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The Fear Economy:
A Theory of Output, Interest, and Safe Assets

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Abstract: This paper presents a fear theory of the economy, based on the interplay between fear of rare disasters and the interest rate on safe assets. To do this, I study the macroeconomic consequences of government-administered interest rates in the neoclassical real business cycle model. When the government has the power to fix the safe real interest rate, the gap between the `sticky real safe rate' and the `neutral rate' can generate far-reaching aggregate distortions. When fear exogenously rises, the demand for safe assets rise and the neutral rate falls. If the central bank does not lower the safe rate by the same amount, savings rise leading to a decline in consumption and aggregate demand. The same mechanism works in reverse, when fear falls. Quantitatively, I show that a single fear factor can simultaneously (i) generate cross-correlations in output, labor, consumption, and investment consistent with the postwar US economy; and (ii) generates variation in equity prices, bond prices, and a large risk premium in line with the asset pricing data. Six novel insights emerge from the model: (1) actively regulating the safe interest rate (in both directions) can mitigate the fluctuations generated by fear cycles; (2) recessions will be deeper and longer when central banks accept the zero lower bound and are unwilling to use negative rates; (3) a commitment to use negative rates in recessions—even if never implemented—raises both the short- and long-run real neutral rates, and moderates the business cycle; (4) counter-cyclical fiscal policy can act as disaster insurance and be expansionary by reducing fear; (5) quantitative easing can be narrowly effective only when fear is high at the lower bound; and (6) when fear is high, especially at the lower bound, policies that boost productivity also help fight recessions.

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September 6, 2022

Abstract

This paper presents a fear theory of the economy, based on the interplay between fear of rare disasters and the interest rate on safe assets. To do this, I study the macroeconomic consequences of government-administered interest rates in the neoclassical real business cycle model. When the government has the power to fix the safe real interest rate, the gap between the 'sticky real safe rate' and the 'neutral rate' can generate far-reaching aggregate distortions. When fear exogenously rises, the demand for safe assets rise and the neutral rate falls. If the central bank does not lower the safe rate by the same amount, savings rise leading to a decline in consumption and aggregate demand. The same mechanism works in reverse, when fear falls. Quantitatively, I show that a single fear factor can simultaneously (i) generate cross-correlations in output, labor, consumption, and investment consistent with the postwar US economy; and (ii) generates variation in equity prices, bond prices, and a large risk premium in line with the asset pricing data. Six novel insights emerge from the model: (1) actively regulating the safe interest rate (in both directions) can mitigate the fluctuations generated by fear cycles; (2) recessions will be deeper and longer when central banks accept the zero lower bound and are unwilling to use negative rates; (3) a commitment to use negative rates in recessions—even if never implemented—raises both the short- and long-run real neutral rates, and moderates the business cycle; (4) counter-cyclical fiscal policy can act as disaster insurance and be expansionary by reducing fear; (5) quantitative easing can be narrowly effective only when fear is high at the lower bound; and (6) when fear is high, especially at the lower bound, policies that boost productivity also help fight recessions.

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1 Introduction

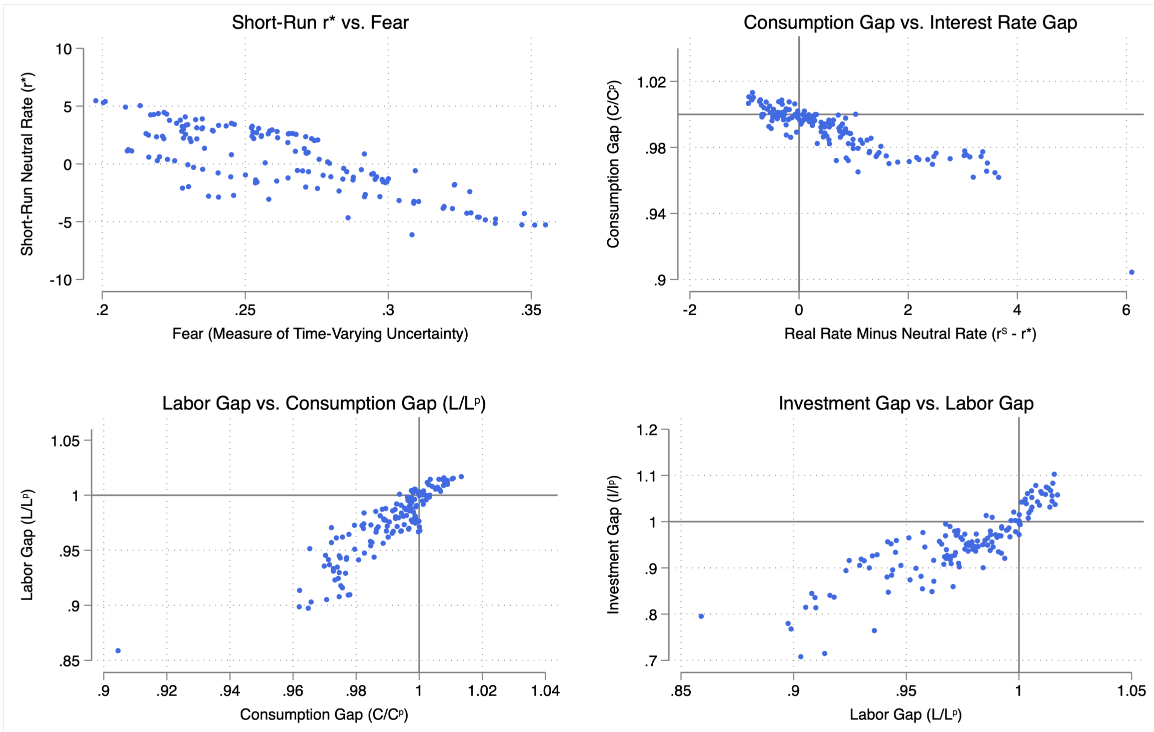
Models of the aggregate economy often aim to explain: (1) why consumption, investment, and labor co-move over the business cycle; and (2) how asset prices co-move with the aggregate economy. In the data, consumption, labor, and investment co-move positively with the business cycle. However, it is often challenging to generate such co-movements in business cycle models, without relying on technology shocks. Further, standard models used for evaluating monetary and fiscal policies, often lack an explicit role for investment, risk, or pricing of assets. Motivated by these issues, this paper presents a fear-based theory of the economy. The model predicts consistent cross-correlations in the key macro variables for the postwar US economy; and using a single fear factor explains several asset pricing puzzles and the joint behavior of key macro variables and asset prices in line with the data. The model also gives us six insights on how to expand the macro policy toolkit to manage the fluctuations generated by exogenous fear cycles.

The paper's theory of aggregate fluctuations is as follows (Figure 1). Fear is a central driver of the economy. When fear rises, it leads to a higher risk premium and raises the demand for safe assets. The neutral rate of interest falls. If the central bank does not lower the safe rate by the same amount, savings rise leading to a decline in consumption and aggregate demand. When aggregate demand weakens, firms scale back investment in capital stock—due to an investment accelerator. This generates a paradox of thrift effect, with both consumption and investment falling, lowering output. The same mechanism works in reverse when fear falls and the safe interest rate is not raised sufficiently, leading to exuberance.

I define *fear* (or uncertainty) as a measure of time-varying rare disaster risk—varying by the magnitude of the disaster. It is measured as the distance between expected consumption and the 'worst-case scenario' consumption.[1] Fear is higher when the perceived worst-case scenario is more severe. The economy's central driving force is the interplay between fear and interest rates on safe assets, which is set by the central bank.[2] From this perspective, the model is linked to two different views of business cycles: the "animal spirits" view (Keynes, 1936) that emphasizes the role of sentiment, and the "Austrian" view (Mises, 1924; Hayek, 1933) that emphasizes the distortionary role of interest rates set by central banks.

[1]This definition draws inspiration from the long line of work in clinical psychology (Ellis, 1962; Beck, 1979), where fear (or anxiety) is considered a reaction to perceived threat. If early humans did not fear and successfully avoid dangers and predators, our species would have likely died out long ago. However, this legacy also has a dark side. Left unchecked, the behavioral expression of fear and anxiety might easily consume a disproportionate portion of our energy and time budgets, to the detriment of crucial activity like obtaining food, reproduction, self-care, or work (Blanchard et al., 2011). A key cognitive process through which excessive fear or clinical anxiety persists is the tendency to overestimate threats (Abramowitz and Blakey, 2020). Such tendency to overestimate threats at an individual-level can have important aggregate implications for the economy, especially with anxiety disorders being the most common mental illness, with symptoms in over 1 in 3 US adults (Vahratian et al., 2021).

[2]An analogy can be illustrative. If a child falls when learning to ride the bicycle, it may develop a fear. The natural response to fear is escape, which is a type of 'safety behavior'. If the parent accommodates the fear and does not encourage the child to re-engage with the feared situation, it may lead to avoidance and chronic fear. Similarly, when the economy is hit by a shock that raises fear, the natural response of agents is flight-to-safety behavior, which raises the demand for safe assets and lowers the neutral rate. Then, the role of the central bank, akin to a 'reasonable parent', is to encourage market participants to re-engage in normal risk-taking behavior by lowering the interest rate on the safe assets. When it fails to do so, it accommodates safety behavior and creates insufficient incentives to spend, which lowers output and can cause fear to persist. By contrast, when there is low fear and thus a higher neutral rate, the central bank may have to curb risky behavior by raising interest rates.



Note: Sample period is Q1:1983-Q1:2022. Potential values are denoted by p. Panel (a) depicts the negative relationship between the neutral rate and the fear factor (derived in this paper). Panel (b) depicts the negative relationship between the interest gap (i.e., actual minus the short-run real rate) and the consumption gap. Panel (c) depicts the positive relationship between the consumption gap and the labor gap. And Panel (d) depicts the positive relationship between the labor gap vs. the investment gap.

Figure 1: Mechanism/Stylized Facts: $Fear \uparrow \Rightarrow r^* \downarrow$, $r^S - r^* \uparrow \Rightarrow \frac{C}{C^P} \downarrow \Rightarrow \frac{L}{L^P} \downarrow, \frac{Y}{Y^P} \downarrow \Rightarrow \frac{I}{I^p} \downarrow$

The model embeds these ideas into the textbook real business cycle model. The key departure from the textbook model is that the government has the power to fix the real interest rate on safe assets. For the government to have the ability to administer the real rate on safe assets it is sufficient to have a rigidity in the price of the numeraire good, for example due to money illusion. Alternatively, the power may arise due to other price rigidities in the economy. The underlying mechanism is taken as exogenous in the main text, while an extension in Appendix B presents an example.[3]

The only other deviation from the textbook model is to allow the labor supply elasticity to be higher in the aggregate than at the individual level. There are various ways to do this, such as using indivisible labor, labor externalities, etc. My setup relies on an externality in labor supply as originally studied by Benhabib and Farmer (2000), such that the people are more willing to work when everyone else is working.

[3]Since prices are typically denominated in units of the numeraire, the price of the numeraire is usually one. So rigidity in the numeraire changes its value relative to all other goods. As per the classification developed by Reis and Watson (2007), 'soft theories of nominal rigidities' assume that prices respond imperfectly to some shocks, leading to monetary non-neutrality, but are able to respond immediately to some other shocks. The classic example is the Lucas (1972) model, where all firms respond immediately to anticipated money changes, but differentially with respect to unanticipated money shocks. Another soft theory is money illusion, dating back to Hume (1752). By contrast, 'strict theories of nominal rigidities' assume that there are always some prices in the economy that cannot respond to current conditions (Fischer, 1977; Sheshinski and Weiss, 1977; Taylor, 1980; Calvo, 1983; Mankiw and Reis, 2002). Overall,Reis and Watson (2007) find evidence in favor of the softer theories.

The paper has three main contributions. First, I develop a theory of aggregate fluctuations, which delivers consistent co-movements in output, consumption, labor and investment—without relying on strict theories of nominal rigidities. In line with recent work (Basu and Bundick (2017); Basu et al. (2021); Angeletos and Lian (2022); Di Tella and Hall (2022)), the model overcomes the co-movement problem faced by a large class of New Keynesian models in response to risk shocks—as originally highlighted by Barro and King (1984). (See Appendix Table A1 for a comparison with other theories.)

Second, I study a novel real rigidity that acts as the key friction in the economy: sticky real interest rates for safe assets. In response to 'fear' shocks (i.e. rise in uncertainty), this friction generates an excess demand for safe assets and dampens economic activity. In contrast to recent work on safe assets (Caballero and Farhi, 2018), the demand for safe assets plays an important role even away from the zero lower bound.

Third, I revive Martin Weitzman's (2007; 2009) notion of subjective uncertainty. A key focus of the literature, building on the work of Rietz (1988); Barro (2006), has been to evaluate the impact of the objective risk of rare disasters in consumption. By contrast, in my framework, tail risks depend on the subjective beliefs of agents. This allows me to back out a model-implied fear factor given observed data. This fear factor can jointly explain both macro and asset pricing fluctuations, and address several asset pricing puzzles. The key macro-finance implications (Section 4) are:

1. A common fear factor provides a unified explanation for the classic asset pricing puzzles, including the equity premium puzzle, the riskfree rate puzzle, and the equity volatility puzzle. Moreover, the observed variation in fear explains the variation in the risk premium over the business cycle.

2. The model highlights that once we account for the bias from persistent output gaps, the long-run neutral rate may have declined by only 0.5 percentage points since the 1980s. However, at the zero lower bound, the long-run neutral rate can decline persistently after recessions due to a sharp rise in long-term fear, which can take several years to unwind.

3. The model generates consistent variation in bond yields over time; and predicts that recessions follow an inverted yield curve because it is associated with higher fear in the future, which is contractionary.

4. The model gives us a novel representation of the price-dividend ratio and expected returns—linked to macro variables and long-run risk. Consistent with the data, I establish that the dividend-price ratio predicts equity returns, with the predictive power greater for longer horizons horizons due to the high degree of persistence in output gap, fear, and the neutral rate of interest.

Six novel insights emerge from the model (Section 5). First, actively regulating the safe interest rate (in both directions) can mitigate the fluctuations generated by fear cycles. Second, recessions will be deeper and longer with an amplified impact of fear, if central banks accept the zero lower bound and are unwilling to use negative rates. Third, a commitment to use negative interest rate policy in recessions raises interest rates over the entire yield curve and moderates the business cycle (even without having to implement negative rates). Fourth, policies to increase the counter-cyclicality of fiscal policy are expansionary; and the effects are amplified at the lower bound or when fear is high. Fifth, quantitative easing is only narrowly effective when fear is high at the lower bound—by satisfying fear-driven demand for short safe debt and by possibly constraining fiscal borrowing in stressed episodes. And sixth, when fear is high, especially at the lower bound, policies that boost productivity also have positive multipliers for fighting recessions.

1.1 Basic Intuition of the SWIM Model

Conceptually, the model I modify the textbook real business cycle (RBC) model in only two ways. First, in a standard separable utility function, I allow the disutility of work to be lower when aggregate employment is higher (as in Benhabib and Farmer (2000)). Since both work and leisure are often a social activities, people may be more willing to work when everyone else is working. Similarly, the cost of working may be lower (with greater work satisfaction) when others are also working—due to better public transport or carpooling options, flexibility in work schedules, availability of suitable jobs, improved match quality, etc. This first modification ensures that consumption and employment are complements in the aggregate. Even with this modification, the aggregate equilibrium conditions look identical to the textbook RBC model with separable utility. However, the parameter governing the aggregate labor supply elasticity conceptually differs from the notion of Frisch labor elasticity at the individual level, due to the nonconvexity from the labor externality.

Second, in such a framework, I show that when the central bank's has the power to exogenously fix the real interest rate on safe assets, it distorts the consumption-savings decisions in the economy. Effectively, the safe interest rate set by the central bank (given the exogenous level of fear) becomes the central driver of the economy, with in turn generates positive co-movements in consumption, labor, investment, and output.

To quantitatively assess the model's predictions for asset prices, I introduce an interest rate rule for the central bank and a stochastic process for technology with time-varying volatility (Section 3). Instead of modeling disaster risk using objective probabilities from historical episodes as popularized by the rare disasters literature (Rietz (1988); Barro (2006)), the model revives the subjective uncertainty framework developed by Weitzman (2007; 2009). Thus, fear corresponds to the tail risk *perceived* by agents in the economy, which can be time-varying. These assumptions allow me to quantify the model-implied fear, and test the model's predictions using macro-financial data.

I refer to the model as the 'SWIM model', as it can be graphically represented using four curves: demand for safety (S), fear (W), investment-savings (IS), and monetary policy (MP). (As a mnemonic device, W stands for worry.) This representation builds on the Old Keynesian IS/LM framework developed by Hicks (1937) and extended by Romer (2000). In the SWIM model, the goods and labor market equilibrium depends on the IS and MP curves. And, instead of the money market, there is a market for safe assets or government bonds, which determines the neutral rate at the intersection of the S and W curves.

1.2 Related Literature

The paper builds on a long line of work at the intersection of macro and finance. The underlying core of the SWIM model is a real business cycle model with market clearing, building on the classic work of Kydland and Prescott (1982); Long and Plosser (1983). While the model deviates from the New Keynesian framework that builds on the insights of Mankiw (1985); Akerlof and Yellen (1985), the central ideas are related and Keynesian in nature. The model relies on a sticky real interest rate to generate aggregate fluctuations. For the government to have such an ability to administer the real rate on safe assets, there must be some underlying mechanism of nominal rigidities (which I take as exogenous). From this perspective, the paper is potentially consistent with both New Keynesian theories of nominal rigidities, and a broader set of price rigidity theories such as those based on nominal illusion (Hume, 1752) or imperfect information (Lucas, 1972) (which Reis and Watson (2007) call 'soft theories of price rigidities').

The paper is also related to the work of Benhabib and Farmer (1994); Farmer and Guo (1994); Benhabib and Farmer (1999, 2000), who highlighted the role of labor externalities. In contrast to that literature, however, this paper's theory of aggregate fluctuations does not rely on increasing returns or on self-fulfilling beliefs (sunspots). Instead, the SWIM model relies on a sticky safe rate—which for a given level of fear pins down the unique equilibrium and becomes the key distortion for the consumption-savings decision.

In response to uncertainty or investment shocks, New Keynesian models can counterfactually predict a negative unconditional correlation between consumption growth and investment growth. The basic idea is that when the marginal product of investment rises, household savings rise, leading to a corresponding decline in consumption. Ascari et al. (2019) call this the "Barro-King curse", highlighting that models built on a real business cycle setup must rely on technology shocks to solve the co-movement problem. They solve this problem by adding intermediate inputs to the production process. A related co-movement problem arises in response to risk shocks. Essentially, an increase in risk premia can raise precautionary savings, potentially generating a counterfactual negative correlation between investment and consumption. As summarized in a table in Appendix A1, several papers address this co-movement problem by using risk-driven shocks by relying on endogenous variations in markups due to sticky prices to generate simultaneous declines in consumption and investment in response to a risk shock (Ilut and Schneider, 2014; Fernández-Villaverde et al., 2015; Basu and Bundick, 2017; Bayer et al., 2019; Caballero and Simsek, 2020). By contrast, Basu et al. (2021); Di Tella and Hall (2022); Angeletos and Lian (2022), generate co-movement with risk shocks without sticky nominal prices. In particular, Basu et al. (2021) emphasize that risk shocks can lead to a flight-to-safety behavior, which may be key to solving the co-movement problem. Their model introduces search frictions in the labor market, which generates reallocations towards safer but less productive jobs in response to risk shocks. In line with this emerging literature, the SWIM model emphasizes the importance of the fear/risk channel. However, my model addresses the co-movement problem by relying on an labor externality based on the preferences developed by Benhabib and Farmer (2000).

Further, the paper draws on the long line of work emphasizing the importance of safe assets including Caballero (2006); Caballero and Krishnamurthy (2009); Bernanke et al. (2011); Gorton (2010); Stein (2012); Gennaioli et al. (2012); Gorton (2017). The paper also builds on my earlier work (Agarwal, 2012), which established the link between supply of safe assets and risk premium, and studied how frictions in securitization can reduce the net supply of safe assets. Caballero and Farhi (2018) study the macro implications of safe asset shortages at the ZLB in an overlapping generations model with Knightian uncertainty. Kocherlakota (2015) and Caballero et al. (2016) discuss environments where issuing safe public debt can stimulate the economy in a liquidity trap. Several authors have studied the asset pricing implications of the specialness of safe public debt (Gürkaynak et al., 2007; Greenwood and Vayanos, 2010; Krishnamurthy and Vissing-Jorgensen, 2012). While the literature has largely emphasized safe assets *shortages* (Caballero et al. (2017)), the SWIM framework focuses on fear *cycles* driven by time-varying safety. For instance, in Caballero and Farhi (2018), safe asset shortages are benign when safe interest rates are positive, with the safe asset shortages having a perverse effect on aggregate output only at the zero lower bound. Further, the literature typically models the demand for safety arising from a convenience yield or Knightian uncertainty. The SWIM model, by contrast, is built on a neoclassical framework with standard CRRA utility preferences prevalent in macro and asset pricing (Lucas, 1978; Mehra and Prescott, 1985; Campbell, 2003; Cochrane, 2009). Then, in line with Weitzman (2007; 2009), the demand for safety in this model arises from exposure to disaster risk. Keynes (1936) was among the first to draw such a connection between safe assets and fear.[4] The model's link between safe assets and the risk premium also draws on rare disasters literature (Rietz (1988); Barro (2006); Barro and Jin (2011); Gourio (2012); Gabaix (2012); Barro et al. (2014)).

The definition of fear used in this paper draws on the psychology literature on fear and anxiety starting from Ellis (1962); Beck (1979). The ideas can also be traced to the Enlightenment philosophers, Hobbes (1651) and Locke (1689), such that a challenge of modern economies is to consistently ensure that the environment is *safe*. And more safe societies will have lower fear.

Sticky safe rates have been indirectly discussed in the context of monetary policy rules (Taylor, 1993), liquidity traps (Krugman, 1998; Werning, 2011; Eggertsson and Krugman, 2012; Eggertsson et al., 2019) and breaking the zero lower bound (Agarwal and Kimball, 2015, 2019). In the SWIM model, the central bank creates distortion in the consumption-savings decision—even away from the lower bound—by fixing the real safe rate different from the neutral rate. Thus, their actions can sometimes be inept while at other times a stabilizing force, as emphasized by Friedman and Schwartz (1963).

[4]Responding to the critics of the general theory, in the Quarterly Journal of Economics, Keynes (1937) wrote: "*Why should anyone outside a lunatic asylum wish to use money as a store of wealth? Because, partly on reasonable and partly on instinctive grounds, our desire to hold Money as a store of wealth is a barometer of the degree of our distrust of our own calculations and conventions concerning the future. Even tho this feeling about Money is itself conventional or instinctive, it operates, so to speak, at a deeper level of our motivation. It takes charge at the moments when the higher, more precarious conventions have weakened. The possession of actual money lulls our disquietude; and the premium which we require to make us part with money is the measure of the degree of our disquietude.*"

2 The Benchmark Case

This section presents the benchmark model, which augments the standard real business cycle model with an externality (5), and an exogenous sticky safe rate set by a central bank (12).

2.1 Preliminaries

There are four sets of actors in the economy: households, a representative firm, the government, and a central bank. Aggregate production is given by a Cobb-Douglas function. As per the aggregate resource constraint, output, Y_t, is used for consumption, C_t, investment, I_t, or government spending G_t. Then:

$$\begin{aligned} C_t + I_t + G_t &= Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \\ c_t + i_t + g_t &= 1 \end{aligned} \tag{1}$$

with c_t, i_t, g_t representing the values of C_t, I_t, G_t scaled by Y_t.

I assume that the government exogenously keeps g_t fixed, such that for a given level of output, we have $G_t = g_t Y_t$. Then, in each period g_t and A_t are given exogenously, while K_t is fixed due to previous period's investment choices. The endogenous variables are $\{L_t, C_t, I_t\}$.

I will study an RBC-type setup. There there will be a unique solution $\{L_t^p, C_t^p, I_t^p\}$ in the frictionless equilibrium (superscript p denotes potential). The only distortion will be on the safe interest rate, r_t. The central bank can exogenously choose a safe real rate r_t^S, which may potentially deviate from the frictionless 'neutral interest rate', r_t^*. Thus, there will be a continuum of unique solution sets $\left\{L_t\left(r_t^S\right), C_t\left(r_t^S\right), I_t\left(r_t^S\right)\right\}$ depending on r_t^S, with the frictionless solution found at $r_t^S = r_t^*$. Then, we get the following definitions.

Definition. *Potential labor, potential consumption, and potential income, $\{L_t^p, C_t^p, I_t^p\}$, represent the frictionless equilibrium values of labor, consumption, and investment, i.e., when the the safe interest, r_t, is freely determined. Potential output is the corresponding level of output: $Y_t^p \equiv A_t K_t^{\alpha} \left(L_t^p\right)^{1-\alpha}$.*

With these definitions, the notional output gap is given by:

$$O_t \equiv \frac{Y_t}{Y_t^p} = \left(\frac{L_t}{L_t^p}\right)^{1-\alpha} \tag{2}$$

Let the scaled values of potential consumption and investment be: $c_t^p \equiv C_t^p / Y_t^p$ and $i_t^p \equiv I_t^p / Y_t^p$. In the model below, $\{L_t^p, c_t^p, p_t^p\}$ will evolve exogenously (as is shown in Appendix A4). I will assume that these are observed and known to all agents (akin to the real-world concept of NAIRU). Then let's assume:

$$\begin{aligned}\Delta L_t^p &= \xi_{L,t} \\ \Delta g_t &= \xi_t \\ \frac{c_t^p}{i_t^p} &= \phi_t\end{aligned} \tag{3}$$

with L_0^p and g_0 known, and $c_t^p + i_t^p + g_t = 1$. (Later, I will setup the model such that $\xi_{L,t} = 0$).

Then, by substituting $i_t^p = \frac{1-g_t}{1+\phi_t}$ into $\frac{I_t}{I_t^p} = \frac{i_t Y_t}{i_t^p Y_t^p}$, the aggregate resource constraint (1) implies:

$$\frac{I_t}{I_t^p} = (1+\phi_t)\left(\frac{Y_t}{Y_t^p}\right) - \phi_t\left(\frac{C_t}{C_t^p}\right) \tag{4}$$

In what follows, I will derive expressions for L_t/L_t^p, C_t/C_t^p and I_t/I_t^p as a function of the output gap, Y_t/Y_t^p and the exogenously given c_t^p, i_t^p. The decentralized problem is described below.

2.2 The Government

The government's spending G_t is financed by lump sum tax T_t or by issuing riskless debt D_t, which pays interest gross return R_t (with $r_t \equiv \ln R_t$). Its spending path $g_t \equiv G_t/Y_t$ is exogenous as per (3).

2.3 Households

Individual-Level Decisions. There is a continuum of households i, who collectively own the firm. Their income is a sum of profits, $\Pi_{i,t}$, interest income on bond holdings, $D_{i,t-1}$, and labor income for given wage W_t. The resources can be used for consumption, or to buy additional government bonds. Then, each household i maximizes the present discounted value of lifetime consumption $C_{i,t}$ less the disutility of labor $L_{i,t}$:

$$\max_{C_{i,t},L_{i,t},D_{i,t}} E_t \sum_{t=0}^{\infty} e^{-\rho t}\left[\frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \left(\frac{\upsilon_t}{L_t^{\omega+\chi}}\right)\cdot\frac{L_{i,t}^{1+\omega}}{1+\omega}\right] \tag{5}$$

subject to

$$C_{i,t} + D_{i,t} \quad \leq \quad W_t L_{i,t} + \Pi_{i,t} - T_{i,t} + R_{t-1} D_{i,t-1} \tag{6}$$

with $C_{i,t}$, $L_{i,t}$, and L_t representing individual consumption, individual labor supply, and aggregate employment respectively. The parameters of the utility function are: the coefficient of relative risk aversion $\gamma > 0$, the discount rate $e^{-\rho}$, the relative cost of labor supply υ_t, the individual-level Frisch elasticity $1/\omega \geq 0$ and a measure of aggregate externality $\chi > 0$. The term $L_t^{\omega+\chi}$ measures the externality from aggregate employment on individual labor supply, with the disutility of labor depending inversely on aggregate employment, L_t. Such a utility function was introduced by Benhabib and Farmer (2000), and subsequently studied by Suarez (2008); Fève et al. (2011); Azariadis et al. (2013). Here υ_t is time-varying, which we will specify later to ensure a balanced growth path, as in Guvenen and Rendall (2015) and ensure that $\Delta L_t^p = 0$.

The household's first order conditions yield:

$$\begin{aligned} C_{i,t}^{-\gamma} &= \exp\left(r_t - \rho\right) \cdot E_t\left[C_{i,t+1}^{-\gamma}\right] \\ C_{i,t}^{-\gamma} &= \frac{\upsilon_t L_{i,t}^{\omega}}{W_t L_t^{\omega+\chi}} \end{aligned} \tag{7}$$

The first order conditions imply that the elasticity of individual labor supply with respect to aggregate employment is given by: $1 + \frac{\chi}{\omega}$. This is bounded below by 1, given our parameter restrictions.[5]

Aggregation. Aggregate consumption and employment are: $C_t = \sum_i C_{i,t}$ and $L_t = \sum_i L_{i,t}$. Similarly, $T_t = \sum_i T_{i,t}$, $D_t = \sum_i D_{i,t}$. Since all households are identical, we have $\frac{C_{i,t}^{-\gamma}}{L_{i,t}^{\chi}} = \frac{C_t^{-\gamma}}{L_t^{\chi}}$ for all i. Thus, we get:

$$\begin{aligned} C_t^{-\gamma} &= \exp\left(r_t - \rho\right) \cdot E_t\left[C_{t+1}^{-\gamma}\right] \\ C_t^{-\gamma} &= \frac{\upsilon_t L_t^{-\chi}}{W_t} \end{aligned} \tag{8}$$

Thus, there is a complementarity between aggregate consumption and labor, with elasticity $\gamma/\chi > 0$. That is, for a given wage rate, a rise in consumption is associated with a rise in aggregate employment.

2.4 The Firm

There is a representative firm, owned by the households. And the firm owns the capital stock.

