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Public Debt and $r - g$ at Risk*

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Abstract

As interest rate-growth differentials ($r-g$) turned negative in many countries, governments consider pursuing fiscal expansion and the potential risks involved. Using a large sample of advanced and emerging economies, our analysis suggests that high public debts can lead to adverse future $r-g$ dynamics. Specifically, countries with higher initial public debt experience (i) a shorter duration of negative $r-g$ episodes and a higher probability of reversal, (ii) higher average $r-g$, and (iii) a more right-skewed $r-g$ distribution, that implies higher down-side risks. Furthermore, high-debt countries experience larger increases in interest rates in response to (iv) an unexpected decline in domestic output and (v) an increase of global volatility. Results are stronger when public debts are denominated in foreign currencies.

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1 Introduction

The interest rate-growth differential ($r-g$) has been declining since the 1980s and turned negative in several economies after the 2008 Global Financial Crisis (GFC). At the same time, interest rates have remained low since the GFC. In [Blanchard \(2019\)](#), these two conditions offer strong arguments to pursue fiscal expansion to spur growth, as a negative long-run $r-g$ implies a more sustainable public debt and countercyclical fiscal policy is arguably more effective in a low-rates environment ([Miyamoto, Nguyen and Sergeyev, 2018](#); [Ramey and Zubairy, 2018](#)). However, even before the Covid-19 pandemic, current debt levels were already at historical high ([Yared, 2019](#); [Mian, Straub and Sufi, 2020](#)) and growth rates had been stagnant in many countries (Figure 1). In this context, a further fiscal expansion could entail significant risks. If history is any guide, there is no guarantee that $r-g$ will remain negative ([Mehrotra and Sergeyev, 2019](#); [Rogoff, 2020](#)).

In this paper, we argue that high public debts can make $r-g$ more likely to rise and turn positive, potentially amplifying the effect of adverse shocks in driving up $r-g$. A surge in $r-g$ is a concern as it can generate large economic costs ([Born, Müller, Pfeifer, Wellmann and Müller, 2020](#)) and lead to sovereign debt distress ([Mauro and Zhou, 2020](#)). Our analysis, based on a large sample of 56 advanced economies (AEs) and emerging markets (EMs), has two distinctive features relative to previous studies. First, we apply the growth-at-risk approach developed by [Adrian, Grinberg, Liang and Malik \(2018\)](#) and [Adrian, Boyarchenko and Giannone \(2019\)](#) to document that a higher level of current public debt is associated with an increase in the likelihood of exceptionally high $r-g$ in the future. Second, we explore the impact of growth forecast errors and changes in the U.S. VIX on interest rates to show that a higher public debt is associated with larger increase in interest rates in response to adverse domestic growth shocks and global volatility shocks.

The main theoretical framework behind this paper is in the same spirit as [Rogoff \(2020\)](#): the Covid-19 pandemic warns that, when disasters do strike, interest rates can rise, increasing the risk of a bad equilibrium and heightening the tension between fiscal stimulus and future debt sustainability. While accommodative monetary policy could further push policy interest rates down, the unprecedented growth slowdown, increased uncertainty and a massive fiscal expansion could push $r-g$ upward and move public debt to an unsustainable path.

The main premise of our empirical exercise is as follows. First, even in normal times, countries with higher outstanding debts are more likely to exhibit higher $r-g$, as higher public debts tend to be associated with higher borrowing costs and could reduce growth through the need for higher primary-balances and lower investment, because of debt overhang and crowding out effects (Bohn, 1998; Aguiar, Amador and Gopinath, 2009; Laubach, 2009; Mauro, Romeu, Binder and Zaman, 2015; Fatas, Ghosh, Panizza and Presbitero, 2020). Second, and more important, high public debt, especially when debt is denominated in foreign currency, can amplify the effect of adverse short-term shocks. These shocks tend to increase risk premia and thus borrowing costs, which in turn can lead to insolvency problems and trigger self-fulfilling crisis (Aguiar, Amador and Gopinath, 2009; Lorenzoni and Werning, 2019; Bocola and Lorenzoni, 2020). As the exposure to this mechanism increases in the level of outstanding debts, countries with higher outstanding debts are more likely to experience an exceptionally high $r-g$ in the future (downside tail risk). The Covid-19 pandemic makes these issues even more prominent.

Our main findings are as follows. First, historical data warn that the current negative- $r-g$ episode may not last long. Over the last 70 years, the duration of negative $r-g$ episodes has been shorter, the higher the initial level of public debt. Consistent with this pattern, high-debt countries are significantly more likely to experience a reversal: moving from negative $r-g$ regime today to positive $r-g$ regime in the future. For example, when a negative (positive) $r-g$ episode is defined as a period of at least two consecutive years of negative (positive) $r-g$, the likelihood of a reversal increases from about 25% for debt to-GDP ratios below the median to more than 75% for countries in the top quartile of the debt-to-GDP distribution.

Second, a higher current public debt is associated with an increase of the conditional mean of $r-g$ in the next two or five years and with a highly right-skewed $r-g$ distribution, such that the probability of exceptionally high $r-g$ in the future—which we refer to as downside risk—increases significantly. We estimate the distribution of $r-g$ as a function of public debt using quantile regressions. The gap between the upper and median quantiles of the average $r-g$ in the next two or five years is positively associated with the level of public debt. For example, as the current debt-to-GDP ratio increases from 40% to 120%, the 90th percentile of the average $r-g$ over the following five years increases from around 0 to 2 percent. At the same time, the median $r-g$ only increases by around 0.8 percentage points. The increase in the downside risk is not compensated by higher upside risk; if anything, higher public debt today is also associated

with a smaller decline in $r-g$ in the very good state (lower quantiles). These patterns hold for interest rates and growth separately, suggesting that both components contribute to the positive association between public debt and $r-g$ at risk.¹

Third, the currency of debt denomination matters: within high-debt countries, a higher share of foreign currency-denominated public debt is associated with higher average $r-g$. This pattern is consistent with the well-documented higher vulnerability attached to foreign-currency liabilities. For example, these debts are mostly denominated in U.S. dollars, which tend to appreciate following adverse shocks, increasing the real value of the outstanding debts and borrowing costs in bad times.

Next, to better understand the mechanisms driving the $r-g$ dynamics, we look at the association between public debt levels and the transmission of adverse shocks to long-term interest rates. We start with estimating the response of long-term interest rates to a negative domestic growth shock, measured by the growth forecast error. We find that countries with high public debts, especially when denominated in foreign currency, experience a large increase in interest rates following a lower-than-expected domestic growth. For instance, a negative growth shock—a realization of GDP growth that is at least 1% lower than expected—is associated with an increase in interest rates by 155 basis points (bps) in countries with high public debt and high share of foreign currency public debt. This result is consistent with the idea that, as risk premia increase in bad times, high-debt countries tend to experience a large increase in their borrowing costs, which limits their capacity to support growth and contributes to explain the worse dynamics of $r-g$.

Finally, using daily data, we find that countries with higher public debt experience a larger (and persistent) increase in interest rates in response to adverse global volatility shocks, measured by the two-day change in the U.S. VIX. In addition, for a given level of debt, the interest-rate response is larger, the higher the share of public debt denominated in foreign currency. For instance, for the average country in the sample, a 5-percent increase in the U.S. VIX is associated with 33 bps increase in long-term interest rates over the next 10 days, but this effect increases to 130 bps for countries with high debt and a high share of foreign currency denominated debt. By contrast, the response of long-term interest rates to changes in the U.S. VIX

¹There is an important caveat to the decomposition exercise: the covariance between r and g matters. Theoretically, public debt may lead to higher downside risk in $r-g$ because high public debt affects both r and g such that they tend to be significantly more negatively correlated in bad times.

shows no correlation with public debt levels for countries that are typically considered as safe havens (the U.S., the UK, Japan, Switzerland, and Germany). This pattern is consistent with investors rebalancing their portfolios towards safe and liquid assets during periods of high global uncertainty (Beber, Brandt and Kavajecz, 2009; Baele, Bekaert, Inghelbrecht and Wei, 2020).

Related literature. Our analysis relates to the growing number of papers that study the behavior of $r-g$ in different countries, motivated by the seminal work of Blanchard (2019) on $r-g$ in the U.S.. While evidence on advanced economies suggests that $r-g$ is likely to be negative in the long run (Barrett, 2018), most of the literature concurs that $r-g$ is highly volatile. Hence, episodes of negative $r-g$ are not necessarily the norm; the past episodes have been concentrated in emerging markets and to some extent driven by financial repression (Reinhart and Sbrancia, 2015; Escolano, Shabunina and Woo, 2017; Jordà, Knoll, Kuvshinov, Schularick and Taylor, 2019; Mauro and Zhou, 2020; Mehrotra and Sergeyev, 2019; Garín, Lester, Sims and Wolff, 2019; Checherita-Westphal, 2019). Relative to this literature, our paper offers new empirical patterns on the relationship between current public debt and the dynamics of future $r-g$: higher initial public debt is associated with shorter duration of negative $r-g$ episodes and higher probability of $r-g$ downside risk.

Our analysis also relates to the large literature that develops multi-equilibria model in which adverse short-term shocks could lead to bad long-term equilibria and self-fulfilling debt crises (Calvo, 1988; Cole and Kehoe, 2000; Chamon, 2007; Aguiar and Amador, 2014; Aguiar, Amador and Gopinath, 2009; Lorenzoni and Werning, 2019), as well as the literature that studies the importance of foreign currency debt in understanding countries' vulnerabilities to external shocks (e.g., Fisher, 1933; Aguiar, 2005; Lane and Shambaugh, 2010; Du and Schreger, 2016; Bocola and Lorenzoni, 2020; Wiriadinata, 2020). Relative to these literatures, this paper offers empirical evidence on the mechanisms through which public debt could amplify the vulnerability of $r-g$ dynamics to external shocks.

Finally, by showing how public debt makes countries' growth and borrowing costs more vulnerable to domestic and global shocks, our results contribute to the extensive empirical literature studying the effects of public debt on growth and sovereign bond yields (Bohn, 1998; Laubach, 2009; Reinhart and Rogoff, 2010; Panizza and Presbitero, 2013; Eberhardt and Presbitero, 2015),

2 Data

We collect information on interest rates, growth, inflation and public debt for a sample of 31 AEs and 25 EMs. Table A1 lists the countries in our sample and, for each country, the sample period for which their data are available.

We start from the Macrohistory database (Jordà, Schularick and Taylor, 2017), which collects a large set of macroeconomic variables, including long-term nominal interest rates, for a sample of 17 AEs.² We update those data to 2019 and construct a *long* sample, that we use to look at the relationship between $r-g$ and debt exploiting mostly the time dimension post World War II. In updating the data, we measure g and public debt using data from the IMF Global Debt Database (Mbaye, Moreno Badia and Chae, 2018), to avoid a discontinuity in the series between 2016 (the last year in the Macrohistory database) and 2017-2019. Data for those years on long-term nominal interest rates are from Bloomberg and refer to the yields on 10-year government bonds. Throughout the analysis, we measure r using the marginal rate on government debt (the interest rate on new government borrowing or on the secondary market) rather than the effective interest rate (the ratio of the interest expenses to government debt), as the former responds faster to changes in global and domestic conditions (Mauro and Zhou, 2020).

We then expand that sample to include other 14 AEs and 25 EMs. We complement data on nominal long-term interest rates from a variety of sources, such as the IMF International Financial Statistics, OECD, Bloomberg, and Thomson Reuters. In these cases, we started using data on 10-year government bonds but, when not available, we used information on other long-term maturities.³

When looking at the role of public debt structure, we collect data on foreign currency denomination, and foreign holdings from the Fiscal Space Dataset (Kose, Kurlat, Ohnsorge and Sugawara, 2017), which we complement with information of foreign currency denomination of sovereign debt from Arslanalp and Tsuda (2014), individual central banks, finance ministries,

²The 17 AEs are Australia, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Norway, Portugal, Spain, Sweden, Switzerland, the UK, and the U.S.

³An important caveat of the empirical analysis is the measurement of r . Since we look at the yields on nominal long-term bonds in local currency, the cost of borrowing includes credit and currency risk and does not take into account that some countries, especially emerging markets, issue bonds in foreign currency. As foreign currency bonds carry lower yields, the ex-ante effective borrowing costs can be lower than what implied by nominal local currency yields. By focusing explicitly on the role of foreign currency denominated public debt, our analysis partially addresses this problem, as it allows for a different exposure to the shock—and therefore a different reaction of r to shocks—depending on the share of debt denominated in foreign currency.

and the Asian Development Bank.⁴

Since we are interested in the response of r to domestic and global shocks, we exclude countries with closed or almost closed capital account, as measured by the Chinn and Ito (2006) financial openness index⁵, since interest rates of these countries are much less likely to be market-driven and more prone to financial repression. Moreover, these countries tend to have fixed exchange rates, which complicate our study on the role of debt denomination as a transmission mechanism. As an illustration, consider an extreme case: a country with a credible peg to the U.S. dollar and no default risk. In this case, both local- and U.S. dollar-denominated debts of the country are practically denominated in dollars, and the interest rates (bond returns) will by and large follow the U.S. interest rates and reflect more of the fundamental of the U.S. economy than the local economy.

