r^* measure with the observed long-term real interest rate, akin to their real rate measure, the positive relationship vanishes in our sample as well. Lastly, coefficient estimates may not be stable over long time periods, something that we will study in the regression analysis.

The expected sign of the relationship between debt levels and neutral rates is ambiguous. An increase in debt can lead to higher demand for savings and therefore a higher level of r^* (Rachel and Summers, 2019). At the same time, an increase in the stock of debt would lead to higher debt service. Mian, Straub and Sufi (2021) argue that, due to non-homothetic preferences along the income distribution, this would imply a redistribution from higher marginal propensity to consume (MPC) individuals to lower MPC ones, leading to a decline in the neutral interest rates. Another consideration is the endogenous response of public primary balances to low interest rates, as low rates might curb consolidation efforts or increase the conceived optimal or tolerable level of public debt. We find a negative relationship of r^* with the public debt to GDP ratio, but its effect is not statistically significant.

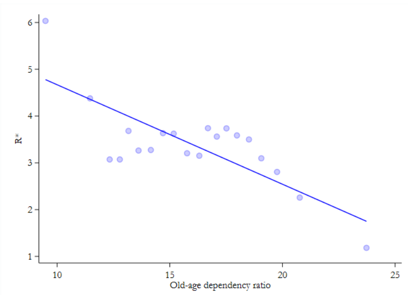
We expect the effect of higher income inequality on r^* to be positive.[18] Increasing inequality due to a larger variance of the earnings process should increase precautionary savings, thus lowering r^*. Another channel works through relatively higher saving rates at the top of the income distribution (Straub, 2019; Mian, Straub and Sufi, 2020). If a larger share of output goes to the affluent, this will shift the savings supply curve outwards, depressing r^*. Moreover, if the poor face more limited access to risk diversification options or are more risk averse, they would increase savings. We find that high income inequality is associated with lower neutral interest rates. However, once we control for factors that are common to all countries in the sample, the relationship is not statistically significant.

The relationship between current account balances, financial openness, and neutral interest rates is not straightforward. The 'savings glut' hypothesis (Bernanke, 2005) maintains that capital exporters like China invest their current account surpluses in safe assets abroad (chiefly the USA), thereby putting downward pressure on interest rates abroad. The impact of this process on the domestic interest rate level of the capital exporter is less clear. Relative to the case of autarky, it could be expected to lead to higher interest rates in the capital exporter country because the excess savings are exported. However, this effect cannot be tested in the data because autarky is a theoretical counterfactual. Comparing countries with current account deficits (capital importers), a higher degree of financial openness should (all else equal) be associated with lower neutral rates because more open economies can be expected to receive more external financing (and/or at more favorable terms). The effect of financial openness among countries with current account surpluses might be more muted as domestic interest rates might not depend strongly on capital inflows. However, a large empirical literature highlights the importance of gross capital flows, suggesting that financial openness might influence neutral rates independently of the current account balance. Developing testable hypotheses on these effects, and thorough empirical investigation, are a promising avenue for future research in international macro. In our analysis, we find a mildly negative relationship between capital account openness and r^*, which is not statistically significant.

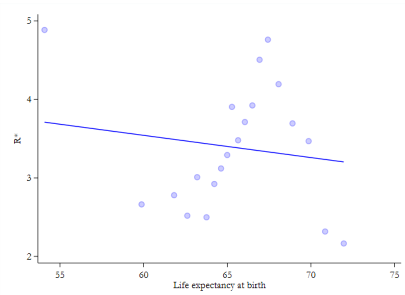
[18] We focus on income inequality as opposed to wealth inequality for two reasons. First, existing theoretical predictions are mainly about income inequality, not wealth inequality. Second, data on wealth inequality is more sparse than data on income inequality.

Figure 7: Relationship of r^* with its determinants

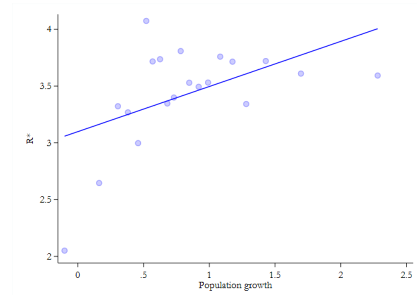
(a) Old-age dependency (Percent)



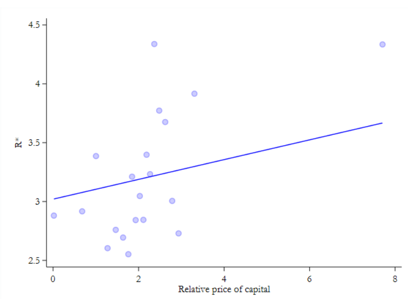
(b) Life expectancy at birth (Years)



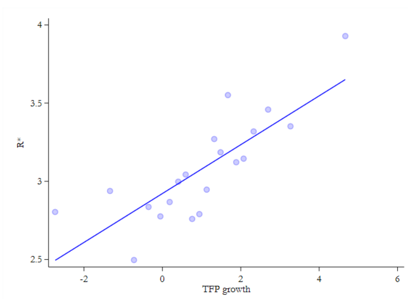
(c) Population growth (Percent)



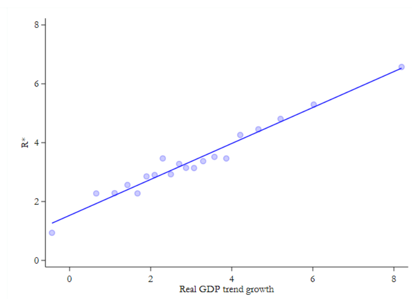
(d) Relative price of capital (CPI deflated)



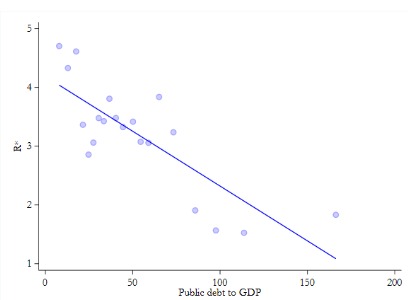
(e) TFP growth (Percent)



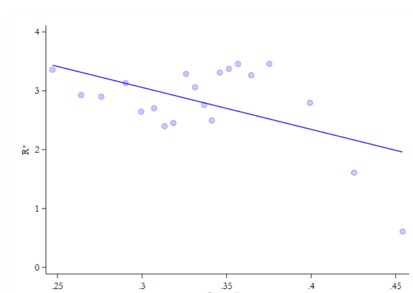
(f) Real GDP trend growth (Percent)



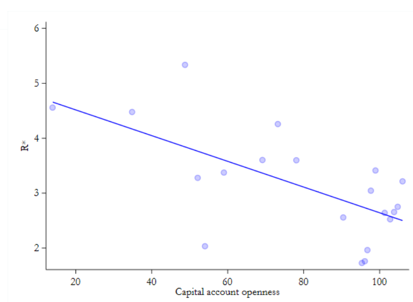
(g) Public debt to GDP (Percent)



(h) Inequality (Income share of top 10 percent)



(i) Capital account openness (Index)



Notes: Each dot represents the average of the variables on the axes. The blue line denote the linear fit. All variables are residualized with respect to country and year fixed effects.