The Firm's Problem. The law of motion of capital is $K_{t+1} = I_t + (1-\delta) K_t$, with depreciation δ. The production function is Cobb-Douglas with with capital share $0 > \alpha > 1$:

$$Y_t \quad = \quad A_t K_t^{\alpha} L_t^{1-\alpha} \tag{9}$$

The firm chooses labor and investment to maximize the present discounted value of profits in each period, $\Pi_t \equiv Y_t - W_t L_t - I_t$, with $\Pi_t = \sum_i \Pi_{i,t}$. The discount rate $m_t \equiv e^{-\rho t}\left(C_t/C_0\right)^{-\gamma}$ is based on the household's stochastic discount factor, $M_{t+1} \equiv m_{t+1}/m_t = e^{-\rho}(C_{t+1}/C_t)^{-\gamma}$. The firm's problem is given by:

$$\max_{L_t, K_{t+1}} E_t \sum_{t=0}^{\infty} m_t\left[Y_t - W_t L_t - K_{t+1} + (1-\delta) K_t\right] \tag{10}$$

[5]To see this, take logs of (7), to get $\ln L_{i,t} = -\frac{1}{\omega}\ln\upsilon_t - \frac{\gamma}{\omega}\ln C_{i,t} + \frac{1}{\omega}\ln W_t + \left(1+\frac{\chi}{\omega}\right)\ln L_t$. The elasticity w.r.t. aggregate labor supply is $1 + (1/\omega)\cdot\chi$, with Frisch elasticity $1/\omega$. One advantage of this setup is that we do not have to rely on a large Frisch elasticity, as several macro models do. Micro studies tend to find small estimates of the Frisch elasticity in the range of 0.0–0.4 ((Chetty et al., 2013; Whalen and Reichling, 2017)), with recent estimates of around 0.02 (Martinez et al., 2021). Thus, if $1/\omega \approx 0$, and χ is not relatively large, we get the individual labor supply elasticity w.r.t. aggregate employment to be just above 1. In the aggregate, we get: $\ln L_t = \frac{1}{\chi}\ln\upsilon_t + \frac{\gamma}{\chi}\ln C_t - \frac{1}{\chi}\ln W_t$, with the consumption-labor elasticity given by $\gamma/\chi > 0$.

With gross return on capital as $R_{K,t+1} \equiv 1 + \alpha Y_{t+1}/K_{t+1} - \delta$, the firm's first order conditions are:

$$W_t = (1-\alpha)\frac{Y_t}{L_t} \tag{11}$$
$$1 = E_t\left[M_{t+1}R_{K,t+1}\right]$$

2.5 The Central Bank

Central Bank Rate. The central bank has the power to fix the safe real rate, r_t, at an exogenous level $r_t^S \equiv \ln R_t^S$. The mechanism by which such a power exists will depend on the underlying theory of nominal rigidities (money illusion, sticky information, etc.), which for now is taken to be exogenous. Then:

$$r_t = r_t^S \tag{12}$$

Shadow Neutral Rate. The shadow neutral rate, r_t^*, is the interest that would prevail when agents expect no central bank intervention today or tomorrow. When $r_t^S = r_t^*$, we have $Y_t = Y_t^p$ or equivalently, $O_t = 1$. Then, setting $r_t = r_t^*$ and $O_t = E_t(O_t) = 1$ in (8), and taking logs we get:

$$r_t^* = \rho - \ln E_t\left[(C_{t+1}/C_t)^{-\gamma} \mid O_t = E_t O_{t+1} = 1\right] \tag{13}$$

2.6 Conjectured Solution

If $r_t^S = r_t^*$ for all t, then the equilibrium conditions are identical to the basic RBC setup, with the labor supply elasticity adjusted for the labor externality (i.e. having $-\chi$ instead of ω in (8)). For situations in which r_t^S deviate from r_t^*, I solve the problem around the exogenously given potential values $\{Y_t^p, C_t^p, I_t^p\}$, by the conjecture and verify method. I conjecture that the solution to the problem is such that, in the aggregate, the consumption gap is proportional to the output gap for exogenous and known C_t^p, Y_t^p:

$$\frac{C_t}{C_t^p} = \left(\frac{Y_t}{Y_t^p}\right)^{\eta} \tag{14}$$

Here η is a coefficient of my conjectured solution, which will be pinned down when I solve the problem. The choice of η must satisfy the optimality conditions for the households and firms in (8) and (11). Substituting C_t from (14) into labor supply condition in (8), which requires $C_t^{-\gamma} = \upsilon_t L_t^{-\chi}/W_t$, and substituting the value for W_t from (11), we get:

$$\left[\left(\frac{Y_t}{Y_t^p}\right)^{\eta} c_t^p Y_t^p\right]^{-\gamma} = \frac{\upsilon_t L_t^{1-\chi}}{(1-\alpha)\, Y_t} \tag{15}$$

When $Y_t/Y_t^p = 1$, this relationship implies: $(c_t^p Y_t^p)^{-\gamma} = \frac{\upsilon_t (L_t^p)^{1-\chi}}{(1-\alpha)Y_t^p}$ Plugging this back into (15), we get: $Y_t/Y_t^p = (L_t/L_t^p)^{\frac{1-\chi}{1-\gamma\eta}}$. Separately, by (2), the output gap is given by: $Y_t/Y_t^p = (L_t/L_t^p)^{1-\alpha}$. Therefore, in order for my conjecture to be verified, η must satisfy: $\frac{1-\chi}{1-\gamma\eta} = 1 - \alpha$. Or, equivalently:

$$\eta = \frac{\chi - \alpha}{\gamma (1-\alpha)} \tag{16}$$

Thus, (14) satisfies the solution as long as (16) holds. The intuition is as follows. In an RBC-type setup, let $\{Y_t, C_t\}$ and $\{Y_t^p, C_t^p\}$ represent the optimal solutions at two different safe interest rates, r_t^S and r_t^* respectively. Then, the relationship between these two solutions must satisfy (14) with η given by (16).

2.7 Aggregate Investment

Then, we can replace $C_t/C_t^p = (Y_t/Y_t^p)^\eta$ from (14) into the resource constraint (4) to get:

$$\frac{I_t}{I_t} = (1+\phi_t)\left(\frac{Y_t}{Y_t^p}\right) - \phi_t \left(\frac{Y_t}{Y_t^p}\right)^\eta \tag{17}$$

Recall that output gap is: $O_t \equiv Y_t/Y_t^p$. Substituting this, and multiplying both sides by $I_t^p \equiv i_t^p Y_t^p$, we get:

$$I_t = \left[(1+\phi_t) O_t - (\phi_t) O_t^\eta\right] \cdot i_t^p \cdot Y_t^p \tag{18}$$

Thus, there is a procyclical relationship between the investment and the output gap (as long as $\eta < 1$). I call this relationship the *investment accelerator*, implied by the resource constraint (4) combined with (14).

2.8 Euler Equation and the Output Gap

As per (14), we have $C_t = C_t^p O_t^\eta$. Substituting this into the household's Euler equation (8) and with r_t^S given by (12), we can solve for the output gap as:

$$O_t^{-\gamma\eta} = \exp\left(r_t^S - \rho\right) \cdot E_t \left(\frac{C_{t+1}^p}{C_t^p} O_{t+1}^{-\eta}\right)^{-\gamma} \tag{19}$$

2.9 Equilibrium

To determine O_t and close the model, we will need to specify the shock processes, which I do in the next section. Before doing so, however, we can study the co-movement properties of the key variables in the model. Suppose there exists a unique $O_t = \hat{O}_t$, that satisfies the Euler equation (19). Then taking $\hat{O}_t$ as given, in a competitive equilibrium, the prices and allocations solve the optimization problems and all markets clear. Three markets need to clear in equilibrium: the government debt market (D_t), the labor market (L_t), and the goods market (Y_t), with their respective prices given by $1/R_t^S$, W_t, and unity. The equilibrium conditions are in Online Appendix A2, and Proposition 1 presents the key results.

Proposition 1. *If there is a unique $O_t = \hat{O}_t$ that satisfies the Euler equation (19), then there exists a Walrasian competitive equilibrium with $\eta = \frac{\chi-\alpha}{\gamma(1-\alpha)}$. In equilibrium, we have:*

$$\begin{aligned} \textit{Consumption}: \quad C_t &= \hat{O}_t^{\eta} \cdot c_t^p \cdot Y_t^p \\ \textit{Investment}: \quad I_t &= \left[(1+\phi_t)\,\hat{O}_t - (\phi_t)\,\hat{O}_t^{\eta}\right] \cdot i_t^p \cdot Y_t^p \\ \textit{Employment}: \quad L_t &= \left(\hat{O}_t\right)^{\frac{1}{1-\alpha}} \cdot L_t^p \\ \textit{Output}: \quad Y_t &= \hat{O}_t \cdot Y_t^p \end{aligned} \tag{20}$$

Potential values for output Y_t^p, investment share $i_t^p = 1 - c_t^p - g_t$, and consumption share $c_t^p \equiv \phi_t i_t^p$ are determined based on the exogenous processes for g_t, ϕ_t, L_t^p in (3) and for technology A_t. Two parameters govern the co-movements: the capital share α, and the consumption elasticity η.

Proof. See Online Appendix A3. □

For intuition, I interpret O_t as a measure of aggregate demand. When households' desire to save goes up exogenously (as per the Euler equation), it lowers consumption and weakens aggregate demand, which in turn lowers investment and employment. Thus, consumption, investment, and labor supply co-move positively.

2.10 Calibration

This calibration exercise focuses on co-movements in the US economy. I feed the observed output gap (O_t) and the exogenous trends in potential values of consumption and investment (c_t^p, i_t^p) with $\phi_t = c_t^p / i_t^p$ into (20), and examine whether the model's predictions can match the observed co-movement in consumption, investment, and labor. Figure 2 presents the correlations (comparing the model to the data), while Figure 3 presents the results. There are only two parameters that govern contemporaneous co-movements: η, α. I choose $\eta = 0.5$ for the consumption elasticity, and $\alpha = 0.33$ for the capital share of income. These parameter values impose a restriction on the underlying parameters, γ and χ, since $\eta = \frac{\chi-\alpha}{\gamma(1-\alpha)}$. (For the asset pricing applications in the next section, we will use $\gamma = 4$, which implies that we need to choose $\chi = 1.67$.)

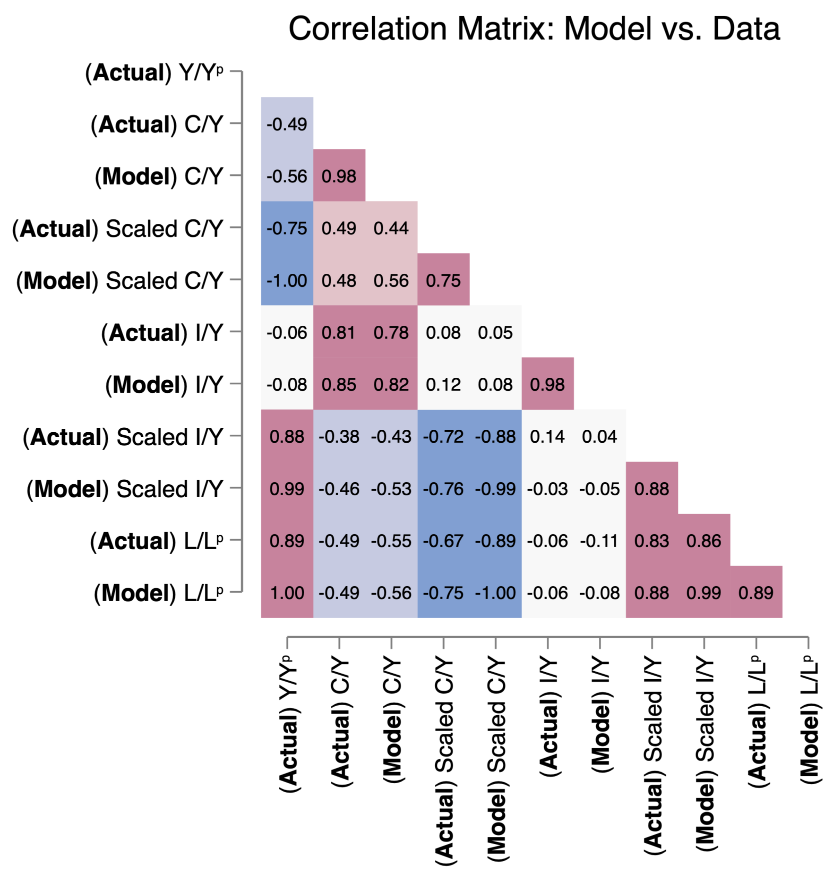
Then the calibration exercise is done in six steps. First, when $O_t \approx 1$, the observed consumption and investment shares at potential are quantified as $c_t^p = C_t^p / Y_t^p$ and $i_t^p = I_t^p / Y_t^p$ respectively, and the series is completed by interpolation. Second, taking the observed output gap, O_t, using CBO estimates, I derive the model-implied consumption ratios, i.e., $C_t / Y_t = c_t^p O_t^{\eta - 1}$(Panel a) and $C_t / Y_t^p = c_t^p O_t^{\eta}$ (Panel b). Third, taking the value of ϕ_t and the observed output gap O_t, I derive the model-implied investment ratio using $I_t / Y_t = \left[1 + \phi_t - \phi_t O_t^{-(1-\eta)}\right] i_t^p$ (Panel c). Fourth, I derive the model implied labor gap using the equation $L_t / L_t^p = O_t^{\frac{1}{1-\alpha}}$ (Panel d). Fifth, Panels (e-f) plot the relation between the I_t / Y_t vs. C_t / Y_t and Y_t / Y_t^p ratios, scaled by their potential values. The scaling allows to account for the underlying trend. Given their trend,

I_t/Y_t and C_t/Y_t are negatively correlated, while I_t/Y_t and the output gap O_t are positively correlated—with the slopes consistent with the model. Overall, taking the observed variation in output gap as given, the model fits the dynamics of the key macro variables, with the directions and magnitudes broadly matching the data. What causes the output gap to fluctuate to generate such co-movement in the economy? As discussed in the next section: a combination of fear and sticky safe real rates.

Definition	Parameter	Value	Rationale
Capital Share of Income	α	0.33	Consistent with standard estimates
Consumption Elasticity	η	0.5	Chosen to match amplifications in the data

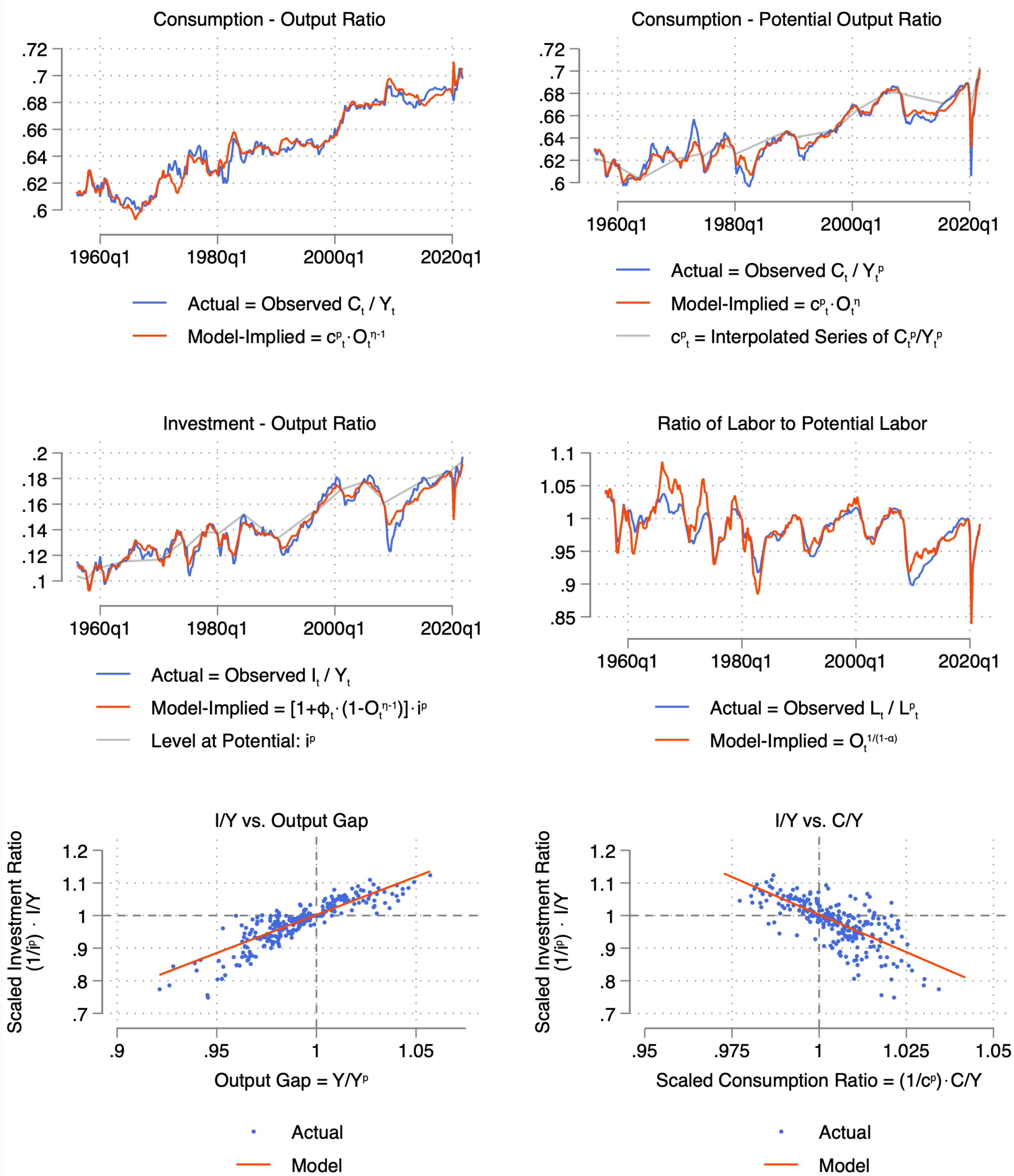
Note: The calibration in Figure 3 relies only on these two parameters. In addition, to be consistent with $\eta = \frac{\chi-\alpha}{\gamma(1-\alpha)}$, the underlying parameters are chosen as: CRRA parameter of $\gamma = 4$ and aggregate labor elasticity as $\chi = 1.67$.

Table 1: Parameter Values



Note: This figure presents the correlation matrix of the frictionless benchmark model for the key macro relationships for the US postwar period 1955–2021 based on both the model in (20) and the real-world data. Overall, the correlations suggest that the model fits the dynamics of the key macro variables, with the directions and magnitudes broadly matching the data. Ascari et al. (2019); Basu et al. (2021) document that standard New Keynesian models generate inconsistent cross-correlations for several key variables.

Figure 2: Correlation Matrix: Model vs. Data



Note: The calibration uses the Congressional Budget Office (CBO) measure of the output gap O_t. Panels (a-b) plot the model-implied consumption shares vs. actual data. Panel (c) plots the investment ratio. Panel (d) plots the labor gap. Panels (e-f) plot the relationship between the investment output ratio vs. either the consumption output ratio or the output gap. The variables are scaled relative to their values at potential to account for trend. When output gap falls both investment and consumption fall, but the fall in investment is greater due to the investment accelerator. This creates a negative relationship between the consumption output ratio and the investment output ratio (given their trend), and a positive relationship between the investment output ratio and the output gap—consistent with the data.

Figure 3: Calibration of the Frictionless Benchmark Model (1955–2021)

2.11 Balanced Growth and Consumption Growth

To study the dynamics of the economy in the next section, it is useful to obtain the conditions for a balanced growth path and also derive a representation of the Euler equation. To do this, I will specify a functional form for the disutility cost of supplying labor υ_t in (5). The functional will also ensure that potential labor supply is not affected by changes in the output gap.

Balanced Growth

In line with Guvenen and Rendall (2015), I assume that the growth in the disutility cost of supplying labor depends on productivity and capital growth, such that: $\upsilon_{t+1}/\upsilon_t = \left(c^p_{t+1}/c^p_t\right)^{-\gamma}\left(\left(A_{t+1}/A_t\right)\left(K_{t+1}/K_t\right)^{\alpha}\right)^{1-\gamma}$. As shown in Online Appendix A4, this assumption ensures that $\frac{d\ln L^p_t}{dt} = 0$. In addition, it ensures that there is a balanced growth path, i.e., capital, output, and productivity grow at constant rates in the steady state.

Consumption Growth

Even if $\frac{d\ln L^p_t}{dt} = 0$, the output gap this period will still affect output in the next period through changes in the capital stock. To make this precise let's introduce some notations.

Notation. When the current output gap is closed, let potential output in the next period be $Y^{pp}_{t+1} \equiv \left[Y^p_{t+1} \mid O_t = 1\right]$ and $C^{pp}_{t+1} \equiv c^p_{t+1} \cdot Y^{pp}_{t+1}$. Let potential consumption growth be: $X_{t+1} \equiv \ln\left[C^{pp}_{t+1}/C^p_t\right]$.

Then we can represent the Euler equation as follows.

Lemma 1 (**Consumption Growth & Euler Equation**). *Consumption growth is a weighted sum of potential consumption growth, the output gap growth, and the dynamic effect of the output gap on next period's capital stock. We can represent consumption growth and the household's Euler equation (8) as:*

$$\begin{aligned} \ln\left[C_{t+1}/C_t\right] &\approx X_{t+1} + \eta \cdot \Delta gap_{t+1} + \nu \cdot gap_t \\ r^S_t &= \rho - \ln E_t\left[\left(C_{t+1}/C_t\right)^{-\gamma}\right] \end{aligned}$$

with $gap_t \equiv \ln O_t$, $\nu \equiv \frac{\alpha a[1+(1-\eta)\phi]}{1+a-\delta}$, *and* ϕ, a *are the average values of* ϕ_t *and the investment-capital ratio.*

Proof. See Online Appendix A4 & A5. □

That is, consumption growth depends on growth in both the potential consumption and the output gap. An increase in the output gap today, also raises the capital stock in the next period through higher investment. This in turn raises next period's potential consumption. The parameter ν accounts for this effect, which is the capital share, α, multiplied by the elasticity of next period's capital to current period's output gap.

3 The SWIM Model

The optimization problem and budget constraints of all agents, and the equilibrium conditions of the SWIM model are identical to the benchmark model, with now the shock processes and the central bank interest rule fully specified. The S, W, and IS curves will simply be a decomposition of the Euler equation in Lemma 1, while the MP curve will be the interest rate condition. Further, the government will provide social insurance to households in crisis scenarios. Finally, the economy will be subject to two types of shocks: a productivity shock that shifts A_t, and an uncertainty or 'fear' shock, which shifts the volatility of A_t.

3.1 Defining Fear & Safety

There are two states of the world: *normal* and *extreme* (Figure 4). The extreme state approximately occurs 1% of the time. I call the boundary point between the two states the 'fail-safe scenario.' The key definitions are below.

Definition 1. ***Fail-Safe Scenario:*** *A_t^{safe} is the (notional) worst 1st percentile outcome for productivity. The fail-safe scenario occurs when $A_t = A_t^{safe}$.*

Definition 2. ***Safety:*** *Safety (s_t) quantifies the expected (log) distance between fail-safe productivity and expected productivity in the next period: $1 - s_t \equiv E_t \ln \left[\frac{A_{t+1}}{A_{t+1}^{safe}}\right]$.*

Definition 3. ***Fear:*** *Fear (w_t) is the expected (log) distance between consumption and fail-safe consumption. That is, $w_t \equiv E_t \ln \left[\frac{C_{t+1}}{C_{t+1}^{safe}}\right]$.*

Definition 4. ***Safe Assets:*** *A safe asset is any security or its tranche that is free of default in normal states. That is, a safe asset guarantees one unit of consumption when $A_t \geq A_t^{safe}$.*

Fear (w_t) represents how much consumption is expected to decline in the fail-case scenario. In other words, if none of the *extreme* events occur, fear measures the maximum decline in consumption relative to

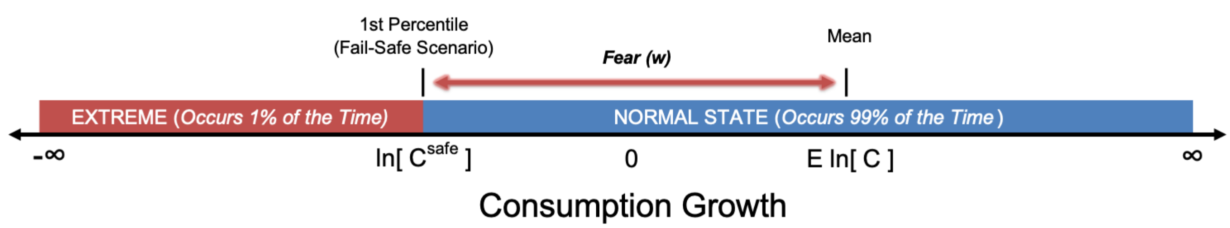


Figure 4: Distribution of Consumption Growth: $\ln \left[\frac{C_{t+1}}{C_t}\right]$ with C_t normalized to 1

expectation. A value of $w_t = 0.25$ implies that consumption in the fail-safe scenario will be about 25% lower than its average. The quantity of safe consumption, C_{t+1}^{safe}, will depend on the risk in the economy, and will be lower when there is greater risk of catastrophes, pandemics, wars, hyperinflation, institutional collapse, etc. From this perspective, w_t is related to the net supply of safe assets. When net supply of safety is higher, fear is lower.

The fundamental idea underpinning these concepts is that one of the goals of modern society is to deliver an environment that is more *safe*. Nothing on our planet is fully safe and protected from *"force majeure"* or *"superior forces"* such as extinction events, catastrophic climate change, pandemics, natural disasters, or the dark side of human nature. Such rare events can possibly wipe out large fractions or all of human resources—even in the most prosperous societies. In this context, a goal of society becomes to strengthen institutional design and technology to reduce the set of possible extreme scenarios and the likelihood of these scenarios, while allowing for the possibility of some breakdown of contracts during the extreme state when superior forces become dominant. The fail-safe scenario will be less bad in more robust and advanced economies, which captures the idea of Locke and Hobbes that more evolved societies are the ones that are less brutish or less subject to the state of nature. And more safe societies will have lower fear or uncertainty. Various innovation activity can then be seen as directed towards making the worst-case scenario less bad by producing more safe assets.

3.2 Setup

3.2.1 Government

The assumption for government's social/disaster insurance role is as follows, which adds structure to the exogenous process for government share, ξ_t, in (3).

Assumption 1 (Social Insurance)**.** *The government's spending path is countercyclical, and depends on the realization of A_{t+1}. That is, $\Delta g_{t+1} = (1 - g_t)\left(1 - \left[\frac{A_t}{A_{t+1}}\right]^{\zeta}\right)$ with $0 \leq \zeta < 1$.*

When $A_{t+1} = A_t$, there is no role for social insurance, and the government spending path is the same as before. However, when realization of A_{t+1} falls, government consumption falls. This can be seen as a social transfer in bad times or disaster scenarios as there is a corresponding decline in the lump sum taxes. (In turn, during bad times, there is an increase in the household's potential consumption share, c_t^p). Note that when $\zeta = 0$ we get $\Delta g_{t+1} = 0$. In addition, I continue to assume no default risk for government debt. Then, the short-safe real rate of return or the 'riskfree' rate (denoted by superscript S) is represented by the one-period government debt, i.e., r_t^S.

3.2.2 Shocks

The only shocks in the economy are the first- and second-moment productivity shocks. And, the variation in the second-moment of productivity growth is taken to be the fundamental driver of the economy. Below, I first define the process for productivity growth, $\ln[A_{t+1}/A_t]$, and then derive the implied processes for potential consumption growth and for consumption growth.

Assumption 2 (**Productivity Process**). *Productivity growth is subject to time-varying volatility:*

$$
\begin{aligned}
\ln A_{t+1}/A_t &= \mu_A + e_{t+1} \\
e_{t+1} &= \vartheta e_t + \sigma_t \cdot \varepsilon_{t+1} \\
\sigma_t &= (\overline{\sigma})^{1-\theta} (\sigma_{t-1})^{\theta} \cdot \sqrt{1+\psi_t}
\end{aligned}
$$

The joint shock process for productivity $\{\varepsilon_t, \psi_t\}$ is stationary with no correlation. The shock ε_t is drawn from a (truncated) student-t distribution with n degrees of freedom, such that $E_t[\varepsilon_{t+1}] = 0$ and $V_t[\varepsilon_{t+1}] = \frac{n}{n-2}$, which implies $\sigma^2_{A,t} = \frac{n}{n-2}\sigma^2_t$. The shock ψ_t is distributed with $E_t[\psi_{t+1}] = 0$ and $V_t[\psi_{t+1}] = \sigma^2_{\psi}$ and bounded below by zero. The parameters $0 \leq \vartheta, \theta \leq 1$ measures the persistence of e_t and σ_t, with long-run mean zero and $\overline{\sigma}$ respectively.

In line with Definitions #1-4, let's imagine that the agents in the economy operate as if the notional 1st percentile outcome for productivity (i.e., the fail-safe scenario) occurs k standard deviations below its mean. I normalize $k = \sqrt{6}$ in line with typical values.[6] Then based on Definition #2, we can represent safety, s_t as:

$$
\frac{1-s_t}{\sqrt{6}} \equiv \sigma_{A,t} = \sqrt{\frac{n}{n-2}} \cdot \sigma_t \tag{21}
$$

As per this representation, s_t is a summary measure of the perceived TFP risk, such that the perceived level of safety is higher when the volatility is lower.[7] The parameter n allows us to have a subjective uncertainty (or Bayesian) interpretation of risk. As (21) shows, the perceived volatility, $\sigma_{A,t}$ depends inversely on n. That is, the perceived volatility, $\sigma_{A,t}$, is higher when the observations (n) available to the agents (or perceived by them as relevant) is smaller. And, as $n \to \infty$ we get $\sigma_{A,t} \to \sigma_t$ from above. This interpretation also allows for the possibility that the econometrician's measure of volatility based on sampled data may differ from the volatility perceived by the agents due to subjective uncertainty. From this perspective,

[6] k is a constant for all distributions in the two-parameter family (or the location-scale family). For normal distributions, k is the z-score associated with the 1st percentile with $k = 2.33$, but can be greater for heavy-tailed distributions. The long-run mean of s_t is given by: $\overline{s} \equiv 1 - k\sqrt{\frac{n}{n-2}} \cdot \overline{\sigma}$, and analogously below, the long-run mean of fear is given by: $\overline{w} \equiv \varrho \cdot (1-\zeta)(1-\overline{s})$.

