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Sudden Stops and Optimal Policy in a Two-agent Economy
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ABSTRACT: We introduce heterogeneity in terms of workers and entrepreneurs in an otherwise standard Fisherian model to study Sudden Stop dynamics and optimal policy. We show that the distinction between workers and entrepreneurs introduces a distributive externality that is absent from the representative-agent setup. While in tranquil times redistribution is driven by the relative marginal utilities of consumption, the planner additionally favors entrepreneurs during Sudden Stops to mitigate Fisherian deation. Although agent-heterogeneity does not add much in explaining the Sudden Stop phenomena, it adds to the understanding of how policies can best be designed to alleviate the negative effects of Sudden Stops.

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1 Introduction

Sudden Stop episodes have characterized many financial crises in both advanced and emerging market economies and are associated with big drops in external financing, as measured by a reversal in the current account, as well as a collapse in asset prices and big drops in consumption and real economic activity. A vast and active literature has been studying the quantitative implications of Sudden Stops by introducing Fisherian deflation dynamics into canonical small open economy macroeconomic models (see Mendoza, 2010, and Mendoza, 2020 for a recent survey). The Fisherian approach features occasionally binding collateral constraints that restrict the ability to borrow up to a fraction of pledged collateral, the value of which depends on market prices. These features can capture Sudden Stop episodes as they generate quantitatively big amplification when the constraint binds, but the probability of such events occurring is small as in the data. Importantly, the presence of market determined prices in the collateral constraint introduces a pecuniary externality as private agents do not internalize how their actions matter for the incidence of binding constraints, which introduces a role for macroprudential policy (see Bianchi, 2011; Bianchi and Mendoza, 2018; and the literature reviewed below).

Our paper builds on the same Fisherian deflation approach to capture Sudden Stops, but attempts to contribute to the literature by introducing agent heterogeneity.[1] In particular, we introduce heterogeneity between entrepreneurs and workers into an otherwise canonical model of Bianchi and Mendoza (2018), whereby the representative agent makes all production, investment, and borrowing decisions as well as supplies labor. In our paper, we split the two functions by having entrepreneurs make decision about production, investment, and borrowing, while workers are the suppliers of labor for which they receive a wage from entrepreneurs. The main novelty of our paper is to show that this distinction has important implications for optimal policy as it introduces a motive for redistribution of resources between the two agents that *meaningfully* interacts with Fisherian deflation dynamics during Sudden Stops. We illustrate this result analytically by solving for the social planner's problem, identifying the externalities that justify a policy intervention, and deriving the optimal taxes that decentralize it. Moreover, we solve the model quantitatively using global solution methods and show the effect of heterogeneity by comparing the optimal policy between the two-agent economy and its representative-agent counterpart.

[1] Most of the literature assumes a representative-agent economy. Mendoza and Quadrini (2010) and Villalvazo (2022) are two exceptions that, nevertheless, consider different types of heterogeneity than in our paper and, more importantly, do not study the implications of agent heterogeneity for optimal policy. We discuss how our paper differs in more detail from theirs when we review the literature below.

We identify three externalities in the two-agent economy. The first externality operates via the price of the asset used as collateral; whereas the other two operate via the wage rate. Pigouvian borrowing and payroll taxes (subsidies), which are set optimally to decentralize the solution of a constrained social planner, tackle these externalities. The externality operating via the price of the asset used as collateral is the same as the one identified in Bianchi and Mendoza (2018). This externality arises as entrepreneurs fail to internalize how borrowing decisions affect asset prices and hence their ability to borrow. The other two externalities operate via the wage rate. Of these, the first one is a pecuniary externality that arises as entrepreneurs fail to internalize that hiring more labor results in higher wages, which in turn, tightens the collateral constraint. This is the case since the inputs of production need to be pre-funded by a loan that requires collateral. This pecuniary externality operates in a similar fashion to the one identified in Bianchi (2016). The second externality that operates via the wage rate is a distributive externality that arises due to the difference in the shadow values of labor income and cost between workers and entrepreneurs, and is introduced in the framework by modeling the two agents separately. As expected, a planner would like to implicitly implement a redistribution of resources to the agent with a higher shadow value of wealth.

The novelty of our analysis is to show that the redistributive motive should not only account for the difference in marginal utilities between workers and entrepreneurs, but also for how entrepreneurial consumption supports asset prices during Sudden Stop episodes. In other words, the planner would like to redistribute more resources back to entrepreneurs during Sudden Stop episodes than what would be justified by the relative marginal utilities of the two agents. The underlying reason for this result is that the price of assets acting as collateral is determined by the stochastic discount factor of entrepreneurs rather than workers, so redistribution can affect the magnitude of Fisherian deflation in the two-agent economy; a mechanism absent in the representative-agent economy.

The quantitative results from solving the model globally also highlight the effect of heterogeneity. In terms of competitive equilibrium outcomes, both the two-agent model and its representative-agent counterpart do a good job in capturing the key aggregate statistics on the incidence and magnitude of Sudden Stops. Both models generate Sudden Stop episodes that occur infrequently with a probability of about 4%, and result in big reversals in the current account and big drops in asset prices and consumption, which are somewhat higher in the two-agent economy. Introducing heterogeneity in the benchmark representative-agent Fisherian model may be useful to study the distributional implications of Sudden Stops, but it does not add considerably much in explaining the aggregate moments underlying the Sudden Stop phenomenon; at least for the type of heterogeneity we consider in this paper. However, we show that even this minimal degree of heterogeneity adds non-trivially to our understanding of how policies can best be designed to alleviate the negative effects of Sudden Stops. Our analytical normative results should be general enough to hold under richer forms of heterogeneity, where there is a redistributive motive.

As in the representative-agent economy, the planner would levy a positive borrowing tax in good times in the two-agent economy in order to lean against credit growth and mitigate the severity of Fisherian deflation dynamics during Sudden Stops. Similarly, the planner uses the payroll tax in both economies to address the pecuniary externality operating though wages when the constraint binds. But, importantly, the planner can use the payroll tax in the two-agent economy to tackle the distributive externality described above. During normal times, the payroll tax is used to implicitly transfer resources to the agent with higher marginal utility facilitating consumption smoothing across agents. During Sudden Stops, the payroll tax is also geared towards helping to address Fisherian deflation by transferring more resources to (less resources away from) entrepreneurs. Naturally, the sign of the payroll tax depends on the social welfare weights assigned to the two agents and the model can quantitatively yield positive or negative payroll taxes. But, irrespective of the calibration, the fact that the payroll tax increases during Sudden Stop episodes to help entrepreneurs is a very general result that highlights the usefulness of this policy to mitigate the negative effects of Sudden Stops ex post. Importantly, using the payroll tax ex post to help entrepreneurs does not remove the need for a (macroprudential) borrowing tax ex ante, as the anticipated redistribution can weaken the precautionary motive of entrepreneurs and urge them to borrow more in normal times. Hence, the combination of borrowing and payroll taxes results in somewhat bigger decrease in the severity of Sudden Stops compared to the decentralized representative-agent economy.

Our analysis has implications for the debate on providing bailouts or subsidies to financially constrained agents (or firms) during crises, echoed also in the analysis in Bianchi (2016). We show that there is a motive to support financially constrained agents, which has been empirically documented in Laeven and Valencia (2013). We also show that this motive is even more pronounced when the constrained agents are the holders of collateral in the economy. Such policy aims to avert a destructive fire-sales—a conclusion in line with the findings in Shleifer and Vishny (2011) and central in Fisherian models of Sudden Stops. It is true that we model the extreme case, where workers do not purchase any capital. However, we show that the distributive externality would still be present, yet attenuated, even if we allowed workers to hold capital, or alternatively, if we introduced additional heterogeneity among entrepreneurs such that some are more and some are less constrained (as, for example, in Villalvazo, 2022).

To further highlight the effect of heterogeneity on optimal policy, we extend the baseline model to incorporate an (anticipated) labor supply shock. Our motivation is to study a shock that would operate similarly to other adverse shocks in the representative-agent economy; but can have different implications in the two-agent economy because it affects workers directly and entrepreneurs indirectly through production and general equilibrium channels. As such, we do not try to argue that labor supply shocks are important ingredients to understand the Sudden Stop phenomenon. But at the same time one could think of a case that a Sudden Stop may be accompanied by a loss of labor force due to immigration to other countries, or lower job creation and higher job destruction as shown in Gallego and Tessada (2012). Such shocks would intuitively make the Sudden Stop more severe as we show by simulating the extended model. Yet, we additionally show that the way the payroll tax adjusts to tackle the labor supply shock is asymmetric with respect to the tightness of the borrowing constraint in the two-agent economy. The planner implements a lower payroll tax when the labor supply shock hits and the constraint does not bind in order to help workers that are directly hit by the shock. By contrast, when the constraint binds, the planner implements a higher payroll tax when the labor shock hits in order to help entrepreneurs and, thus, support asset prices by supporting their consumption. This result further speaks to the importance of the redistribution channel for Sudden Stops that we highlighted above.

Literature review—Our paper relates to the literature studying optimal policy in economies with financial frictions more broadly, and to the literature studying optimal policy in response to Sudden Stops more specifically. Our paper most closely relates to Bianchi and Mendoza (2018), which derives optimal macroprudential regulation that can be used to alleviate the negative effects of Fisherian deflation. As mentioned earlier, we extend their model by introducing heterogeneity in terms of workers and entrepreneurs. This modification gives rise to a distributive externality that operates through wages and is absent from the representative-agent framework. While the macroprudential policy recommendations remain largely unaltered between the representative-agent and the two-agent economy, the optimal payroll tax has an additional role in the two-agent economy framework as it serves to redistribute resources ex post between the agents.

Another closely related paper is Bianchi (2016). The model in that paper, similarly to ours, features a pecuniary externality arising from the fact that firms do not internalize how their labor demand affects the tightness of the collateral constraint. This externality is addressed by a payroll tax. But, the crucial difference from our paper is that our model additionally features a distributive externality, which interacts in an important way with the Fisherian deflation mechanism, absent in Bianchi (2016).

Two other papers that consider agent heterogeneity, albeit different from ours, are Mendoza and Quadrini (2010) and Villalvazo (2022). Mendoza and Quadrini (2010) features savers, who own financial intermediaries that lend to producers subject to a collateral constraint. Contrary to our paper, there is no labor supplied by savers and production just uses capital. Hence, the distributive externality we describe in our paper in absent in the analysis. Villalvazo (2022) considers a Bewley-type model of Sudden Stops, in which the economy is populated by a continuum of households that differ in their income endowment. In his model, unconstrained households can buy assets from constrained households, dampening the effect of Sudden Stops on asset prices. As we hinted above, this dampening effect would attenuate the distributive motive towards entrepreneurs. In our model, workers do not hold capital and, thus, supporting entrepreneurs consumption is always desirable to mitigate the drop in asset prices during Sudden Stops. Importantly, neither of the two aforementioned papers derives the optimal policy in the presence of heterogeneity in a Fisherian model, which is the focus of our paper.

Our paper, more broadly, relates to the literature studying the dynamics of Sudden Stop events and the optimal policy responses designed to mitigate the effects of such boom-bust cycles. Bianchi and Mendoza (2020) offers an exhaustive survey of the empirical and theoretical literature on this topic. Many well-known empirical contributions include Edwards (2004), Rothenberg and Warnock (2006), Forbes and Warnock (2012), Calvo, Izquierdo and Talvi (2006), Eichengreen, Gupta and Masetti (2018), which mainly focus on performing event analysis in a cross-country panel datasets using one or more filters to identify and analyze Sudden Stop episodes. This empirical work also complements a related empirical literature documenting the deep recessions and price corrections that follow the collapse of credit booms (e.g., Mendoza and Terrones, 2012; Schularick and Taylor, 2009). On the theoretical side, Uribe and Schmitt-Grohè (2017) provides a useful textbook presentation of models with Fisherian approach to Sudden Stops. These models typically feature a borrowing constraint (e.g. Aiyagari and Gertler, 1999; Kiyotaki and Moore, 1997), which can be derived from a limited enforcement problem or costly state verification (e.g., Bianchi and Mendoza, 2018; Mendoza and Quadrini, 2010). Different specifications of borrowing constraints—either in the form of a Loan-to-Value or Debt-to-Income ratio—have been extensively used in Fisherian models of Sudden Stops.[2] From these papers, those that are more closely related to ours include Bianchi (2011), Benigno, Chen, Otrok, Rebucci and Young (2013), Bengui and Bianchi (2018), Hernandez and Mendoza (2017), and Mendoza and Rojas (2019) given their focus on optimal policy analysis; but none of these papers studies the role of policy in tackling a distributive externality in normal times and during Sudden Stops.

More generally, our paper relates to the theoretical literature studying the implications of pecuniary externalities that arise due to the presence of financial frictions. Some examples of these papers include Lorenzoni (2008), Stein (2012), and Jeanne and Korinek (2020). These papers have mainly focused on credit policies to tackle the externalities. We differ from these papers because, apart from credit policies, we also focus on labor policies to address the distributive externality we identify.

2 Model Economy

The economy comprises two types of distinct agents: workers and entrepreneurs. Entrepreneurs own a firm that utilizes labor, intermediate goods, and capital to produce a consumption good. They also have access to international credit markets where they can borrow pledging their capital as collateral. Workers supply labor to the firm and only consume out of their labor income.[3] We proceed by outlining the model and defining the optimality conditions for each agents. Some derivations are relegated to the Appendix at the end of the paper, while we have also included additional analytical and quantitative results in an Online Appendix.

2.1 Workers

The economy is populated by a unit mass of identical workers whose preferences are represented by the utility function

[2] This list of papers includes models with with capital accumulation and working-capital financing added to the credit constraint (Mendoza, 2010), models of reserve accumulation (Durdu et al., 2009; Arce et al., 2019), macroprudential policy (Bianchi, 2011), real-exchange-rate stabilization policies (Benigno et al., 2013), ex-post intervention with industrial policy (Hernandez and Mendoza, 2017), self-fulfilling crises (Schmitt-Grohè and Uribe, 2020), imperfect enforcement in capital-flow management policies (Bengui and Bianchi, 2018), models with banks intermediating capital inflows (Mendoza and Rojas, 2019), and models of exchange-rate policy with nominal rigidities and credit frictions (Ottonello, 2021; Coulibaly, 2018; Farhi and Werning, 2016).

[3] We opt for hand-to-mouth workers that cannot pledge their labor income to borrow inter-temporally in order to represent a segment of the population that does not have access to credit markets and the possibility to smooth consumption over time. Kaplan, Violante and Weidner (2014) document that one-third of all US households live hand-to-mouth. The model can also be extended to allow for workers' access to credit markets at the cost of complicating the analysis and making the derivation of optimal policy more convoluted. We elaborate later in Section 3.3 what the implications of such an extension would be.

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - G(h_t)), \tag{1}$$

where $E(\cdot)$ denotes the expectations operator and β is the subjective discount factor. The utility function $U(\cdot)$ is standard concave, twice continuously differentiable, and satisfies the Inada conditions. It depends on consumption, c_t, and labor supply, h_t, which are combined in a composite commodity $c_t - G(h_t)$, defined by Greenwood, Hercowitz, and Huffman (1988). The term $G(h_t)$ is a convex, strictly increasing, and continuously differentiable function, measuring the disutility of labor. This form of a utility function removes the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crisis. We consider a standard CRRA utility function $U(c_t-G(h_t)) = [(c_t-G(h_t))^{(1-\sigma)}-1]/(1-\sigma)$ with risk aversion coefficient σ, while $G(h_t) = \psi h^{1+\phi}/(1+\phi)$ with $1/\phi$ being the Frisch elasticity of labor supply.

As workers are hand-to-mouth agents, their budget constraint is given by

$$c_t \leq w_t h_t, \tag{2}$$

where w_t denotes the wage they receive for the labor supplied.

