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Financial Amplification of Labor Supply Shocks*

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Abstract

We study how financial frictions amplify labor supply shocks in a macroeconomic model with occasionally binding financing constraints. Workers supply labor to entrepreneurs who borrow to purchase factors of production. Borrowing capacity is restricted by the value of capital, generating a pecuniary externality when financing constraints bind. Additionally, there is a distributive externality operating through wages. The planner's allocation can be decentralized with two instruments: a credit tax/subsidy and a labor tax/subsidy. Labor shocks, such as the COVID-19 shock, amplify the policy responses, which critically depend on whether financing constraints bind or not.

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1 Introduction

The COVID-19 pandemic has resulted in a complete shutdown of many sectors of the economy with adverse effects for employment and production. The nature of the shock resembles a big drop in labor supply, which has been substantial, but in all hope, temporary. At the same time, such supply shock can adversely affect demand through many channels including the financial sector, and in particular financial frictions can play an important role in amplifying the initial shock.[1] For example, a drop in output due to a labor supply shock can weaken borrowers' balance sheets, impeding their ability to obtain financing.

The effect of financial frictions critically depends on financial conditions among which is the willingness of lenders to extend credit. In the aftermath of the COVID-19 outbreak, firms world-wide increased their borrowing to tackle the negative effects of the shock. Li, Strahan and Zhang (2020) show that firms in the US massively increased their borrowing, and the 2020Q2 ECB Bank Lending Survey showed a similar pattern for the Eurozone. However, as the pandemic persists, lenders are expected to impose stricter lending standards resulting in tighter financial conditions; see 2020Q2 ECB Bank Lending Survey and 2020Q2 Senior Loan Officer Opinion Survey on Bank Lending Practices.[2]

To study the impact of pandemic shock under different financial conditions, we extend the Bianchi and Mendoza (2018) model, which features occasionally binding collateral constraints, to account separately for entrepreneurs and workers, as well as to incorporate the possibility of a labor supply shock. The reason for doing so is that we want to study both credit and labor policies, and investigate their distributional impact across the two types of agents. Both entrepreneurs and workers are affected by the labor supply shock; entrepreneurs because their ability to produce and borrow is curtailed, and workers because their labor income goes down.

The labor supply shock—capturing the economic effects from the pandemic—is modeled following Guerrieri, Lorenzoni, Straub and Werning (2020).[3] We opt for this approach as the model herein generates an endogenous decline in demand once collateral constraints bind, hence we refrain from adding an exogenous shock in demand through a preference shock. This assumption seems to square well with the data. Brinca, Duarte and Faria-e Castro (2020) find that two thirds of the drop in hours worked in April 2020 was due to a drop in labor supply, while Bekaert, Engstrom and Ermolov (2020) also find using real-time analysis that two thirds

[1] Admittedly, a drop in demand could also materialize independent of a supply shock if, for example, a rise in uncertainty reduces individuals' willingness to consume and firms' willingness to invest (i.e. this can happen even if no lockdown/shutdown is in place).

[2] Federal Reserve Bank of St. Louis, series DRTSCIS.

[3] In the rest of the paper we refer to labor supply, COVID-19, and pandemic shock interchangeably.

of the drop in 2020Q2 GDP was due to a labor supply shock.[4] Importantly, policies that aim at stimulating borrowing and demand are much harder to justify under supply shocks as Guerrieri, Lorenzoni, Straub and Werning (2020) point out. Our analysis is complementary to theirs as we highlight how labor supply shocks are amplified by financial frictions and how they influence optimal policy rather than derive the conditions under which a labor supply shock generates demand externalities. In our framework the externalities are present even in the absence of a labor supply shock, but can be amplified by it.

In the model, entrepreneurs finance consumption, inputs to production, and investment in new capital by revenues from production and new loans from external financiers. Due to their inability to commit to repay loans, their borrowing capacity is limited by a collateral constraint depending on the market value of private assets. Workers supply labor to entrepreneurs inelastically and do not have any other sources of income apart from the wages they earn. Still, the ability of entrepreneurs to borrow will matter for workers through the equilibrium wage level. In the model, collateral constraints bind endogenously because of negative shocks to productivity, stricter lending standards (imposing stricter loan-to-value ratios), or, particularly in our case, labor supply shocks.

The occasionally binding collateral constraints setup is particularly important as it allows to study the impact of COVID-19 shocks under loose and tight financial conditions, which are captured by non-binding and binding collateral constraints.[5] Intuitively, transitory supply shocks such as the COVID-19 shock would have detrimental effects on current production and welfare, but they could be smoothed out to an extent by inter-temporal borrowing, as long as collateral constraints are loose. However, in a second phase, should the pandemic continue, collateral constraints can become binding because of impaired borrower balance sheets or tightening in lending standards, thereby limiting their borrowing ability while the pandemic persists. In this situation, the original labor supply shock generates financial amplification due to unfavorable financial conditions.

In this framework, two types of externalities are at play, justifying policy intervention. The first is a pecuniary externality operating via the price of the asset used as collateral. This is the same externality identified in Bianchi and Mendoza (2018). The second is a distributive externality that arises due to the difference in the shadow values of labor income and cost

[4]Others have modeled the macroeconomic effects of COVID-19 as a negative shock to the growth rate (for example, Fornaro and Wolf, 2020) or as a negative shock to the utility of consumption (for example, Faria-e-Castro, 2020).

[5]It is important to note the difference between loose and tight lending standards vs loose and tight financial conditions. The former refer to the exogenous imposed loan-to-value (LTV) ratio (strictness of lending standards); the latter refers to the endogenous tightness of the collateral constraint.

between workers and entrepreneurs, and is introduced in the framework by modeling the two agents separately. Corrective taxes (subsidies) on borrowing and labor, set optimally by a Ramsey planner, tackle the pecuniary externality and the distributive externality, respectively.

The pecuniary externality arises as entrepreneurs fail to internalize how borrowing decisions affect asset prices and hence their ability to borrow. The planner internalizes the effects of borrowing decisions on the incidence and tightness of a binding collateral constraint, and chooses a different level of borrowing—by setting a tax (subsidy) on borrowing—to address the pecuniary externality. If collateral constraints do not bind at t, but a COVID-19 shock hits the economy, the planner opts for a tax on borrowing that is higher than the one set in absence of a COVID-19 shock, in order to preempt the financial amplification induced by the COVID-19 shock should the collateral constraint bind in the future.

The distributive externality arises because the shadow value of labor income of workers and the shadow value of labor cost of entrepreneurs are not equalized.[6] By choosing a labor tax (subsidy), the planner can manipulate allocations to induce a price change at which agents trade labor—wages—in order to improve the terms of the transaction of those agents with relatively higher shadow value. As long as the collateral constraint does not bind, the planner would reduce labor taxes or implement a subsidy to redistribute more resources to workers when a COVID-19 shock hits the economy. If the collateral constraint binds, the planner also needs to take into account the pecuniary externality from the borrowing decision.[7]

We calibrate the model economy and solve it quantitatively employing a non-linear, global solution algorithm. In order to assess the model's ability to generate crisis and the effectiveness of the policy instruments at reducing the severity of the crisis, we simulate the economy and examine the behavior across three events: (I) when collateral constraints bind, (II) when a COVID-19 shock hits, and (III) when collateral constraints bind and a COVID-19 shock hits.

Our key quantitative results can be summarized as follows. First, the model generates financial amplification of labor supply shocks. When a labor supply shock hits the economy, agents

[6] Dávila and Korinek (2017) define distributive externalities as the externalities arising when the marginal rates of substitution between dates/states differ across agents, and a planner can improve the allocations by affecting the relative prices at which agents trade. In our framework, workers do not have access to assets and the planner can address the distributive externality by affecting the wage. Hence, considering a utilitarian planner, the externality arises if the per period shadow values of income are not equated across the two agents.

[7] Bianchi (2016) also identifies an externality, which operates through wages, and shows that it can be addressed by a payroll tax. In short, in his model, firms do not internalize how their labor demand affects the equilibrium wage and, thus, over-demand labor and under-invest when (equity-financing) constraints bind. This externality, which is pecuniary in nature, differs from the one we investigate herein for two reasons. First, our externality has distributional implications because we model entrepreneurs and workers as different agents rather than having firms be owned by workers as in Bianchi (2016). This creates a trade-off in setting the optimal labor tax in response to labor supply shocks depending on which agent is favored. Second, in the baseline model we have assumed an inelastic supply of labor, thus the externality described in Bianchi (2016) is absent from our analysis. In the Appendix, we investigate the case of an elastic labor supply and derive the optimal payroll that tackles both the distributive aspect as well as the pecuniary aspect of the externality operating through the wage.

would like to smooth consumption. As long as the collateral constraint is loose, they can do so by increasing borrowing. But when the collateral constraint binds and borrowing ability is curtailed, agents have to cut consumption, which exacerbates the pecuniary externality. Second, the planner's economy is characterized by a lower volatility than the competitive economy across all three events. In particular, the planner's economy experiences a more attenuated drop in borrowing as the planner manages to alleviate the negative effects of the pecuniary externality operating via the asset price.

In terms of optimal policy, our quantitative results suggest that the tax on borrowing used to lean against future pecuniary externalities is higher when a COVID-19 shock hits the economy, which can be interpreted as a means to preempt the financial amplification induced by the labor supply shock should the collateral constraint bind in the future. In absence of a COVID-19 shock, the planner always sets a positive labor tax in our baseline calibration. However, if the economy is hit by a COVID-19 shock while collateral constraints are loose, the planner reduces the tax on labor as workers experience a large drop in income by not being able to supply labor. Yet, if the economy is hit by a COVID-19 shock while the collateral constraint binds, the ability of entrepreneurs to borrow and produce is lower, resulting in lower entrepreneurs' profits compared to workers' labor income, calling for a higher tax on labor.