4.2 Multivariate regression analysis

To examine more formally the relationship between the neutral rate of interest and its determinants, we estimate the following panel regression

$$r^*_{i,t} = \alpha_i + \tau_t + \beta^x X_{i,t-1} + \epsilon_{i,t} \tag{7}$$

where $r^*_{i,t}$ denotes the neutral interest rate of country i in year t, estimated as described in Section 2; α_i are country fixed effects that control for all time-invariant country characteristics; τ_t are year fixed effects that purge the effect of factors that are common to all countries in any given year; $X_{i,t-1}$ is a vector of r^* determinants, which we include with a lag to mitigate obvious endogeneity; and $\epsilon_{i,t}$ is the error term. The coefficients of interest, β, merely describe the association between the neutral interest rate and its determinants rather than the causal relationship.[19]

We first include only the demographic variables, which are available going back to before 1900. The results in column (1) of Table 1 indicate that increases in the old-age dependency ratio are associated with lower neutral interest rates. The coefficient on life expectancy is also negative, but it is statistically insignificant over such a long period. When we include variables that are related to the factors of production in column (2), our sample effectively starts in the late 1950s. We find that population growth and TFP growth are positively associated with r^*, while the relative price of capital is not significant. For this recent period, life expectancy acquires statistical significance and is negatively related to neutral interest rates as expected. In column (3), we substitute the variables proxying the factors of production with a catch-all measure—the real GDP trend growth—to preserve the long sample. Also, we drop life expectancy, which was insignificant over such a long period. As expected, real GDP trend growth turns out positively and significantly related to r^*. In column (4), we add the share of public debt to GDP, which is negatively associated with the neutral interest rates. Finally, in columns (5) and (6) we add our inequality measure and the capital account openness index, respectively. However, only the latter is significant, taking the expected positive sign.

All in all, the results of the multivariate regressions confirm the insights from the univariate correlations. Demographic trends and long-run growth appear to be important elements closely related to neutral interest rates. Among other variables, we confirm the

[19]In the HLW model, r^* is assumed to follow a random walk process. Our regression framework is consistent with this assumption as long as the stochastic trend in r^* is common across countries, in which case it will be absorbed by the year fixed effects. We find this a reasonable approach to keep the analysis simple and given that the trends in r^* and many of its determinants are indeed common across countries.

Table 1: Multivariate regressions of r^*

	(1)	(2)	(3)	(4)	(5)	(6)
Old-age dependency ratio	-0.251***	-0.224***	-0.189***	-0.103**	-0.148***	-0.127***
	(0.042)	(0.068)	(0.047)	(0.046)	(0.027)	(0.034)
Life expectancy	-0.055	-0.390***				
	(0.113)	(0.088)				
Population growth		0.431***				
		(0.142)				
TFP growth		0.113***				
		(0.033)				
Relative price of capital		-0.024				
		(0.091)				
Real GDP trend growth			0.454***	0.376***	0.458***	0.391***
			(0.086)	(0.063)	(0.087)	(0.061)
Public debt to GDP				-0.017**	-0.011***	-0.016**
				(0.007)	(0.002)	(0.007)
Inequality					-2.702	
					(1.985)	
Capital account openness						0.013***
						(0.004)
Period	1878–2020	1956–2020	1878–2020	1878–2020	1989–2020	1878–2020
Observations	1,834	974	1,831	1,741	1,000	1,744
R^2	0.713	0.867	0.776	0.817	0.892	0.825

Notes: The table reports the results from regressions of the neutral interest rates on lagged determinants. All regressions include country and time fixed effects. Standard errors clustered at the country level are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

roles of public debt and the capital account openness.[20]

We argued in Section 2 that proxying r^* with observed long-term real interest rates can be misleading. While convenient, we find that regression results using such proxy are generally less consistent with theory and more unstable. Table 2 reports estimates of the specifications in Table 1 but uses observed real rates as the dependent variable and restricts the sample to be the same. We find no evidence that population aging—measured with the old-age dependency ratio—has a negative effect on this proxy of neutral interest rates. Life expectancy, instead, has a negative effect in the long period as shown in column (1), but not in the short one as reported in column (2). When we introduce variables for the factors of production in column (2), we only find some counterintuitive results: a negative association between the long-run real interest rates and the relative price of capital and TFP growth (even though the coefficient on the latter is only borderline significant). Among the other variables introduced in the other columns, we only find a positive relationship with real GDP trend growth in one instance and a negative association with inequality. This evidence lends further support for using measures of r^* rather than simply proxying it with observed long-term real rates.

[20] Appendix Table A.3 shows that our results are robust in the exclusion of any country.

Table 2: Multivariate regressions of r

	(1)	(2)	(3)	(4)	(5)	(6)
Old-age dependency ratio	0.054	0.053*	0.033	0.031	0.052	0.058
	(0.054)	(0.025)	(0.052)	(0.067)	(0.053)	(0.060)
Life expectancy	-0.137**	-0.290				
	(0.048)	(0.191)				
Population growth		-0.443				
		(0.310)				
TFP growth		-0.090*				
		(0.050)				
Relative price of capital		-0.344***				
		(0.111)				
Real GDP trend growth			0.103	0.107	0.230**	0.093
			(0.098)	(0.080)	(0.098)	(0.080)
Public debt to GDP				0.000	0.002	-0.000
				(0.006)	(0.003)	(0.006)
Inequality					-5.448**	
					(2.492)	
Capital account openness						-0.013
						(0.009)
Period	1878–2020	1956–2020	1878–2020	1878–2020	1989–2020	1878–2020
Observations	1,834	974	1,831	1,741	1,000	1,741
R^2	0.561	0.508	0.527	0.530	0.668	0.536

Notes: The table reports estimates of regressions of observed long-term real interest rates on lagged determinants. All regressions include country and year fixed effects. Standard errors clustered at the country level are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Real interest rate is constructed as nominal yield on long term bond (10y when available) minus 10 year trailing moving average of inflation.