Workers maximize the utility function (1) subject to their budget constraint (2), which yields the following consumption and labor optimality conditions, respectively,

$$c_t : \quad u_{c,t} = \lambda_t^w, \tag{3}$$

$$h_t : \quad G_{h,t} = w_t, \tag{4}$$

where λ_t^w denotes the Lagrange multiplier associated with workers' budget constraint (2).For notation convenience, we have scaled all Lagrange multipliers throughout the paper by β^t. For example, the multiplier on the worker's budget constraint is $\lambda_t^w = \beta_t \hat{\lambda_t^w}$ in the Lagrangean. We adopt the same notation for all other Lagrange multipliers. $X_{i,t}$ denotes derivatives of a function $X(\cdot)$ with respect to a variable i at time t. This notation for denoting derivatives of functions will be preserved throughout the paper.

2.2 Entrepreneurs

In addition to workers, the economy is populated by a unit mass of identical entrepreneurs whose preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(x_t), \tag{5}$$

where x_t is consumption, β is the time discount factor—equal to the one of workers—and $U(x) = [x^{(1-\sigma)} - 1]/(1-\sigma)$ is a standard CRRA utility function with the same risk aversion coefficient σ to workers.

Entrepreneurs produce $y_t = F(z_t, k_t, l_t, v_t)$ each period. $F(\cdot)$ is a Cobb-Douglas production function, which combines labor, l_t, with the stock of capital purchased in the previous period, k_t, and an intermediate good, v_t; z_t is an aggregate productivity shock. Aggregate capital is in unit fixed supply: $K = 1$. The intermediate good is traded in competitive world markets at a fixed exogenous price, p^v. The budget constraint of entrepreneurs is given by

$$x_t + b_t + p^v v_t + w_t l_t + q_t k_{t+1} = y_t + \frac{b_{t+1}}{R} + q_t k_t, \tag{6}$$

where b_t denotes the beginning-of-period borrowing using one-period, non-state contingent bonds issued last period, q_t is the price of capital, and R is the world-determined gross real interest rate taken as given in the small open economy. Entrepreneurs' consumption, x_t, equals output net of the outlays for the factors of production, v_t and l_t, the net capital expenditure, $q_t(k_{t+1} - k_t)$, and the net debt issuance, $b_{t+1}/R - b_t$.

We assume that entrepreneurs cannot raise equity and that their borrowing decision is limited by a collateral constraint, *endogenously* derived from a renegotiation problem between borrowers and lenders (see section (A) in the Appendix). Entrepreneurs obtain two types of loans: An inter-temporal, b_{t+1}/R, and an intra-temporal loan. They need the latter to finance, ahead of production, a portion θ^v of the intermediate good purchases and a portion θ^l of the labor expenses. Hence, the total liabilities at the beginning of the period comprise of $b_{t+1}/R + \theta^v p^v v_t + \theta^l w_t l_t$. While b_{t+1} is an inter-temporal loan and bears an interest payment, $\theta^v p^v v_t + \theta^l w_t l_t$ does not as it is repaid within the same period (we relax this assumption in the Online Appendix). All borrowed funds can be diverted, a situation that is precluded by imposing the following collateral constraint

$$\frac{b_{t+1}}{R} + \theta^v p^v v_t + \theta^l w_t l_t \leq \kappa_t q_t k_t. \tag{7}$$

Constraint (7) limits the size of total borrowing to a fraction κ_t of the beginning of period holdings of physical capital. κ_t follows a two-state, regime-switching Markov process with $\kappa^l < \kappa^h$, where κ^h and κ^l represent looser and tighter lending standards, respectively (see Section 4 for details about the specification of the process).

Entrepreneurs maximize (5) subject to (6) and (7). This maximization problem leads to the following optimality conditions

$$U_{x,t} = \lambda_t^e, \tag{8}$$

$$F_{v,t} = p^v(1 + \theta^v \mu_t), \tag{9}$$

$$F_{l,t} = w_t(1 + \theta^l \mu_t), \tag{10}$$

$$U_{x,t}(1 - \mu_t) = \beta R E_t U_{x,t+1}, \tag{11}$$

$$q_t U_{x,t} = \beta E_t[U_{x,t+1}(F_{k,t+1} + q_{t+1}) + \kappa_{t+1} U_{x,t+1} \mu_{t+1} q_{t+1}], \tag{12}$$

where λ_t^e denotes the Lagrange multiplier on entrepreneurs' budget constraint and $U_{x,t}\mu_t$ denotes the Lagrange multiplier on the collateral constraint scaled by entrepreneurs' marginal utility.

The presence of the collateral constraint distorts both the optimal inter- and intra-temporal marginal decisions when binding. Conditions (9) and (10), defining entrepreneurs' optimal choice of the intermediate good and labor, embed an additional cost, i.e. the cost of collateral financing equal to $\theta^v \mu_t p^v$ and $\theta^l \mu_t w_t$, respectively. In addition, both Euler equations—with respect to borrowing and capital—are distorted. The Euler equation for borrowing (11) implies that the marginal benefit from increasing borrowing today is equal to the expected future marginal cost from repaying the debt plus the cost of tightening the collateral constraint today, given by the shadow value μ_t. Similarly, the Euler equation with respect to capital (12), equating the marginal cost of an extra unit of capital today with its future marginal benefit, embeds an additional benefit obtained by relaxing the future collateral constraints, valued at $\kappa_{t+1} U_{x,t+1} \mu_{t+1} q_{t+1}$.

2.3 Competitive Equilibrium

The competitive equilibrium of the economy can be defined as follows.

Definition 1. *For given initial values of the endogenous state variable, b_0, and exogenous processes $\{z_t, \kappa_t\}_{t=0}^{\infty}$, a competitive equilibrium for the two-agent economy is a sequence of allocations $\{c_t, x_t, v_t, h_t, l_t\}_{t=0}^{\infty}$, an asset profile $\{k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, and a price system $\{q_t, w_t, p^v\}_{t=0}^{\infty}$, such that*

1. *Given the price system $\{q_t, w_t, p^v\}_{t=0}^{\infty}$, the allocations and the asset profile solve workers' and entrepreneurs' optimization problems as defined in sections 2.1 and 2.2, respectively,*

and

2. *Labor, asset, and goods markets clear, satisfying conditions*

$$h_t = l_t \quad \forall t, \tag{13}$$

$$k_t = K = 1 \quad \forall t, \tag{14}$$

$$c_t + x_t + b_t + p^v v_t = y_t + \frac{b_{t+1}}{R}, \quad \forall t, \tag{15}$$

$$c_t = w_t l_t, \quad \forall t. \tag{16}$$

3 Optimal Policy

In this section, we first define the optimization problem of a constrained social planner, and then discuss the properties of the optimal policies that implement the planner's allocations.

3.1 Social Planner's Economy

We formulate the social planner's problem in a similar manner to Bianchi and Mendoza (2018), who follow the "primal approach" to optimal policy analysis. The planner chooses allocations in order to maximize a social welfare function subject to agents' budget, implementability, and collateral constraints. We assume that, when choosing allocations, the planner lacks the ability to commit to future policies. Therefore, it chooses policy rules at any given period while taking as given the policy rules that represent future planners' decisions.

Formally, the optimization of the constrained social planner can be defined as follows.

Definition 2. *The planner maximizes the infinite weighted-sum of agents' future discounted utilities,* $\sum_{t=0}^{\infty} \beta^t [\omega U(c_t - G(l_t)) + U(x_t)]$*, subject to all conditions comprising the competitive equilibrium,* (2)-(4)*,* (6)-(9)*, and* (12)*. The exceptions are the Euler condition with respect to borrowing,* (11)*, and the optimal decision with respect to entrepreneurs' labor demand,* (10)*, which are omitted since policy is chosen such that these conditions do not represent binding constraints for the planner.* ω *denotes the relative welfare weight on workers' utility, which is assigned exogenously.*[4] *Then, the planner's maximization problem is given by*

[4]We consider alternative values for ω in our quantitative exercises.

$$\max_{c_t,x_t,b_{t+1},l_t,v_t,q_t} E_0 \sum_{t=0}^{\infty} \beta^t [\omega U(c_t - G(l_t)) + U(x_t)]$$

$$x_t + b_t + p^v v_t + G_{l,t} l_t \leq F(z_t, 1, l_t, v_t) + \frac{b_{t+1}}{R} \quad (\lambda_t^{SP,e}) \tag{17}$$

$$c_t = G_{l,t} l_t \quad (\lambda_t^{SP,w}) \tag{18}$$

$$\frac{b_{t+1}}{R} + \theta^v p^v v_t + \theta^l G_{l,t} l_t \leq \kappa_t q_t \quad (\mu_t^{SP}) \tag{19}$$

$$U_{x,t} q_t = \beta E_t U_{x,t+1} \left\{ F_{k,t+1} + q_{t+1} \left[1 + \frac{\kappa_{t+1}}{\theta^v} (\frac{F_{v,t+1}}{p^v} - 1) \right] \right\} \quad (\xi_t) \tag{20}$$

where condition (4) *has been used to substitute* w_t *in constraints* (17), (18), *and* (19)*; and condition* (9) *has been used to substitute* μ_{t+1} *in constraint* (20).[5] *Lagrange multipliers associated with each constraint are given in parentheses.*

Equations (17) and (18) denote the budget constraints of entrepreneurs and workers, respectively. Equation (19) is entrepreneurs' collateral constraint, and (20) is the implementability condition of the planner, which reflects the fact that the planner has to respect competitive asset pricing in the economy. It is through this equation that the planner internalizes how private agents' choices affect equilibrium asset pricing. Moreover, through the optimal labor supply condition, (10), which has been used to substitute wages in equations (17), (18) and (19), the planner internalizes how private agents' choices affect equilibrium wages, which, in turn, matter for the implicit income distribution. Finally, capital is set to its equilibrium aggregate value, $K = 1$; and, labor markets clear, $h_t = l_t$.

The first order optimality conditions of the planner take the following form

$$c_t : \quad \lambda_t^{SP,w} = \omega U_{c,t}, \tag{21}$$

$$x_t : \quad \lambda_t^{SP,e} = U_{x,t} - \xi_t U_{xx,t} q_t, \tag{22}$$

$$b_{t+1} : \quad \lambda_t^{SP,e} = \beta R E_t (\lambda_{t+1}^{SP,e} - \xi_t \Omega_{t+1}) + \mu_t^{SP}, \tag{23}$$

$$l_t : \quad \omega U_{c,t} G_{l,t} = (\lambda_t^{SP,w} - \lambda_t^{SP,e} - \mu_t^{SP} \theta^l)(G_{ll,t} l_t + G_{l,t}) + \lambda_t^{SP,e} F_{l,t}, \tag{24}$$

$$v_t : \quad \mu_t^{SP} = \frac{\lambda_t^{SP,e}}{\theta^v} \left(\frac{F_{v,t}}{p^v} - 1 \right), \tag{25}$$

$$q_t : \quad \xi_t = \frac{\kappa_t \mu_t^{SP}}{U_{x,t}}. \tag{26}$$

[5] The Lagrange multipliers on the collateral constraints in the competitive and the planner's problem, μ_t and μ_t^{SP}, are different but connected in equilibrium as shown below.

where Ω_{t+1} collects all partial derivatives with respect to b_{t+1} on the right-hand side of the capital-Euler equation, capturing the impact of the planner's choice of b_{t+1} on the actions of future planners (reflecting the "time-consistency" nature of the policy rule).[6]

The allocations in the social and the competitive equilibria differ in three main respects. First, unlike the private agents, the planner internalizes how consumption and borrowing choices affect asset prices, and hence the borrowing ability in states in which the collateral constraint binds. In this respect, our findings replicate those of Bianchi and Mendoza (2018). Second, the planner internalizes labor decisions affect the payroll expenses funded by the working capital loan, and hence the borrowing ability in states in which the collateral constraint binds. In this respect, we introduce the mechanism in Bianchi (2016) in the framework of Bianchi and Mendoza (2018). Third, the planner internalizes the difference in the shadow costs of wealth between workers and entrepreneurs and can improve on the allocations by affecting the relative price of labor, i.e. the wages, at which agents trade. In representative-agent frameworks, the latter motive for redistribution is obviously absent given that there is only one agent in the economy. To highlight in further detail the differences between the competitive and social equilibria in the two-agent economy, we proceed by comparing their optimality conditions. We also derive the solution to the representative-agent counterpart economy in section B in the Appendix to facilitate the comparison between the two economies.

First, we compare the first order conditions with respect to entrepreneurs' consumption in the competitive and social equilibria for the two-agent economy. The condition in the competitive equilibrium is $U_{x,t} = \lambda_t$, where λ_t is the Lagrange multiplier on the budget constraint of entrepreneurs (6), while the corresponding condition of the planner is given by equation (22). The key difference between these two equations is that the shadow value of wealth in the planner's solution does not only incorporate the marginal utility from current consumption, but also the amount by which an additional unit of consumption relaxes the collateral constraint through its effect on prices (-$\xi_t U_{xx,t} q_t$). The latter is not accounted for in the competitive equilibrium.[7] Hence the private agents do not internalize how their consumption choice affects the asset price, q_t, as well as the tightness of the collateral constraint. This equation is at the core of the pecuniary externality that operates via the price of collateral (see Bianchi and Mendoza (2018) for details).

[6]Note that $U_{x,t}, F_{k,t+1}, q_{t+1}, \mu_{t+1}$ are all functions of the endogenous state variable b_{t+1} as well as the exogenous state variables at t.

[7]To clarify this point, note that condition (22) shows that there is a positive social benefit from relaxing the implementability constraint at times when the collateral constraint binds at t for the social planner. Moreover, conditions (22) and (26) combined, yielding $\lambda_t^{SP} = U_{x,t} - \kappa_t \mu_t^{SP} U_{xx,t} q_t / U_{x,t}$, show that when the collateral constraint binds, an additional unit of consumption generates a positive marginal social benefit of wealth by raising the equilibrium asset price, which, in turn, relaxes the collateral constraint.

Second, using the competitive and planner's optimality conditions with respect to the intermediate good, (9) and (25) respectively, we get the following condition that connects the Lagrange multipliers on the collateral constraints in the competitive and social equilibria

$$\mu_t = \frac{\mu_t^{SP}}{\lambda_t^{SP,e}}. \tag{27}$$

Hence, μ_t and μ_t^{SP} are either both positive or zero.

Third, we compare the Euler equations for bonds in the competitive and social equilibria. The latter can be written as follows by combining conditions (22) and (23),

$$U_{x,t} = \beta R E_t(U_{x,t+1} - \xi_{t+1} U_{xx,t+1} q_{t+1} - \xi_t \Omega_{t+1}) + \xi_t U_{xx,t} q_t + \mu_t^{SP}. \tag{28}$$

This comparison highlights the pecuniary externalities operating through the current and future price of capital, initially identified in Bianchi and Mendoza (2018). For a more detailed discussion of the mechanism, we refer the interested reader to their paper and herein we provide only a brief description. Consider first the case when the collateral constraint does not bind at t, such that the Lagrange multipliers on the collateral constraints equal zero, $\mu_t = \mu_t^{SP}$=0, as well as $\xi_t = 0$ due to condition (26). In this case, the planner's solution features a higher marginal cost of borrowing at t than the one of the private agents by an amount $\beta R E_t[\xi_t + U_{xx,t+1} q_{t+1}]$. This term implies that the planner, through the implementability constraint (20), internalizes that larger debt at t reduces the borrowing capacity at $t+1$ through its adverse effect on the price of capital, q_{t+1}, when the collateral constraint binds at $t+1$.[8] Now consider the case that the collateral constraint binds at t (and may also bind at $t+1$). Then there are two opposing effects resulting from the borrowing decision that the planner needs to consider. On one hand, higher borrowing accompanied by higher consumption at t increases the price of capital q_t and relaxes the collateral constraint. On the other, more borrowing and higher consumption at t may result in lower consumption and lower price of capital at $t+1$ if the collateral constraint continues to bind in the future. Hence, the planner faces a trade off between choosing allocations such that it increases current prices, q_t, at the cost of potentially decreasing future prices, q_{t+1}.

