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A Conceptual Model for the Integrated Policy Framework

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Abstract

In the Mundell-Fleming framework, standard monetary policy and exchange rate flexibility fully insulate economies from shocks. However, that framework abstracts from many real world imperfections, and countries often resort to unconventional policies to cope with shocks, such as COVID-19. This paper develops a model of optimal monetary policy, capital controls, foreign exchange intervention, and macroprudential policy. It incorporates many shocks and allows countries to differ across the currency of trade invoicing, degree of currency mismatches, tightness of external and domestic borrowing constraints, and depth of foreign exchange markets. The analysis maps these shocks and country characteristics to optimal policies, and yields several principles. If an additional instrument becomes available, it should not necessarily be deployed because it may not be the right tool to address the imperfection at hand. The use of a new instrument can lead to more or less use of others as instruments interact in non-trivial ways.

JEL Classification Numbers: F31, F32, F38, F41

Keywords: exchange rate flexibility, monetary policy, capital controls, foreign exchange intervention, macroprudential regulation

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1 Introduction

The Mundell-Fleming model remains the workhorse framework for the analysis of small open economies in the policy space. In that model, flexible exchange rates are optimal, and can deliver full employment together with low inflation after domestic and external shocks. This is partly because of a powerful expenditure-switching channel that operates both on the import side and on the export side. However, the world is more complex than modeled in the Mundell-Fleming framework: the rigidity of prices can take different forms, and international and domestic financial imperfections are prevalent. Such complexities can make monetary policymaking more involved than setting a policy rate and allowing the exchange rate to flexibly adjust. This paper provides an analytical framework to guide policymakers on the joint configuration of monetary policy, capital controls, foreign exchange (FX) intervention, and macroprudential policy.

Our work is motivated by two observations. First, the empirical evidence is inconsistent with some of the assumptions underlying the Mundell-Fleming framework regarding both trade and finance. Many emerging markets have dollar invoicing shares above 80 percent. This empirical fact implies that we need to consider a dominant currency pricing paradigm where export prices are sticky in a dominant currency, which is most often the dollar and in some cases the euro. Similarly, on the financial side, a vast literature establishes an array of imperfections in international and domestic capital markets. Foreign currency borrowing is prevalent, and generates a link between the exchange rate and the macroeconomy through currency mismatches and external borrowing constraints. Financial intermediaries operating in foreign exchange markets generally have limited appetite for taking on emerging markets' currency exposure and hence, the uncovered interest parity condition breaks down. Those countries which intervene heavily in foreign exchange markets during depreciation episodes tend to be the countries where balance sheet concerns prevail, and where financial markets are not deep enough to provide hedging opportunities. In addition, imperfections in domestic financial markets, particularly in the housing sector, are well-established for both emerging and advanced markets.

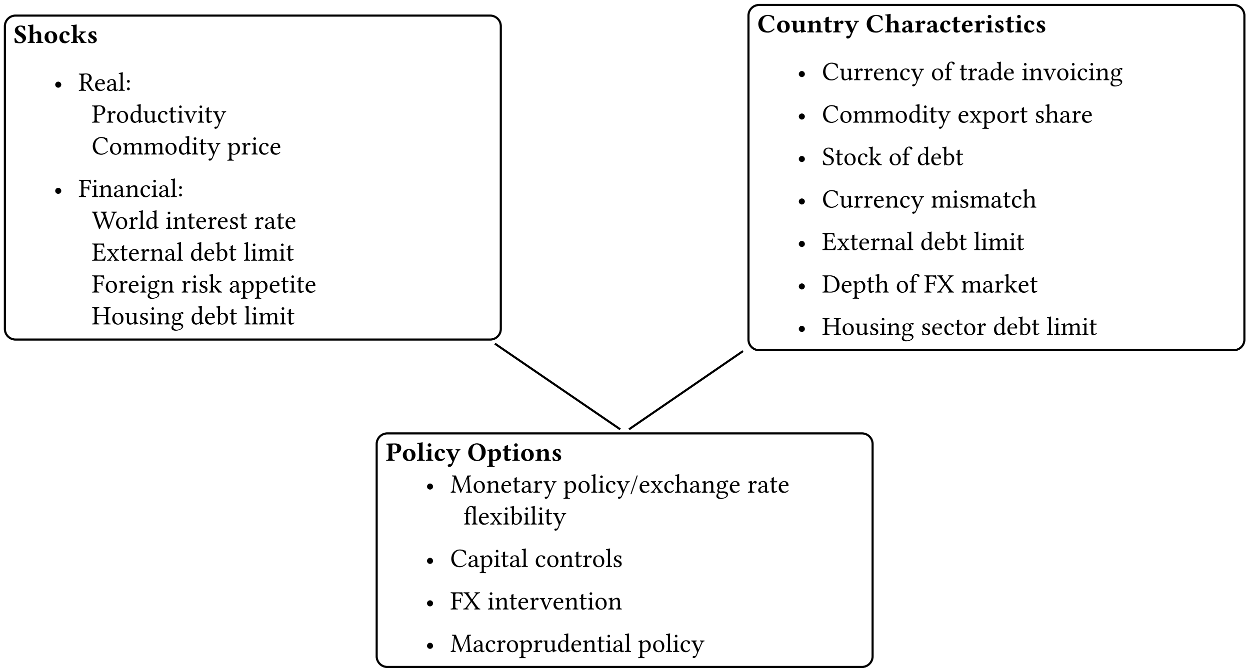
The second observation is that many small open economies adopt more eclectic approaches than standard interest rate setting and floating exchange rates to cope with shocks. For example, during the ongoing COVID-19 epidemic, many emerging markets faced capital outflow pressures and used multiple tools to achieve macroeconomic stabilization, with some variation across countries. While most countries lowered policy rates and eased macroprudential regulation, some complemented it with sales of foreign exchange reserves to lean against the depreciation. At the same time, a few countries relaxed their restrictions on capital inflows. Given their frequent and often heterogeneous use, one of the objectives of this paper is to better understand these alternative tools: what they do, how they interact, the trade-offs involved, and the characterization of policy counterfactuals. In other words, this paper aims to establish under what conditions the standard prescription of flexible exchange rates still holds, and when it may instead be optimal to rely on other tools. Doing so requires going beyond the Mundell-Fleming framework.

The novelty of our analysis is that we develop an integrated model that jointly considers the role of monetary policy, capital controls, foreign exchange intervention, and macroprudential regulation in small open economies.[1] We characterize the use of these policy instruments as a function of shocks and both real and nominal frictions. The diagram in figure 1 visually outlines this agenda. The top left box in the diagram lists the real and financial shocks that our model incorporates. In the top right box are the country characteristics that our model aims to capture. The objective of our analysis is then to map the combinations of shocks and country characteristics to the optimal policy mix, listed in the box at the bottom of the diagram.

To achieve these objectives, we develop a model of a small open economy with three sectors: the tradable differentiated goods sector, the commodity sector, and the nontradable housing sector. The tradable differentiated good is produced by firms with pricing power, and its price may be sticky according to either the producer currency pricing (PCP) or the dominant currency pricing (DCP)

[1]We do not consider optimal fiscal policy, i.e., we assume that fiscal policy is generally not flexible enough to respond to shocks within the horizon considered. That said, fiscal policy and public debt levels can influence the initial conditions and can make certain shocks more likely.

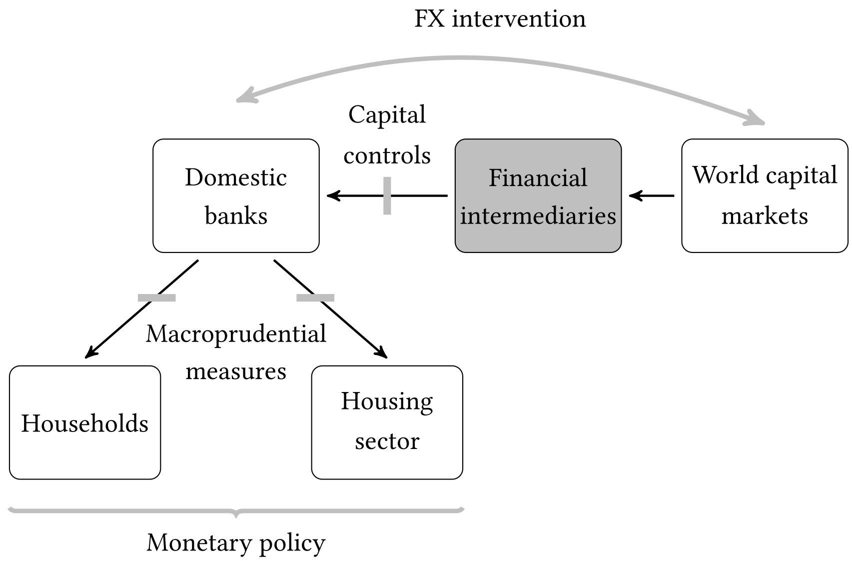
Figure 1: Modeling Agenda



paradigms. The country is a price taker in commodity markets and can face shocks to the price of commodities. Housing services are produced by firms using land as the input into production, and rental prices are set flexibly.

Figure 2 displays the financial structure of our model. Domestic banks borrow from global financial intermediaries and lend to the domestic households and the housing sector firms. An occasionally-binding borrowing constraint limits the banks' borrowing from the financial intermediaries to a fraction of the domestic price of the differentiated tradable good. Another occasionally-binding borrowing constraint requires the housing sector firms to post a fraction of the value of their land holdings as collateral. These constraints are not binding in normal times, but they become binding after sufficiently large adverse shocks. Financial intermediaries borrow in foreign currency on the world market and satisfy the domestic banks' domestic currency borrowing needs. The intermediaries are partly foreign and partly owned by domestic agents: this domestic ownership is the source of the economy's currency mismatch. The extreme case where the financial intermediaries are owned entirely domestically corresponds to the highest degree of currency mismatch. The other

Figure 2: Structure of the Financial Market



extreme with no domestic ownership of the intermediaries is the same as the country borrowing only in its own currency. We assume that the intermediaries are constrained in their ability to bear the country's currency exposure, which leads to deviations from the uncovered interest parity condition. We refer to this inefficiency as *shallow FX markets*.

With these ingredients in hand, we consider optimal policies under full commitment that aim to address the following externalities.

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- Households do not internalize the impact of their consumption decisions on aggregate demand, a problem which arises from our assumption of sticky prices for the tradable differentiated good and paves the way to the well-known Keynesian *aggregate demand externality*. This is typically the key friction in models of monetary policy.

- As is standard in open economy models of monetary policy, our assumption of firms having pricing power and facing a downward-sloping export demand schedule gives rise to a *terms of trade externality*, where individual firms do not take into account that their production decisions impact the position of the aggregate economy on the export demand schedule. In other words, firms tend to produce too much and set prices too low, and do not exploit their pricing power to benefit the aggregate economy. Even though this externality arises naturally in our setting, it is not clear that it is relevant for policymaking in the real world, so we focus on results that do not hinge on it, and in some cases we set parameterizations that neutralize it.

- Banks' borrowing constraints combined with the economy's FX exposure through the ownership of financial intermediaries generate a *pecuniary aggregate demand externality*, as households and banks do not internalize the effects of their individual actions on aggregate demand, the exchange rate, and on the tightness of the constraint ex post. This externality leads to overborrowing and overly appreciated exchange rates ex ante, and to too little borrowing and overly depreciated exchange rates ex post after adverse shocks that make the constraint bind.
- Housing sector firms' borrowing constraints lead to a *pecuniary production externality*, since these firms do not internalize the effects of their borrowing and production decisions on land prices. After adverse shocks, their production and demand for land may be too low, leading to depressed land prices, further exacerbating the initial shock and tightening their borrowing constraint.
- What we call a *financial terms of trade externality* arises when FX markets are shallow, because households and banks do not take into account that their borrowing decisions impact the external premium that the economy as a whole needs to pay to the financial intermediaries.

Our model incorporates several roles for monetary policy and exchange rate flexibility, capital controls, FX intervention, and macroprudential regulation. Monetary policy, working through changes in the policy rate, affects the interest rate faced by domestic agents when they make consumption, production, and borrowing decisions, as well as the rate that the domestic banks offer to the financial intermediaries. In our integrated framework, it can influence most externalities, one of which is the aggregate demand externality. As in standard open economy models of monetary policy, flexible exchange rates have expenditure-switching benefits whereby an exchange rate depreciation makes imports more expensive relative to home-produced goods. Households therefore switch away from consuming imports towards consuming home goods. Under PCP, expenditure switching is also operational through exports: an exchange rate depreciation boosts demand by making exports more competitive. Under DCP, exchange rate adjustment becomes a weaker tool: while it continues to affect import consumption, it no longer affects the competitiveness of exports on world markets, as dollar prices remain unchanged.

Following the vast literature on emerging market capital flows and sudden stops, we model capital controls in the form of state contingent taxes on capital inflows, as depicted in figure 2.[2] Prudential capital controls work to prevent overborrowing by curbing debt and consumption ex ante, and shift aggregate demand from normal times to distressed periods. This is desirable under several conditions.[3] First, as shown extensively in the literature, curbing debt ex ante reduces the risk of sudden stops where a depreciation can make the banks' borrowing constraint bind by worsening the country's balance sheet through currency mismatches. When FX markets are shallow, another reason to curb debt through capital controls arises because doing so reduces the losses incurred due to the inefficiency in the intermediation of debt. This use of capital controls can happen ex ante (prudential) or ex post. On the downside, capital controls can distort capital flows relative to the efficient benchmark, which may generate welfare losses since those flows can be beneficial for the recipient countries. It is also important to note that prudential capital controls need to be raised counter-cyclically during booms in external debt and reduced during busts.

Macroprudential tools are conceptually similar to capital controls in that they also curb overborrowing by affecting debt flows. We model them as taxes on domestic banks' lending to domestic agents that can differ across households and the housing sector. As we discuss in more detail later, macroprudential consumer taxes can sometimes work as substitutes for prudential capital controls

[2]Quantity restrictions instead of taxes would have identical implications. In addition, taxes or subsidies on capital outflows, instead of inflows, would work similarly, in the absence of any further costs associated with regulating outflows.

[3]In the IMF's taxonomy, capital controls are called capital flow management measures (CFMs). This taxonomy also allows for a category called CFM/MPMs, which are macroprudential policies (MPMs) that discriminate between residents and nonresidents or are designed to limit capital flows for financial stability reasons. Since *prudential* capital controls in our framework mainly curb overborrowing in an ex ante manner to prevent sudden stops or fire sales in housing markets, they correspond most closely to CFM/MPMs. Capital controls used ex post in our model map most closely to CFMs.

in the absence of housing sector frictions as they affect external debt through affecting households' demand for domestic debt from the banks. In other cases, macroprudential consumer taxes and prudential capital controls are complements. Macroprudential housing taxes are an important tool for reducing the risk of fire sales in housing markets.

Sterilized FX intervention circumvents the inefficiency of financial intermediaries in countries with shallow FX markets. It achieves this objective by changing the quantity of external debt that needs to be absorbed by the financial intermediaries and therefore the associated premium. After an adverse shock to the foreign appetite for domestic currency debt, FX sales reduce the need for the policy rate to be increased, and in that sense can enhance monetary autonomy. Alleviating the pressure for the policy rate to increase also prevents a decline in land prices, and thereby avoids the shock spilling over to the housing sector. However, reserve accumulation involves buying low-return foreign currency bonds and selling high-return domestic currency bonds, which incurs a carry cost.