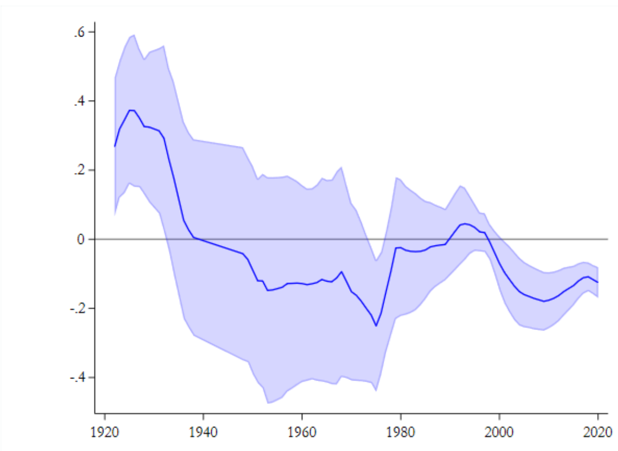
To provide an overview of how the relationship between r^* and the explanatory variables changes over time, we estimate a set of rolling regressions using a 40-year window. Figure 8 reports the results of the rolling regressions. Some of the coefficients show significant variation, suggesting that the period covered by the sample can drastically change the results. The coefficient old-age dependency ratio, for example, was positive in the pre-WWI period but moved into negative territory after that. However, it remained statistically insignificant for most of the sample length, suggesting that the aging acceleration of the 90s was crucial for its identification. The coefficient on real GDP trend growth remained always positive and generally statistically significant. Yet, its magnitude varies from about 0.1 to 0.6. The relationship with the public debt to GDP ratio is more stable, with a coefficient hovering between -0.005 to -0.018 which turns out to be statistically significant most of the time. Finally, the index proxying capital account openness remained statistically insignificant for a large part of the sample, except during the period in which most of the capital account liberalizations took place.

Appendix Table A.4 shows an extension of the multivariate regressions of this section by including the current account balance as a regressor. Column 1 reproduces the

Figure 8: Time-varying coefficients from rolling regressions

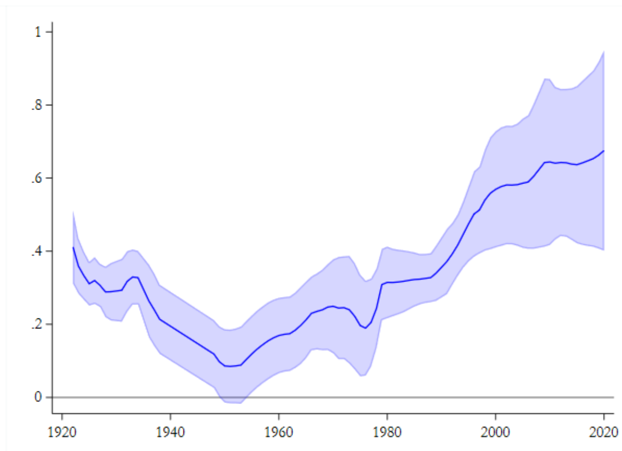
(a) Old-age dependency

(Percent)



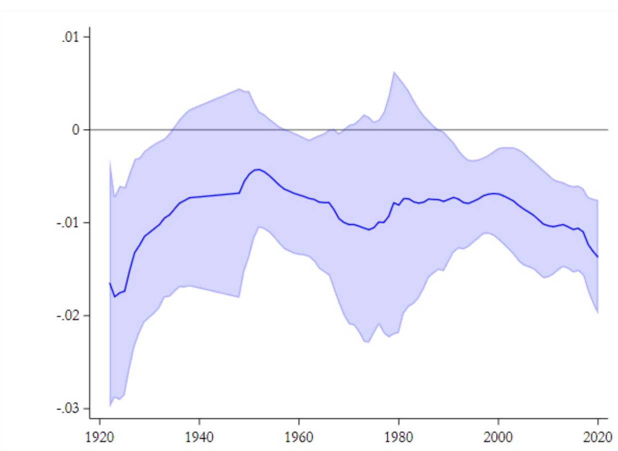
(b) Real GDP trend growth

(Percent)



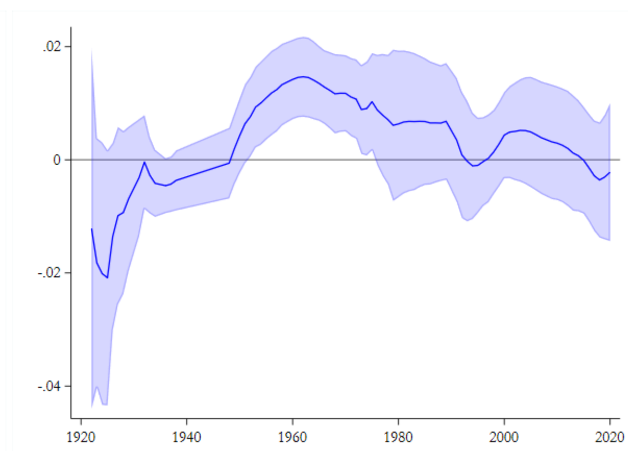
(c) Public debt to GDP

(Percent)



(d) Capital account openness

(Index)



Notes: The panels plot the coefficients on the explanatory variables included in the baseline specification (i.e., column (6) of Table 1), based on rolling regressions using a 40-year moving window. The coefficient estimates are reported in correspondence of the last year of the moving window. The shaded areas denote the 90 percent confidence intervals.

baseline specification of Table 1. Column 2 adds the variable current account balance (% of GDP). A positive value corresponds to a current account surplus. The coefficient on the current account balance is insignificant, while all other coefficients stay nearly unchanged. The period since 1973, following the collapse of the Bretton Woods system, is characterized by intensified globalization. This motivates us to check if the results are different for the period after 1973. Column 3, therefore, interacts the current account variable with a post-1973 dummy. The current account interacted with the post-1973 dummy is positive and significant at a 10 percent level. In the recent period of globalization, a more positive current account is associated with higher r^*. This result is consistent with the global savings glut hypothesis that states that large capital inflows to the U.S. (current account deficit) are partially responsible for a low r^* in the US. Finally, we are interested in the interaction of the current account and capital account openness. Column 4 adds an interaction term of current account and the capital account openness index. We find that, after 1973, the interaction of current account and capital account openness are positively related to r^*. This suggests that in countries with more openness, the association of current account imbalances and r^* are stronger.[21] In conclusion, results when adding current account openness are consistent with our expectations. The coefficient on capital account openness stays nearly unaffected, which confirms that the capital account openness index and current account balance capture different features of the data.

4.2.1 Robustness

To ensure that our results are robust, we re-estimate our preferred specification in column (6) of Table 1 using alternative regression approaches. We report the results in Table 3. We start from the simplest approach which consists of pooling together all country-year observations as if it was a single cross section. The results in column (1) indicate that only the old-age dependency ratio and the real GDP trend growth turn out statistically significant, suggesting that time-invariant and factors common to all countries are important for the identification of the other coefficients. When we purge time-invariant factors in column (2), the ratio of public debt to GDP becomes significant with a magnitude close to the one in our baseline specification in column (3), which also includes time fixed effects. In column (4) we account for the estimation uncertainty of our r^* estimates by using the inverse of the squared standard errors as weights for the least squares estimation. The

[21] The interaction of current account and the post 1973 dummy has a negative sign. However, a careful interpretation of the association of higher current account on r^* has to take into account the level of capital openness. The relation between current account and r^* is positive for capital openness index larger than 79%, which is the case in our post-1973 sample for about 66% of country-year observations.

results are close to the baseline ones.

We also test the robustness of our results to alternative ways of computing the standard errors. In column (5) we double cluster the standard errors at the country and year level. In column (6) we compute the Driscoll and Kraay (1998) standard errors, which correct for cross-sectional correlation. Again, results appear robust to these variations. Finally, in column (7) we include the lagged dependent variable to account for the inertia in neutral interest rates. The lag of r^* is as large as 0.9, suggesting a high degree of persistence. Its inclusion, however, leads to an insignificant coefficient on the old-age dependency ratio, which is the variable showing the steadiest trends over time.