Finally, we compare the optimality condition with respect to labor of the planner, (24), with that of the private agents,

$$F_{l,t} - G_{l,t}(1 + \theta^l \mu_t) = 0, \tag{29}$$

[8]This can easily be seen by iterating forward and substituting (26) in (28).

obtained by combining the optimal labor conditions of workers and entrepreneurs, (4) and (10), respectively. Condition (29) indicates that the marginal product of labor in the competitive equilibrium equals the wage, given by $w_t = G_{l,t}$, plus the collateral cost due to intra-period borrowing. However, in the planner's equilibrium the marginal product of labor exceeds the wage and the collateral cost by an amount $\left[\theta^l \mu_t + (\lambda_t^{SP,e} - \lambda_t^{SP,w})/\lambda_t^{SP,e}\right] G_{ll,t}$. This can be seen by rewriting equation (24) as follows

$$F_{l,t} - G_{l,t}(1 + \theta^l \mu_t) = \left(\theta^l \mu_t + \frac{\lambda_t^{SP,e} - \lambda_t^{SP,w}}{\lambda_t^{SP,e}}\right) G_{ll,t} l_t, \tag{30}$$

which is obtained by substituting conditions (22) and (27) in (24). The first component of the wedge, $\theta^l \mu_t G_{ll,t} l_t$, pertains to the *pecuniary externality* that operates via wages: Private agents do not internalize that an additional unit of labor increases wages, $w_t = G_{l,t}$, which, in turn, tightens the collateral constraint. Bianchi (2016) identifies a similar externality, which is also present in the representative-agent framework (see the corresponding equation for the representative-agent economy, (63), in section B.2 in the appendix). This externality exists in both the representative- and the two-agent economies because the wage enter in the collateral constraint; had it been absent, this externality would not present. The second component of the wedge, $(\lambda_t^{SP,e} - \lambda_t^{SP,w})/\lambda_t^{SP,e} G_{ll,t} l_t$, pertains to the *distributive externality*: The planner internalizes the difference in the shadow value of labor income between the two agents and, therefore, would like to perform a redistribution between them. This is the key difference between the two-agent and the representative-agent economies. In the former, the planner takes into account the implications of the optimal labor decision for the tightness of the collateral constraint (wage pecuniary externality) as well as the difference in the valuations of labor income between the two agents (distributive externality). By contrast, in the representative-agent economy, the planner only internalizes the effect of the labor decision on the tightness of the collateral constraint.

To further understand the workings of the distributive externality, using the planner's first order conditions with respect to workers' and entrepreneurs' consumption, (21) and (22), respectively, equation (30) can be written as

$$F_{l,t} - G_{l,t}(1 + \theta^l \mu_t) = \left(\theta^l \mu_t + \frac{U_{x,t} - \omega U_{c,t}}{U_{x,t} - \xi_t U_{xx,t} q_t} - \frac{\xi_t U_{xx,t} q_t}{U_{x,t} - \xi_t U_{xx,t} q_t}\right) G_{ll,t} l_t. \tag{31}$$

Equation (31) suggests that the distributive externality comprises two components. The first component, $(U_{x,t} - \omega U_{c,t})/(U_{x,t} - \xi_t U_{xx,t} q_t)$, arises due to the difference in the marginal utilities of consumption between the two agents. The second component, $-\xi_t U_{xx,t} q_t / (U_{x,t} - \xi_t U_{xx,t} q_t)$, incorporates the pecuniary externality that operates via the price of the asset.

This decomposition highlights that, if the collateral constraint does not bind, the planner only accounts for the wedge between agents' marginal utilities of consumption, captured by the first component mentioned above. This is a well-known result in the literature, suggesting that, when a gap exists among agents' marginal utilities, the planner would aim to close that gap by choosing allocations that would implicitly redistribute resources among the agents. In the current case, the planner optimally chooses labor, and hence wages, that account for the gap between the marginal utilities of consumption of workers and entrepreneurs. However, if the collateral constraint binds, the planner also accounts for the effect of the redistribution on asset prices, which introduces an interaction between the distributive externality, operating via the wage, and the pecuniary externality, operating via the asset price. Through this interaction, the planner internalizes that entrepreneurs' consumption, which matters for asset prices and for the tightness of the collateral constraint, is affected by the motives for redistribution. Therefore, when the collateral constraint binds, the planner would like to redistribute more resources to entrepreneurs than what would be justified by just comparing the relative marginal utilities of the two agents. This result is novel and emphasizes the link between the distributive motive of policy and Fisherian deflation.

3.2 Decentralized Economy

We assume that the planner has access to a distortionary borrowing tax/subsidy (as in Bianchi and Mendoza, 2018) and a distortionary payroll tax/subsidy. The distortionary instruments are Pigouvian in nature with the tax revenues being rebated back to the private agents.[9] We assume that all tax payments and rebates are settled at the end of the period after production has taken place. Given the absence of direct lump-sum transfers, the planner will utilize the policy at hand, i.e. the distortionary instruments, not only to tackle the inefficiencies arising from the presence of the collateral constraint, but also to perform redistribution of resources between the two types of agents.

With the policy instruments in place, the budget constraint of entrepreneurs in the decentralized economy takes the following form

[9] The planner needs to respect the per-period budget constraint, which means tax transfers are funded within the same period lump-sum. Alternatively, we could allow the planner to borrow inter-temporally (presumably with looser collateral requirements than the private agents) to raise revenues for tax transfers. This modification would arguably, strengthen the effects of borrowing and payroll subsidies when collateral constraints bind as agents would not need to finance tax transfers because resources are not subtracted in the same period.

$$x_t + (1+\tau_{t-1}^b)b_t + p^v v_t + (1+\tau_t^l)w_t l_t + q_t k_{t+1} \leq y_t + \frac{b_{t+1}}{R} + q_t k_t + T_t. \tag{32}$$

where τ_t^b is the tax on new borrowing b_{t+1} determined at t but levied at $t+1$ when debt is repaid, τ_t^l is the payroll tax, and $T_t = \tau_{t-1}^b b_t + \tau_t^l l_t$ is the total rebate from the tax on borrowing and the payroll tax. Moreover, the Euler condition with respect to borrowing and the optimal labor demand decision of entrepreneurs in the decentralized economy read, respectively,

$$U_{x,t}(1-\mu_t) = \beta R(1+\tau_t^b)E_t U_{x,t+1}, \tag{33}$$

$$F_{l,t} = w_t(1+\tau_t^l + \theta^l \mu_t). \tag{34}$$

All other equilibrium conditions in the decentralized economy remain the same as outlined in section 2.

3.3 Optimal Tax Rates

This section derives the optimal borrowing and payroll taxes.

The optimal tax on borrowing can be derived by combining the Euler equation for borrowing of the planner, (28), with the corresponding equation of the private agents incorporating the tax rate, (33), and takes the following form

$$\tau_t^b = \frac{1}{\beta R E_t U_{x,t+1}}\left[\mu_t^{SP} - U_{x,t}\mu_t + \xi_t U_{xx,t} q_t - \beta R \xi_t E_t \Omega_{t+1}\right] - \frac{1}{E_t U_{x,t+1}} E_t\left[\xi_{t+1} U_{xx,t+1} q_{t+1}\right]. \tag{35}$$

We find that, the optimal tax on borrowing, (35), takes the exact same formula as the one in Bianchi and Mendoza (2018) (also presented by equation (68), in section B.3 in the Appendix), and as in their case, it consists of two components that match the pecuniary externalities operating via the current and future price of capital, q_t and q_{t+1}, identified in the planner's Euler equation. This suggests that introducing agent heterogeneity into the framework does not affect the functional form of the tax on borrowing, which continues to tackle the pecuniary externality arising from occasionally binding collateral constraints.

In the case that the collateral constraint does not bind at t, i.e. $\mu_t = \mu_t^{SP} = \xi_t = 0$, the tax rate reduces to

$$\tau_t^{MP} = -\frac{1}{E_t U_{x,t+1}} E_t \left[\xi_{t+1} U_{xx,t+1} q_{t+1}\right] = -\frac{1}{E_t U_{x,t+1}} E_t \left[\kappa_{t+1} \mu_{t+1}^{SP} q_{t+1} \frac{U_{xx,t+1}}{U_{x,t+1}}\right], \tag{36}$$

which can easily be shown to be always positive, pushing for a *tax* on borrowing, as long as the collateral constraint binds only in expectation. As discussed in detail in Bianchi and Mendoza (2018), the tax rate tackles the pecuniary externality operating via q_{t+1}, and obtains a macro-prudential interpretation since it is levied during good times (i.e. when the collateral constraint does not bind), to allow for more borrowing during bad times (i.e. when the collateral constraint binds in the future).

In the case when the collateral constraint also binds at t, the term $1/(\beta R E_t U_{x,t+1})[\mu_t^{SP} - U_{x,t}\mu_t + \xi_t U_{xx,t} q_t - \beta R \xi_t \Omega_{t+1}]$ in (35) is non-zero, and it addresses the externality operating via the current price of capital, q_t. Higher borrowing at t supports higher asset prices q_t and relaxes the collateral constraint, pushing for a *subsidy on borrowing*. However, higher borrowing at t would require a repayment at $t+1$, resulting in lower asset prices q_{t+1} and a tighter collateral constraint, pushing for a *tax on borrowing*. Hence, when choosing the optimal tax on borrowing, the planner balances these two effects.

Now we turn to the optimal payroll tax. This tax rate can be derived by combining the planner's optimal decision for labor, (24), and the corresponding condition of the private agents that incorporates the tax rate, (34). Noting that $\lambda^{SP,w} = \omega U_{c,t}$ and that $\mu_t = \mu_t^{SP}/\lambda_t^{SP,e}$ (obtained using the optimality conditions with respect to the intermediate good in the competitive and planner's equilibria, (9) and (25), respectively), the payroll tax takes the following form

$$\tau_t^l = \left(\theta^l \mu_t + \frac{\lambda_t^{SP,e} - \lambda_t^{SP,w}}{\lambda_t^{SP,e}}\right) \frac{G_{ll,t}}{G_{l,t}} l_t. \tag{37}$$

The payroll tax tackles precisely the wedge that exists between the optimal labor decisions in the competitive and social equilibria, highlighted in section 3.1. To reiterate, the tax rate comprises two terms.

The first term on the right hand-side in (37) tackles the pecuniary externality operating via the wage rate. The planner internalizes that hiring more labor increases the cost of labor, which, in turn, tightens the collateral constraint as labor costs are financed via intra-period borrowing. Given that $\theta^l \mu_t G_{ll,t} l_t > 0$ (since $G_{ll,t} l_t > 0$), to relax the collateral constraint, the planner would like to increase the payroll tax or decrease the subsidy in order to discourage labor demand and, thus, the cost that need to be prefunded by the loan.

The second term on the right hand-side in (37) tackles the distributive externality described in section 3.1, which, as mentioned, consists of two components. To see how each of these components affect the tax rate, we can rewrite (37) as

$$\tau_t^l = \left(\theta^l \mu_t + \frac{U_{x,t} - \omega U_{c,t}}{U_{x,t} - \xi_t U_{xx,t} q_t} - \frac{\xi_t U_{xx,t} q_t}{U_{x,t} - \xi_t U_{xx,t} q_t}\right) G_{ll,t} l_t \tag{38}$$

by substituting in equations (21) and (22). This decomposition suggests that, as long as the collateral constraint does not bind at t, i.e. $\xi_t = 0$, the planner would set the payroll tax to tackle the gap that exists between agents' marginal utilities of consumption. In particular, if $U_{x,t} > \omega U_{c,t}$, the planner would like to impose a tighter payroll tax (or a lower payroll subsidy), which would discourage labor demand, effectively reducing the wage, and acting as an implicit redistribution of resources from workers to entrepreneurs. On the contrary, if $U_{x,t} < \omega U_{c,t}$, the planner would like to impose a lower payroll tax (or higher payroll subsidy), which would incentivize labor demand and push wages up. Such policy would act as a resource redistribution from entrepreneurs to workers.

However, if the collateral constraint binds at t such that $-\xi_t U_{xx,t} q_t/(U_{x,t} - \xi_t U_{xx,t} q_t) > 0$, the planner has an additional consideration.[10] Through this component of the tax rate, the planner internalizes the interaction of the pecuniary externality, operating via the asset price, and the distributive externality, operating via wages. By imposing a higher tax (lower subsidy) on payroll (since $-\xi_t U_{xx,t} q_t/(U_{x,t} - \xi_t U_{xx,t} q_t) > 0$), the planner internalizes that higher entrepreneurial consumption supports asset prices, which can loosen the collateral constraint; hence, the optimal payroll tax can be used to mitigate the adverse effects of Fisherian deflation ex post, i.e. during Sudden Stop episodes.[11]

Finally, we compare the optimal policy in the two-agent economy to the one in the representative-agent economy, which we derive in Section B.3 in the Appendix. The tax on borrowing and the macroprudential tax in the two-agent economy, shown in equation (35) and (36), take the same form as those in the representative-agent economy, shown in equations (68) and (69). However,

[10]Note that $-\xi_t U_{xx,t} q_t/(U_{x,t} - \xi_t U_{xx,t} q_t)$ is positive because $(U_{x,t} - \xi_t U_{xx,t} q_t) = \lambda_t^{SP,e} > 0$ and $U_{xx,t} q_t < 0$.

[11]In our framework, the payroll tax achieves an implicit redistribution between workers and entrepreneurs because it affects the demand for labor and the equilibrium wage. Hence, it is a tool to indirectly transfer resources to entrepreneurs during Sudden Stops to mitigate the pecuniary externalities from falling asset prices. In principle, the payroll tax could also be used as a direct bailout tool if, for example, payroll-tax payments had to be also funded with the working capital loan and, thus, appeared in the left-hand side of the collateral constraint. In our framework, this is not the case because taxes are paid at the end of the period and, thus, entrepreneurs do not borrow intra-period to fund them (see Appendix A for details). We believe that this is a reasonable assumption, which also simplifies the derivation of optimal policy; if payroll taxes appeared in the collateral constraint, we would need to assume a timing mismatch between tax payment and tax rebates as well as follow the dual approach to the Ramsey problem. The tax in (38) would, then, include additional terms, but the channel we describe would still be present. The derivation of optimal policy under this alternative framework is presented in Section C of the Online Appendix.

there is an important difference between the payroll taxes in the two economies, given by equations (37) (two-agent economy) and (67) (representative-agent economy). In both economies, the payroll tax tackles the pecuniary externality operating via the wage, accruing from the fact that a portion θ^l of the payroll costs are funded with the working capital loan. For $\theta^l = 0$, the collateral constraint would collapse to the one in Bianchi and Mendoza (2018) and there would be no role for a payroll tax in the representative-agent economy. Yet, the payroll tax would still be useful in the two-agent economy to tackle the distributive externality, incorporating both the difference in the marginal utilities of consumption between workers and entrepreneurs and the effect of entrepreneurial consumption on the asset price. To be able to connect our results to other papers in the literature, where the labor decision directly affects the collateral constraint (Bianchi, 2016; Mendoza, 2012), we have opted to derive the optimal payroll tax for $\theta^l \geq 0$ (instead of $\theta^l = 0$), and highlight what part of the payroll tax stems from $\theta^l > 0$ versus the introduction of heterogeneity.

As an aside, we should also mention that the component of the distributive externality, which captures the interaction with the asset price externality, pushes unambiguously for a payroll tax. However, one might argue that this result accrues from our assumption that workers do not participate in the market for capital and, thus, do not hold capital even if they wanted to in equilibrium.[12] This component of the payroll tax would still be present even if workers held capital, but it would capture the relative contribution of workers' and entrepreneurs' drop in consumption during Sudden Stops on asset prices depending on their respective capital holdings.

3.4 Additional Tools

The analysis thus far was kept close to Bianchi and Mendoza (2018), where the planner takes as given the pricing of capital in the competitive economy, given by equation (12). Incorporating this constraint in the planner's problem guarantees that the planner's allocations are implementable in the decentralized economy, where the planner does not have direct control over the price of capital but internalizes how other decisions matter for it. A natural question that arises is whether the planner could more directly affect how binding the constraint is by influencing the equilibrium asset price, for example by levying distortionary taxes/subsidies on capital purchases. This question does not arise from the introduction of heterogeneity, but it is also valid for the framework in Bianchi and Mendoza (2018). The idea of such instrument is

[12]The only reason that workers may have wanted to hold capital would be to pledge it as collateral in order to be able to borrow, given that they cannot use it for production. This motive would accrue from a strong precautionary demand for savings as workers would need to sacrifice current consumption to purchase capital.

to directly affect/choose the price of collateral, similarly to Benigno, Chen, Otrok, Rebucci and Young (2016). We relegate the detailed analysis in Section B of the Online Appendix and we discuss below the key takeaways.