[7] Such time-varying volatility has been studied by several papers in the literature (Justiniano and Primiceri, 2008; Jaimovich and Rebelo, 2009; Bloom et al., 2018; Basu et al., 2021). Here I simply give it a Bayesian interpretation with the representation based on the 1st percentile outcomes instead of the standard deviation (or the scale parameter).

the difference between $\sigma_{A,t}$ and σ_t could be seen as the subjective-perception based volatility versus the 'objective' volatility that would be measured in the data.[8]

Potential Consumption. Combining Assumptions 1 & 2 with Lemma 1, we can represent potential consumption growth, $X_{t+1} \equiv \ln\left[C_{t+1}^{pp}/C_t^p\right]$ as:

$$X_{t+1} = b + (1-\zeta) \cdot \ln\left[A_{t+1}/A_t\right] \tag{22}$$

where $b \equiv \alpha \ln(1 + a - \delta)$ represents the contributions from capital growth, with $E_t\left[X_{t+1}\right] \equiv \mu_t = b + (1-\zeta)(\mu_A + \vartheta e_t)$ and $V_t\left[X_{t+1}\right] \equiv \sigma_{X,t}^2 = (1-\zeta)^2 \sigma_{A,t}^2$, and long-run mean $\overline{\mu} = b + (1-\zeta)\mu_A$. (See Online Appendix A4 for derivations). Thus, greater social insurance (i.e., higher ζ) helps shield consumption volatility for a given amount of volatility in productivity.

Consumption. Finally, based on Assumption 2, we can represent the consumption process as follows.

Lemma 2 (**Consumption Process**). *The mean and variance of consumption growth are:* $E_t \ln\left[C_{t+1}/C_t\right] = \mu_t + \eta \cdot E_t\left[gap_{t+1}\right] - (\eta - \nu) \cdot gap_t$ *and* $V_t \ln\left[C_{t+1}/C_t\right] = V_t\left[X_{t+1} + \eta \cdot gap_{t+1}\right]$. *In equilibrium, we will have* $V_t\left[gap_{t+1}\right] \approx \left(\frac{\eta}{\eta(1-\beta)-\nu}\right)^2 \sigma_{X,t}^2$, *which implies* $V_t \ln\left[C_{t+1}/C_t\right] \approx \frac{w_t^2}{6}$ *with* $w_t \equiv \varrho \cdot (1-\zeta)(1-s_t)$ *and* $\varrho \equiv 1 + \frac{\eta\vartheta}{\eta(1-\beta)-\nu}$.

Proof. See Online Appendix A5. □

This lemma established that our measure of fear, $w_t \equiv \varrho \cdot (1-\zeta)(1-s_t)$, governs the volatility of consumption growth. And, in turn, fear depends on the safe technology (and the degree of negative correlation between the output gap and potential consumption). The more quantities of safe output available in the fail-safe scenario (i.e., lower tail risk), the less worry there is about disaster scenarios in the next period. Thus, s_t corresponds to both the aggregate supply of safe assets in the economy and also drives the quantity of fear. Further, greater persistence in technology shocks (ϑ) or in central bank interest rates (β) heightens fear since it leads to amplification of technology shocks.

Moreover, under this representation, fear, w_t, is the summary measure of risk in the economy. The econometricians studying the economy or the agents operating in the economy do not need to know the degrees of freedom n, the nature of time-varying disaster risks, the variance of the processes, as long as they have a measure of the perceived fear w_t.

[8] See Weitzman (2007) for a rigorous discussion of the distinction between a Bayesian framework with subjective uncertainty and a rational expectations equilibrium (REE). Equation (21) also sheds light on the distinction between the Rietz (1988); Barro (2006) tradition of modeling disaster risks and Weitzman (2007; 2009) subjective uncertainty framework. Roughly speaking, the Rietz (1988); Barro (2006) approach raises the quantity of risk in macro-finance models by accounting for objective probabilities of rare disasters, which directly raises the value of σ_t (and of the higher moments of A_t). By contrast, the subjective uncertainty approach of Weitzman (2007; 2009) starts from the premise that there is parameter uncertainty about the true underlying value of σ_t, such that the posterior predictive distributions have fat tails and with n remaining finite even with infinite observations due to periodic shocks to the underlying parameters governing volatility.

3.2.3 The Central Bank

The price of goods is fully flexible in the SWIM model and normalized to unity. We do not consider the role of inflation here. (The extension in Online Appendix B adds inflation to the model). The only role of the central bank is that it that they have the power to control the real rate of return in the safe assets market. The central bank sets the interest rate, r_t^S, before observing the realization of the shock ε_t in the current period. The central bank sets rate to equal the expected neutral rate of return r_t^* (defined below) plus a persistence term. These assumptions are summarized below.[9]

Assumption 3 (Monetary Policy Rule)**.** *Goods prices are fully flexible and normalized to unity. There is a central bank that sets the real rate of return in the safe asset market such that* $r_t = r_t^S$*, with* $r_t^S = E_{t-1}\left[r_t^*\right] + \beta\left[r_{t-1}^S - r_{t-1}^*\right]$ *with* $0 \leq \beta < 1$*.*

3.3 Market Clearing Conditions

We now discuss the market clearing conditions of the SWIM model to explain how the equilibrium arises. We have three markets that need to clear in equilibrium: the goods market, the labor market, and the safe government debt market (i.e., the "safe assets market"). Along the MP curve, the safe assets market will be in equilibrium. Along the IS curve, *both* the goods and labor markets will be in equilibrium. Thus, at the intersection of these two curves, all markets will be in equilibrium. What is the role of the other two curves? The S curve and the W curve will play the role of *shadow* market clearing conditions for the safe assets market, in the hypothetical scenario that the output gap is closed in equilibrium. The intersection of these two curves will determine the so-called *neutral rate of interest,* which the IS curve will depend on.

3.3.1 The S Curve (Demand for Safe Assets)

The S curve represents the demand for safe assets. The neutral rate (r_t^*) is defined as the short-term interest rate for a riskless asset that pays off 1 unit of consumption in the future, when the output gap is expected to be closed today and tomorrow (as in (13)). That is, $r_t^* = \left[r_t \mid gap_t = E_t gap_{t+1} = 0\right]$. Then, using Lemmas 1 & 2, we can represent the S curve as:

$$r_t^* = \rho + \gamma\mu_t - \frac{\gamma^2}{12} w_t^2 \tag{23}$$

See Online Appendix A7 for the derivation, which follows from the moment-generating function of consumption growth. The first term, ρ, is the pure rate of time preference. When households are more impatient they will demand a higher compensation for delaying consumption from today to tomorrow, leading to a

[9]The functional form in Assumption 3 implies that $r_t^S = (1-\beta)E_{t-1}\left[r_t^*\right] + \beta\left[r_{t-1}^S - \left(r_{t-1}^* - E_{t-1}\left[r_t^*\right]\right)\right]$. Alternatively, one could consider a closely-related interest-smoothing rule of the form: $r_t^S = (1-\beta)E_{t-1}\left[r_t^*\right] + \beta\left[r_{t-1}^S\right]$, which yields similar results but adds additional algebra terms in the equilibrium conditions—without necessarily changing the core insights. Hence, in the interest of simplicity, I have opted for the functional form represented in Assumption 3.

higher risk-free rate. The second term is the product of the coefficient of relative risk aversion γ and the expected growth in disposable income. Note that with CRRA utility, γ is the inverse of the intertemporal elasticity of substitution. When γ is high, the households are less willing to accept swings in consumption over time, and they will demand a higher compensation to delay consumption. Thus, in the second term, γ measures the *price* of intertemporal substitution, and the mean growth in consumption (when the output gap is closed) measures the *quantity* of intertemporal substitution. The third term measures risk in the next period. The greater fear there is about potential consumption in the next period the more households are willing to transfer resources from today to tomorrow, leading to a lower risk-free rate. The *price* of this risk depends on the coefficient of relative risk aversion γ. And the *quantity* of risk depends on the variance of consumption growth $V_t\left[\ln C_{t+1}/C_t\right] \approx w_t^2/6$, which depends on fear (w_t).[10]

3.3.2 The W Curve (Fear or Uncertainty)

The W curve represents the *quantity of fear*, which depends on the *net supply of safe assets*. Recall that fear (w_t) is defined as the log distance between expected consumption and fail-safe consumption, such that: $w_t \equiv E_t \ln\left[\frac{C_{t+1}}{C_{t+1}^{safe}}\right]$. Using the results of Lemma 2, we can write the W curve as:

$$w_t = \varrho \cdot (1-\zeta) \cdot (1-s_t) \tag{24}$$

with $\varrho \equiv 1 + \frac{\eta\vartheta}{\eta(1-\beta)-\nu}$. That is, fear depends on the supply of safe assets as measured by the safe tranche of productivity, s_t, the degree of social insurance provided by the government as measured by ζ, and the persistence in technology shocks (ϑ) and in central bank interest rates (β).

3.3.3 The MP Curve (Interest Rate Rule)

The MP curve is based on Assumption 3, such that the safe real interest rate fixed by the central bank such that $r_t = r_t^S$ in each period t, which gives us:

$$r_t^S = E_{t-1}\left[r_t^*\right] + \beta\left[r_{t-1}^S - r_{t-1}^*\right] \tag{25}$$

with the expected neutral rate given by: $E_{t-1}\left[r_t^*\right] = \rho + \gamma\left[a_t + (1-\zeta)\left(\mu_A + \vartheta^2 e_{t-1}\right)\right] - \frac{\gamma^2}{12}\cdot\left(w_{t-1}^2\right)^{\theta}\left(\overline{w}^2\right)^{1-\theta}$. Thus, we have 'monetary non-neutrality' due to the central bank role in the market for safe assets, by its ability to pin down the short-term safe rate, i.e., due to *sticky safe rates*.

[10] Note that if we move from a truncated student-t distribution to an untruncated one, Weitzman (2007; 2009) showed that $r_t^* \to -\infty$. Weitzman's result emphasizes that small changes in where the tails are truncated (which here can be seen as changes in w_t) can lead to large changes in the riskfree rate.

3.3.4 The IS Curve (Euler Equation)

Using the Euler equation representation in Lemma 1 with the equation for consumption growth in Proposition 2, substituting r_t^* using (23), we get:

$$r_t^S = r_t^* - \gamma (\eta - \nu)\, gap_t + \gamma\eta E_t [gap_{t+1}] \tag{26}$$

See Online Appendix A6 for the derivation. Here r_t^* denotes the neutral rate of interest, as discussed above. The IS curve equation presents a synthesis between macroeconomics and finance, which appears in one form or another in much of modern macro and finance theory. From the asset pricing perspective, it tells us that given the household's potential consumption choice and given the output gap today and in the future, what gross return we should expect on the safe asset. From the macro perspective, it tells us given gross return on the safe asset and given household's consumption choice, what variation in output gap we should expect. The macro literature often takes the stochastic discount factor as given. By contrast, the asset pricing literature often takes the household's consumption choices as given and works within an endowment economy environment. What allows us to merge the two approaches here and get a simple solution is the tractability of the frictionless benchmark and the definitions of safety and fear.

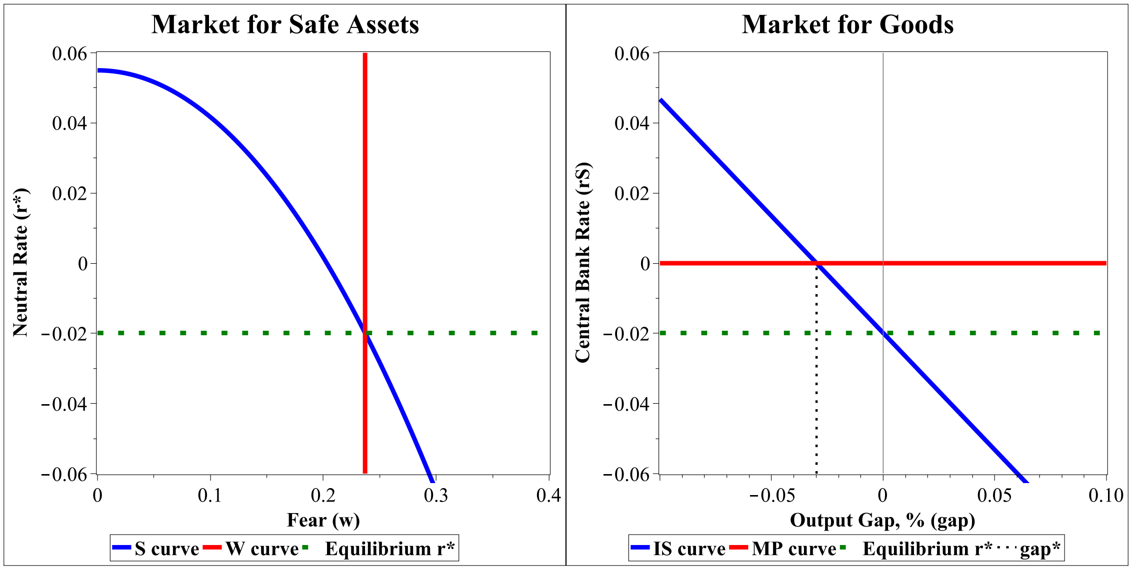
The S, W, and IS curves are simply a decomposition of the household's Euler equation (7). The intersection of the S curve (23) and the W curve (24) determine the neutral rate r_t^*, which the IS curve (26) takes as given. Whenever shifts in the S and W curves lead to a change in r_t^*, it shifts the intercept of the IS curve.

Finally, we can eliminate the expectation term from the IS curve. Suppose at some future time $t + J$, we have $E_t (gap_{t+J}) = 0$. Then, (26) implies: $gap_{t+J-1} = \frac{1}{\gamma(\eta-\nu)} \left(r_{t+J-1}^* - r_{t+J-1}^S\right)$. From (25) we get $E_{t-1}\left[r_t^* - r_t^S\right] = \beta\left[r_{t-1}^* - r_{t-1}^S\right]$ for all t. Then iterating backward, taking expectations, and letting $J \to \infty$, we can represent the IS curve ('investment-savings' curve) as:

$$r_t^S = r_t^* - \gamma [\eta (1 - \beta) - \nu]\, gap_t \tag{27}$$

3.4 Equilibrium

We study a standard competitive equilibrium in which prices and allocations solve the household's and firm's optimization problems and all markets clear. Three markets need to clear in equilibrium: the government debt market (D_t), the labor market (L_t), and the goods market (Y_t), with their respective prices given by $1/R_t^S$, W_t, and unity. Given potential output Y_t^p and the shock processes for productivity $\left\{\varepsilon_t, \psi_t^2\right\}$, the equilibrium market clearing conditions are characterized by four equations:



Note: The figure depicts the general equilibrium using the four curves of the SWIM model. In this example, the neutral rate of interest is found at the intersection of the S and W curves in the left panel ($r_t^* = -0.02$), depicted by the green dotted line that can be traced from the left panel to the right panel. However, the central bank has set the real rate at $r_t^S = 0$ (red curve in the right panel). Given that $r_t^S > r_t^*$, we have a negative output gap, which is found at the intersection of the IS and MP curves in the right panel ($gap_t \approx -.03$).

Figure 5: The SWIM Model: An example in which the central bank has set the interest rate 'too high' relative to the neutral rate

$$\begin{aligned} S\ Curve: &\quad r_t^* = \rho + \gamma\mu_t - \frac{\gamma^2}{12}\cdot w_t^2 \\ W\ Curve: &\quad w_t = \varrho\cdot(1-\zeta)\cdot(1-s_t) \\ IS\ Curve: &\quad r_t^S = r_t^* - \gamma\left[\eta\left(1-\beta\right)-\nu\right]gap_t \\ MP\ Curve: &\quad r_t^S = E_{t-1}\left[r_t^*\right] + \beta\left[r_{t-1}^S - r_{t-1}^*\right] \end{aligned} \tag{28}$$

The following five steps describes the equilibrium, which is also depicted in Figure 5.

1. First, the realization of $\{\varepsilon_t, \psi_t\}$ pins down expected productivity growth (μ_t) and safety (s_t) in each period t. The S curve and the W curve correspond to the demand and supply of safe assets respectively. Given the realization of $\{\varepsilon_t, \psi_t\}$, the intersection of the S and W curves determine the neutral rate of interest the neutral rate of interest r_t^*. It represents the 'shadow' safe rate.

2. Second, the safe assets market clearing condition is given by the MP curve, which fixes the price of of government debt $1/r_t^S$. In equilibrium, all quantities of debt supplied by the government must be held by the households at that fixed price.

3. Third, the goods market equilibrium determines the combination of output Y_t and the safe rate r_t^S, such that investment must equal total savings in the economy. When the price of the safe assets fixed

by the central bank deviates from the neutral rate, households must adjust their consumption-savings decision such that in equilibrium they are willing to hold the fixed quantity of safe assets supplied. The market clearing condition for the goods market is given by the IS curve, which is the combination of output and r_t^S necessary at a given neutral rate r_t^* to clear the goods market.

4. Fourth, as per the IS curve, the goods market equilibrium pins down the output gap ($gap_t \equiv \ln O_t$). The combination of gap_t and the exogenously given potential labor L_t^p, corresponds to a unique level of total labor supply L_t by households. In equilibrium, the price of labor, W_t, adjusts to ensure that the firm employs all quantities of labor supplied by households L_t, thereby clearing the labor market.

5. Thus, along the IS curve both the goods market and the labor market are in equilibrium for a given neutral rate r_t^*, and along the MP curve the safe assets market is in equilibrium. Then for all the markets to be in equilibrium, the system must simultaneously be at the intersection of the S curve and W curve on the one hand (which determines the neutral rate), and the IS curve and MP curve on the other (which determines the safe rate and the quantities of output and labor). Therefore, the four equations in (28) represent a dynamic general equilibrium of the economy. Figure 5 represents the four key equations using side-by-side panels that are termed the safe assets market and the goods market.

For intuition, note that if there were no sticky safe rates, i.e., the safe rate would equal the neutral rate ($r_t^S = r_t^*$), then the IS curve would be vertical with $gap_t = 0$. However, when the safe rate deviates from the neutral rate as per the MP curve, the output gap can deviate from zero. Using the IS curve we can solve for the output gap as $gap_t = -\frac{r_t^S - r_t^*}{\gamma[\eta(1-\beta)-\nu]} = -\frac{r_t^S - \left(\rho + \gamma\mu_t - \frac{\gamma^2}{12} w_t^2\right)}{\gamma[\eta(1-\beta)-\nu]}$ and plugging in the MP curve we get:

$$gap_t = \beta \cdot gap_{t-1} + \left(\frac{\varrho - 1}{\eta\varrho}\right) w_{t-1} \cdot \varepsilon_t - \varrho_2 w_{t-1}^{2\theta} \cdot \psi_t \tag{29}$$

with $\varrho_2 \equiv \frac{\gamma \cdot \overline{w}^{2(1-\theta)}}{12[\eta(1-\beta)-\nu]}$.

Given the output gap, we can pin down the rest of the variables as per Proposition 1. This equation allow us to characterize the model's theory of output fluctuations below.

Proposition 2 (**Role of Fear in Output, Interest & Safe Assets**). *The output gap is given by (29).*

(i): *The output gap declines when (i) fear increases, (ii) the central bank's sets the interest rate higher than the neutral rate, or (iii) there is a negative productivity shock. The contractionary effect of fear is bigger when the central bank is slow to react to changes in the neutral rate (i.e., when β is high).*

(ii): *The neutral rate declines when (i) productivity growth falls; (ii) patience rises; or (iii) fear rises.*

(iii): *Fear rises when (i) there are fewer safe assets in the reasonable worst-case scenario (i.e., s_t is low); or when (ii) the social insurance provided by governments (ζ) is smaller.*

Proof. See Online Appendix A8. □

4 Mapping the SWIM Model to Macro-Financial Data

4.1 Deriving the Model-Implied Fear Factor & Testing the Model Using Asset Prices

Based on the parameters in Table 2, I first derive the model-implied fear factor using (28). Then, I compare the model-implied fluctuations in equity and bond prices against the monthly data. The steps are as follows:

1. *Extract the model-implied neutral-rate* r_t^*: By using the IS curve in (28) using a measure of the observed gap gap_t and a measure of expected potential consumption growth μ_t. The measure of gap is twice the unemployment gap as measured by the CBO using BLS data (i.e., CBO's estimate of NAIRU less the BLS reported unemployment rate). And the measure of expected potential consumption growth is the long-term moving average of the per capita real personal consumption expenditure (PCE) growth. The neutral rate is compared to the observed safe rate r_t^S, measured as the Fed Funds rate less the trimmed mean PCE inflation rate. (See Panel a).

2. *Extract the model-implied fear* w_t: Plugging r_t^* from step 1 into the S curve in (28). (See Panel b).

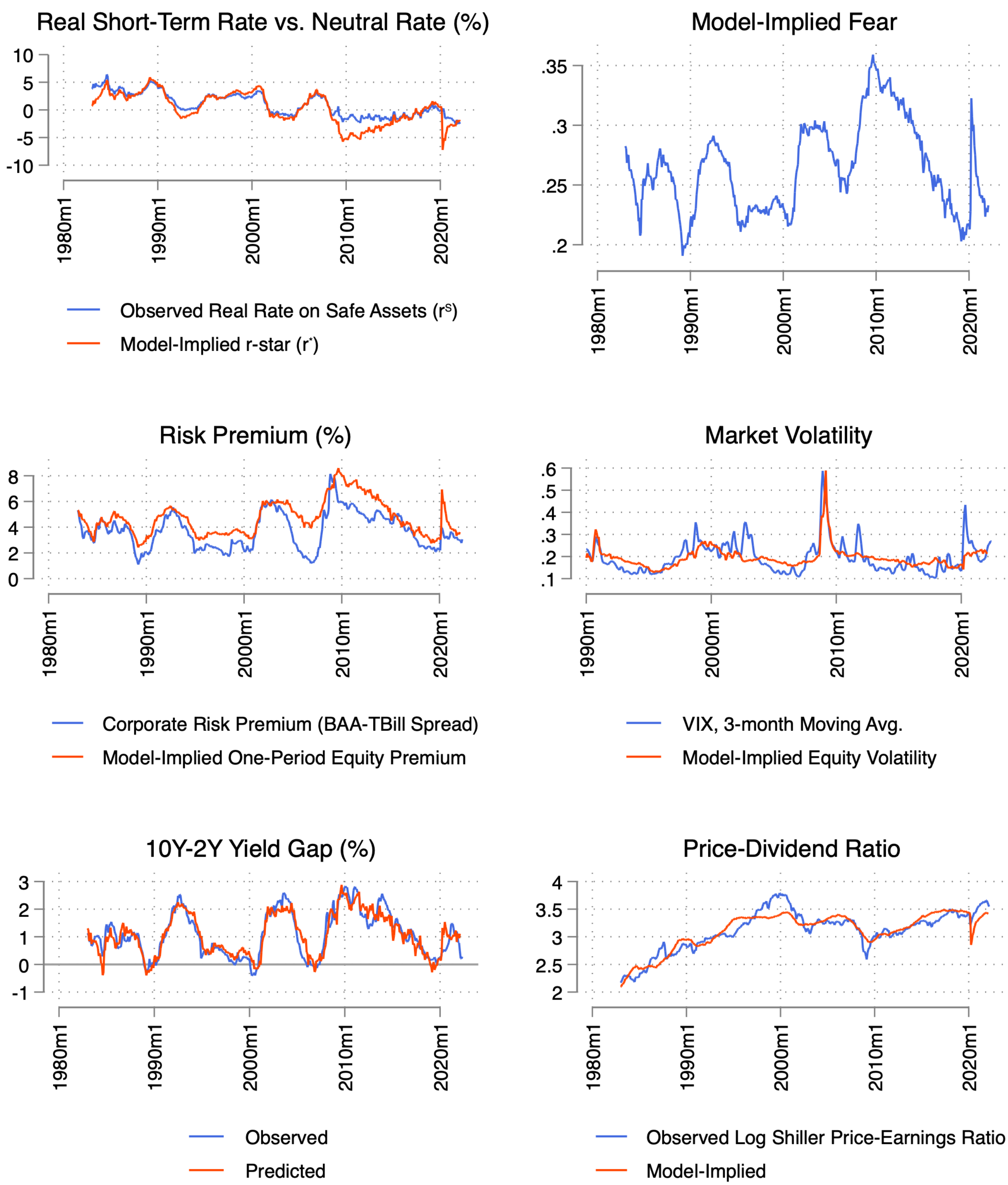
3. *Derive the one-period equity premium* ep_t *and equity volatility*: By plugging w_t from step 2 into the formulae for the equity premium formula and the volatility of multi-period equity, both of which are shown in (30). This is compared to the observed corporate risk premium, measured as the Baa-TBill spread and the Chicago Board Options Exchange's CBOE Volatility Index (VIX). (See Panel c-d).

4. *Derive the slopes of the yield curve*: By plugging values of gap_t and μ_t from Step 1, and w_t from Step 2 into equation (36). (See Panel e).

5. *Derive the price-dividend ratio*: The model-implied log price-dividend ratio, based on the derivation in (41), is compared to the Shiller Price-Earnings ratio. (See Panel f)

The results are plotted in Figure 6. Overall, the model appears to provide a good fit to the observed variation in the risk premium and the yield curve. The model generates a high (unlevered) one-period equity premium of around 4–6%, which co-moves with the observed risk spread. The model-implied equity premium is in line with (Barro et al., 2014) who use a heterogeneous agent model with disaster risk.

This calibration exercise is essentially telling us that a common fear factor may hold the promise of providing a unified explanation of key macro-finance patterns. First, the fear factor broadly explains the variations in asset prices and the variations in the output gap as per (28). And, in turn, the fear-induced variations in the output gap explain the contemporaneous co-movements of key macro variables as per (20).



Note: Panel a plots the observed short real rate (blue) vs. the model-implied r_t^* as per the IS curve (red). Panel b plots the model-implied fear, w_t as per the S curve. Panel c plots the model-implied one-period equity premium (γw_t^2), and the observed corporate risk spread. Panel d plots the model-implied equity volatility and the observed volatility (as measured by CBOE VIX). Panel e plots the model-implied 10–2 year yield spread and the observed spread for the same maturities. And Panel f plots the model-implied log price-dividend ratio and the observed Shiller price-earnings ratio.

Figure 6: Calibration of the SWIM Model

Definition	Parameter	Value	Rationale
Coefficient of Relative Risk Aversion	γ	4	Consistent with standard estimates
Discount rate	ρ	0.015	Consistent with standard estimates
Elasticity of Capital w.r.t. the Output Gap	ν	0.05	Implied by underlying parameters
Persistence of the Interest Rate Rule	β	0.67	Based on simple linear estimation

Note: In addition, $\eta = 0.5$ is chosen the same as in Section 2.

Table 2: Additional Parameter Values

4.2 The Joint Behavior of Asset Prices and the Economy

4.2.1 Three Classic Asset Pricing Puzzles (The Risk-Free Rate, Equity Premium, and Equity Volatility Puzzles)

The risk-free rate puzzle asks: why is the real short-run risk-free rate so low (in the range of 1%)? The equity premium puzzle asks: why are the real stock returns so high (in the range of 8%) compared to the real short-run risk-free rate? And the equity volatility puzzle asks: why is the standard deviation of stock returns so high (in the range of 16%) compared to the volatility of consumption growth? Quantitatively, solving the three puzzles requires finding a risk-free rate of around 1%, a market Sharpe Ratio (i.e., the expected excess return of equities relative to its standard deviation) of around 40%, and a standard deviation of equity returns of about 15%. We can examine these three, respectively, in the SWIM model:

$$
\begin{array}{rlll}
\textit{Risk Free Rate}: & r_t^* & \approx \rho + \gamma\mu_t - \dfrac{\gamma^2}{2}\cdot\left(\dfrac{w_t^2}{6}\right) & \sim 1\% \\
\textit{Market Sharpe Ratio}: & \dfrac{E_t\left(R^e_{t+1}\right) - R^S_t}{\sigma_t\left(R^e_{t+1}\right)} & \approx \gamma\sigma_t\left(\Delta \ln C_{t+1}\right) = \dfrac{\gamma w_t}{\sqrt{6}} & \sim 40\% \\
\textit{Equity Volatility}: & \sigma_t\left(\ln R^e_{t+1}\right) & \approx \dfrac{w_t}{\sqrt{6}}\left[2 - \left(\dfrac{C_t}{P^e_t}\right) + \left(\dfrac{C_t}{P^e_t}\right)^2\right]^{\frac{1}{2}} & \sim 15\%
\end{array}
\qquad (30)
$$

See Appendix A9 for the derivations. Typical estimates for the relevant parameters (on an annual basis) are $\rho \approx 1.5$ percent, $\gamma \approx 4$, and $\mu_t \approx 2$ percent. And the (unlevered) market dividend-price ratio would suggest $C_t/P^e_t \approx 0.03$.

Let's start with the first puzzle. The quantity of risk depends on the variance of consumption growth $V_t\left[\ln C_{t+1}/C_t\right] \approx w_t^2/6$, which depends on fear (w_t). The empirically-observed values of the variance are: $V_t\left[\ln C_{t+1}/C_t\right] \approx .04$ percent. Plugging this into equation (30), would give us $r_t^* \approx 6$ percent, which compared to the actual real-world risk-free rate of 1 percent would be too high. This gap between the theoretical formula and the real-world risk-free rate is what the literature calls the 'risk-free rate puzzle.'

The literature has recently adopted two-related ways to solve the risk-free rate puzzle based on fat tails or disaster risk. First, Weitzman (2007) relies on subjective uncertainty, which due to fat tails leads $w^2/6 \to \infty$, and he shows that there exists a truncation of the student-t distribution, which generates a reasonable value of the variance to solve the risk-free rate puzzle. Second, Barro (2006) adds a third term on the right-hand-side of (30) to explicitly measure disaster risk. He considers a disaster probability of 1.5–2 percent of declines in per capita GDP ranging between 15 and 64 percent.

My paper builds on the Weitzman (2007) approach, and explicitly quantifies the subjective uncertainty as fear (w_t). Given the persistence in the potential consumption growth and in central bank interest rates, Then, an average value of $w_t \approx 0.25$, gives us $r_t^* \approx 1$ percent, solving the risk-free rate puzzle.

This common risk factor w_t also allow us to solve the other classic asset pricing puzzles. Plugging $w_t = 0.25$ into the second equation of (30) gives us a Sharpe Ratio of 40 percent. And plugging it into the third equation gives us $\sigma_t\left(\ln R^e_{t+1}\right) \approx 15\%$.

Thus, the SWIM model presents a unified explanation of the three classic asset pricing puzzles using a common fear (or risk) factor, w_t. In addition, the model goes further than the Weitzman (2007) approach, as it also explains the time-variation in the risk-free rate, risk premium, and equity volatility over the business cycle, which matches with the observed trend in the data (Figure 6).