Literature review—Our paper relates to two strands in the literature. The first is the newly emerged literature on the macroeconomic effects of COVID-19, and the second is the literature studying optimal policy in economies with financial frictions.

In the literature studying the economic effects of COVID-19, Guerrieri, Lorenzoni, Straub and Werning (2020) is mostly related to ours. Like them, we model the pandemic as a labor supply shock, but we focus on the financial amplification of the shock from occasionally binding collateral constraints and derive optimal credit and labor policies to tackle the externalities in the competitive economy. In addition, within this newly emerged literature, a series of recent papers study the macroeconomic effect of COVID-19 pandemic featuring epidemiological dynamics (e.g. SIR models), multi sector economies, and/or network linkages among others (Farhi and Baqaee, 2020; Eichenbaum et al., 2020; Bodenstein et al., 2020; and Acemoglu et al., 2020). Compared to these papers, the framework herein is much simpler and models the COVID-19 shock as an exogenous sudden decline in labor supply. The simplicity of the framework enables us to study and quantify the non-linear dynamics generated by the interaction between the COVID-19 shock and financing constraints. These non-linearities are hard to examine in the more elaborate frameworks mentioned above.

In the literature of optimal policy in models with financial frictions, our paper most closely relates to Bianchi and Mendoza (2018). Other related papers studying the effects of pecuniary externalities arising from financial frictions include Lorenzoni (2008) and Jeanne and Korinek (2020). These papers have mainly focused on credit policies to tackle the externalities. Bianchi (2016) investigates the implication of a pecuniary externality operating via the wage and derives optimal credit and payroll taxes to provide bailout to firms. We differ from these papers because apart from credit policies we also focus on labor policies to address a distributional externality we identify, and because we apply the framework to study the financial amplification of labor supply shocks.

2 Model Economy

The model economy comprises of two types of agents: workers and entrepreneurs. Entrepreneurs are modeled following Bianchi and Mendoza (2018). We extend their model to include workers, who are modeled as hand-to-mouth supplying labor inelastically, in order to study labor policies.[8] The labor supply shock in the economy is modeled as a sudden drop in labor hours. We proceed by outlining the model economy and define the optimality conditions for each agents.

2.1 Workers

The model is populated by a unit mass of identical hand-to-mouth workers whose preferences are represented by the utility function

$$E_t \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1}$$

where c_t denotes consumption and $U(c) = c^{(1-\sigma)}/(1-\sigma)$ is a standard CES utility function with a risk aversion parameter σ. Each worker is endowed with $\bar{h}_t \in \{\bar{h}_n, \bar{h}_p\} > 0$ units of labor that are supplied inelastically, with h_n denoting the labor supply in normal times, while h_p denoting the labor supply during the pandemic. Since agents are hand-to-mouth consumers and supply all their endowed labor, their consumption equals their labor income $w_t\bar{h}_t$ with w_t denoting wages.

[8] We opt for hand-to-mouth workers that cannot pledge their labor income to borrow inter-temporally in order to represent a segment of the population that does not have access to credit markets and smooth consumption over time. Kaplan, Violante and Weidner (2014) document that one-third of all US households live hand-to-mouth. Moreover, Baker, R.A., Meyer, Pagl and Yannelis (2020) argue that households with lower incomes, greater income drops, and lower levels of liquidity display stronger responses to fiscal stimulus measures during the COVID-19 crisis. The model can also be extended to introduce another set of workers with access to credit markets.

We model the COVID-19 shock following Guerrieri, Lorenzoni, Straub and Werning (2020) as a sudden drop in the labor supply. This suggests that due to the pandemic, it is unsafe for some workers to report to work because their job requires close interaction with the public; hence they stay at home either by choice or by governments' imposing quarantines or social distancing policies. Capturing the economic effects of COVID-19 through a labor supply shock makes sense since (i) lockdowns to deal with the pandemic are a direct shock on labor supply, (ii) its persistence, which can be set from low to moderate, determines the speed with which the economy can bounce back (given that the pandemic is of transitory nature), (iii) the reduction in employment is the source rather than the outcome of the recession (unlike a regular business cycle or financial recession), and (iv) the drop in unemployment is larger than in a standard recession.

2.2 Entrepreneurs

The model is populated by a unit mass of identical entrepreneurs whose preferences are denoted by the utility function

$$E_t \sum_{t=0}^{\infty} \beta^t U(x_t), \tag{2}$$

where x_t is consumption. Their utility function takes the same form as the one of workers.

Entrepreneurs produce $y_t = F(z_t, k_t, l_t, v_t)$ each period. $F(\cdot)$ is a Cobb-Douglas production function, which combines labor, l_t, with the stock of capital purchased in the previous period, k_t, and an intermediate good, v_t; z_t is an aggregate productivity shock. Aggregate capital is in unit fixed supply: $K_t = 1$. The intermediate good is traded in competitive world markets at a fixed exogenous price, p^v. The flow budget constraint of entrepreneurs is given by

$$x_t + b_t + p^v v_t + w_t l_t + q_t k_{t+1} = y_t + \frac{b_{t+1}}{R} + q_t k_t, \tag{3}$$

where b_t denotes the beginning-of-period borrowing from one-period non-state contingent bonds issued last period (a negative b_t implies positive net holdings of bonds), q_t is the price of capital, and R is the world-determined gross real interest rate taken as given in the small open economy. Entrepreneurs' consumption, x_t, equals output net of the outlays for the factors of production, v_t and l_t, the net capital expenditure, $q_t(k_{t+1} - k_t)$, and the net debt issuance, $b_{t+1}/R - b_t$.

We assume that entrepreneurs cannot raise equity and that their borrowing decision is limited by a collateral constraint, which can *endogenously* be derived from a limited commitment problem similar to Jermann and Quadrini (2012) and Bianchi and Mendoza (2018). Entrepreneurs obtain two types of loans: An inter-temporal (b_{t+1}/R) and an intra-temporal loan. They need the latter to finance ahead of production a portion $\theta \leq 1$ of the intermediate good purchases and labor wages. Hence, the total liabilities at the beginning of the period comprise of $\theta(p^v v_t + w_t l_t) + b_{t+1}/R$. While b_{t+1} is an inter-temporal loan and bears an interest payment, $\theta p^v v_t + \theta w_t l_t$ does not as it is repaid within the same period. All borrowed funds can be diverted, a situation which is precluded by imposing the following collateral constraint

$$\frac{b_{t+1}}{R} + \theta(p^v v_t + w_t l_t) \leq \kappa_t q_t k_t. \tag{4}$$

Constraint (4) limits the size of total borrowing to a fraction κ_t of the beginning of the period asset holdings.

Entrepreneurs maximize (2) subject to (3) and (4). This maximization problem leads to the following optimality conditions for each date $t = 0, ..., \infty$

$$F_{v,t} = p^v(1 + \theta\mu_t), \tag{5}$$

$$F_{l,t} = w_t(1 + \theta\mu_t), \tag{6}$$

$$U_{x,t}(1 - \mu_t) = \beta R E_t U_{x,t+1}, \tag{7}$$

$$q_t U_{x,t} = \beta E_t[U_{x,t+1}(F_{k,t+1} + q_{t+1}) + \kappa_{t+1} U_{x,t+1} \mu_{t+1} q_{t+1}], \tag{8}$$

where $U_{x,t}\mu_t$ denotes the Lagrange multiplier on the collateral constraint scaled by entrepreneurs' marginal utility and $X_{i,t}$ denote the first derivatives of a function $X(i)$ with respect to a variable i at time t.

The presence of the collateral constraint distorts both the optimal inter- and intra-temporal marginal decisions when binding. Conditions (5) and (6), defining entrepreneurs' optimal choice of the intermediate good and labor, embed an additional cost, i.e. the cost of collateral financing equal to $\theta\mu_t p^v$ and $\theta\mu_t w_t$, respectively . In addition, both Euler equations are distorted. The Euler equation for borrowing (7) implies that the marginal benefit from increasing borrowing today outweighs the expected future marginal cost by an amount equal to the shadow price, μ_t, of relaxing the collateral constraint. Similarly, the Euler equation with respect to capital (8), equating the marginal cost of an extra unit of capital with its marginal benefit, embeds an additional benefit obtained by relaxing the collateral constraint, valued at $\kappa_{t+1} U_{x,t+1} \mu_{t+1} q_{t+1}$. As we will show and discuss later, this equation is at the core of the mechanism through which the pecuniary externality operates: The choice of borrowing and consumption today influence the price of the asset used as collateral, which in turn affects the tightness of the collateral constraint.

2.3 Competitive Equilibrium

The competitive equilibrium in this economy is defined as follows.

Definition 1. *For given initial values of the endogenous state variable, b_0, and exogenous processes $\{z_t, \kappa_t, h_t\}_{t=0}^{\infty}$, a competitive equilibrium for the economy with a collateral constraint is a sequence of allocations $\{c_t, x_t, v_t\}_{t=0}^{\infty}$, an asset profile $\{k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, and a price system $\{q_t, w_t, p^v\}_{t=0}^{\infty}$, such that*

1. *Given the price system $\{q_t, w_t, p^v\}_{t=0}^{\infty}$, the allocations and the asset profile solve workers' and entrepreneurs' optimization problems, and*

2. *Labor, asset and goods markets clear, satisfying conditions*

$$\bar{h}_t = l_t \quad \forall t, \tag{9}$$

$$k_t = 1 \quad \forall t, \tag{10}$$

$$c_t + x_t + b_t + p^v v_t = y_t + \frac{b_{t+1}}{R} \quad \forall t. \tag{11}$$

3 Optimal Policy

In this section we first define the optimization problem of the social planner, and then discuss the properties of the optimal policies that implement the planner's solution.