Table 3: Alternative estimation approaches

	Pooled OLS	Only country fixed effects	Baseline	Weighted least squares	Double clustered SE	DK SE	DK SE dynamic
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Old-age dependency ratio	-0.083**	-0.083***	-0.127***	-0.072**	-0.128***	-0.128***	-0.014
	(0.030)	(0.022)	(0.034)	(0.033)	(0.033)	(0.018)	(0.012)
Real GDP trend growth	0.576***	0.498***	0.389***	0.458***	0.389***	0.389***	0.071***
	(0.117)	(0.047)	(0.061)	(0.074)	(0.063)	(0.031)	(0.015)
Public debt to GDP	-0.006	-0.017***	-0.016**	-0.023**	-0.016**	-0.016***	-0.001*
	(0.004)	(0.006)	(0.007)	(0.010)	(0.007)	(0.001)	(0.000)
Capital account openness	-0.002	0.002	0.013***	0.013***	0.013***	0.013***	0.002***
	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)	(0.001)
Lag dependent variable							0.883***
							(0.034)
Observations	1,741	1,741	1,741	1,741	1,741	1,741	1,656
R^2	0.463	0.786	0.825	0.854	0.825	0.825	0.970

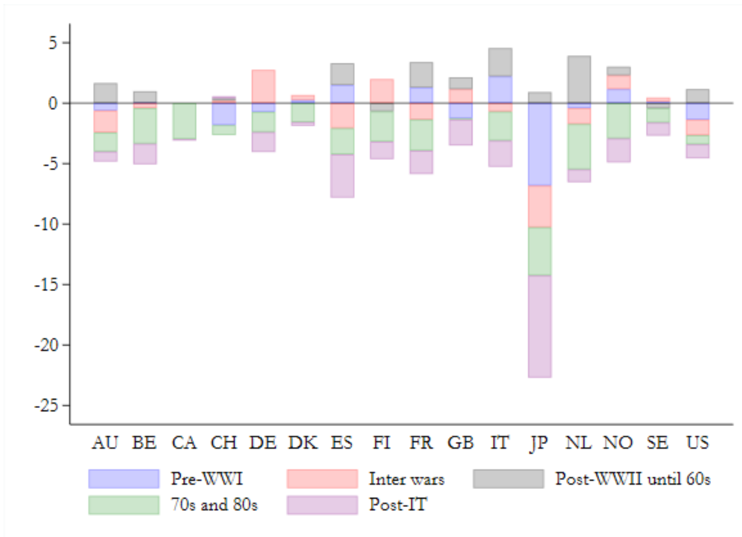
Notes: The table reports the results from regressions of the neutral interest rates on lagged determinants. WLS in column (4) stands for weighted least squares, with the weights equal to the inverse of the squared standard errors of the R^* estimates. Double clustered standard errors in column (5) are at the country and year level. DK in columns (6) and (7) stands for Driscoll-Kraay standard errors (Driscoll and Kraay, 1998), which correct for cross-sectional dependence. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

4.3 Contributions to changes in r^*

As documented in Section 2, changes in neutral interest rates over the long period present many commonalities across countries, but also some differences. Figure 9 plots the country-specific changes in r^* over 5 periods of interest: i) pre-WWI (1885–1913), ii) inter wars (1919–1923), iii) post-WWII until 60s (1946–1969), iv) 70s and 80s (1970–1989), and v) post-inflation targeting (1990–2020).

Cross-country co-movements are evident. Increases and decreases in neutral rates are about balanced in the period leading up to WWI, while there tend to be more declines in the interwar period. Then, they generally increased after WWII up until the 60s. Since then they fell in all countries both in the 70s and 80s and in the most recent post-IT period. Yet, there is some heterogeneity in the magnitude of the fluctuations. Japan stands out as the country with the largest declines, especially before WWI and in the post-IT period. To provide a reference, the fall of r^* in Japan is about a third of the average decline (excluding Japan). Other countries with the largest post-60s declines are Belgium, France, Italy, the Netherlands, and Spain; in contrast, Canada, Denmark, and the United States experienced the smallest declines.

Figure 9: Changes in r^* by country and period
(Percentage points)



Notes: The stacked bars correspond to the changes in r^* in each of the following periods: pre-WWI (1885–1913), inter wars (1919–1923), post-WWII until 60s (1946–1969), 70s and 80s (1970–1989), post-IT (1990–2020). To compute the changes, the start (end) value is computed as the average over the previous (following) 5 years, including the reported year.

To understand which factors account for these patterns in the neutral interest rates, we compute country-specific contributions to changes in r^* between year t and t' as

$$C^x_{i,t,t'} = \hat{\beta}^x (x_{i,t'} - x_{i,t}) \tag{8}$$

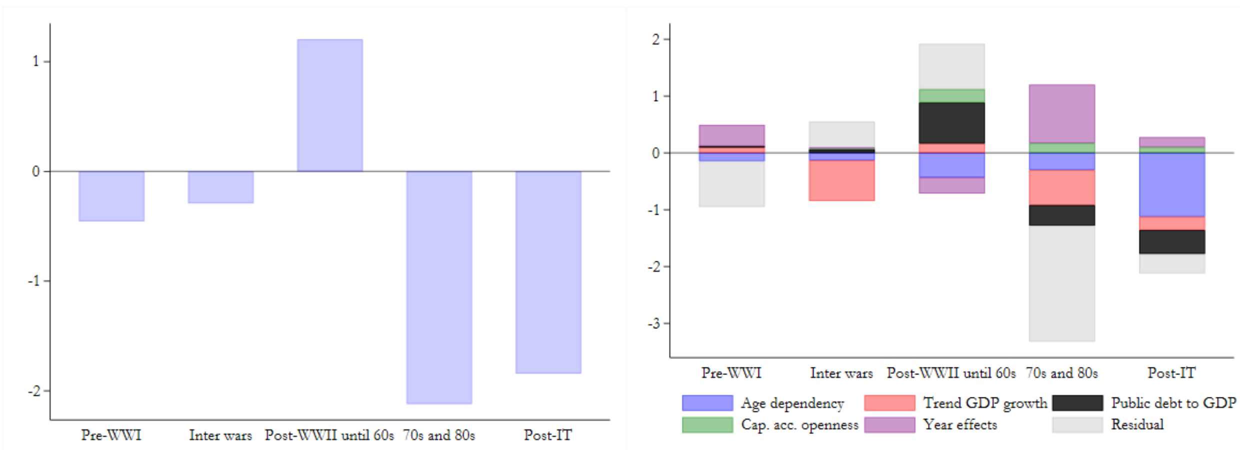
where $\hat{\beta}^x$ is the estimated coefficient for variable x from column (6) of Table 1, and t and t' identify the first and last year of the 5 periods of interest. However, instead of using the value of the variables in single years, we take 5-year averages to avoid that fluctuations in specific years affect our calculations.[22]

[22]For year t, we actually use the average between $t-5$ and t; and for year t', we use the average between

The left panel of Figure 10 plots the cross-country averages of the changes in the neutral interest rates by period. In the right panel, we decompose these average changes by the contribution of each variable included in our baseline specification. We are unable to attribute a large part of the changes in neutral interest rates that took place in the pre-WWI period to any of the explanatory variables. The residual in fact is negative and as large as one percentage point. For the inter-war period, the decline in neutral rates is largely accounted for by lower trend GDP growth and to a much smaller degree by an increase in the age dependency ratio. After WWII, declining public debt stocks and a liberalization of the capital account contributed to increases in r^*. These contributions, however, are partially offset by an increasing old-age dependency ratio.