Result 1 (A common fear factor provides a unified explanation for the classic asset pricing puzzles): *An average value of $w_t = 0.25$ for fear in (30) can match the empirically-observed risk-free rate, the market Sharpe Ratio, and the volatility in equity markets. Moreover, variation in w_t helps explain variation in these three values over the business cycle.*

4.2.2 Why are Long-Run Interest Rates Low? (The Low-for-Long Puzzle)

Why did the long-run neutral rate apparently decline after 2007 (compared to the Great Moderation period of 1983–2006)? To examine this, let's first define the long-term neutral rate and map it to the SWIM model. For this exercise let the unit of time, t, be years. Define the average real short rate N period aheads as:

$$r^S_{t,N} = (1/N)E_t\left[r^S_t + r^S_{t+1} + ... + r^S_{t+N-1}\right] \tag{31}$$

And let the real yield ($Yield_t$) of the riskfree bond be defined as the notional, constant, known, interest rate that justifies the quoted price of the riskfree bond, $P_t^{(N)}$. Denote $y_t^{(N)}$ as the log yield. And denote the one-period (log) forward rate is the rate for lending between period $t+N-1$ and $t+N$ by $f_t^{(N)}$. Then, the N period yield can be represented as a sum of the average real-short rates and a term premium, with its corresponding forward rate as:

$$\begin{aligned} y_t^{(N)} &= r^S_{t,N} + risk\,premium_t^{(N)} \\ f_t^{(N)} &= N \cdot y_t^{(N)} - (N-1)\cdot y_t^{(N-1)} \end{aligned} \tag{32}$$

where $risk\, premium_t^{(N)}$ is the term premium for horizon N at time t.

Using the equilibrium conditions in (28) we can represent the expected short-rate N years ahead as:

$$E_t\left[r_{t+N}^S\right] = E_t\left[r_{t+N}^*\right] - \gamma\left[\eta\left(1-\beta\right)-\nu\right]\beta^N E_t\left[gap_t\right] \tag{33}$$

When the degree of persistence in central bank interest rates are high, i.e. a high value of β, then there may be a gap between the expected short-rate and the expected neutral rate, even for several years ahead. That is, $E_t\left[r_{t+N}^S\right] \neq E_t\left[r_{t+N}^*\right]$ for N relatively small. And the gap between the two can be particularly large around recessions, since $gap_t << 0$ making the second term non-negligible.

Thus, choosing N sufficiently large is important to extract the information about the real neutral rates from the market implied short forward rates. A value of $N \approx 10$ when time t is measured in years, could be a reasonable value such that: $E_t\left[r_{t+10}^S\right] = E_t\left[r_{t+10}^*\right] - \gamma\left[\eta\left(1-\beta\right)-\nu\right]\beta^{10} E_t\left[gap_t\right] \approx E_t\left[r_{t+10}^*\right]$, since $\beta^{10} \approx 0$ for typical values of the persistence parameter (measured at an annual frequency). Thus, we get:

$$E_t\left[r_{t+10}^*\right] \approx 11\left(r_{t,11}^S\right) - 10\left(r_{t,10}^S\right) = f_t^{(10)} + 11\left(risk\, premium_t^{(11)}\right) - 10\left(risk\, premium_t^{(10)}\right) \tag{34}$$

The definition for the *long-run neutral real rate* typically used in the literature is the 5–10 year expected short-rate: $r_{t,literature}^{*,LR} = \frac{1}{5}\cdot E_t\left[r_{t+6}^S + r_{t+7}^S + \ldots + r_{t+10}^S\right]$. However, as the discussion here highlights, if there is high persistence in central bank interest rates, the 5-10 year expected short rate will contain a bias. In particular, in periods of sizable recessions or booms (i.e., when gap_t is sizably different zero today) or when central bank rates differ significantly from the neutral rate, this bias can be large (see Figure 7). Therefore, a preferred measure of the long-run neutral rate could be $r_t^{*,LR} \equiv E_t\left[r_{t+10}^S\right] \approx E_t\left[r_{t+10}^*\right]$, given $\beta^{10} \approx 0$. This can be characterized as:

$$r_t^{*,LR} \equiv E_t\left[r_{t+10}^S\right] \approx E_t\left[r_{t+\infty}^*\right] = \rho + \gamma\overline{\mu} - \frac{\gamma^2}{12}\cdot\overline{w}^2 \tag{35}$$

This is often interpreted as the long-run real rate consistent with an economy operating at its potential.

With these concepts defined, we can turn to the question of why have the the long-run rates declined. The literature has discussed at least two potential drivers. First, a decline in the long-run growth rate combined with an increase in savings rate due to aging and other factors. This essentially corresponds to a decline in the long-run mean of consumption growth, $\overline{\mu}$, such that $-\frac{dr_t^{*,LR}}{d\overline{\mu}} = -\gamma < 0$. A second driver that has been discussed is increase in demand for safe assets. In the SWIM model framework this corresponds to an increase in long-run fear, $\overline{w}$, such that $\frac{dr_t^{*,LR}}{d\overline{w}} = -\frac{\gamma^2}{6}\overline{w} < 0$.

But why would the long-run fear be higher after the Great Recession? Recall, long-term fear is given by: $\overline{w} \equiv \varrho\cdot\left(1-\zeta\right)\left(1-\overline{s}\right)$. The long-run fear can rise if government disaster insurance declines (lower ζ), or if expected long-run safety of the production technology declines (lower $\overline{s}$), or if the central bank's reaction

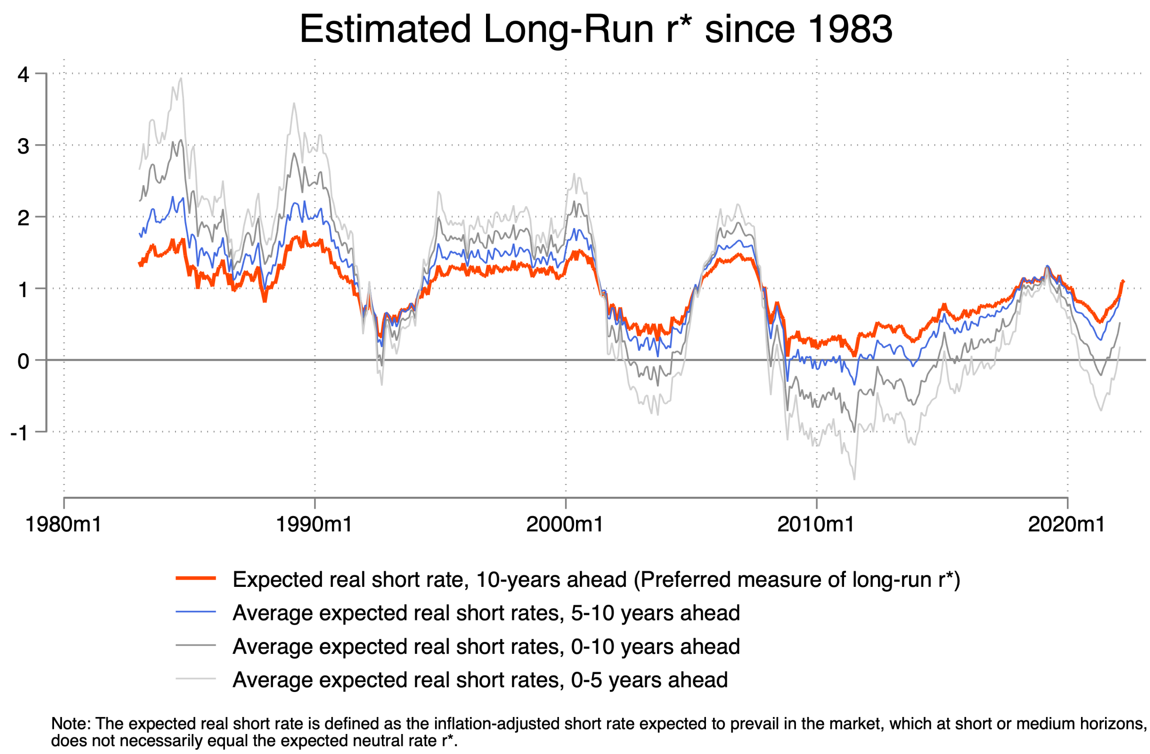


Figure 7: Estimated long-run neutral rate $r_t^{*,LR}$

function becomes more sluggish—for instance due to the effective lower bound (higher β, with $\partial\varrho/\partial\beta > 0$). Empirically, the long-run fear tends to rise during recessions when central banks are unwilling to cut rates into deeper territory, suggesting that variation in β may be an important driver of long-term fear.

Having discussed the various mechanisms that may lead to a decline in $r_t^{*,LR}$, I now present a decomposition of the two effects ($\overline{\mu}$ vs. $\overline{w}$) to explain the underlying drivers of the long-run neutral rate. Estimating $r_t^{*,LR}$ requires quantifying the forward rates (quasi-observable), forward expected inflation (unobservable), and the term premium (unobservable). I rely on the following steps for the decomposition:

1. Use the Fed Board's staff estimates of the yield curve (based on Gürkaynak et al. (2007)), to obtain the 10-year *nominal* yields.

2. Use the Cleveland Fed's inflation model estimates of average expected inflation (based on Haubrich et al. (2012)) and subtract it from #1 to obtain the implied real yields, $y_t^{(N)}$, and forward rates, $f_t^{(N)}$.

3. Use the New York Fed's staff model (based on Adrian et al. (2013)) estimates of the term premiums, and the estimates of $f_t^{(N)}$from #2, obtain an estimate of the market-implied long-run neutral rate, $r_t^{*,LR} \approx E_t\left[r^*_{t+10}\right]$ as per (34).

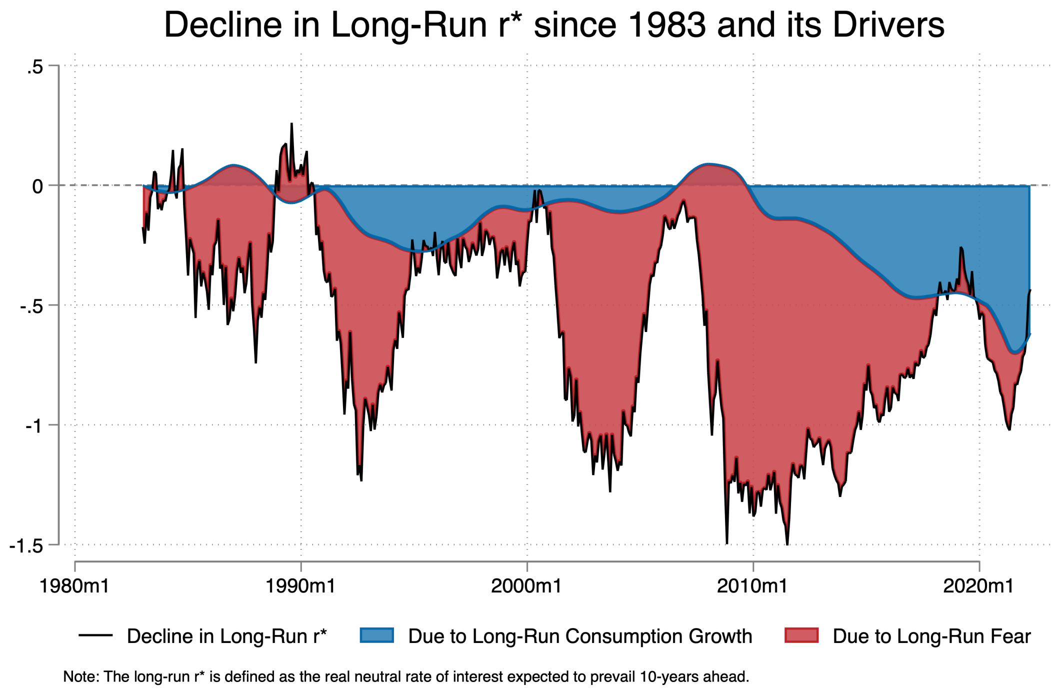


Figure 8: Drivers of the decline in the long-run neutral rate $r_t^{*,LR}$

4. Estimate a linear regression of the form: $r_t^{*,LR} = \alpha_0 + \alpha_1 (\mu_t - \mu_0) + \alpha_2 \left(w_t^2 - w_0^2\right) + \varepsilon_t$, under the assumption that long-run values of $\left\{\overline{\mu}, \overline{w}^2\right\}$ co-vary with their short-term values ($R^2 \approx 0.9$). Here $\left\{\mu_0, w_0^2\right\}$ correspond to values in 1983.

5. Use the estimated regression to decompose the decline in $r_t^{*,LR}$ between changes in long-run consumption growth vs. long-run fear (plus residual).

The results of the exercise are presented in Figures 7 and 8. The decomposition suggests that the neutral rate tends to decline persistently after recessions largely due to a sharp and persistent rise in long-term fear.[11] The key findings are summarized below.

Result 2 (The long-run neutral rate, $r_t^{*,LR}$, tends to decline persistently after recessions due to a sharp rise in long-term fear, which can take several years to unwind—especially at the lower bound. Accounting for persistent output gaps, $r_t^{*,LR}$ may have declined by only* 0.5 *percentage points since the 1980s): *The decomposition exercise suggests that (a) between 1983 and 2022, the market-implied long-run neutral rate, $r_t^{*,LR}$ has fallen by about* 0.5 *percentage points once accounting for the bias as per (34), (b) the decline was sharpest immediately after the Great Recession between 2008–13*

[11]The data-implied macro-financial pattern of a sharp increase in long-term fear followed by a gradual decline over a longer period has parallels to the anxiety response and flight-to-safety behavior in humans and other mammals when exposed to a feared situation.

when the size of the decline was about 1.25 percentage points, which partly reversed over the subsequent 5 years, (c) between 2008–10 nearly all the decline can be attributed to a rise in long-term fear, (d) while by 2019 the long-term fear effects had zeroed out, with nearly all the remaining decline (of 0.5 percentage point.) explained by a decline in long-run consumption growth, and (e) the COVID-19 shock led to a temporarily decline in $r_t^{*,LR}$ due to rise in long-term fear, which lasted only until early 2022.

4.2.3 Why Does the Yield Curve Slope Predict Recessions? (Yield Curve Inversion Puzzle)

The slope of the yield curve for two bonds that matures $T+N$ and N periods ahead for $T > N > 0$ is given by $y_t^{(T+N)} - y_t^{(N)}$:

$$y_t^{(T+N)} - y_t^{(N)} \approx \Lambda_\beta \cdot \gamma \left[\eta\left(1-\beta\right) - \nu\right] \cdot \left(gap_t\right) - \Lambda_\vartheta \cdot \gamma \cdot \left(\mu_t - \overline{\mu}\right) + \Lambda_\theta \cdot \frac{\gamma^2}{12} \cdot \left(w_t^2 - \overline{w}^2\right) + \left(risk\, premium\right) \tag{36}$$

with $\Lambda_x \equiv \frac{1}{1-x}\left[\frac{\left(1-x^N\right)}{N} - \frac{\left(1-x^{T+N}\right)}{T+N}\right] \geq 0$. See Appendix A10 for derivations. That is, the slope of the yield curve is steeper when fear is high, when the output gap is positive, and when potential consumption growth is low.

So the condition for inverted yield curve when $gap_t > 0$ and around $\mu_t \approx \overline{\mu}$ is:

$$\Lambda_\theta \cdot \frac{\gamma}{12} \cdot \left(\overline{w}^2 - w_t^2\right) + \Lambda_\vartheta \cdot \left(\mu_t - \overline{\mu}\right) > \Lambda_\beta \cdot \left[\eta\left(1-\beta\right) - \nu\right] \cdot \left(gap_t\right) \tag{37}$$

So when fear is sufficiently low and/or productivity growth is relatively high (compared to their long-term average), we can have an inversion of the yield curve. That is, interest rates in the future will be expected to be lower since fear is expected to rise and productivity is expected to fall. In addition, when β is higher, i.e., there is more persistence in the central bank interest rates, the right-hand-side is smaller for a positive output gap. This could happen when the central bank is in a tightening cycle, therefore less likely to switch gears if the real rate declines in the future.

Why does it predict recessions? Note that the expected future output gap is given by: $E_t\left[gap_{t+N}\right] = -\frac{E_t\left[r_{t+N}^S - r_{t+N}^*\right]}{\gamma\left[\eta(1-\beta)-\nu\right]}$. Under the conditions that the yield curve inverts, the real rates r_{t+N}^* in the future are expected to be lower (with lower μ_t and higher w_t). And sluggish adjustment of the central bank (due to β) could mean that interest rate gaps in the future are higher, i.e., $r_{t+N}^S - r_{t+N}^*$, hence leading to contractions.

Result 3 (An inverted yield curve predicts recessions because it is associated with higher fear in the future, especially when central banks are in tightening cycles): *Under the conditions that the yield curve inverts (37), the real rates r_{t+N}^* in the future are expected to be lower (with lower μ_t and higher w_t). And sluggish adjustment of the central bank (due to β) could mean that interest rate gaps in the future are higher, i.e., $r_{t+N}^S - r_{t+N}^*$, hence leading to macroeconomic contractions.*

4.2.4 Why Do Price-Dividend Ratios Predict Stock Market Returns? (Predictability Puzzle)

Campbell and Shiller (1988) derived an accounting identity, which implied that the log dividend-price ratio is approximately equal to a constant plus the sum of present values of future log returns minus the sum of present values of future log dividend growths. This implies that current log dividend-price ratio should be able to predict future log returns, or log dividend growth rates, or both. The SWIM model enables us to derive a different derivation of the log dividend-price ratio and for expected returns.

Let dividends be $D_t \equiv C_t^{\lambda}$ with $\lambda \geq 1$ measuring the degree of leverage. The N period return on equity is $R_{t+N}^e = \frac{P_{t+N}^e + D_{t+1} + \ldots + D_{t+N}}{P_t^e}$. In Appendix A11, I show that by taking a Taylor approximation, the identity for log returns can be represented as:

$$\ln R_{t+N}^e \approx \ln \frac{D_{t+N}}{D_t} + \frac{1}{f} \ln \left(\frac{P_{t+N}^e / D_{t+N}}{P_t^e / D_t} \right) + \frac{1}{f} \left(N - \frac{N(N-1)}{2} g_{t,N} \right) \cdot \frac{D_t}{P_t^e} \tag{38}$$

for a constants $f \geq 1$. Here $g_{t,N} \approx \frac{2}{N(N+1)} \sum_{k=1}^{N} k \cdot g_{t+j}$ is a weighted arithmetic mean of future growth rates, such that distant growth rates having a greater weight than near-term growth rates. (For typical values of $g_{t,N}$, and the price-dividend ratio, $f \approx \frac{3}{2}$ provides a good approximation). See Appendix A11 and A12 for the derivation.[12] In the relevant range of the approximation (i.e., for N not too large), we have $\frac{d^2 \ln R_{t+N}^e}{d(D_t/P_t^e)dN} > 0$. That is the slope coefficient on the dividend-price ratio is increasing with horizon N.

Further, in the SWIM model, we can add structure to this identity, which allows us to re-write it as:

$$E_t \left[\ln R_{t+N}^e \right] \approx \Theta_0 - \Theta_\beta \cdot \left[\eta (1-\beta) - \nu \right] gap_t + \Theta_\vartheta \cdot (\mu_t - \overline{\mu}) - \Theta_\theta \cdot \left(w_t^2 - \overline{w}^2 \right) + \Theta_g \cdot \frac{D_t}{P_t^e} \tag{39}$$

with $\Theta_0 \equiv N\lambda\overline{\mu}$, $\Theta_x \equiv \left[\lambda + \frac{1}{f}(\gamma - \lambda) \right] \left(\frac{1-x^N}{1-x} \right)$ for $x = \beta, \vartheta$. And with $\Theta_\theta = \frac{1}{12} \left[\lambda + \frac{1}{f}(\gamma - \lambda)^2 \right] \left(\frac{1-\theta^N}{1-\theta} \right)$, and $\Theta_g = \frac{1}{f} \left(N - \frac{N(N-1)}{2} E_t \left[g_{t,N} \right] \right)$. Similarly, with some tedious algebra it can be shown that in the SWIM model the log dividend-price ratio can be written as:

$$\ln \left[\frac{P_t^e}{D_t} \right] \approx - \ln k_0 + \frac{(\gamma - \lambda) \left[\eta (1-\beta) - \nu \right]}{1-\beta} gap_t - \frac{\gamma - \lambda}{1 - \vartheta} (\mu_t - \overline{\mu}) + \frac{1}{12} \frac{(\gamma - \lambda)^2}{1-\theta} \left(w_t^2 - \overline{w}^2 \right) \tag{40}$$

with a constant k_0 defined as $k_0 \equiv \rho \cdot \exp \left\{ \frac{1}{\rho} (\gamma - \lambda) \overline{\mu} - \frac{1}{12\rho} (\gamma - \lambda)^2 \overline{w}^2 \right\}$. See Appendix A11 for the derivations. This representation of expected returns allows us to establish a few results. First, the dividend-price ratio predicts expected equity returns, in line with past evidence and what has been called the 'returns predictability puzzle.' Second, the predictive power of the dividend-price ratio for future returns is small

[12] The key steps of the derivation are as follows. By re-arranging the return on equity identity, we get $R_{t+N}^e = \frac{D_{t+N}}{D_t} f \left[1 - \left(\frac{f-1}{f} \right) + \frac{1}{f} \left(\frac{P_{t+N}^e / D_{t+N} - P_t^e / D_t}{P_t^e / D_t} \right) + \frac{1}{f} \left(\frac{1 + g_{t,N}}{g_{t,N}} \right) \left(1 - (1 + g_{t,N})^{-N} \right) \cdot \frac{D_t}{P_t^e} \right]$, for a constant $f > 1$. For small X we have the result: $\ln(1+X) \approx X$. Then, we can approximate: $\left(\frac{1+g_{t,N}}{g_{t,N}} \right) \left(1 - (1+g_{t,N})^{-N} \right) \approx N - \frac{N(N-1)}{2} g_{t,N}$ and $\ln f - \left(\frac{f-1}{f} \right) \approx 0$. Plugging these in allows us to get the identity in (38).

around $N = 1$, and rises with the horizon (for N not too large). Third, the price-dividend ratio co-moves positively with the output gap and fear, and negatively with potential consumption growth. Why? To see the intuition behind these results, we can re-write the log price-dividend ratio by plugging in our values of the short-run and long-run neutral rate:

$$\ln\left[\frac{P_t^e}{D_t}\right] \approx -\ln k_0 - k_1\left(r_t^* - r_t^{*,LR}\right) + k_2 \cdot gap_t - k_3\left(w_t^2 - \overline{w}^2\right) \tag{41}$$

with positive constants $k_1 \equiv \frac{1}{\gamma}\left(\frac{\gamma-\lambda}{1-\vartheta}\right)$, $k_2 \equiv \frac{(\gamma-\lambda)[\eta(1-\beta)-\nu]}{1-\beta}$, and $k_3 \equiv \frac{\gamma-\lambda}{12}\left[\frac{\gamma}{1-\vartheta} - \frac{\gamma-\lambda}{1-\theta}\right]$. This representation shows that a higher value of output gap today raises the log price-dividend ratio as it is associated with higher expected output in the future, and thus expected dividends. Further, a higher slope of the real neutral rate curve $\left(r_t^{*,LR} - r_t^*\right)$ raises the log price-dividend ratio, as it is associated with stronger growth in potential consumption, and therefore in the trend productivity of the economy. And finally, the price-dividend ratio is lower when fear is high today (relative to it's long-run value). And we also get that $E_t \ln\left[\frac{P_{t+\infty}^e}{D_{t+\infty}}\right] = -\ln(k_0)$, such that the price-dividend ratio reverts to its long-term mean. Therefore, the dividend-yield contains predictive information for expected equity returns, with the predictive power greater for longer horizons due to the high degree of persistence in output gap, fear, and the neutral rate of interest. Using the representation in (41) and taking exponents and re-arranging, we can also derive a modified Gordon growth model type equation for the fear-economy with a tight link to macro variables:

$$P_t^e \approx \frac{D_t}{k_0\left(1 + k_1\left[r_t^* - r_t^{*,LR}\right]\right)\left(1 - k_2 \cdot gap_t\right)\left(1 + k_3\left[w_t^2 - \overline{w}^2\right]\right)} \tag{42}$$

This modified Gordon growth equation implies a tight link between the macroeconomy and asset prices.

I test these equity relationships of the SWIM model using monthly data for equity returns, the price-dividend ratio, and the macro variables. The results are presented in Table 3. Overall, the evidence strongly confirms these predictions.

Result 4 (The dividend-yield predicts equity returns, with the predictive power greater for longer horizons due to persistence in output gap, fear, and the neutral rate of interest): *First, the dividend-price ratio predicts expected equity returns. Second, the predictive power of the dividend-price ratio for future returns is small around $N = 1$, and rises with the horizon, due to persistence in macro variables as shown in the modified Gordon growth equation (42). Third, the price-dividend ratio co-moves positively with the output gap and the* slope of the real neutral rate curve, *and negatively with fear.*

Table 3: Regression Table for the Model's Asset Pricing Predictions

	(1) 2-0Y Spread	(2) 10-2Y Spread	(3) 30-10Y Spread	(4) 6M Eq. Return	(5) 1Y Eq. Return	(6) Log P/D	(7) Log P/D
Output Gap	0.131***	0.152***	0.044***	-0.024***	-0.026***	0.082***	0.087***
	(0.018)	(0.013)	(0.007)	(0.004)	(0.006)	(0.009)	(0.009)
Cons. Gr.	-1.564***	-1.534***	-0.521***	-0.021	-0.136***	-0.128***	
	(0.101)	(0.070)	(0.039)	(0.020)	(0.029)	(0.047)	
Fear	36.972***	64.169***	21.360***	-2.501***	-1.695**	3.391***	-0.754
	(2.743)	(1.908)	(1.049)	(0.562)	(0.777)	(1.291)	(1.009)
D/P Ratio				1.708**	5.094***		
				(0.822)	(1.136)		
Short - Long r*							-0.044***
							(0.014)
Observations	458	458	458	455	449	458	458
R^2	0.433	0.842	0.769	0.195	0.299	0.579	0.581

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table examines the different asset pricing predictions of the model. The first set of results (columns 1, 2, and 3) correspond to the predictions of the model with respect to Treasury bond yield spreads in equation (36), at 2-0 year, 10-2 year, and 30-10 year maturities respectively. The next set of results (columns 4 and 5) correspond to the predictions of the model with respect to equity returns in equation (39), for 6 months and 1 year ahead respectively. And the final set of results (columns 6 and 7), correspond to the model's predictions with respect to the log price-dividend ratio in equations (40) and (41). Here, the sample is monthly time series data for the US between 1984 and 2022. The dependent variables for the three sets of results are the bond yield spreads, realized future equity returns, and the log price-dividend ratio. The bond yield spreads are computed using data from the Fred database of the St. Louis Federal Reserve, while the equity returns and price-dividend ratio are proxied using the online data provided by Robert Shiller, which uses his calculation for realized monthly real equity returns and the cyclically-adjusted price earnings ratio. The right-hand-side variables used are model-implied fear, consumption growth, a measure of output gap based on the unemployment rate (as discussed in Section 4.1), a measure of the price dividend ratio and the model-implied spread between the short-run neutral rate and the long-run neutral rate. The regressions are estimated using OLS, controlling for a linear time trend. The coefficients for the constant term and the linear time trend are not reported. All results broadly match the empirical predictions, except the coefficient of consumption growth in the realized 1 year equity returns regression in column 5.

5 Six Implications

In this section I present six novel insights for macro policy based on the SWIM model.

5.1 Fear Cycles & Recessions

Fear shocks in the model are recessionary. We can quantify the negative impact of a fear shock on the output gap today using (29) such that:

$$\frac{d(gap_t)}{d(w_t)} = -\frac{\gamma \cdot w_t}{6\left[\eta\left(1-\beta\right)-\nu\right]} < 0 \tag{43}$$

This result demonstrates that as fear goes up output gap falls. Further, to understand how the fear shocks can be amplified we can examine the cross-derivatives:

$$\begin{aligned}
\frac{d^2(gap_t)}{d(w_t)d(\beta)} &= -\frac{\gamma\eta \cdot w_t}{6\left[\eta\left(1-\beta\right)-\nu\right]^2} - \frac{\gamma \cdot \left(1-\zeta\right)\left(1-s_t\right)\eta^2\vartheta}{6\left[\eta\left(1-\beta\right)-\nu\right]^3} < 0 \\
\frac{d^2(gap_t)}{dw_t^2} &= -\frac{\gamma}{6\left[\eta\left(1-\beta\right)-\nu\right]} < 0
\end{aligned} \tag{44}$$

Thus, the fear shocks are amplified when the central bank is sluggish in adjusting the interest rate on safe assets in response to shifts in the neutral rate of interest (higher β). And, the negative relationship between output gap and fear (w_t) is concave, suggesting an increase in fear in this model can have a disproportionately large adverse impact on the output gap when the existing level of fear is high.

We can also rely on the calibration exercise (Figure 6) to examine the variation in uncertainty over time. The calibration exercise suggests that during the Great Recession of 2007–09 there was a sharp rise in fear (w_t). This could have happened due to a change in the market's expectation of aggregate safe assets available in the worst-case scenario (i.e., s_t became low) for instance due to a shock to the real estate sector and to the aggregate production technology more generally. Or instead, it could have happened due to a deterioration in social insurance available to consumers, reducing the effective supply of safety. In either case, we would see a rightward shift in the W curve, leading to a lower neutral rate of interest, and raising the possibility of getting stuck at the Federal Reserve's effective lower bound (Figure 9).