3.1 Planner's Economy

The policy design follows the Ramsey approach, which consists of the planner choosing policies, prices, and allocations in order to maximize the economy's wide social welfare function. In doing so, the planner has to respect all equilibrium conditions of the competitive economy to ensure that the allocations chosen can be implemented as allocations in the competitive economy.

Unlike in the standard Ramsey literature, where the planner optimally chooses distortionary policies intended to finance government expenditure, the planner here chooses policies to alleviate the inefficiencies arising from pecuniary externalities.

We assume that the planner has access to two types of policy instruments: A tax/subsidy on borrowing (as in Bianchi and Mendoza, 2018) and tax/subsidy on labor. Both instruments are Pigouvian in nature with the tax revenues being rebated lump-sum back to the private agents, T_t. This way the policy has a role to correct a distorted decision that arises because of the presence of the financial frictions.[9]

With the policy instruments in place, the budget constraint of entrepreneurs in the decentralized economy takes the following form

$$x_t + (1 + \tau^b_{t-1})b_t + +p^v v_t + (1 + \tau^h_t)w_t l_t \leq y_t + \frac{b_{t+1}}{R} + T_t, \tag{12}$$

where τ^b_t is the tax on new borrowing b_{t+1} determined at t but levied at $t+1$ when debt is repaid, τ^h_t is the labor tax, and T_t is the lump-sum transfer, which in equilibrium is equal to $\tau^b_{t-1}b_t + \tau^h_t w_t l_t$.

Moreover, the Euler conditions with respect to borrowing and labor become, respectively,

$$U_{x,t}(1 - \mu_t) = \beta R(1 + \tau^b_t)E_t U_{x,t+1}, \tag{13}$$

$$F_{l,t} = w_t(1 + \tau^h_t + \theta\mu_t) \tag{14}$$

All other equilibrium conditions remain the same as outlined in section 2.

In the formulation of the Ramsey problem, we do not impose as constraints the policy distorted optimality conditions, (13) and (14), in the planner's optimization problem since it can easily be shown that τ^b_t and τ^h_t are chosen such that the Lagrange multipliers on these two conditions are zero.[10] Also, note that the optimality condition, (5), with respect to the intermediate good is not distorted by a tax instrument and will hold with equality in the planner's problem. Thus, we use it to solve for the Lagrange multiplier μ_t,

$$\mu_t = \frac{1}{\theta}\left(\frac{F_{v,t}}{p^v} - 1\right), \tag{15}$$

and substitute its value directly into the planner's problem to solve for μ_{t+1} in equation (8).

[9]The planner needs to respect the per-period budget constraint, which means tax transfers are funded within the same period lump-sum. Alternatively, we could allow the planner to borrow inter-temporally (presumably with looser collateral requirements than the private agents) to raise revenues for tax transfers. This modification would strengthen the effects of credit and labor subsidies that we discuss later when collateral constraints bind as agents would not need to finance tax transfers because resources are not subtracted in the same period.

[10]Given that the planner internalizes that $T_t = \tau^b_{t-1}b_t + \tau^h_t w_t l_t$, the only places that the taxes appear are the distorted Euler conditions. In other words, the planner uses (13) and (14) to compute the level of the taxes needed to implement her allocations as a competitive equilibrium, but she is not constrained by these two Euler conditions.

Finally, we assume that the planner does not have the technology to commit to future policies.[11] Therefore, we solve for the optimal time-consistent macroprudential policy, taking into account the effects of the planner's current period choices on future planners' choices. As a result, the planner does not have an incentive to deviate from policy rules of previous social planners.

We follow the utilitarian approach under which the planner wants to maximize the infinite weighted-sum of agents future discounted utilities, $\sum_{t=0}^{\infty} \beta^t[\omega U(c_t)+U(x_t)]$. The relative welfare weight on the worker, ω, is assigned exogenously and we consider alternative values for it in our quantitative exercises.

The planner's maximization problem is given by

$$\max_{c_t, x_t, b_{t+1}, v_t, q_t, w_t} E_t \sum_{t=0}^{\infty} \beta^t[\omega U(c_t) + U(x_t)]$$

$$x_t + b_t + p^v v_t + w_t \bar{h}_t \leq F(z_t, 1, v_t, \bar{h}_t) + \frac{b_{t+1}}{R} \quad (\lambda_t^{SP,e}) \tag{16}$$

$$c_t = w_t \bar{h}_t \quad (\lambda_t^{SP,w}) \tag{17}$$

$$\frac{b_{t+1}}{R} + \theta p^v v_t + \theta w_t \bar{h}_t \leq \kappa_t q_t \quad (\mu_t^{SP}) \tag{18}$$

$$U_{x,t} q_t = \beta R E_t U_{x,t+1} \left[F_{k,t+1} + q_{t+1} + \frac{\kappa_t}{\theta} (\frac{F_{l,t+1}}{p^v} - 1) q_{t+1} \right] \quad (\xi_t) \tag{19}$$

where the Lagrange multipliers associated with each constraint are given in parentheses.

Equation (16) denotes the budget constraint of entrepreneurs after accounting for the Pigouvian taxes/subsidies and lump-sum transfers. Similarly equation (17) denotes the budget constraint of workers. Equation (18) is the economy's collateral constraint, and (19) is the implementability condition of the planner, which reflects the fact that the planner has to respect competitive asset pricing in the economy. It is through this equation that the planner internalizes how private agents' choices affect equilibrium asset pricing. Note that we have substituted for μ_{t+1} appearing in the right-hand side of (19) using (15) for $t+1$.[12] Finally, capital and labor are set to their equilibrium aggregate values, $K = 1$ and $l_t = \bar{h}_t$, respectively.

[11] Bianchi and Mendoza (2018) show that the optimal policy under commitment is time inconsistent since asset prices are determined by a dynamic condition linking present and future (expected) marginal utilities of consumption. They follow the time-consistent approach under which a planner cannot commit at t to the whole path of future policy choices as we also do here.

[12] The Lagrange multipliers on the collateral constraints in the competitive and the planner's problem, μ_t and μ_t^{SP}, are different, but connected in equilibrium as shown below.

The first order optimality conditions of the planner take the following form

$$x_t: \quad \lambda_t^{SP,e} = U_{x,t} - \xi_t U_{xx,t} q_t, \tag{20}$$

$$c_t: \quad \lambda_t^{SP,w} = \omega U_{c,t}, \tag{21}$$

$$b_{t+1}: \quad \lambda_t^{SP,e} = \beta R E_t(\lambda_{t+1}^{SP,e} - \xi_t \Omega_{t+1}) + \mu_t^{SP}, \tag{22}$$

$$v_t: \quad \lambda_t^{SP,e} F_{v,t} = p^v(\lambda_t^{SP,e} + \theta \mu_t^{SP}), \tag{23}$$

$$q_t: \quad \kappa_t \mu_t^{SP} = \xi_t U_{x,t}, \tag{24}$$

$$w_t: \quad \lambda_t^{SP,e} + \theta \mu_t^{SP} = \lambda_t^{SP,w}, \tag{25}$$

where Ω_{t+1} collects all partial derivatives with respect to b_{t+1} on the right-hand side of the capital-Euler equation, capturing the impact of the planner's choice of b_{t+1} on the actions of future planners (reflecting the "time-consistency" nature of the policy rule).[13]

The allocations of the planner and the competitive economy differ in two main respects. First, unlike the private agents, the planner internalizes how consumption and borrowing choices affect asset prices and hence the borrowing ability in states in which the collateral constraint binds. Second, the planner internalizes the difference in the shadow costs of wealth between workers and entrepreneurs and can improve on the allocations by affecting the relative price, i.e. wages, at which agents trade. To outline these differences, we compare the optimality conditions in the two economies.

First we compare the first order condition with respect to entrepreneurs' consumption in the competitive and the planner's economy. The competitive economy condition is $U_{x,t} = \lambda_t$, where λ_t is the Lagrange multiplier on the budget constraint of entrepreneurs (3), while the corresponding condition of the planner is given by equation (20). The key difference between these two equations is that the shadow value of wealth in the planner's solution do not only incorporate the marginal utility from current consumption, but also the amount by which an additional unit of consumption relaxes the collateral constraint through its effect on prices (-$\xi_t U_{xx,t} q_t$). The latter is not accounted for in the competitive equilibrium.[14] Hence the private

[13]Note that $U_{x,t}, F_{k,t+1}, q_{t+1}, \mu_{t+1}$ are all functions of the endogenous state variable b_{t+1} as well as the exogenous state variables at t.

[14]To clarify this point, note that condition (20) shows that there is a positive social benefit from relaxing the implementability constraint at times when the collateral constraint binds at t for the social planner. Moreover, conditions (20) and (24) combined—$\lambda_t^{SP} = U_{x,t} - \kappa_t \mu_t^{SP} U_{xx,t} q_t / U_{x,t}$—show that when the collateral constraint binds, an additional unit of consumption generates a positive marginal social benefit of wealth by raising the equilibrium asset price, which in turn relaxes the collateral constraint.

agents do not internalize how their consumption choice affect the asset price, q_t, as well as the tightness of the collateral constraint. This equation is at the core of the pecuniary externality present in the model.

Next we compare the planner's optimality condition with respect to the intermediate good (23) and the corresponding condition in the competitive economy (5). The only difference between the two is that the former is not scaled by the shadow cost of wealth, while the latter is, since the shadow cost of wealth in the planner's condition incorporates the pecuniary externality. Using (15) and (23) we get the following condition that connects the Lagrange multipliers on the collateral constrained between the competitive economy and the planner's solution

$$\mu_t = \frac{\mu_t^{SP}}{\lambda_t^{SP,e}}. \tag{26}$$

Hence, μ_t and μ_t^{SP} are either both positive or zero.