Figure 10: Contributions to changes in r^*
(Percentage points)

(a) Average changes (b) Contributions



Notes: In the left panel, the bars denote the cross-country average changes in r^* by period. In the right panel, the stacked bars correspond to the cross-country average contributions of each explanatory variable in column (6) of Table 1 to the average changes in r^*. The periods considered are: pre-WWI (1885–1913), inter wars (1919–1923), post-WWII until 60s (1946–1969), 70s and 80s (1970–1989), post-IT (1990–2020). To compute the changes, the start (end) value is computed as the average over the previous (following) 5 years, including the reported year.

Coming to the most recent decades, slowing GDP growth, population aging, and debt accumulation appear behind the fall in neutral rates during the 70s and the 80s. At the same time, the continued efforts in opening the capital account and, more importantly, factors common to all countries played a positive role. All in all, however, the residual is as large as 2 percentage points, indicating that there are other factors that were associated

t' and $t' + 5$.

with the observed decline in neutral interest rates. The acceleration of demographic aging in the 90s—that is, in the post-IT period—is the main factor behind the fall in r^*. Debt accumulation and slowing trend GDP growth also contributed to that.

5 Conclusions

This paper estimates the neutral rate of interest for 16 advanced economies over the period 1878 to 2019, using the most widely cited estimation approach (i.e., Laubach-Williams). Our main contribution is to provide r^* estimates which are comparable across a wide range of countries and over a long sample period. We also provide suggestive evidence on the drivers of r^* over the very long run.

Our r^* series exhibits three distinct phases over the last 150 years. From the 1870s to World War Two (WWII), neutral rates were relatively stable or slightly declining. After WWII up until the 1960s, r^* was increasing. Since the 1960s, r^* has declined steadily. For the median country, the decline since the 1960s' peak is 4.5 percentage points, bottoming out at 0.5 percent in 2019. These patterns are broadly shared across countries. There is considerable cross-country comovement throughout the sample period and particularly so since the 1980s, which is consistent with greater capital market integration.

Our findings on the determinants of r^* are broadly consistent with economic theory. Population aging and improvements in the healthcare system that increased the old-age dependency ratio and life expectancy are associated with lower neutral interest rates. At the same time, variables proxying the evolution of production factors (i.e., population growth, TFP growth, as well as real GDP trend growth) correlate positively with r^*. These findings stand in contrast to earlier studies, which conclude that traditional r^* determinants play no or only a minor role (Borio et al., 2017; Rogoff, Rossi and Schmelzing, 2022; Lunsford and West, 2019). We attribute the different findings mainly to our semi-structural estimation approach as opposed to using observed real rates as r^* proxies.

We also find that variables which commonly do not play a central role in discussions about the r^* decline are significantly related to it. First, higher public debt-to-GDP ratios are associated with lower neutral rates. While we cannot claim a causal relationship, our findings suggest a closer look into theories that can explain such a negative link (Mian, Straub and Sufi, 2021). Second, countries with more open capital accounts tend to have higher neutral interest rates. These findings suggest that broad macroeconomic policy regimes might be important to explain secular shifts in r^*.