Consider that the planner can levy a distortionary tax τ_t^k on capital holdings k_{t+1} with the proceeds rebated back to entrepreneurs lump-sum. Then, the new implementability condition governing the pricing of capital becomes

$$(1+\tau_t^k) q_t U_{x,t} = \beta E_t U_{x,t+1} \left\{ F_{k,t+1} + q_{t+1} \left[1 + \frac{\kappa_{t+1}}{\theta^v} \left(\frac{F_{v,t+1}}{p^v} - 1 \right) \right] \right\}, \tag{39}$$

where we have substituted $\mu_{t+1} = (F_{v,t+1}/p^v - 1)/\theta^v$ on the right hand-side of the equation. Without any constraints on τ_t^k the planner can choose a capital tax/subsidy to implement any q_t, which means, in practice, that the planner can implement an equilibrium where the collateral constraint never binds. Yet, there is a natural lower bound on τ_t^k imposed by the transversality condition as we show in the Online Appendix. Alternative, there may be other considerations that limit the ability of the fiscal authority to issue subsidies, which can accrue, for example, from funding or political constraints.[13] The exact microfoundations of such constraint and how they relate to the natural lower bound imposed by the transversality condition are beyond the scope of our analysis. Instead, we abstractly consider that the planner faces an additional constraint given by $\tau_t^k \geq \underline{\tau}$, where $\underline{\tau} < 0$ is the lower bound.

We show that the planner would still like to levy a tax on borrowing as long as there are states where the tax on capital hits its lower bound. That means that the macroprudential tax on borrowing—a central result in Bianchi and Mendoza (2018) which carries over in our framework—can continue to be positive and useful, though arguably smaller in magnitude. More specifically for our model, the fact that a capital subsidy may not be able to eliminate the collateral constraint in all eventualities means that the borrowing and payroll taxes will continue to be a useful tools to tackle Fisherian deflation when the collateral constraint binds. In our baseline quantitative analysis, we consider the case that the planner does not have access to a tax on capital similarly to Bianchi and Mendoza (2018). Yet, we report quantitative results for different levels of $\underline{\tau}$ in Section B of the Online Appendix and we acknowledge that having a model to determine $\underline{\tau}$ may be an interesting extension to study the interactions of capital and borrowing taxes to tackle Sudden Stop episodes. Overall, we show that capital subsidies

[13] It is true that such constraints should also apply for borrowing and payroll subsidies, but they would just reduce their effectiveness rather than eliminate them from the optimal policy mix. Moreover, we show, in our quantitative analysis, that any such subsidies are not excessive and unreasonable. Thus, for simplicity, we abstract from introducing lower bounds for τ_t^b and ^l_t in our baseline analysis. See the Online Appendix for quantitative comparison between the levels of capital and payroll subsidies.

are beneficial and welfare increase even further, but they create adverse incentives to take more debt ex ante amplifying the role of the borrowing and payroll tax policies rather than eliminating their usefulness.

4 Quantitative Analysis

This section presents a quantitative analysis of the economy, which includes the computation and discussion of the policy functions as well as numerical simulations for a baseline calibration. The first part describes the calibration and the rest discusses the quantitative results.

To solve the model, we use a global, non-linear solution algorithm. The competitive economy solution is obtained by iterating over the first-order conditions, and the SP problem solution is obtained by applying a value function iteration algorithm. Our algorithm is similar to the one in Bianchi and Mendoza (2018) and Mendoza and Villalvazo (2020) adjusted for heterogeneity. In order to obtain the solution for the competitive economy, we iterate the (competitive) Euler equation for borrowing, which does not incorporate the pecuniary externality. Value function iteration incorporates the effect of pecuniary externalities on welfare and, hence, yields the planner's solution. Given that we solve for time-consistent policies, we use a nested fixed point algorithm for the value function iteration. Section C in the appendix discusses the details of the numerical solution method.

4.1 Calibration and Summary Statistics

Given the resemblance of our model to Bianchi and Mendoza (2018), the majority of the model parameters are calibrated similar to them.[14] Each time period should be interpreted as a year. We deviate from their calibration in the following ways. First, we abstract from modeling a stochastic process for the global interest rate in order to limit, for simplicity, the number of exogenous states. Instead, we opt for a fixed rate set to $R = 1.02$, which is slightly higher than the long-term average level of 1.01 in Bianchi and Mendoza (2018), but consistent with the fixed rate used in Bianchi and Mendoza (2020).[15] Second, we use the same value θ for θ^v and θ^l equal to 0.09 instead of 0.16 in their paper in order to obtain a share of working capital loan to GDP equal to 0.13 as in the data. The reason for doing this is that in our model portions of both the payroll costs and the intermediate goods purchases need to be funded by a working

[14] For details, we refer the reader to section III.A of their paper.

[15] Had we set $R = 1.01$, the probability of a crisis (sudden stop) would be still low and about 0.1 percentage points higher in the representative and two-agent economies than the ones reported in Table 2.

capital loan contrary to only some of intermediate goods purchases in their paper.[16] Third, we set the transition probability from a regime with looser financial conditions, κ^h, to a regime with tighter financial condition, κ^l, equal to 5 percent instead of 10 percent in Bianchi and Mendoza (2018). We do so to match a probability of a crisis (Sudden Stop) equal to 4%, as in their paper and consistent with the empirical literature (see below for details on how Sudden Stop episodes are defined).[17] Lastly, for our baseline calibration we assume a welfare weight $\omega = 1$, i.e. we consider a strictly utilitarian social welfare function according to which the planner values every agent equally given that there is a unit mass of workers and a unit mass of entrepreneurs. We perform comparative statics with respect to ω and, as we show later in section 4.4, our results are qualitatively unaffected but quantitatively the use of policy for redistribution is apparent. All other parameters are the same as in Bianchi and Mendoza (2018) and are reported in Table 1.

Table (1) Calibration

Parameter	Value
Risk aversion	$\sigma = 1$
Labor disutility coefficient	$\psi = 0.352$
Frisch elasticity of labor supply	$1/\phi = 2$
Share of intermediate good in output	$\alpha_v = 0.45$
Share of labor in output	$\alpha_l = 0.352$
Share of assets in output	$\alpha_k = 0.008$
Interest rate	$R = 1.02$
TFP process	$\rho_z = 0.78$
	$\sigma_\epsilon = 0.01$
Discount factor	$\beta = 0.95$
Working capital coefficient	$\theta^v = \theta^l = 0.09$
Tight credit regime	$\kappa^l = 0.75$
Normal credit regime	$\kappa^h = 0.90$
Transition probability, κ^h to κ^l	$P_{h,l} = 0.05$
Transition probability, κ^l to κ^l	$P_{l,l} = 0.00$
Welfare weight on workers	$\omega = 1$

Table 2 reports the aggregate statistics of the simulated competitive representative- and two-agent economies. The probability of a crisis is captured by the probability that a Sudden Stop occurs in the model. We define a Sudden Stop event as a period during which the drop in long-term borrowing (b) between two periods over output, i.e., the current-account-to-GDP ratio, exceeds two standard deviations of its long-term distribution, similar to Bianchi and

[16]Setting $\theta^v = \theta^l$ suffices for our purposes, but distinguishing between them could allow a finer calibration of the relative contributions of intermediate goods purchases and labor payroll costs in working capital.

[17]We target the probability of a Sudden Stop for the two-agent competitive economy to be consistent with the data and we use the same calibration to compute the probability of a Sudden Stop in the counterpart representative-agent competitive economy. Although the representative-agent economy is very close to Bianchi and Mendoza (2018), the inclusion of the payroll costs in the working capital loan and the fixed interest rate would require a higher probability for staying in the loose regime to match the probability of a crisis.

Mendoza (2018).[18] Sudden Stops are infrequent events accompanied by large reversals in the current account and credit crunches, and are accompanied by a binding collateral constraint 99.5 percent of the time. However, the opposite is not true, i.e. a binding collateral constraint is not sufficient for the occurrence of a Sudden Stop, as there can be instances that the constraint binds with multipliers very close to zero implying small corrections in credit. As shown in Table 2, the probability that the constraint binds is higher than the probability of a Sudden Stop event materializing, which is true for both the representative and two-agent economies—consistent with the findings of the existing literature on Sudden Stops that uses representative-agent models—but somewhat more pronounced for the former.

Focusing on the competitive equilibrium outcomes, the probabilities of a Sudden Stop episode in the competitive representative- and two-agent economies are similar and consistent with the data. The same is true for the ratios of short-term borrowing (working capital loan) and long-term borrowing over GDP.[19] Other business cycle statistics for consumption, asset prices, and wages, reported in Panel B in Table 2, are also suggestive that introducing heterogeneity between workers and entrepreneurs does not materially affect the key aggregate moments produced by the model for the competitive economy. Yet, modeling heterogeneity is still useful for positive analysis to understand the distributional effects of Sudden Stops on different agents in competitive economies, which we discuss in Section 4.3. Yet, even this minimal degree of heterogeneity alters the dynamics of the economy and the optimal policy responses, which has important implications for the ability of the planner to reduce the probability and severity of Sudden Stops. We discuss these issues in greater detail in the following subsections, where we first compare the policy functions for borrowing and the tax rates in the decentralized representative and two-agent economies and then perform event analysis around Sudden Stop episodes. Finally, we also analyze the performance of the model in presence of exogenous labor shocks because they directly affect the decision to supply labor and, thus, should propagate differently in the two-agent compared to the representative-agent economy.

As an aside, an additional difference between our model and Bianchi and Mendoza (2018) is that a portion of the payroll costs need to be funded with a working capital loan, which tightens

[18] Bianchi and Mendoza (2020) and Korinek and Mendoza (2014) identify Sudden Stops as events where the increase in the current account as percentage of GDP is large enough to be in the 95th percentile of the frequency distribution of annual changes in the current-account-to-GDP ratio. The way we identify the episodes is also consistent with this criterion: 99.6 percent and 100 percent of the Sudden Stop events in the representative-agent and two-agent economies, respectively, are characterized by changes in the current account as percentage of GDP that are high enough to be in the 95th percentile.

[19] As reported by Bianchi and Mendoza (2018), in the data, short-term borrowing (working capital loans) as percent of GDP is 13.3%, and long-term borrowing, measured by the NFA (net foreign asset position), as percent of GDP is 25%. Their estimates are based on 2013 US data. Recall that we have targeted the probability of Sudden Stops and working capital loan over GDP in the two-agent competitive economy and we use the same calibrated parameter when solving for the representative-agent competitive equilibrium.

the collateral constraint. As we discussed in Section 3.3, we do so for generality and to compare with other models in which labor decisions are directly affecting the collateral constraint. In order to study the quantitative implications of this modification for Sudden Stops, we have solved for both $\theta^l = 0$ and $\theta^l = 0.09$ using the same calibration otherwise. Section A of the Online Appendix reports the results, which are in line with what we described above, along with detailed sensitivity analysis for many other parameters in the model. Our results continue to hold under those alternative parameter choices.

Table (2) Summary Statistics

	Repr. agent		Two-agent	
	CE	SP	CE	SP
PANEL A				
Probability of Sudden Stop	4.2	3.1	4.0	2.7
Probability of binding constraint	13.8	15.7	4.3	4.7
Average Working capital loan / GDP	13.1	13.1	13.1	13.7
Standard deviation	*0.002*	*0.001*	*0.002*	*0.002*
Average Credit-to-GDP (b_{t+1}) / GDP	24.4	16.4	21.1	17.0
Standard deviation	*0.038*	*0.020*	*0.033*	*0.016*
PANEL B				
Average aggregate consumption	0.26	0.27	0.26	0.29
Standard deviation	*0.017*	*0.014*	*0.015*	*0.015*
Average asset price	0.11	0.09	0.11	0.10
Standard deviation	*0.008*	*0.005*	*0.009*	*0.004*
Average wage	0.28	0.28	0.28	0.29
Standard deviation	*0.005*	*0.005*	*0.006*	*0.006*

Note: Panels A and B report numbers in percent and levels, respectively. We report unconditional averages over the simulation horizon. Credit-to-GDP is the the ratio of the long-term loan (b_{t+1}) to GDP. CE=Competitive Economy, SP=Social Planner.

4.2 Policy Functions Analysis and Optimal Taxes

In this section, we outline the differences between the two-agent and the representative-agent economies by comparing their respective optimal (private and social) borrowing decisions as well as the optimal borrowing and payroll taxes.

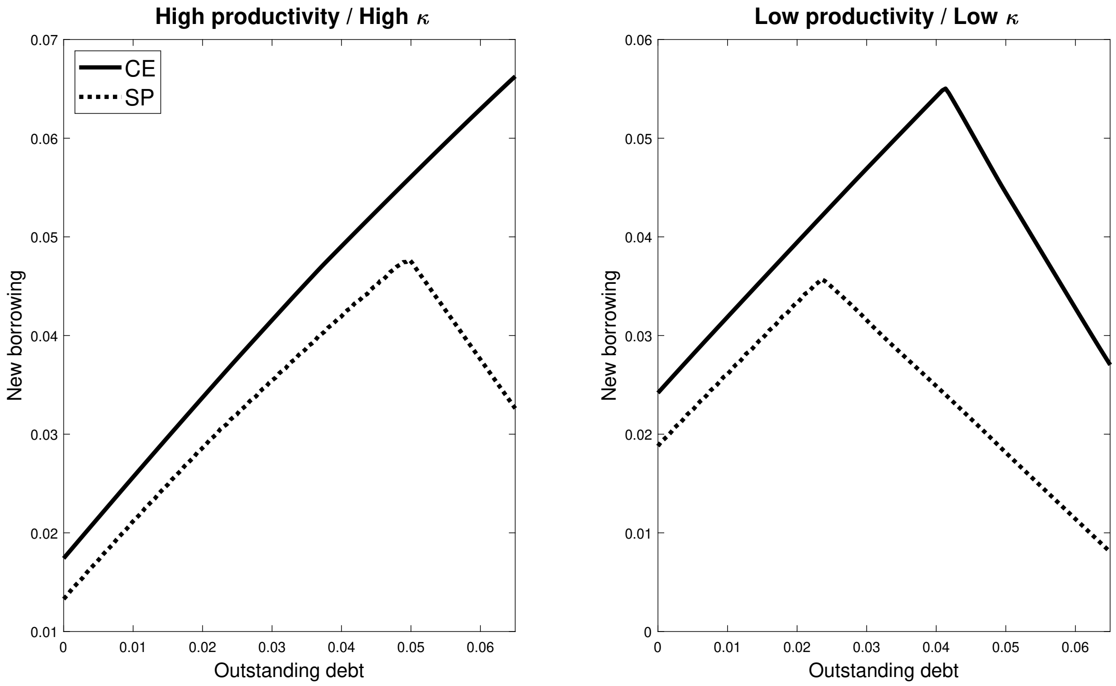
Figure 1 shows new borrowing, b_{t+1}, as a function of the outstanding debt level, b_t, when financial conditions are favorable and productivity is high ($\kappa_t = \kappa^h$ and $z_t = z^h$) and when they are low ($\kappa_t = \kappa^l$ and $z_t = z^l$). The top panels show the economy with a representative agent, and the bottom panels show the economy with workers and entrepreneurs. The solid (dotted) lines correspond to the private agents' (social planner's) policy rules.

A couple of differences between the competitive (CE) and social (SP) equilibria can be noted. First, in both economies, the planner chooses a lower level of new borrowing b_{t+1} compared to private agents before the borrowing constraint turns binding at t, which is at the (kink) point when new borrowing starts to decline. This happens because the planner internalizes the externalities, and mitigates their negative impact on consumption, asset prices, and welfare. Moreover, in both economies, the borrowing constraint turns binding in the planner's equilibrium for lower levels of debt than for the private agents. This is because lower levels of borrowing and consumption put downward pressure on collateral values, thereby tightening the borrowing constraint.