After having discussed the role of each policy tool individually, we next lay out further insights on their joint use. Integrating a range of monetary and financial policies as well as externalities enables us to derive novel results relative to the literature, which typically bundles a single policy along with a single friction. We study the effects of policy tools on frictions beyond their typical bundling, and the interaction of tools and frictions with each other. This way, we develop a more comprehensive understanding of the implications of each tool, used in isolation or jointly with other tools, in a framework with several externalities.

For a class of shock and country characteristic combinations, we find the Mundell-Fleming prescription of fully flexible exchange rates to be optimal. While pricing in the dominant currency reduces the benefits of exchange rate flexibility and generally features under- or over-exporting, we find flexible exchange rates to be optimal in absence of other frictions. In the case of deep FX markets without borrowing constraints, i.e., without any financial frictions, policies such as capital controls do not improve efficiency beyond the terms of trade externality. The reason is that capital controls do not address the stickiness of export prices in the dominant currency. At the same time, under most shocks, the DCP economy is characterized by more volatile exchange rates than the PCP economy: under DCP, achieving benefits from exchange rate flexibility comparable to PCP requires *larger* exchange rate movements. Therefore, in the absence of financial frictions, the DCP economy stabilizes aggregate demand through larger exchange rate movements.

When future shocks can lead to a binding borrowing constraint for the banks, the pecuniary aggregate demand externality creates a role for prudential capital controls in normal times, with both the incidence and intensity of their use depending on the pricing paradigm. Adverse shocks to commodity prices or to banks' debt limits can make external constraints bind ex post and lead to overborrowing ex ante. Such shocks alter the trade-offs for monetary policy: depreciating the exchange rate after an adverse shock becomes costlier as a depreciation would further tighten the constraint. Thus, monetary policy weighs the macro benefits of depreciation against the financial costs. The consideration of relaxing the borrowing constraint leads to a more appreciated exchange rate relative to the case without such considerations, which means that aggregate demand is depressed ex post relative to the level that stabilizes price pressures. Prudential capital controls are needed to shift demand intertemporally from normal times to the period of distress, by curbing demand before the shock and stimulating it afterwards. For economies vulnerable to commodity price shocks, a wider set of unhedged external debt levels can justify prudential capital controls under DCP than PCP; and for economies vulnerable to commodity price or debt limit shocks, prudential capital controls are larger under DCP than PCP.

Capital controls and macroprudential regulations on consumers are substitutes under some conditions, but generally, they are both needed. Domestic macroprudential taxes on consumer debt are perfect substitutes for capital controls when macroprudential taxes cover the entire economy. As displayed in figure 2, since all external flows are channeled through domestic banks, taxing domestic banks' borrowing at the border is isomorphic to taxing the domestic debt of every domestic agent.[4] But this result no longer holds when some domestic agents can borrow directly from

[4]Given our modeling of domestic banks, these two cases are the same in terms of the banks' balance sheets. This may not be the case with a richer modeling of domestic banks.

abroad. If these agents with direct access to borrowing from abroad are outside the macroprudential perimeter, macroprudential taxes become an imperfect substitute for capital controls. If, instead, these agents are subject to macroprudential taxes but can circumvent them, then capital controls and macroprudential taxes need to be deployed together and move in tandem, because otherwise, these agents would borrow either entirely domestically or entirely externally depending on which taxes are smaller.

One way to reduce the banks' vulnerability to debt limit shocks is to ban FX exposures, but this policy can make FX markets shallower. Since the country's FX exposure is due to the ownership of the intermediaries, it seems sensible to ban open FX positions for those intermediaries which are domestically owned and let domestic currency debt be absorbed entirely by foreign-owned intermediaries. Such bans on FX exposures are optimal under deep markets because they address the pecuniary aggregate demand externality and eliminate the need for prudential capital controls. However, under shallow FX markets, the ban leads to fewer investors willing to finance external debt and makes FX markets even shallower. As a result, it increases the vulnerability to foreign appetite shocks, i.e., shocks to the foreigners' willingness to hold domestic currency debt. In turn, it may make the economy more dependent on FX intervention, by increasing the marginal value of FX intervention.

Under shallow FX markets, FX intervention and capital controls improve monetary autonomy after foreign appetite shocks. After a decline in foreigners' willingness to hold domestic currency debt, the country needs to offer higher external premia to foreigners to finance its debt. When the only available tool is monetary policy, this shock leads to a depreciation on impact to reduce imports and generate an expected appreciation. However, monetary policy alone cannot balance domestic price pressures and also target premia. Instead, households excessively deleverage their debt. FX sales can cushion the adverse shock, allowing the policy rate and domestic macro outcomes to change less. Capital inflow subsidies (or reductions in inflow taxes) also allow the policy rate to respond less while containing the aggregate demand externality.[5] Finally, if FX intervention and capital controls (or consumer macroprudential taxes if they can substitute for capital controls) are used together, monetary policy rate is more detached from the shock and monetary autonomy further enhanced than when the policies are used one at a time.

When we add frictions in the housing sector, we find that foreign appetite shocks can lead the housing sector constraint to bind and thus require macroprudential housing taxes to be used ex ante. As explained above, in the face of adverse foreign appetite shocks, capital controls and FX intervention improve monetary autonomy by reducing the need for the policy rate to be increased. If capital controls are not available, consumer macroprudential taxes can be used. But in that case, at the time of the adverse shock, a cut in consumer macroprudential taxes needs to be accompanied with an increase in the policy rate. This is because in absence of capital controls, the policy rate is tightly connected to the external premia and a higher policy rate is necessary to offer a higher premium for foreigners. At the same time, a higher policy rate causes a decline in land prices and can make the housing constraint bind. As a result, foreign appetite shocks can make the domestic housing market constraints bind.[6] A direct implication of this result is that macroprudential housing taxes may be needed in anticipation of not just domestic but also external shocks. Given the tension between external and internal stabilization, it becomes optimal for the policy rate to be increased by less than would be necessary to fully stabilize the premium and the exchange rate.

In the presence of housing frictions, a new role for exchange rate flexibility arises after some kinds of shocks. The existing literature shows that closed economies with high housing debt should impose macroprudential housing debt taxes in normal times, and relax them (together with monetary policy)

[5] While they have similar macro effects in response to this shock, FX intervention and capital controls work through different channels. FX sales reduce the total effective outflow that the financial intermediaries need to absorb, decreasing the necessary external premia. Capital controls detach the external premia from the policy rate, so that a loosening of controls can provide higher returns to foreigners without a change in the policy rate.

[6] This scenario with foreign appetite shocks in countries with shallow FX markets and housing frictions provides another example of a case where capital controls and consumer macroprudential taxes are not substitutes. Specifically, capital controls effectively detach the domestic economy, including the policy rate and the housing market, from foreign appetite shocks. If macroprudential consumer debt taxes are used instead, the required interest rate increase to generate larger external premia can cause housing constraints to bind, leading to a fundamental difference between capital controls and macroprudential consumer debt taxes.

to support land prices when the housing constraints bind. Our framework features this standard mechanism. In addition, a new channel arises for open economies: an exchange rate depreciation generates expenditure switching not only towards the domestically produced traded good but also towards housing services. This increased demand for housing services bolsters rents and land prices, and relaxes housing borrowing constraints that are set in domestic currency. This channel is most apparent when we remove all of the policy instruments but let the exchange rate move flexibly in the face of a shock to the housing debt limit. Comparing that case with the one where the exchange rate is fixed reveals that the exchange rate depreciates and relaxes the housing sector constraint, even possibly to the extent of making macroprudential housing debt taxes unnecessary. The desired depreciation required to relax the housing constraint may not match with the desired size of the policy rate reduction to ease the constraint. If the policy rate reduction depreciates the exchange rate beyond the desired size of depreciation, capital inflow subsidies (or reductions in inflow taxes) or FX sales can contain the depreciation associated with the policy rate cut and avoid excessive expenditure switching.

While the larger exchange rate volatility in DCP aggravates FX borrowing constraints, it eases domestic currency borrowing constraints. Recall that, everything else equal, there is higher exchange rate volatility under DCP. This feature of DCP is beneficial in the case of borrowing constraints in domestic currency. The exchange rate depreciation can boost housing sector consumption and relax the housing constraint. The aggregate-demand-destabilizing effects of the depreciation are smaller under DCP because of weaker expenditure switching. Thus, the DCP economy faces an easier trade-off between relaxing the housing constraint and demand stabilization. This property translates into smaller ex ante macroprudential housing taxes under DCP, since the exchange rate can be used more forcefully ex post.

Considering the borrowing constraint of the housing sector and the banks at the same time, we find that a housing crisis can trigger an external constraint and vice versa. For example, when there is an adverse shock to the housing sector's debt limit, as discussed above, the exchange rate depreciates to help relax the constraint, which is in domestic currency. But when the banks' borrowing constraint is also relevant, the depreciation lowers their debt limit in FX terms and tightens their constraint. In this scenario, interest rates and capital controls should be used ex ante to reduce the interest burden on inherited housing sector debt and to limit external FX debt, and ex post, it becomes optimal for the exchange rate not to depreciate by as much as it would in the absence of the banks' borrowing constraint.

Symmetrically, the banks' debt limit shock may cause the housing constraint to bind. The external constraint is associated with a large cut in the policy rate and an exchange rate depreciation which tends to increase the domestic currency value of rents and the land price. However, it is also associated with an increase in the borrowing rate for domestic households and the housing sector, and a decrease in household consumption. These latter factors tend to reduce rents and the land price. If the latter effects are larger than the former ones, the housing constraint may bind. Similar to above, in this situation, ex ante policy actions such as ex ante housing macroprudential taxes become optimal. Since the exchange rate depreciation is larger and the increase in the domestic borrowing rate is smaller under DCP after external debt limit shocks, the housing market is better insulated under DCP.

How FX intervention is used when both constraints bind depends on whether all or part of the economy is subject to a sudden stop. The more standard use of FX intervention to affect the exchange rate by distorting premia in shallow FX markets does not help when the borrowing constraint binds for all banks. In fact, we find that in those cases, the only role for ex post FX intervention is to absorb external premia. The reason is that once the external constraint binds, the policy rate becomes available to manage the exchange rate costlessly, as it can no longer affect the domestic agents' decisions. But if only some domestic banks are subject to a sudden stop, there may be a case for FX intervention to manage the exchange rate ex post. In such cases, the policy rate should be reduced to relax the domestic housing constraint, but on its own, it has an unfortunate side-effect of causing a depreciation which tightens the external constraint. FX sales can then help limit the depreciation, improving the trade-off between relaxing the housing constraint and the external constraint.

Our results make a strong case for the integration of monetary and financial policies, and support the following broad principles. First, policy instruments are not created equal: they operate through different margins. If an additional tool becomes available, it does not mean one should use it because it just may not be the right tool. A good example is the case of under- or over-exporting and employment destabilization under DCP where capital controls are not useful. Second, instruments generally affect multiple imperfections, so the use of an existing policy may be reduced or increased after a new tool becomes available, that is, tools may be substitutes or complements. This is well exemplified by the ambiguous effects of prudential capital controls on the use of monetary policy under deep FX markets. Third, there is no strict assignment of domestic policies (policy rate and macroprudential debt taxes) to domestic shocks and domestic frictions or external policies (capital controls and FX intervention) to external shocks and external frictions. An example of this case arises when the housing sector firms face domestic currency borrowing constraints, yet optimal policies involve not just domestic instruments but also capital controls and FX intervention, regardless of whether the underlying source of shocks is domestic or external.

The rest of this paper proceeds as follows. First, section 2 provides a review of the literature we build on in this paper. Next, section 3 lays out the model environment. The subsequent sections present our findings by gradually turning on additional frictions. Section 4 describes the results derived from the smallest version of our model, with sticky prices and banks' borrowing constraints under different shocks and pricing paradigms. Section 5 allows for shallow FX markets and lays out our findings for that case. Section 6 zooms in on our results for the case that also feature frictions in the housing sector. Finally, section 7 concludes.

2 Related Literature

Our paper contributes to four strands of literature. First, we build on the insights developed by Gopinath (2015), Casas et al. (2016) and Gopinath et al. (2020) on the dominant currency paradigm.

Following Casas et al. (2016), we compare and contrast the monetary policy implications of producer and dominant currency pricing for a small open economy. But unlike Casas et al. (2016), we consider a smaller scale model that can be solved nonlinearly, and we develop a rich financial market structure that allows us to analyze policies other than interest rate policy. In this vein, our work is also related to Egorov and Mukhin (2019) who look at optimal monetary policy and the use of capital controls under DCP but differently from us, they use a setup without pecuniary externalities or shallow FX markets.

Second, our paper contributes to the literature on aggregate demand and pecuniary externalities, and the joint analysis of monetary and macroprudential policies. Similar to Farhi and Werning (2016), we build a small scale model with nominal rigidities and monetary policy to form the backbone of our model environment. Also similar to Farhi and Werning (2016), we benefit from the findings of the vast literature on pecuniary externalities and the use of macroprudential policies, exemplified by Mendoza (2010), Jeanne and Korinek (2010), Bianchi (2011) and Benigno et al. (2012). We build on these papers by considering alternative pricing paradigms as well as additional financial frictions and policy tools.

Third, our paper borrows elements from the literature on exchange rate determination and FX intervention in the presence of inefficient financial intermediation. The intermediation inefficiency we consider follows that developed by Gabaix and Maggiori (2015). Similar to Fanelli and Straub (2019) and Cavallino (2019), this inefficiency provides a rationale for FX intervention and at the same time determines its degree of effectiveness. Unlike these papers, we nest the intermediation inefficiency into a larger setting that features other frictions and policies.

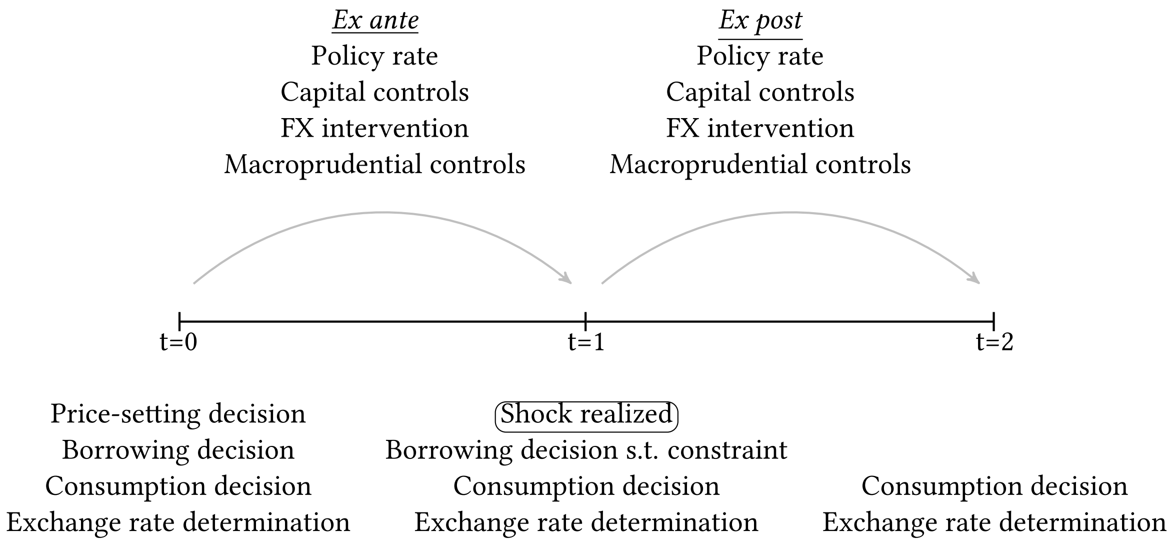
Finally, our modeling of the frictions in the housing sector borrows elements from Kiyotaki and Moore (1997). Specifically, we assume a similar externality, whereby the use of land depends on a constraint that includes the land price, but we consider an occasionally-binding rather than an always-binding constraint, allowing us to explore how different shocks might cause the constraint to bind. Our work is related to Korinek and Sandri (2016) who also aim to capture the differences between capital controls and macroprudential tools using distinct sources of pecuniary externalities. Differently from them, we build a unified model nesting rationales for capital controls and macroprudential policies, while maintaining a representative household setup and allowing for general equilibrium interactions across sectors and policies. Our work on the interaction between domestic and external constraints is related to the theoretical work of Caballero and Krishnamurthy (2001) and empirical findings of Kaminsky and Reinhart (1999). Differently from them, we focus on a setup with sticky prices, different forms of constraints, and more policies.