Is there any recognizable evidence of such a shift in the effective supply of safe assets available to households? One source of indirect evidence is the evolution of broad money supply during this period, which can be roughly seen a quasi-safe asset available to savers (households, firms, or institutions). For such an exercise, it is useful to rely on a broad notion of money that includes money creation done by the shadow bank. In this context, a useful benchmark is the M3 measure which includes money creation done by the money market mutual funds (MMMFs), that play a key role in channelling resources from corporates, large institutions, universities, sovereign wealth funds, high net-worth individuals, etc. to the broader shadow

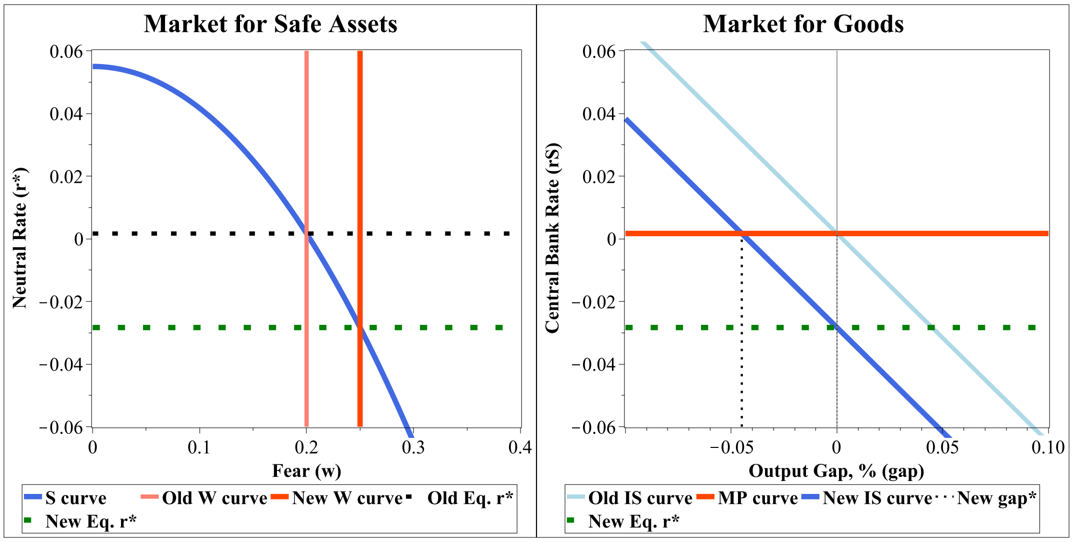


Figure 9: SWIM Model with a fear shock (a rightward shift of the W curve & leftward shift of the IS curve)

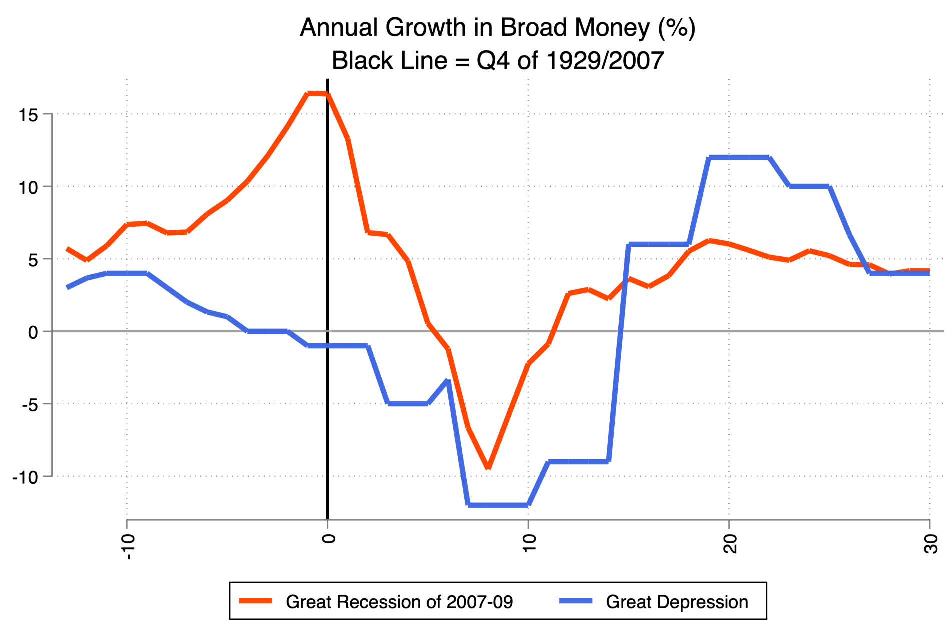


Figure 10: Money Supply during the Great Depression and the Great Recession

banking system.[13] The M3 measure of money shows that the information content of M3 is non-negligible and may help reframe the historical narrative about the crisis. In particular, the data suggests that the broad measure of money (M3) declined by about 10 percent during the Great Recession of 2007–09 similar in magnitudes to the declines in money supply observed during the Great Depression (Figure 10). However, during the same period, M2 balances continued to grow, albeit at a slightly slower pace. The significant difference in growth patterns of M2 and non-M2 components of M3 illustrates this contraction in money supply was felt much harder by the institutional investors compared to the retail investors—hampering credit creation in the shadow banking system.[14] Overall—both in the Great Depression and the Great Depression—the growth in broad money supply declined sharply with a few months and went into negative territory in about two years. This points to the possibility that the rise in fear (i.e., decline in w_t) or perceived decline in safety during both the Great Depression and the Great Recession could have been associated with a shock to the supply of broad money (Brunnermeier, 2009; Kacperczyk and Schnabl, 2010; Gorton and Metrick, 2012; Gennaioli et al., 2012; Agarwal, 2012).[15]

Implication 1 (Actively regulating the interest on safe assets (in either direction) can mitigate the fluctuations generated by fear cycles): *Exogenous fear cycles offer a common explanation for asset price movements and macro outcomes as in (30) and (29). The fear-driven contractions as shown in (43) are amplified at higher levels of initial fear, and when interest rates are more sluggish (i.e., a high β).*

[13] In the U.S. the M2 monetary aggregate includes currency held by the public, transaction deposits at depository institutions, savings deposits, and small-denomination time deposits (i.e., of amounts less than \$100,000). The M2 monetary aggregate also includes retail money market mutual fund shares. Thus, a part of the money creation done through the shadow banking system appears in M2. One way to interpret M2 then is that it includes much of cash or cash-like instruments that regular households and small firms hold, but does not capture much of cash or cash-like instruments held by larger institutions or clients. That is, much of the money creation done outside the shadow banking system is not captured in the M2 monetary aggregate. In the past, the Fed used to also publish the M3 monetary aggregate, which has been discontinued since March 2006. The M3 included all M2 components plus balances in institutional MMMFs, large-denomination time deposits (i.e., of amounts of \$100,000 or more), repos of depository institutions, and Eurodollars held at foreign branches of U.S. banks worldwide. That is, the M3 monetary aggregate of the U.S. did include some important components of the money created by the shadow banking system. In particular, it captured all the money creation done by the institutional MMMFs, and also large time deposits. This makes the M3 aggregate a useful benchmark to measure amount of money creation that is being done in the U.S. banking system. The Federal Reserve has continued to publish data on institutional MMMFs and large time deposits, which made up about 75 percent of the non-M2 component of M3 in 2006, which makes it possible to estimate the M3 series after 2006.

[14] The sharp decline in M3 has not received much attention. Instead, several observers have focused only on M2 balances, and have reached the conclusion that money supply expanded during the global financial crisis (as opposed to the sharp decline in money supply during the Great Depression). Eichengreen and O'Rourke (2009) write: "C*learly, monetary expansion was more rapid in the run-up to the 2008 crisis than during 1925–29, which is a reminder that the stage-setting events were not the same in the two cases. Moreover, the global money supply continued to grow rapidly in 2008, unlike in 1929 when it leveled off and then underwent a catastrophic decline.*" However, Figure 10 shows that the story looks quite different when one compares growth of M3 money supply during the Great Recession to the growth of money supply during the Great Depression.

[15] An extension of the SWIM model with banks could address such a scenario, in which (in addition to the government) the bank deposits can also provide a disaster insurance role. In such a setup a shock to the liquidity or safety of bank deposits can lead to a rise in uncertainty in line with these two episodes.

5.2 Longer & Deeper Business Cycles with Stagnation Episodes

The Great Recession and its aftermath (2007–2015) was one of the deepest recessions in history followed by a painful and slow recovery. Such stagnation episodes may become more frequent, in part due to the zero lower bound or the effective lower bound. Before zero lower bound became an issue, advanced country central banks typically cut nominal rates by 5–6 percentage points to restore the economy to its full potential. Given expected inflation, a cut in nominal rates translates into a cut in real rates that increases spending both by tilting incentives toward spending now rather than later and via a constellation of wealth effects in favor of the borrowers in every borrower-lender relationship (see Agarwal & Kimball, 2015; 2019). However, due to a variety of factors, including an aging population that has raised the savings rate, a decline in productivity, and greater demand for safe assets, the "natural" real rate of interest has trended downward worldwide. The downward trend in nominal interest rates has made zero lower bounds serious obstacles to monetary policy. During the Great Recession several advanced country central banks cut their policy rates to near-zero and kept them there for years. A few even went further by cutting rates into mildly negative territory. But since the rate cuts were not sufficiently large, the downturn lasted much longer than previous ones, and recovery was slow. Recently, the COVID-19 pandemic led to even more central banks cutting rates to near-zero. But the interest rate cuts were bigger for countries further away from the zero lower bound (Agarwal & Kimball, 2022), suggesting that central banks that cut rates less felt constrained by zero lower bounds.

Why is there a zero lower bound? It is not a law of nature, but a policy choice. The zero lower bound arises when a government issues pieces of paper (or coins) guaranteeing a zero nominal interest rate, over all time horizons, that can be obtained in unlimited quantities in exchange for money in the bank. This acts as an interest rate floor, making people unwilling to lend at significantly lower rates. In the SWIM model, when a central bank chooses to not cut rates deeper into negative territory and accepts the zero lower bound, it corresponds to an increase in the persistence term, β, in the MP curve. We will call a higher value of β a 'more binding lower bound.' That is, any deviation of the short rate from the neutral rate takes longer to correct since the central bank is unwilling to cut rates into the negative territory. What is the impact of a higher β on the economy? An increase in β implies a deeper recession for the same interest differential and level of fear. Let's call $r_t^S - r_t^*$ the interest rate gap. From the IS curve we have the contractionary effect of an increase in the interest rate gaps as $\frac{d(gap_t)}{d(r_t^S - r_t^*)} = -\frac{1}{\gamma[\eta(1-\beta)-\nu]}$. Thus a more sluggish central bank function amplifies the contractionary effects of an increase in the interest rate gap:

$$\frac{d^2(gap_t)}{d(r_t^S - r_t^*)d(\beta)} = -\frac{\eta}{\gamma\left[\eta\left(1-\beta\right)-\nu\right]^2} < 0 \tag{45}$$

And duration of recessions and booms are longer when β is higher. That is, from (29) we get that $E_t\left[gap_{t+N}\right] = -\frac{\beta^N}{\gamma[\eta(1-\beta)-\nu]}\left[r_t^S - r_t^*\right] = \beta^N \cdot gap_t$ for $N \geq 0$. Let N_{half} denote the expected half-life of recessions, that is the time required for a recession or boom to reduce to half its value. It is given by:

$\frac{1}{2} = \beta^{N_{half}}$,or $N_{half} = \frac{\ln 2}{-\ln \beta}$. And its first two derivatives with respect to β are:

$$\begin{aligned} \frac{dN_{half}}{d\beta} &= \frac{\ln 2}{\beta \cdot (\ln \beta)^2} > 0 \\ \frac{d^2 N_{half}}{d\beta^2} &= \frac{\ln 2 \cdot [2 + \ln \beta]}{-\beta^2 \cdot (\ln \beta)^3} > 0 \end{aligned} \tag{46}$$

Thus, the half-life of recessions or booms is strictly increasing for all values of β and convex for $\beta > \frac{1}{e^2} \approx 0.14$. The relationship is highly non-linear for higher values of β. If we take each period to be a quarter, typical values for β is around 0.8, but higher after the Great Recession. An increase of β from 0.7 to 0.8 leads the half-life of recessions to increase by about 1 quarter. But an increase from 0.8 to 0.9 (roughly what we observed after 2009) can lead the half life to increase by 3.5 quarters. This result is consistent with simulations of the Fed's main macroeconometric model suggest that relying on the policy rules developed before the Great Recession would result in short-term rates being constrained by zero as much as one-third of the time, with costlier and longer recessions (Kiley and Roberts, 2017).

The effective lower bound only binds when $r_t^S > r_t^*$, but the results presented here also works in reverse, with a high β corresponding to a situation in which the central bank is too sluggish to raise rates when the natural rate rises (as observed during the high inflation episodes around the world after the COVID-19 pandemic starting in 2021). That is, a more sluggish central bank function amplifies the expansionary effects of a decrease in the interest rate gap, i.e., $\frac{d^2(gap_t)}{d(r_t^* - r_t^S)d(\beta)} > 0$. However, a consequence of the zero lower bound is that the actual response of central banks has been highly asymmetric above and below the 2 percent target—with central banks tolerating inflation below 2 percent but acting as if the welfare costs of inflation above 2 percent are high (Agarwal and Kimball, 2022a). This is why one can expect the problem of a high β to matter much more when $r_t^S > r_t^*$, leading to a world of relatively more frequent long-lasting stagnations. However, if central banks' dislike for inflation gets suppressed given the enduring long-term impact of the pandemic, uncertainty about the recovery, and the temptation to inflate away higher debt burdens globally, then we could also experience longer-lasting boom/inflationary episodes Agarwal and Kimball (2022a; 2022b; 2022c). Further, while not modeled here, the joint distribution of ε_{t+1} and ψ_t could be such that the distribution of output gap is negatively skewed with long left tails. Such a distribution could also make secular stagnation type outcomes more frequent, with longer and deeper recessions.

Implication 2 (Recessions will be deeper & longer when central banks accept the lower bound and are unwilling to use negative rates): A more sluggish central bank function—such as the one experienced at the lower bound—amplifies the size of recessions as shown in (45). And, the half-life of recessions (or booms) is strictly increasing in the degree of sluggishness in central bank interest rates (β), and the relationship between β and the half life is highly non-linear as shown in (46).

5.3 Power of Negative Interest Rate Policy to Overcome Low-for-Long Rates

Over the last decade, the neutral rate of interest has fallen and many expect it to stay low. Why are interest rates expected to stay low for longer? The expected rate on safe assets J periods in the future is given by $E_t\left[r^*_{t+J}\right] = \rho + \gamma\mu_{t+J} - \frac{\gamma^2}{12}\cdot w^2_{t+J}$. To examine expected rates far into the future, one can let $J \to \infty$, such that $w^2_{t+\infty} \to \overline{w}^2$ and thus $E_t\left[r^*_{t+\infty}\right] = \rho + \gamma\mu - \frac{\gamma^2}{12}\cdot\overline{w}^2$ where μ is the long-run value of μ_{t+J}. To evaluate the effect of the effective lower bound (as measured by greater persistence in the monetary policy reaction function, β) we can differentiate the expected rate with respect to β. Note that $\frac{d\varrho}{d\beta} > 0$, and we get:

$$\frac{dE_t\left[r^*_{t+\infty}\right]}{d\beta} = -\frac{\gamma^2\left(1-\zeta\right)\left(1-\overline{s}\right)\eta^2\vartheta}{12\left[\eta\left(1-\beta\right)-\nu\right]^2} < 0 \tag{47}$$

That is, when central banks set interest rates different from the real rate today, it can have persistent effect on the yields far into the future when the persistence parameter β is high. Moreover, since $\frac{d^2E_t\left[r^*_{t+\infty}\right]}{d\beta^2} < 0$, the magnitude of this effect is non-linear and can get quantitatively large at higher values of β.

At the effective lower bound, a reduction in the value of β essentially means that the central bank is committing to use deeper negative rates. This may appear to be paradoxical. Committing to use deeper negative rates to fight recessions in the future can lead to *higher* longer-term rates. In other words, pushing down the perceived effective lower bound may raise rather than lower long-term rates. Or equivalently, long-term real rates may become persistently low if the central bank signals to the market that they are reluctant to use negative rates to fight future recessions. The intuition behind this result is that if markets' perception of the effective lower bound is low enough, the downward pressure on long-term rates from fear that central banks will be stuck for a long time at the effective lower bound is alleviated. This leads to a "hope for the return of a normal yield curve," which could act to raise the long-term rate. See Agarwal and Kimball (2015; 2019) for further discussion.

So even in absence of low growth today, as long as agents expect fear to be higher during periods of low growth in the future, long rates can fall if the market believes the central bank does not have the ability to cut interest rates in the future to bring it in line with the neutral rate. Thus, this result demonstrates a novel way in which monetary policy can be non-neutral with respect to even long-term rates in the economy. That is, the effective lower bound is a real constraint in the economy (not a nominal constraint) since it increases the likelihood of future recessions. Therefore, interest rate policy of the central bank—in particular adopting a zero lower bound policy vs. convincing the market that they are open to implementing negative interest rates to fight future recessions—can have a sizable impact even on long-term rates.

Breaking the zero lower bound requires eliminating the arbitrage opportunity between cash that guarantees a zero nominal interest rate and money in the bank that would earn negative interest rates if policy rates were below zero. All mechanisms to break zero lower bounds modify paper currency policy, making electronic money more central to the monetary system (Agarwal and Kimball, 2022c). One way is through

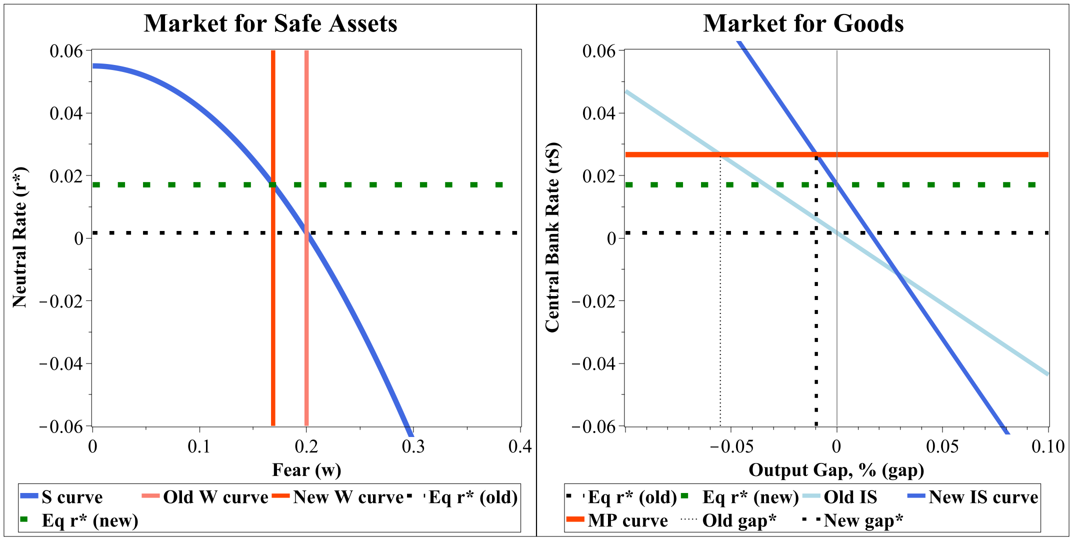


Figure 11: SWIM Model with a commitment to use negative interest rates whenever needed (a leftward shift of the W curve and a steepening of the IS curve)

the combination of (a) adopting or strengthening an electronic standard in which electronic money is the unit of account and (b) implementing a time-varying interest rate (or more generally, rate of return) on paper currency (cash). Then, as the interest rate on cash moves in line with the official policy rates, there is no arbitrage between cash and money in the bank. While operationally this can be done while remaining quite close to the current monetary system, there are several legal, communication and political challenges to transitioning to such an electronic money standard (Agarwal and Kimball 2015; 2019).

Moreover, a commitment to use negative rates today—even without actually using it—can reduce the magnitude of recessions (and booms). This is because such a commitment is akin to having a lower β today and in the future, which leads to a steepening of the IS curve (as depicted in Figure 11). Thus, the central bank is able to use interest rate policy more vigorously in either direction in line with movements in the real interest rate, thereby reducing the volatility of the output gap (i.e., shallower recessions and smaller booms).

> ***Implication 3 (Committing to use negative interest rate policy in recessions raises the real neutral rate over the entire yield curve and moderates the business cycle):*** *There is long-run monetary non-neutrality. A sluggish central bank reaction function can have persistent effect on the yields far into the future when the persistence parameter β is high. Committing to use deeper negative rates to fight recessions in the future can lead to higher longer-term rates across the entire yield curve, and reduce the amplitude of the business cycle (due to a steeper IS curve). This effect holds even away from the zero lower bound, as long as the central bank can commit to use negative rates whenever needed (including when fear rises sharply).*

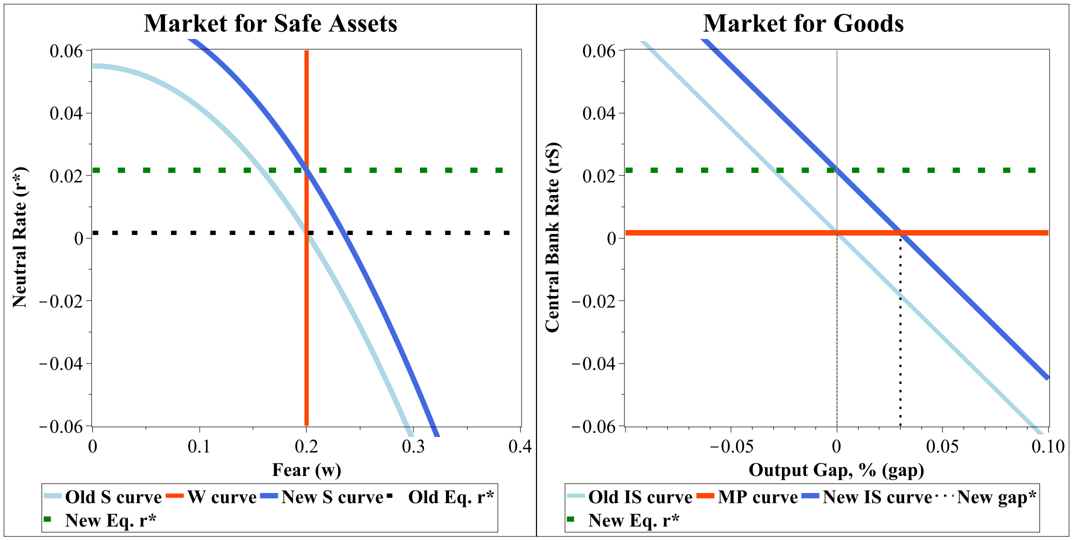


Figure 12: SWIM Model with an increase in counter-cyclicality of fiscal policy (a rightward shift of the S curve and the IS curve)

5.4 The Fear-Mitigation Role of Fiscal Policy

A novel role of fiscal policy in the SWIM model is the fear-reduction role. When the degree of social insurance provided by the government is greater (i.e., ζ is higher), fear is lower and therefore the neutral rate of interest is higher. Thus, any policy action to raise the counter-cyclicality of fiscal policy by raising ζ can be expansionary. And the benefit of raising ζ is higher when safety is low. To see this note that from the S curve we have $w_t = \varrho \cdot (1-\zeta) \cdot (1-s_t)$. So, $\frac{dw_t}{d\zeta} > 0$. Using this result and the result from Section 4.1, we examine the output effects of a policy action that raises ζ:

$$\frac{d(gap_t)}{d\zeta} = \left(\frac{1}{1-\zeta}\right) \cdot \frac{\gamma \cdot w_t^2}{6\left[\eta\left(1-\beta\right)-\nu\right]} > 0 \tag{48}$$

Such a case is depicted in Figure 12, which corresponds to a rightward shift in the S curve, raising the neutral rate r_t^* (left panel), and therefore leads to a rightward shift of the IS curve (right panel), since the IS curve intersects the zero output gap line now at the higher value of r_t^*.

Further, note that the fear-mitigation role of fiscal policy is more potent at the effective lower bound, i.e., when β is high or when fear is high. That is:

$$\begin{aligned} \frac{d^2(gap_t)}{d(\zeta)d(\beta)} &= \left(\frac{1}{1-\zeta}\right) \cdot \frac{d}{d\beta}\left(\frac{\gamma \cdot w_t^2}{6\left[\eta\left(1-\beta\right)-\nu\right]}\right) > 0 \\ \frac{d^2(gap_t)}{d(\zeta)d(w_t)} &= \left(\frac{1}{1-\zeta}\right) \cdot \frac{\gamma \cdot w_t}{3\left[\eta\left(1-\beta\right)-\nu\right]} > 0 \end{aligned} \tag{49}$$

Graphically, a higher value of fear means, the

Implication 4 (Policies to increase the counter-cyclicality of fiscal policy are expansionary; and the effects are amplified at the lower bound or when fear is high): *Any policy action to raise the counter-cyclicality of fiscal policy by raising ζ can be expansionary. And the benefit of raising ζ is higher at the lower bound or when fear is high.*

5.5 The Fear Theory of Quantitative Easing

In this section we evaluate the effect of policies that substitute long-term government debt with short-maturity instruments. These include quantitative easing policies in which the central bank's purchase long-term Treasuries in exchange for reserves; and Treasury operations which raise the supply of short-term government debt while simultaneously lowering the supply of long-term debt ('Operation Twist'). The fear theory of quantitative easing is about the demand for short-maturity safe debt and the constraining role of debt maturity on fiscal spending.

First, the yield curve equation in (36) can be used to derive the excess demand for short-safe debt (with maturity $N = 1$) vs. very long-maturity safe debt (with maturity $T \to \infty$), such that:

$$y_t^{(\infty)} - y_t^{(1)} \approx \gamma \left[\eta \left(1-\beta\right) - \nu\right] \cdot \left(gap_t\right) - \gamma \cdot \left(\mu_t - \overline{\mu}\right) + \frac{\gamma^2}{12} \cdot \left(w_t^2 - \overline{w}^2\right) + \left(risk\, premium\right) \tag{50}$$

That is, the long-term slope of the yield curve is steeper when fear is high, implying demand for short-term safe asset (relative to long-maturity safe assets) spikes up when fear is high: $\frac{d\left(y_t^{(\infty)} - y_t^{(1)}\right)}{dw_t} = \frac{\gamma^2}{6} w_t > 0$. Moreover, this yield steepening effect is highest for the short-term debt vs. the very long-term debt. That is: $\frac{d^2\left(y_t^{(T)} - y_t^{(1)}\right)}{dw_t dT} > 0$.

Do we see evidence for this in the data? Yes. Using the US Treasury Monthly Statement of the Public Debt, Figure 13 presents the share of Treasury Bills (T-Bills) held by the public relative to total marketable US Treasury debt held by the public since 2001. This can be seen as the demand for short-term safe debt satisfied by the government by changing the maturity profile of debt outstanding. The figure compares this to the model-implied fear (w_t) and the yield spread between a 10-year TIPS bond and the Fed Funds rate. Both during the Great Recession of 2007–09 and the COVID-19 pandemic in 2020 we observe a simultaneous rise in fear, the yield spreads, and the share of T-Bills held by the public (relative to total Treasury debt held by them).

Second, the quantity of debt maturing in a given period may also have a constraining effect on fiscal policy. This argument is similar to the one developed by Cochrane (2022), who argues that when there the central bank raises interest rates, fiscal policy must tighten as well. This is in part because a rise in interest rates raises interest costs on the debt. And, the government must pay those higher interest costs, by raising tax revenues and cutting spending, or by credibly promising to do so in the future.

In each period, the government's gross financing needs are given by maturing debt plus new borrowing

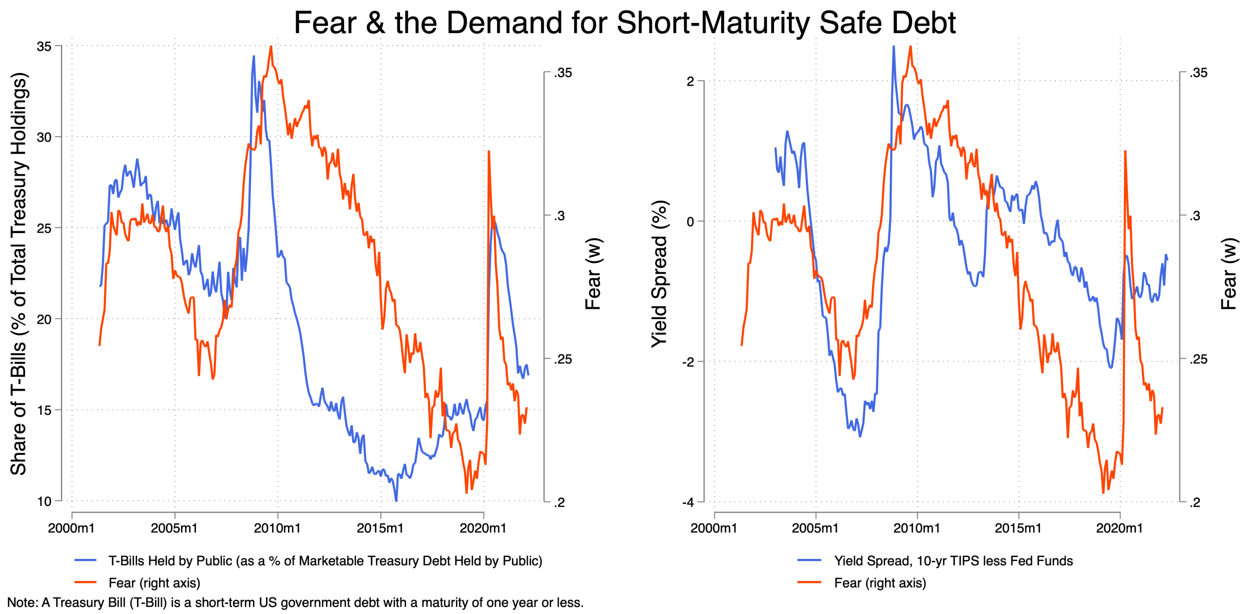


Figure 13: Relationship between Fear and Demand for Short-Maturity Safe Debt

requirement. In practice, governments raise financing by issuing sovereign bonds at an auction. And, the demand curve for sovereign debt in the auctions is not always flat, and can be upward sloping. That is, sovereign bond yields tend to rise when the demand for the new issue at sovereign auctions is smaller relative to its supply. Then, larger quantities of debt maturing in a given period can potentially limit total financing raised in a given period, which in turn can constrain new borrowing for fiscal spending. This constraining effect of maturing debt may be non-binding in normal times, but could potentially become binding during crises (Beetsma et al., 2016). This could be due to primary dealers requiring greater compensation for inventory risk when market uncertainty is larger. Thus, one way in which quantitative-easing-type maturity transformation (substituting long-term debt with short-term debt) can work is that it constrains fiscal authorities from spending in crises scenarios therefore raising the counter-cyclicality of fiscal spending (i.e., a higher ζ). The effectiveness of quantitative easing policies then may depend on the extent to which it credibly shifts ζ.

As discussed in Section 5.4, the expansionary effects of raising ζ is relatively small when fear is low. But can become sizable when fear is high or when the economy is stuck at the effective lower bound. Let the share of total debt maturing in a given period be given by $Maturing\ Debt_t$. Recall that the process for government consumption share of output is $\Delta g_{t+1} = (1-g_t)\left(1-\left[\frac{A_t}{A_{t+1}}\right]^{\zeta}\right)$. For maturing debt to play a constraining role in crisis outcomes (i.e., when consumption is significantly low) we need $\frac{d\zeta}{d(Maturing\ Debt_{t+1})} \geq 0$. So, for this to hold we would need $\frac{dg_{t+1}}{d(Maturing\ Debt_{t+1})} \leq 0$ in crises if during normal times we have $\frac{dg_{t+1}}{d(Maturing\ Debt_{t+1})} \approx 0$.