A notable difference arises between the representative and the two-agent economies when productivity is low ($z_t = z^l$) and financial conditions are tight ($\kappa_t = \kappa^l$), shown in the two right-hand-side panels in Figure 1. While in the representative-agent economy, the Online Appendix and social equilibrium borrowing follow very similar dynamics, this is not the case in the two-agent economy. In particular, in the the Online Appendix equilibrium, the decline in new borrowing is sharp once the borrowing constraint starts to bind, whereas the decline in the social planner's equilibrium is smoother. This result arises from the fact that, in the two-agent economy, the planner is also able to redistribute resources from workers to entrepreneurs when they are most needed, i.e. when financing constraints turn binding. This way, the planner is able to provide more support to entrepreneurs by allowing their borrowing ability to recover faster following a tightening of the collateral constraint. Subsequently, we analyze the optimal tax rates used to decentralize the planner's allocations, which corroborate this finding.

The top chart in Figure 2 presents the optimal borrowing tax for high productivity ($z_t = z^h$) and favorable financial conditions ($\kappa_t = \kappa^h$) in the representative-agent (solid line) and the two-agent (dashed line) economies. In both economies, the tax rate is about zero for low levels of current period borrowing, but as borrowing starts to increase, the tax rate goes up before it starts decreasing, which is at the point when the collateral constraint turns binding at period t. The difference between the tax rate in the two economies is the sharper decline of the tax rate in the representative-agent compared to the two-agent economy, in which the decline is smoother. When the constraint turns binding, the planner would like to support borrowing, and the only

(a) Representative-agent economy



(b) Two-agents economy

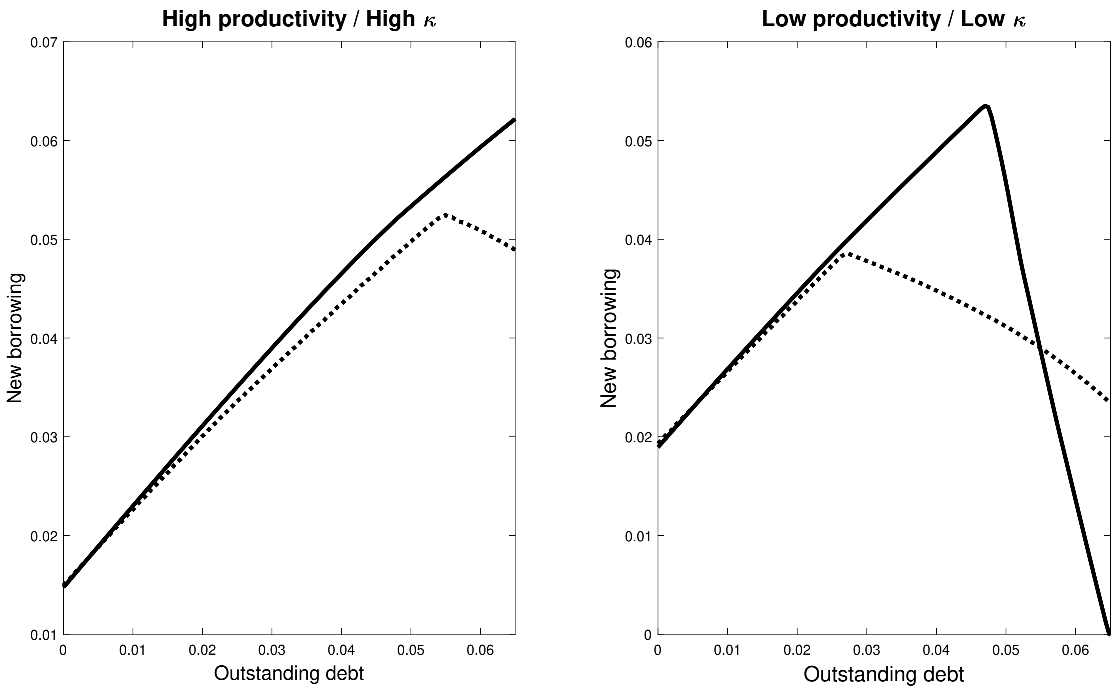


Figure (1) Note: The figure plots new borrowing b_{t+1} as a function of outstanding debt b_t for $z_t = z^h$ and $\kappa_t = \kappa^h$ (left-hand-side panels) and $z_t = z^l$ and $\kappa_t = \kappa^l$ (right-hand-side panels) for the representative-agent economy (top panels) and the two-agent economy (bottom panels). The solid (dotted) lines correspond to the private agents' (social planner's) policy rules.

way to do so in the representative-agent framework is by reducing the tax rate. On the contrary, in the two-agent framework, borrowing can also be supported by transferring resources from workers to entrepreneurs, which occurs through the payroll tax as we discuss next.

The two bottom charts in Figure 2 present the optimal payroll tax for high productivity and favorable financial conditions (left panel) and low productivity and unfavorable financial conditions (right panel) in the representative-agent (solid line) and the two-agent (dashed line) economies. In the left-hand-side chart, the payroll tax is zero in the representative-agent economy and turns slightly positive when the borrowing constraint starts to bind (on the right side of the kink point). On the contrary, in the two-agent economy, the payroll tax is negative when the borrowing constraint is slack, but starts becoming less negative and gets close to zero as the constraint turns binding (on the right side of the kink point). The difference in the policies arises because in the representative-agent economy, the payroll tax is only used to address the pecuniary externality operating through wages; whereas in the two-agent economy, the payroll tax is additionally used to address the distributive externality. A higher payroll tax discourages labor demand and helps relax the binding constraint, implicitly redistributing resources to entrepreneurs; while a lower payroll tax encourages labor demand, implicitly redistributing resources to workers, but further tightens the constraint. In the two-agent economy, the planner initially wants to redistribute resources to workers and levies a negative tax. The tax becomes less negative for higher levels of outstanding debt because entrepreneurs' net worth is lower and, hence, the redistributive motive is weaker. Once the constraint binds, the planner further decreases the payroll subsidy to additionally address the distributive externality.

We observe similar dynamics in the right-hand-side chart, where the optimal payroll tax is plotted for low productivity ($z_t = z^l$) and unfavorable financial conditions ($\kappa_t = \kappa^l$). But, in addition, this case highlights more clearly the redistributive motive: Not only does the planner reduce the payroll subsidy for higher levels of indebtedness, but also starts levying increasingly higher positive taxes in order to redistributive resources from workers to entrepreneurs. In other words, the underlying motive for the positive payroll tax does not only come from a desire to address the pecuniary externality operating through the wage, but also to transfer resources to the very constrained and indebted entrepreneurs supporting asset prices, and enabling borrowing and economic expansion.

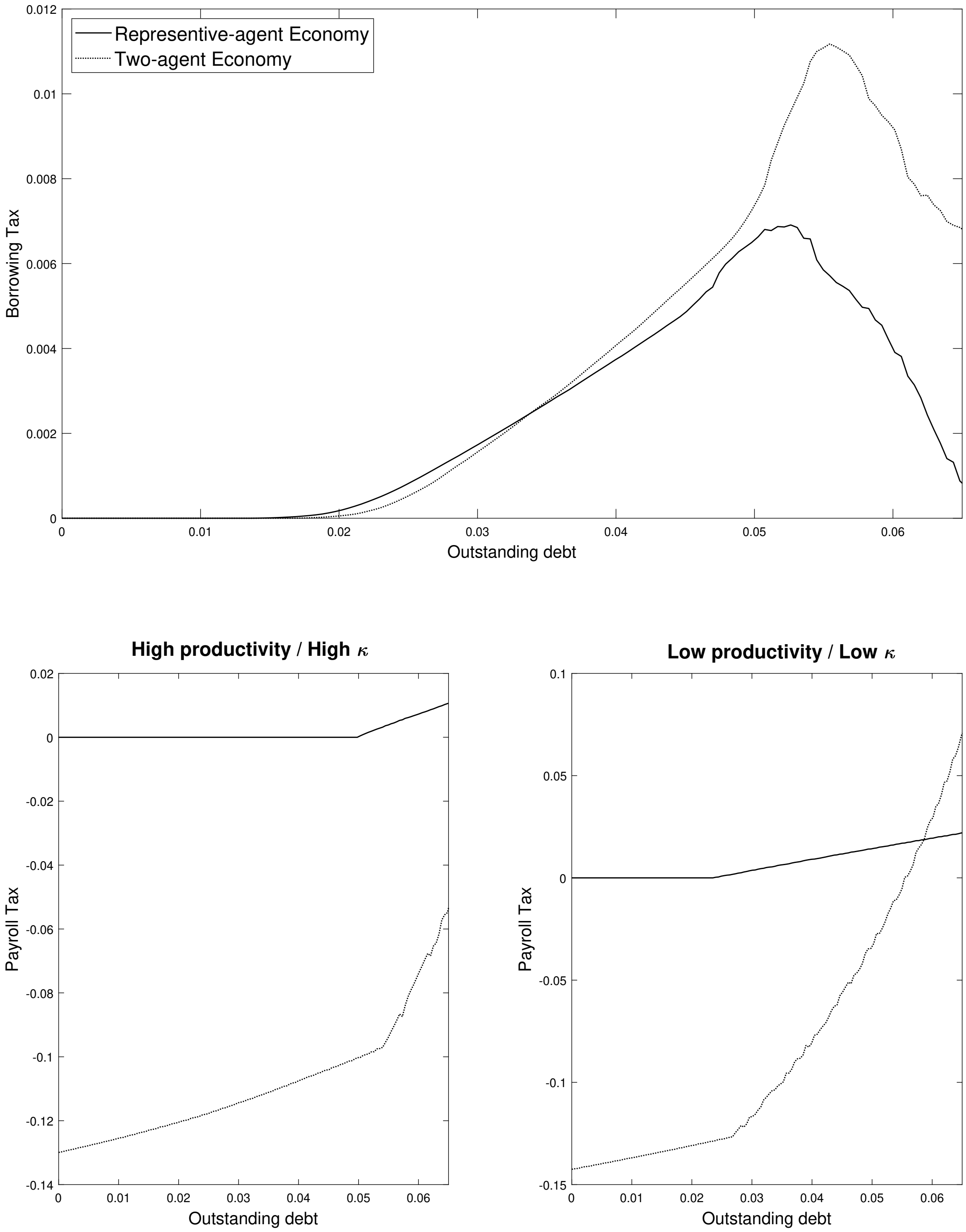


Figure (2) Note: The figure plots the tax rates as functions of outstanding debt b_t, for $z_t = z^h$ and $\kappa_t = \kappa^h$ (top panel for tax on borrowing and left-side, bottom panel for tax on payroll) and $z_t = z^l$ and $\kappa_t = \kappa^l$ (right-side bottom panel for tax on payroll). The solid and dashed lines correspond to the representative-agent and two-agent economies' policy rules, respectively.

4.3 Event Analysis and Welfare

In order to assess the effectiveness of optimal policy at reducing the severity of Sudden Stops, we perform an event analysis of the competitive and planner's equilibria during a Sudden Stop episode. We identify Sudden Stop episodes separately for both the competitive and the planner's equilibria, but the comparison of the dynamics between the two equilibria in Figure 3 is performed at the Sudden Stop episodes of the former.

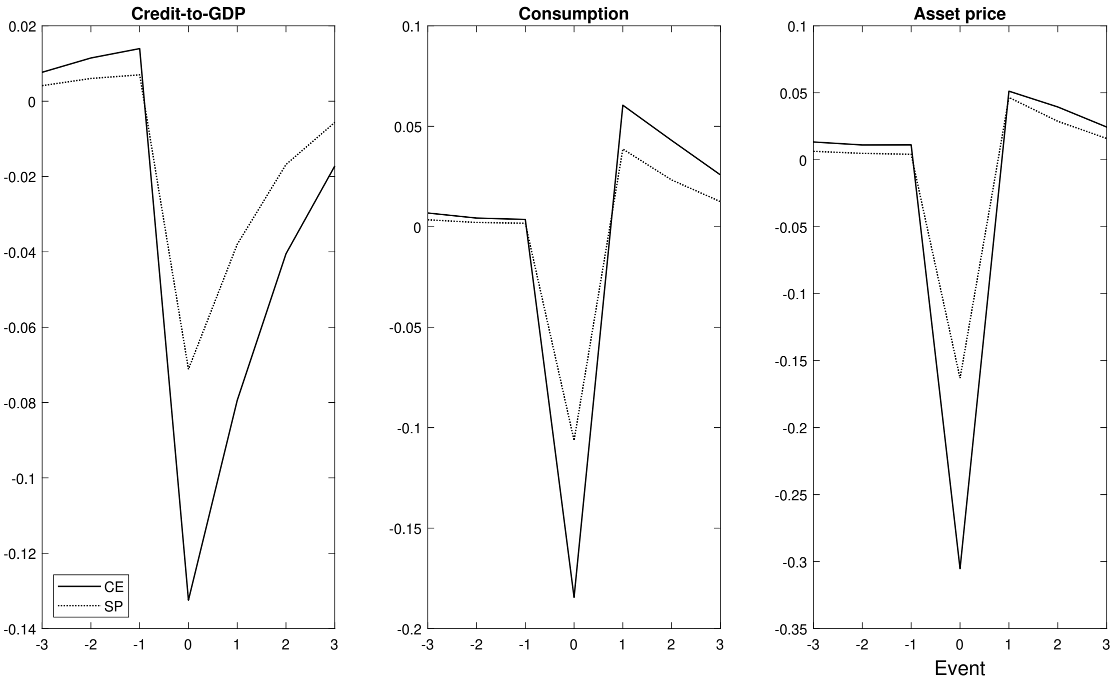
The events are constructed as follows. First, the competitive and planners' equilibria are simulated for 100,000 periods and the Sudden Stop events are identified based on the criteria discussed earlier. Next, we construct 5-year event windows centered around the year when the event materializes and compute averages for each variable across the cross section of events at each date. These steps generate the dynamics plotted in Figure 3. The competitive and the planner's equilibria start from the same level of borrowing in the initial period and go through the same simulated path of shocks for the exogenous variables. The results are presented in terms of deviations from the long-term average.

Figure 3 shows the dynamics for credit-to-GDP, (entrepreneurs') consumption, and asset prices in the competitive (solid line) and social (dashed line) equilibria under a Sudden Stop scenario. The top panels are for the representative-agent economy, while the bottom panels are for the two-agent economy.[20]

When a Sudden Stop episode materializes (solid line in Figure 3), the borrowing ability in both the competitive and planner's equilibria is curtailed, which leads to a drop in credit-to-GDP, consumption, and asset prices. In both representative-agents and two-agent economies, the competitive and planner's equilibria show qualitatively similar dynamics following a Sudden Stop episode, but the magnitudes are somewhat different. The collapse in the asset price in the competitive equilibrium in both economies is substantial, amplifying the drop in credit and consumption via the Fisherian deflation dynamics. By internalizing the pecuniary externalities when the constraint binds, the planner mitigates the effect of Fisherian deflation during Sudden Stops. Compared to the competitive equilibrium, the planner can achieve a correction (drop) in the asset price that is about 14 and 24 percentage points smaller in the representative-agent and two-agent economies, respectively. As a result, the correction in credit-to-GDP (consumption) in the planner's economy is about 6 (8) and 7 (21) percentage points smaller in the representative- and two-agent economies, respectively. It is interesting that the planner can achieve relatively

[20]Section A in the Online Appendix reports the exact numerical values for the changes in the variables reported in Figure 3. Moreover, we report the amplification during Sudden Stops for the representative- and two-agent economies with $\theta^l = 0$, which we do not discuss in the paper, but for which similar conclusion apply.