3 A Three-Period Small Open Economy

We construct a three-period model of a small open economy composed of households, a government, tradable sector firms, housing sector firms, domestic banks, and international financial intermediaries, a fraction of which is owned by domestic households. Tradable sector firms use labor to produce tradable goods. Housing sector firms use land to produce nontradable housing services; a subset of these firms operates a linear technology and another subset uses a concave technology. The economy receives an endowment of commodities that are exported. Tradable good prices are sticky, and following Gopinath (2015), export prices of home-produced tradable goods may follow producer currency pricing (PCP, i.e., exports are invoiced in domestic currency) or dominant currency pricing (DCP, i.e., exports are invoiced in dollars). Under both PCP and DCP, import and commodity prices are denominated in dollars.

An occasionally-binding borrowing constraint limits domestic banks' debt to a fraction of the domestic price of the tradable good, in the spirit of Mendoza (2010), Bianchi (2011) and Farhi and Werning (2016). Another occasionally-binding borrowing constraint limits the debt of linear firms in the housing sector to a fraction of the value of their landholdings, following Kiyotaki and Moore (1997). There are two noncontingent assets—a domestic currency bond and a dollar bond—and asset market segmentation: domestic agents can only trade the domestic currency bond, while international financial intermediaries can trade in both bonds. This segmentation in international financial

Figure 3: Timeline of Events



markets follows Gabaix and Maggiori (2015). The constrained social planner maximizes households' welfare while taking as given the decisions of private agents.

The financial structure of our model is shown in figure 2 and a stylized timeline of events is shown in figure 3. A variety of shocks strike in period 1, after which all uncertainty is resolved. The planner can implement policies either in period 0 in anticipation of possible shocks (i.e., prudential or ex ante policy) or in period 1 after the shock has been realized (i.e., ex post policy):

- Monetary policy. The planner sets the policy rate, which is equal to the interest rate on domestic currency bonds, between periods 0 and 1 and between periods 1 and 2.

- Capital inflow controls. The planner can set taxes/subsidies on inflows which generate a spread between the policy rate and the interest rate earned by international financial intermediaries. Such controls can be used in a prudential fashion, between periods 0 and 1, or in an ex post fashion, between periods 1 and 2. Prudential capital controls are similar to those studied by Bianchi (2011), Korinek (2011), and Farhi and Werning (2016).

- FX intervention. The planner can intermediate between the domestic currency bond and the dollar bond, circumventing the financial intermediaries and their inefficiency, between periods 0 and 1 and between periods 1 and 2. Such intervention is similar to that in Gabaix and Maggiori (2015), Cavallino (2019), and Fanelli and Straub (2019).

- Macroprudential controls. The planner can set taxes/subsidies separately on the borrowing of households and of the linear housing sector firms. Such controls can be used in a prudential fashion, between periods 0 and 1, or in an ex post fashion, between periods 1 and 2.

Next, we lay out the environment for the private sector agents and derive their optimal decisions before turning to the constrained social planner problem.

Households

Households maximize a welfare function which follows the Cole and Obstfeld (1991) formulation over consumption, and the Gali and Monacelli (2005) special case of linear disutility of labor:

$$\mathbb{E}_0\left[\sum{}_{t=0}^{2}\beta^t U\left(C_{Ht}, C_{Ft}, C_{Rt}, N_t\right)\right]$$

where $U\left(C_{Ht}, C_{Ft}, C_{Rt}, N_t\right) = \alpha_H \log C_{Ht} + \alpha_F \log C_{Ft} + \left(1-\alpha_H-\alpha_F\right)\log C_{Rt} - N_t$

Their maximization is subject to a budget constraint:

$$\begin{aligned} &W_t N_t + \Pi_{Tt} + \Pi_{Bt} + \lambda\Pi_{FIt} + T_{FXIt} - T_{Gt} + T_{Rt} + E_t P^*_{Zt} Z_t + D_{HHt+1} \\ &\geq P_{Ht}C_{Ht} + E_t P^*_{Ft} C_{Ft} + P_{Rt}C_{Rt} + \left(1+\theta_{HHt-1}\right)\left(1+\rho_{t-1}\right) D_{HHt}. \end{aligned} \tag{1}$$

Starting with the right hand side of the budget constraint, P_{Ht} and C_{Ht} are the domestic price and consumption of the tradable good, E_t is the exchange rate in units of domestic currency per dollar, $E_t P^*_{Ft}$ (i.e., the exchange rate multiplied by the dollar price of imports) is the domestic currency price of imports, C_{Ft} is the consumption of imports, P_{Rt} and C_{Rt} are the rental price and consumption of nontradable housing services, D_{HHt+1} is the domestic currency debt at the end of period t, θ_{HHt} is the consumer macroprudential tax, and ρ_t is the interest rate offered by domestic banks on domestic currency debt in period t, which applies between periods t and $t+1$.

On the left hand side of the budget constraint, N_t is labor supply, W_t is the wage, Π_{Tt} is the profit from tradable sector firms, Π_{Bt} is the profit from domestic banks, λ is the fraction of international financial intermediaries owned by domestic households while Π_{FIt} is the profit of each of them, T_{FXIt} is the profit of the planner from FX operations, T_{Gt} is the lump-sum tax levied by the planner, T_{Rt} is the transfer from housing firms (made only in period 2), $E_t P^*_{Zt}$ (i.e., the exchange rate multiplied by the dollar price of commodities) is the domestic price of commodity exports, and Z_t is the endowment of commodities, which are entirely exported.

The households' first order conditions (FOCs) lead to the following intratemporal conditions:

$$\frac{\alpha_H}{C_{Ht}P_{Ht}} = \frac{\alpha_F}{E_t P^*_{Ft} C_{Ft}} = \frac{\alpha_R}{C_{Rt}P_{Rt}} = \frac{1}{W_t} \tag{2}$$

$$\Rightarrow C_{Rt} = \frac{\alpha_H}{\alpha_F} p_{Rt} C_{Ft} \text{ where } p_{Rt} = \frac{E_t P^*_{Ft}}{P_{Rt}}$$

$$\text{and } C_{Ht} = \frac{\alpha_H}{\alpha_F} p_{Ht} C_{Ft} \text{ where } p_{Ht} = \frac{E_t P^*_{Ft}}{P_{Ht}},$$

where p_{Rt} is the price of foreign goods relative to domestic rents, and p_{Ht} is the price of foreign goods relative to home tradable goods. The FOCs also yield the Euler conditions:

$$\frac{\alpha_F}{P^*_{F0}C_{F0}} = \beta\left(1+\theta_{HH0}\right)\left(1+\rho_0\right)\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right] \text{ and } \frac{\alpha_F}{P^*_{F1}C_{F1}} = \beta\left(1+\theta_{HH1}\right)\left(1+\rho_1\right)\frac{E_1}{E_2}\frac{\alpha_F}{P^*_{F2}C_{F2}}. \tag{3}$$

Tradable sector firms

Tradable sector firms are monopolistically competitive and set prices at the beginning of period $t = 0$, after which prices are fully rigid (so we can remove the time subscripts on tradable good prices).[7] Following the New Keynesian tradition, we assume that they produce a variety $j \in [0, 1]$ of tradable goods, $Y_{Tt}(j)$, using labor, $N_t(j)$. The varieties may be consumed domestically, $Y_{Ht}(j)$, or

[7]This price-setting assumption keeps the model tractable. Under this assumption, we can interpret the optimal exchange rate policy as being related to the planner's desire to mitigate static price pressures, i.e., to ensure that the domestic price level is at an appropriate level relative to the price level of foreign goods. However, this assumption prevents us from considering the welfare costs of price dispersion and inflation dynamics that may damage credibility.

exported, $Y_{Xt}(j)$:

$$Y_{Tt}(j) = Y_{Ht}(j) + Y_{Xt}(j) = A_t N_t(j). \tag{4}$$

Firms face downward-sloping demands for their output from domestic consumption and from export demand. Domestic consumption involves combining the tradable varieties into an aggregate tradable good:

$$Y_{Ht} = \left(\int_0^1 Y_{Ht}(j)^{(\varepsilon-1)/\varepsilon}\, dj\right)^{\varepsilon/(\varepsilon-1)}.$$

The corresponding domestic demand curve is:

$$Y_{Ht}(j) = \left(\frac{P_H(j)}{P_H}\right)^{-\varepsilon} Y_{Ht},$$

where $P_H(j)$ is the sticky domestic currency price of each variety and P_H is the price index for the aggregate tradable good. We assume that the export demand curve follows the same form over each traded variety and a unit-elastic expression for the aggregate traded good:

$$Y_{Xt}(j) = \left(\frac{P_X(j)}{P_X}\right)^{-\varepsilon} Y_{Xt} \text{ and } Y_{Xt} = \omega p_{Xt},$$

where $P_X(j)$ is the price fixed by firms for each exported variety, P_X is the corresponding price index for the exported tradable good, and p_{Xt} is the relative price of foreign goods to exports. The denomination of $P_X(j)$ and P_X and the algebraic definition of p_{Xt} depend on the pricing paradigm.

Under PCP, firms set identical domestic currency prices for all of their output, regardless of whether the good is consumed domestically or exported, i.e., $P_X(j) = P_H(j)$. In other words, the law of one price holds, as in Gali and Monacelli (2005). Under DCP, firms set a domestic currency price, $P_H(j)$, for the domestically-consumed portion of the tradable good, and a separate dollar price, $P_X(j)$, for the exported portion of the good. As a result, the relative price of foreign goods to exports, i.e., the terms of trade, follows separate definitions under PCP and DCP:

$$p_{Xt}^{PCP} = \frac{E_t P_{Ft}^*}{P_H} \text{ and } p_{Xt}^{DCP} = \frac{P_{Ft}^*}{P_X}.$$

Profit maximization for firms *under PCP* is given by:

$$\max \Pi_{Tt}(j) = \Pi_{Ht}(j) + \Pi_{Xt}(j)$$

$$\begin{aligned}
&= \max \mathbb{E}_0 \left[\sum_{t=0}^{1} \Lambda_t \left[P_H(j) \left(Y_{Ht}(j) + Y_{Xt}(j) \right) - (1+\phi) W_t N_t(j) \right] \right] \\
&= \max \mathbb{E}_0 \left[\sum_{t=0}^{1} \Lambda_t \left[P_H(j) - (1+\phi) \frac{W_t}{A_t} \right] \left(Y_{Ht} + Y_{Xt} \right) \left(\frac{P_H(j)}{P_H} \right)^{-\varepsilon} \right],
\end{aligned}$$

where ϕ is a constant labor tax applied on all home production of tradable goods. We assume that firms have perfect access to dollar debt markets, so we set their discount factors as follows: $\Lambda_0 = 1$, $\Lambda_1 = \frac{1}{(1+i_0^*)} \frac{E_0}{E_1}$, and $\Lambda_2 = \frac{1}{(1+i_0^*)(1+i_1^*)} \frac{E_0}{E_2}$.[8] The FOC of the above expression produces an equation for $P_H(j)$—and, since all varieties are identical, for P_H—as a function of home demand, export demand, and the labor tax:

$$P_H = P_H(j) = (1+\phi) \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_0 \left[\sum_{t=0}^{2} \Lambda_t \frac{W_t}{A_t} \left(Y_{Ht} + Y_{Xt} \right) \right]}{\mathbb{E}_0 \left[\sum_{t=0}^{2} \Lambda_t \left(Y_{Ht} + Y_{Xt} \right) \right]} \tag{5}$$

The optimal price trades off the profit-maximizing positions the firm wants to target on the two separate home and export demand schedules. By changing the labor tax, ϕ, the planner can control the domestic price level and the export price level, both given by P_H.

Profit maximization for firms *under DCP* follows:

$$\max \Pi_{Tt}(j) = \Pi_{Ht}(j) + \Pi_{Xt}(j)$$

where

$$\Pi_{Ht}(j) = \mathbb{E}_0 \left[\sum_{t=0}^{1} \Lambda_t \left[P_H(j) - (1+\phi) \frac{W_t}{A_t} \right] Y_{Ht} \left(\frac{P_H(j)}{P_H} \right)^{-\varepsilon} \right]$$

[8] This assumption means that while households own tradable sector firms, the discount factor of tradable sector firms differs from those of the representative household. The reason for this assumption is our goal to de-emphasize the terms of trade externality. If the tradable sector firms have the same discount factors as households, the terms of trade externality would produce a motivation under shallow FX markets for the planner to distort exchange rates in order to alter the firms' discount factors and thereby influence the production of tradable goods.

$$\Pi_{Xt}(j)=\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\left[E_tP_X(j)-(1+\phi)\frac{W_t}{A_t}\right]Y_{Xt}\left(\frac{P_X(j)}{P_X}\right)^{-\varepsilon}\right].$$

The fact that the labor tax is commonly applied on all home production of tradable goods, and not differentiated across goods according to their final destination, imposes a connection between the domestic price, P_H, and the export price, P_X, in equilibrium. Taking FOCs of the above expressions and rearranging:

$$P_H=P_H(j)=(1+\phi)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Ht}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tY_{Ht}\right]},\; P_X=P_X(j)=(1+\phi)\frac{\varepsilon}{\varepsilon-1}\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Xt}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tE_tY_{Xt}\right]}$$

$$\begin{aligned}\Rightarrow P_X&=P_H\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Xt}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tE_tY_{Xt}\right]}\frac{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_tY_{Ht}\right]}{\mathbb{E}_0\left[\sum_{t=0}^{2}\Lambda_t\frac{W_t}{A_t}Y_{Ht}\right]}\\&=P_X\left(P_H,C_{F0},\{C_{F1}\},\{C_{F2}\},E_0,\{E_1\},\{E_2\}\right).\end{aligned}\tag{6}$$

The planner needs to take into account that the expression for the export price, P_X, is not an independent choice variable, but rather a function of the domestic price, P_H, and the levels of tradable consumption and exchange rates in all periods and states. The algebraic expression for P_X is provided in Appendix A.1.