Next we compare the Euler equation for bonds in the competitive economy to the corresponding equation of the planner, which can be written as follows by combining entrepreneurs' optimal consumption and borrowing decisions, (20) and (22),

$$U_{x,t} = \beta R E_t(U_{x,t+1} - \xi_{t+1}U_{xx,t+1}q_{t+1} - \xi_t\Omega_{t+1}) + \xi_t U_{xx,t}q_t + \mu_t^{SP}. \tag{27}$$

This comparison highlights the pecuniary externalities that operate through the future and current price of capital.

Consider that the collateral constraint does not bind at t, such that the Lagrange multipliers on the collateral constraint in the competitive, $\mu_t = 0$, and the planner's economy, $\mu_t^{SP} = \xi_t = 0$, equal zero. In this case, based on the Euler equations for borrowing, (7) and (27), the planner's economy features a higher marginal cost of borrowing at t than the competitive economy by an amount $\beta R E_t[\xi_t + U_{cc^e,t+1}q_{t+1}]$. This term implies that the planner, through the implementability constraint (19), internalizes the impact that a larger debt at t has on reducing the borrowing capacity at $t+1$ by lowering the price of capital, q_{t+1}, when the $t+1$ constraint binds.[15] In other words, the planner understands that more borrowing at t will need to be repaid at $t+1$, which would reduce consumption if the future borrowing capacity is curtailed. This would result in higher future marginal utility and, thus, a lower asset price q_{t+1}, which further tightens the collateral constraint, reduces the borrowing capacity, and requires an even bigger drop in consumption and asset prices.

[15] Similar as before, this can easily be seen by iterating forward and substituting (24) in (27).

Now consider that the collateral constraint binds at t (and may also bind at $t+1$). Then there are two opposing effects resulting from the borrowing decision that the planner needs to consider. One the one hand, higher borrowing accompanied by higher consumption at t, increases the price of capital q_t and relaxes the collateral constraint. On the other hand, more borrowing and higher consumption at t may result in lower consumption and lower price of capital at $t+1$ if the collateral constraint continues to bind in the future. Hence, the planner faces a trade off between choosing allocations such that she increases current prices, q_t, at the cost of potentially decreasing future prices, q_{t+1}.

Finally, we consider the optimality condition (25) with respect to wages, linking the shadow cost of labor income and labor cost between workers and entrepreneurs. Note that this condition is absent from the competitive economy; yet the planner would like to equalize the shadow values of the two agents, while accounting for the possibility of a binding collateral constraint. The reason why this condition does not hold in the competitive economy is twofold. First, private agents do not internalize that the shadow value of labor income/cost depends on the pecuniary externality (i.e. $\lambda_t^{SP,e}$ is a function of ξ_t). Needless to say, this will only be the case as long as the collateral constraint binds. Second, even absent a binding collateral constraint, the shadow values of labor income/cost will still not be equalized between the two agents as there is no equation in the competitive economy equalizing the two. The unequalized shadow values of labor income/cost generate the distributive externality, augmented by a pecuniary externality when constraints bind. Hence, the planner chooses allocations in a way to redistribute income such that the shadow costs/benefits of labor income/cost of the two agents get closer.

3.2 Optimal Tax Rates

This section derives the optimal credit and labor tax rates. The optimal tax on credit can be derived by combining the Euler equation for borrowing of the planner (27) with the corresponding equation of the agents incorporating the credit tax (13), and takes the following form

$$\tau_t^b = \frac{1}{\beta R E_t U_{x,t}} \left[\mu_t^{SP} - U_{x,t}\mu_t + \xi_t U_{xx,t} q_t - \beta R \xi_t \Omega_{t+1}\right] - \frac{1}{E_t U_{x,t}} E_t \left[\xi_{t+1} U_{xx,t+1} q_{t+1}\right]. \quad (28)$$

The optimal credit tax consists of two components that match the pecuniary externalities operating via the prices of capital, q_t and q_{t+1}, identified in the planner's Euler equation. For simplicity, first assume that the collateral constraint does not bind at t in both economies, i.e.

$\mu_t = \mu_t^{SP} = \xi_t = 0$. Then, the tax rate reduces to

$$\tau_t^{MP} = -\frac{1}{E_t U_{x,t}} E_t \left[\xi_{t+1} U_{xx,t+1} q_{t+1}\right], \tag{29}$$

which can easily be shown to be always positive as long as the collateral constraint binds in expectation. The tax rate tackles the pecuniary externality operating via q_{t+1}, and has a macro-prudential interpretation as it is levied during good times (i.e. when the collateral constraint does not bind), to allow for more borrowing during bad times (i.e. when the collateral constraint binds in the future).

Now assume that the collateral constraint also binds at t. In this case, $1/(\beta R E_t U_{x,t})[\mu_t^{SP} - U_{x,t}\mu_t + \xi_t U_{xx,t} q_t - \beta R \xi_t \Omega_{t+1}]$ is non-zero, tackling the externality operating via q_t. This part of the tax rate pushes for a subsidy on credit as higher borrowing at t supports higher asset prices q_t and relaxes the collateral constraint. However, more borrowing at t would require a repayment at $t+1$, resulting in lower asset prices q_{t+1} and a tighter collateral constraint. When choosing the optimal tax rate, the planner balances this two effects.

Although we cannot show analytically how the labor shock affects the pecuniary externality, we discuss its implications intuitively first and corroborate the discussion with our quantitative results presented in section 4. The labor supply shock will operate distinctly through the period t and $t+1$ channels described above. If the collateral constraint does not bind at t, the first order effect of the labor supply shock will be lower current production and higher borrowing to smooth consumption. The planner understands that higher indebtedness can exacerbate the pecuniary externality when constraints bind in the future and, thus, levies a higher macroprudential tax at t than otherwise via equation (29). If the collateral constraint binds at t, the first order effect of the labor supply shock will not only lower production, but also considerably lower consumption given the inability to increase borrowing. In turn, this suppresses asset prices resulting in even tighter collateral constraint and borrowing. As a result, the role of the pecuniary externality from binding constraints today—the first term in (28)—is enhanced, while the role of the pecuniary externality from binding constraints in the future—the second term in (28)—is weakened given that current borrowing and indebtedness go down. These considerations would urge the planner to levy a higher credit subsidy at t than otherwise. Overall, the labor supply shock exacerbates the policy response in opposite directions depending on whether the collateral constraint binds in the present. We verify this line of thinking when we present the quantitative results in section 4 and discuss in more detail the policy implications.

Now we turn to the optimal tax on labor. This tax rate can be derived by combining the planner's optimal decision for wages (25), which can be rewritten as $1+\theta\mu_t = \lambda_t^{SP,w}/\lambda_t^{SP,e}$ using (26), and the privately optimal labor decision (14) that incorporates the tax rate. Moreover, from (20), (24), and (26), $\lambda_t^{SP,e}$ can be re-written as $\lambda_t^{SP,e} = U_{x,t}/(1+\mu_t\kappa_t U_{xx,t}/U_{x,t}q_t)$, while $\lambda_t^{SP,w} = \omega U_{c,t}$ from (21). Using the above conditions, substituting for wages from the budget constraint of workers (17), and employing market clearing in the labor market ($l_t = \bar{h}_t$), the tax on labor takes the following form

$$\tau_t^h = \frac{F_{l,t}\bar{h}_t}{c_t} - \frac{\omega U_{c,t}}{U_{x,t}}\left(1 + \mu_t\kappa\frac{U_{xx,t}}{U_{x,t}}q_t\right). \tag{30}$$

The labor tax consists of two terms, the first is the labor share over workers' consumption, and the second term is the ratio of the marginal utilities of workers and entrepreneurs augmented by a term that captures the pecuniary externality. The planner uses the tax (subsidy) on labor to tackle the distributive externality described in section 3.1. To reiterate the intuition, the objective of the planner is to manipulate the wage such that to equate the shadow values of labor income and cost between agents while taking into account binding collateral constraints.

To fix ideas, assume for now that the collateral constraint does not bind, i.e. $\mu_t^{SP} = \mu_t = 0$. Then, equation (25) reduces to $\lambda_t^{SP,e} = \lambda_t^{SP,w}$, and the first order conditions with respect to entrepreneurs' and workers' consumption in the planner's economy, (20) and (21), respectively, imply $U_{x,t} = \omega U_{c,t}$. This means that the planner would like to redistribute income between the two agents in order to equate their marginal utilities. She will do so by manipulating wages. In turn, this means that the wage is not equal to the marginal product of labor in the planner's economy, hence the ratio $F_{l,t}\bar{h}_t \neq c_t$.[16] If the labor share is lower than workers' consumption, $F_{l,t}\bar{h}_t < c_t$, then the planner levies a higher tax (lower subsidy) on labor, which suppresses wages and helps entrepreneurs. The opposite is true, i.e. the planner levies a lower tax (higher subsidy) on labor, if $F_{l,t}\bar{h}_t > c_t$. Hence, the labor tax is levied to implement the wage the planner chooses to address this distributive externality.[17]

Next, suppose that $\mu_t^{SP} > 0$, and $\mu_t > 0$ from (26).[18] Then, in addition to manipulating

[16]Note that in the competitive economy $F_{l,t}\bar{h}_t = c_t$ when the collateral constraint does not bind. This can easily be seen by substituting the budget constraint of workers in the optimal labor demand decision of entrepreneurs and setting μ_t=0.

[17]Since labor is in fixed supply, the planner can manipulate the wage to implement an income redistribution without affecting the equilibrium hours going to production. This would not be the case under flexible labor supply as the planner also needs to respect workers' incentives to supply labor. We derive the case of the flexible labor supply in the Appendix and show that there is still room for redistribution. As the same forces are in play, we opt to present the case of a fixed labor supply, which better characterizes the labor supply shocks we have in mind.