Representative-agent economy



Two-agents economy

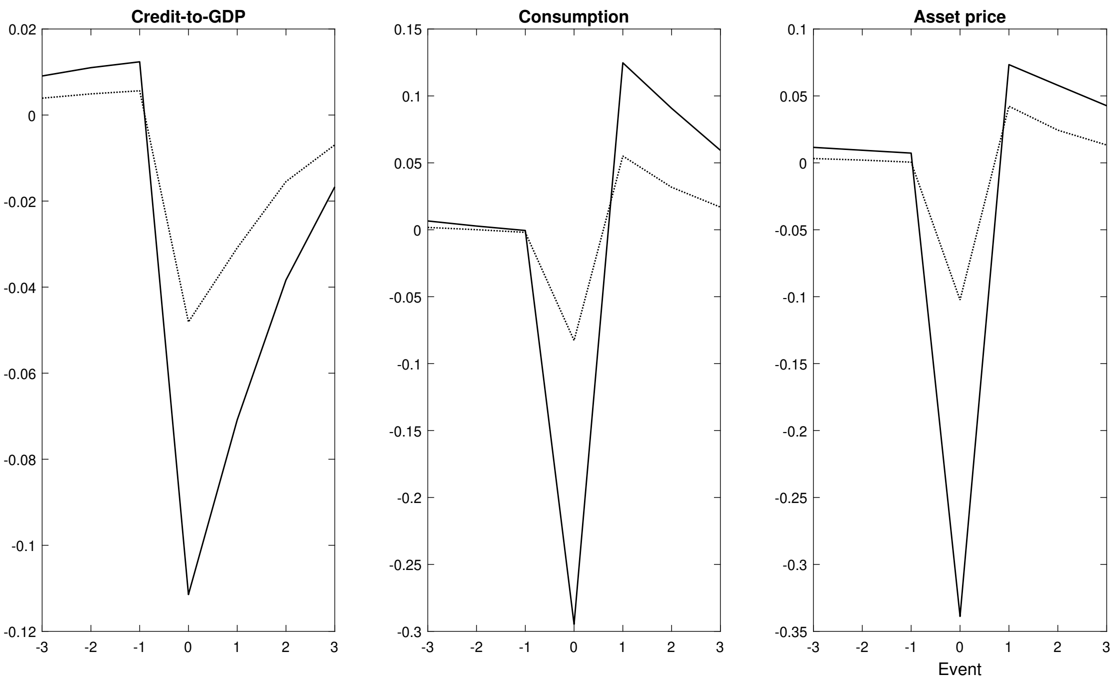


Figure (3) Note: The figure plots responses of credit-to-GDP (new borrowing over GDP), consumption, and asset prices in the representative- and the two-agent economies. The solid line denotes the dynamics of the competitive economy and the dashed line denotes those of the planner during a Sudden Stop episode ($T = 0$ in the figure). The response of credit-to-GDP is in terms of percentage points deviation from the long-term average across all simulations. The other two responses are in terms of percent deviations from their long-term averages.

smaller improvements, especially in the asset price and consumption, in the representative-agent economy. As we discuss below, one reason for the relative difference in magnitudes between the two economies is that in the two-agent economy the planner can use the payroll tax to redistribute resources to entrepreneurs during Sudden Stop episodes.

As we argue, the motive for redistribution is important due to the cross-sectional effects of Sudden Stops. For our calibration, entrepreneurs' and workers' consumption decrease by about 30 and 10 percent, respectively, during a Sudden Stop compared to their long-run averages in the competitive two-agent economy, while in the competitive representative-agent economy consumption drops by 19 percent. Hence, constrained agents are mostly hit, a result also echoed in Villalvazo (2022). The planner accounts for the cross-sectional effects of Sudden Stops and implements policies that result in relatively less severe drops in consumption for entrepreneurs versus workers. As mentioned, the drop in entrepreneurial consumption is about 21 percentage points smaller in the the planner's equilibrium, while the drop in workers consumption is about 3 percentage points larger in the planner's equilibrium indicating the redistribution from workers to entrepreneurs during Sudden Stops. We should note that the type of heterogeneity we consider is not as rich as in Villalvazo (2022) to obtain cross-sectional distributions in the competitive economy, but it is tractable enough to allow a clean normative analysis, absent in his paper. Thus, we focus the rest of our discussion on how the social planner implements the desired redistribution while accounting for the pecuniary externalities from asset prices.

Table (3) Decentralized Representative & Two-agent Economies

	Representative	Two agents
Mean Payroll tax (unconditional)	0.1	-9.9
Mean Payroll tax (conditional on SS)	1.0	-4.4
Mean Tax on borrowing (unconditional)	1.4	1.8
Mean Tax on borrowing (conditional on SS)	-0.1	-0.3
Mean Welfare gains	0.3	1.1

Note: All numbers are in percent. SS=Sudden Stop.

Table 3 reports the average tax rates that decentralize the planners' allocations for the representative-agent and the two-agent economies, unconditional over the simulation horizon and conditional on a Sudden Stop episode. The first notable difference between the taxes in the two economies is that the unconditional average of the payroll tax in the representative-agent economy is close to zero compared to a payroll subsidy of about 10 percent in the two-agent economy.[21] This result corroborates the findings discussed earlier when describing Figure 2, and implies that, for our calibration, the planner would generally like to redistribute resources to workers. Yet, conditional on a Sudden Stop episode, the planner substantially curtails the average payroll subsidy. There are two reasons for doing so. First, during a Sudden Stop episode, the pecuniary externality operating via the wage is amplified, which the planner addresses by reducing the payroll subsidy (increasing the payroll tax). Second, a lower payroll subsidy helps the constrained entrepreneurs, because it implies a smaller redistribution towards workers as the planner recognizes the positive effect of (higher) entrepreneurial consumption on asset prices and on the tightness of the constraint; this latter redistributive motive is quantitatively important and results in a decrease in the payroll subsidy that is much higher than what is required to just address the pecuniary externality operating via the wage. By contrast, the change in the average payroll tax, conditional on a Sudden Stop, is relatively small in the representative-agent economy where the redistributive motive is absent and the payroll tax only tackles the pecuniary externality operating via the wage.

The usefulness of the payroll tax during a Sudden Stop in the two-agent economy is also apparent from the somewhat limited scope of using the borrowing tax to support borrowing during a Sudden Stop episode. Although the planner sets a borrowing tax close to zero in both economies to mitigate the pecuniary externality operating via the asset price in a Sudden Stop, the planner does not introduce big borrowing subsidies to further support borrowing because the optimal policy balances the current and future externalities from binding constraints, which push in opposite directions (see analysis in Section 3.3). Compared to unconditional average tax of borrowing of 1.4 percent and 1.8 percent in the representative-agent and two-agent decentralized economies, the average borrowing tax, conditional on a Sudden Stop, is -0.1 percent and -0.3 percent, respectively. Therefore, given the limitation of levying big borrowing subsidies during a Sudden Stop, using the payroll tax for redistribution makes it an important ex post policy tool.

Nonetheless, the payroll tax is still complementary to the ex ante (macroprudential) borrowing tax and the optimal policy mix includes the joint use of both. In other words, the ability to intervene ex post does not eliminate the need to also intervene ex ante, as the anticipated redistribution can weaken the precautionary motive of entrepreneurs and urge them to borrow more in normal times. Indicative of a weakened precautionary motive is the fact that the plan-

[21] We should note that obtaining a negative payroll tax depends on the way we calibrated workers' parameters in the two-agent economy to keep it close to the representative-agent economy. Doing so resulted in a higher shadow value of income for workers most of the time. Under a different calibration we could have, instead, obtained a positive payroll tax unconditionally. Yet, the (more important) results we describe below on the directional changes in the payroll tax and the way they are used to perform redistribution during and outside Sudden Stops are very general and should continue to hold.

ner levies a slightly higher macroprudential borrowing tax in the two-agent economy than in the representative-agent one in order to discourage further borrowing.

We conclude the analysis by presenting the welfare gains achieved by the planner in the simulated representative-agent and two-agent economies. Table 3 reports the percentage compensating consumption variation such that the social welfare of the competitive equilibrium equals the welfare in the planner's equilibrium. For the two-agent economy, we compute a compensating consumption variation, γ, equally obtained by both agents using the following formula (the formula is similar for the representative-agent economy but only accounts for the consumption of the representative agent):

$$
\begin{aligned}
&E_0 \sum_{t=0}^{\infty} \beta^t \left[U\left((1+\gamma) c_t^{CE} - G(l_t^{CE}) \right) + U\left((1+\gamma) x_t^{CE} \right) \right] \\
=&E_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(c_t^{SP} - G(l_t^{SP}) \right) + U\left(x_t^{SP} \right) \right]. \qquad (40)
\end{aligned}
$$

where the superscripts CE and SP on workers' consumption, labor, and entrepreneurs' consumption stand for the values they take in the competitive and social planner's equilibria, respectively. Then, mean welfare gain is the average γ computed with the ergodic distribution.

As explained earlier, Sudden Stops have a strong adverse effect on consumption and asset prices as well as on the ability to fund working capital for purchasing factors of production. Hence, the lower frequency of Sudden Stops in both the representative-agent and two-agent decentralized economies generates welfare gains from both a reduction in the volatility of consumption and an increase in production efficiency. However, as reported in Table 3, the welfare gains are considerably higher in the two-agent economy than in the representative-agent economy, which is traditionally characterized by low welfare gains from policy intervention.

The representative-agent economy is typically characterized by low welfare gains because absent Sudden Stop events, which are very rare, it behaves a lot like the Real Business Cycle small open economy model.[22] The main reason that welfare gains in the two-agent economy are higher is that the tax system allows for redistribution of resources from one agent to another. As discussed above, the planner decreases substantially the payroll subsidy in the two-agent economy during a Sudden Stop episode to help entrepreneurs and support asset prices. The probability of a Sudden Stop is also somewhat lower compared to the representative-agent economy, owing partly

[22] The welfare gains we report for the representative-agent economy are consistent with those reported in Bianchi and Mendoza (2018). Also, note that the comparison of the welfare gains between the two economies should not be misinterpreted as a comparison of their absolute levels of welfare, which would not be meaningful.

to the ability to intervene ex post and mitigate Sudden Stop episodes. Thus, it is reasonable to obtain somewhat higher welfare gains in the two-agent economy as both less frequent and severe Sudden Stops imply a lower variability of consumption over time, between normal and crisis times, and higher productive efficiency.

Yet, there is an additional subtle reason why welfare gains are considerably higher, which goes beyond the effect of policy on Sudden Stops. In particular, the planner can use the payroll tax for redistribution even in normal times when the constraint does not bind, which allows to further reduce consumption volatility of the two agents and to redistribute resources more efficiently. Indicative of this is the fact that the correlation of entrepreneurial consumption and the payroll tax is close to minus one conditional on Sudden Stop episodes in both the representative-agent and two-agent economies, implying that when entrepreneurial consumption goes down, the payroll tax increases to discourage labor demand. On the contrary, outside Sudden Stop episodes, the correlation is close to zero (-0.05) in the representative-agent economy, whereas it remains negative, but smaller (-0.39) in the two-agent economy. This result suggests that the planner in the two-agent economy continues to rely on the payroll tax during normal times.[23]

Lastly, the evidence showing the use of the payroll tax to help workers outside Sudden Stops in the two-agent economy may also explain why the probability of a binding constraint is slightly higher in the social planner's than in the competitive equilibrium despite the big decrease in the probability of a Sudden Stop in the former as reported in Panel A in Table 2. A redistribution of resources from entrepreneurs to workers could tighten the collateral constraint while the reversal in borrowing is still not high enough to be classified as a Sudden Stop. As a result, because of this redistribution of resources, the probability of a collateral constraint binding may be slightly higher in the social planner's than the competitive equilibrium in the two-agent economy, while the probability of a Sudden Stop is still considerably lower.

4.4 Payroll tax and planner's weight on workers

The results presented above are for equal weights for entrepreneurs and workers in the social welfare function, i.e. $\omega = 1$. In this section, we perform comparative statics with respect to ω and show how the policy functions for the payroll tax as well as the average payroll taxes over

[23]The non-zero correlation absent a Sudden Stop in the representative-agent economy is due to the fact that the collateral constraint may bind, implying a positive payroll tax, even if the fall in external borrowing is not high enough to classify these events as Sudden Stops. Indeed, if we consider more generally events that the collateral constraint does not bind, then the correlations of the payroll tax with entrepreneurial consumption are 0 and -0.40 in the representative- and two-agent economies, respectively, highlighting further our result.

the simulation period change. We present only result for the payroll tax for conciseness and to highlight the redistribution between workers and entrepreneurs. The rest of the results are qualitatively the same as for $\omega = 1$.

Figure 4 presents the optimal payroll tax in the two-agent economy for high productivity and favorable financial conditions (left panel) and low productivity and unfavorable financial conditions (right panel) in the representative-agent (solid line) and the two-agent (dashed line) economies for three different levels of ω. As expected, the payroll tax is higher (subsidy is smaller) for lower weight ω put on workers as the motive of the planner to redistribute resources to workers is weaker, and vice versa.

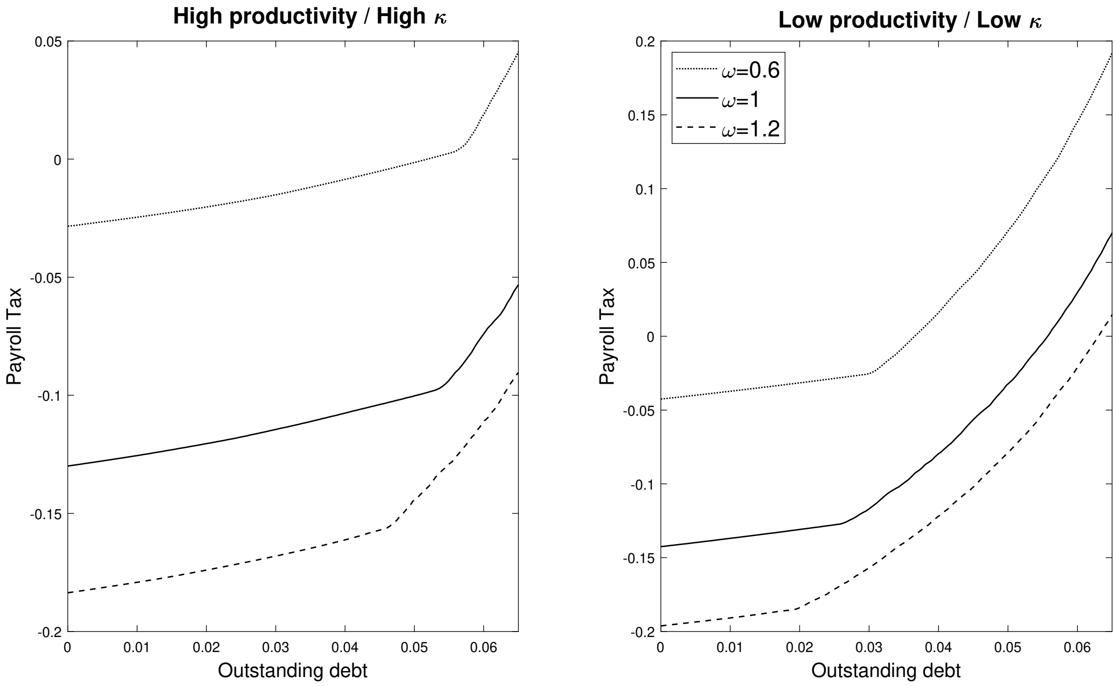


Figure (4) Note: The figure plots the payroll tax rates as functions of outstanding debt b_t, for $z_t = z^h$ and $\kappa_t = \kappa^h$ (left panel) and $z_t = z^l$ and $\kappa_t = \kappa^l$ (right panel) for three levels of ω.

Table 4 reports the unconditional and conditional on a Sudden Stop average payroll for the simulated two-agent economy under different ω's. In all cases, the average payroll tax is higher (subsidy is smaller) conditional on a Sudden Stop as the planner would like to engage is smaller redistribution to workers in order to support entrepreneurs and mitigate the Fisherian deflation dynamics. For $\omega = 0.6$, the payroll tax is positive signalling an actual redistribution of resources from workers to entrepreneurs, while it is even more negative for $\omega = 1.4$. These results are not surprising. Yet, they highlight the usefulness of the payroll tax to implement a redistribution between workers and entrepreneurs.

Table (4) Payroll taxes in two-agent economy for different ω's

	$\omega = 0.6$	$\omega = 1$	$\omega = 1.4$
Mean Payroll tax (unconditional)	0.3	-9.9	-15.6
Mean Payroll tax (conditional on SS)	7.8	-4.4	-11.2

Note: All figures are in percent. SS=Sudden Stop.