Housing sector firms

Housing sector firms are perfectly competitive and take rental prices as given, which are flexible in every period. Following Kiyotaki and Moore (1997), there are two housing subsectors, one with a linear production function and another with a concave production function. Firms in subsector $k\in\{Linear,Concave\}$ purchase land, L_t^k, in period t in order to produce housing services, Y_{Rt+1}^k, in period $t+1$:

$$Y_{Rt+1}^k=\left\{\begin{array}{cc}L_t^k & \text{for } k=Linear\\ G\left(L_t^k\right) & \text{for } k=Concave\end{array}\right\}$$

where $G' > 0$, $G'' < 0$, and $G'(0) = 1$. They maximize expected profits given by:

$$\mathbb{E}_t \Pi^k_{Rt+1} = \mathbb{E}_t \left[P_{Rt+1} Y^k_{Rt+1} + q_{t+1} L^k_t \right] - \left(1 + \theta^k_{Rt}\right) (1 + \rho_t) \, q_t L^k_t,$$

where P_{Rt+1} is the rental price of housing, q_t is the price of land, θ^k_{Rt} is the housing macroprudential tax applied to each subsector, and ρ_t is the interest rate offered by domestic banks.

Housing sector firms finance their operations by borrowing from domestic banks and remit their final asset position to households in period 2. The domestic currency debt of subsector k evolves as follows:

$$D^k_{Rt+1} = \left(1 + \theta^k_{Rt-1}\right) (1 + \rho_{t-1}) \, D^k_{Rt} + q_t L^k_t - \left[P_{Rt} Y^k_{Rt} + q_t L^k_{t-1} \right] - T^k_{MPt} + T^k_{Rt}.$$

The first term on the right hand side is accumulated debt including interest payments, the second term is the financing of land purchases via additional debt, the third term in square brackets is the repayment of debt using rental income and the resale value of the land purchased in the previous period, the fourth term, T^k_{MPt}, is a lump-sum transfer to each subsector,[9] and the final term, T^k_{Rt}, is a lump-sum transfer made to the households in period 2.

The linear subsector is subject to a borrowing constraint between periods 1 and 2:

$$D^{Linear}_{R2} \leq \kappa_{L1} q_1 L^{Linear}_1,$$

where κ_{L1} is a parameter governing the pledgability of land value between periods 1 and 2. The right hand side of the constraint becomes tighter when the land price declines.[10] The linear subsector's

[9]We allow the planner to rebate the proceeds from macroprudential taxes back to the agents that have been taxed to begin with. This assumption allows us to abstract from the income effects and to focus only on the substitution effects of the taxes, in parallel with the literature studying capital controls and macroprudential taxes in representative agent models.

[10]We assume that the current price of land enters the constraint, rather than its future price as in Kiyotaki and Moore (1997). This assumption accounts for the finite horizon in our model instead of the infinite horizon in theirs, sacrificing some of the amplification and persistence from shocks in their model while preserving the pecuniary externality we wish to highlight.

optimality conditions are:

$$\frac{\mathbb{E}_0\left[P_{R1}+q_1\right]}{\left(1+\theta_{R0}^{Linear}\right)\left(1+\rho_0\right)}=q_0 \text{ and } \frac{P_{R2}+q_2}{\left(1+\theta_{R1}^{Linear}\right)\left(1+\rho_1\right)}\left\{\begin{array}{ll}=q_1 & \text{if } D_{R2}^{Linear}<\kappa_{L1}q_1L_1^{Linear} \\ \geq q_1 & \text{if } D_{R2}^{Linear}=\kappa_{L1}q_1L_1^{Linear}\end{array}\right\} \tag{7}$$

The concave subsector does not face a borrowing constraint. It satisfies the FOCs:

$$\frac{G'\left(L_0^{Concave}\right)\mathbb{E}_0\left[P_{R1}\right]+\mathbb{E}_0\left[q_1\right]}{\left(1+\theta_{R0}^{Concave}\right)\left(1+\rho_0\right)}=q_0 \text{ and } \frac{G'\left(L_1^{Concave}\right)P_{R2}+q_2}{\left(1+\theta_{R1}^{Concave}\right)\left(1+\rho_1\right)}=q_1. \tag{8}$$

Market clearing in the land market requires:

$$L_t^{Linear}+L_t^{Concave}=1. \tag{9}$$

Market clearing in the market for nontradable housing services requires:

$$C_{Rt}=Y_{Rt+1}^{Linear}+Y_{Rt+1}^{Concave}. \tag{10}$$

The planner's proceeds from macroprudential taxes on each subsector are rebated back to the same subsector via a lump-sum transfer:

$$T_{MPt}^{k}=\theta_{Rt-1}^{k}\left(1+\rho_t\right)D_{Rt}^{k}. \tag{11}$$

In Kiyotaki and Moore (1997), the two subsectors are not regulated. If the planner can impose separate macroprudential taxes on both subsectors, it can neutralize the linear subsector's borrowing constraint. In this paper, we will mainly focus on the more interesting case where macroprudential taxes are allowed on the linear subsector, i.e., $\theta_{Rt}^{Linear}\in\mathbb{R}$, while the concave subsector is unregulated, i.e., $\theta_{Rt}^{Concave}\equiv 0$.

Domestic banks

The total debt position of the economy sums over household and housing sector debts:

$$D_{t+1}=D_{HHt+1}+D_{Rt+1}^{Linear}+D_{Rt+1}^{Concave}.$$

Domestic banks lend to households and the housing sector by transferring funds in domestic currency from international financial intermediaries. They maximize profits:

$$\Pi_{Bt+1} = (\rho_t - i_t)\, D_{t+1}$$

subject to the external borrowing constraint between periods 1 and 2:

$$D_2 \leq \kappa_{H1} P_{H1}. \tag{12}$$

This constraint takes a simple form: κ_{H1} is a parameter governing the pledgability of domestic tradable goods between periods 1 and 2, and it multiplies the domestic currency price P_{H1}, which means that in dollar terms, the constraint becomes tighter when the exchange rate depreciates. This formulation aligns our model with the practical concerns of policymakers around the world, and it also echoes the constraint in Farhi and Werning (2016).[11] If banks' constraints do not bind, competition between banks ensures that households and the housing sector can borrow and lend at the policy rate: $\rho_t = i_t$. If banks' constraints do bind, the borrowing rate ρ_t rises above the policy rate i_t in order to clear the domestic debt market.

International financial intermediaries

International financial intermediaries take positions of q_{t+1} in domestic currency bonds and $-\frac{q_{t+1}}{E_t}$ in dollar bonds in period t in order to maximize their dollar profits subject to a balance sheet friction echoing the one considered by Gabaix and Maggiori (2015):[12]

$$\max_{q_{t+1}} \frac{1}{(1+i_t^*)} \frac{q_{t+1}}{E_t} \mathbb{E}_t \left[(1-\varphi_t)\,(1+i_t)\,\frac{E_t}{E_{t+1}} - (1+i_t^*) \right]$$

[11]In subsection 5.3 of Farhi and Werning (2016), the nontradable good has a sticky price, and households can borrow up to a specific fraction of the value of nontradable output. Instead, in our model, we assume that households borrow from banks, and those banks can borrow up to a specific fraction of the sticky price of the home-produced tradable good. Both constraints become tighter when the exchange rate depreciates.

[12]We assume that intermediaries maximize the dollar value of profits, not the domestic currency value of profits. This assumption means that in the absence of balance sheet frictions (i.e., if $\Gamma = 0$), the intermediaries' uncovered interest parity (UIP) condition can be written in a form that clearly parallels the households' Euler condition. As a result, when combining the UIP and Euler conditions (as in equation (22), for example), there is no case for prudential capital controls if households' consumption levels across period-1 states are identical and unaffected by period-1 shocks.

$$\text{subject to } \frac{1}{(1+i_t^*)}\frac{q_{t+1}}{E_t}\mathbb{E}_t\left[(1-\varphi_t)(1+i_t)\frac{E_t}{E_{t+1}}-(1+i_t^*)\right] \geq \frac{1}{(1+i_t^*)}\Gamma\left(\frac{q_{t+1}}{E_t}\right)^2,$$

where $\Gamma_t \geq 0$ captures the severity of the balance sheet friction, and φ_t is the capital inflow tax announced in period t and applies to the repayments made to the financial intermediaries in period $t+1$. A fraction λ of the intermediaries are owned by domestic households and the remaining fraction $(1-\lambda)$ are owned by foreigners. Capital controls distort the decisions of all intermediaries, but since the planner rebates all tax revenues to households, only the foreign-owned fraction of the intermediaries ends up paying taxes in net terms.

The constraint for financial intermediaries always binds. We can derive the intermediaries' demand for domestic currency bonds:

$$\frac{Q_{t+1}}{E_t} = \frac{1}{\Gamma}\mathbb{E}_t\left[(1-\varphi_t)(1+i_t)\frac{E_t}{E_{t+1}}-(1+i_t^*)\right].$$

The intermediaries' realized profit in domestic currency in period $t+1$ is:

$$\Pi_{FIt+1} = Q_{t+1}\left[(1-\varphi_t)(1+i_t)-(1+i_t^*)\frac{E_{t+1}}{E_t}\right].$$

We assume that there is a separate group of non-optimizing foreign intermediaries who have exogenous and stochastic demands for domestic currency debt. Their exogenous debt holdings are L_{t+1} in domestic currency bonds, amounting to $S_t = \frac{L_{t+1}}{E_t}$ in dollar value. They are not subject to the balance sheet friction described above, and their decisions to purchase domestic currency debt do not depend on the expected returns.

In our model, FX intervention involves the planner taking a position of O_{t+1} in local currency bonds and $FXI_t = -\frac{O_{t+1}}{E_t}$ in dollar bonds. The realized profit for the planner from this transaction is:

$$T_{FXIt+1} = O_{t+1}\left[(1+i_t)-(1+i_t^*)\frac{E_{t+1}}{E_t}\right]. \tag{13}$$

Market clearing in the domestic currency debt market requires:

$$Q_{t+1} = D_{t+1} - O_{t+1} - L_{t+1},$$

which produces the "Gamma equations" that relate expected excess premia to capital inflows:

$$\Gamma\left(\frac{D_1}{E_0}+FXI_0-S_0\right)=\mathbb{E}_0\left[\eta_1-(1+i_0^*)\right] \tag{14}$$

$$\Gamma\left(\frac{D_2}{E_1}+FXI_1-S_1\right)=\eta_2-(1+i_1^*)\,, \tag{15}$$

where we define the gross return on domestic assets in dollar terms:

$$\eta_{t+1}=(1-\varphi_t)\,(1+i_t)\,\frac{E_t}{E_{t+1}}>0.$$

Since the gross external return is a combination of the ex ante policy rate, the ex ante capital controls, and the ex ante and ex post exchange rates, it must inherit the contingency properties of its constituent components. Using H and L superscripts for the values of variables after the period-1 realizations of high and low shocks respectively, we derive the following "contingency constraint:"

$$\frac{\eta_1^H}{\eta_1^L}=\frac{E_1^L}{E_1^H}\Rightarrow E_1^H\eta_1^H=E_1^L\eta_1^L. \tag{16}$$

The planner's proceeds from labor taxes, capital inflow taxes, and consumer macroprudential taxes are distributed to households via a lump-sum transfer:

$$T_{Gt+1}+\phi W_{t+1}N_{t+1}+\varphi_t\,(1+i_t)\,(Q_{t+1}+L_{t+1})+\theta_{HHt}\,(1+\rho_t)\,D_{HHt+1}=0. \tag{17}$$

Competitive equilibrium

Definition A competitive equilibrium for this economy is a set of quantities $\{C_{Ht},C_{Ft},C_{Rt},N_t,$ $L_t^{Linear},L_t^{Concave},Y_{Ht},Y_{Xt},Y_{Rt}^{Linear},Y_{Rt}^{Concave}\}_{t=0}^{2}$ and prices $\left\{P_H,P_X,\{\rho_t\}_{t=0}^{1},\{W_t,E_t,P_{Rt},q_t\}_{t=0}^{2}\right\}$ that satisfy the households' constraints and FOCs (1)-(3), the tradable sector firms' production and price-setting decisions (4) and either (5) or (6), the housing sector firms' production decisions (7) and (8), the land and housing services market clearing conditions (9) and (10), the banks' borrowing constraint (12), the domestic currency bond market clearing conditions (14)-(15), the contingency constraint for gross external returns (16), and the lump sum transfer constraints (11), (13), and (17), taking as given the planner's choice of the policy instruments $\{i_t, \varphi_t, \theta_{HHt}, \theta_{Rt}^{Linear}, FXI_t\}_{t=0}^{1}$.

Substituting the competitive equilibrium equations into the households' budget constraints, we obtain the economy-wide resource constraint for tradable goods:

$$D_{t+1} \geq -E_t P_{Ft}^* \left[\omega C_t^* - C_{Ft}\right] - E_t P_{Zt}^* Z_t - (1-\lambda) O_t \left[\left(1+\widehat{i}_{t-1}\right) - \left(1+i_{t-1}^*\right) \frac{E_t}{E_{t-1}}\right] + \lambda \left(1+i_{t-1}^*\right) \frac{E_t}{E_{t-1}} D_t + (1-\lambda) \left(1+\widehat{i}_{t-1}\right) D_t, \tag{18}$$

where $\left(1+\widehat{i}_t\right) = (1-\varphi_t)(1+i_t)$. Combining capital controls and the policy rate into an "effective foreigners' interest rate" is useful for analytical simplicity. Once the effective rate is pinned down, the planner can decompose it into the two separate policy instruments using the households' Euler condition and the information on whether the banks' borrowing constraint is binding or not.

The resource constraint highlights the importance of the parameter λ. Households own a fraction λ of the intermediaries, and those intermediaries borrow in dollars to purchase the domestic currency debt that is issued by households and the housing sector. Therefore, when considering the economy-wide external debt position, the fraction λ of the domestic currency debt position nets out to generate a net dollar exposure. If $\lambda > 0$, households' income moves as if households and the housing sector have issued some dollar bonds themselves: there is a currency mismatch, and a depreciation in the exchange rate increases the domestic currency value of the households' external debt repayments, which may tighten the banks' borrowing constraint. This connection of the banks' constraint to the exchange rate becomes more evident when the constraint is written in dollar terms:

$$\frac{D_2}{E_1} \leq \kappa_{H1} \frac{P_H}{E_1}. \tag{19}$$

The remaining fraction $(1-\lambda)$ represents the domestic currency portion of external debt.

Substituting the competitive equilibrium equations and the housing sector conditions into the linear housing subsector's borrowing constraint, we obtain a single equation which summarizes the contribution of the housing sector to the competitive equilibrium:

$$\begin{aligned}
D_{R2}^{Linear} &= (1+\rho_0)\left[(1+\rho_{-1})\,D_{R0}^{Linear} - P_{R0}L_{-1}^{Linear}\right] \\
&+ \left\{\frac{G'\left(1-L_0^{Linear}\right)\mathbb{E}_0\left[P_{R1}\right]}{\left(1+\theta_{R0}^{Concave}\right)} + \mathbb{E}_0\left[\frac{G'\left(1-L_1^{Linear}\right)P_{R2}+q_2}{\left(1+\theta_{R0}^{Concave}\right)\left(1+\theta_{R1}^{Concave}\right)(1+\rho_1)}\right]\right\}\left(L_0^{Linear}-L_{-1}^{Linear}\right) \\
&- P_{R1}L_0^{Linear} + \frac{G'\left(1-L_1^{Linear}\right)P_{R2}+q_2}{\left(1+\theta_{R1}^{Concave}\right)(1+\rho_1)}\left((1-\kappa_{L1})\,L_1^{Linear}-L_0^{Linear}\right) \leq 0 \qquad (20)
\end{aligned}$$

$$\text{where } P_{Rt} = \frac{\alpha_R P_H C_{Ht}}{\alpha_H\left[L_{t-1}^{Linear} + G\left(1-L_{t-1}^{Linear}\right)\right]}.$$

This inequality condition is slack if the planner can regulate both housing subsectors, i.e., if both $\theta_{Rt}^{Linear} \in \mathbb{R}$ and $\theta_{Rt}^{Concave} \in \mathbb{R}$ are allowed, or if $\theta_{R0}^{Concave} \equiv 0$ but κ_{L1} is high enough such that the flexible adjustment of rents and land prices after shocks poses no financing problems for the linear housing subsector. If so, the non-housing-sector quantities and prices are not affected by the existence of the housing sector. If the constraint does bind in equilibrium, owing to rents and house prices becoming excessively depressed after specific shocks, then the housing sector does distort the competitive equilibrium. Rents and housing prices fall after shocks which decrease the pledgability of land, κ_{L1}, or which decrease domestic aggregate demand, causing a reduction in the consumption of home tradable goods, C_{Ht}.