Other macroeconomic data are taken from the IMF World Economic Outlook. We end up with a *large* sample of 56 countries. To avoid having our results driven by changes in the sample composition, we restrict the sample period to the last 20 years (2000-2019). In this way we define an almost balanced sample, as all countries have at least 12 years of data for the key variables and 41 countries have data for the entire period.

The final part of the analysis uses daily data on nominal 10-year local currency government bond yields for 50 countries between January 2000 and December 2018, obtained from Bloomberg and Thomson Reuters. We measure global volatility shocks using daily U.S. VIX data, from the Federal Reserve Economic Data (FRED).

Finally, we winsorize our main variables at the 1 and 99 percentiles to minimize the impact of outliers. We compute country i 's $r-g$ in year t ($r-g_{i,t}$) as nominal local currency long-term rates $r_{i,t}$ minus nominal local currency annual growth rates $g_{i,t}$. Ideally, we would like to match the duration of the nominal interest rate and growth, that is, if we use annualized 10-year rate then we use the annualized expected growth over the same period, since theoretically the long-term rate is a function of both growth today and growths in the future. A matched duration also means that computing expected inflation is unnecessary in measuring $r-g$ since the expected inflation will fully cancel out: the nominal $r-g$ equals the real $r-g$. Given our $r-g$ measurement,

⁴We use the data from the Asian Development Bank for HongKong, Indonesia, Japan, Malaysia, the Philippines, Singapore, South Korea, Thailand, and Vietnam. We use the data from individual central banks and finance ministries for Chile, Denmark, Hungary, Israel, New Zealand, Poland, and Turkey.

⁵In particular, we compute the average of the financial openness index since 2000 (using the latest update, available at: http://web.pdx.edu/~ito/Chinn-Ito_website.htm) and we exclude countries with a value of the index below 0.2. These countries are: China, Ghana, India, Namibia, Pakistan, Sierra Leone, South Africa and Tanzania.

the $r-g$ used in this paper may deviate from the true $r-g$. However, the direction of the deviation is not clear and may cancel out on average.

3 Public Debt and $r-g$ at Risk

In this section we show a set of stylized facts to illustrate the association between public debt and $r-g$ dynamics, including the tail risks in $r-g$, and the role of the currency of debt denomination.

3.1 Public Debt, Negative $r-g$ Spells, and Reversals

Given that the current discussion is about the possibility that $r-g$ will remain negative for a relatively long period of time, we start by looking at the duration of negative $r-g$ spells over the past decades. With the caveats that there are reasons to argue that *this time is different*⁶, it is still instructive to start by looking at what history tells us.

Our first finding—illustrated in panel A of Figure 2—is that episodes of negative $r-g$ are on average shorter, the higher the *initial* public debt-to-GDP ratio. In the panel dataset of 17 AEs from 1950 to 2019⁷, the average duration of the 72 negative $r-g$ episodes is 7.7 years, but the length of the negative $r-g$ spells has been decreasing over time (it is 5.3 years since 1980). Interestingly, the length of these spells is decreasing with the initial public debt-to-GDP ratio, measured in the year before the start of the episode. When the debt-to-GDP ratio is 10-percentage-points higher, the duration of negative $r-g$ episodes is, on average, almost 5 months shorter.⁸

We then look at the probability that $r-g$ turns from negative to positive in the same long sample of 17 advanced economies. The average probability that $r-g$ turns positive over a five-year period, conditional on the current (and previous year) $r-g$ being negative, is around 30 percent, similar to the value calculated by Mehrotra and Sergeev (2019). An interesting pattern suggesting a role for public debt in affecting the duration of $r-g$ is that the probability of a

⁶The literature has proposed several factors to explain the decline of the natural interest rate (Caballero, Farhi and Gourinchas, 2017; Del Negro, Giannone, Giannoni and Tambalotti, 2017; Eggertsson, Mehrotra and Robbins, 2019; Kozlowski, Veldkamp and Venkateswaran, 2019; Mian, Straub and Sufi, 2020).

⁷This part of the analysis can be done only on a smaller set of 17 AEs for which the time coverage is long enough to look at negative $r-g$ spells.

⁸A regression of the length of the negative $r-g$ episodes against the initial debt-to-GDP ratio, absorbing country fixed effects, gives a point estimate on the debt ratio of -0.043 (p-value = 0.04). This pattern holds true controlling or not for country fixed effects.

reversal increases with the level of public debt, from about 25% for countries with debt-to-GDP ratios below the sample median to almost 75% for countries in the top quartile of the public debt-to-GDP distribution (Figure 2, panel B).

These general trends would suggest that one cannot take for granted the current environment of low (and negative) $r-g$, especially because the interest rate-growth differential is endogenous to the size and dynamics of public debt (Wyplosz, 2019). The short duration of the $r-g$ spells suggests that even if the long-run average of $r-g$ is negative, one cannot rule out the possibility that adverse shocks can drive $r-g$ to become positive, at least in the short run. In that case, public debt sustainability should hinge on a belief that $r-g$ would come back to its long-run average. However, the benefit of this belief would be smaller with a stronger rise in $r-g$. In the next two subsections, we look more deeply into the facts regarding the connection between the level and currency composition of public debt and the distribution of $r-g$.

3.2 Public Debt and $r-g$ Downside Risk: Quantile Regressions

Our second set of results show that a higher public debt now is associated with a higher probability of an exceptionally high $r-g$ in the future, as the $r-g$ distribution becomes more right-skewed. To quantify the relationship between public debt and the downside risk of $r-g$, we apply the growth-at-risk approach developed by Adrian, Boyarchenko and Giannone (2019) and estimate the distribution of future $r-g$ as a function of the current level of public debt.

Before moving to the quantile regressions, we could already observe the pattern in the raw data. Figure 3 and Table A2 show and summarize the distributions of annual $r-g$ for different groups of countries constructed based on the quartiles of the *ex-ante* debt-to-GDP ratio (left panels) and the 3-year change in the debt-to-GDP ratio (right panels). Moving from low- to high-debt groups, the conditional mean of $r-g$ increases, and, importantly, the probability of exceptionally high $r-g$ increase. As reported in Table A2, the shares of $r-g$ observations that are positive, higher than 2%, and higher than 4% are all increasing monotonically from the lowest to highest debt groups. For instance, in the large sample, the probability of a positive $r-g$ is 24% for the lowest quartile group but around 65% for the top 5% debt group. Related to the tail risks, the share of $r-g$ higher than 4% increases from 7% to 23% moving from the first quartile to the top 5% debt group. This share increases even more, from 3% to 48% for the groups constructed based on 3-year changes debt. In general, the patterns are more striking in the

groups are based on the change in public debt, consistent with the evidence that the dynamics of the public debt-to-GDP ratio plays a critical role for market access (Bassanetti, Cottarelli and Presbitero, 2018).⁹

Quantile regressions: methodology and specification. We estimate the conditional $r-g$ distribution using quantile regressions, in the same spirit of Adrian, Grinberg, Liang and Malik (2018). Suppose the τ^{th} quantile of $r-g$ distribution as a function of X is $X\beta_\tau$, the quantile regression coefficient β_τ is the value that minimizes the τ -weighted mean absolute deviation:

$$\beta_\tau = \arg \min_{\beta \in \mathbb{R}^k} \sum_{i=1}^N \rho_\tau(\underbrace{y_i - X_i\beta_\tau}_{\epsilon_i}), \text{ where } \rho_\tau(\epsilon) = \tau \cdot \max(\epsilon, 0) + (1 - \tau) \cdot \max(-\epsilon, 0). \quad (1)$$

We estimate the distribution of future $r-g_{t+k}$ as a function of a constant, the current public debt-to-GDP ratio ($debt_t$), and a set of control variables. In the baseline, we take the two-year and five-year averages of future $r-g$ as dependent variables. The set control variables include: the local-currency nominal interest rate r_t , the local-currency nominal growth rate g_t , the local-currency inflation π_t , and a dummy variable for large recessions in year t ($\mathbb{D}_{g,t}$).¹⁰

Estimated conditional distribution of $r-g$ as a function of public debt. The predicted value from the quantile regression of quantile τ gives the estimated τ^{th} percentile of $r-g$ distribution as a function of debt. Figure 4 plots the estimated 10th, 50th, and 90th percentiles of two-year (panel a) and five-year (panel b) average future $r-g$ for different level of the current public debt-to-GDP ratio. The charts show that a higher public debt is associated with higher future $r-g$ across quantiles, and, importantly, the increase of the 90th percentile of $r-g$ is significantly higher than the increase of the median $r-g$. This means that, as current public debt increases, the $r-g$ distribution becomes more right-skewed and entails a higher downside risk. Also, it is interesting to note that the increase in the downside risk is not compensated by a higher upside risk: in the good state (e.g., around the 10th percentile), countries with higher public debt also experience higher $r-g$. In other words, countries with high public debt suffer more when the

⁹An important concern is that these patterns are driven by short-term fluctuations in $r-g$ and, as long as there is mean reversion, they would have no consequences for the long-run path of $r-g$ and thus do not matter for debt sustainability. To address and alleviate this concern, we replicate the same exercise taking 5-year non-overlapping averages of $r-g$ and find the same patterns (Figure A1).

¹⁰The dummy variable for large recession equal to 1 if current period growth is in the lower two deciles of growth distribution. This condition helps control for low future $r-g$ simply driven by growth reversal. Results are robust to dropping this variable.

very bad state materializes and gain less when the very good state materializes. Note that both the increase in the downside risk and decline in the upside risk lead to a more right-skewed $r-g$ distribution.

To illustrate the economic significance of the coefficient, Figure 4 shows that, as the current debt-to-GDP ratio increases from 40% to 120%, the 90th percentile of average $r-g$ over the next five years increases from around zero to 2 percent. Meanwhile, the median and 10th percentile also increase, by around 0.8 and 1.2 percentage points, respectively. The $r-g$ over the next two years exhibits the same patterns. Between the two distributions, the decline in the upside risk is more pronounced in the $r-g$ over the next two years.

Figure 5 plots that point estimates and confidence intervals of the coefficients of the debt-to-GDP ratio in the quantile regressions at different quantiles, from the 5th to the 95th quantiles, for the 2-year (panel a) and 5-year (panel b) horizons. Indeed, as shown by the slopes in Figure 4, debt coefficients at the 90th quantile are much higher than at the median. For the $r-g$ over the next two years, debt coefficients at the 10th quintiles also are much higher than the median. Importantly, the figure shows our results hold over a wide range of lower and upper quintile definitions, not only at the 10th and 90th quintiles. Figure 5 also shows that the debt coefficient at the 95th percentile of $r-g$ over the next five years is larger than over the next two years. This pattern is consistent with the idea that higher government spending may boost the economy in the short term (and thus a smaller β_{debt}), but the full costs of debt (such as high risk premium and low long-term growth) often surface only in medium term.¹¹

Estimated conditional distribution of r and g separately. Figure A3 shows the same patterns hold when we apply the quintile regressions on interest rates and growths separately. Both future interest rates and growths exhibit higher downside risks and/or lower upside risks when current public debts are higher. The patterns in Figure A3 suggest both r and g contribute to the relation between public debt and $r-g$ tail risks.

However, note that the sum of the debt coefficients of the separate r and g quantile regressions are unlikely to equal to the debt coefficient in the $r-g$ regression since the covariance between r and g is relevant. Theoretically, public debt may lead to higher downside risk in $r-g$

¹¹Standard OLS panel regressions also show the significance of the relationship between public debt and $r-g$ (Table A3). It is important to note the relation between public debt and $r-g$ downside risk identified in the quintile regressions means the standard OLS-regression analysis may considerably underestimate the implication of high public debt on country's vulnerability.

especially because public debt affects both r and g such that they tend to move in opposite directions. For example, the interest rate tends to be high following large adverse growth shocks and growth tends to be low following large adverse shocks on risk premia. If interest rates and growth behave differently in response to small shocks or in normal times in general¹², then this exercise would underestimate the relationship between public debt and $r-g$ at risk.