4.5 Labor shocks

To further highlight the implications of the two-agent framework for optimal policy, we extend the model to include a labor supply shock, which is modeled as a shock to the disutility from supplying labor. This is meant to capture that during certain periods of time, workers may be more or less willing to supply labor. The shock has two main effects on the equilibrium outcome. First, it directly affects workers welfare. Second, through production and general equilibrium effects, it affects entrepreneurs' welfare and the asset price. The planner would like to alleviate the adverse effects of a negative labor supply shock by using the taxes on borrowing and payroll. As we will show below the use of the payroll tax is asymmetric with respect to the tightness of the borrowing constraint in the two-agent economy when comparing the states with and without the labor shock. This result further speaks to the importance of the redistribution channel that we have highlighted earlier.

In presence of the labor supply shock, workers preferences become

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - G(h_t; \psi_t)), \tag{41}$$

with ψ_t denoting the shock that follows a two-state, regime-switching Markov process, which can take values, ψ_n and ψ_l, representing the value of the shock during "normal times" and "crisis times," respectively. The transition probabilities are given by $P(\psi_{t+1} = \psi_l | \psi_t = \psi_n) = 5\%$ and $P(\psi_{t+1} = \psi_l | \psi_t = \psi_l) = 0\%$, i.e. during normal times there is a 5 percent probability of a negative labor supply shock after which the economy transitions back to the normal state for the labor supply shock. ψ_n takes the same value as in the benchmark economy equal to 0.352, while we calibrate $\psi_l = 0.369$ such that labor supply declines by 4% on average in the simulated economy when the negative labor supply shock materializes. The calibration for the probability and the magnitude of the labor supply shocks are chosen to match the the probability of a 4 percent deviation from a Hodrick-Prescott trend computed from annual OECD data on total hours worked for the period 1980-2019. Such events belong to the 5th percentile of the distribution of percent-deviations from a Hodrick-Prescott trend.

Table 5 reports the statistics for the simulated two-agent economy with the labor shock. The qualitative results we described earlier for the economy without the possibility of labor shocks continue to hold. Yet, the realization of a labor shock amplifies the magnitude of the effects as evident by the larger drops in credit as a percentage of GDP, entrepreneurial consumption, and asset prices. In short, the labor shock amplifies the Fisherian deflation dynamics. The effects are similar for the representative-agent economy, which we do not report for conciseness. More generally, the labor supply shock propagates, on average, similarly to other adverse shocks in both economies, but as we show below the optimal policy response exhibits some interesting asymmetries in the two-agent economy.

Table (5) Statistics for two-agent economy with labor shocks

	CE	SP
Probability of Sudden Stop	4.1	2.4
Probability of Sudden Stop & Labor Shock	0.4	0.2
Drop in credit-to-GDP in Sudden Stop	-11.2	-4.6
Drop in credit-to-GDP in Sudden Stop & Labor Shock	-12.6	-4.4
Drop in entr. consumption in Sudden Stop	-29.6	-8.0
Drop in entr. consumption in Sudden Stop & Labor Shock	-36.9	-10.3
Drop in asset price in Sudden Stop	-34.6	-10.3
Drop in asset price in Sudden Stop & Labor Shock	-41.4	-12.6
Payroll tax (unconditional)	-	-10.1
Payroll tax (conditional on Sudden Stop)	-	-5.1
Payroll tax (conditional on Sudden Stop & Labor Shock)	-	-4.4
Borrowing tax (unconditional)	-	1.9
Borrowing tax (conditional on Sudden Stop)	-	0.13
Borrowing tax (conditional on Sudden Stop & Labor Shock)	-	-0.03
Mean welfare gains	-	1.2

Note: All figures are in percent apart from credit-to-GDP which is expressed in percentage points as the ratio of the long-term loan to GDP. CE=Competitive Economy; SP=Social Planner.

Figure 5 presents the optimal payroll (left panel) and borrowing (right panel) taxes as functions of outstanding debt for high productivity and favorable financial conditions. The top left and right charts correspond to the representative-agent economy, while the bottom left and right charts correspond to the two-agent economy.

For the representative agent, the effect of a labor shock increases the tightness of the borrowing constraint, requiring a somewhat higher payroll tax to tackle the pecuniary externality operating through wages. By contrast, in the two-agent economy, the planner has an additional degree of freedom as the payroll tax can be used to perform redistribution on top of tackling the pecuniary externality operating through wages. Looking at the bottom-left figure, we can observe that the planner implements a higher payroll subsidy (more negative payroll tax) when the labor supply shock hits but the borrowing constraint is not yet binding, which corresponds to debt levels before the kink point. Doing so allows the planner to support the workers who are directly affected by the shock. By contrast, when the borrowing constraint binds (debt levels after the kink), transferring resources from entrepreneurs to workers has a negative impact on asset prices. The planner takes these effects into consideration and limits the support to workers by reducing the payroll subsidy.[24] Table 5 reports the unconditional, conditional on a Sudden Stop event, and conditional on a Sudden Stop and negative labor shock event, averages of the payroll tax over the simulation horizon for the two-agent economy. As expected, the average payroll tax conditional on a Sudden Stop and negative labor shock is the highest, which again highlights the use of this policy instrument for redistributive purposes.

In contrast to its effect on the payroll taxes, the labor shock does not introduce an asymmetry in the optimal borrowing tax between the two economies. In both the representative-agent and two-agent economies, the tax on borrowing is higher in presence of a labor shock and a non-binding borrowing constraint compared to when labor shock events are absent. The reason for this is that the planner would like to lean against future pecuniary externalities, which can be interpreted as a means to preempt the financial amplification induced by the labor shock should the collateral constraints bind in the future. Similarly, when the constraint binds the planner cuts the borrowing tax by more when the labor shock hits in order to help alleviate its negative effects.

5 Conclusion

We study how agent heterogeneity alters the optimal policy recommendations to tackle Sudden Stops compared to a representative-agent framework. To this end, we extend the model of

[24] Note that the asymmetric effect does not simply arise from the pecuniary externality operating through wages between the labor and no-labor shock states. The asymmetry is still present even after netting out the part of the payroll tax that tackles this pecuniary externality(not shown in the charts). Thus the redistribution is the important driver for the asymmetric effect in the payroll tax.

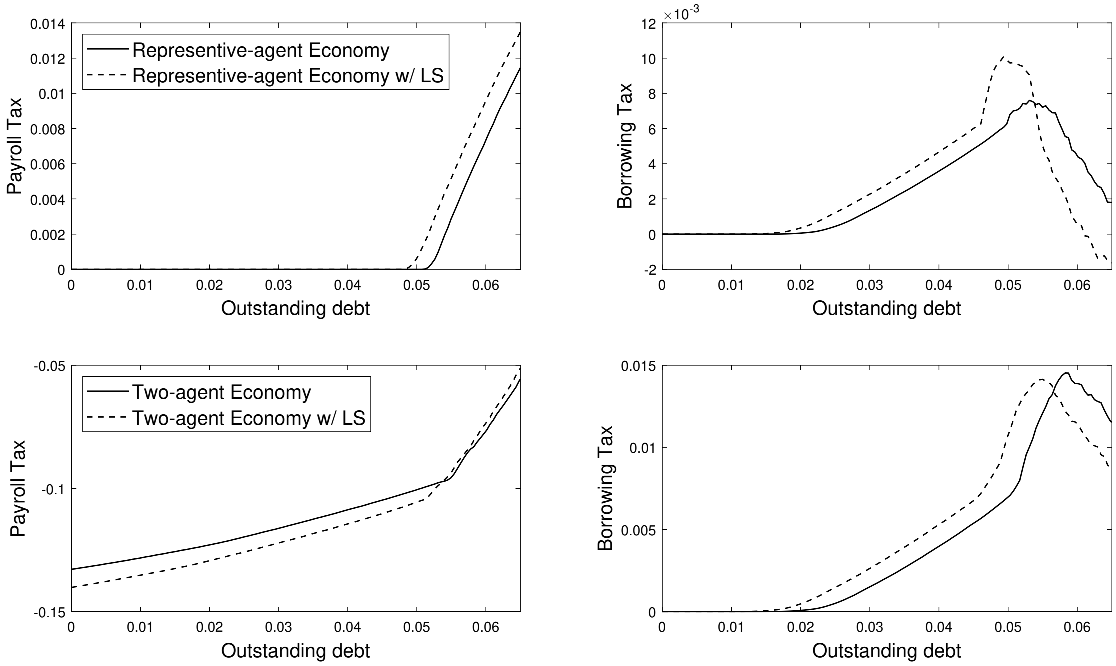


Figure (5) Optimal Policy and Labor Shocks

Bianchi and Mendoza (2018) to distinguish between workers and entrepreneurs, and perform optimal Ramsey policy analysis. Our paper shows that this distinction has important implications for optimal policy.

Our normative analysis indicates that there is a distributive externality on top of the typical pecuniary externality that operates via the price of the asset used as collateral. The novelty of our paper is to show that the distributive externality interacts with the pecuniary externality in a meaningful way during Sudden Stops, which has implications for optimal policy. While in tranquil times the motive for redistribution is driven by the relative marginal utilities of consumption, the planner additionally favors entrepreneurs during Sudden Stops to boost asset prices and mitigate Fisherian deflation.

Our analysis highlights the need for both ex ante and ex post interventions, in terms of a macroprudential tax on borrowing and a payroll tax, respectively. The former aims to limit borrowing during "good times," when financing constraints are loose, whereas the latter aims to reallocate resources to entrepreneurs and support asset prices during "bad times," when financing constraints are tight.

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Appendix

A Derivation of the Collateral Constraint

The collateral constraint can be derived from a renegotiation of debt problem between borrowers and lenders. At the beginning of period t, after previous period borrowing, b_t, has been repaid, the total liabilities of the borrower are $b_{t+1}/R + \theta^l w_t l_t + \theta^v p^v v_t$, which comprise of the inter-period debt plus the intra-period debt to fund working capital. Before production and investment in new capital take place, the borrower can decide to divert the borrowed funds. Using the threat to divert, the borrower can try to renegotiate the debt. But, if diversion does not happen at the specific point in time, there will be no opportunity for the borrower to divert within the same period. If the lender does not agree to renegotiate the debt, it can seize and liquidate the collateral of the borrower, which yields $\kappa_t q_t k_t$. κ_t is the liquidation value of the borrower's assets and captures the financial conditions in the economy. Following Jermann and Quadrini (2012), we assume that the borrower has full negotiation power.

Value from renegotiation. If the borrower decides to renegotiate the debt, the value from renegotiation is given by

$$V^R = \frac{b_{t+1}}{R} + \theta^l w_t l_t + \theta^v p^v v_t - \kappa_t q_t k_t + \beta E_t V(k_{t+1}, b_{t+1}), \tag{42}$$

where the first four terms on the right hand-side of the equation denote the net benefit from debt renegotiation. This is equal to the borrowed funds that can be diverted minus the outside option of lenders, equal to the liquidation value of collateral; borrowers would need to compensate lenders their outside option to avoid liquidation. The last term denotes the continuation value of the entrepreneur as they can gain access to credit markets in the following period.

Value from honoring debt obligations. The value from avoiding renegotiation of debt is given by

$$V^{NR} = \beta E_t V(k_{t+1}, b_{t+1}), \tag{43}$$

which is equal to the continuation value of the entrepreneurs without any renegotiation being attempted and with all obligations being repaid to lenders.

The incentive compatibility constraint requires that the net renegotiation value is smaller than or equal to the value from honoring the debt obligation, $V^R \leq V^{NR}$. This incentive compatibility constraint gives rise to the collateral constraint $b_{t+1}/R + \theta^l w_t l_t + \theta^v p^v v_t \leq \kappa_t q_t k_t$.

Note that for the decentralized economy with payroll taxes, the derivation of the collateral constraint, (7) is the same as above, because we assume that any payroll taxes on $\theta^l w_t l_t$ are paid/settled at the end of the period, so that the borrower does not need to fund them with the working capital loan. As a result, the actual funds that the borrower can divert and that enter into the renegotiation problem do not include $\tau_t^l \theta^l w_t l_t$, where τ_t^l is the payroll tax. Taxes may accrue earlier, but are due and, thus, need to funded at the end of the period. Of course, tax obligation that accrue early may affect how the funds from the liquidation of collateral are split, conditional on diversion, between borrowers and the tax authority. But, the level of borrowed funds that the entrepreneur can divert and the value of the liquidation thread are the same, implying the same collateral constraint described above. The derivation of the collateral constraint is also analogous for the constrained social planner's economy. Hence, the same collateral constraint applies in the competitive and the social planner's equilibria.

B Representative-agent Economy

In this section, we derive the optimality conditions characterizing the representative-agent economy. We first solve for the allocations of the competitive equilibrium and then for those of the planner. Finally, we derive the optimal borrowing and payroll taxes that decentralize the planner's allocations.

B.1 Competitive Economy

The economy is populated by a representative household, who maximizes its utility function subject to a budget constraint, (45)

$$\max_{c_t, l_t} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - G(l_t)), \tag{44}$$

$$c_t = w_t l_t + d_t. \tag{45}$$

$U(\cdot)$ and $G(\cdot)$ are characterized by the same properties outlined in section 2.1. c_t and l_t denote consumption and labor, respectively, at period t. d_t denotes the dividends that households earn at period t from owning shares in the representative firm.

The first order optimality conditions of the household read

$$U_{c,t} = \lambda_t^h, \tag{46}$$

$$G_{l,t} = w_t, \tag{47}$$

where λ_t^h denotes the Lagrange multiplier on the borrowing constraint, (45).

The representative firm, owned by households, maximizes the discounted sum of future dividends subject to budget constraint, (49), and collateral constraint, (50),

$$\max_{b_{t+1},k_{t+1},l_t,v_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_{c,t+1}}{Uc,t} d_t, \tag{48}$$

$$d_t \equiv \frac{b_{t+1}}{R} + q_t k_t + F(z_t, k_t, l_t, v_t) - b_t - p^v v_t - q_t k_{t+1} - w_t l_t, \tag{49}$$

$$\frac{b_{t+1}}{R} + \theta^v p^v v_t + \theta^l w_t l_t \leq \kappa_t q_t k_t, \tag{50}$$

where $\beta^t \frac{U_{c,t+1}}{Uc,t}$ is the stochastic discount factor of the household and $F(\cdot)$ is the production function as defined in section 2.2. b is borrowing, k_t is capital (at t), l_t is labor, and v_t is the intermediate good; q_t is the price of capital (at t), w_t is the wage, and p^v is the price of the intermediate good v; R is the exogenous gross interest rate on inter-period borrowing and z_t is the exogenous productivity process. θ^v and θ^l denote the shares of the intermediate good and labor financed by the intra-period loan.

The first order optimality conditions of the firm read

$$F_{v,t} = p^v(1 + \theta^v \mu_t^f), \tag{51}$$

$$F_{l,t} = w_t(1 + \theta^l \mu_t^f), \tag{52}$$

$$U_{c,t}(1 - \mu_t^f) = \beta E_t U_{c,t+1}, \tag{53}$$

$$q_t U_{c,t} = \beta E_t \left[U_{c,t+1}(q_{t+1} + F_{k,t+1}) + \kappa_{t+1} U_{c,t+1} \mu_{t+1}^f q_{t+1} \right], \tag{54}$$

where μ^f denotes the Lagrange multiplier associated with the collateral constraint, (50).