Constrained Efficient Allocations

We can write the indirect utility function in period t as follows:

$$V\left(C_{Ft}, p_{Ht}, p_{Xt}, L_{t-1}\right) = U\left(\frac{\alpha_H}{\alpha_F}p_{Ht}C_{Ft}, C_{Ft}, L_{t-1}^{Linear} + G\left(1-L_{t-1}^{Linear}\right), \frac{\frac{\alpha_H}{\alpha_F}p_{Ht}}{A_t}C_{Ft} + \frac{\omega p_{Xt}}{A_t}C_t^*\right)$$

$$\begin{aligned}
\text{where } V_{Ft} &= \frac{\alpha_F}{C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\left(1-\frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}\right)\right], V_{p_H t} = \frac{\alpha_H}{p_{Ht}}\left(1-\frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}\right), \\
V_{p_X t} &= -\frac{\omega}{A_t}C_t^*, \text{ and } V_{Lt} = \alpha_R\frac{1-G'\left(1-L_{t-1}^{Linear}\right)}{L_{t-1}^{Linear}+G\left(1-L_{t-1}^{Linear}\right)}.
\end{aligned}$$

Next, we define four wedges which summarize the distance of the allocation from the efficient frontier. We identify the key externalities related to each wedge, with the proviso that in our integrated framework, the wedges are jointly determined as a result of all the externalities.

The first wedge is for home consumption, as in Farhi and Werning (2016), and arises from the stickiness of the tradable-good price when sold for domestic consumption:

$$\tau_{Ht} = 1 + \frac{1}{A_t}\frac{U_{Nt}}{U_{Ht}} = 1 - \frac{1}{A_t}\frac{C_{Ht}}{\alpha_H}.$$

This "aggregate demand (AD) wedge" is positive if P_H is too high relative to domestic aggregate demand. There are *aggregate demand externalities* because households do not internalize the impact of their consumption decisions on the time path of aggregate demand, which determines the appropriateness of the pre-set domestic price, P_H. There are also *pecuniary aggregate demand externalities* because they do not internalize the impact of their decisions on the level of the exchange rate E_1 which enters the banks' borrowing constraint.

The second wedge relates to export production and varies depending on the price-setting paradigm:

$$\tau_{Xt}^{PCP} = \left(1 - \frac{\omega p_{Xt}^{PCP}}{p_{Xt}^{PCP}\frac{d}{dp_{Xt}^{PCP}}\left(\omega p_{Xt}^{PCP}\right)}\right) + p_{Xt}^{PCP}\frac{1}{A_t}\frac{U_{Nt}}{U_{Ft}} = -p_{Xt}^{PCP}\frac{1}{A_t}\frac{C_{Ft}}{\alpha_F}$$

$$\tau_{Xt}^{DCP} = \left(1 - \frac{\omega p_{Xt}^{DCP}}{p_{Xt}^{DCP}\frac{d}{dp_{Xt}^{DCP}}\left(\omega p_{Xt}^{DCP}\right)}\right) + p_{Xt}^{DCP}\frac{1}{A_t}\frac{U_{Nt}}{U_{Ft}} = -p_{Xt}^{DCP}\frac{1}{A_t}\frac{C_{Ft}}{\alpha_F}.$$

This "terms of trade (TOT) wedge" highlights that there is a *TOT externality* because while firms do take into account that the demand curve for their own export variety is downward-sloping, they do not internalize that the demand curve for the aggregate export good is also downward-sloping. Under the unit elastic demand assumption for export demand, the first term in the above expressions is zero. These wedges are always negative because firms set the export price, P_X, lower than the level that maximizes the economy-wide TOT. In other words, the planner wishes to push the economy to an allocation with a higher export price, P_X, and a lower export volume, Y_{Xt}, while earning the same dollar value of export revenues.

Together, the first and second wedges capture what we call "static AD and price pressures": whether excess/insufficient domestic and external demand for home-produced traded goods would tend to push the prices of these goods up or down relative to the pre-set rigid level. In sections 4-6, we show that constrained welfare maximization produces a motivation for the planner to minimize overall price pressures, i.e., to minimize a weighted sum of the above wedges, unless other wedges need to be addressed at the same time. Therefore, while we assume rigid prices for analytical tractability, optimal policies from our framework achieve the major price stabilization motive of the traditional New Keynesian framework. However, we do not capture inefficiencies related to price dispersion or credibility-damaging inflation dynamics.

The third wedge captures the deviation of the gross external return from the level that would prevail if all households could borrow using dollar bonds:

$$\tau_{\Gamma t} = \eta_t - \left(1 + i^*_{t-1}\right).$$

This "UIP wedge" enters the economy-wide resource and borrowing constraints. If the wedge is positive in a particular state and some of the intermediaries are foreign-owned, i.e., $\lambda \in [0, 1)$, then there is a net loss of resources from the domestic economy to those foreign-owned intermediaries in that state. There is what we call a *financial TOT externality* because households do not internalize the impact of their borrowing decisions on the external returns that other households must pay.

The fourth wedge captures the deviation of housing services production from its maximum level:

$$\tau_{Rt} = 1 - G'\left(1 - L^{Linear}_{t-1}\right)$$

This "housing wedge" is positive if land usage is shifted from the linear to the concave subsector of the housing market. The production of housing services is maximized when the linear subsector uses all of the land in the economy for its production. Production is reduced in the presence of borrowing constraints and/or macroprudential taxes for the linear subsector. There is a *pecuniary production externality* because housing sector firms do not internalize the impact of their land usage decisions on the land price q_1 which enters their borrowing constraint.

In this economy, the first best allocation is not feasible: all the wedges $\{\tau_{Ht}, \tau_{Xt}, \tau_{\Gamma t}, \tau_{Rt}\}$ cannot be equal to zero in every state. In a deterministic closed economy, the planner can set the labor tax, ϕ, to manipulate the domestic price level, P_H, such that the distortion owing to monopolistic competition is perfectly eliminated and the AD wedge is zero, i.e., $\tau_{Ht} = 0$. In a deterministic open economy under PCP, the planner cannot perfectly eliminate this distortion. Instead, it has to set the labor tax to balance the AD and TOT wedges, and, as in Gali and Monacelli (2005), some of both distortions remain in equilibrium, i.e., $\tau_{Ht} \neq 0$ and $\tau_{Xt} \neq 0$. The same principle applies under DCP because of the fact that the export price, P_X, is still tied to the domestic price, P_H. The addition of shocks to the economy, as well as the introduction of the UIP wedge, $\tau_{\Gamma t}$, and housing wedge, τ_{Rt}, reinforces the result that all distortions cannot be entirely eliminated.

This observation leads us to focus on deriving constrained efficient allocations.

Definition A constrained efficient allocation is a set of quantities $\left\{C_{Ht}, C_{Ft}, C_{Rt}, N_t, L_t^{Linear}, L_t^{Concave},\right.$ $\left.Y_{Ht}, Y_{Xt}, Y_{Rt}^{Linear}, Y_{Rt}^{Concave}\right\}_{t=0}^{2}$, prices $\left\{P_H, P_X, \{\rho_t\}_{t=0}^{1}, \{W_t, E_t, P_{Rt}, q_t\}_{t=0}^{2}\right\}$, and policy instruments $\left\{i_t, \varphi_t, FXI_t, \theta_{HHt}, \theta_{Rt}^{Linear}\right\}_{t=0}^{1}$ which solve under full commitment:

$$\max_{\left\{C_{Ft}, P_H, E_t, \eta_{t+1}, FXI_t, L_{t-1}^{Linear}\right\}} \left\{ \begin{array}{lr} \mathbb{E}_0\left[\sum_{t=0}^{2} \beta^t V\left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{E_t P_{Ft}^*}{P_H}, L_{t-1}^{Linear}\right)\right] & \text{if PCP} \\ \mathbb{E}_0\left[\sum_{t=0}^{2} \beta^t V\left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{P_{Ft}^*}{P_X}, L_{t-1}^{Linear}\right)\right] & \\ \text{with } P_X = P_X\left(C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}, P_H\right) & \text{if DCP,} \end{array} \right\}$$

subject to the restriction that the allocation constitutes a competitive equilibrium. The full set of equations is listed in Appendix A.2, which uses the dollar forms of all the constraints, fixes the dollar values of all initial debt stocks and the period-2 land price, and sets $\theta_{Rt}^{Concave} \equiv 0$.

The joint consideration of the above policy instruments and wedges nests many important results from the literature and also allows us to establish several results that are novel relative to the literature. We describe these results in the following sections, adding one set of frictions at a time to gradually build towards a bigger model. Our framework allows us to determine whether policies which have been highlighted in the literature as being useful to minimize specific wedges after specific shocks can also be used to address other wedges after other shocks. We are also able to analyze whether policies which have been recommended to reduce specific wedges in the literature may in fact exacerbate other wedges when economies suffer from multiple frictions.

To assess the complementarity and substitutability of instruments, and to facilitate the use of the model for practical policy advice, we can derive optimal policies when different sets of instruments are available in the planner's policy toolkit. Every time an instrument is removed from the toolkit, additional constraints need to be added onto the planner problem:

- If FX intervention is not permitted, we set $FXI_0 = FXI_1 = 0$ and remove the FOCs with respect to FX intervention, FXI_t.
- If neither capital controls nor consumer macroprudential controls are permitted, we add the household Euler conditions (3) as constraints with the capital control and macroprudential control terms set to zero.
- If housing macroprudential controls are not permitted, we set $L_0 = 1$.
- If the domestic policy rate cannot be used, we set it equal to the foreign interest rate, i.e., $i_t = i_t^*$, in the expression for the gross external premium, η_{t+1}.
- If the exchange rate is pegged, we set the exchange rate in all periods and states to the initial value E_0.

Our solution approach is as follows. We assume that the constrained planner problem is convex in the region of interest, and correspondingly, we derive the FOCs for the problem in Appendix A.2. In the next two sections, we summarize the salient properties of these FOCs, indexing our results by the pricing paradigm:

$$\mathbb{I}^{PCP} = \left\{ \begin{array}{ll} 1 & \text{if PCP} \\ 0 & \text{if DCP} \end{array} \right\} \text{ and } \mathbb{I}^{DCP} = \left\{ \begin{array}{ll} 0 & \text{if PCP} \\ 1 & \text{if DCP} \end{array} \right\}.$$

Then in each section, we explain our results using a mix of analytical and numerical results to qualitatively characterize the optimal integrated use of policies.

4 Deep Foreign Exchange Markets

To present our results most clearly, we start with the smallest integrated model and gradually add frictions one at a time. In this section, we abstract from frictions in FX markets and the housing sector, and focus on the optimal integrated use of the policy rate and capital controls under different pricing paradigms when borrowing constraints are present. Most advanced economies and a few emerging markets have deep FX markets, with their currencies being traded by a substantial number of financial intermediaries, except possibly during episodes of severe global financial stress such as the ongoing COVID-19 crisis.

As we explain below, we study the case with deep FX markets by setting $\Gamma = 0$ (no intermediary frictions), $\lambda = 1$ (households own all intermediaries), and either perfect housing sector regulation or $\kappa_{L1} \to \infty$ (no binding housing frictions), and by removing FX intervention from the planner's toolkit.

4.1 Policy Instruments and Wedges

The deep FX markets case formally corresponds to setting $\Gamma = 0$ in the constraints and FOCs summarized in Appendix A.2. Financial intermediaries face no balance sheet constraints, so their capacity to hold domestic currency debt is unlimited, and the country's external debt position does not matter for the country's gross external return, η_t. The Gamma equations therefore reduce to the UIP conditions:

$$\mathbb{E}_0\left[\tau_{\Gamma 1}\right] = 0 \text{ and } \tau_{\Gamma 2} = 0. \tag{21}$$

UIP wedges, $\tau_{\Gamma t}$, paid by the domestic economy to intermediaries generate welfare losses if a fraction of the intermediaries are foreign-owned, i.e., $\lambda \in [0, 1)$. However, since the UIP wedges average out to zero, the average external premium is zero. Therefore, to simplify the algebra, we ignore foreign ownership of the intermediaries in this section and set $\lambda = 1$. Note that if domestic households own all financial intermediaries, the economy's liabilities are effectively entirely in dollars. As a further simplification, we assume that housing frictions do not bind. As described in section 3, this result follows either from regulation of both housing subsectors, i.e., both $\theta_{Rt}^{Linear} \in \mathbb{R}$ and $\theta_{Rt}^{Concave} \in \mathbb{R}$ are allowed, or from high housing sector debt capacity, i.e., $\kappa_{L1} \to \infty$.

The absence of binding housing sector frictions means that one of the two instruments of capital controls and consumer macroprudential taxes may become redundant, because both affect the economy via altering external debt.[13] In this section, we first focus on optimal capital controls and then explain whether or not they can be substituted by consumer macroprudential taxes. Equation (21) establishes that FX intervention does not affect the exchange rate, so we defer further consideration of FX intervention to section 5. In addition, we defer further consideration of housing macroprudential taxes to section 6.

The households' Euler conditions can be rewritten as:

$$\frac{\alpha_F}{P_{F0}^* C_{F0}} = \beta \frac{(1+i_0^*)}{(1-\varphi_0)} \frac{1}{\mathbb{E}_0\left[\frac{E_0}{E_1}\right]} \mathbb{E}_0\left[\frac{E_0}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}}\right] \text{ and } \frac{\alpha_F}{P_{F1}^* C_{F1}} \geq \beta \frac{(1+i_1^*)}{(1-\varphi_1)} \frac{\alpha_F}{P_{F2}^* C_{F2}}. \tag{22}$$

These Euler conditions demonstrate that capital controls are effective instruments under deep FX markets, as they raise the domestic policy rate above the foreign interest rate and thereby reduce domestic borrowing. Exchange rates E_0 and E_1 enter the Euler condition between periods 0 and 1 because households have access to domestic currency bonds only, and not dollar bonds, and the extent of possible risk-sharing depends on the contingency of the available bonds.[14] They do not enter the Euler condition between periods 1 and 2 because there is no uncertainty between those periods.

Next, we turn to the conditions characterizing the constrained efficient allocation, to understand which externalities arise under deep FX markets and how policies should be used to alleviate them.

[13] Both capital controls and consumer macroprudential taxes to address pecuniary AD externalities as envisaged in this section would be labeled as CFM/MPMs in the IMF's taxonomy, because as will be clear below, they are both designed to limit capital flows for financial stability reasons.