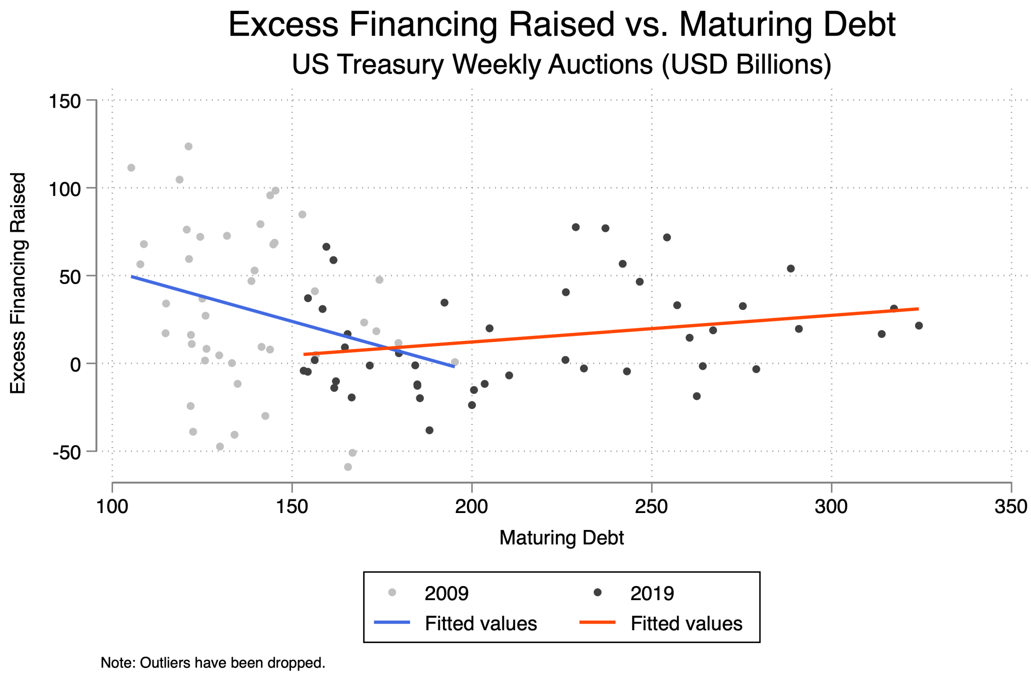


Figure 14: Relationship between Excess Financing Raised and Maturing Debt

Is there evidence supporting this? For the euro area, Beetsma et al. (2016) show that the secondary-market yields on Italian public debt increase in anticipation of auctions of new issues and decrease after the auction, while no or a smaller such effect is present for German public debt. And, these yield movements on the Italian debt are largely confined to the period of the crisis since mid-2007. There is suggestive evidence for a similar mechanism in the US. This can be seen by comparing the amount of financing raised by the US Treasury in 2009 and 2019 as a function of the quantity of debt maturing in the given period (Figure 14). That is, during normal times the quantity of maturing debt is not necessarily a constraint on nonessential government consumption, while it may act as a constraint during crisis periods. Thus, quantitative easing may work in some instances by reducing the chance of more government borrowing/spending in future stressed episodes. However, there are limits to QE under the fear theory of quantitative easing. First, if government borrowing constraint is less likely to bind once a given amount of maturing debt has been taken away from public hands then there may be strong diminishing returns to the policy, i.e., $\frac{d^2\zeta}{d(Maturing\ Debt_{t+1})^2} < 0$. Second, as discussed in Section 5.4, the expansionary effects of raising ζ are smaller when fear (w_t) is low or when the effective lower bound is not as binding (i.e., a lower β). Thus, QE policies that shorten the maturity of government debt in public hands can be effective in special circumstances, but the real effects may be small in normal circumstances.

There is a real-world finance counterpart to the fear theory of quantitative easing. If the secondary-market yields on government debt increase in stressed episodes, then financial market participants face 'market risk' or 'interest rate risk' from holding longer maturity bonds—especially if they have to potentially to sell it during stressed episodes (i.e., the bonds are not intended to be held to maturity). Holding short-maturity bonds (as opposed to long-maturity bonds) gives market participants an option to convert their assets into liquidity (for 'consumption') in stressed episodes without worrying about the market risk associated with selling government bonds when yields are high (Vayanos and Vila, 2021; Greenwood and Vayanos, 2010). And the value of that option is greater the higher the fear of catastrophes in the immediate future. The fear theory of QE suggests that the market risk (or duration risk) can be endogenously lower when the quantity of debt maturing in a given period is higher, by limiting how much debt issuance the government can do in a given stressed period. This effect can also be seen in the slope of the yield curve as shown in (50). In periods of high fear such as during the Great Recession of 2007–09 and the COVID-19 pandemic, the demand for short-term debt vs. long-term debt rises when fear spikes, leading to a steepening of the real yield curve. Policy action to substitute long-term debt with short-term debt is likely to be effective precisely in these cases. This is exactly what the US government did during these two shock episodes (Figure 13).

> ***Implication 5 (Quantitative easing has limits but may be narrowly effective when fear is high at the lower bound—by satisfying demand for safe short-term debt when fear is high and by credibly constraining fiscal borrowing in stressed episodes):*** The effectiveness of quantitative easing depend on whether fear-driven demand for safe short debt exists, and also the extent to which it credibly raises the counter-cyclicality of fiscal spending (i.e., a higher ζ). In certain circumstances, it may work by reducing the chance of more government borrowing/spending in future stressed episodes. However, if government borrowing constraint is less likely to bind once a given amount of maturing debt has been taken away from public hands then there may be strong diminishing returns to the policy. Further, its real effects are smaller when fear (w_t) is low or when the effective lower bound is not as binding (i.e., a lower β).

5.6 The Structural Reforms Multiplier at the Effective Lower Bound

When fear is high, structural reforms can also have positive cyclical effects. Consider structural reform policies that boosts productivity boost (ε_t) or raises the average investment-capital ratio (a).

The effect of an increase in productivity today on future output gap can be given by:

$$\frac{dgap_{t+N}}{d\varepsilon_t} = \beta^N \left(\frac{\varrho - 1}{\eta\varrho} \right) w_{t-1} \tag{51}$$

There is a "business cycle multiplier" such that a boost to productivity also increases the output gap, not just trend output. Note that the derivative is zero when $\beta = 0$, implying that the multiplier is zero when central banks rates are not sluggish. And, the size of the multiplier is rising in current fear levels $w_{t-1}$and in β, such that:

$$\begin{array}{c} \frac{dgap_{t+N}}{d(\varepsilon_t)d(w_{t-1})} = \beta^N \left(\frac{\varrho-1}{\eta\varrho}\right) > 0 \\ \frac{dgap_{t+N}}{d(\varepsilon_t)d(\beta)} = N\beta^{N-1}\left(\frac{\varrho-1}{\eta\varrho}\right)w_{t-1} + \beta^N \frac{\eta}{\vartheta}\left(\frac{\varrho-1}{\eta\varrho}\right)^2 w_{t-1} > 0 \end{array} \tag{52}$$

Therefore, the beneficial effect of improving productivity is greater when the effective lower bound is more binding (that is when β is high) or when fear today is high.

In addition, the benefit of the productivity boost is higher when the investment-capital ratio is higher. That is:

$$\frac{dgap_{t+N}}{d(\varepsilon_t)da} = \frac{\beta^N w_{t-1}}{\eta\varrho^2}\left(\frac{\varrho-1}{\eta\varrho}\right)^2 \frac{d\nu}{da} > 0 \tag{53}$$

since $d\nu/da > 0$. Blanchard (2022) and others have emphasized the benefits of public investment and green investments in low interest rate environments. The results in (53) show that potentially there may also exist a cyclical benefit of a higher investment ratio when combined with a productivity boost.

They key results are summarized below.

> ***Implication 6 (When fear is high, especially at the lower bound, policies that boost productivity also have positive multipliers for fighting recessions):*** *When fear is high there is a non-negligible "business cycle multiplier" such that the boosting productivity increases the output gap, not just trend output. Moreover, the beneficial is higher at the effective lower bound or when the investment-capital ratio is higher.*

Overall, the six implications highlight the scope for expanding the macro-financial policy toolkit based on the insights of the fear economy framework.

6 Discussion

This paper presents a fear-based theory of output, interest rates, and safe assets. The model relies on the combination of fear, sticky safe rates, and an investment-based notion of the output gap to generate aggregate fluctuations in the economy. The paper's three main contributions are (i) developing a closed-form dynamic general equilibrium model of the economy that generates consistent cross-correlations in the key macro variables in the postwar US economy; (ii) presenting a unified framework for analyzing both the macroeconomy and financial markets with a common fear factor being the underlying driving force, thereby explaining several asset pricing puzzles and generating variation in equity prices, bond prices, and the risk premium in line with the data; and (iii) based on this framework, deriving six insights on how to expand the macro policy toolkit to manage the 'fear economy', including to address some of the new challenges we have been facing since the Great Recession and the COVID-19 pandemic.

The six key policy implications are: (1) fear is key driver of business cycles and asset prices, which central banks can manage by regulating the interest rate on safe assets; (2) recessions will be deeper and longer when central banks accept the self-imposed zero lower bound and are unwilling to use negative rates; (3) a commitment to use negative rates in recessions—even if never implemented—raises both the short- and long-run real neutral rates, and moderates the business cycle (as seen during the Great Moderation); (4) counter-cyclical fiscal policy provides disaster insurance and is expansionary by reducing fear; (5) quantitative easing can be narrowly effective when fear is high at the lower bound; and (6) when fear is high, especially at the lower bound, policies that boost productivity also have positive multipliers for fighting recessions.

Given the model's simplicity, it can be adopted to different country and economic settings. For instance, the model can be extended to include a banking system to study macro-financial linkages, which can potentially generate financial cycles with boom-bust dynamics, due to the link between banking shocks and safety. It can also be extended to open economy settings to generate implications for exchange rates and capital flows. Such extensions can be a fruitful avenue for further research.

In addition, the SWIM model can be enriched to consider subjective uncertainty to integrate insights from clinical psychology into macro. In the main text of the paper, I broadly worked within the rational expectations equilibrium framework. However, as shown in Online Appendix C, when this assumption is relaxed by adding structure on how subjective beliefs are formed, we can get a downward sloping W curve with the possibility of multiple equilibria and 'fear breeds fear' dynamics. In this specific extension, the safe rate of return acts as a signal about how bad the reasonable worst-case scenario can be. In such a setup, the model can generate fear cycles with episodes of euphoria followed by episodes of panic.

The framework developed in this paper is one attempt to weave together some of the emerging themes in policy debates, building on the growing literature at the intersection of macroeconomics and finance. I hope that over time, and with collective research effort, such frameworks can become useful for practitioners to evaluate key policy issues and for students to understand macro-finance phenomena.

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Online Appendix A: Proofs and Derivations of Results

A1. A Select Overview of Business Cycle Theories

See Table A1 below.

A2. Equilibrium Conditions of the Frictionless Benchmark

We can represent the equilibrium conditions of the frictionless benchmark model as:

$$\begin{aligned}
C_t^{-\gamma} &= e^{-\rho} E_t\left[C_{t+1}^{-\gamma}\right]\left(1+r_t^S\right) \\
C_t^{-\gamma} &= \upsilon_t L_t^{-\chi} \ / W_t \\
C_t &= \left(\frac{Y_t}{Y_t^p}\right)^{\eta} c_t^p Y_t^p \\
W_t &= (1-\alpha) Y_t / L_t \\
1+r_t^S &= \frac{E_t\left[r_{t+1}\right]}{I_t/I_t^p - Cov\left(M_{t+1}, r_{t+1}\right)} \\
K_t &= I_{t-1} + (1-\alpha) K_{t-1} \\
Y_t^p &= A_t K_t^{\alpha} \left(L_t^p\right)^{1-\alpha} \\
\frac{I_t}{Y_t} &= i_t^p \left[1 + \phi_t - \phi_t \left(\frac{Y_t}{Y_t^p}\right)^{-(1-\eta)}\right] \\
Y_t &= O_t Y_t^p \\
Y_t &= C_t + I_t + G_t
\end{aligned}$$

with $r_{t+1} \equiv \left[\alpha Y_{t+1}/K_{t+1} - \delta\right]$, $M_{t+1} = \left(C_{t+1}/C_t\right)^{-\gamma}$ and $\eta \equiv \frac{\chi-\alpha}{\gamma(1-\alpha)}$.

A3. Proof of Proposition 1

Suppose there exists a unique $O_t = O_t^*$ consistent with the household's Euler equation. From the conjectured solution, we get that optimal consumption is given by:

$$C_t^* = c_t Y_t^* = \left(\frac{Y_t^*}{Y_t^p}\right)^{\eta} c_t^p Y_t^p$$

with the objects (c_t^p, Y_t^p) not depending on O_t^*. Then, substituting the value for wages in the labor-leisure choice first order condition we get:

$$C_t^{*-\gamma} = \frac{\upsilon L_t^{*-\chi}}{W_t} = \frac{\upsilon_t L_t^{*1-\chi}}{(1-\alpha) Y_t^*}$$

Theory	Rigidities	Source of Shocks	Non-Standard Preferences or Beliefs	Production or Consumption Externality	Capacity Utilization or Adj. Costs	Special Features Needed for Positive Co-Movements in Output, Consumption, Investment, & Labor
The Real Business Cycle Model	None	TFP	None	None	None	Technology shocks (Kydland and Prescott, 1982; Long and Plosser, 1983)
News Shocks	None	Expectations	Internal Habits or Confidence Multipliers	None	Yes	Requires capacity utilization with internal habits for positive wealth effect (Jaimovich and Rebelo, 2009), or firm-level inattention (Angeletos and Lian, 2022)
Uncertainty & TFP Shocks	None	Uncertainty + TFP	Inelastic Labor Supply	None	Yes	Requires first moment TFP shocks (Bloom et al., 2018) with uncertainty shocks
Uncertainty & Idiosyncratic Risk	None	Uncertainty + Idio. Risk	None	Consumption Externality	None	Requires inefficient risk-sharing (Di Tella and Hall, 2022)
Increasing Returns with Sunspots (A)	None	Self-Fulfilling Beliefs	None	Production Externality	Yes	Requires production externality & capacity utilization (Benhabib and Wen (2004))
Increasing Returns with Sunspots (B)	None	Self-Fulfilling Beliefs	Labor Externality	Production Externality	None	Requires production externality with an upward sloping labor demand curve (Benhabib and Farmer, 1996, 2000)
The SWIM Model	Sticky Real Interest Rates	Uncertainty	Labor Externality	None	None	Requires a labor externality and sticky real interest rates (*This paper*)
Uncertainty Shocks with Sticky Prices	Sticky Prices	Uncertainty	Epstein-Zin Preferences	None	Yes	Requires countercyclical markups and sticky prices (Basu and Bundick, 2017)
Uncertainty Shocks with Sticky Wages	Sticky Real Wages	Uncertainty	Inelastic Labor Supply & Epstein-Zin	None	Yes	Requires inelastic labor plus labor market imperfections (Basu et al., 2021)
News Shocks with Sticky Wages	Sticky Prices & Wages	Expectations	Ambiguity & Knightian Uncertainty	None	Yes	Requires capacity utilization with ambiguity, Knightian uncertainty, and sticky wages (Ilut and Saijo, 2021)
New Keynesian Models with Sticky Prices	Sticky Prices/ Wages	Monetary or Investment	Positive Wealth Effects of Positive Shocks	None	Yes	Requires non-standard preferences to mute wealth effects on labor supply (Christiano et al., 2005; Khan and Tsoukalas, 2011; Eusepi and Preston, 2015), or roundabout production (Ascari et al., 2019)

Table A.1: Select Business Cycle Theories with Consistent Co-Movements and Sizable Aggregate Fluctuations where L_t^* denotes the optimal labor supply at the conjectured equilibrium. Plugging in the optimal value for C_t^*, gives us:

$$\left[c_t^p Y_t^p \left(\frac{Y_t^*}{Y_t^p}\right)^{\eta}\right]^{-\gamma} = \frac{\upsilon_t L_t^{*1-\chi}}{(1-\alpha)Y_t^*}$$

By re-arranging, we can express the optimal labor supply as a function of other variables:

$$L_t^* = \left[(\frac{1}{\upsilon_t})(1-\alpha)Y_t^*\left[c_t^p Y_t^p \left(\frac{Y_t^*}{Y_t^p}\right)^{\eta}\right]^{-\gamma}\right]^{\frac{1}{1-\chi}} = \left[(\frac{1}{\upsilon_t})(1-\alpha)\left(Y_t^*\right)^{1-\gamma}\left(\frac{Y_t^*}{Y_t^p}\right)^{\gamma(1-\eta)}\left(c_t^p\right)^{-\gamma}\right]^{\frac{1}{1-\chi}}$$

At potential we get $L_t^p = \left[(\frac{1}{\upsilon_t})(1-\alpha)\left(Y_t^p\right)^{1-\gamma}\left(c_t^p\right)^{-\gamma}\right]^{\frac{1}{1-\chi}}$, such that:

$$\frac{L_t^*}{L_t^p} = \left[\left(\frac{Y_t^*}{Y_t^p}\right)^{1-\gamma}\left(\frac{Y_t^*}{Y_t^p}\right)^{\gamma(1-\eta)}\right]^{\frac{1}{1-\chi}} = \left(\frac{Y_t^*}{Y_t^p}\right)^{\frac{1-\eta\gamma}{1-\chi}}$$

Separately, from the production function we have: $Y_t^* = A_t K_t^{\alpha}\left(L_t^*\right)^{1-\alpha}$,which implies $\frac{Y_t^*}{Y_t^p} = \left(\frac{L_t^*}{L_t^p}\right)^{1-\alpha}$, or:

$$\frac{L_t^*}{L_t^p} = \left(\frac{Y_t^*}{Y_t^p}\right)^{\frac{1}{1-\alpha}}$$

So as long as $\frac{1-\eta\gamma}{1-\chi} = \frac{1}{1-\alpha}$, or equivalently $\eta = \frac{\chi-\alpha}{\gamma(1-\alpha)}$, the households' and firm's optimality conditions are satisfied, the firms hire all labor supplied by the household, and the conjectured solution holds. And, imposing $K_{t+1}^* = K_{t+1}$ such that $K_{t+1}^* - (1-\delta)K_t = I_t^*$, we ensure that the firm utilizes all the capital stock. Then, for a given $O_t = O_t^*$, we can represent the equilibrium conditions as function of the output gap O_t^*, such that:

$$\begin{aligned}
Y_t^* &= A_t K_t^{\alpha}\left(L_t^*\right)^{1-\alpha} \\
L_t^* &= \left(O_t^*\right)^{\frac{1}{1-\alpha}} L_t^p \\
L_t^p &\equiv \left[(1-\alpha)\left(\frac{1}{\upsilon_t}\right)\left(c_t^p\right)^{-\gamma}\left(A_t K_t^{\alpha}\right)^{(1-\gamma)}\right]^{\frac{1}{(1-\alpha)\gamma+\alpha-\chi}} \\
Y_t^p &\equiv A_t K_t^{\alpha}\left(L_t^p\right)^{1-\alpha} \\
Y_t^* &= O_t^* \cdot Y_t^p \\
C_t^* &= \left(O_t^*\right)^{\eta} c_t^p Y_t^p \\
I_t^* &= \left[1+\phi_t-\phi_t\left(O_t^*\right)^{-(1-\eta)}\right] i_t^p O_t^* Y_t^p
\end{aligned}$$

A4. The Balanced Growth Path and Proof of Lemma 1

First on balanced growth. We want to choose υ_t such that two conditions hold: (1) there is a balanced growth path in steady state, and (2) the potential labor supply is indifferent to the output gap.

From the household's first order conditions we have:

$$C_t^{-\gamma} = \frac{\upsilon_t L_t^{-\chi}}{W_t} = \frac{\upsilon_t L_t^{1-\chi}}{(1-\alpha)Y_t}$$

When $Y_t = Y_t^p$, we have: $L_t = L_t^p$. Plugging that in and substituting $Y_t^p = A_t K_t^\alpha \left(L_t^p\right)^{1-\alpha}$ and $C_t^p = c_t^p Y_t^p$, we get:

$$\left(L_t^p\right)^{\alpha-\chi} = \left(\frac{1}{\upsilon_t}\right)(1-\alpha)\, A_t K_t^\alpha \left(C_t^p\right)^{-\gamma} = \left(\frac{1}{\upsilon_t}\right)(1-\alpha)\left(A_t K_t^\alpha\right)^{(1-\gamma)}\left(L_t^p\right)^{-(1-\alpha)\gamma}\left(c_t^p\right)^{-\gamma}$$

Or re-writing:

$$L_t^p = \left[(1-\alpha)\left(\frac{1}{\upsilon_t}\right)\left(c_t^p\right)^{-\gamma}\left(A_t K_t^\alpha\right)^{1-\gamma}\right]^{\frac{1}{(1-\alpha)\gamma+\alpha-\chi}}$$

Therefore, the growth in potential labor is given by:

$$\frac{L_{t+1}^p}{L_t^p} = \left[\left(\frac{\upsilon_t}{\upsilon_{t+1}}\right)\left(\frac{c_{t+1}^p}{c_t^p}\right)^{-\gamma}\left(\frac{A_{t+1}K_{t+1}^\alpha}{A_t K_t^\alpha}\right)^{1-\gamma}\right]^{\frac{1}{(1-\alpha)\gamma+\alpha-\chi}}$$

For a balanced growth path, we want to normalize potential labor such that $\frac{d\ln L_t^p}{dt} = 0$, i.e., $\frac{L_{t+1}^p}{L_t^p} = 1$. Plugging that in, and solving for υ_t, we get:

$$\frac{\upsilon_{t+1}}{\upsilon_t} = \left(\frac{c_{t+1}^p}{c_t^p}\right)^{-\gamma}\left(\frac{A_{t+1}K_{t+1}^\alpha}{A_t K_t^\alpha}\right)^{1-\gamma}$$

Let's interpret A_t as a measure of labor-augmenting productivity z_t, such that: $A_t \equiv z_t^{1-\alpha}$. Then in steady state, we get $\frac{\upsilon_{t+1}}{\upsilon_t} = \left(\frac{z_{t+1}}{z_t}\right)^{1-\alpha}$. To see this note that in the steady state, we have: $\frac{d\ln O_t}{dt} = 0$, $\frac{d\ln L_t^p}{dt} = 0$, and $\frac{d\ln c_t^p}{dt} = 0$. And having $\frac{d\ln z_t}{dt} = \frac{d\ln K_t}{dt}$ in the steady state ensures that $\frac{d\ln Y_t}{dt} = \frac{d\ln z_t}{dt} = \frac{d\ln K_t}{dt} = \frac{d\ln C_t}{dt}$, which gives us a balanced growth path. And plugging these into the equation for $\frac{\upsilon_{t+1}}{\upsilon_t}$, in steady state, we get: $\frac{d\ln \upsilon_t}{dt} = (1-\gamma)\frac{d\ln z_t}{dt}$, which ensures potential labor growth continues to be zero in the steady state.

Thus, imposing the condition above for the growth of υ_t in the household's utility function ensures (a) potential labor does not grow over time, and (b) there is a balanced growth path in which the key variables grow at a constant rate in the steady state.

Turning to Lemma 1. Note $gap_t \equiv \ln O_t$. We want to derive $\frac{d\ln Y_t^p}{dgap_{t-1}}$, which is given by:

$$\frac{d\ln Y_t^p}{dgap_{t-1}} = \alpha\frac{d\ln K_t}{dgap_{t-1}}$$

since $\frac{d\ln L_t^p}{d\ln O_{t-1}} = 0$, and A_t is exogenous. In equilibrium, we have: $I_{t-1} = \left[(1+\phi_{t-1}) O_{t-1} - \phi_{t-1} O_{t-1}^{\eta}\right] I_{t-1}^p$. Then, defining $a_t \equiv I_t^p / K_t$, we get: $K_t = \left[\left\{(1+\phi_{t-1}) O_{t-1} - \phi_{t-1} O_{t-1}^{\eta}\right\} a_{t-1} + (1-\delta)\right] K_{t-1}$. (This implies $K_{t+1}/K_t \mid_{O_t=1} = 1 + a_t - \delta$). Taking a first-order Taylor expansion around $O_{t-1} = 1$, $a_t = a$, and $\phi_t = \phi$ (where a and ϕ are average values of a_t and ϕ_t), we get: $\ln K_t \approx \ln K_{t-1} + \ln(1 + a - \delta) + \left(\frac{a[1+(1-\eta)\phi]}{1+a-\delta}\right) \ln O_{t-1}$. Thus:

$$\frac{d\ln Y_t^p}{dgap_{t-1}} = \alpha \frac{d\ln K_t}{dgap_{t-1}} \approx \alpha \frac{a\left[1+(1-\eta)\phi\right]}{1+a-\delta} \equiv \nu$$

Finally, let $Y_{t+1}^{pp} \equiv Y_{t+1}^p \mid Z_t = 1$ denote the potential output in period $t+1$ conditional on output gap being closed in period t such that $Y_{t+1}^p \equiv Y_{t+1}^{pp} \cdot j(O_t)$, and $C_{t+1}^{pp} \equiv c_{t+1}^p \cdot Y_{t+1}^{pp}$. Here $j(O_t)$ is a function that captures the dynamic effect of the output gap in period t on capital in period $t+1$. Then we get:

$$C_{t+1}^p = c_{t+1}^p Y_{t+1}^p = c_{t+1}^p Y_{t+1}^{pp} \cdot j(O_t) = C_{t+1}^{pp} \cdot j(O_t) \approx C_{t+1}^{pp} \cdot O_t^{\nu}$$

Thus consumption growth is given by:

$$\frac{C_{t+1}}{C_t} \approx \left(\frac{C_{t+1}^{pp}}{C_t^p}\right)\left(\frac{O_{t+1}}{O_t}\right)^{\eta} (O_t)^{\nu}$$

Finally, using $X_{t+1} \equiv \ln\left[C_{t+1}^{pp}/C_t^p\right]$, we can represent consumption growth as:

$$\ln\left[\frac{C_{t+1}}{C_t}\right] \approx X_{t+1} + \eta \ln\left[\frac{O_{t+1}}{O_t}\right] + \nu \ln\left[O_t\right]$$

with ν measuring the dynamic effect of the output gap on next period's potential output. This is the expression in Lemma 1.

Finally, I show that c_t^p is exogenous under the assumptions of υ_t. To see this start with the household's first order condition (in the aggregate) for labor $C_t^{-\gamma} = \frac{\upsilon_t L_t^{-\chi}}{W_t}$, plug in $W_t = (1-\alpha) Y_t / L_t$, and divide both sides by Y_t to get:

$$\frac{C_t}{Y_t} = \left[\frac{\upsilon_t L_t^{1-\chi}}{(1-\alpha)}\right]^{-\frac{1}{\gamma}} Y_t^{\frac{1}{\gamma}-1}$$

At potential we get:

$$c_t^p = \frac{C_t^p}{Y_t^p} = \left[\frac{\upsilon_t \left(L_t^p\right)^{1-\chi}}{(1-\alpha)}\right]^{-\frac{1}{\gamma}} Y_t^{\frac{1}{\gamma}-1}$$

So taking the ratio of c_{t+1}^p / c_t^p, and raising both sides to the power $-\gamma$, we get:

$$\left(\frac{c_{t+1}^p}{c_t^p}\right)^{-\gamma} = \left[\frac{\upsilon_{t+1}}{\upsilon_t}\right]\left(\frac{Y_{t+1}}{Y_t}\right)^{\gamma-1}$$

Re-arranging this expression we get:

$$\frac{\upsilon_{t+1}}{\upsilon_t} = \left(\frac{c^p_{t+1}}{c^p_t}\right)^{-\gamma} \left(\frac{A_{t+1}K^\alpha_{t+1}}{A_t K^\alpha_t}\right)^{1-\gamma}$$

Which is the exact express of $\upsilon_{t+1}/\upsilon_t$ we have chosen above for the balanced growth bath. Therefore, an exogenous change in c^p_{t+1}/c^p_t is consistent with our choice of $\upsilon_{t+1}/\upsilon_t$. Thus, with g_t exogenous c^p_t exogenous, we also get i^p_t exogenous given that $c^p_t + i^p_t + g_t = 1$.

A5. Proof of Lemma 2

The growth in potential consumption can be represented as:

$$X_{t+1} = \ln\left[C^{pp}_{t+1}/C^p_t\right] = \ln\left[\frac{c^p_{t+1}}{c^p_t}\frac{Y^{pp}_{t+1}}{Y^p_t}\right]$$

From A5, we have $\frac{L^p_{t+1}}{L^p_t} = 1$, and $\frac{K_{t+1}}{K_t} \mid_{O_t=1} = 1 + a_t - \delta$. For simplicity we can consider the case where $a_t = a$ is rough constant over short horizons. So we can write, $b \equiv \alpha \ln(1 + a - \delta)$, with a representing the potential investment capital ratio. In addition, potential consumption is given by $\frac{c^p_{t+1}}{c^p_t} = \frac{\phi_{t+1}}{\phi_t}\left(\frac{1+\phi_t}{1+\phi_{t+1}}\right)\left(\frac{1-g_{t+1}}{1-g_t}\right)$. For simplicity, we can consider the case where ϕ_t is roughly constant over short horizons, such that $\frac{c^p_{t+1}}{c^p_t} = \frac{1-g_{t+1}}{1-g_t}$ over short horizons. Then, based on Assumption 3 we can represent the growth in the potential consume share as: $\frac{c^p_{t+1}}{c^p_t} = \frac{1-g_{t+1}}{1-g_t} = \left[\frac{A_t}{A_{t+1}}\right]^\zeta$. Substituting these we can re-write X_{t+1}:

$$X_{t+1} = b + (1-\zeta)\ln\left(\frac{A_{t+1}}{A_t}\right)$$

such that: $E_t[X_{t+1}] = \mu_{A,t}$ and $V_t[X_{t+1}] = (1-\zeta)^2\sigma^2_{A,t}$. (For applications with longer horizons one can easily make b have a time-varying trend). This is the first result presented in Lemma 1.