Robustness checks. The left panel of Figure A4 shows that results are robust to excluding countries with unusually large deficit or level of debt (e.g., Greece and Japan). The results also hold if we drop from the sample one country at the time (for brevity, results are not reported in the paper). In addition, the right panel of the figure shows that results are robust to the exclusion of the crisis dummy $D_{g,t}$ from the set of control variables.

3.3 Public Debt Structure and $r-g$.

In our sample of AEs and EMs, we find evidence of the importance of currency denomination in capturing $r-g$ variation. Figure 6 shows a motivating pattern in the raw data. Specifically, it shows the average of $r-g$ in three groups constructed based on: i) the median of public debt-to-GDP ratio in the overall sample (used to define the low- and high-debt groups), and ii) the median of share of foreign currency denominated debt in the high-debt sample (used to define the high-debt and low-FX share, and the high-debt and high-FX share groups).

Consistent with Figure 3, we find that low-debt countries exhibit lower average and median values of $r-g$ than high-debt countries. What is also interesting is comparing observations in the high-debt sample, separating between those in which the share of foreign currency denominated debt is below and above the median. While high-debt countries with a low share of foreign currency denominated debt still have low and negative interest rate-growth differentials, it is when countries combine a high debt with a high share of foreign currency denomination that $r-g$ becomes positive and significantly larger than in the rest of the sample.

Table A4 confirms the importance of foreign currency denominated public debt to explain the variation of $r-g$. A simple regression of $r-g$ against public debt, decomposed into the local and foreign currency parts, shows that the dynamics of the interest rate-growth differential is driven by foreign currency denominated debt, especially in the within-country dimension

¹²For example, it is possible that r and g are less negatively correlated, weakly correlated, or even positively correlated in normal times.

(columns 2 and 4).

4 Public Debt and $r-g$ Exposure to Shocks

Having assessed in a large sample of countries that high and growing public debts are a source of risk for the dynamics of the interest rate-growth differential, in this section we look at a potential important mechanism behind these stylized facts. Given the importance of negative shocks for the dynamics of $r-g$, especially in EMs (Kose, Nagle, Ohnsorge and Sugawara, 2020), we study the importance of public debt in amplifying the impact of adverse domestic and global shocks. As high public debt levels could trigger liquidity events and the stylized facts clearly point to a connection between public debt levels and the distribution and dynamics of $r-g$, finding that high debts amplify the impact of adverse shocks on $r-g$ should raise concerns about the recent (and current) increase in public debt.¹³

Specifically, we examine whether ex-ante public debt explains how interest rates respond to: i) a substantial lower-than-expected domestic growth, and ii) a large change in the U.S. VIX. Mechanically, a negative growth surprise can increase the public debt-to-GDP ratio and could trigger concerns about debt sustainability through to a feedback loop: higher debt-GDP ratio may lead to lower growth, which in turn further drives up the debt ratio. This mechanism makes it more likely that public debt affects the transmission of growth shocks. Through a similar argument, the impact of U.S. VIX shock could also depends on public debt levels.

4.1 Domestic Negative Growth Shocks

Specification. Growth shocks are computed as the difference between realized GDP and its expectation as published by the Consensus Forecast. In particular, a growth shock of country i in year t is the difference between country i 's realized year- t GDP growth and the expected year- t GDP growth as of October in year $t-1$.

We estimate the following equation:

$$r_{i,t} = \alpha r_{i,t-1} + \sum_{c=1}^4 \beta_c (1_c \times \epsilon_{i,t}^g \times debt_{i,t-1}^c) + \gamma_g \epsilon_{i,t}^g + \sum_{c=1}^4 \gamma_c (1_c \times debt_{i,t-1}^c) + \eta_i + \tau_t + \omega_{i,t}, \quad (2)$$

¹³One caveat is important to keep in mind: in the data, rising public debt went together with a secular decline in interest rate, and our analysis does not consider that it becomes easier to finance public debt when interest rates are lower. However, if such a secular trend of interest rate is behind our results, it should make it harder rather than easier for us to detect a role of public debt in exacerbating undesired consequences of adverse shocks.

where $r_{i,t}$ denote country i 's long-term interest rate in year t ; 1_c denotes a dummy indicating whether a country falls into group c —one of four country sub-groups that are defined based on the level and currency composition of public debt; $\epsilon_{i,t}^g$ is a dummy equal to 1 for negative growth shocks of 1% or larger (in absolute value), and zero otherwise; and $debt_{i,t-1}^c$ is the public-debt-to-GDP ratio in year $t - 1$, divided in four debt group ($c = 1, \dots, 4$). First, we split the observations in low- and high-debt depending on being below or above the median, respectively. Then, within each group, we define the low- and high-foreign currency share, again across the sample median. In this way, we estimate four β_c coefficients that measure the effect of a negative growth surprise on long-term interest rates, conditional on past debt level and structure. As a baseline, we also estimate a simple model in which we have the negative growth surprise dummy as a standalone explanatory variable to measure its average effect on r , controlling for the lagged public debt-to-GDP ratio and interest rates. The country (η_i) and time (τ_t) fixed effects absorb time invariant unobserved country characteristics and common shocks across countries which can explain part of the variation in interest rates. Standard errors are clustered within country.

Results. Our findings are summarized in Figure 7, which shows that, on average, interest rates increase in response to a contemporaneous negative domestic growth shock. The point estimate indicates that interest rates increase by 75 bps in response to a negative growth shock of at least 1 percent. More important for our analysis, this average effect increases for countries with higher ex-ante public debt, especially if a large part of the debt is denominated in foreign currency, while the response of r to a growth shock is muted for countries with a low share of foreign currency denominated public debt. In particular, a negative growth shock raises interest rates by 72 bps in countries with high public debt and a low share of foreign currency denominated public debt, but by 155 bps when high public debt is associated with a high share of foreign currency denominated debt. These results are not driven by the negative growth shocks associated with the global financial crisis (GFC). Excluding 2008 and 2009 from the sample delivers qualitatively similar results (with magnitudes moderately larger, see Figure A5). In addition, results hold also within the sample of advanced countries, see Figure A6).¹⁴ These findings are consistent with the hypothesis that, as risk premia increase after bad news,

¹⁴The full set of results are show in the Appendix, see Table A5.

countries with high public debt experience significant increases in borrowing costs, leading to an unfavorable $r-g$ dynamics.

4.2 Global Volatility Shocks

Specifications. When looking at global volatility shocks, defined as changes in the U.S. VIX over a 2-day period, we can take advantage of daily data and estimate a more flexible specification to trace out the effect of the shock on long term nominal interest rates over a 20-day horizon. In particular, we estimate the following equation, based on a standard local projection method (Jordà, 2005):

$$\Delta r_{i,t+h} = \beta_h \epsilon_t^{VIX} + \eta_i + \omega_{i,t}, \quad (3)$$

where $\Delta r_{i,t+h}$ denote the change in long-term nominal interest rates of country i from day t to $t+h$, ϵ_t^{VIX} is a measure of global volatility shocks and it is equal to the two-day change in the VIX, computed between $t-2$ to t . To capture differences in the level of interest rates across countries we include country fixed-effects (η_i are). The coefficients β_h trace the effect of the U.S. VIX shock on interest rates over the 20-day window.

To look at the role of public debt we augment equation 3 by including the interaction term between ϵ_t^{VIX} and two dummies that identify country-year observations for which public debt and the share of debt in foreign currency are: 1) both low, and 2) both high.¹⁵ In this way, we can trace out the impact of a global shock separately for low-debt and low-foreign currency debt countries and for high-debt and high-foreign currency debt countries.

Results. Long-term nominal interest rates increase in response to a negative global shock. This effect is statistically significant, economically large and persistent (Figure 8, panel A). On average, an increase in the U.S. VIX of 5 percent over a 2-day window (which is close to the top quartile of the sample distribution) is associated with 33 bps increase in log-term interest rates after 10 days.

This average response, however, hides large differences, depending on the level and structure of public debt. In particular, in periods of global uncertainty, interest rates actually decline

¹⁵As in the previous exercise, we first split the countries in low- and high-debt depending on being below or above the median of the yearly sample distribution of the public debt-to-GDP ratio, respectively. Then, within each group, we define the low- and high-foreign currency share, again across the sample median for each year.

for countries with low debt levels and a low share of foreign currency denominated public debt. By contrast, when debt is high and mostly denominated in foreign currency, interest rates are particularly sensitive to changes in the VIX. The same 5 percent increase in the U.S. VIX over a 2-day window leads to a response of interest rates almost 4 times larger than the average, with long-term nominal rates increasing by about 130 bps (Figure 8, panel B).

Finally, to better measure the importance of public debt for the sensitivity of interest rates to global shocks, we estimate equation 3 separately for each country-year pair (hence, dropping the country fixed effect) and fixing the horizon to compute the change in interest rates to 10 days ($h = 10$). In this way, we can retrieve a set of country-year specific coefficients that measure the response of interest rates to changes in the U.S. VIX after 10 days. Plotting these coefficients against the initial public debt-to-GDP ratio and absorbing country and year fixed effects clearly shows a positive relationship (Figure 9, panel A): when public debt is higher, so it is the elasticity of r to global shocks. The positive slope is economically meaningful, as the elasticity of r to a VIX shock (which is equal to 0.085 on average) increases by 0.06 when the debt-to-GDP ratio increases by 10 percentage points. However, this positive relationship vanishes for safe haven countries (the U.S., the UK, Japan, Switzerland, and Germany). In those countries, bond yields generally decline in response to increases in the U.S. VIX (the average elasticity of r to a VIX shock is equal to -0.09)—consistent with a flight to safety during periods of global uncertainty—and their debt ratios do not explain the response of interest rate to global uncertainty (Figure 9, panel B).

5 Conclusion

Our results suggest that a high public debt can make countries more vulnerable to an increase in $r-g$, even in a low interest rates environment. Hence, some degree of caution is needed when assuming that the interest rate-growth differential will remain negative, especially when public debt is high (and growing) and in countries where public debt is largely denominated in foreign currency. To support these claims, we highlight three empirical patterns. First, negative $r-g$ episodes are not the norm and they may not last long. Moreover, reversals from negative to positive values of the differential are more common when public debt is higher. Second, a higher (and increasing) public debt is associated with higher average $r-g$, and higher probability of extremely high $r-g$, increasing downside risks. Third, and more important, higher public

debts amplify the vulnerability of the dynamics of $r-g$ to domestic and global shocks, especially when the share of public debt denominated in foreign currency is large.

There is a widespread consensus that countercyclical fiscal policy is less costly and more effective at the zero lower bound. Our findings do not question this argument, but they show the potential risks of loose fiscal policy amid the low interest rates environment. As the current pandemic crisis shows, large negative shocks happen and a sound fiscal stance can help to weather the crisis. In particular, while safe haven countries may face very limited risks of surges in $r-g$ and have more to gain from expansionary fiscal policy, the balance of risks is tilted to the downside for countries with high public debts and a high share of foreign currency denominated public debt.