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Appendix

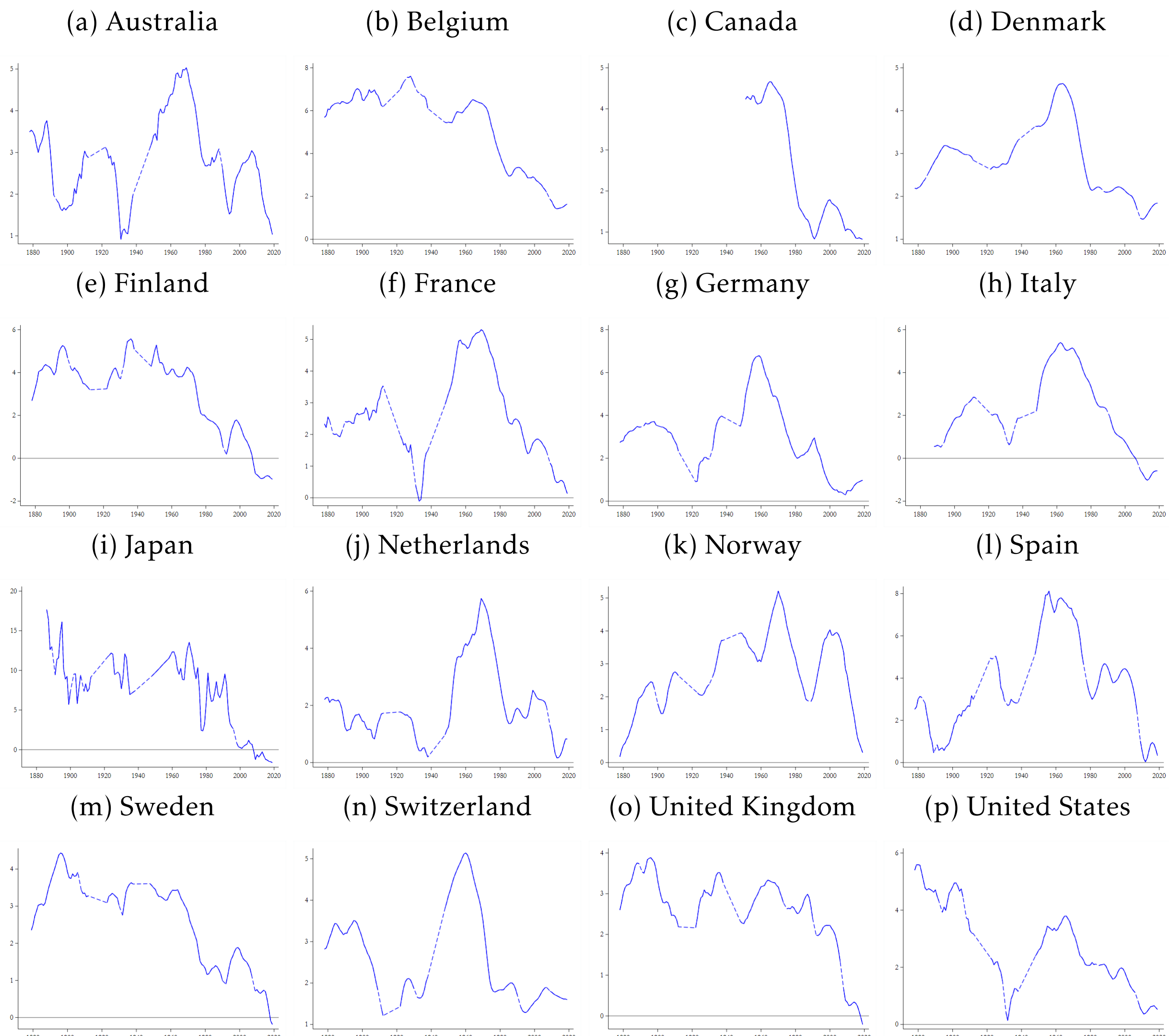
A Additional results

Table A.1: Values imposed for error term variances

	σ^2_{ϵ,y^*}	$\sigma^2_{\epsilon,\tilde{y}}$	$\sigma^2_{\epsilon,\pi}$	$\sigma^2_{\epsilon,g}$	$\sigma^2_{\epsilon,z}$	λ_g	λ_z	Source
Australia	0.330	0.125	0.625	0.015	0.023	0.045	0.000	HLW GB*
Belgium	0.200	0.077	0.950	0.005	0.050	0.025	0.018	HLW Euro
Canada	0.330	0.250	0.625	0.015	0.023	0.045	0.000	HLW CA*
Denmark	0.200	0.077	0.950	0.005	0.050	0.025	0.009	HLW Euro
Finland	0.330	0.125	0.625	0.015	0.023	0.045	0.017	HLW GB*
France	0.200	0.077	0.950	0.005	0.050	0.025	0.011	HLW Euro
Germany	0.630	0.200	0.625	0.035	0.033	0.055	0.015	HLW Euro*
Italy	0.200	0.231	1.900	0.005	0.050	0.025	0.002	HLW Euro*
Japan	0.723	0.185	2.005	0.023	0.410	0.031	0.094	FIMNS 2016
Netherlands	0.200	0.077	0.950	0.005	0.050	0.025	0.002	HLW Euro
Norway	0.330	0.125	0.625	0.015	0.023	0.045	0.017	HLW GB*
Spain	0.150	0.044	1.002	0.013	0.054	0.083	0.053	HLW Euro
Sweden	0.330	0.125	0.625	0.015	0.023	0.045	0.017	HLW GB*
Switzerland	0.200	0.154	0.950	0.005	0.050	0.025	0.001	HLW Euro*
United Kingdom	0.330	0.125	0.625	0.015	0.023	0.045	0.001	HLW GB*
United States	0.330	0.375	0.625	0.015	0.023	0.045	0.008	HLW US*

Notes: The table reports the values of the error term variances used in the estimation of the Holston, Laubach and Williams (2017) model: $\lambda_g = \frac{\sigma_{\epsilon,g}}{\sigma_{\epsilon,y^*}}$ and $\lambda_z = \frac{\sigma_{\epsilon,z}}{\sigma_{\epsilon,\tilde{y}}} \times \frac{a_r}{\sqrt{2}}$, where a_r is the initial value of the coefficient on the real rate gap in the IS curve. The sources are shown in the last column. HLW refers to Holston, Laubach and Williams (2017). We use the values reported in the replication files. FIMNS 2016 refers to Fujiwara et al. (2016). The asterisk denotes that we started at the values from the source, but manually adjusted the values in order to get convergence of the algorithm.

Figure A.1: r^* estimates of individual countries
(Percent)



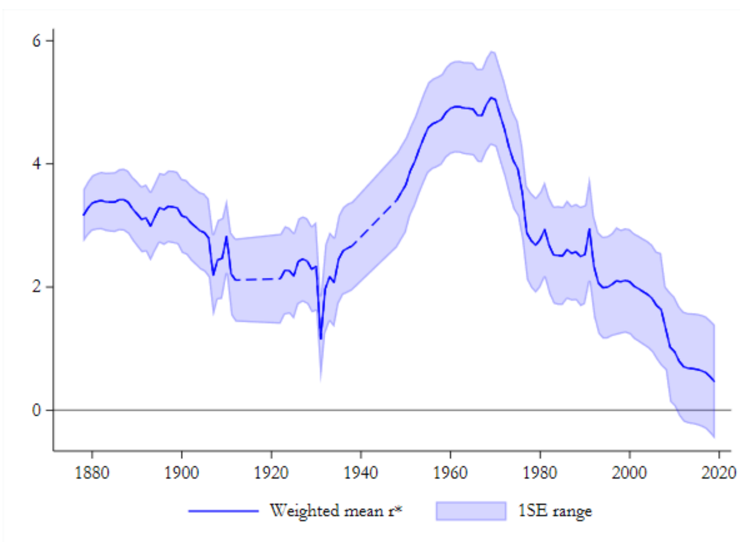
Notes: The figure plots the time series of r^* under the baseline for each individual country. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*.

Table A.2: Descriptive statistics

	Obs	Mean	SD	Min	Max
r*	1,857	3.1	2.2	-2.1	17.7
Old-age dependency ratio	1,983	15.8	7.1	3.0	48.0
Life expectancy at birth	1,952	64.5	14.0	29.5	84.4
Population growth	1,913	0.9	0.6	-2.1	4.2
Relative price of capital	1,097	2.2	1.9	0.9	17.4
TFP growth	1,017	1.0	1.8	-6.9	10.4
Real GDP trend growth	1,978	3.1	2.0	-5.8	12.9
Public debt to GDP	1,859	53.0	38.6	1.9	239.6
Inequality	1,114	0.3	0.0	0.2	0.6
Capital account openness	1,996	79.6	26.4	0.0	100.0

Notes: The Old-age dependency ratio is defined as the share of the population of age 65 and older relative to the share of the population of age 15 to 64. Inequality refers to the share of pre-tax national income held by top 10 percent. Capital account openness is an index from Quinn (2003).

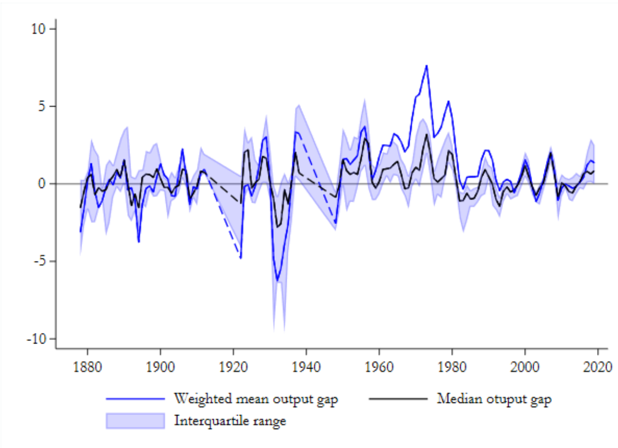
Figure A.2: Uncertainty around r* estimates
(Percent)



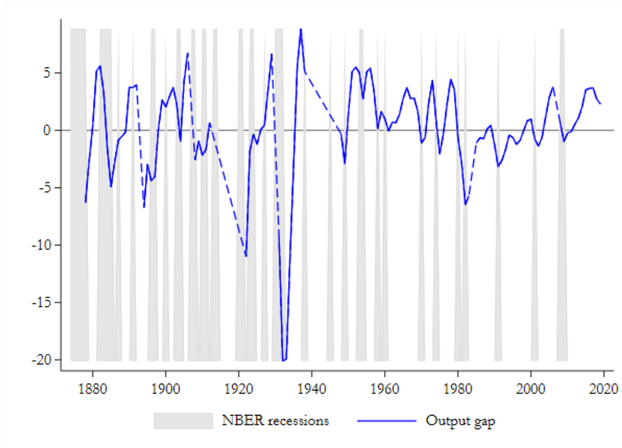
Notes: The weighted mean of r^* is constructed using PPP GDP weights. The 1SE range is calculated as the PPP GDP-weighted mean of the standard errors of the r^* estimates across all countries. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*.