[18]Note that if the planner had access to lump-sum transfers, then she could achieve $\lambda_t^{SP,e} = \lambda_t^{SP,w}$, which together with (26) would imply that $\mu_t^{sp} = 0$. In other words, the planner would be able to completely relax the collateral constraint with a set of borrowing tax, labor tax, and lump-sum transfers (see Biljanovska, 2019, for a detailed discussion in a model with always binding collateral constraints). Yet, a labor tax is still needed to

the ratio between labor share and workers' consumption to balance the marginal utilities of consumption between the two agents, the planner also needs take into consideration the effect of the pecuniary externality. A binding collateral constraint pushes the tax rate in opposite directions. On the one hand, it pushes for a lower tax (higher subsidy) on labor, as $\omega U_{c,t} > U_{x,t}$ when constraints bind and the planner would like to redistribute more resources towards workers.[19] On the other, it pushes for a higher tax (lower subsidy) on labor due to the presence of the pecuniary externality, captured by the term $\mu_t \kappa U_{xx,t}/U_{x,t} q_t < 0$, resulting in a higher redistribution of resources towards entrepreneurs.

Finally, it is important to understand how the labor supply shock affects the tax rate. It is straightforward that workers would suffer more from the labor shocks as they do not have any other means to consume. As long as the collateral constraint does not bind, the planner would reduce labor taxes or implement a subsidy to redistribute more resources to workers. However, if the collateral constraint binds, then the labor shock may more severely affect entrepreneurs, who could experience a rapid decrease in their consumption. This would not only exacerbate the pecuniary externality, but would also push the distributive externality more in favor of entrepreneurs. Thus, the planner would want to increase the labor tax not only to tackle the pecuniary externality, but also to redistribute more resources to entrepreneurs. We verify this line of thinking when we present the quantitative results in section 4 and discuss in more detail the policy implications.

4 Quantitative Analysis

This section outlines the quantitative analysis of the model economy by conducting numerical simulations and policy functions analysis for a baseline calibration. The first part describes the calibration and the rest discusses the results.

To solve the model, we use a global, non-linear solution algorithm. The competitive economy solution is obtained by iterating over the first-order conditions, and the SP problem solution is obtained by applying a value function iteration algorithm. In order to obtain the solution for the competitive economy, we iterate the (competitive) Euler equation for borrowing, which does not incorporate the pecuniary externality. Value function iteration incorporates the effect of pecuniary externalities on welfare and, hence, yields the planner's solution. Given that we solve

achieve the desired redistribution; without it the planner would need to respect the private optimality condition (6) and, thus, (26) would not obtain.

[19] This can easily be shown by combining (20), (21), (24), and (25), which yields $U_{x,t} + \mu_t^{SP}(\theta - \kappa_t \frac{U_{xx,t}}{U_{x,t}} q_t) = \omega U_{c,t}$, i.e. $\omega U_{c,t} > U_{x,t}$ for $\mu_t^{SP} > 0$.

for time-consistent policies, we use a nested fixed point algorithm for the value function iteration. Section B in the Appendix discusses the details of the numerical solution method.

4.1 Calibration

Given the similarity of the model, the majority of the model parameters are calibrated following Bianchi and Mendoza (2018), who calibrate their model to OECD data.[20] Each time period should be interpreted as a year. We deviate from their calibration for a subset of the parameters. First, the global interest rate in their model follows an AR(1) process. In order to limit the exogenous number of states in the model, we opt for a fixed interest rate $R = 1.01$, set at the long-term average level of the rate in Bianchi and Mendoza (2018). Second, we calibrate the fixed labor supply in normal times to the long-run average value in Bianchi and Mendoza (2018), i.e. $\bar{h}_n = 0.6$. The rest of the parameters are reported in Table 1.

The COVID-19 shock represents a 15 percent drop in labor supply, i.e. $\bar{h}_p = 0.85\bar{h}_n = 0.51$. Brinca, Duarte and Faria-e Castro (2020) show that the labor supply shock in April 2020 resulted in a 12 percent drop in the growth rate of total private hours worked in the U.S. Using high-frequency Automatic Data Processing (ADP), Autor, Cho, Crane, Goldar, Lutz, Montes, Peterman, Ratner, Villar and Yildirmaz (2020) show that employment declined by about 14 percent for both big and small firms through the beginning of the crisis into April 2020. During the time while preparing this draft, data for hours worked were not yet available for the OECD average, but unemployment is forecasted to climb to about 13 percent under the adverse scenario from 5 percent before the COVID-19 outbreak.[21] The qualitative results and the optimal policy we derive are not affected by the level of the labor supply shock. We have opted to calibrate the labor supply shock to about 15 percent based on the studies using high-frequency data of hours worked. The probability of such shock materializing is set to once in one hundred years. Moreover, the labor shock persists in the following period with probability 20 percent which implies a 14-month length for the pandemic since its outbreak.

4.2 Financial Amplification Dynamics

In order to assess the model's ability to generate crisis and the effectiveness of the policy instruments at reducing the severity of the crisis, we perform an event analysis of the competitive and planner's economies. We do so by examining long-term averages across three events: (I) when

[20] For details, we refer the reader to section III.A of their paper.

[21] Using unemployment to gauge the size of the labor supply shock is not ideal because its dependence on policy interventions and cross-country heterogeneity in job separation laws.

Table (1) Calibration

Parameter	Value
Risk aversion	$\sigma = 1$
Share of intermediate good in output	$\alpha_v = 0.45$
Share of labor in output	$\alpha_l = 0.352$
Share of assets in output	$\alpha_k = 0.008$
Interest rate	$R = 1.01$
TFP process	$\rho_z = 0.78$
	$\sigma_\epsilon = 0.01$
Discount factor	$\beta = 0.95$
Working capital coefficient	$\theta = 0.16$
Tight credit regime	$\kappa^l = 0.75$
Normal credit regime	$\kappa^h = 0.90$
Transition probability, κ^h to κ^l	$P_{h,l} = 0.10$
Transition probability, κ^l to κ^l	$P_{l,l} = 0.00$
Labor supply in normal times	$\bar{h}_n = 0.60$
Labor supply in pandemic times	$\bar{h}_p = 0.51$
Transition probability, $\bar{h}_n$ to $\bar{h}_p$	$P_{n,p} = 0.01$
Transition probability, $\bar{h}_p$ to $\bar{h}_p$	$P_{p,p} = 0.20$

the collateral constraint binds, (II) when a COVID-19 shock hits, and (III) when the collateral constraint binds and a COVID-19 shock hits. Event (II) could be interpreted as the first phase of the pandemic, during which financing constraints do not bind as the drop in output due to the labor supply shock has still not weakened firms' ability to borrow either due to deteriorated firm balance sheet or tighter lending standards; and event (III) could be interpreted as the second phase of the pandemic, during which the pandemic continues and financing constraints become binding.

The events are constructed as follows. First, the competitive and the planner's economy are simulated for 100,000 periods and the three events are identified following the approach of event study analysis in the empirical literature.[22] Next, we construct 5-year event windows centered around the year when the event materializes, i.e. T. We compute averages for each variable across the cross section of crisis events at each date. These steps generate the dynamics plotted in figures 1 and 2. The competitive and the planner's economies start from the same level of borrowing in the initial period and go through the same simulated path of shocks for the exogenous variables. The results are presented in terms of deviations from the long-term mean.

Figure 1 shows the dynamics for new borrowing, output and asset prices in the competitive economy under the three scenarios. First, the solid line shows the responses of the variables when an adverse shock, other than a COVID-19, results in binding collateral constraints and curtails the ability to borrow. Second, the dashed line shows the response of the variables to a COVID-19 shock when collateral constraints do not bind. Finally, the dotted line shows the

[22] In doing this, we follow the approach in Bianchi and Mendoza (2018).

responses of the variables to a COVID-19 shock and binding collateral constraints.

When collateral constraints start to bind (solid line in Figure 1), the borrowing ability of agents is curtailed, which leads to a drop in asset prices and output. The reduction in output is not very severe as labor and capital supply are not impeded. On the contrary, when the COVID-19 shock hits (akin to phase one of the pandemic), output falls by about 15 percent within the same year (dashed line in Figure 1). Agents smooth consumption by borrowing, which needs to be repaid in the following period, resulting in a decrease in consumption and asset prices in the following period. Finally, when collateral constraints bind and the COVID-19 shock hits (akin to phase two of the pandemic), agents are no longer able to smooth consumption as borrowing is constrained (dotted line Figure 1), leading to a more severe drop in asset prices, output and borrowing.

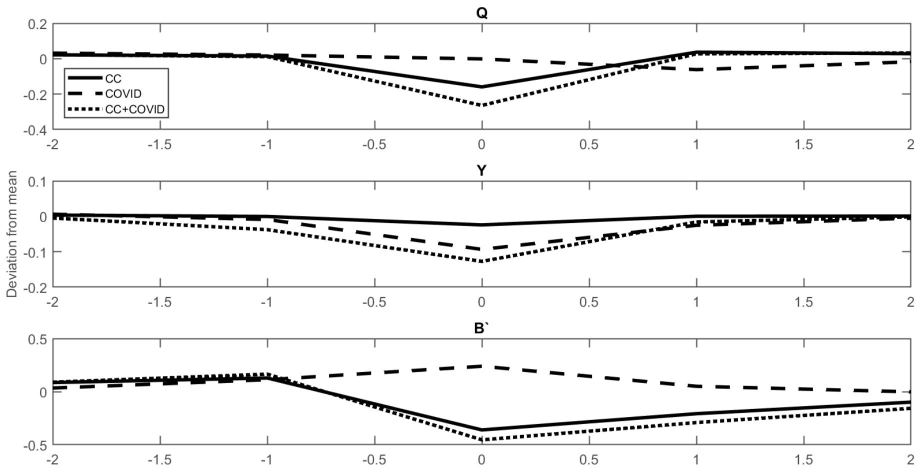

Figure (1) Comparison of competitive economy across events

Figure 2 compares the dynamics of the same set of variables in the competitive (solid line) and the social planner's (dashed line) economy under the three scenarios. The first row corresponds to event (I), the second corresponds to event (II), and the third corresponds to event (III). Three observations are worth noting.