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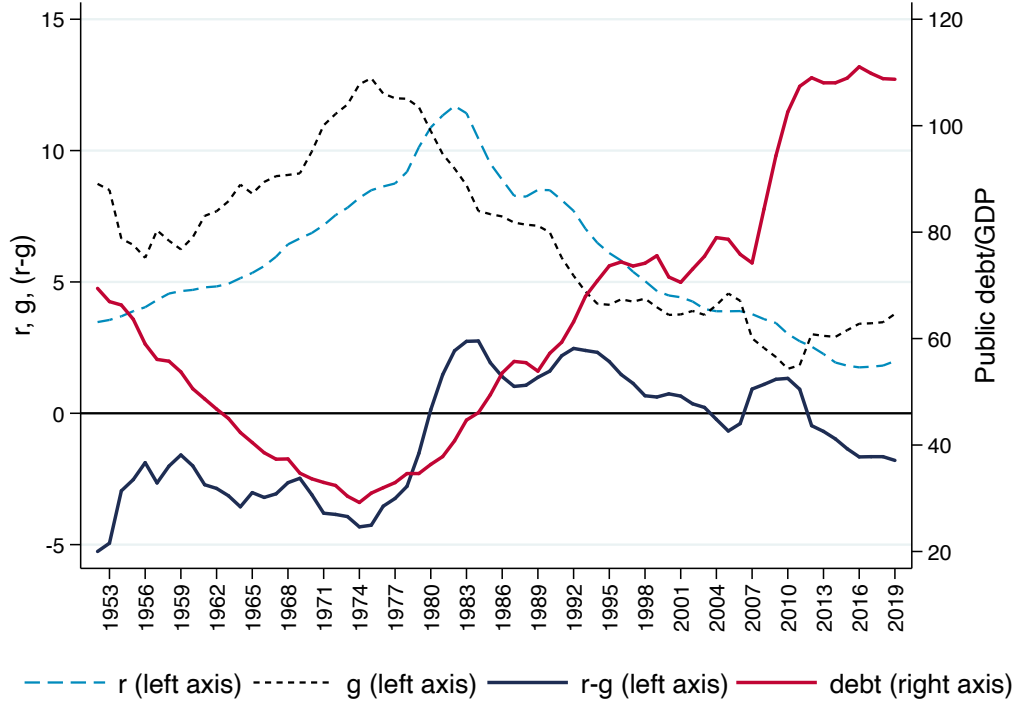
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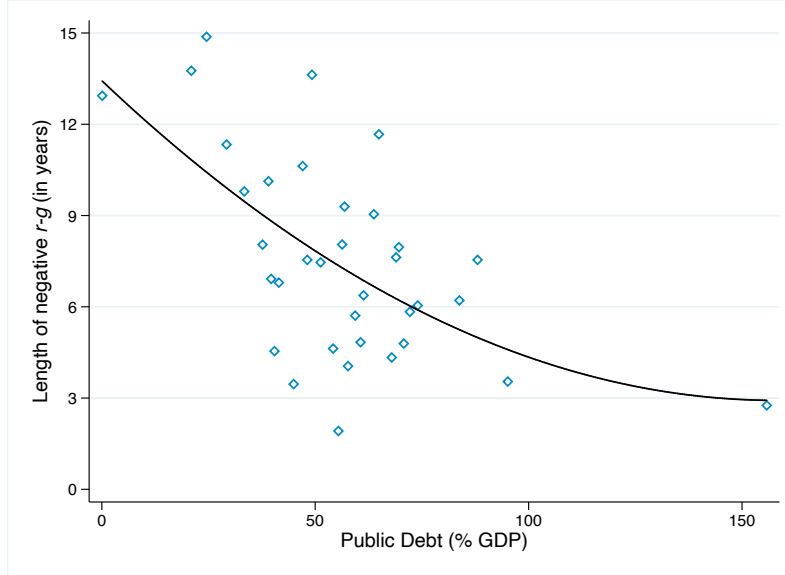
Figures

Figure 1: Interest Rate, Growth, and Public Debt

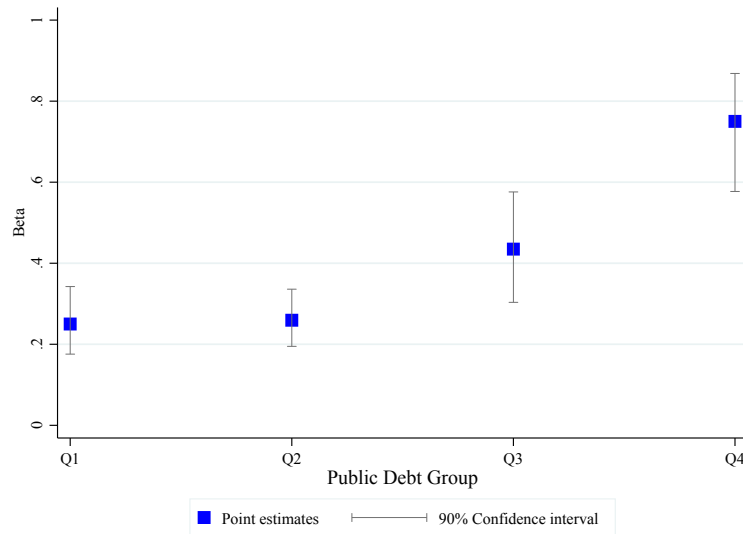


Notes: This figure plots the time series of the world's interest rate r , growth rate g , interest-growth differential $r-g$, and public debt-to-GDP ratio based on 17 advanced economies over the period 1950-2019. The world's values are GDP-weighted averages. All variables are 5-year moving averages. $r-g_t$ are computed as nominal local currency long-term rates r_t minus nominal local currency annual growth rates g_t .

Figure 2: Negative $r-g$ Spell, Reversals, and Public Debt



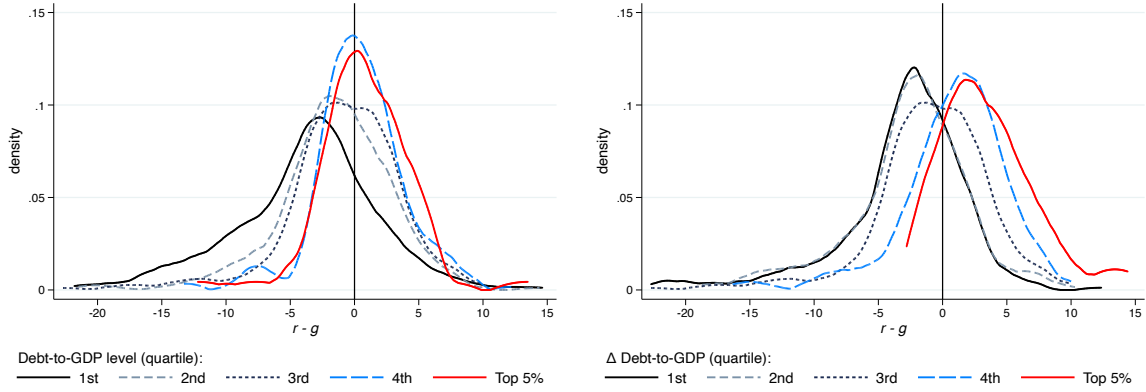
(a) Negative $r-g$ spells



(b) Reversals

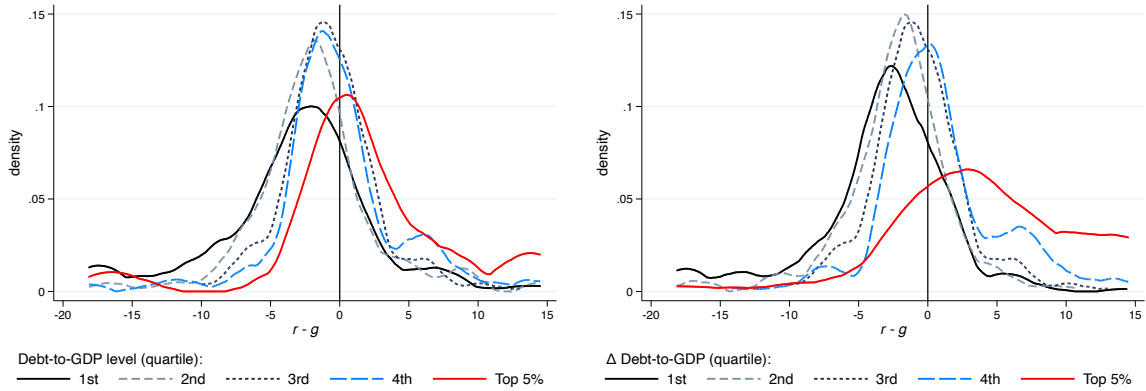
Notes: Panel A shows the lengths (in years) of negative $r-g$ episodes as a function of the initial public debt-to-GDP ratios $Debt$, measured in the year before the episodes. A negative $r-g$ episode is defined as consecutive periods (three years or longer) of negative $r-g$. All data are annual. $r-g_t$ are computed as nominal local currency long-term rates r_t minus nominal local currency annual growth rates g_t . The sample consists of 17 AEs over the period 1950-2019, during which there are 72 episodes starting from 1951 onward, with an average length of 7.7 years, minimum length of 3 years, and maximum length of 28 years. A linear regression of the length of the negative $r-g$ spells against the public debt-to-GDP ratio, controlling for country and year fixed effects, gives a coefficient of -0.043 (s.e.= 0.020). Using the same sample, Panel B plots the estimated probabilities (and the associated 90% confidence intervals) that the average $r-g$ over the next five years ($r-g_{t+5}$) is positive, given at least two consecutive years of negative $r-g$ (in year $t-1$ and t), as a function of the current public debt-to-GDP ratio. The probabilities are estimated using a logit model and regressing an indicator of positive future $r-g_{t+5}$ on an indicator of negative current $r-g_t$, indicators of debt groups based on the quartiles of the public debt-to-GDP distribution, and their interaction terms. $r-g_t$ denotes annual $r-g$ in year t and $r-g_{t+5}$ denotes annualized 5-year average $r-g$ from year $t+1$ to $t+5$.

Figure 3: $r-g$ Distribution and Public Debt



(a) Debt levels, 17 AEs

(b) Debt changes, 17 AEs

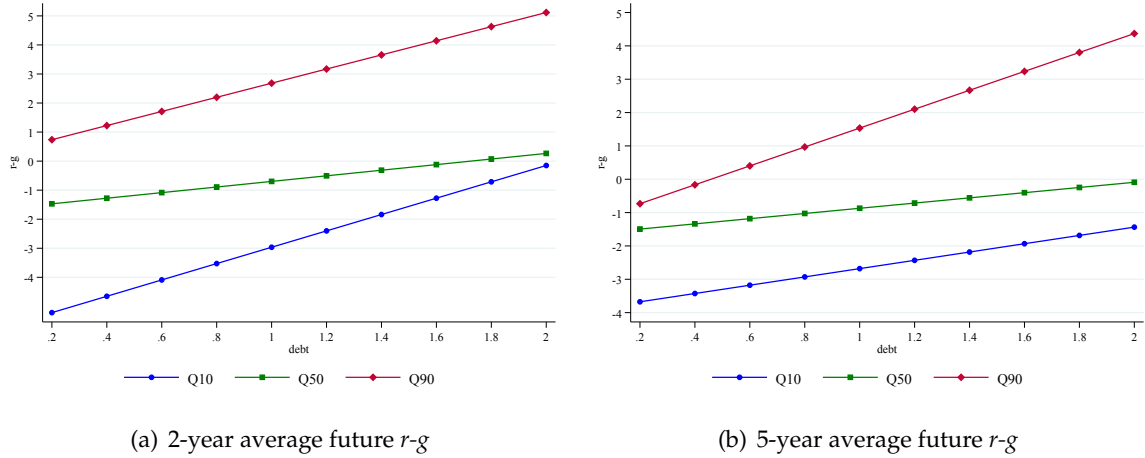


(c) Debt levels, 56 AEs and EMs

(d) Debt changes, 56 AEs and EMs

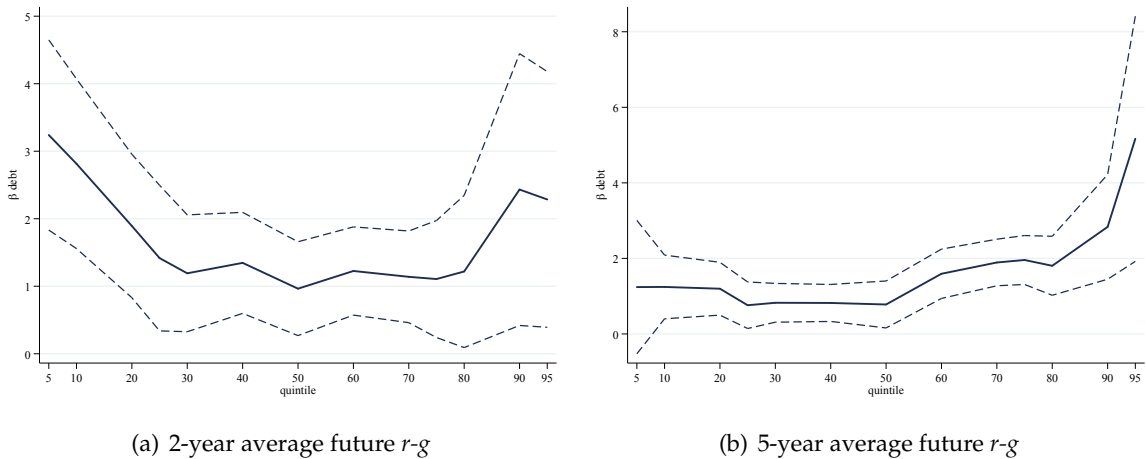
Notes: These figures show $r-g$ distributions of different debt groups constructed based on the quartiles of the sample distribution of debt-to-GDP ratio (panels a and c) or the 3-year change in the debt-to-GDP ratio (panels b and d) across time and country (*debt*). For example, $r-g$ of country i in year t is allocated to the low-debt group when $debt_{i,t}$ is below the 25th percentile of across-time-and-country *debt* distribution. The data in the 4th-quartile group are further split between the top 5 percent and the rest. All data are annual. $r-g_{i,t}$ is computed as nominal local currency long-term rates $r_{i,t}$ minus nominal local currency annual growth rates $g_{i,t}$. The sample includes 17 AEs from 1950 to 2009 in panels (a) and (b), and 31 AEs and 25 EMs over the period 2000-2019 in panels (c) and (d).