Figure A.3: Output gap estimates
(Percent of potential output)

(a) Sample



(b) United States



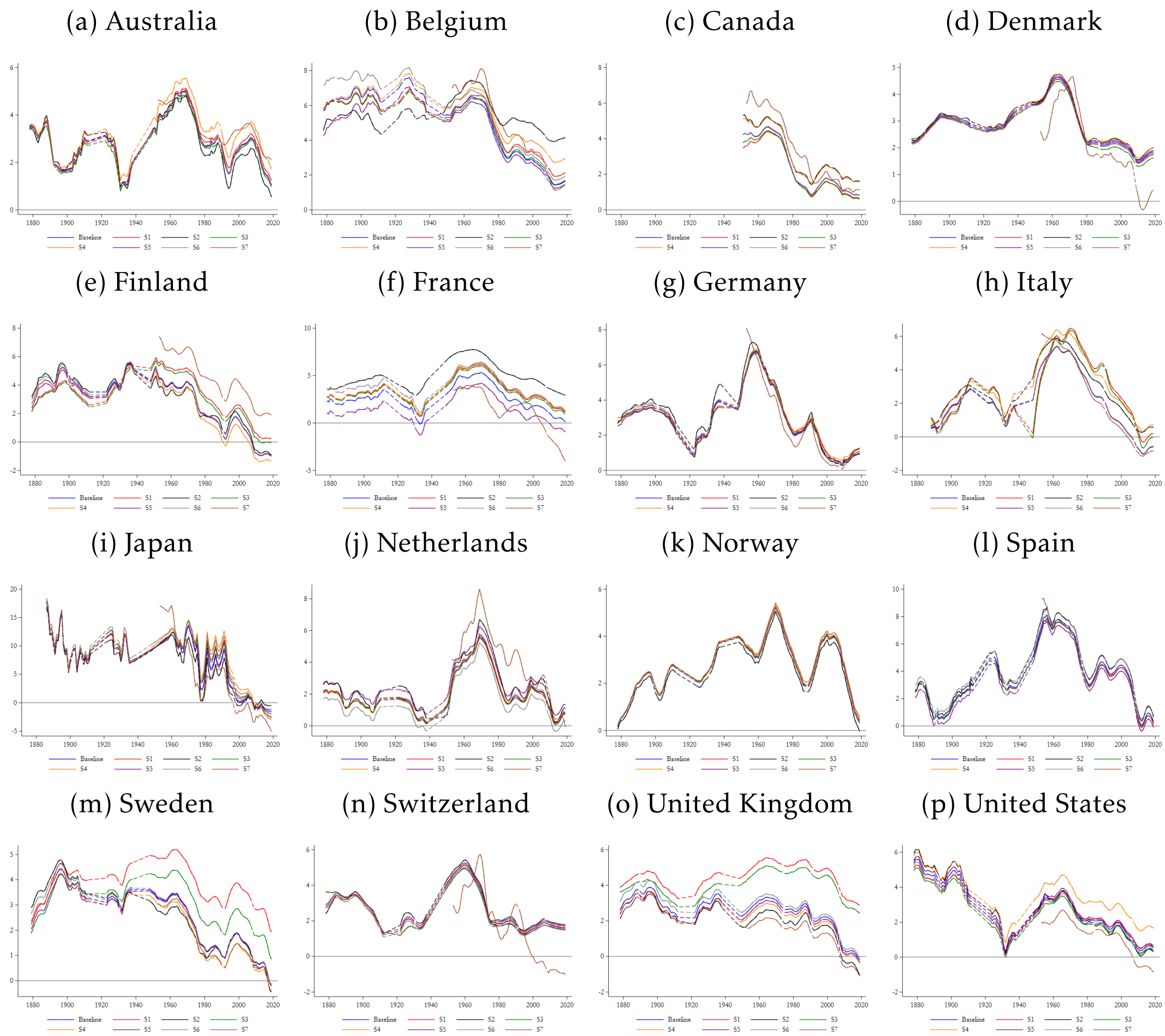
Notes: The figure plots the time series of the output gap estimates obtained in the baseline r^* estimation. The weighted mean is constructed using PPP GDP weights. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*.

Table A.3: Multivariate regressions of r^* excluding one country at a time

Excluded country:	(1) Australia	(2) Belgium	(3) Canada	(4) Denmark	(5) Finland	(6) France	(7) Germany	(8) Italy
Old-age dependency ratio	-0.128***	-0.122***	-0.130***	-0.123***	-0.129***	-0.127***	-0.134***	-0.137***
	(0.034)	(0.033)	(0.035)	(0.037)	(0.036)	(0.037)	(0.037)	(0.035)
Real GDP trend growth	0.399***	0.435***	0.389***	0.389***	0.382***	0.390***	0.406***	0.389***
	(0.061)	(0.049)	(0.061)	(0.063)	(0.064)	(0.065)	(0.068)	(0.063)
Public debt to GDP	-0.015**	-0.014*	-0.016**	-0.016**	-0.016**	-0.016*	-0.016**	-0.016**
	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.008)	(0.007)	(0.007)
Capital account openness	0.015***	0.012***	0.013***	0.012***	0.013***	0.013***	0.013***	0.013***
	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Observations	1,623	1,625	1,673	1,635	1,655	1,628	1,632	1,635
R^2	0.828	0.823	0.823	0.826	0.821	0.823	0.823	0.821
Excluded country:	(9) Japan	(10) Netherlands	(11) Norway	(12) Spain	(13) Sweden	(14) Switzerland	(15) UK	(16) United States
Old-age dependency ratio	-0.118***	-0.125***	-0.113***	-0.150***	-0.134***	-0.120***	-0.104**	-0.122***
	(0.033)	(0.033)	(0.035)	(0.030)	(0.034)	(0.037)	(0.038)	(0.037)
Real GDP trend growth	0.375***	0.391***	0.381***	0.350***	0.382***	0.408***	0.385***	0.379***
	(0.067)	(0.066)	(0.066)	(0.055)	(0.063)	(0.064)	(0.056)	(0.063)
Public debt to GDP	-0.011**	-0.015**	-0.017**	-0.015**	-0.016**	-0.017**	-0.022***	-0.017**
	(0.005)	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)	(0.006)	(0.007)
Capital account openness	0.010***	0.013***	0.011**	0.012**	0.012**	0.012***	0.014***	0.013***
	(0.003)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)
Observations	1,633	1,621	1,625	1,631	1,624	1,626	1,624	1,625
R^2	0.817	0.822	0.835	0.826	0.826	0.832	0.841	0.829