B.2 Planner's Economy

The planner's optimization problem consists of maximizing the utility function of the household, (44), subject to (i) the economy's resource constraint, obtained by combining the budget constraint of the household, (45), and the budget constraint of the firm, (49), (ii) the collateral constraint, (50), which will incorporate the optimal labor decision of households, (47), to substitute for w_t, and (iii) the implementability constraint, (54), which will incorporate the optimal decision with respect to the intermediate good, v_t, (51), to express μ_{t+1}^f. Equation (52) and (53) are omitted from the planner's problem as policy will be chosen such that they do not represent binding constraints for the planner. Then, the planner's problem reads as follows

$$\max_{c_t,b_{t+1},l_t,v_t,q_t} E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - G(l_t))$$

$$c_t = \frac{b_{t+1}}{R} + F(z_t, 1, l_t, v_t) - b_t - p^v v_t \quad (\lambda_t^{SP,r}) \tag{55}$$

$$\frac{b_{t+1}}{R} + \theta^v p^v v_t + \theta^l G_{l,t} l_t \leq \kappa_t q_t \quad (\mu_t^{SP,r}) \tag{56}$$

$$U_{c,t} q_t = \beta E_t U_{c,t+1} \left\{ F_{k,t+1} + q_{t+1} \left[1 + \frac{\kappa_{t+1}}{\theta^v} (\frac{F_{v,t+1}}{p^v} - 1) \right] \right\} \quad (\xi_t^r), \tag{57}$$

where Lagrange multipliers associated with each constraint are given in parenthesis. The optimality conditions of the maximization problem are as follows

$$\lambda_t^{SP,r}{}_t = U_{c,t} - \xi_t^r U_{cc,t} q_t, \tag{58}$$

$$\lambda_t^{SP,r}{}_t = \beta R (\lambda_{t+1}^{SP,r} - \xi_t^r \Omega_{t+1}) + \mu_t^{SP,r}, \tag{59}$$

$$U_{c,t} G_{l,t} = \lambda_t^{SP,r} F_{l,t} - \theta^l \mu_t^{SP,r} (G_{ll,t} l_t + G_{l,t}) + \xi_t^r U_{cc,t} G_{l,t} q_t, \tag{60}$$

$$\mu_t^{SP,r} = \frac{\lambda_t^{SP,r}}{\theta^v} (\frac{F_{v,t}}{p^v} - 1), \tag{61}$$

$$\xi_t^r = \frac{\kappa_t \mu_t^{SP,r}}{U_{c,t}}. \tag{62}$$

To derive an expression for the optimal labor decision that is similar to equation (30), we proceed as follows. First, we combine equation (61) and (51) to establish a link between $\mu_t^{SP,r}$ and μ_t^f, given by $\mu_t^{SP,r} = \lambda_t^{SP,r} \lambda_t^f$. Then, we substitute this expression and equation (58) in (60) to obtain

$$F_{l,t} - G_{l,t}(1 + \theta^l \mu_t^f) = \theta^l \mu_t^f G_{ll,t} l_t. \tag{63}$$

B.3 Decentralized Economy and Optimal Tax Rates

With the policy instruments in place, the budget constraint of the firm in the decentralized representative-agent economy takes the following form

$$d_t \equiv \frac{b_{t+1}}{R} + q_t k_t + F(z_t, k_t, l_t, v_t) - (1 + \tau_{t-1}^{b,r}) b_t - p^v v_t - q_t k_t - (1 + \tau_t^{l,r}) w_t l_t + T_t^{b,r} + T_t^{l,r}, \tag{64}$$

where $\tau^{b,r}$ is the tax on borrowing, $\tau^{l,r}$ is the payroll tax; $T_t^r = \tau_{t-1}^{b,r} b_t + \tau^{l,r} l_t$ are the rebates from the tax on borrowing and payroll.

Moreover, the Euler condition with respect to borrowing and the optimal labor demand decisions of the firm in the decentralized economy, respectively, become

$$U_{c,t}(1-\mu_t^f) = \beta R(1+\tau_t^{b,r}) E_t U_{c,t+1}, \tag{65}$$

$$F_{l,t} = w_t(1+\tau_t^{l,r} + \theta^l \mu_t^f). \tag{66}$$

All other remaining conditions are the same as outlined in section B.1.

The optimal payroll tax can be derived by combining the planner's optimal labor decision, (63), with the corresponding condition in the the Online Appendix economy that incorporates the tax rate, (66). The payroll tax in the representative-agent framework then reads

$$\tau_t^{l,r} = \theta^l \mu_t^f \frac{G_{ll,t}}{G_{l,t}} l_t. \tag{67}$$

The tax on borrowing and the macroprudential tax take the following forms, respectively, and can be derived following the same steps outlined in section 3.3

$$\tau_t^{b,r} = \frac{1}{\beta R E_t U_{c,t+1}} \left[\mu_t^{SP,r} - U_{c,t}\mu_t^f + \xi_t^r U_{cc,t} q_t - \beta R \xi_t^r E_t \Omega_{t+1}\right] - \frac{1}{E_t U_{c,t+1}} E_t \xi_{t+1}^r U_{cc,t+1} q_{t+1}, \tag{68}$$

$$\tau_t^{MP,r} = -\frac{1}{E_t U_{c,t+1}} E_t \xi_{t+1}^r U_{cc,t+1} q_{t+1}. \tag{69}$$

C Numerical Algorithm

This section outlines the steps for the numerical algorithm for computing globally the competitive and social planner's equilibria in the two-agent economy without labor supply shocks. The steps are similar under the presence of labor supply shock, which simply requires to expand the exogenous state-space. The steps for the representative-agent economy are also similar but simpler (see Bianchi and Mendoza, 2018).

Competitive equilibrium. We solve for the competitive economy (CE) using an Euler-equation iteration algorithm. In each iteration, we solve the system of equations presented below in a recursive form for each of 900 grid points: 150 values of debt (b), and 6 states (3 states for productivity $\times$ 2 states for pledgeable fraction of collateral). Formally, we solve for the policy functions $\{\tilde{b}(b,\Xi), x(b,\Xi), c(b,\Xi), l(b,\Xi), q(b,\Xi), w(b,\Xi), v(b,\Xi), \mu(b,\Xi)\}$, where Ξ is the tuple of exogenous state variables, such that the equilibrium conditions below are satisfied

$$x(b,\Xi) + b + p^v v(b,\Xi) + w(b,\Xi)l(b,\Xi) = F(z,1,v(b,\Xi),l(b,\Xi)) + \frac{\tilde{b}(b,\Xi)}{R}, \tag{70}$$

$$\frac{\tilde{b}(b,\Xi)}{R} + \theta^v p^v v(b,\Xi) + \theta^l w(b,\Xi)l(b,\Xi) \leq \kappa(b,\Xi)q(b,\Xi), \tag{71}$$

$$\mu(b,\Xi) = 1 - \beta R E_{\Xi'|\Xi} \frac{U_x(x(b',\Xi'))}{U_x(x(b,\Xi))}, \tag{72}$$

$$\begin{aligned} q(b,\Xi)U_x(x(b,\Xi)) = {} & \beta E_{\Xi'|\Xi}[U_x(x(b',\Xi'))\times \\ & (q(b',\Xi') + F_k(z',1,v(b',\Xi'),l(b,\Xi)) + \kappa(b',\Xi')\mu(b',\Xi')q(b',\Xi'))], \end{aligned} \tag{73}$$

$$c(b,\Xi) = w(b,\Xi)l(b,\Xi), \tag{74}$$

$$F_l(z,1,v(b,\Xi),l(b,\Xi)) = w(b,\Xi)(1+\theta\mu(b,\Xi)), \tag{75}$$

$$G_h(l(b,\Xi)) = w(b,\Xi), \tag{76}$$

$$F_v(z,1,v(b,\Xi),l(b,\Xi)) = p^v(1+\theta\mu(b,\Xi)), \tag{77}$$

where $\tilde{b}(b,\Xi)$ is the new borrowing, and y' denotes the next period realization of variable y.

The algorithm proceeds in the following steps:

1. For each grid point in b, conjecture future policy functions $b' = \tilde{b}(b,\Xi)$, $c(b',\Xi')$, $q(b',\Xi')$, $l(b',\Xi')$, $v(b',\Xi')$, $\mu(b',\Xi')$. For the first iteration use a guess. For further iterations define future polices as the solution to the current policy functions from the previous iteration (see step 3 below).

2. Taking future policies from step 1 as given, for each grid point in b, solve (70)-(77) to obtain current policy functions $\tilde{b}(b,\Xi)$, $x(b,\Xi)$, $c(b,\Xi)$, $q(b,\Xi)$, $l(b,\Xi)$, $v(b,\Xi)$, $\mu(b,\Xi)$. We distinguish between cases that the collateral constraint binds and does not bind in the present:

   i. First, assume that the collateral constraint (71) binds and solve for the current policy functions. Then, check that $\mu(b,\Xi) > 0$ using equation (72). If this is true, proceed to step 3; otherwise move to substep ii.

   ii. If for a given grid point the collateral constraint in the present does not bind, solve the system of equations above for the current policy functions by setting $\mu(b,\Xi) = 0$.

3. Use the optimal policy functions from substeps 2-i or 2-ii to update the (conjectured) future policy functions in step 1.

4. Stop when convergence is achieved, i.e. when for two consecutive iterations $i-1$ and i it holds that $\sup_{b,\Xi} ||y_i(b,\Xi) - y_{i-1}(b,\Xi)|| < \varepsilon$, where $y = \tilde{b}, c$. We set $\varepsilon = 10^{-3}$, but we also confirm that the results do not change if we choose a stricter convergence criterion.

Social planner. We solve for the policy functions of the social planner (SP) using a value function iteration, nested fixed point algorithm. In each iteration we solve for the value function using a fixed-grid optimization procedure as an inner loop. In the outer loop, we update future policies given the solution to the Bellman equation from the inner loop. As in Klein, Krusell and Ríos-Rull (2008) and Bianchi and Mendoza (2018), this procedure delivers time-consistent policies. The detailed steps are described below.

The value function representation of the SP's optimization problem is:

$$V(b,\Xi) = \max_{\tilde{b},c,x,w,v,l,q,\mu} \big(\omega U(c(b,\Xi) - G(l(b,\Xi))) + U(x(b,\Xi)) + \beta E_{\Xi'|\Xi}[V(b',\Xi')]\big) \tag{78}$$

subject to (79)-(83):

$$x(b,\Xi) + b + p^v v(b,\Xi) + w(b,\Xi)l(b,\Xi) = F(z,1,v(b,\Xi),l(b,\Xi)) + \frac{\tilde{b}(b,\Xi)}{R} \tag{79}$$

$$\frac{\tilde{b}(b,\Xi)}{R} + \theta^v p^v v(b,\Xi) + \theta^l w(b,\Xi)l(b,\Xi) \le \kappa(b,\Xi)q(b,\Xi) \tag{80}$$

$$q(b,\Xi)U_x(x(b,\Xi)) = \beta E_{\Xi'|\Xi}[U_x(x(b',\Xi'))\times$$
$$(q(b',\Xi') + F_k(z',1,v(L',\omega'),l(b,\Xi)) + \kappa(b',\Xi')(F_v(z',1,v(b',\Xi),l(b,\Xi))/p^v - 1)/\theta q(b',\Xi'))], \tag{81}$$

$$G_h(l(b,\Xi)) = w(b,\Xi), \tag{82}$$

$$c(b,\Xi) = w(b,\Xi)l(b,\Xi). \tag{83}$$

Moreover, use the optimality conditions (21)-(26) along with (9) and (27) to derive the following policy functions:

$$\lambda^{SP,w}(b,\Xi) = \omega U_c(c(b,\Xi) - G(l(b,\Xi))), \tag{84}$$

$$\lambda^{SP.e}(b,\Xi) = \frac{U_x(x(b,\Xi))}{1 + \kappa(b,\Xi)\frac{F_v(z,1,v(b,\Xi),l(b,\Xi))/p^v - 1}{\theta}\frac{U_{xx}(x(b,\Xi))}{U_x(x(b,\Xi))}q(b,\Xi)}. \tag{85}$$

The algorithm proceeds in the following steps:

1. In the outer loop, define future policies $V(b',\Xi'),\tilde{b}(b',\Xi')$, $x(b',\Xi')$, $c(b',\Xi')$, $q(b',\Xi')$, $w(b',\Xi')$, $l(b',\Xi')$, $v(b',\Xi')$, as the solution to current policy functions from the previous iteration (see step 3 below) or the policy functions from the CE solution for the first iteration. Similarly, take $l(b,\Xi)$ from previous iteration (or CE solution for the first iteration) as the new conjecture and use it to compute conjectures for $w(b,\Xi)$, $\lambda^{SP,w}(b,\Xi)$, and $\lambda^{SP,e}(b,\Xi)$, using (82), (84), and (85).

2. In the inner loop, for each grid point of b, solve for new policy functions $V(b,\Xi)$, $\tilde{b}(b,\Xi)$, $x(b,\Xi)$, $c(b,\Xi)$, $v(b,\Xi)$ that satisfy (78) - (83) given future policies as well as the last conjecture for $l(b,\Xi)$, $w(b,\Xi)$, and $q(b,\Xi)$, from the outer loop (step 1). The objective is to find the level of $\tilde{b}(b,\Xi)$ that maximizes (78). We distinguish between cases that the collateral constraint (80) binds or not:

    i. Assume that the collateral constraint does not bind and set the Lagrange multiplier on (80), which yields $(F_v(z,1,v(b,\Xi),l(b,\Xi))/p^v - 1) = \mu(b,\Xi) = 0$. Using this condition and the conjecture for $l(b, \Xi)$ and $w(b, \Xi)$ from the outer loop in step 1, solve for $v(b, \Xi)$. Then, using this solution for $v(b, \Xi)$, and given conjectures for $V(b', \Xi')$, $l(b, \Xi)$, and $w(b, \Xi)$ from the outer loop in step 1, calculate corresponding values of $x(b, \Xi)$ and $c(b, \Xi)$ satisfying equations (79) and (83), for each point on the subgrid of 5000 values for $\tilde{b}$. Then, choose the value for $\tilde{b}(b, \Xi)$ with the highest $V(b, \Xi)$ among the many grid points: $\tilde{b}$ matters for $V(b, \Xi)$ not only because it determines current utility $\omega U(c(b, \Xi) - G(l(b, \Xi))) + U(x(b, \Xi))$, but also because it is the future state variable, i.e. $b' = \tilde{b}(b, \Xi)$. Thus, its choice determines the level of the continuation value $V(b', \Xi')$. The policy function $V(b', \Xi')$ assigning a value for different values b' is taking as given from the outer loop in step 1. But, in the inner loop in step 2, we choose the value of b'—$\tilde{b}$—that maximizes the sum of current utility and the continuation value. Using the same conjectures for $q(b, \Xi)$, $l(b, \Xi)$, and $w(b, \Xi)$ as well as the new solution for $v(b, \Xi)$, we can compute the maximum possible value for $\tilde{b}$, denoted by $\sup \tilde{b}$ using the collateral constraint 80 and assuming it binds for $\sup \tilde{b}$. If the computed $\tilde{b}$ that maximizes (78) is less than $\sup \tilde{b}$, proceed to step 3; otherwise proceed to substep ii. Note that $q(b, \Xi)$ does not matter for deriving the optimal $\tilde{b}$ when the collateral constraint does not bind, but it is important to verify that the constraint indeed is slack.

ii. Assume that the collateral constraint binds. For the same subgrid of 5000 values for $\tilde{b}$ as in step 2i and given conjectures for $V(b', \Xi')$, $q(b, \Xi)$, $l(b, \Xi)$, and $w(b, \Xi)$ from the outer loop, compute, for each grid point, the corresponding $v(b, \Xi)$, $x(b, \Xi)$, and $c(b, \Xi)$, from (80), (79), and (83). Then, choose the value for $\tilde{b}(b, \Xi)$ with the highest sum of current utility and the continuation value among the many grid points. Finally, compute the Lagrange multiplier on collateral constraint (in the decentralized equilibrium) from $\mu(b, \Xi) = (F_v(z, 1, v(b, \Xi), l(b, \Xi))/p^v - 1)$ and restrict it to be non-negative.

3. Derive a new $l(b, \Xi)$ as a solution to optimality condition (24) using the policy functions for $\lambda^{SP,w}(b,)$ and $\lambda^{SP,e}(b, \Xi)$ from step 1 and the values for $v(b, \Xi)$ and $\mu(b, \Xi)$ from step 2. Use the new $l(b, \Xi)$ to update the conjectures for $c(b,)$, $x(b, \Xi)$, $w(b, \Xi)$, $q(b, \Xi)$.

4. Stop when convergence is achieved, i.e. when for two consecutive iterations $i - 1$ and i it holds that $\sup_{b,\Xi} ||V_i(b, \Xi) - V_{i-1}(b, \Xi)|| < \varepsilon$, where $\varepsilon = 10^{-3}$. Otherwise, move to step 1.

PUBLICATIONS

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