Turning to the second result. We have $\ln[A_{t+1}/A_t]$ is distributed with a truncated student-t. Thus, its moment generating function exists. And we have: $1 - s_t \equiv E_t \ln[A_{t+1}] - \ln\left[A^{safe}_{t+1}\right] = -\ln\left[A^{safe}_{t+1}/A_t\right] + E_t \ln[A_{t+1}/A_t]$. For For any distribution in the scale-location family we can represent the ratio of $1 - s_t$ to the standard deviation of $\ln[A_{t+1}/A_t]$ as the quantile function, which is a constant:

$$\frac{1-s_t}{\sigma_{A,t}} = -\frac{\ln\left[A^{safe}_{t+1}/A_t\right] - E_t\ln[A_{t+1}/A_t]}{\sigma_{A,t}} = -\Phi^{-1}(.01) \tag{A.1}$$

where $\Phi^{-1}(.01)$ is the inverse CDF at the 1st percentile (or the 'standard score' corresponding to the 1 percentile outcome). Define the value of the inverse CDF at .01 is given by $-\Phi^{-1}(.01) \equiv k$. For normal distributions $k = 2.33$, but it can be much higher for heavy-tailed distributions. Thus, we can represent the relationship between s_t and $\sigma_{A,t}$ as:

$$1 - s_t = k\sigma_{A,t} \tag{A.2}$$

Note that consumption growth depends linearly on productivity growth, i.e., $X_{t+1} = b+(1-\zeta)\cdot\ln\left(\frac{A_{t+1}}{A_t}\right)$. Therefore, it has the same underlying distribution as that of A_{t+1}/A_t, since the distribution of A_{t+1}/A_t belongs to the scale-location family. Its mean is: $\mu_t = b+(1-\zeta)\,\mu_{A,t}$ and its variance is: $\sigma^2_{X,t} = (1-\zeta)^2\,\sigma^2_{A,t}$.

Then from Lemma 1, we know $\ln[C_{t+1}/C_t] \approx X_{t+1} + \eta\cdot\Delta gap_{t+1} + \nu\cdot gap_t$. So, $V_t\ln[C_{t+1}/C_t] = V_t[X_{t+1} + \eta\cdot gap_{t+1}]$. From Appendix A7 we know that in equilibrium $V_t[gap_{t+1}] \approx \left(\frac{\eta}{\eta(1-\beta)-\nu}\right)^2\sigma^2_{X,t}$. Thus, we get: $V_t\ln[C_{t+1}/C_t] \equiv \sigma^2_{C,t} = \left(1+\frac{\eta\vartheta}{\eta(1-\beta)-\nu}\right)^2\sigma^2_{X,t}$. Similarly, we get: $E_t\ln[C_{t+1}/C_t] \equiv \mu_{C,t} = \mu_t + \eta\cdot E_t[gap_{t+1}] - (\eta-\nu)\cdot gap_t$.

Fear is defined as $w_t \equiv E_t\ln\left[\frac{C_{t+1}}{C^{safe}_{t+1}}\right]$. Then, using the quantile function, we can represent the ratio of w_t to the standard deviation of consumption growth as:

$$\frac{w_t}{\sigma_{C,t}} = -\frac{\ln\left[C^{safe}_{t+1}/C_t\right] - \mu_{C,t}}{\sigma_{C,t}} = k$$

Thus, we get: $w_t = k\sigma_{C,t} = k\cdot(1-\zeta)\cdot\sigma_{A,t}\cdot\left(1+\frac{\eta\vartheta}{\eta(1-\beta)-\nu}\right) = (1-\zeta)(1-s_t)\left(1+\frac{\eta\vartheta}{\eta(1-\beta)-\nu}\right)$. This concludes the proof.

A6. Derivation of the IS Curve

The first order condition of the household's problem with respect to any financial asset is given as:

$$1 = E_t\left[R^i_t e^{-\rho}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right] = E_t\left[\underbrace{R^i_t}_{Return\ on\ Asset\ i}\quad\underbrace{e^{-\rho}\left(\frac{C^{pp}_{t+1}}{C^p_t}\right)^{-\gamma}}_{'Neutral'\ Discount\ Factor}\cdot\underbrace{\left(O^\nu_t\left(\frac{O_{t+1}}{O_t}\right)^\eta\right)^{-\gamma}}_{Output\ Gap\ Terms}\right]$$

Taking $i = S$ to be the safe asset, substituting $X_{t+1} \equiv \ln\left[C^{pp}_{t+1}/C^p_t\right]$, and $r^S_t \equiv \ln R^S_t$, and taking logs we get:

$$r^S_t = \rho - \ln E_t\left[\exp\left\{-\gamma\left(X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln[O_t]\right)\right\}\right]$$

Then, using the cumulant generating function $\ln E_t[\exp(tX)] = \frac{t}{1!}\kappa_1 + \frac{t^2}{2!}\kappa_2 + ...$, with $\kappa_1 = E[X]$, $\kappa_2 = V[X]$, and ignoring the higher order cumulants κ_j for $j > 2$, we get:

$$r^S_t \approx \rho + \gamma E_t\left[X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln[O_t]\right] - \frac{\gamma^2}{2}V_t\left[X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln[O_t]\right]$$

Or

$$r_t^S = \rho + \gamma E_t \left[X_{t+1} + \eta \ln \left[\frac{O_{t+1}}{O_t} \right] + \nu \ln \left[O_t \right] \right] - \frac{\gamma^2}{2} V_t \left[X_{t+1} + \eta \cdot gap_{t+1} \right]$$

Taking $i = S$ to be the riskless safe asset, and defining $r_t^* \equiv \ln R_t^* \equiv \left[r_t^S \mid O_t = 1, E_t(O_{t+1}) = 1 \right] = \rho + \gamma E_t[X_{t+1}] - (\gamma^2/2) V_t[X_{t+1} + \eta \cdot gap_{t+1}]$, we get:

$$r_t^S = r_t^* - \gamma \left(\eta - \nu \right) gap_t + \gamma \eta E_t \left[gap_{t+1} \right]$$

A7. Derivation of the S Curve

We use the neutral rate equation (r^*) to represent the demand for safe assets curve by households. Here r^* is defined as the rate at which the output gap is closed currently and in the future, and it represents the prevailing interest rate for a riskless asset that pays off 1 unit of consumption in the future in all states of the world. This is the rate of interest implied by the household's Euler equation when the output gap is closed. The neutral rate here is simply the equation for risk-free rate that appears through stochastic-growth interest-rate theory and the asset pricing literature. Plugging the first two moments of X_{t+1} values into the r_t^* equation allows us the represent the S curve in (23), with the neutral rate given by: $r_t^* = \rho + \gamma E_t \left[X_{t+1} \right] - (\gamma^2/2) V_t \left[X_{t+1} + \eta \cdot gap_{t+1} \right]$, since the correlation is assumed to be zero.

Under our assumptions, $V_t \left[gap_{t+1} \right] \approx \left(\frac{\vartheta}{\eta(1-\beta)-\nu} \right)^2 \sigma_{X,t}^2 + \left(\frac{\gamma \left(w_t^2 \right)^\theta \left(\overline{w}^2 \right)^{1-\theta}}{12[\eta(1-\beta)-\nu]} \right)^2 \sigma_\psi^2 \approx \left(\frac{\vartheta}{\eta(1-\beta)-\nu} \right)^2 \sigma_{X,t}^2$ as the second-term is second-order and negligible in size. Thus noting that by assumption the correlation between ψ_t^2 and ε_t is zero, we get:

$$V_t \left[X_{t+1} + \eta \cdot gap_{t+1} \right] \approx V_t \left[X_{t+1} \right] + \eta^2 V_t \left[gap_{t+1} \right] + 2\eta \cdot Cov \left(X_{t+1}, gap_{t+1} \right)$$

Or plugging in values for $V_t \left[gap_{t+1} \right]$, we get:

$$V_t \left[X_{t+1} + \eta \cdot gap_{t+1} \right] \approx \left[1 + \frac{2\vartheta\eta}{\eta \left(1 - \beta \right) - \nu} + \left(\frac{\vartheta\eta}{\eta \left(1 - \beta \right) - \nu} \right)^2 \right] \sigma_{X,t}^2$$

Noting that $1 + \frac{2\vartheta\eta}{\eta(1-\beta)-\nu} + \left(\frac{\vartheta\eta}{\eta(1-\beta)-\nu} \right)^2 = \left(1 + \frac{\vartheta\eta}{\eta(1-\beta)-\nu} \right)^2$, and $\sigma_{X,t}^2 = (1-\zeta)^2 \sigma_{A,t}^2 = \frac{(1-\zeta)^2 (1-s_t)^2}{6}$, we can re-write the variance as:

$$V_t \left[X_{t+1} + \eta \cdot gap_{t+1} \right] \approx \left[1 + \frac{2\vartheta\eta}{\eta \left(1 - \beta \right) - \nu} + \left(\frac{\vartheta\eta}{\eta \left(1 - \beta \right) - \nu} \right)^2 \right] \sigma_{X,t}^2 = \left(1 + \frac{\vartheta\eta}{\left[\eta \left(1 - \beta \right) - \nu \right]} \right)^2 \sigma_{X,t}^2$$

Or plugging in $w_t = k\sigma_{X,t} \left(1 + \frac{\eta\vartheta}{\eta(1-\beta)-\nu} \right) = (1-\zeta)(1-s_t) \left(1 + \frac{\eta\vartheta}{\eta(1-\beta)-\nu} \right)$ with $k^2 = 6$ from Appendix A5, we get:

$$V_t \left[X_{t+1} + \eta \cdot gap_{t+1} \right] \approx \frac{w_t^2}{6}$$

And thus, with $E_t[X_{t+1}] = \mu_t$, the neutral rate (S curve) can be represented as:

$$r_t^* \approx \rho + \gamma\mu_t - \frac{\gamma^2}{12} w_t^2$$

A8. Proof of Proposition 3

From Lemma 1, we have: $\ln[C_{t+1}/C_t] \approx X_{t+1} + \eta \cdot \Delta gap_{t+1} + \nu \cdot gap_t$. And using Assumption 2 and Lemma 2, we get:

$$X_{t+1} = b + (1-\zeta) \cdot \ln[A_{t+1}/A_t] = b_t + (1-\zeta) \cdot (\mu_A + e_{t+1}) = \overline{\mu} + (1-\zeta)\, e_{t+1} = \overline{\mu} + (1-\zeta)\left(\vartheta e_t + (1-s_t)\, \varepsilon_{t+1}\right)$$

with $\overline{\mu} \equiv b + (1-\zeta)\,\mu_A$. From Lemma 2, we have $\mu_t = \overline{\mu} + \vartheta\,(1-\zeta)\, e_t$, such that

$$X_{t+1} = \overline{\mu} + \vartheta\,(1-\zeta)\, e_t + (1-\zeta)\,(1-s_t)\,\varepsilon_{t+1} = \mu_t + (1-\zeta)\,(1-s_t)\,\varepsilon_{t+1}$$

And plugging $w_t \equiv (1-\zeta)\,(1-s_t)\,\varrho$, with $\varrho \equiv 1 + \frac{\eta\vartheta}{[\eta(1-\beta)-\nu]}$, we get:

$$X_{t+1} = \mu_t + \left(\frac{1}{\varrho}\right) w_t \varepsilon_{t+1}$$

Using the IS curve we can solve for the output gap in equilibrium as: $gap_t = -\frac{r_t^S - r_t^*}{\gamma[\eta(1-\beta)-\nu]}$ and plugging in the MP curve we get $gap_t = -\frac{\beta\left[r_{t-1}^S - r_{t-1}^*\right] + E_{t-1}[r_t^*] - r_t^*}{\gamma[\eta(1-\beta)-\nu]}$. That is:

$$gap_t = -\frac{\beta\left[r_{t-1}^S - r_{t-1}^*\right] - \gamma\vartheta\left(\frac{1}{\varrho}\right) w_{t-1}\,[\varepsilon_t] + \frac{\gamma^2}{12}\left(w_{t-1}^2\right)^{\theta}\left(\overline{w}^2\right)^{1-\theta}[\psi_t]}{\gamma\,[\eta\,(1-\beta)-\nu]}$$

with $\varrho \equiv 1 + \frac{\eta\vartheta}{\eta(1-\beta)-\nu}$. We can substitute $\frac{\vartheta}{[\eta(1-\beta)-\nu]} = \frac{\varrho-1}{\eta}$, and iterate one period ahead to get:

$$gap_{t+1} \approx \beta \cdot gap_t + \left(\frac{\varrho-1}{\eta\varrho}\right) w_t \varepsilon_{t+1} - \frac{\gamma\left(w_t^2\right)^{\theta}\left(\overline{w}^2\right)^{1-\theta}}{12\,[\eta\,(1-\beta)-\nu]}[\psi_t]$$

So plugging the values into the consumption growth equation from Lemma 1, we get:

$$\ln\left[\frac{C_{t+1}}{C_t}\right] \approx \mu_t - [\eta\,(1-\beta)-\nu]\, gap_t + \left(\frac{1}{\varrho}\right) w_t \varepsilon_{t+1} + \left(\frac{\varrho-1}{\varrho}\right) w_t \varepsilon_{t+1} - \frac{\gamma\eta\left(w_t^2\right)^{\theta}\left(\overline{w}^2\right)^{1-\theta}}{12\,[\eta\,(1-\beta)-\nu]}[\psi_t]$$

Or simplifying:

$$\ln\left[\frac{C_{t+1}}{C_t}\right] \approx \mu_t - [\eta\,(1-\beta)-\nu]\, gap_t + w_t \varepsilon_{t+1} - \frac{\gamma\eta\left(w_t^2\right)^{\theta}\left(\overline{w}^2\right)^{1-\theta}}{12\,[\eta\,(1-\beta)-\nu]}[\psi_t]$$

These representations for the output gap and consumption growth are shown in Proposition 3, and one can then easily derive implications (i)-(iii) by taking the derivative of the output gap with respect to the different shocks. This concludes the proof.

A9. Derivation of the Risk Premium

These derivations are in line with the literature on consumption-based asset pricing (Lucas, 1978; Hansen and Singleton, 1983; Mehra and Prescott, 1985; Campbell, 1986; Cochrane and Hansen, 1992; Jermann, 1998; Abel, 1999; Cochrane, 2009). Let the stochastic discount factor (SDF) be given as $M_{t+1} \equiv e^{-\rho}\left[C_{t+1}/C_t\right]^{-\gamma} = e^{-\rho}\left[(C^{pp}_{t+1}/C^{p}_t)(O_{t+1}/O_t)^{\eta}(O^{\nu}_t)\right]^{-\gamma}$. Thus, the fundamental pricing theorem gives us the price of any security i at time t with one-period ahead payoff F^i_{t+1}as: $P^i_t = E_t\left\{M_{t+1}F^i_{t+1}\right\}$. Then, as worked out in Weitzman (2007), the return on equity (R^{1e}_t) is given by the return on a hypothetical asset that pays the next period's consumption divided by its price, i.e.:

$$R^{1e}_{t+1} = \frac{C_{t+1}}{P^e_t} = \frac{C_{t+1}/C_t}{e^{-\rho}E_t\left[\left(C_{t+1}/C_t\right)^{1-\gamma}\right]} = \frac{\left(C^{pp}_{t+1}/C^{p}_t\right)\left(O_{t+1}/O_t\right)^{\eta}(O^{\nu}_t)}{e^{-\rho}E_t\left[\left(\left(C^{pp}_{t+1}/C^{p}_t\right)\left(O_{t+1}/O_t\right)^{\eta}(O^{\nu}_t)\right)^{1-\gamma}\right]}$$

And, so the equity premium is the log difference in the return on this asset and the riskfree rate: $ep_t = \ln E_t\left[R^{1e}_{t+1}\right] - \ln R^S_t$, which can be written as:

$$ep_t = \ln\left\{\frac{E_t\left[\exp\left(X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln\left[O_t\right]\right)\right]E_t\left[\exp\left\{-\gamma\left(X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln\left[O_t\right]\right)\right\}\right]}{E_t\left[\exp\left\{(1-\gamma)\left(X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln\left[O_t\right]\right)\right\}\right]}\right\}$$

Then, in line with the standard asset pricing result, using the cumulant generating function $\ln E_t\left[\exp\left(tX\right)\right] = \frac{t}{1!}\kappa_1 + \frac{t^2}{2!}\kappa_2 + ...$, with $\kappa_1 = E\left[X\right]$, $\kappa_2 = V\left[X\right]$, and ignoring the higher order cumulants κ_j for $j > 2$, we get:

$$ep_t = \frac{1}{2}\left[1+\gamma^2-(1-\gamma)^2\right]V_t\left[X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln\left[O_t\right]\right] = \gamma V_t\left[X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln\left[O_t\right]\right] \approx \gamma\frac{w^2_t}{6}$$

We can also derive the Sharpe ratio for one-period equity. And from the fundamental pricing theorem we have $E_t\left[R^{1e}_{t+1}\right] - R^S_t = -\rho_{M,R^e}\cdot\frac{\sigma_t[M_{t+1}]}{E_t[M_{t+1}]}\sigma_t\left[R^e_{t+1}\right]$, with ρ_{M,R^e} representing the correlation between the SDF and equity return. When the equity represents the aggregate market, the correlation is -1, and thus the market Sharpe ratio is given by:

$$\frac{E_t\left[R^{1e}_{t+1}\right] - R^S_t}{\sigma_t\left[R^{1e}_{t+1}\right]} = \frac{\sigma_t\left[M_{t+1}\right]}{E_t\left[M_{t+1}\right]} = \frac{\sigma_t\left[\left(C_{t+1}/C_t\right)^{-\gamma}\right]}{E_t\left[\left(C_{t+1}/C_t\right)^{-\gamma}\right]} \approx \gamma\sigma_t\left(\Delta\ln C_{t+1}\right) \approx \frac{\gamma w_t}{\sqrt{6}}$$

Finally, to derive the volatility of multi-period equity returns, R^e_{t+1}, note that for dividends $D_t \equiv C^{\lambda}_t$

with $\lambda \geq 1$, we get:

$$R^e_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t} = \left(\frac{C_{t+1}}{C_t}\right)^\lambda \left[\frac{1 + P^e_t/D_t}{P^e_t/D_t} + \frac{P^e_{t+1}/D_{t+1} - P^e_t/D_t}{P^e_t/D_t}\right]$$

Or taking logs, with $\frac{P^e_{t+1}/D_{t+1} - P^e_t/D_t}{P^e_t/D_t} \approx \ln \frac{P^e_{t+1}/D_{t+1}}{P^e_t/D_t}$, and the log approximation $\ln(1 + X) \approx X$ for small X, we get:

$$\ln R^e_{t+1} = \lambda \ln \left[\frac{C_{t+1}}{C_t}\right] + \ln \left[\frac{1 + P^e_t/D_t}{P^e_t/D_t} + \frac{P^e_t/D_t - P^e_t/D_t}{P^e_t/D_t}\right] \approx \lambda \ln \left[\frac{C_{t+1}}{C_t}\right] + \frac{D_t}{P^e_t} + \ln \frac{P^e_{t+1}/D_{t+1}}{P^e_t/D_t}$$

So

$$V_t \left[\ln R^e_{t+1}\right] \approx \lambda^2 \frac{w_t^2}{6} + V_t \left[\ln \frac{P^e_{t+1}/D_{t+1}}{P^e_t/D_t}\right]$$

And

$$V_t \left[\ln \frac{P^e_{t+1}/D_{t+1}}{P^e_t/D_t}\right] = \left(\frac{D_t}{P^e_t}\right)^2 V_t \left[\ln \frac{P^e_{t+1}}{D_{t+1}}\right] = \left(\frac{D_t}{P^e_t}\right)^2 V_t \left[\ln \frac{D_t}{D_{t+1}} + \ln \frac{P^e_{t+1}}{D_t}\right]$$

Or

$$V_t \left[\ln \frac{P^e_{t+1}/D_{t+1}}{P^e_t/D_t}\right] = \left(\frac{D_t}{P^e_t}\right)^2 \left[\lambda^2 \frac{w_t^2}{6} + \left(\frac{P^e_t}{D_t}\right)^2 V_t \left[\ln \frac{P^e_{t+1}}{P^e_t}\right] + \left(\frac{P^e_t}{D_t}\right) cov \left(\ln \frac{D_t}{D_{t+1}}, \ln \frac{P^e_{t+1}}{P^e_t}\right)\right]$$

It can be shown that $V_t \left[\ln \frac{P_{t+1}}{P_t}\right] \geq \lambda^2 \frac{w_t^2}{6}$. So we approximate it with this lower value to get:

$$V_t \left[\ln \frac{P^e_{t+1}/D_{t+1}}{P^e_t/D_t}\right] \approx \lambda^2 \frac{w_t^2}{6} \left[\left(\frac{D_t}{P^e_t}\right)^2 - \left(\frac{D_t}{P^e_t}\right) + 1\right]$$

So

$$V_t \left[\ln R^e_{t+1}\right] \approx \lambda^2 \frac{w_t^2}{6} + V_t \left[\ln \frac{P^e_{t+1}/D_{t+1}}{P^e_t/D_t}\right] \approx \lambda^2 \frac{w_t^2}{6} \left[2 - \left(\frac{D_t}{P^e_t}\right) + \left(\frac{D_t}{P^e_t}\right)^2\right]$$

And for $\lambda = 1$, we get the value presented in the main text.

A10. Derivation of the Term Structure

The fundamental pricing theorem gives us the price of any security i at time t with one-period ahead payoff F^i_{t+1}as: $P^i_t = E_t \left\{M_{t+1} F^i_{t+1}\right\}$, with stochastic discount factor $M_{t+1} \equiv e^{-\rho} \exp\left\{-\gamma \ln(C_{t+1}/C_t)\right\}$. Let the time t price of a riskfree bond that matures N periods ahead be: $P^{(N)}_t$. Then one-period ahead payoff of this bond is $P^{(N-1)}_{t+1}$, and with price at time t:

$$P^{(N)}_t = E_t \left\{M_{t+1} P^{(N-1)}_{t+1}\right\}$$

Let the yield ($Yield_t$) of a the riskfree bond be defined as the notional, constant, known, interest rate that justifies the quoted price of the riskfree bond such that: $P_t^{(N)} = \frac{1}{\left[Yield_t^{(N)}\right]^N}$. Denote y_t^N as the log yield, then by iterating forward, we can represent the yield of the riskfree bond with maturity N at time t as:

$$y_t^{(N)} = -\frac{1}{N}\ln\left(P_t^{(N)}\right) = -\frac{1}{N}\ln E_t\left[\prod_{j=0}^{N-1} E_{t+j}M_{t+j+1}\right] = \rho - \frac{1}{N}\ln E_t\left[\exp\left\{-\gamma\ln\left(C_{t+N}/C_t\right)\right\}\right]$$

Plugging in values for the stochastic discount factor we get:

$$y_t^{(N)} \approx \rho + \frac{\gamma}{N}E_t\ln\left[C_{t+N}/C_t\right] - \frac{\gamma^2}{2N}V_t\ln\left[C_{t+N}/C_t\right]$$

Consumption growth is given by: $\ln C_{t+1} = \ln C_t + X_{t+1} + \eta\ln\left[\frac{O_{t+1}}{O_t}\right] + \nu\ln\left[O_t\right]$. For multi-period consumption growth we start by 2-period ahead consumption:

$$\ln C_{t+2} = \ln C_{t+1} + X_{t+2} + \eta\ln\left[\frac{O_{t+2}}{O_{t+1}}\right] + \nu\ln\left[O_{t+1}\right]$$

By plugging in $\ln C_{t+1}$, we get:

$$\ln C_{t+2} = \ln C_t + X_{t+1} + X_{t+2} + \eta\ln\left[\frac{O_{t+2}}{O_t}\right] + \nu\left\{\ln\left[O_t\right] + \ln\left[O_{t+1}\right]\right\}$$

Iterating forward we get:

$$\ln\left[\frac{C_{t+N}}{C_t}\right] = \sum_{j=1}^{N} X_{t+j} + \eta\left[gap_{t+N} - gap_t\right] + \nu\sum_{j=0}^{N-1} gap_{t+k}$$

We have $X_{t+1} = b + (1-\zeta)\left(\mu_A + \vartheta e_t\right) + w_t\varepsilon_{t+1}$. And $E_t u_{t+k} = \vartheta^k e_t$. And so

$$E_t\ln\left[\frac{C_{t+N}}{C_t}\right] = E_t\sum_{j=1}^{N} X_{t+j} + \eta E_t\left[gap_{t+N} - gap_t\right] + \nu E_t\sum_{j=0}^{N-1} gap_{t+j}$$

Or

$$E_t\ln\left[\frac{C_{t+N}}{C_t}\right] = N\cdot\overline{\mu} + (1-\zeta)\,\vartheta E_t\sum_{j=1}^{k} u_{t+j-1} + \eta\left[\beta^k gap_t - gap_t\right] + \nu E_t\sum_{j=0}^{k-1}\beta^j gap_t$$

Or with $E_t\sum_{j=1}^{k} u_{t+j-1} = e_t E_t\sum_{j=0}^{k-1}\vartheta^j$, and $E_t\sum_{j=0}^{k-1}\beta^j gap_t = gap_t E_t\sum_{j=0}^{k-1}\beta^j$, substituting $(1-\zeta)\,\vartheta e_t = \mu_t - \overline{\mu}$, and solving for the finite sums we get:

$$E_t\ln\left[\frac{C_{t+N}}{C_t}\right] = N\cdot\overline{\mu} + \left(\frac{1-\vartheta^N}{1-\vartheta}\right)\left[\mu_t - \overline{\mu}\right] - \left[\eta\left(1-\beta\right) - \nu\right]\left(\frac{1-\beta^N}{1-\beta}\right)\cdot gap_t$$

And by ignoring the variance of ψ_{t+1} we can approximate the multi-period consumption growth as:

$$\ln\left[\frac{C_{t+N}}{C_t}\right] \approx N\overline{\mu} - \left[\eta\left(1-\beta\right)-\nu\right]\left(\frac{1-\beta^N}{1-\beta}\right)\cdot gap_t + \left(\frac{1-\vartheta^N}{1-\vartheta}\right)\left[\mu_t-\overline{\mu}\right] + \sum_{j=1}^{N} w_{t+j-1}\varepsilon_{t+j}$$

So, the multi-period consumption growth variance is approximately:

$$V_t \ln\left[\frac{C_{t+N}}{C_t}\right] \approx V_t\left(\sum_{j=1}^{N} w_{t+j-1}\varepsilon_{t+j}\right) \approx \frac{1}{6}\left[N\overline{w}^2 + \left(\frac{1-\theta^N}{1-\theta}\right)\left(w_t^2-\overline{w}^2\right)\right] + (risk\,premium)$$

where the risk premium depends on the covariance in $w_{t+j-1}\varepsilon_{t+j}$ across periods, which can be positive or negative. Plugging these into the yield equation, we get:

$$y_t^{(N)} \approx r^*_{t+\infty} - \frac{\gamma}{N}\left[\eta\left(1-\beta\right)-\nu\right]\left(\frac{1-\beta^N}{1-\beta}\right)\cdot gap_t + \frac{\gamma}{N}\left(\frac{1-\vartheta^N}{1-\vartheta}\right)\left[\mu_t-\overline{\mu}\right] - \frac{\gamma^2}{12N}\left(\frac{1-\theta^N}{1-\theta}\right)\left(w_t^2-\overline{w}^2\right) + (risk\,premium)$$

And therefore, the slope of the yield curve is given by:

$$y_t^{(T+N)} - y_t^{(N)} \approx \Lambda_\beta\cdot\gamma\left[\eta\left(1-\beta\right)-\nu\right]\cdot\left(gap_t\right) - \Lambda_\vartheta\cdot\gamma\cdot\left(\mu_t-\overline{\mu}\right) + \Lambda_\theta\cdot\frac{\gamma^2}{12}\cdot\left(w_t^2-\overline{w}^2\right) + (risk\,premium)$$

with $\Lambda_x \equiv \frac{\left(1-x^N\right)}{N(1-x)} - \frac{\left(1-x^{T+N}\right)}{(T+N)(1-x)} > 0$.