Figure 4: Public Debt and $r-g$ Risk: Estimated Quantile Regression Distributions



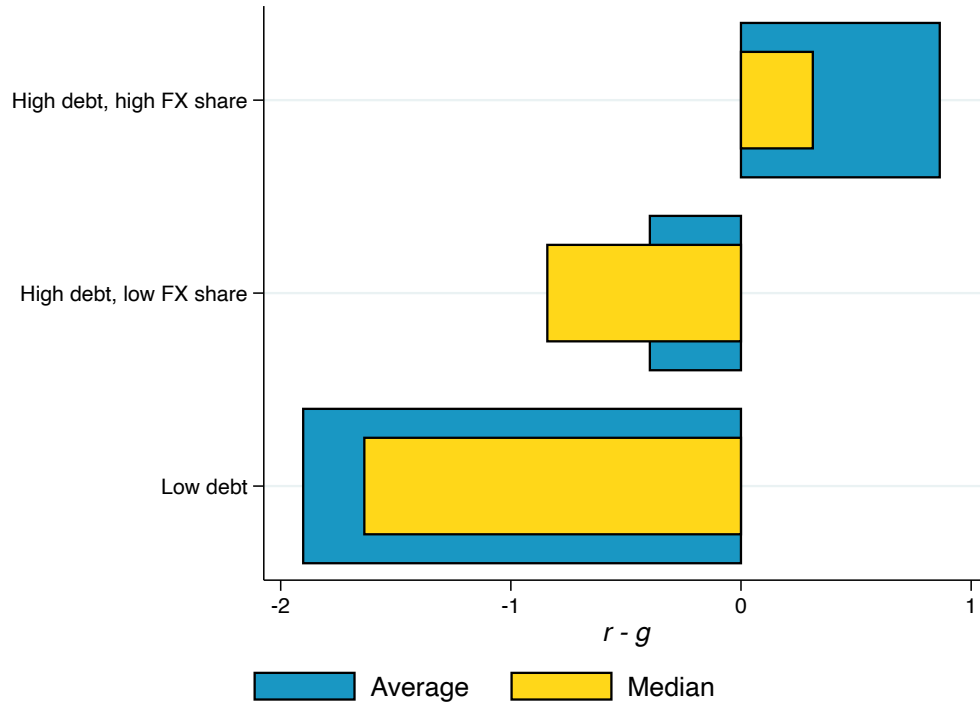
Notes: The figure shows the estimated future $r-g$ distribution as a function of current public debt based on the quantile regressions. See specification 1 for details; the model regresses k -year average future $r-g$ (the annualized average from year $t+1$ to $t+k$) on public-debt-to-GDP ratio in year t . $r-g_t$ is annual local-currency nominal rate in year t minus local-currency growth rate in year t . The sample includes 31 AEs and 25 EMs over the period 2000-2019.

Figure 5: Public Debt and $r-g$ Risk: Estimated Quantile Regression Coefficients



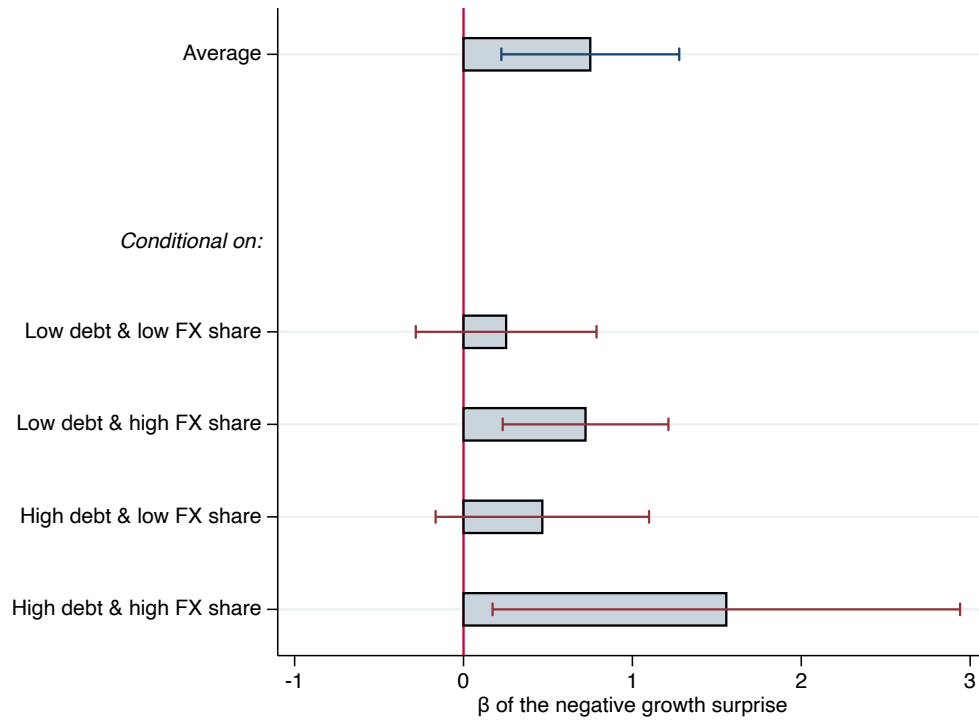
Notes: The figure plots the estimated coefficients of public debt (y axis) obtained from quantile regressions for different quantile values (x axis). See specification 1 for details; the model regresses k -year average future $r-g$ (the annualized average from year $t+1$ to $t+k$) on public-debt-to-GDP ratio in year t . Standard errors are obtained from bootstrapping techniques with 1,000 replications. The dotted lines represent the 90% confidence intervals. The sample includes 31 AEs and 25 EMs over the period 2000-2019.

Figure 6: $r-g$ and Foreign Currency Denominated Public Debt



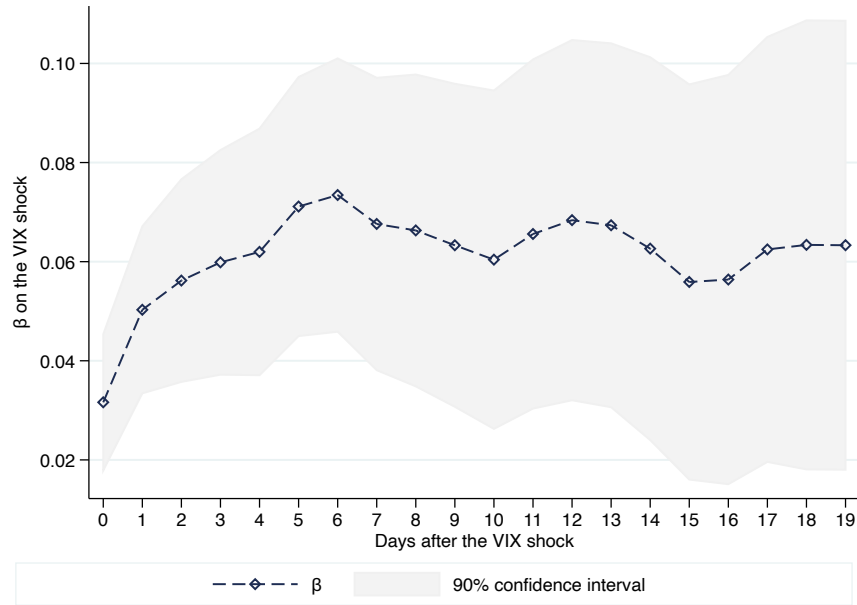
Notes: This figure plots the average and the median values of $r - g$ for three different group of country-year pairs, constructed based on the ex-ante debt-to-GDP ratios and shares of foreign currency denominated debt. Low debt includes all the observation in the bottom 75% of the sample distribution of the debt-to-GDP ratio. Then, the high-debt observations (top quartile) are further split into high and low share of foreign currency denominated public debt based on the median of the sample distribution. The data are annual. The sample includes 31 AEs and 25 EMs over the period 2000-2019.

Figure 7: Interest Rate Exposures to Negative Growth Shocks and Public Debt Structure

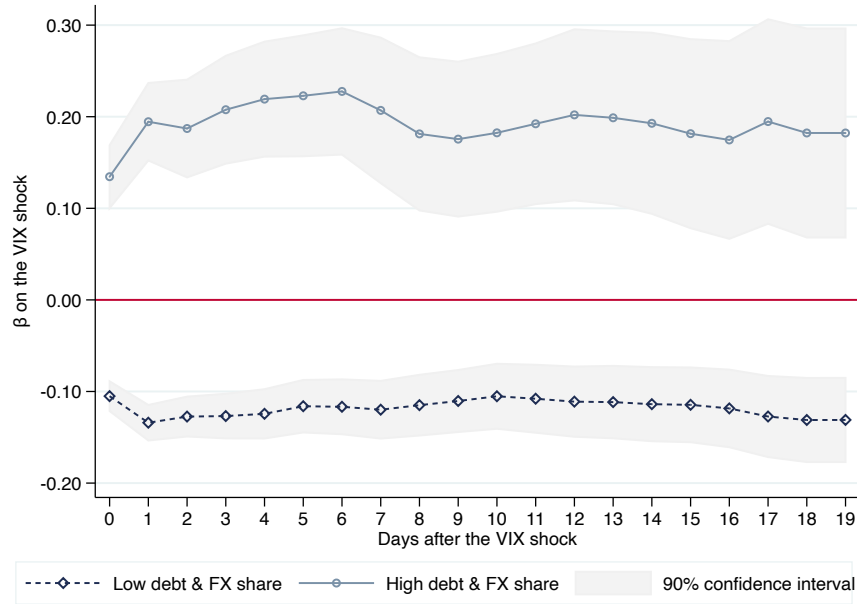


Notes: This figure shows the interest-rate exposures to large negative growth shocks for countries with different debt levels and different shares of foreign currency denominated debt. Specifically, the figure shows the estimated coefficients of regressing nominal long-term interest rates on the growth shock, defined as a dummy equal to 1 if the country realized GDP growth in year t is 1% or more lower than the forecast in October of year $t - 1$, and zero otherwise. This is the baseline 'average' bar. Then, the other bar shows the coefficients of the interaction terms between the growth shock and four different debt categories, defined splitting the observation in low- and high-public debt and then, within each category, between low- and high-foreign currency denominated public debt. The low vs high split is done along the sample median. The lines show the 90% confidence intervals of the point estimates. See equation 2 for details. The data are annual. The sample includes 29 AEs and 21 EMs over the period 2000-2019.

Figure 8: Interest Rate Response to Global Shocks



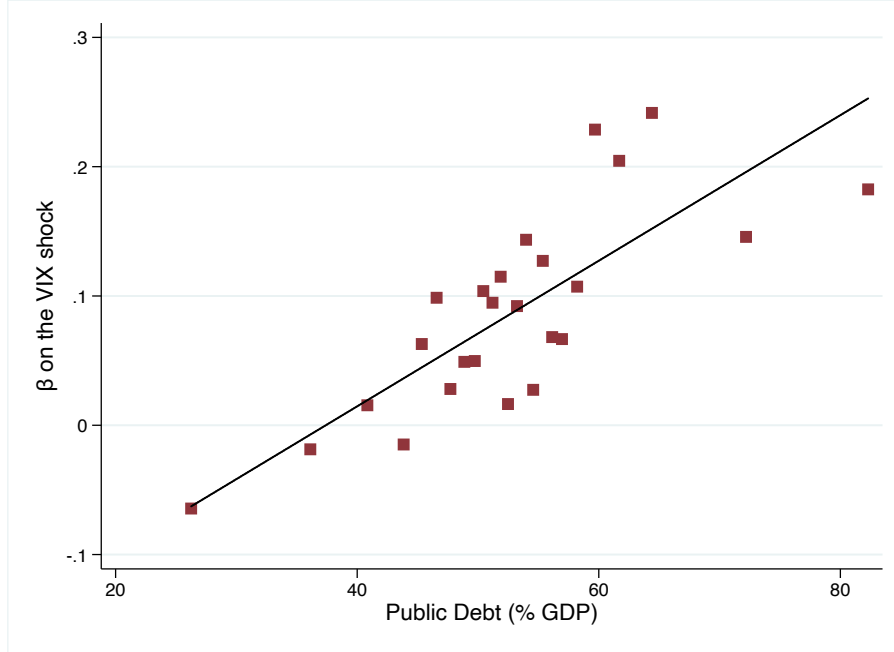
(a) All countries, pooled



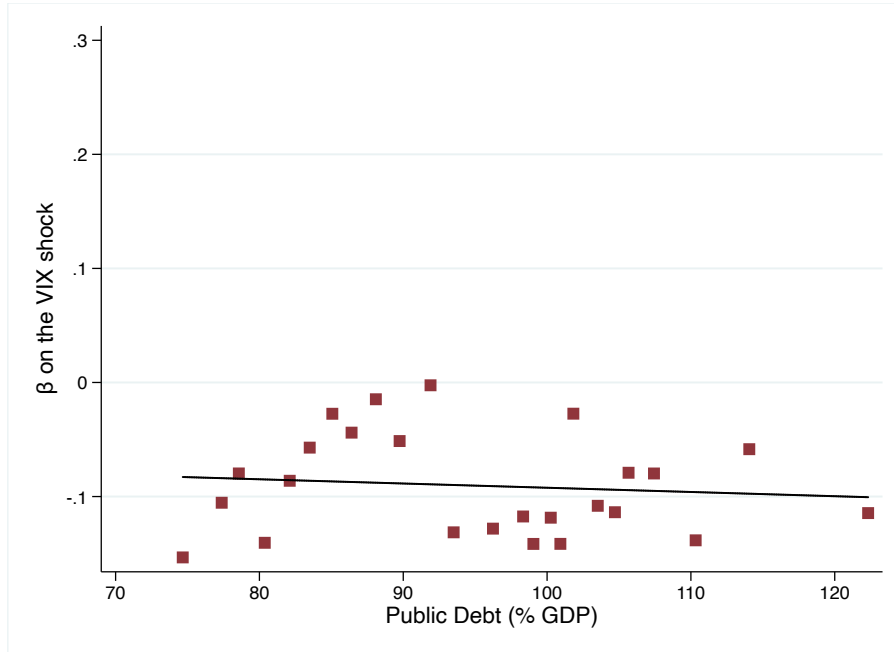
(b) High vs low public debt and foreign currency denomination