First, Figure 2 shows that the planner's economy is characterized by a lower volatility than the competitive economy across all crisis events. This can be seen by the lower deviation from the long-term averages for borrowing and asset prices for the planner than for the private agents.

Second, when the collateral constraint binds (event I and III), the planner's economy experiences a much lower drop in borrowing as she manages to alleviate the negative effects of the pecuniary externality operating via the price of capital. As a result, also asset prices during these crisis events experience a smaller drop. The planner achieves this by providing credit subsidies

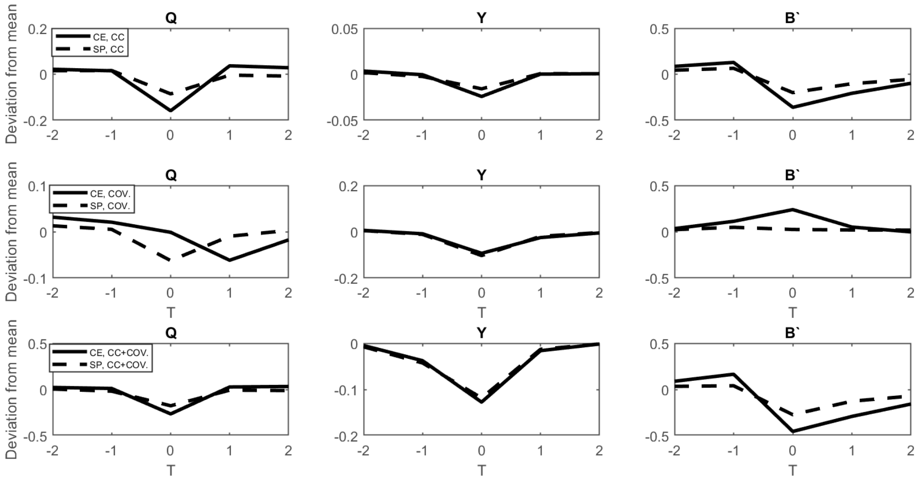

Figure (2) Comparison competitive economy and social planner

(see Table 3 and discussion in section 4.3 for details).

Third, when the COVID-19 shock hits but the collateral constraint does not bind, private agents would like to borrow to smooth consumption. The planner would like to do the same, but unlike the private agents she takes into account that higher borrowing today limits the borrowing ability tomorrow. Conditional on a COVID-19 shock, the probability that the collateral constraint binds is higher once the COVID-19 shock hits. The planner internalizes this, hence she restricts the increase in current period's borrowing (compared to the competitive economy) to avoid becoming too indebted such that pecuniary externalities are exacerbated in the future. The planner achieves this by imposing a minimal credit subsidy. However, when the COVID-19 shock is accompanied by binding collateral constraints, the planner would levy a bigger credit subsidy to support borrowing and tackle not only the adverse consequences of the COVID-19 shock, but also the pecuniary externalities from binding constraints in the present (see Table 3 and discussion in section 4.3 for details).

Before turning to the implementation of the planner's solution with tax instruments, we present the welfare gains achieved by the planner. Table 2 shows the percentage compensating consumption variation, γ^{Event}, equally obtained by both agents such that the social welfare of the competitive economy equals the welfare in the planner's economy. We calculate both unconditional and conditional on the three events welfare gains using the following formula:

$$
\begin{aligned}
&E_{0|Event} \sum_{t=0}^{\infty} \beta^t \left[U((1+\gamma^{Event})c_t) + U((1+\gamma^{Event})x_t) \right] \\
=&E_{0|Event} \sum_{t=0}^{\infty} \beta^t \left[U(c_t^{SP}) + U(x_t^{SP}) \right], \qquad (31)
\end{aligned}
$$

where *Event* corresponds to (i) all observations in the simulations (unconditional welfare gain), or (ii) one of the three events (conditional welfare gain). Then mean welfare gain is the average $\gamma^{Event}(B, s)$ computed with the ergodic distribution.

Table 2 shows that the planner's economy always achieves a higher welfare than the competitive economy (with both agents realizing positive gains).

Table (2) Welfare gains

Unconditional	4.19%
Conditional on CC	4.17%
Conditional on COVID	4.20%
Conditional on CC & COVID	4.23%

Note: Welfare gains are reported in terms of average compensating consumption variations.

4.3 Tax Instruments

In this section, we examine the quantitative features of the policy instruments and their welfare implications.

The top-left panel in Figure 3 shows the optimal borrowing tax as a function of the current level of debt for looser lending standards, i.e. $\kappa_t = \kappa^h$, and a low productivity realization.[23] The chart verifies the findings in section 3.2, confirming an established result in the literature: For low levels of debt, the collateral constraint does not bind either today or tomorrow and the tax on borrowing is zero. As debt levels increase, the probability of a binding collateral constraint tomorrow increases, requiring a positive tax on borrowing to tackle the pecuniary externalities (operating via q_{t+1}) from binding collateral constraints in the future (Bianchi and Mendoza, 2018). However, for higher levels of debt, binding collateral constraints today reverse the direction of policy, requiring a subsidy on borrowing to tackle the pecuniary externalities (operating via q_t) from binding collateral constraints in the present.

These qualitative results continue to hold under a COVID-19 shock, but the quantitative response differs in interesting ways. First, the macroprudential tax used to lean against pecuniary

[23]The optimal policy decisions look similar for medium and high productivity realizations, hence we do not report them.

externalities in the future is higher, which can be interpreted as a means to preempt the financial amplification induced by the COVID-19 shock should the collateral constraints bind in the future. Second, the borrowing subsidy when collateral constraints bind in the present is higher when accompanied by a COVID-19 shock. This result can be interpreted as a way to deal with the current financial amplification of the COVID-19 shock. Third, both the borrowing tax and subsidies kick in for much lower levels of existing debt suggesting a higher need for policy intervention during the pandemic.

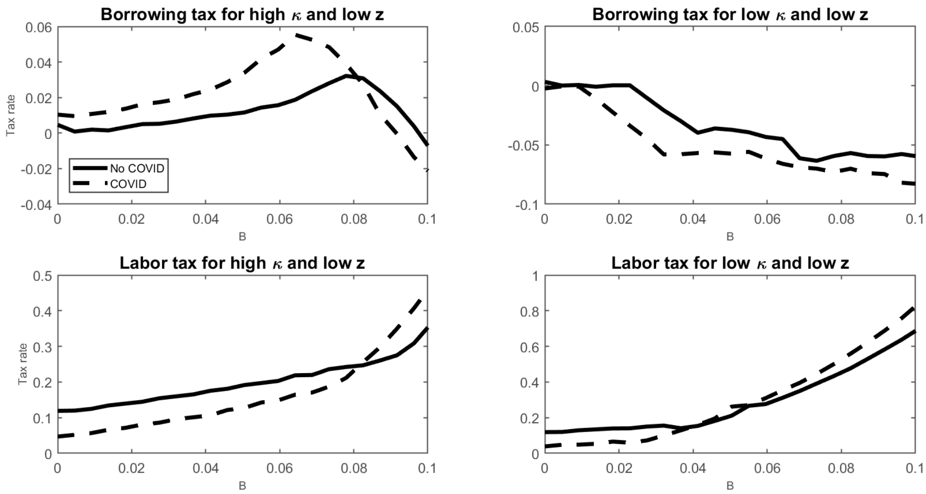

Figure (3) Tax Policy Functions

The top right chart in Figure 3 presents the optimal borrowing tax for tighter lending standards, i.e. $\kappa_t = \kappa^l$, and a low productivity realization. As above, the borrowing subsidy is higher and kicks in faster under a COVID-19 shock. An important difference is that there is no scope for preemptive macroprudential policy, which is reasonable given the curtailed ability to borrow due to the unfavorable financial conditions. This result could potentially suggests that preemptive policy is only possible during the first phase of the pandemic when firms can still obtain credit at more favorable conditions. But during the second phase, accommodative policies are needed even for lower levels of indebtedness.

These results are verified in the average borrowing tax across the three events in the simulated economy presented in Table 3. When financial conditions are favorable (κ^h), the tax on borrowing is very close to zero prescribing a minimal role for intervention.[24] This result implies that, across all three events, it is optimal to balance the negative impact of current versus future externalities from borrowing decisions. The former dominates when current financial conditions

[24]Note that, in our economy there are no events where, for κ^l, the COVID-19 shock is accompanied by non-binding collateral constraints. This is intuitive as financial conditions are unfavorable and a pandemic shock hits, collateral constraints are likely to bind.

are unfavorable (κ^l), pushing for a higher borrowing subsidy, and vice versa.

The bottom left chart in Figure 3 presents the optimal labor tax for favorable financial conditions and low realization of productivity. As discussed in the analytical part in section 3.2, the labor tax effectively represents a redistribution across agents by affecting the wage rate.[25] In general, if workers' (entrepreneurs') consumption is lower, the planner will levy a lower (higher) labor tax. Moreover, when a COVID-19 shock hits the economy, the planner further reduced the tax on labor.[26]. This result is reasonable given that workers experience a large drop in income by not being able to supply labor. However, for high levels of debt, the ability of entrepreneurs to borrow and produce is lower, resulting in lower entrepreneurs' profits compared to workers' labor income, which manifests in a higher labor tax. Due to the presence of pecuniary externalities, these tax rates are amplified if collateral constraints are binding in the present, as we show analytically in section 3.2. As discussed, this amplification comes from the desire to tackle the pecuniary externality on top of engaging in redistribution between workers and entrepreneurs. The amplification is much more severe under a COVID-19 shock, resulting in a labor policy that is even less favorable for workers as entrepreneurs are constrained to borrow.