A11. Derivation of the Price-Dividend Ratio

Supposed dividend is $D_t = C_t^\lambda$ for $\lambda \geq 1$. In the pricing equation we get:

$$P_t^e = C_t^\gamma \sum_{k=1}^{\infty} e^{-k\rho} E_t\left[C_{t+k}^{\lambda-\gamma}\right]$$

Or

$$\frac{P_t^e}{D_t} = \frac{P_t^e}{C_t^\lambda} = C_t^{-(\lambda-\gamma)}\sum_{k=1}^{\infty} e^{-k\rho}E_t\left[C_{t+k}^{\lambda-\gamma}\right] = \sum_{k=1}^{\infty} e^{-k\rho}E_t\left[\left(\frac{C_{t+k}}{C_t}\right)^{\lambda-\gamma}\right] = \sum_{k=1}^{\infty} e^{-k\rho}E_t\left[\exp\left\{(\lambda-\gamma)\ln\left(\frac{C_{t+k}}{C_t}\right)\right\}\right]$$

Let $\varpi \equiv e^{-\rho}$. Then, using the moment generating function, the price-dividend ratio of levered equity is:

$$\frac{P_t^e}{D_t} \approx \sum_{k=1}^{\infty} e^{-k\rho}\left\{1 + (\lambda-\gamma)E_t\left[\ln\left(\frac{C_{t+k}}{C_t}\right)\right] + \frac{(\lambda-\gamma)^2}{2}V_t\left[\ln\left(\frac{C_{t+k}}{C_t}\right)\right]\right\}$$

From Appendix A10, we have: $E_t \ln\left[\frac{C_{t+k}}{C_t}\right] = k\cdot\overline{\mu} + \left[\mu_t-\overline{\mu}\right]\left[\frac{1-\vartheta^k}{1-\vartheta}\right] - \left[\eta\left(1-\beta\right)-\nu\right]\cdot gap_t\left[\frac{1-\beta^k}{1-\beta}\right]$ and $V_t \ln\left[\frac{C_{t+k}}{C_t}\right] = \frac{1}{6}\left[k\overline{w}^2 + \left(\frac{1-\theta^k}{1-\theta}\right)\left(w_t^2-\overline{w}^2\right)\right]$. In addition, we have the following properties for finite sums: $\sum_{k=1}^{\infty}\varpi^k = \frac{\varpi}{1-\varpi}$, and $\sum_{k=1}^{\infty}\varpi^k\cdot k = \frac{\varpi}{(1-\varpi)^2}$, and $\sum_{k=1}^{\infty}\varpi^k\cdot\left(\frac{1-\theta^k}{1-\theta}\right) = \frac{\varpi}{(1-\varpi)(1-\varpi\theta)}$. So, the summation to infinity simplifies to:

$$\frac{P_t^e}{D_t} \approx \frac{\varpi}{1-\varpi}\left[1+(\gamma-\lambda)\left(\frac{\left(\frac{(\gamma-\lambda)}{12}\overline{w}^2-\overline{\mu}\right)}{1-\varpi}-\frac{\mu_t-\overline{\mu}}{1-\varpi\vartheta}+\frac{\left[\eta\left(1-\beta\right)-\nu\right]gap_t}{1-\varpi\beta}\right)+\frac{\frac{(\gamma-\lambda)^2}{12}\left[w_t^2-\overline{w}^2\right]}{1-\varpi\theta}\right]$$

Note that for small ρ, we have: $\frac{\varpi}{1-\varpi} \approx \frac{1}{\rho}$ and $\frac{1}{1-\varpi} \approx \frac{1}{\rho}$, and $\frac{1}{1-\varpi x} \approx \frac{1}{1-x}$ for $x=\beta,\vartheta,\theta$. We can re-write it as:

$$\frac{P_t^e}{D_t} \approx \frac{1}{\rho}\left[1+(\gamma-\lambda)\left(\frac{\frac{(\gamma-\lambda)}{12}\overline{w}^2-\overline{\mu}}{\rho}+\frac{\left[\eta\left(1-\beta\right)-\nu\right]gap_t}{1-\beta}-\frac{\left[\mu_t-\overline{\mu}\right]}{1-\vartheta}\right)+\frac{\frac{(\gamma-\lambda)^2}{12}\left[w_t^2-\overline{w}^2\right]}{1-\theta}\right]$$

Or taking logs we get:

$$\ln\left[\frac{P_t^e}{D_t}\right] \approx -\ln\rho-\frac{1}{\rho}\left[(\gamma-\lambda)\,\overline{\mu}-\frac{1}{12}\left(\gamma-\lambda\right)^2\overline{w}^2\right]+\frac{(\gamma-\lambda)\left[\eta\left(1-\beta\right)-\nu\right]}{1-\beta}gap_t-\frac{\gamma-\lambda}{1-\vartheta}\left(\mu_t-\overline{\mu}\right)+\frac{1}{12}\frac{(\gamma-\lambda)^2}{1-\theta}\left(w_t^2-\overline{w}^2\right)$$

We have $r_t^*-r_t^{*,LR}=\gamma\left(\mu_t-\overline{\mu}\right)-\frac{\gamma^2}{12}\cdot\left(w_t^2-\overline{w}^2\right)$. So:

$$-\frac{\gamma-\lambda}{1-\vartheta}\left(\mu_t-\overline{\mu}\right)+\frac{1}{12}\frac{(\gamma-\lambda)^2}{1-\theta}\left(w_t^2-\overline{w}^2\right)=-\frac{1}{\gamma}\left(\frac{\gamma-\lambda}{1-\vartheta}\right)\left(r_t^*-r_t^{*,LR}\right)-\frac{(\gamma-\lambda)}{12}\left[\frac{\gamma}{1-\vartheta}-\frac{\gamma-\lambda}{1-\theta}\right]\left(w_t^2-\overline{w}^2\right)$$

Plugging this into the log price-dividend ratio formula, we get:

$$\ln\left[\frac{P_t^e}{D_t}\right] \approx -\ln k_0-k_1\left(r_t^*-r_t^{*,LR}\right)+k_2\cdot gap_t-k_3\left(w_t^2-\overline{w}^2\right)$$

with positive constants $k_0 \equiv \rho\cdot\exp\left\{\frac{1}{\rho}\left(\gamma-\lambda\right)\overline{\mu}-\frac{1}{12\rho}\left(\gamma-\lambda\right)^2\overline{w}^2\right\}$, $k_1 \equiv \frac{1}{\gamma}\left(\frac{\gamma-\lambda}{1-\vartheta}\right)$, $k_2 \equiv \frac{(\gamma-\lambda)[\eta(1-\beta)-\nu]}{1-\beta}$, and $k_3 \equiv \frac{\gamma-\lambda}{12}\left[\frac{\gamma}{1-\vartheta}-\frac{\gamma-\lambda}{1-\theta}\right]$. Or taking exponents:

$$P_t^e \approx \frac{D_t}{k_0\cdot\left(1+k_1\left[r_t^*-r_t^{*,LR}\right]\right)\cdot\left(1-k_2\cdot gap_t\right)\cdot\left(1+k_3\left[w_t^2-\overline{w}^2\right]\right)}$$

This is the representation of the modified Gordon growth model in the main text. And finally the difference in expected log price-dividend ratio can be represented as:

$$E_t\ln\left[\frac{P_{t+N}^e}{D_{t+N}}\right]-\ln\frac{P_t^e}{D_t} \approx$$

$$\approx -\left[\eta\left(1-\beta\right)-\nu\right]\left(\gamma-\lambda\right)\left(\frac{1-\beta^N}{1-\beta}\right)gap_t+\left(\gamma-\lambda\right)\left(\frac{1-\vartheta^N}{1-\vartheta}\right)\left(\mu_t-\overline{\mu}\right)-\frac{1}{12}\left(\gamma-\lambda\right)^2\left(\frac{1-\theta^N}{1-\theta}\right)\left(w_t^2-\overline{w}^2\right)$$

A12. Derivation of the Expected Equity Returns

The N period return on equity is:

$$R^e_{t+N} = \frac{P^e_{t+N} + D_{t+1} + ... + D_{t+N}}{P^e_t}$$

Let $f > 1$ be a constant. We can multiply and divide the right-hand side by $\frac{D_{t+N}}{D_t} f$, to represent the equity return as:

$$R^e_{t+N} = \frac{D_{t+N}}{D_t} f \left[\frac{1}{f} \frac{P^e_{t+N}/D_{t+N} + (D_{t+1}/D_{t+N} + ... + D_{t+N}/D_{t+N})}{P^e_t/D_t} \right]$$

Or by adding and subtracting P^e_t/D_t in the numerator we get:

$$R^e_{t+N} = \frac{D_{t+N}}{D_t} f \left[1 - \left(\frac{f-1}{f} \right) + \frac{1}{f} \left(\frac{P^e_{t+N}/D_{t+N} - P^e_t/D_t}{P^e_t/D_t} \right) + \frac{1}{f} \frac{(D_{t+1}/D_{t+N} + ... + D_{t+N}/D_{t+N})}{P^e_t/D_t} \right]$$

Or by multiplying and dividing the last term by D_t/D_{t+N} we get:

$$R^e_{t+N} = \frac{D_{t+N}}{D_t} k \left[1 - \left(\frac{f-1}{f} \right) + \frac{1}{f} \left(\frac{P^e_{t+N}/D_{t+N} - P^e_t/D_t}{P^e_t/D_t} \right) + \frac{1}{f} \frac{D_t}{P^e_t} \frac{D_t}{D_{t+N}} \left(\frac{D_{t+1}}{D_t} + ... + \frac{D_{t+N}}{D_t} \right) \right]$$

Let dividend growth be g_t, such that $D_{t+1} = (1 + g_t) D_t$ such that:

$$\frac{D_t}{D_{t+N}} \left(\frac{D_{t+1}}{D_t} + ... + \frac{D_{t+N}}{D_t} \right) = \frac{\sum_{k=1}^{N} \prod_{j=0}^{k-1} (1 + g_{t+j})}{\prod_{j=0}^{N-1} (1 + g_{t+j})}$$

Define $g_{t,N}$ as the average dividend growth such that:

$$\frac{\sum_{k=1}^{N} \prod_{j=0}^{k-1} (1 + g_{t+j})}{\prod_{j=0}^{N-1} (1 + g_{t+j})} \equiv \frac{\sum_{k=1}^{N} \prod_{j=0}^{k-1} (1 + g_{t,N})}{\prod_{j=0}^{N-1} (1 + g_{t,N})} = \left(\frac{1 + g_{t,N}}{g_{t,N}} \right) \left(1 - \frac{1}{(1 + g_{t,N})^N} \right)$$

Note, for small g_{t+j}, this means $g_{t,N} \approx \frac{2}{N(N+1)} \sum_{k=1}^{N} k \cdot g_{t+j}$ is a weighted arithmetic mean of future growth rates, with distant growth rates having more weight than near-term growth rates. So

$$R^e_{t+N} = \frac{D_{t+N}}{D_t} f \left[1 - \left(\frac{f-1}{f} \right) + \frac{1}{f} \left(\frac{P^e_{t+N}/D_{t+N} - P^e_t/D_t}{P^e_t/D_t} \right) + \frac{1}{f} \left(\frac{1 + g_{t,N}}{g_{t,N}} \right) \left(1 - \frac{1}{(1 + g_{t,N})^N} \right) \cdot \frac{D_t}{P^e_t} \right]$$

Taking logs of both sides we get:

$$\ln R^e_{t+N} = \ln \frac{D_{t+N}}{D_t} + \ln f + \ln [\ldots]$$

We can substitute $\frac{P^e_{t+N}/D_{t+N} - P^e_t/D_t}{P^e_t/D_t} \approx \ln \left[\frac{P^e_{t+N}/D_{t+N}}{P^e_t/D_t} \right]$, with the relationship being exact in continuous time. And for small $g_{t,N}$ and N not too large (e.g., less than 50 when $g_{t,N} \approx .02$), a good first-order approximation is: $\left(\frac{1+g_{t,N}}{g_{t,N}}\right)\left(1-\frac{1}{(1+g_{t,N})^N}\right) \approx N - \frac{N(N-1)}{2} g_{t,N}$. For small X, we have the result that: $\ln(1+X) \approx X$. So plugging this in and with $\ln f - \left(\frac{f-1}{f}\right) \approx 0$, we can re-write the log return as:

$$\ln R^e_{t+N} \approx \ln \frac{D_{t+N}}{D_t} + \frac{1}{f} \ln \left(\frac{P^e_{t+N}/D_{t+N}}{P^e_t/D_t} \right) + \frac{1}{f} \left(N - \frac{N(N-1)}{2} g_{t,N} \right) \cdot \frac{D_t}{P^e_t}$$

(For typical values of $g_{t,N}$ and D_t/P_t, $f \approx \frac{3}{2}$ can provide a good approximation). From Appendix A10 we have the values for $E_t \ln \left[\frac{P^e_{t+N}}{D_{t+N}}\right] - \ln \frac{P^e_t}{D_t}$, and from Appendix A9 we have the values for $E_t \ln \left[\frac{D_{t+N}}{D_t}\right]$. So plugging this in to the expected returns formula above, we get:

$$E_t \left[\ln R^e_{t+N}\right] \approx \Theta_0 - \Theta_\beta \cdot \left[\eta(1-\beta) - \nu\right] gap_t + \Theta_\vartheta \cdot (\mu_t - \overline{\mu}) - \Theta_\theta \cdot \left(w_t^2 - \overline{w}^2\right) + \Theta_g \cdot \frac{D_t}{P^e_t} \tag{A.3}$$

with $\Theta_0 \equiv N\lambda\overline{\mu}$, $\Theta_x \equiv \left[\lambda + \frac{1}{f}(\gamma - \lambda)\right]\left(\frac{1-x^N}{1-x}\right)$ for $x = \beta, \vartheta$. And with $\Theta_\theta = \frac{1}{12}\left[\lambda + \frac{1}{f}(\gamma-\lambda)^2\right]\left(\frac{1-\theta^N}{1-\theta}\right)$, and $\Theta_g = \frac{1}{f}\left(N - \frac{N(N-1)}{2} E_t\left[g_{t,N}\right]\right)$.

This is the representation presented in the main text.

Online Appendix B: The SWIM Model with Inflation (SWIM-P)

In this section we add inflation in the SWIM model by introducing a Phillips Curve (or the 'P curve'). This appendix is mainly to lay down the ingredients of a simple setup with inflation (π_t). In this extension the good prices are no longer fully flexible. For simplicity consider the case of a backward-looking Phillips curve (that arises due to adaptive expectations about inflation):

$$\pi_t = \pi_t^e + \tau \cdot gap_t + \varepsilon_{\pi,t}$$

such that expected inflation equals previous period's inflation, $\pi_t^e = \pi_{t-1}$, and $\varepsilon_{\pi,t}$ is a white noise shock. If price setting was done under adaptive expectations consistent with such a Phillips Curve, then that would be one possible micro-foundation for why the central bank can fix the real interest rate on safe assets as studied in the basic SWIM model. (Note, for more realistic inflation dynamics, one could instead consider a setup with sticky information (Mankiw and Reis, 2002). Alternatively, we could consider softer theories of nominal rigidity in which there is some rigidity in the price of the numeraire good as in Lucas (1972) or due to money illusion, as discussed in Reis and Watson (2007).)

Assume that with $\kappa \geq 0$, the MP curve is now given by:

$$r_t^S = E_{t-1}\left[r_t^*\right] + \beta\left[r_{t-1}^S - r_{t-1}^*\right] + \kappa E_{t-1}\left[\pi_t - \pi^*\right]$$

The S, W, and IS conditions are the same as before. Then after some tedious algebra it can be shown that the equilibrium is characterized by the following five equations:

$$\begin{aligned}
S\ Curve: &\quad r_t^* = \rho + \gamma\mu_t - \frac{\gamma^2}{12}w_t^2 \\
W\ Curve: &\quad w_t = \varrho \cdot (1-\zeta)\cdot(1-s_t) \\
IS\ Curve: &\quad r_t^S = r_t^* - \kappa\varphi_3\left[\pi_{t-1} - \pi^* + \varepsilon_{\pi,t}\right] - \left(\frac{1}{\varphi_1} + \kappa\tau\varphi_3\right) gap_t \\
MP\ Curve: &\quad r_t^S = E_{t-1}\left[r_t^*\right] + \beta\left(1-\kappa\tau\varphi_1\right)\left[r_{t-1}^S - r_{t-1}^*\right] + \kappa\left(1-\tau\varphi_2\right)\left[\pi_{t-1} - \pi^*\right] \\
P\ Curve: &\quad \pi_t = \pi_{t-1} + \tau\cdot gap_t + \varepsilon_{\pi,t}
\end{aligned}$$

with $\varphi_1 \equiv \frac{1}{\gamma[\eta(1-\beta)-\nu]}$, $\varphi_2 \equiv \frac{(\eta-\nu)}{\eta[\gamma(\eta-\nu)-1]\gamma[\eta(1-\beta)-\nu]}$, and $\varphi_3 \equiv \frac{1}{\gamma(\eta-\nu)-1}$.

Note that when $\kappa = 0$ we are back to the equilibrium conditions in the SWIM model.

We can derive the equilibrium output gap as:

$$gap_t = -\varphi_1\left\{\beta\left(1-\kappa\tau\varphi_1\right)\left[r_{t-1}^S - r_{t-1}^*\right] + \kappa\left(1-\tau\varphi_2+\varphi_3\right)\left[\pi_{t-1}-\pi^*\right] + \kappa\varphi_3\left[\varepsilon_{\pi,t}\right] + \frac{\gamma^2}{12}\left[w_t^2 - E_{t-1}\left(w_t^2\right)\right]\right\}$$

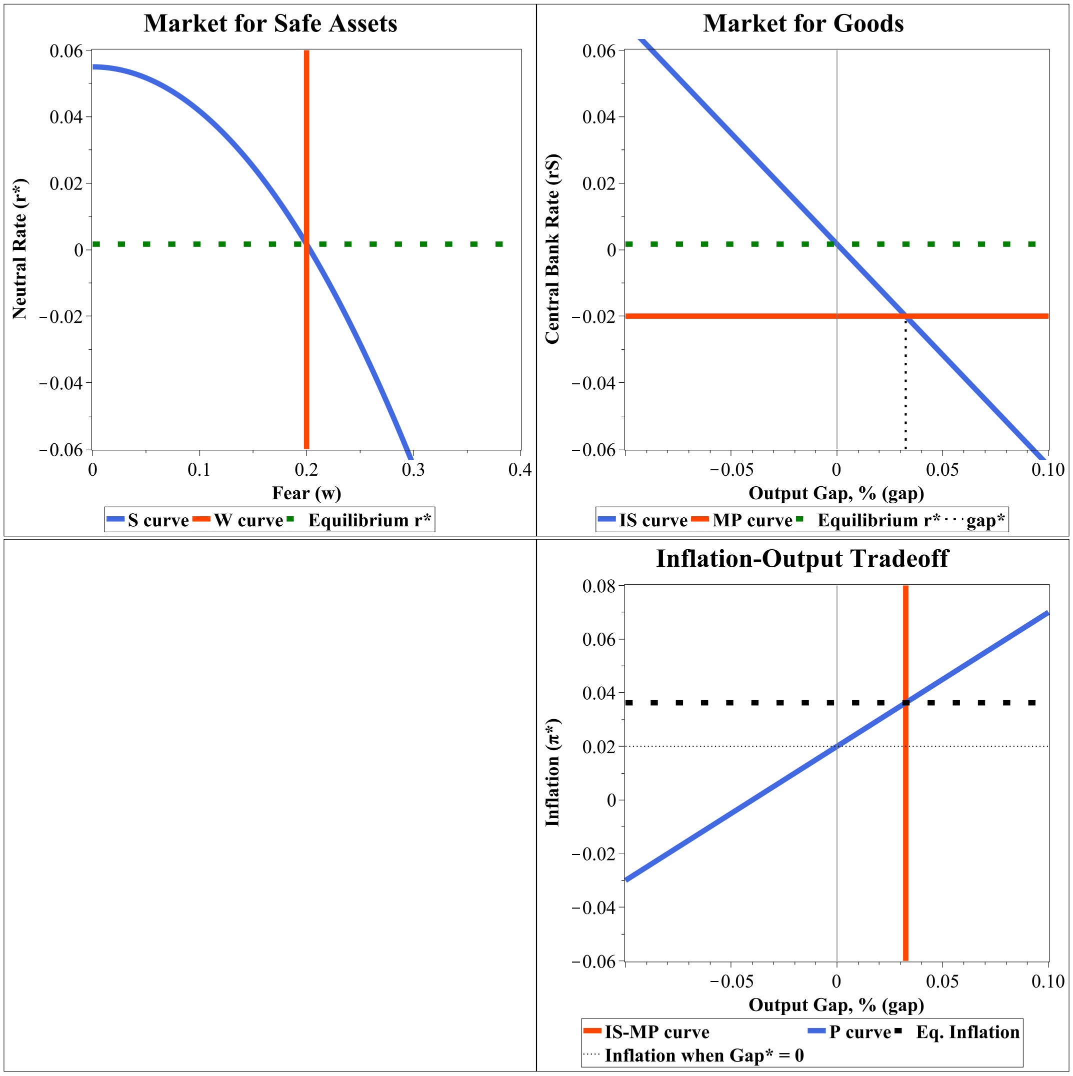


Figure .1: SWIM Model with inflation (SWIM-P): An example in which the MP curve sets interest rates too ‘low’ compared to the r_t^*, leading to a positive output gap and higher inflation.

This can be graphically represented with three panels (Figure B.1), with an additional panel depicting the classic inflation-output tradeoff.

Online Appendix C: The Fear-Avoidance Model of Uncertainty

This section considers a case in which the parameter uncertainty about consumption growth depends on the observed real rate of interest. This will allow us to build on various insights in Weitzman (2007), which considers a non-REE environment. We do this in a very reduced-form way, although future work could explore how to ground these insights within a Bayesian framework with subjective priors as in Weitzman (2007).

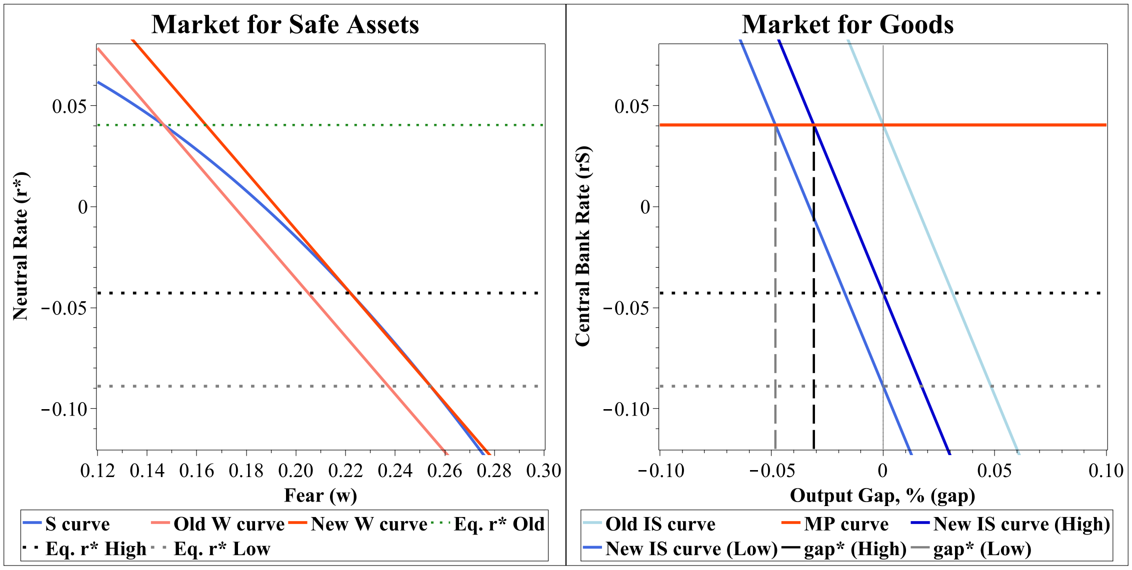
Here, conceptually, I draw on a key idea from clinical psychology– *catastrophizing*—introduced by Ellis (1962) and later adapted by Beck (1979) and others (Gellatly and Beck, 2016; Quartana et al., 2009). Ellis described catastrophizing as the tendency to magnify a perceived threat and overestimate the seriousness of its potential consequences.[16] Catastrophizing is recognized as a common feature in overreactions to the stressors of life in otherwise healthy individuals, and also to play a causal role in a wide variety of disorders. In particular, I build on the clinical psychology literature on intolerance of uncertainty (Dugas et al., 2004; Ladouceur et al., 2000) and the model of pain catastrophizing called the Fear-Avoidance Model (Lethem et al., 1983; Vlaeyen and Linton, 2012; Vlaeyen et al., 2016). The model extension demonstrates that when the safe rate of return acts as a signal about how bad the worst-case scenario can be, you can get fear cycles or an animal spirits effect.

We consider the case in which the agent's estimate of w_t is bounded between $[\underline{w}, \overline{w}]$. And suppose for a non-negative constant $\varphi \geq 0$, we have the case that:

$$m_t^2 = \frac{1}{6}\left(\frac{n-2}{n}\right)(1 - \varphi \cdot r_t^*)^2 \tag{C.1}$$

Such a subjective belief may arise when agents look to the macro-economy to infer the state of the underlying economy, whereby a low real interest rate would signal a greater risk of underlying weakness in the economy. To see this, first note that, if the agents observed infinite observations, then one could have a setup in which, m converges as the degree of freedoms go to infinity. That is, as $n \to \infty$ we have $m_t \to \frac{1}{\sqrt{6}}$. But the observations available to the agents (or perceived by them as relevant) is a finite n, leading to m being higher, thereby increasing the perceived volatility. Thus, the econometrician may measure something different than what is perceived by the agents due to subjective uncertainty. From this perspective, the difference between ε_{t+1} and $e_{t+1}/\sqrt{6}$ could, respectively, be seen as the subjective-perception based volatility versus the 'objective' volatility that would be measured in the data. So m_t is assumed to depend inversely on the neutral rate essentially implies that the confidence of the agents in their subjective believe about the volatility process is low when the real interest rates are low, thereby leading to a higher perceived uncertainty.

[16] In the early development of cognitive behavioral therapy, Ellis was influenced by the classical ideas of Epictetus (125 C.E.) who stated, "Men are not disturbed by the things which happen, but their opinions about the things."



Note: The figure depicts the general equilibrium using the four curves of the SWIM model, but now with a downward sloping W curve. In this example, when fear rises (as captured by a rightward shift in the W curve), there is a possibility of two equilibria: with a high and a low value of the neutral rate of interest found at the two intersections of the S and W curves in the left panel, depicted by the black and gray dotted lines respectively. The black and gray lines can be traced from the left panel to the right panel to find the intersection point of the two IS curves corresponding to the two equilibria (depicted by the dark blue and blue curves). Given that $r_t^S > r_t^*$, we have two possible outcomes for the negative output gap, which is found at intersections of the MP curve and the two new IS curves in the right panel.

Figure C.1: The SWIM Model with Fear-Avoidance Dynamics (i.e., a downward sloping W curve)

Then with this modification, the modified W curve in the SWIM model becomes $w_t = \varrho \cdot (1-\zeta)(1-s_t)(1-\varphi \cdot r_t^*)$. So we can represent the equilibrium as:

$$\begin{aligned} S\ Curve: \quad & r_t^* = \rho + \gamma\mu_t - \frac{\gamma^2}{12} w_t^2 \\ W\ Curve: \quad & w_t = \varrho \cdot (1-\zeta)(1-s_t)(1-\varphi \cdot r_t^*) \\ IS\ Curve: \quad & r_t^S = r_t^* - \gamma\left[\eta(1-\beta) - \nu\right] gap_t \\ MP\ Curve: \quad & r_t^S = E_{t-1}\left[r_t^*\right] + \beta\left[r_{t-1}^S - r_{t-1}^*\right] \end{aligned} \tag{C.2}$$

Solving for w_t we get that:

$$w_t = \frac{6}{\gamma^2 \varphi \varrho (1-\zeta)(1-s_t)} \pm \sqrt{\left(\frac{6}{\gamma^2 \varphi \varrho (1-\zeta)(1-s_t)}\right)^2 - \frac{12\left[1-\varphi(\rho+\gamma\mu_t)\right]}{\gamma^2\varphi}}$$

Or equivalently:

$$w_t = S_t \pm \sqrt{(S_t)^2 + Resilience_t} \tag{C.3}$$

with $Resilience_t \equiv \frac{2k^2}{\varphi\gamma^2}\left[\varphi(\rho+\gamma\mu_t) - 1\right]$ and a measure of safety defined as $S_t \equiv \frac{k^2}{\gamma^2 \varphi w_{t,0}}$, such that

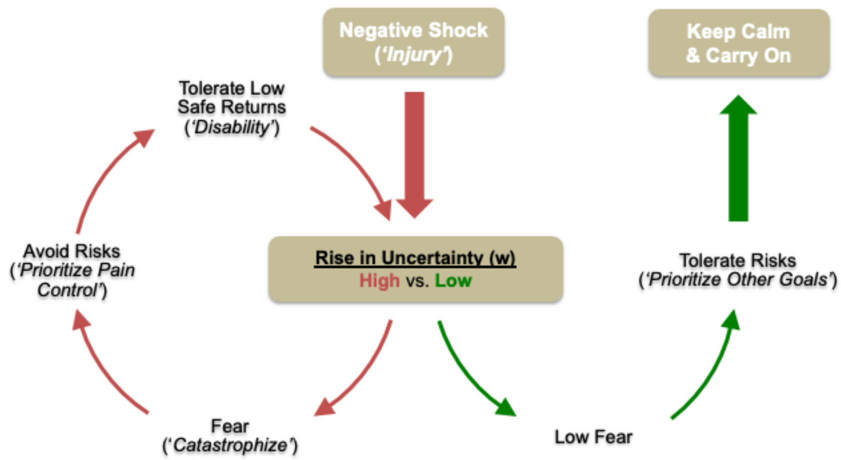


Figure C.2: The Fear-Avoidance Model Adapted to Choice Under Uncertainty

$w_{t,0} \equiv [w_t \mid r_t^* = 0] = \varphi\varrho(1-\zeta)(1-s_t)$ is the level of fear when the neutral rate is zero, and $k^2 = 6$ (by assumption as per Assumption 1 in the main text). Here, $Resilience_t$ can be seen as an summary measure of the underlying robustness of consumption growth for the agents. It moves positively with mean potential growth, μ_t, and with k, which measures the size of tail risk (i.e. how many standard deviations below the mean does the 1 percentile outcome occur for consumption growth). And it moves negatively with γ, which is the coefficient of relative risk aversion. And, S_t can be seen as an inverse measure of subjective tail risk. It is positively associated with k, and negatively with: the degree of risk aversion γ, the level of baseline fear, $w_{t,0}$, and the degree to which subjective beliefs about fear depend on the neutral rate of interest as measured by φ.

As an initial starting point we consider the case in which the upper bound on w_t eliminates the possibility of multiple equilibria such that $S_t + \sqrt{(S_t)^2 + Resilience_t} > \overline{w}$. In this case we have an unique equilibrium with $w_t = S_t - \sqrt{(S_t)^2 + 2 \cdot Resilience_t}$. This case is represented in Figure C.1 as the initial conditions.

A key implication of this model extension is the presence of a 'fear breeds fear' dynamic. First note that when $w_t = S_t - \sqrt{(S_t)^2 + Resilience_t}$, fear falls when $Resilience_t$ rises, i.e., $dw_t/dResilience_t < 0$. Second, note that fear is convex with respect to $Resilience_t$, such that $d^2w_t/dResilience_t^2 > 0$. That is, for high levels of $Resilience_t$, the level of uncertainty is low, and a decline in $Resilience_t$ has a relatively small impact on the equilibrium amount of fear. By contrast, at low levels of $Resilience_t$, the level of fear is high to start with, and therefore a decline in $Resilience_t$ leads to a relatively larger increase in fear. Essentially, in the background, the neutral rate to fall, which in turn raises fear disproportionately. This is what is referred to here as the 'fear breeds fear' dynamic, and provides a macro-economic parallel to the Fear-Avoidance Model of Pain (See Figure C.2). In particular, just like in the Fear-Avoidance Model of Pain, the original level of resilience/safety in the economy, determines whether a negative shock (i.e., a decline in $Resilience_t$) gets amplified or not.

A second key implication of the model extension is how multiple equilibria may arise. Recall, for high levels of $Resilience_t$ and S_t, the upper bound on the posterior distribution of w_t ensures a unique equilibrium in the model, i.e., $S_t - \sqrt{(S_t)^2 + Resilience_t} > \overline{w}$. However, when $Resilience_t$ or S_t is relatively low, the condition will no longer hold, opening up the possibility of having two possible equilibria in the model. In particular, note that $S_t + \sqrt{(S_t)^2 + Resilience_t} > S_t - \sqrt{(S_t)^2 + Resilience_t}$. Thus, at sufficiently low levels of $Resilience_t$ or S_t, we may have a situation in which the level of uncertainty can increase significantly by jumping up from the lower value of w_t to a higher value of w_t. This possibility of multiple equilibria is a second way in which we may have a 'fear breeds fear' dynamic. This possibility would occur, for instance, if the W curve shifted rightwards (as depicted in Figure C.1.)

Proposition (The Fear Avoidance Model of Uncertainty)**.** *When the agent's subjective beliefs about fear (w_t) depend inversely on the neutral rate of interest as given in (C.1) with bounds $[\underline{w}, \overline{w}]$, then the equilibrium market clearing conditions of the SWIM model are given by (C.2). In this model, there are two types of 'fear breeds fear' dynamic. First, uncertainty w_t is a convex and negative function of the underlying resilience in the economy, $Resilience_t$ as given in (C.3). Second, this SWIM model extension features the possibility of multiple equilibria, with the chance of the economy jumping to a perverse fearful equilibrium becoming higher when the underlying resilience or safety in the economy deteriorates.*

PUBLICATIONS

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