[14] If the shock is such that period-1 imports, C_{F1}, are perfectly stabilized across high and low realizations of the shock, or if the period-1 exchange rate, E_1, is perfectly stabilized across realizations, then the state-contingency of the bond is no longer important, and the exchange rates do not appear in the Euler conditions. The Euler conditions become identical to the Euler conditions of households who are able to participate directly in the dollar bond market subject to capital controls.

The planner's Euler conditions for $t \in \{0,1\}$ are:

$$\begin{aligned}
&\frac{\alpha_F}{P^*_{Ft} C_{Ft}} \left[1 + \frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] - \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt}\right] \frac{1}{\beta^t \pi_t} \frac{1}{P^*_{Ft}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft}} \right\} \\
&= \beta \left(1 + i^*_t\right) \mathbb{E}_t \left\{ \frac{\alpha_F}{P^*_{Ft+1} C_{Ft+1}} \left[1 + \frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right] \right\} + \Psi_{Bt} \frac{1}{\beta \left(1 + i^*_{t-1}\right)} \\
&- \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt}\right] \mathbb{E}_t \left[\frac{(1 + i^*_t)}{\beta^t \pi_{t+1}} \frac{1}{P^*_{Ft+1}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft+1}} \right] \right\}, \qquad (23)
\end{aligned}$$

where Ψ_{Bt} is the multiplier on the banks' borrowing constraint. The first term on the left hand side represents the marginal utility of consumption in period t, taking into account the AD wedge. The second term on the left hand side captures an effect which only arises under DCP: the impact of the period-t consumption decision on welfare via the period-0 export-price-setting decision. The first term on the right hand side represents the marginal utility of consumption in period $t+1$, taking into account the AD wedge. The second term on the right hand side captures the distortion in the Euler conditions if the borrowing constraint binds. The third term on the right hand side captures the impact of the period-$t+1$ consumption decision on welfare via the period-0 export-price-setting decision.

Comparing the household and planner Euler conditions, we can see that two of the wedges, τ_{Ht} and τ_{Xt}, may provide a rationale for policy intervention in the deep FX markets case.

We have mentioned in section 3 that it is not feasible for these two wedges to be equal to zero in every state. Nevertheless, for illustrative purposes, we observe that if we set all wedges to zero, then both the households' and the planner's Euler conditions would reduce to:

$$\frac{\alpha_F}{P^*_{Ft} C_{Ft}} = \beta \left(1 + i^*_t\right) \mathbb{E}_t \left[\frac{\alpha_F}{P^*_{Ft+1} C_{Ft+1}}\right] \text{ for } t \in \{0,1\},$$

which is identical to the Euler condition of households who are able to participate without restriction in the dollar bond market. In this case, the planner would set a domestic policy rate consistent with zero capital controls, and would allow full flexibility of the exchange rate.

When the wedges are not zero, there may be a case for the planner to move the domestic policy rate in a different manner, and also to add capital controls into the toolkit in order to stabilize the wedges over time.

The FOCs for exchange rates in each state are:

$$\underbrace{\alpha_H \tau_{Ht}}_{\text{Stabilize demand for home goods}} = \underbrace{\begin{array}{c} -\mathbb{I}^{PCP} \cdot \left\{ \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{PCP} \right\} \\ -\mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP} \right] \frac{1}{\beta^t \pi_t} \frac{E_t}{P_X} \left(-\frac{\partial P_X}{\partial E_t} \right) \right\} \end{array}}_{\text{Optimize TOT on export goods}}$$
$$+ \underbrace{\frac{\Psi_{Bt}}{\beta^t \left(1 + i_{t-1}^*\right)} \kappa_{H1} \frac{P_H}{E_t}}_{\text{Relax bank constraint}} . \tag{24}$$

The planner sets the exchange rate to balance price pressures and binding borrowing constraints within each state. The first term, which includes the AD wedge τ_{Ht}, represents the benefit of moving the exchange rate to generate import substitution and stabilize domestic demand for the home-produced tradable good. The second term, which includes the TOT wedges, τ_{Xt}, represents the benefit of moving the exchange rate to optimize the TOT on export goods by altering the export volume—either just within a specific state (under PCP), or on average across all states via the impact on the period-0 export-price-setting decision (under DCP). Price pressures are balanced if, for example, the first term is positive because prices are too high for consumption purposes, but the second term (including the minus sign in front) is also positive because prices are too low for export purposes. The third term represents the effect of exchange rate movements on the tightness of the borrowing constraint. In line with condition (21), the domestic policy rate moves inversely to the expected exchange rate depreciation.

The expression for capital controls is:

$$\varphi_t = \begin{cases} 1 - \dfrac{\frac{1}{\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\right]}\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\right]\left[\begin{array}{c}\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] \\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{\beta^t\pi_t}\frac{1}{P^*_{Ft}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft}}\right\}\end{array}\right]}{\frac{\alpha_F}{P^*_{Ft}C_{Ft}}\left[\begin{array}{c}\mathbb{E}_t\left\{\frac{\alpha_F}{P^*_{Ft+1}C_{Ft+1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right]\right\} \\ -\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_t\left[\frac{1}{\beta^{t+1}\pi_{t+1}}\frac{1}{P^*_{Ft+1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{Ft+1}}\right]\right\}\end{array}\right]} \\ \text{if } \Psi_{Bt}=0 \\ \\ 0 \text{ if } \Psi_{Bt}>0, \end{cases} \tag{25}$$

where capital controls are ineffective, and therefore set to zero, when the banks' borrowing constraint binds.

Capital controls are non-zero if and only if the numerator and denominator of the fraction in equation (25) are unbalanced. The expressions for the numerator and denominator are obtained by substituting for the AD wedge, τ_{Ht}, using equation (24). Therefore, there are two possible rationales for capital controls in this version of our model.

The first potential rationale for capital controls arises if there is a pecuniary AD externality from an occasionally-binding borrowing constraint, i.e., $\Psi_{Bt} > 0$, which captures the concerns of many emerging-market policymakers. Households do not internalize that their borrowing in period t may generate lower aggregate demand and a more depreciated exchange rate in period $t+1$, making the banks' borrowing constraint binding. When the borrowing constraint binds, equation (24) indicates that the AD wedge, τ_{Ht}, is optimally kept higher than the TOT wedges, τ_{Xt}, would justify, i.e., the exchange rate is more appreciated and the domestic policy rate is kept higher in order to address the pecuniary externality and relax the constraint. Therefore, monetary policy and exchange rate flexibility no longer fully address the AD externality. As a result, prudential capital controls become optimal. This finding captures Farhi and Werning's (2016) insights regarding the case for capital controls with occasionally-binding borrowing constraints. Our model additionally allows for the size of the externality to be related to the pricing paradigm (PCP versus DCP).

The second potential rationale for capital controls arises from the TOT externality. Capital controls are non-zero if the weighted TOT wedges, τ_{Xt}, are not balanced over time (as in Costinot, Lorenzoni, and Werning, 2014). Our model nests the results of Farhi and Werning (2014) and extends them to the DCP case. This rationale naturally arises in any open-economy framework with price-setting, but policymakers do not emphasize this channel, so we focus on insights that do not hinge on this motive.

Equations (23)-(25) demonstrate that an integrated model is necessary to characterize the optimal use of multiple instruments: the use of each policy instrument affects several wedges and, as a result, the optimal use of other policy instruments. Specifically, we can see that the level of the domestic policy rate affects exchange rates and thereby the optimal use of capital controls, and vice versa.

Next, we illustrate how the constrained efficient policy response varies with different shocks, and for each shock, we highlight both the impact of the pricing structure (PCP or DCP) and other structural characteristics such as commodity dependence and the level of external debt.

In subsections 4.2-4.3, consumer macroprudential taxes, θ_{HHt}, are substitutes for the capital controls, φ_t, described above. Their optimal use follows the following expression, in which either of them can be used by the planner while the other can be set to zero:

$$\frac{(1-\varphi_t)}{(1+\theta_{HHt})} = \begin{cases} \frac{\eta_{t+1} E_{t+1} \beta \mathbb{E}_t \left\{ \frac{1}{E_{t+1}} \frac{\alpha_F}{P^*_{Ft+1} C_{Ft+1}} \right\}}{\frac{\alpha_F}{P^*_{Ft} C_{Ft}}} & \text{if } \Psi_{Bt} = 0 \\ 1 & \text{if } \Psi_{Bt} > 0. \end{cases} \tag{26}$$

However, in subsection 4.4, when we consider the possibility of unregulated sectors and/or circumvention of policy instruments, the two instruments become complements instead of substitutes, and they need to be used together.

4.2 Real Shocks

In this subsection, we consider domestically- and externally-generated shocks which affect the real income of domestic households. Some of the policy recommendations are conditioned on $B_t \left(\equiv \frac{D_t}{E_{t-1}}\right)$, the total domestic currency debt stock at the beginning of period t converted into a dollar value. Since this debt is entirely sold to international financial intermediaries, and domestic households own all financial intermediaries, B_t also represents the representative household's effective exposure to external dollar-denominated debt.

4.2.1 Productivity Shocks

A permanent productivity shock is a shock to the value of $A_1 = A_2$ in the production function of the home tradable good. Since we would like to focus on insights which do not hinge on intertemporal TOT externalities, we set a parameterization of the shock to neutralize this motive:[15]

$$\mathbb{E}_0 \left[\frac{A_0}{A_1}\right] = 1.$$

Under PCP, it is optimal to let the exchange rate move to absorb the shock without moving the level of any policy instruments. Figure 5 shows that the policy rate remains unchanged, and both ex ante and ex post capital controls are zero. After the high realization of the productivity shock, there is an increase in domestic aggregate supply. To close the gap between aggregate demand and aggregate supply, the exchange rate depreciates and makes exports more competitive. Given the unit elastic demand for exports, there is no increase in the dollar value of export revenues, so domestic households are not more wealthy in dollar terms, leaving imports unchanged regardless of the realization of the shock. Households consume more of the home tradable good after positive productivity shocks, as those goods become cheaper relative to imports. The exchange rate depreciates by the same percentage as the increase in productivity, so $\frac{E_t}{A_t}$ is constant across states of nature. Employment is perfectly stabilized across high and low shocks. The AD and TOT wedges are fully

[15] The PCP result on the optimality of capital controls does not depend on this parameterization (consistent with Farhi and Werning, 2014), while the DCP result does.

stabilized across shocks and over time, which means that there is no case for capital controls. This result does not depend on the level of initial external debt, B_0.

Under DCP, the key difference is that employment and the TOT wedges are no longer stabilized across high and low shocks by the exchange rate (figure 6). Nevertheless, capital controls do not become optimal. Just because there is an unstabilized wedge and an extra available instrument does not mean that the instrument should be used. The instrument should only be used if it can help stabilize the wedge. The imperfect macro stabilization under DCP arises from the stickiness of the dollar price of exports across states. Capital controls alter the level of borrowing and the time path of consumption between periods, but they do not address the source of the stabilization problem.

Result 1. *Countries facing permanent productivity shocks should rely solely on exchange rate flexibility, under both PCP and DCP.*

Remark 1. *After permanent productivity shocks, TOT wedges are stabilized under PCP and not under DCP, but capital controls are optimally zero in both cases.*

4.2.2 Commodity Price Shocks

A permanent commodity price shock is a shock to the value of $P^*_{Z1} = P^*_{Z2}$, the dollar price of commodities, and is relevant for countries with large commodity sectors. Since commodities are undifferentiated while tradable goods are differentiated across countries, the effect of the shock should be different from the effect of the productivity shock. In our simulations, we compare two commodity exporters with different initial conditions: one with no unhedged external FX debt, $B_0 = 0$, and one with high unhedged external FX debt, $B_0 > 0$, assuming that both countries have the same level of $\kappa_{H1} \in (0, 1)$.[16]

Under PCP and $B_0 = 0$, capital controls are only optimal insofar as the intertemporal TOT externality is relevant. Figure 7 shows that after commodity price shocks, it is optimal to allow exchange

[16] Our goal in this exercise is to compare safer versus more externally vulnerable countries. An alternative exercise with the same qualitative features would be to fix B_0 and instead compare safer countries with no borrowing constraints, i.e., $\kappa_{H1} \to \infty$, against more vulnerable countries which face occasionally binding borrowing constraints, i.e., $\kappa_{H1} \in (0, 1)$.

rate flexibility without moving the policy rate, and there are small ex ante capital controls arising solely from the intertemporal TOT externality. After the low realization of the shock, there is a decrease in aggregate demand, and domestic households become poorer in dollar terms. The planner allows an exchange rate depreciation which generates a decrease in imports. The higher export demand from the depreciation exactly offsets the wealth-driven decrease in domestic consumption of the home traded good, so that employment in the tradable sector does not change. Indeed, tradable sector employment is stabilized across high and low shocks, and insulated from the commodity sector.

When B_0 is high enough, however, capital controls become optimal to address pecuniary AD externalities. Figure 8 shows that after the low realization of the shock, the borrowing constraint becomes binding, because the associated exchange rate depreciation causes a contraction in the dollar value of the debt limit. The forced deleveraging of households causes a further contraction in aggregate demand. The planner recognizes that depreciation tightens the constraint (19). Therefore, following equation (24), the planner keeps the policy rate higher, and the exchange rate more appreciated, than the price pressures from the AD and TOT wedges would justify. As a result, the domestic consumption of the home good, and employment in the traded sector, both decline. The planner resorts to ex ante capital controls to address pecuniary AD externalities in the presence of occasionally-binding borrowing constraints. However, there is no case for ex post capital controls.

Under DCP, the exchange rate is more volatile and employment in the tradable sector is destabilized. Figure 9 shows that in the absence of the borrowing constraint, it is optimal to implement the same decrease in imports under DCP as under PCP after the low realization of the shock. However, when the exchange rate depreciates under DCP, there is no increase in export demand, and the planner tries to induce domestic households to consume more home goods. Inducing households to consume the same level of imports as under PCP, but more home goods, means that the depreciation must be larger under DCP. In the end, the depreciation is large enough that tradable sector employment increases.

As figure 10 shows, commodity-exporting countries with DCP are more vulnerable to binding borrowing constraints. The larger depreciation after the low realization of the shock means that the debt limit contracts more under DCP. As a result, for the same shock and initial FX debt B_0, borrowing constraints bind more severely under DCP than PCP; and as a corollary, the constraints bind for a larger set of shocks and initial conditions, i.e., for smaller shocks and for lower initial FX debt. Therefore, it is more likely that countries with DCP impose ex ante capital controls to address pecuniary AD externalities than countries with PCP, and the planner sets larger ex ante capital controls under DCP.

Result 2. *Countries with large commodity sectors should not impose capital controls to address pecuniary AD externalities if there are no borrowing constraints, but they should impose positive ex ante capital controls for this purpose if there are borrowing constraints and external FX debt is high. Ex post capital controls are not desirable.*

Remark 2. *After commodity price shocks, exchange rates are more volatile under DCP than PCP.*

Remark 3. *For countries with large commodity sectors, borrowing constraints, and high initial external FX debt, ex ante capital controls to address pecuniary AD externalities are more frequently optimal, and they are larger in magnitude, under DCP than PCP.*

4.3 Financial Shocks

In this subsection, we consider shocks which affect borrowing/lending transactions between domestic banks and the rest of the world. We wish to capture a range of shocks which unpack the global financial cycle—shocks to world interest rates, foreign appetite for domestic currency assets, and the FX debt limit—because different instruments may be necessary to address different dimensions of the cycle. Each of the shocks could also strike on a country-specific basis.