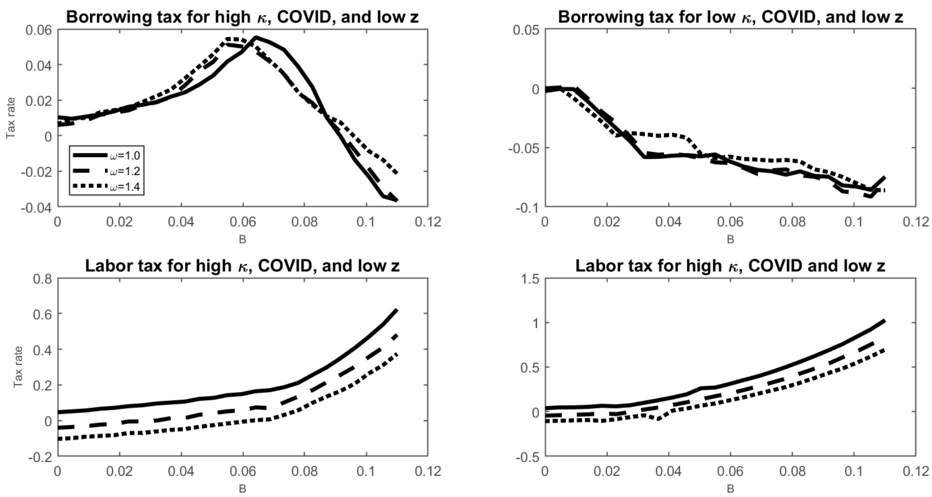

Figure (4) Tax policy functions for different values of ω

The bottom right chart in Figure 3 presents the optimal labor tax for unfavorable financial conditions and low realizations of productivity. The same qualitative results, as under favorable financial conditions, continue to hold. Quantitatively the difference is that financial amplification is more pronounced under a COVID-19 shock calling for higher labor taxes to support

[25]A labor tax (subsidy) induces entrepreneurs to offer a lower (higher) wage to workers. Because labor is in fixed supply affecting the wage has a first-order redistirbutive effect

[26]Naturally this depends on the weight ω placed on workers' utility, which for the baseline simulation is set to 1, but for higher values of ω, a subsidy can be obtained (see Figure 4). The weight put on workers has a first-order effect on the labor tax, which tackles the distributive externality, while it does not matter a lot for the tax on borrowing, which tackles the pecuniary externality.

Table (3) Tax rates across events

	CC		COVID		CC & COVID	
	κ^h	κ^l	κ^h	κ^l	κ^h	κ^l
τ^b	-0.3%	-6.2%	-0.4%		0.8%	-6.8%
τ^h	30.4%	57.1%	34.9%		30.6%	67.0%

entrepreneurs, especially for higher levels of indebtedness. This finding is important for comparing the first and second phase of the pandemic as policies to support labor income may be tougher to implement in the latter. The average tax rates presented in Table 3 confirm these findings: When financial conditions are unfavorable, the tax on labor is always higher than when they are positive.

5 Conclusion

We study how financial frictions amplify labor supply shocks in a model with occasionally binding collateral constraints. The model extends the framework of Bianchi and Mendoza (2018) in two ways, first by introducing heterogeneity across agents—workers vs entrepreneurs—and, second, by introducing a labor supply shock generated by the COVID-19 pandemic. In the framework, two types of externalities are present, justifying policy intervention. A pecuniary externality that operates via the asset price of capital used as collateral, and a distributive externality that arises due to the difference in the shadow values of income across agents and operates via the labor wage. We derive optimal credit and labor policy following the Ramsey approach.

The tax on borrowing is set to tackle the pecuniary externality, while the tax on labor tackles the distributive externality. Labor shocks, such as the COVID-19 shock, amplify the policy responses, which critically depend on whether financing constraints bind or not, i.e on the state of financial conditions. If financial conditions are favorable, but a labor supply shock hits the economy, the tax on borrowing is set higher than in absence of a labor supply shock. The reason is that firms increase their borrowing massively in a effort to smooth the labor supply shock, which implies that future collateral constraints will bind with higher probability and severity. A higher preemptive borrowing tax is needed to address the elevated risk from pecuniary externalities in the future. On the contrary, under unfavorable financial conditions, a subsidy on borrowing ameliorates current pecuniary externalities and is especially helpful during transitionary labor supply shocks, which tighten the constraint more.

The labor tax is levied to implement a redistribution of resources between workers and entrepreneurs. The labor supply shock can be detrimental to workers that have no other sources of income and lowering the labor tax or even subsidizing labor is optimal. Yet, if a labor supply shock hits the economy when collateral constraints bind, the optimal labor tax is higher than otherwise in order to support collateral constrained entrepreneurs and avoid pecuniary externalities from binding constraints. This comes at the expense of workers.

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Appendix

A Flexible Labor Supply

This section shows that the qualitative insights of the model featuring labor in exogenous fixed supply extend to a model with a flexible endogenous labor supply.

A.1 Competitive Economy

There is a competitive market for labor, where workers (entrepreneurs) supply (demand) h_t (l_t) hours at the price w_t. Workers optimal choice depends on the gain in consumption versus the loss in leisure an additional unit of labor generates. Under this scenario, the optimization problem of entrepreneurs remains the same as in section 2.2, whereas the one of the workers becomes

$$\max_{c_t,h_t} \sum_{t=0}^{\infty} \beta^t U(c_t) - V(h_t)$$

subject to

$$c_t = w_t h_t. \tag{32}$$

The optimal labor supply and market clearing condition are respectively given by

$$w_t = \frac{V_{h,t}}{U_{c,t}}, \tag{33}$$

$$h_t = l_t. \tag{34}$$

A.2 Planner's Economy

With endogenous labor, the planner no longer takes h_t as given, and solves the following optimization problem

$$\max_{c_t^e, c_t^w, l_t, b_{t+1}, v_t, q_t, w_t} E_t \sum_{t=0}^{\infty} \beta^t [U(c_t^e) + \omega U(c_t^w) - \omega V(l_t)]$$

$$c_t^e + b_t + p^v v_t + w_t l_t \leq z_t F(1, v_t, l) + \frac{b_{t+1}}{R} \quad (\lambda_t^{SP,e}) \tag{35}$$

$$c_t^w = w_t l_t \quad (\lambda_t^{SP,w}) \tag{36}$$

$$\frac{b_{t+1}}{R} + \theta p^v v_t + \theta w_t l_t \leq q_t \quad (\mu_t^{SP}) \tag{37}$$

$$U_{c^e,t} q_t = \beta R E_t U_{c^e,t+1} [z_{t+1} F_{k,t+1} + q_{t+1} + \frac{\kappa q_{t+1}}{\theta} (z_{t+1} F_{l,t+1} / p_v - 1)/] \quad (\xi_t) \tag{38}$$

$$w_t = \frac{V_{h,t}}{U_{c,t}} \quad (\psi_t), \tag{39}$$

where the objective function is augmented by workers' dis-utility of labor, $V(h_t)$, and the optimal labor supply equation, $w_t = V_{h,t}/U_{c,t}$, is added as a constraint in the planner's problem. Lagrange multipliers associated with each constraint are given in parentheses.

The optimality conditions with respect to x_t, b_{t+1}, v_t, q_t remain the same as in section 3.1, while those with respect to c_t, l_t, and w_t become, respectively

$$c_t : \quad \lambda_t^{SP,w} + \psi_t U_{cc,t} w_t = \omega U_{c,t}, \tag{40}$$

$$l_t : \quad \omega V_{l,t} = \lambda_t^{SP,e} F_{l,t} - w_t (\lambda_t^{SP,e} + \theta \mu_t^{SP} - \lambda_t^{SP,w}) + \psi_t \frac{V_{ll,t}}{U_{c,t}}, \tag{41}$$

$$w_t : \quad \lambda_t^{SP,e} + \theta \mu_t^{SP} - \lambda_t^{SP,w} = -\frac{\psi_t}{l_t}. \tag{42}$$

A.3 Optimal Policy

The tax on borrowing remains unchanged. Under flexible labor supply, the optimal tax on labor obtains a different functional form, which can be derived using the optimal labor decision from the competitive economy that embeds the labor tax

$$F_{l,t} = w_t (1 + \tau_t^h + \theta \mu_t), \tag{43}$$

and the planner's optimality conditions with respect to l_t, (41), and w_t, (42). Combining these three equations, and substituting equation (39) for wages—after some algebraic manipulations—yields

$$\frac{\omega V_{l,t}}{\lambda_t^{SP,e}} = \frac{V_{l,t}}{U_{c,t}}(1+\theta\mu_t+\tau_t^h) - \left(\frac{V_{l,t}}{U_{c,t}} + \frac{V_{ll,t}}{U_{c,t}}l_t\right)\left(1+\theta\frac{\mu_t^{SP}}{\lambda_t^{SP,e}} - \frac{\lambda_t^{SP,w}}{\lambda_t^{SP,e}}\right). \tag{44}$$

Finally, using the equality $\mu_t = \mu_t^{SP}/\lambda_t^{SP,e}$ (obtained from the first order condition with respect to the intermediate good in the competitive and the planner's economy) and the first order condition with respect to workers' consumption, (42), the tax on labor is given by

$$\tau_t^h = \left(1+\theta\mu_t - \frac{\lambda_t^{SP,w}}{\lambda_t^{SP,e}}\right)\left(\frac{U_{cc,t}}{U_{c,t}}V_{l,t} + \frac{V_{ll,t}}{V_{l,t}}l_t\right)l_t. \tag{45}$$

Similarly as under fixed labor supply, the tax on labor depends on the planner's relative shadow values of wealth of the two agents ($\frac{\lambda_t^{SP,w}}{\lambda_t^{SP,e}}$), the tightness of the collateral constraint (μ_t), and the pecuniary externality (embedded in $\lambda_t^{SP,e}$). Under the fixed labor supply case, the planner chooses wages in an attempt to equate the shadow values of wealth across agents, while taking into account the pecuniary externality. Under the flexible labor case, while attempting to equate the shadow values of wealth, the planner also needs to account for the second-order derivatives, $U_{cc,t}$ and $V_{ll,t}$, capturing the pecuniary effect of agents' decisions on the wage. These difference derives from the fact that under flexible labor supply, the planner chooses l_t (instead of it being fixed), while still respecting the optimal labor supply decision of workers, (39). Nonetheless, the desire for redistribution across agents, which crucially depends on the relative values of $\lambda_t^{SP,w}$ and $\lambda_t^{SP,e}$ remains and does not hinge upon the fixed labor supply assumption.