Notes: The chart in the top panel plots the impulse response function of regressing daily changes in long-term nominal interest rates computed over an 20-day period against the previous 2-day change in the U.S. VIX. The chart in the bottom panel plots the impulse response functions separately for countries that have low public debt and a low share of foreign currency denominated debt (dashed line) and for those that have high public debt and a high share of foreign currency denominated debt (solid line). See equation 3 for details. The groups are constructed based on the debt-to-GDP ratios and shares of foreign currency denominated debt in the year before the events. In each year t , the high and low classifications are based on the respective cross-country median values in year t . The shaded areas show the 90% confidence intervals. The interest rates are the 10-year local currency government bond yield, from Bloomberg. The sample includes 50 countries over the period January 2000-December 2018.

Figure 9: Public Debt and the Interest Rate Response to Global Shocks



(a) All countries (excluding safe havens)



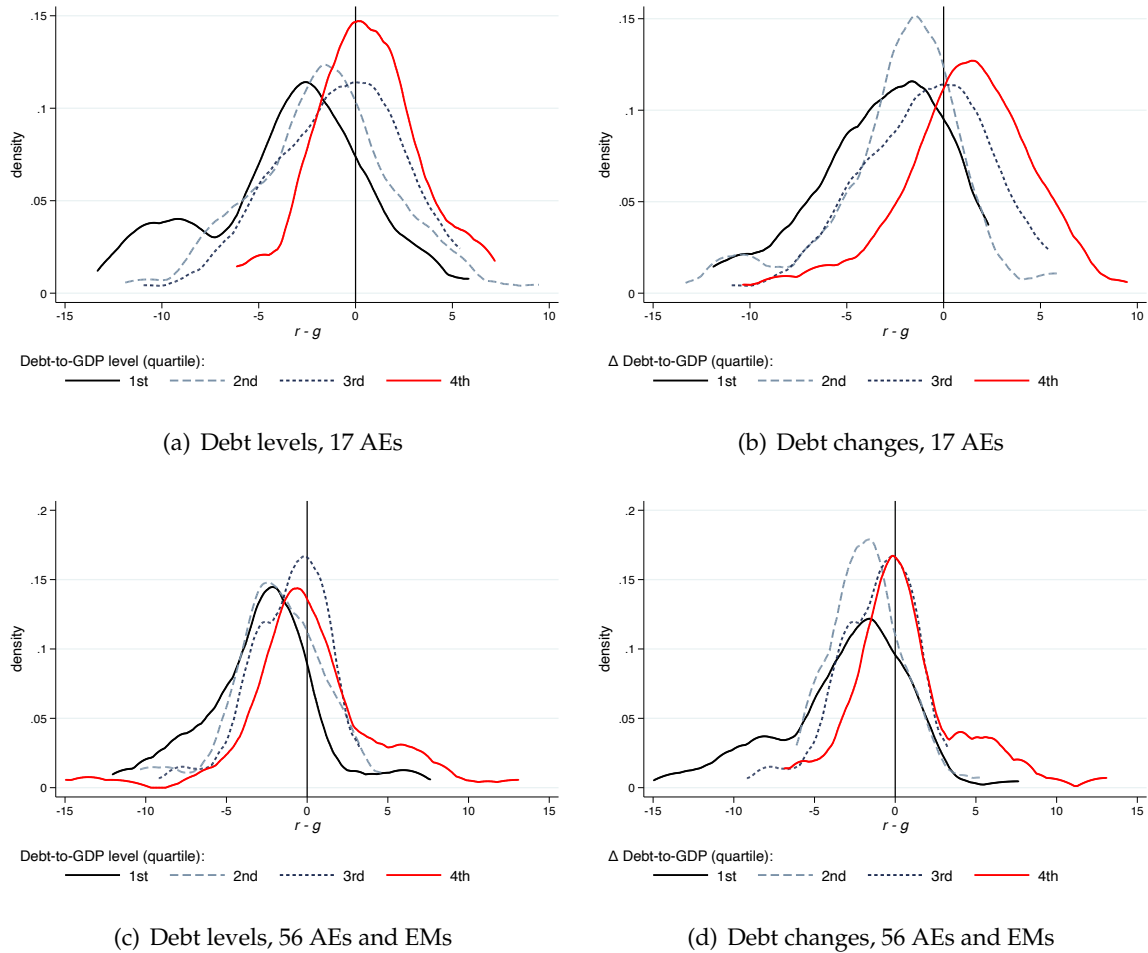
(b) Safe haven countries

Notes: These figures show a binned scatterplot of the interest-rate exposures to a 2-day change in the U.S. VIX against the country's *ex-ante* public-debt-to-GDP ratios. Specifically, the interest-rate exposures to a 2-day change in the VIX are the coefficients obtained by regressing 10-day interest rate changes on the two-day change in the VIX, estimated with country-year-level regressions. These country-year coefficients are then winsorized at the 1st and 99th percentiles and plotted against the debt-to-GDP ratios measured in year $t - 1$, absorbing country and year fixed effects. The sample includes 50 countries (of which 5 safe haven countries—the U.S., the UK, Japan, Switzerland, and Germany) over the period January 2000-December 2018. Panel A plots the relationship for the 45 countries (excluding safe havens), while Panel B plots the relationship for safe haven countries only. The point estimate on the debt-to-GDP ratio is equal to 0.006 (standard error = 0.002) in Panel A and -0.004 (standard error = 0.008) in Panel B.

Online Appendix—Not for Publication

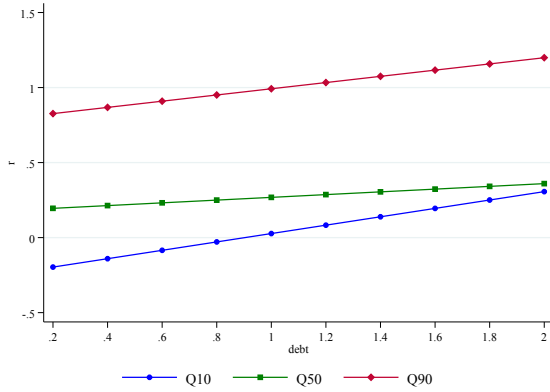
Additional Figures

Figure A1: $r-g$ Distribution and Public Debt, 5-year Non-overlapping Averages

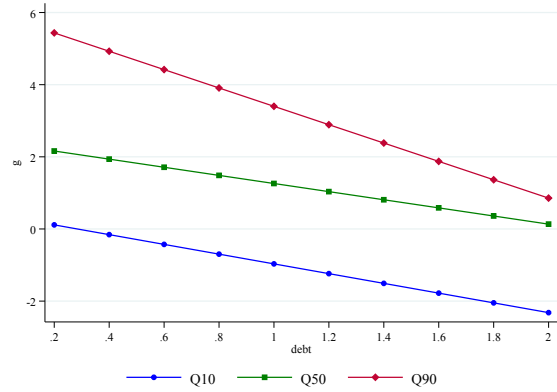


Notes: These figures show $r-g$ distributions of different debt groups constructed based on the quartiles of the sample distribution of debt-to-GDP ratio (panels a and c) or the 3-year change in the debt-to-GDP ratio (panels b and d) across time and country (*debt*). For example, $r-g$ of country i in year t is allocated to the low-debt group when $debt_{i,t}$ is below the 25th percentile of across-time-and-country *debt* distribution. The data in the 4th-quartile group are further split between the top 5 percent and the rest. All data are 5-year non-overlapping averages. $r-g_{i,t}$ is computed as nominal local currency long-term rates $r_{i,t}$ minus nominal local currency annual growth rates $g_{i,t}$. The sample includes 17 AEs from 1950 to 2019 in panels (a) and (b), and 31 AEs and 25 EMs over the period 2000-2019 in panels (c) and (d).

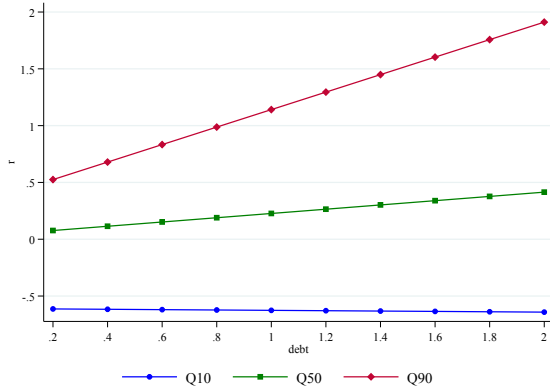
Figure A2: Estimated Quantile Regression Distributions of r and g



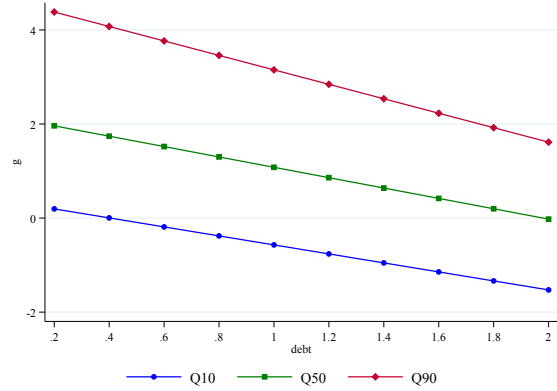
(a) 2-year average future r



(b) 2-year average future g



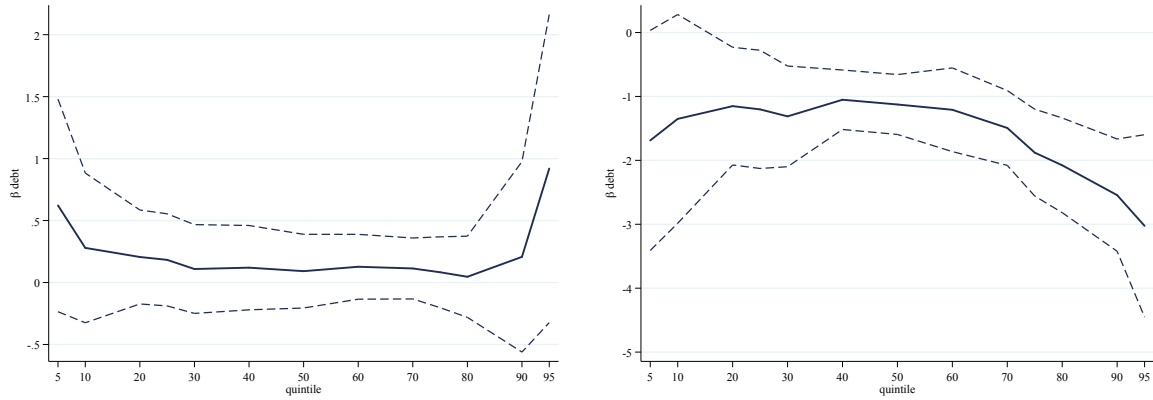
(c) 5-year average future r



(d) 5-year average future g

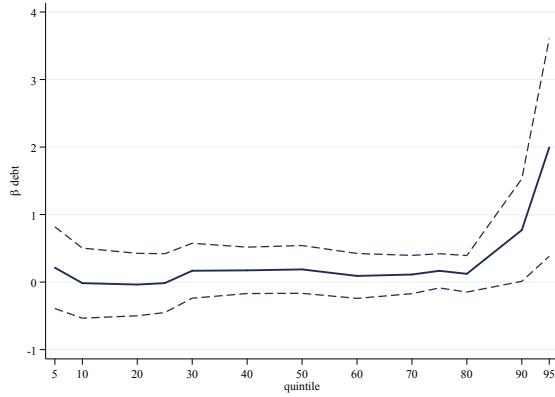
Notes: The figure shows the estimated distributions of future real interest rate r and real growth g as a function of current public debt based on quantile regressions. See specification 1 for details; the model regresses k -year average future y *var* (the annualized average from year $t+1$ to $t+k$) on public-debt-to-GDP ratio in year t . Panels (a) and (c) plot the results for future r , while panels (b) and (d) plot the results for future g . The top panels (a) and (b) refer to the 2-year window, while the bottom panels (c) and (d) refer to the 5-year window. Real interest rate r_t is annual local-currency nominal rate in year t minus local-currency inflation rate in year t . The sample includes 31 AEs and 25 EMs over the period 2000-2019.