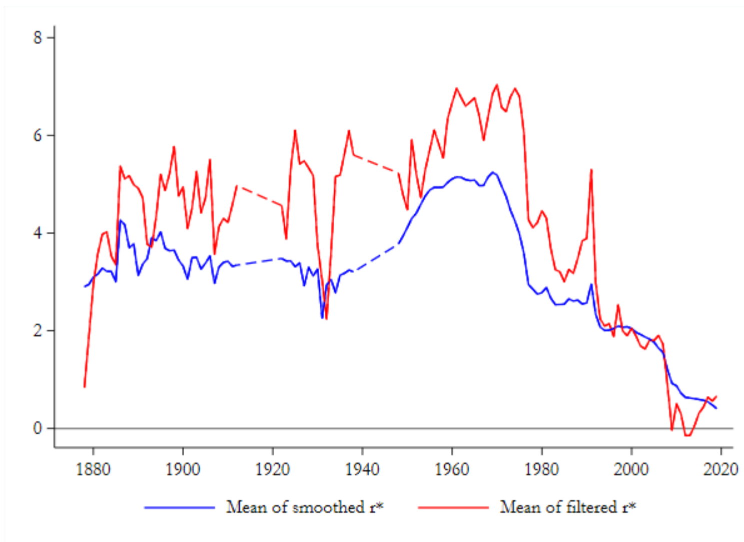
Notes: The table reports the results from regressions of the neutral interest rates on lagged determinants. All regressions include country and time fixed effects. Standard errors clustered at the country level are reported in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Figure A.4: Sensitivity checks for r^* estimates of individual countries
(Percent)



Notes: The Figure shows the mean of the baseline r^* estimates along with the results of sensitivity checks S1 to S7. S1: increase error term variances by 50 percent; S2: decrease error term variances by 50 percent; S3: force error term variances to fall by 50 percent after 1990; S4: force Phillips curve to flatten by 50 percent; S5: change the initial value of r^* by -50 percent; S6: change the initial value of r^* by 50 percent; S7: estimate only from 1950 onwards. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*.

Figure A.5: Smoothed and filtered r^* estimates
(Percent)



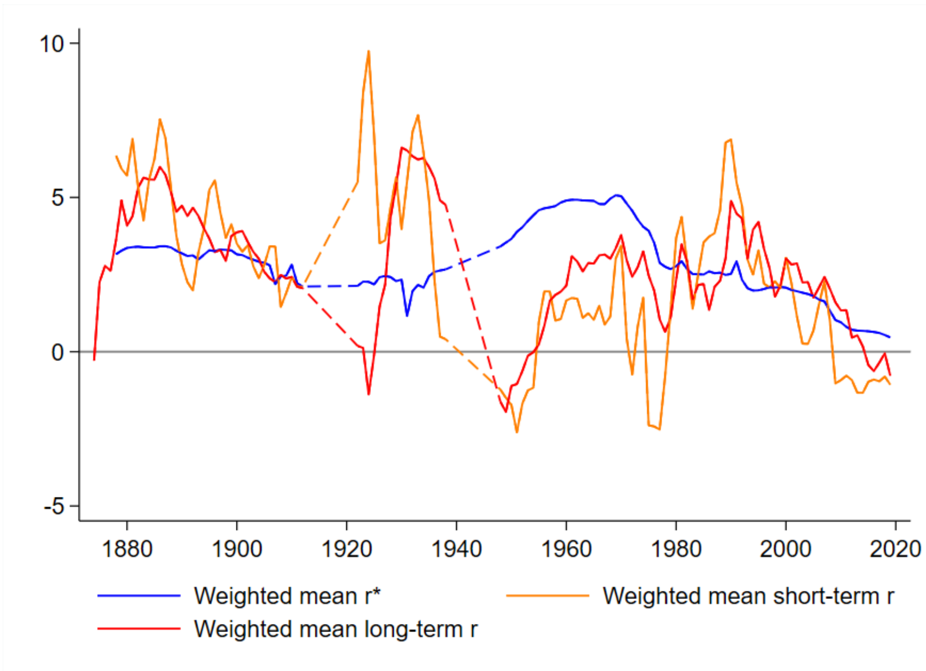
Notes: The Figure plots the mean of r^* extracted using a two-sided filter (smoothed) and a one-sided filter (filtered). Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*.

Table A.4: Multivariate regressions of r^* including current account

	(1)	(2)	(3)	(4)
Old-age dependency ratio	-0.128***	-0.123***	-0.120***	-0.118***
	(0.034)	(0.036)	(0.036)	(0.034)
Real GDP trend growth	0.389***	0.399***	0.405***	0.404***
	(0.061)	(0.063)	(0.062)	(0.062)
Public debt to GDP	-0.016**	-0.016**	-0.016**	-0.016**
	(0.007)	(0.007)	(0.007)	(0.007)
Capital account openness	0.013***	0.012***	0.012***	0.012***
	(0.004)	(0.004)	(0.004)	(0.003)
Openness × Post73				-0.003
				(0.010)
Current account balance		-0.001	-0.033	0.040
		(0.022)	(0.030)	(0.048)
CA × Post73			0.070*	-0.268*
			(0.039)	(0.140)
Openness × CA				-0.001
				(0.001)
Openness × CA × Post73				0.004**
				(0.002)
Period	1978-2020	1978-2020	1978-2020	1978-2020
Observations	1,741	1,700	1,700	1,700
R^2	0.825	0.827	0.829	0.832

Notes: The table reports the results from regressions of the neutral interest rates on lagged determinants. All regressions include country and time fixed effects. Standard errors clustered at the country level are reported in parentheses. Openness refers to capital account openness. CA refers to current account balance. Post73 refers to a dummy variable that is one if year is after 1973. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Figure A.6: r^* versus observed short-term and long-term real rates
(Percent)



Notes: The real long-term interest rate is constructed as the nominal yield on long-term government bonds (10y when available) minus the 10-year trailing moving average of inflation. The real short-term interest rate is constructed as the nominal central bank rate minus the four-year trailing moving average of inflation. Dashed line portions correspond to observations during the war periods (1913–1921 and 1939–1947), which are excluded from the estimation of r^*. Weighted means are constructed using PPP GDP weights. The correlation coefficient between the two measures is 6.8 percent; excluding outliers (i.e., removing observations outside the [-5,5] interval for r and r^*) brings the correlation to 18.3 percent.

PUBLICATIONS

Low for (Very) Long? A Long-Run Perspective on r* across Advanced Economies

Working Paper No. [WP/2023/###]