B Numerical Algorithm

Competitive equilibrium. We solve for the CE using an Euler-equation iteration algorithm. In each iteration, we solve the system of equations presented below in a recursive form for each of 300 gridpoints: 25 values of debt (b), and 12 states (3 states for productivity $\times$ 2 states for pledgeable fraction of collateral $\times$ 2 states for the labor supply). Formally, we solve for the policy functions $\{\tilde{b}(b,\psi), x(b,\psi), c(b,\psi), q(b,\psi), w(b,\psi), v(b,\psi), \mu(b,\psi)\}$, where ψ is the tuple of exogenous state variables, such that the equilibrium conditions below are satisfied

$$x(b,\psi) + b + p^v v(b,\psi) + w(b,\psi)\bar{h}_t = F(z,1,v(b,\psi),\bar{h}_t) + \frac{\tilde{b}(b,\psi)}{R}, \tag{46}$$

$$\frac{\tilde{b}(b,\psi)}{R} + \theta p^v v(b,\psi) + \theta w(b,\psi)\bar{h}_t \le \kappa q(b,\psi), \tag{47}$$

$$\mu(b,\psi) = 1 - \beta R E_{\psi'|\psi} \frac{U_x(x(b',\omega'))}{U_x(x(b,\psi))}, \tag{48}$$

$$q(b,\psi)U_x(x(b,\psi)) = \beta E_{\psi'|\psi}[U_x(x(b',\psi'))\times$$
$$(q(b',\psi') + F_k(z',1,v(b',\psi'),\bar{h}_t') + \kappa'\mu(b',\psi')q(b',\psi'))], \tag{49}$$

$$c(b,\psi) = w(b,\psi)\bar{h}_t, \tag{50}$$

$$F_l(z,1,v(b,\psi),\hat{h}) = w(b,\psi), \tag{51}$$

$$F_v(z,1,v(b,\psi),\hat{h}) = p^v(1+\theta\mu(b,\psi)), \tag{52}$$

where $\tilde{b}(b,\psi)$ is the new borrowing, and y' denotes the next period realization of variable y.

The algorithm proceeds in the following steps:

1. For each gridpoint in b, conjecture future policy functions $b' = \tilde{b}(b,\psi)$, $c(b',\psi')$, $q(b',\psi')$, $w(b',')$, $v(b',\psi')$, $\mu(b',\psi')$. For the first iteration use a guess. For further iterations define future polices as the solution to the current policy functions from the previous iteration (see step 3 below).

2. Taking future policies from step 1 as given, for each gridpoint in b, solve (46)-(52) to obtain current policy functions $\tilde{b}(b,\psi)$, $x(b,\psi)$, $c(b,\psi)$, $q(b,\psi)$, $w(b,\psi)$, $v(b,\psi)$, $\mu(b,\psi)$. We distinguish between cases that the collateral constraint binds and does not bind in the present:

   i. First, assume that the collateral constraint (47) binds and solve for the current policy functions. Then, check that $\mu(b,\psi) > 0$ using equation (48). If this is true, proceed to step 3; otherwise move to substep ii.

   ii. If for a given gridpoint the collateral constraint in the present does not bind, solve the system of equations above for the current policy functions by setting $\mu(b,\psi) = 0$.

3. Use the optimal policy functions from substeps 2i or 2ii to update the (conjectured) future policy functions in step 1.

4. Stop when convergence is achieved, i.e. when for two consecutive iterations $i-1$ and i it holds that $\sup_{b,\psi} ||y_i(b,\psi) - y_{i-1}(b,\psi)|| < \varepsilon$, where $y = \tilde{b}, c$. We set $\varepsilon = 10^{-3}$, but we also confirm that the results do not change if we choose a stricter convergence criterion.

Social planner. We solve for the SP policy functions using a value function iteration, nested fixed point algorithm. In each iteration we solve for the value function using a fixed-grid optimization procedure as an inner loop. In the outer loop, we update future policies given the solution to the Bellman equation from the inner loop. As in Klein, Krusell and Ríos-Rull (2008) and Bianchi and Mendoza (2018), this procedure delivers time-consistent policies. The detailed steps are described below.

The value function representation of the SP's optimization problem is:

$$V(b,\psi) = \max_{\tilde{b},c,x,w,v,q,\mu} \big(\omega U(c(b,\psi)) + U(x(b,\psi)) + \beta E_{\psi'|\psi}[V(b',\psi')]\big) \tag{53}$$

subject to (54)-(57):

$$x(b,\psi) + b + p^v v(b,\psi) + w(b,\psi)\bar{h}_t = F(z,1,v(b,\psi),\bar{h}_t) + \frac{\tilde{b}(b,\psi)}{R} \tag{54}$$

$$\frac{\tilde{b}(b,\psi)}{R} + \theta p^v v(b,\psi) + \theta w(b,\psi)\bar{h}_t \leq \kappa q(b,\psi) \tag{55}$$

$$\begin{aligned} q(b,\psi)U_x(x(b,\psi)) = \beta E_{\psi'|\psi}[U_x(x(b',\psi'))\times \\ (q(b',\psi') + F_k(z',1,v(L',\omega'),\bar{h}_t') + \kappa'(F_v(z',1,v(b',\psi),\bar{h}_t')/p^v - 1)/\theta q(b',\psi'))], \end{aligned} \tag{56}$$

$$c(b,\psi) = w(b,\psi)\bar{h}_t, \tag{57}$$

The algorithm proceeds in the following steps:

1. In the outer loop, define future policies $V(b',\psi'), \tilde{b}(b',\psi')$, $x(b',\psi')$, $c(b',\psi')$, $q(b',\psi')$, $w(b',\psi')$, $v(b',\psi')$, $\mu(b',\psi')$ as the solution to current policy functions from the previous iteration (see step 3 below) or the policy functions from the CE solution for the first iteration.

2. In the inner loop, for each gridpoint of b, solve for policy functions $V(b,\psi)$, $\tilde{b}(b,\psi)$, $x(b,\psi)$, $c(b,\psi)$, $q(b,\psi)$, $w(b,\psi)$, $v(b,\psi)$, $\mu(b,\psi)$ that satisfy (53) - (57) given future policies from step 1. We distinguish betwee cases that the collateral constraint (55) binds or not:

   i. First, assume that the collateral constraint binds. The objective is to find the level of $\tilde{b}(b,\psi)$ that maximizes (53). For a subgrid of 100 values of $\tilde{b}$ and given conjectures for $V(b',\psi')$, $q(b,\psi)$ and $w(b,\psi)$ from the outer loop, compute the corresponding $v(b,\psi)$, $x(b,\psi)$, and $c(b,\psi)$, from (55), (54), and (57). Then, choose the value for $\tilde{b}(b,\psi)$ with the highest $V(b,\psi)$ among the many grid points: $\tilde{b}$ matters for $V(b,\psi)$ not only because it determines current utility $\omega U(c(b,\psi)) + U(x(b,\psi))$, but also because it is the future state variable, i.e. $b' = \tilde{b}(b,\psi)$. Thus, its choice determines the level of the continuation value $V(b',\psi')$. The policy function $V(b',\psi')$ assigning a value for different values b' is taken as given from the outer loop in step 1, but, in the inner loop, we choose the value of b' ($\tilde{b}$) that maximizes the sum of current utility and the continuation value. Finally, compute if the Lagrange multiplier on (55) is positive, for which it suffices that $(F_v(z,1,v(b,\psi),\bar{h}_t)/p^v - 1) > 0$ (see (26)). If this is true, proceed to step 3; otherwise move to substep ii.

   ii. Set the Lagrange multiplier on (55), which yields $(F_v(z,1,v(b,\psi),\bar{h}_t)/p^v - 1) = 0$. Use this condition to solve for $v(b,\psi)$. For each point on the subgrid of $\tilde{b}$, using the solution for $v(b,\psi)$, and given conjectures for $V(b',\psi')$ and $w(b,\psi)$ from the outer loop, calculate corresponding values of $x(b,\psi)$ and $c(b,\psi)$ satisfying equations (54) and (57). Finally, choose the level of $\tilde{b}$ for which $V(b,\psi)$ is the highest similar to substep i above. Note that $q(b,\psi)$ does not matter when the collateral constraint does not bind, but it is compute to verify that the constraint indeed is slack.

3. Use the optimal policy functions from substeps 2i or 2ii to update the (conjectured) future policy functions in step 1.

4. Stop when convergence is achieved, i.e. when for two consecutive iterations $i-1$ and i it holds that $\sup_{b,\psi} ||V_i(b,\psi) - V_{i-1}(b,\psi)|| < \varepsilon$, where $\varepsilon = 10^{-2}$.