Figure A3: Public Debt and $r-g$ Risk: Estimated Quantile Regression Coefficients

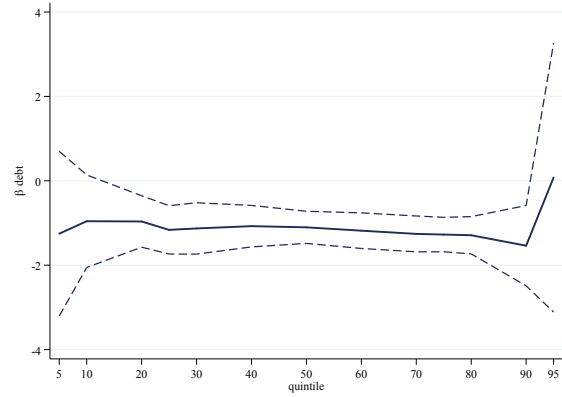


(a) 2-year average future r

(b) 2-year average future g



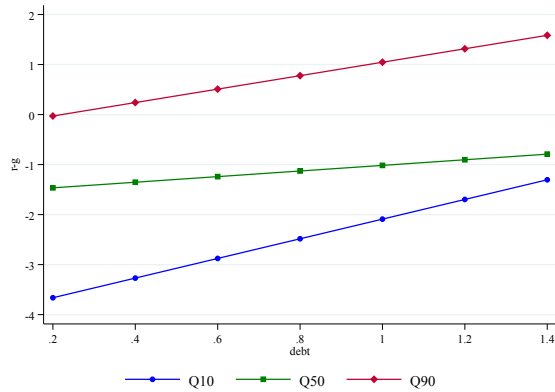
(c) 5-year average future r



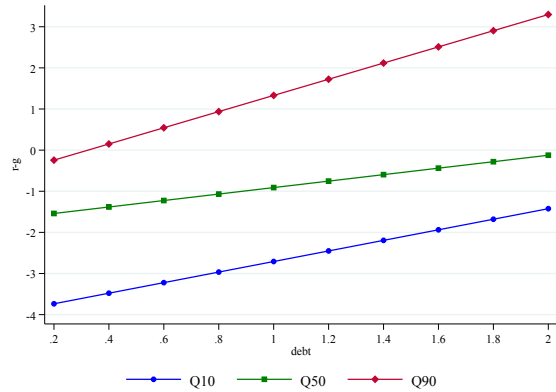
(d) 5-year average future g

Notes: The figure plots the estimated coefficients of public debt (y axis) obtained from quantile regressions for different quantile values (x axis). See specification 1 or details; the model regresses k -year average future r and g (the annualized average from year $t+1$ to $t+k$) on public-debt-to-GDP ratio in year t . Panels (a) and (c) plot the results for future r , while panels (b) and (d) plot the results for future g . The top panels (a) and (b) refer to the 2-year window, while the bottom panels (c) and (d) refer to the 5-year window. Standard errors are obtained from bootstrapping techniques with 1,000 replications. The dotted lines represent the 90% confidence intervals. The sample includes 31 AEs and 25 EMs over the period 2000-2019.

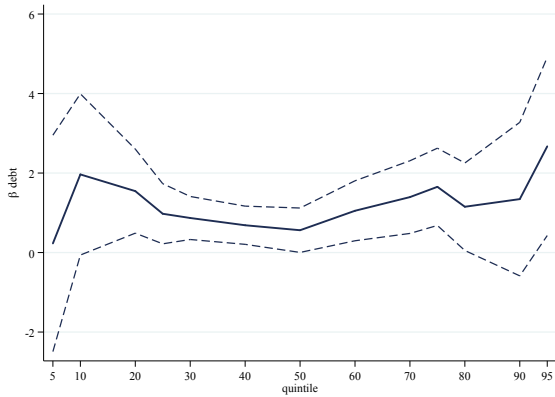
Figure A4: Estimated Quantile Regression Distributions (Robustness Checks)



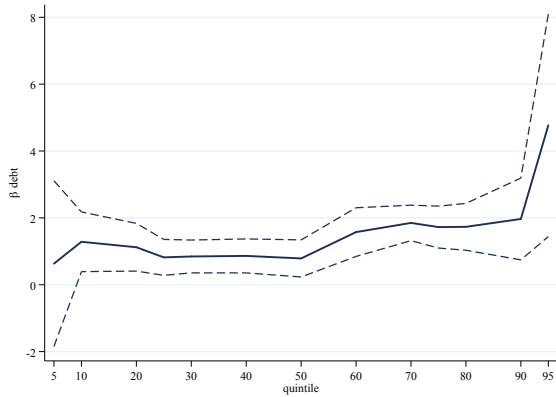
(a) Alternative sample



(b) Alternative specification



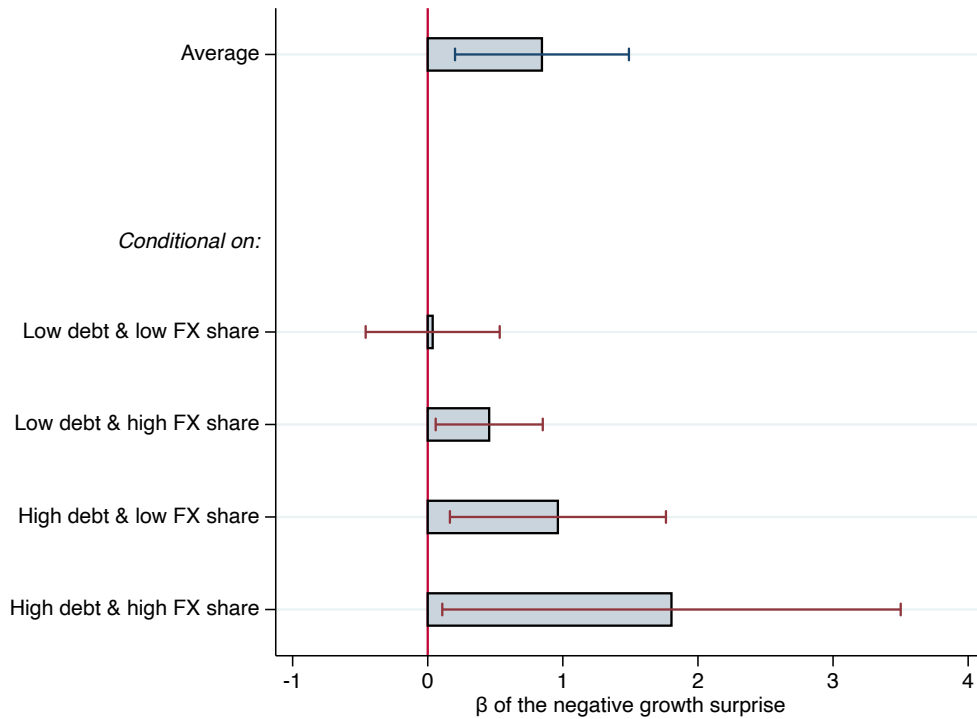
(c) Alternative sample



(d) Alternative specification

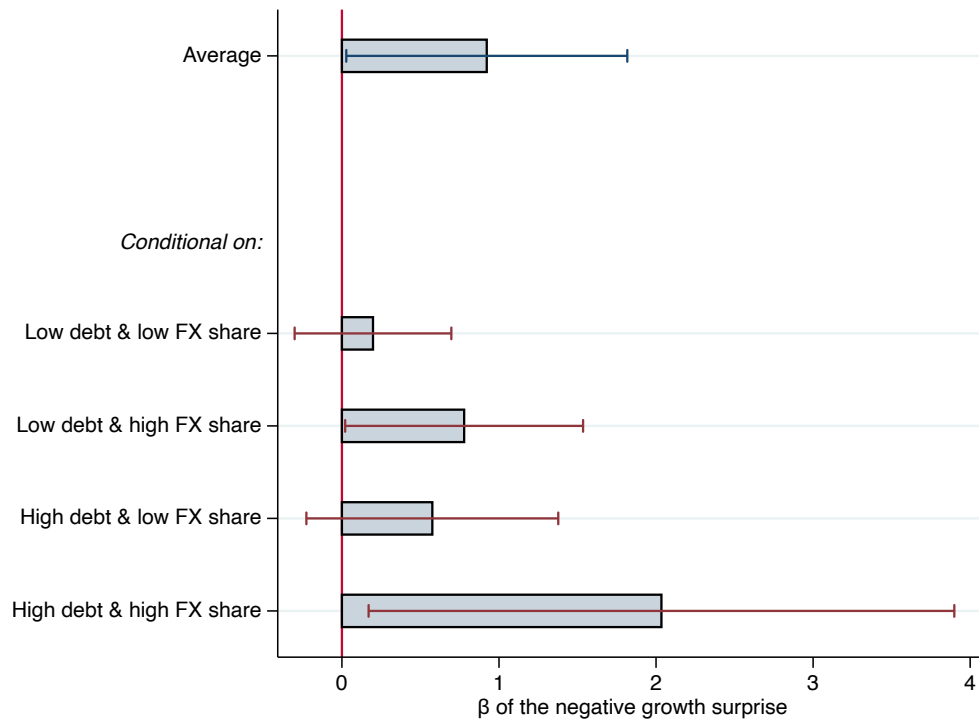
Notes: The figure shows robustness results of the baseline quantile regressions, exclude Greece and Japan from the sample (panels a and c) and excluding the crisis dummy from the baseline specification (panels b and d). Panels (a) and (b) show the estimated future $r-g$ distribution as a function of current public debt based on the quantile regressions. Panels (c) and (d) plot the corresponding estimated coefficients of public debt (y axis) obtained from quantile regressions for different quantile values (x axis). See specification 1 or details; the model regresses 5-year average future $r-g$ (the annualized average from year $t+1$ to $t+5$) on public-debt-to-GDP ratio in year t . $r-g_t$ is annual local-currency nominal rate in year t minus local-currency growth rate in year t . The sample includes 31 AEs and 25 EMs over the period 2000-2019.

Figure A5: Interest Rate Exposures to Negative Growth Shocks and Public Debt Structure, Excluding the Global Financial Crisis



Notes: This figure shows the interest-rate exposures to large negative growth shocks for countries with different debt levels and different shares of foreign currency denominated debt, excluding the years of the Global Financial Crisis. Specifically, the figure shows the estimated coefficients of regressing nominal long-term interest rates on the growth shock, defined as a dummy equal to 1 if the country realized GDP growth in year t is 1% or more lower than the forecast in October of year $t - 1$, and zero otherwise. This is the baseline 'average' bar. Then, the other bar shows the coefficients of the interaction terms between the growth shock and four different debt categories, defined splitting the observation in low- and high-public debt and then, within each category, between low- and high-foreign currency denominated public debt. The low vs high split is done along the sample median. The lines show the 90% confidence intervals of the point estimates. See equation 2 for details. The data are annual. The sample includes 29 AEs and 21 EMs over the period 2000-2019, excluding the years 2008 and 2009.

Figure A6: Interest Rate Exposures to Negative Growth Shocks and Public Debt Structure, Advanced Economies



Notes: This figure shows the interest-rate exposures to large negative growth shocks for countries with different debt levels and different shares of foreign currency denominated debt, excluding emerging markets from the sample. Specifically, the figure shows the estimated coefficients of regressing nominal long-term interest rates on the growth shock, defined as a dummy equal to 1 if the country realized GDP growth in year t is 1% or more lower than the forecast in October of year $t - 1$, and zero otherwise. This is the baseline 'average' bar. Then, the other bar shows the coefficients of the interaction terms between the growth shock and four different debt categories, defined splitting the observation in low- and high-public debt and then, within each category, between low- and high-foreign currency denominated public debt. The low vs high split is done along the sample median. The lines show the 90% confidence intervals of the point estimates. See equation 2 for details. The data are annual. The sample includes 29 AEs over the period 2000-2019.