4.3.1 World Interest Rate Shocks

A world interest rate shock is a shock to the value of $(1 + i_1^*)$. It could be triggered by a change in U.S. monetary policy, expected global growth, or global risk appetite.

Under PCP, it is optimal to use the domestic policy rate combined with exchange rate flexibility, and capital controls are only optimal insofar as the intertemporal TOT externality is relevant. Consistent with the households' Euler equation between periods 1 and 2, figure 11 shows that if the realization of the world interest rate is high, then it is optimal for imports to decline in period 1 and increase in period 2. There is a decline in aggregate demand in period 1, which the planner cushions by allowing an exchange rate depreciation (the planner does increase the domestic policy rate, but less than the increase in the world interest rate), which stimulates export demand and generates import substitution. Employment is stabilized across the high and low shocks.

A jump in the world interest rate on its own does not tend to make borrowing constraints binding in our simulations. The reason is that although the depreciation does cause the debt limit to contract, the desired debt level falls more rapidly.

Under DCP, the main difference is that the exchange rate is more volatile under DCP than PCP (figure 12), for the same reason as discussed above for the commodity price shock: export demand is no longer boosted by exchange rate movements, so import substitution must play a larger role. This exchange rate volatility is achieved by keeping the policy rate unchanged in response to the shock. Employment becomes destabilized by the shock. The ex post capital controls which were optimal under PCP owing to the intertemporal TOT externality disappear under DCP, while the same motive generates small ex ante capital controls.

Result 3. *Countries facing world interest rate shocks should not use capital controls to address AD externalities under PCP or DCP.*

Remark 4. *After world interest rate shocks, exchange rates are more volatile under DCP than PCP.*

4.3.2 Foreign Appetite Shocks

A foreign appetite shock is a shock to the value of S_1. For the case of deep FX markets, allocations are not affected by this shock. We defer further consideration of this shock to section 5.

Result 4. *Countries with deep FX markets do not need to respond to shocks to the foreign appetite for domestic currency debt.*

4.3.3 Bank Debt Limit Shocks

A bank debt limit shock is a shock to the value of κ_{H1}, the pledgability parameter in the banks' external borrowing constraint. A decline in κ_{H1} reflects either a downgrade in the perceived creditworthiness of banks in a particular borrower country, or a reversal in the willingness of international banks to extend credit to banks in all borrower countries.

Under PCP, when the initial external FX debt B_0 is high enough, it is optimal to impose ex ante capital controls to address pecuniary AD externalities. Figure 13 shows that after a low realization of the shock, the borrowing constraint becomes binding. The planner lowers the policy rate to support aggregate demand. However, as described above in the example of the commodity price shock with borrowing constraints, the planner keeps the policy rate higher, and the exchange rate more appreciated, than the price pressures from the AD and TOT wedges would justify. Ex ante capital controls are used to address the pecuniary AD externalities.

Under DCP, the exchange rate is more depreciated after the debt limit shock and yet the ex ante capital controls are larger. The reason for these results is as follows. Figure 14 shows the allocations when the planner is able to use both monetary policy and capital controls. We already know from above that the exchange rate is a weaker tool for import/export substitution under DCP than PCP. If there is little cost associated with exchange rate volatility, as with the productivity and interest rate shocks above, then the planner's solution is simply to implement the preferred allocations using higher exchange rate volatility. However, if an exchange rate depreciation may tighten external debt limits, the planner weighs the benefits of depreciation (smaller under DCP than PCP) against the costs of tightening the constraint (equal under DCP and PCP). The end result is that there is still a greater depreciation after the shock under DCP than PCP, but the planner imposes higher ex ante capital controls under DCP as it places substantial weight on relaxing the constraint in period 1, and indeed, the multiplier on the external borrowing constraint is lower under DCP than PCP. There is no case for ex post capital controls during the sudden stop when inflows are constrained already, as controls have no effect.

Allowing ex ante capital controls increases the ex post policy rate in our simulation owing to a combination of two effects. The "financial channel" pushes for a lower ex post policy rate, as capital controls reduce external debt and thereby the need for a high ex post policy rate to relax the external constraint. The "macro channel" pushes for a higher ex post policy rate, as capital controls shift aggregate demand from period 0 to period 1. In practice, the effect on the ex post policy rate may be ambiguous.

Result 5. *Countries with high external FX debt whose banks are vulnerable to external debt limit shocks should impose positive ex ante capital controls.*

Remark 5. *For countries with high external FX debt and vulnerability to external debt limit shocks, ex ante capital controls are larger under DCP than PCP.*

Remark 6. *Imposing ex ante capital controls shifts both sides of the macro-financial trade-off for monetary policy, with ambiguous results on whether the policy rate and capital controls are substitutes or complements.*

4.4 Extension: Limits to Regulation

The above subsections assumed that the housing sector frictions are not relevant, all households borrow only from domestic banks, and these banks are the sole domestic counterparties of the international financial intermediaries. If so, then consumer macroprudential taxes are able to achieve full coverage of all relevant debt transactions and are substitutes for capital controls, following equation

(26). In this subsection, we consider the joint use of consumer macroprudential taxes and capital controls when there is imperfect coverage.

We consider two forms of imperfect coverage. First, consumer macroprudential taxes may cover a subset of households while the remainder of them borrow directly from international financial intermediaries instead of borrowing from domestic banks. Transactions with the intermediaries are beyond the perimeter of domestic macroprudential regulations. For simplicity, we assume that households who borrow directly from intermediaries face an external borrowing constraint of their own with the same formulation as the banks' constraint.

In this case, consumer macroprudential taxes are an imperfect substitute for capital controls: they reduce the external debt of regulated households, but not that of unregulated households. The planner should set consumer macroprudential taxes, θ_{HHt}, for domestically-regulated households of a magnitude pinned down by setting $\varphi_t = 0$ in the expression (26), and should set capital controls, φ_t, on intermediaries lending to the other households of a magnitude pinned down by setting $\theta_{HHt} = 0$ in that expression.

Second, all households may officially be regulated by consumer macroprudential taxes, but they can circumvent the taxes and issue debt directly to international financial intermediaries. For simplicity, we again assume that households and banks face an identical external borrowing constraint.

In this case, consumer macroprudential taxes and capital controls are perfect complements. If the planner imposes consumer macroprudential taxes, households would conduct all their borrowing directly with financial intermediaries, and the taxes would have no effect at all on macro allocations. For there to be any effect, these taxes need to be complemented by capital controls. In other words, the planner must set consumer macroprudential taxes, θ_{HHt}, as above on all households' borrowing from domestic banks, and additionally impose capital controls, φ_t, as above on any intermediaries lending directly to households.

More generally, imperfect coverage by consumer macroprudential taxes should be remedied by the use of capital controls, and vice versa.

Result 6. *Capital controls and consumer macroprudential taxes are imperfect substitutes if macroprudential taxes do not cover all household borrowing, and they are perfect complements if households can circumvent the macroprudential taxes via cross-border transactions which are not intermediated through the domestic banking system.*

4.5 Extension: FX Swap Lines

In this extension, we consider the possibility that the planner may have access to bilateral or multilateral FX swap lines. The planner can draw on these facilities in period 1, convert the FX into domestic currency, transfer the proceeds to domestic banks, and then make repayments at the world interest rate on those facilities in period 2 by collecting domestic currency repayments from banks in line with the domestic interest rate.[17] The overall impact of this set of transactions is to augment the banks' debt limit from equation (12) to the following expression:

$$D_2 \leq \kappa_{H1} P_{H1} + F_2,$$

where F_2 is the total available FX summed over all swap lines.

Access to swap lines is isomorphic to an increase in κ_{H1}. Drawing on these facilities is valuable during period-1 states when the banks' borrowing constraint is binding (i.e., from the above discussions, after large commodity price declines or a substantial tightening of banks' external debt limits), and reduces the severity of the constraint. A reduction in the severity of the constraint in period 1 means that there would be less need for ex ante capital controls.

4.6 Summary

For countries with deep FX markets, we have identified several shocks and structural characteristics which justify a deviation from the traditional Mundell-Fleming prescription of relying solely on the domestic policy rate and exchange rate flexibility. Focusing on the pecuniary-AD-externality

[17] The UIP condition (21) between periods 1 and 2 ensures that the domestic currency funds raised from banks are sufficient to make the necessary FX repayments.

rationale for capital controls, countries should impose positive ex ante capital controls to reduce ex ante overborrowing if they have high external FX debt relative to their debt limits, combined with an exposure to commodity price and/or debt limit shocks. If external FX debt limits are irrelevant, there is no case for the use of capital controls to address AD externalities for countries facing productivity shocks, commodity price shocks, world interest rate shocks, and foreign appetite shocks.

DCP on its own does not justify capital controls to address AD externalities, but in combination with FX mismatches and external debt limits, DCP increases the incidence and severity of binding borrowing constraints. Countries with DCP require ex ante capital controls for a larger range of initial external FX debt levels than countries with PCP, and ex ante capital controls are larger under DCP than PCP.

Mapping the global financial cycle to our model

Consider a retrenchment of the global financial cycle which encompasses three component shocks: a rise in world interest rates, a decline in foreign appetite for domestic currency debt, and a contraction in FX debt limits. We can derive lessons for the policy rate and capital controls.

The optimal policy rate response is heterogeneous across different types of countries and retrenchment episodes. The planner accommodates the world interest rate shock by allowing the exchange rate to depreciate and cushion aggregate demand. This outcome is accomplished by raising the policy rate less than the increase in the world interest rate under PCP, and by having minimal change in the policy rate under DCP. The planner ignores the foreign appetite shock and keeps the policy rate unchanged. After the debt limit shock, the planner reduces the policy rate, but keeps it high enough to partially defend the exchange rate. Given that each global retrenchment episode has a different blend of component shocks, and that countries have heterogeneous vulnerabilities to the different component shocks, there should be heterogeneous directions of policy rate responses in the cross-section of countries in response to global financial cycle shocks.

Capital controls to address pecuniary AD externalities are used in only one direction, but not evenly across countries and episodes. Ex ante capital controls may be justified in some but not all countries to address pecuniary AD externalities. The tax is welfare-improving to the extent that the debt-limit-contraction component of the global financial cycle is relevant, and is only used by countries with high FX external debt relative to their debt limits. Consumer macroprudential taxes are substitutes for capital controls if they have perfect coverage of all relevant debt transactions, but they become imperfect substitutes or even complements if there is imperfect coverage. In the latter case, consumer macroprudential taxes and capital controls should be used together. Access to FX swap lines reduces the necessary ex ante capital controls and macroprudential taxes.

5 Shallow Foreign Exchange Markets

Next, we consider the additional friction of shallow FX markets and focus on the optimal integrated use of the policy rate, capital controls, and FX intervention. The shallow FX markets case is relevant for most emerging markets, as their currencies tend to be traded by a limited set of financial intermediaries. Even in normal times in the absence of shocks, these countries may only be able to finance their external debt by offering a premium to foreign investors. Additionally, these countries are vulnerable to risk-on/risk-off phases of the global financial cycle, as the willingness of foreigners to participate in the domestic currency debt market exhibit boom-bust dynamics.

5.1 Policy Instruments and Wedges

The case with shallow FX markets corresponds to setting $\Gamma > 0$ in the constraints and FOCs summarized in Appendix A.2. The relevant household Euler conditions and Gamma equations are equations (3) and (14)-(15). As in section 4, we continue to assume that housing frictions do not bind.[18]

Under shallow FX markets, the UIP conditions in equation (21) are violated, with the level of gross external returns depending on the quantity of domestic currency debt that financial intermediaries must be induced to hold on their balance sheets. These gross external returns must be provided

[18] This assumption ensures that the substitutability/complementarity of capital controls and macroprudential taxes follows the same logic as in section 4. With shallow FX markets, however, there emerges a new rationale for ex post capital controls to stabilize macro allocations after shocks, which would be labeled as CFMs rather than CFM/MPMs in the IMF's taxonomy.

through a combination of the domestic policy rate (which also sets the returns available to households), capital controls, and the expected exchange rate movements (the latter two of which create a gap between households' and intermediaries' returns). For the shallowness of the FX market to matter in welfare terms, we impose that domestic households do not own all the intermediaries, i.e., $\lambda \in [0, 1)$.

Moving from the deep to the shallow FX markets case, the instrument of FX intervention becomes effective through the portfolio balance channel as in Gabaix and Maggiori (2015), Cavallino (2019), and Fanelli and Straub (2019). Under shallow FX markets, the planner can use FX intervention to absorb some of the debt inflows and outflows, thereby altering the equilibrium exposure of financial intermediaries to domestic currency debt. In this manner, FX intervention changes the necessary level of the gross external returns on this debt, which in turn alters exchange rates and allocations.

Specifically, the Gamma equations (14)-(15) establish that the planner should set $(B_{t+1} + FXI_t - S_t) = 0$ if the sole purpose of FX intervention is to reduce the expected external premia, $\mathbb{E}_t \tau_{\Gamma t+1}$, to zero for any given level of debt, B_{t+1}, and foreign appetite shock, S_t. By contrast, the planner should set $(B_{t+1} + FXI_t - S_t) \neq 0$ if it wishes to influence the exchange rate at the cost of allowing some premia to occur between periods t and $t+1$.

Next we turn to the FOCs for the constrained efficient allocation to understand which additional externalities emerge as we move from deep to shallow FX markets. The planner's Euler conditions for $t \in \{0, 1\}$ are now:

$$\begin{aligned}
&\frac{\alpha_F}{P^*_{Ft} C_{Ft}} \left[1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht}\right] - \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt} \right] \frac{1}{\beta^t \pi_t} \frac{1}{P^*_{Ft}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft}} \right\} \\
&= \beta \mathbb{E}_t \left\{ \frac{\alpha_F \left[(1+i^*_t) + (1-\lambda)\, \tau_{\Gamma t+1}\right]}{P^*_{Ft+1} C_{Ft+1}} \left[1 + \frac{\alpha_H}{\alpha_F} \tau_{Ht+1}\right] \right\} + \frac{\Psi_{Bt}}{\beta I_{t-1}} + \left(\frac{1}{\beta}\right)^t \Gamma \Omega_t \\
&- \mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C^*_t \frac{\alpha_F}{C_{Ft}} \tau^{DCP}_{Xt} \right] \mathbb{E}_t \left[\frac{(1+i^*_t) + (1-\lambda)\, \tau_{\Gamma t+1}}{\beta^t \pi_{t+1}} \frac{1}{P^*_{Ft+1}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft+1}} \right] \right\}. \qquad (27)
\end{aligned}$$

Three wedges, $\{\tau_{Ht}, \tau_{Xt}, \tau_{\Gamma t}\}$, now enter the planner's Euler conditions and generate divergences from the households' Euler conditions. Relative to the deep FX markets Euler condition (23), there are two main additions: the term $\left(\frac{1}{\beta}\right)^t \Gamma\Omega_t$, where Ω_t is the multiplier on the Gamma equation; and the dependence of the discount factor in period $t+1$ on the UIP wedges in the same period, $\tau_{\Gamma t+1}$.