Tables

Table A1: Countries in Sample and Sample Period

Group	Country	Long sample	Baseline sample	FX sample	Daily rate sample
AE	Australia	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Austria	-	2000 - 2019	2000 - 2019	2000 - 2018
AE	Belgium	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Canada	1950 - 2019	2000 - 2019	2000 - 2011	2000 - 2018
AE	Cyprus	-	2000 - 2019	2000 - 2019	-
AE	Czech Republic	-	2000 - 2019	2000 - 2019	2000 - 2018
AE	Denmark	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Finland	1950 - 2019	2000 - 2019	2000 - 2009	2000 - 2018
AE	France	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Germany	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Greece	-	2000 - 2019	2000 - 2019	2000 - 2018
AE	Hong Kong SAR	-	2001 - 2019	2009 - 2019	2002 - 2018
AE	Iceland	-	2000 - 2019	2000 - 2019	2002 - 2018
AE	Ireland	-	2000 - 2019	2000 - 2019	2000 - 2018
AE	Israel	-	2000 - 2019	2000 - 2019	2002 - 2018
AE	Italy	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Japan	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Korea	-	2000 - 2019	2000 - 2019	2000 - 2018
AE	Latvia	-	2001 - 2019	2001 - 2019	2004 - 2018
AE	Lithuania	-	2001 - 2018	2001 - 2018	2003 - 2018
AE	Luxembourg	-	2000 - 2019	2000 - 2019	-
AE	Netherlands	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	New Zealand	-	2000 - 2019	2000 - 2019	2000 - 2018
AE	Norway	1950 - 2019	2000 - 2019	2000 - 2009	2000 - 2018
AE	Portugal	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Singapore	-	2000 - 2019	2000 - 2019	2000 - 2018
AE	Slovak Republic	-	2000 - 2019	2000 - 2019	2002 - 2018
AE	Spain	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Sweden	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	Switzerland	1950 - 2019	2000 - 2019	2000 - 2009	2000 - 2018
AE	United Kingdom	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
AE	United States	1950 - 2019	2000 - 2019	2000 - 2019	2000 - 2018
EM	Bosnia and Herzegovina	-	2000 - 2019	2000 - 2019	-
EM	Brazil	-	2000 - 2019	2000 - 2019	2000 - 2018
EM	Bulgaria	-	2003 - 2019	2003 - 2019	2006 - 2018
EM	Chile	-	2004 - 2019	2004 - 2019	2007 - 2018
EM	Colombia	-	2002 - 2019	2002 - 2019	2002 - 2018
EM	Croatia	-	2007 - 2019	2007 - 2019	2008 - 2018
EM	Egypt	-	2010 - 2019	2010 - 2019	2010 - 2018
EM	Hungary	-	2000 - 2019	2000 - 2019	2002 - 2018
EM	Indonesia	-	2003 - 2019	2003 - 2019	2003 - 2018
EM	Malaysia	-	2000 - 2019	2000 - 2012	2000 - 2018
EM	Mauritius	-	2000 - 2019	2000 - 2019	-
EM	Mexico	-	2000 - 2019	2000 - 2019	2001 - 2018
EM	Nigeria	-	2007 - 2018	2007 - 2018	2007 - 2018
EM	Panama	-	2000 - 2019	2000 - 2019	-
EM	Peru	-	2007 - 2019	2007 - 2019	2007 - 2018
EM	Philippines	-	2000 - 2019	2000 - 2019	2000 - 2018
EM	Poland	-	2000 - 2019	2000 - 2019	2000 - 2018
EM	Romania	-	2005 - 2019	2005 - 2019	2007 - 2018
EM	Russia	-	2000 - 2019	2000 - 2019	2001 - 2018
EM	Seychelles	-	2004 - 2019	2004 - 2019	-
EM	Thailand	-	2000 - 2019	2000 - 2019	2001 - 2018
EM	Turkey	-	2000 - 2019	2000 - 2019	2000 - 2018
EM	Vietnam	-	2007 - 2019	2007 - 2019	2007 - 2018
OT	Kenya	-	2004 - 2018	2009 - 2018	2004 - 2018

Notes: The main variables required to define the sample are: long-term nominal interest rates, GDP growth, and the public debt-to-GDP ratio.

Table A2: Summary Statistics of $r - g$ vs. Public Debt

(a) Debt

Debt group	Panel A: 17 AEs					Panel B: 56 AEs & EMs				
	Q1	Q2	Q3	Q4	Top 5%	Q1	Q2	Q3	Q4	Top 5%
Mean $r-g$	-3.7	-1.5	-0.8	0.4	0.8	-3.1	-1.8	-0.7	-0.2	1.7
Median $r-g$	-3.1	-1.3	-0.4	0.3	0.7	-2.6	-1.9	-0.7	-0.6	1.2
Sh. ($r-g > 0\%$)	22	36	44	54	60	24	24	35	43	65
Sh. ($r-g > 2\%$)	12	20	24	30	33	12	13	15	21	40
Sh. ($r-g > 4\%$)	6	8	10	12	17	7	9	8	15	23
Mean $Debt$	18	37	55	87	158	21	41	59	90	163
Mean $\Delta Debt$	-1.0	-0.1	1.1	3.8	6.2	-1.4	-1.2	0.7	4.8	13.4
No. Obs.	298	297	298	237	60	288	288	288	230	57

(b) Change in Debt

Debt group	Panel A: 17 AEs					Panel B: 56 AEs & EMs				
	Q1	Q2	Q3	Q4	Top 5%	Q1	Q2	Q3	Q4	Top 5%
Mean $r-g$	-2.2	-2.5	-1.2	0.9	3.5	-3.3	-2.2	-1.6	0.8	4.3
Median $r-g$	-2.0	-2.1	-1.1	1.2	3.0	-2.9	-2.0	-1.3	0.2	3.5
Sh. ($r-g > 0\%$)	27	28	38	65	86	22	21	27	53	81
Sh. ($r-g > 2\%$)	11	11	20	43	66	7	7	13	29	62
Sh. ($r-g > 4\%$)	3	5	7	19	40	3	4	6	20	48
Mean $Debt$	55	37	48	64	101	51	47	55	64	110
Mean $\Delta Debt$	-	-2.6	2.7	11.2	28.1	-	-2.1	3.2	11.7	34.9
	10.4					12.8				
No. Obs.	289	289	289	231	58	286	286	286	227	58

Notes: This table reports the summary statistics of different debt groups constructed based on the quartiles of $Debt$ (Panel A) and 3-year-changes-in- $Debt$ (Panel B) across-time-and-country distributions. The data in the 4th-quartile group are further split between the top 5 percent and the rest. All data are annual. $r-g_{i,t}$ are computed as nominal local currency long-term rates $r_{i,t}$ minus nominal local currency annual growth rates $g_{i,t}$.

Table A3: $r-g$ and Public Debt

Dep. var.: $(r - g)_t$	(1)	(2)	(3)	(4)
Panel A: 17 AEs, 1950-2019				
$Debt_{t-1}$	2.66*** (0.83)	3.51*** (0.94)	1.52*** (0.51)	2.09*** (0.45)
$\Delta Debt_{t-1}$	1.56*** (0.37)	1.81*** (0.36)	0.52** (0.20)	0.81*** (0.16)
Observations	1,190	1,190	1,190	1,190
R^2	0.13	0.22	0.50	0.56
Panel B: 31 AEs and 25 EMs, 1950-2019				
$Debt_{t-1}$	2.43* (1.19)	0.72 (2.66)	2.74** (1.09)	1.68 (1.89)
$\Delta Debt_{t-1}$	1.24*** (0.23)	1.16*** (0.29)	1.02*** (0.27)	0.88*** (0.30)
Observations	958	958	958	958
R^2	0.14	0.29	0.33	0.48
Panel C: 17 AEs, 1950-2019, 5-year non-overlapping averages				
$Debt_{t-1}$	2.91** (1.21)	3.93** (1.31)	1.73** (0.59)	2.58*** (0.46)
$\Delta Debt_{t-1}$	2.37*** (0.62)	2.95*** (0.55)	1.08** (0.47)	1.82*** (0.37)
Observations	238	238	238	238
R^2	0.26	0.43	0.58	0.70
Time FE	No	No	Yes	Yes
Country FE	No	Yes	No	Yes

Notes: Panels A and B report the results of regressing country i 's interest-growth rate differentials $r-g_{i,t}$ on previous year's public-debt-to-GDP ratio $Debt_{i,t-1}$ and lagged one-year change in debt $\Delta Debt_{i,t-1}$, for a long sample of 17 advanced economies and a large sample of advanced economies and emerging markets, respectively. Panel C reports the results of the same regression, but based on non-overlapping 5-year averaged variables for the long sample of 17 advanced economies. $r-g_t$ are computed as nominal local currency rates r_t minus nominal local currency annual growth rates g_t , where r_t are the 10-year local currency government bond yield. $\Delta Debt_{t-1}$ is computed as $Debt_{t-1}$ minus $Debt_{t-2}$ and normalized so that its coefficient estimate reflects that of one standard deviation change in $\Delta Debt_{i,t-1}$. All data are annual. Standard errors in parenthesis are clustered at the year and country level. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table A4: $r-g$ and Foreign Currency Denominated Public Debt

Dep. Var.: $(r - g)_t$	(1)	(2)	(3)	(4)
$LC Debt_{t-1}$	2.77* (1.37)	2.26 (1.99)	2.90** (1.24)	1.48 (2.43)
$FX Debt_{t-1}$	3.95* (2.11)	15.55*** (3.41)	3.04 (2.17)	13.18*** (3.50)
Observations	856	856	856	856
R^2	0.30	0.48	0.41	0.58
Year FE	No	No	Yes	Yes
Country FE	No	Yes	No	Yes

Notes: This table reports the results of regressing country i 's interest-growth rate differentials $r-g_{i,t}$ on previous year's public-debt-to-GDP ratio, splitting it into $Debt FX$ and $Debt LC$, which refer to public debt denominated in foreign currencies and domestic currencies, respectively, as a ratio to GDP. $r-g_t$ are computed as nominal local currency rates r_t minus nominal local currency annual growth rates g_t , where r_t are the 10-year local currency government bond yield. Each regression also control for $\Delta Debt_t$, computed as $Debt_t$ minus $Debt_{t-1}$. All data are annual. The sample includes 31 AEs and 25 EMs over the period 2000-2019. Standard errors in parenthesis are clustered at the year and country level. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Table A5: Interest Rate Exposures to Negative Growth Shocks and Public Debt Structure

Dep. Var.: r_t	(1)	(2)	(3)	(4)	(5)	(6)
	Whole sample		No GFC		No EMs	
r_{t-1}	0.6049*** (0.045)	0.6186*** (0.037)	0.5890*** (0.043)	0.6087*** (0.034)	0.6476*** (0.042)	0.6784*** (0.024)
ϵ_t^g	0.7507** (0.314)		0.8456** (0.384)		0.9223* (0.526)	
$\epsilon_t^g \times debt_t^{1stQ}$		0.2525 (0.319)		0.0367 (0.296)		0.1978 (0.293)
$\epsilon_t^g \times debt_t^{2ndQ}$		0.7226** (0.293)		0.4551* (0.236)		0.7781* (0.445)
$\epsilon_t^g \times debt_t^{3rdQ}$		0.4668 (0.377)		0.9636** (0.477)		0.5755 (0.471)
$\epsilon_t^g \times debt_t^{4thQ}$		1.5563* (0.826)		1.8042* (1.012)		2.0345* (1.096)
Observations	718	718	620	620	443	443
R^2	0.857	0.860	0.870	0.874	0.795	0.808
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: This table reports the results of regressing country i 's long-term interest rate on its lagged value, a growth surprise (ϵ_t^g) and the lagged value of the public-debt-to-GDP ratio (columns 1 and 3). In columns 2 and 4, the growth surprise is interacted with 4 debt dummy variables, which identify country year pair with low vs high public debt ratios and low vs high shares of foreign currency denominated public debt. See equation 2 and the text for details. All data are annual. The sample includes 29 AEs and 21 EMs over the period 2000-2019. Columns 3 and 4 exclude the years 2008 and 2009. Columns 5 and 6 exclude emerging markets. Standard errors in parenthesis are clustered at the year and country level. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.