Focusing first on the $\left(\frac{1}{\beta}\right)^t \Gamma\Omega_t$ term, we observe that the multiplier on the Gamma equation is positive when the external debt level in period t is forcing the UIP wedge, $\tau_{\Gamma t+1}$, to be higher than the planner would otherwise like it to be. We refer to this term as the financial TOT externality, which arises owing to the following channel: when deciding on their level of borrowing, each household takes returns as given, without internalizing the impact of its borrowing decision on the returns facing all households. It does not internalize that since the economy as a whole is the sole supplier of domestic currency bonds to the financial intermediaries, the level of debt determines the UIP wedge in equilibrium. High UIP wedges lower welfare because they constitute excessive premia paid by domestic households to the foreign-owned fraction of the financial intermediaries.

Turning next to the UIP wedges, $\tau_{\Gamma t+1}$, we observe that if households do not own all of the financial intermediaries, i.e., $\lambda \in [0, 1)$, then their external liabilities are effectively partially in domestic currency. The planner can improve welfare by redistributing resources across states using the exchange rate: specifically, the planner should depreciate away the dollar value of repayments on external liabilities in states when economy-wide dollar resources are reduced by shocks, and increase the dollar value of repayments when economy-wide dollar resources are enhanced by shocks.

The expression for capital controls changes relative to the deep FX markets case as follows:

$$\varphi_t = \begin{cases} 1 - \dfrac{\beta \frac{(1+i_t^*)+\mathbb{E}_t \tau_{\Gamma t+1}}{\mathbb{E}_t\left[\frac{E_t}{E_{t+1}}\right]} \mathbb{E}_0 \left[\frac{E_t}{E_{t+1}} \frac{\alpha_F}{P^*_{Ft+1} C_{Ft+1}}\right] \left[\begin{array}{c} \frac{\alpha_F}{P^*_{Ft} C_{Ft}} \left[1 + \frac{\alpha_H}{\alpha_F}\tau_{Ht}\right] \\ -\mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP}\right] \frac{1}{\beta^t \pi_t} \frac{1}{P^*_{Ft}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft}} \right\} \end{array} \right]}{\frac{\alpha_F}{P^*_{Ft} C_{Ft}} \left[\begin{array}{c} \beta \mathbb{E}_t \left\{ \frac{\alpha_F [(1+i_t^*)+(1-\lambda)\tau_{\Gamma t+1}]}{P^*_{Ft+1} C_{Ft+1}} \left[1 + \frac{\alpha_H}{\alpha_F}\tau_{Ht+1}\right] \right\} + \left(\frac{1}{\beta}\right)^t \Gamma \Omega_t \\ -\mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP}\right] \mathbb{E}_t \left[\frac{(1+i_t^*)+(1-\lambda)\tau_{\Gamma t+1}}{\beta^t \pi_{t+1}} \frac{1}{P^*_{Ft+1}} \frac{1}{P_X} \frac{\partial P_X}{\partial C_{Ft+1}}\right] \right\} \end{array} \right]} \\ \text{if } \Psi_{Bt} = 0 \\ \\ 0 \text{ if } \Psi_{Bt} > 0, \end{cases} \tag{28}$$

This expression captures both the financial TOT externality and the UIP wedge arguments from above. On the financial TOT externality, if Ω_t is positive, the level of capital controls tends to be larger, as the planner discourages households from borrowing in order to reduce the UIP wedge. On the UIP wedges, the terms $\tau_{\Gamma t+1}$ are multiplied by $(1-\lambda)$ in the denominator but not in the numerator, showing that there is a role for the planner to use capital controls to redistribute resources across states, because households take the domestic policy rate as given, while the planner recognizes that it depends on the endogenous UIP wedges, taking into account the domestic ownership of financial intermediaries.

Exchange rate determination now follows the below expression:

$$\underbrace{\alpha_H \tau_{Ht}}_{\text{Stabilize demand for home goods}} = \underbrace{\begin{array}{c} -\mathbb{I}^{PCP} \cdot \left\{ \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{PCP} \right\} \\ -\mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP}\right] \frac{1}{\beta^t \pi_t} \left(-\frac{E_t}{P_X}\frac{\partial P_X}{\partial E_t}\right) \right\} \end{array}}_{\text{Optimize TOT on export goods}}$$
$$+ \underbrace{\frac{\Psi_{Bt}}{\beta^t \left[\left(1+i_{t-1}^*\right) + (1-\lambda)\,\tau_{\Gamma t}\right]} \kappa_{H1} \frac{P_H}{E_t}}_{\text{Relax bank constraint}} \pm \underbrace{\frac{1}{\beta^t \pi_t} \Lambda E_t \left[\left(1+i_{t-1}^*\right) + \tau_{\Gamma t}\right]}_{\text{Prevent excessive contingency of exchange rate}}. \tag{29}$$

This expression is similar to the expression under deep FX markets, equation (24). Relative to that equation, the discounting of the bank constraint is altered depending on the size of the UIP wedge, and there is an additional final term which ensures that the optimizations by the planner over exchange rates and UIP wedges are connected, i.e., the distribution of exchange rates and UIP wedges must respect the non-contingency of the domestic policy rate.

The following trade-off determines the optimal UIP wedge, $\tau_{\Gamma t+1}$:

$$\underbrace{\Omega_t}_{\text{Ability to borrow more today}} = \underbrace{(1-\lambda)\frac{\Phi_{t+1}+\Psi_{Bt+1}}{\Pi_{s=0}^{t} I_s}\left(B_{t+1}+FXI_t\right)}_{\text{Higher repayments tomorrow}} \pm \underbrace{\frac{1}{\pi_{t+1}}\Lambda E_{t+1}}_{\text{Prevent excessive contingency of premium}}$$
$$+ \underbrace{(1-\lambda)\,\Omega_{t+1}\Gamma\left(B_{t+1}+FXI_t\right)}_{\text{Higher premium tomorrow owing to rollover needs}}, \tag{30}$$

where $B_{t+1} \equiv \frac{D_{t+1}}{E_t}$ is the dollar value of debt, and Φ_t, Ψ_{HHt}, and Λ_t are the multipliers on the resource constraint, household borrowing constraint, and contingency-check equation in period t.

The planner understands that under shallow FX markets, increasing the UIP wedge in order to allow a higher level of debt in period t worsens consumption in period $t+1$ owing to higher external debt repayments, and also requires a higher UIP wedge in period $t+1$ if the debt is rolled over. In addition, the possibility of FX intervention alters the expression for the gross external debt position: when the planner borrows in the domestic currency debt market in order to accumulate dollar assets abroad, a fraction $(1-\lambda)$ of the debt ends up on the balance sheet of the foreign-owned intermediaries, so it constitutes external debt, and the FXI_t terms enter the equation above.

The following trade-off determines the optimal level of FX intervention:

$$\underbrace{\Gamma\Omega_t}_{\text{Lower premium today}} \underbrace{+(1-\lambda)\,\mathbb{E}_t\left[\frac{\Phi_{t+1}+\Psi_{Bt+1}}{\Pi_{s=0}^{t} I_s}\tau_{\Gamma t+1}\right]}_{\text{Change in carry cost}} + \underbrace{(1-\lambda)\,\Gamma\mathbb{E}_t\left[\Omega_{t+1}\tau_{\Gamma t+1}\right]}_{\text{Change in premium tomorrow owing to change in carry cost}} = 0. \tag{31}$$

By absorbing some of the capital inflow or outflow, FX intervention can reduce the external debt that foreign-owned intermediaries have to absorb, and it can thereby reduce the UIP wedge. This benefit should be combined with any carry costs incurred by the FX intervention, taking into account that higher carry costs incurred between periods t and $t+1$ may result in higher external debt, which needs to be rolled over in period $t+1$. The planner should intervene until the net marginal benefit of intervention is pushed down to zero.

Moving from the deep FX markets case to the shallow FX markets case, one would expect to find a greater role for capital controls and a case for FX intervention. The equations above reveal several interactions between FX intervention and capital controls. FX intervention has two effects on the optimal capital control expression: (i) intervention affects the UIP wedge, $\tau_{\Gamma t}$, which alters the time path of consumption and the AD wedge, τ_{Ht}; and (ii) intervention alters the multiplier on the Gamma equation, Ω_t. Capital controls have two effects on the optimal level of FX intervention: (i) capital controls reduce gross external returns, η_t, which affect the carry cost of intervention; and (ii) they reduce households' debt, which alters the incentive to absorb the debt via FX intervention.

5.2 Real Shocks

In this subsection, we consider domestically- and externally-generated shocks which affect the real income of domestic households. As in the deep FX markets case, we define $B_t \equiv \frac{D_t}{E_{t-1}}$. In the shallow FX markets case, the representative household's effective exposure to dollar-denominated debt at the beginning of period 0 is given by λB_0. At the beginning of any other period t, the effective dollar-denominated-debt exposure is given by $\lambda B_t - (1-\lambda) FXI_{t-1}$ while the effective domestic-currency-denominated debt exposure is given by $(1-\lambda)(B_t + FXI_{t-1})$.

5.2.1 Productivity Shocks

We consider permanent shocks to the value of productivity, $A_1 = A_2$. The parameterization of the shock which neutralizes the intertemporal TOT externality changes to the following condition:[19]

$$\mathbb{E}_0 \left[\frac{A_0}{A_1} \frac{\lambda (1+i_0^*) + (1-\lambda)\eta_1}{(1+i_0^*)} \right] = 1.$$

[19] As in the deep FX markets case, the PCP result on the optimality of capital controls does not depend on this parameterization, while the DCP result does. The new condition under shallow FX markets is more complex than under deep FX markets, because it relates the exogenous shock A_1 to the endogenous variable η_1. However, the condition is easily implemented numerically as follows: (i) we simulate the model with $\lambda = 1$ and our choice of A_1^H, and use the result to fix the values of η_1 and thereby A_1^L; (ii) then we verify that allowing λ to vary within $[0,1)$ results in no change to the values of η_1, so the condition for A_1 continues to be satisfied.

Under PCP, figure 15 shows that as in the case of deep FX markets, it is optimal to rely on exchange rate flexibility to absorb the shock. Since imports can be kept stable between periods 1 and 2 after a permanent shock, households do not issue external debt for consumption-smoothing purposes in period 1. Without new domestic currency debt being issued, the depth of the FX market—i.e., the ability of intermediaries to absorb new debt without requiring a premium—is not relevant for ex post policy responses. The smoothing of imports and employment between periods 0 and 1 and across period-1 states means that ex ante capital controls and FX intervention are zero as well.

Under DCP, figure 16 shows that exchange rate flexibility is again the main tool to absorb the shock. In both period-1 states, $(B_2 + FXI_1)$ is kept at zero, establishing that FX intervention is used solely to minimize external premia, and capital controls are set to zero as well. As in the deep FX markets case, employment is destabilized across states under DCP.

From the ex ante perspective, capital controls and FX intervention are close to zero but not exactly so, owing to this ex post destabilization of employment. The logic from the deep FX markets case continues to apply, pushing both instruments towards zero. However, there is a new channel owing to the presence of less-than-full currency mismatch, i.e., $\lambda < 1$: the "repayment contingency" motive. Some of any ex ante external debt remains effectively denominated in domestic currency, so the dollar value of ex post repayments is altered by ex post exchange rate movements. Since employment is destabilized by the shock under DCP, the planner wishes to generate an ex ante domestic-currency-denominated asset position so that the ex post exchange rate movements transfer resources in dollar terms from high to low states.

The desired asset position, i.e., $(1 - \lambda)(B_1 + FXI_0) < 0$, is necessarily small owing to the desire to smooth imports between periods 0 and 1. In our simulations, we find that it is implemented ex ante via an elevated policy rate combined with small capital inflow subsidies and FX sales. In practice, the contingency of ex post debt repayments is realistic, but the desire to distort the ex ante debt level is not, so we do not focus on ex ante policy recommendations which hinge on this motive.

Result 7. *Countries facing permanent productivity shocks should rely solely on exchange rate flexibility,* *under both PCP and DCP, even if FX markets are shallow.*

Remark 7. *Permanent productivity shocks do not rationalize ex post capital controls. FX intervention should minimize premia while allowing the same degree of exchange rate flexibility as in the deep FX markets case.*

5.2.2 Commodity Price Shocks

We consider permanent shocks to the dollar price of commodities, $P^*_{Z1} = P^*_{Z2}$. We again compare two commodity exporters with different initial levels of unhedged external FX debt, B_0, and the same level of $\kappa_{H1} \in (0, 1)$.

Under both PCP and DCP, for $B_0 = 0$, figures 17 and 18 show that the optimal policies are identical between the deep and shallow FX markets cases. The insight from the productivity shock analysis continues to hold: since new domestic currency debt should not be issued after permanent shocks, the depth of the FX market is not relevant for ex post policy responses, while the smoothing of imports and employment between periods 0 and 1 and across period-1 states pins down the ex ante policy responses.

Under both PCP and DCP, for B_0 high enough for banks' external constraints to bind after the low realization of the shock, figures 19 and 20 show that the optimal policies are identical between the deep and shallow FX markets cases except the steady-state level of FX intervention. Under shallow FX markets, the existence of a positive domestic currency debt level for financial intermediaries to absorb means that there are steady-state financial TOT externalities and gross external premia paid to foreigners, which reduce welfare. Since we have assumed that the planner has an unrestricted ability to intermediate in the domestic currency debt market and reduce the premia, it does so until the premia go to zero. In practice, the ability of the planner to intermediate may be more limited.

It is important to note that this steady-state intermediation motive is the only rationale for intervention in our simulations. In both period-1 states, $(B_2 + FXI_1)$ is kept at zero, establishing that FX intervention is used solely to minimize external premia, and not to otherwise manage the level of exchange rate, after permanent commodity price shocks. In fact, managing premia is consistent with the exchange rate having the same degree of flexibility as in the deep FX markets case.

Result 8. *Countries with large commodity sectors should not impose capital controls to address pecuniary AD externalities if there are no borrowing constraints, but they should impose positive ex ante capital controls for this purpose if there are borrowing constraints and initial external FX debt is high.*

Remark 8. *Permanent commodity price shocks do not rationalize ex post capital controls. FX intervention should minimize premia while allowing the same degree of exchange rate flexibility as in the deep FX markets case.*

Remark 9. *For countries with large commodity sectors, borrowing constraints, high initial external FX debt, and either deep and shallow FX markets, ex ante capital controls to address pecuniary AD externalities are more frequently optimal, and they are larger in magnitude, under DCP than PCP.*

5.3 Financial Shocks

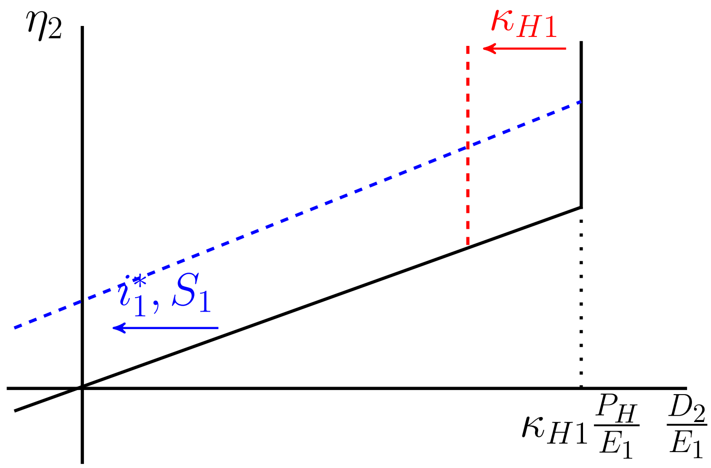
Under shallow FX markets, financial shocks interact with both the occasionally-binding debt limits and the always-binding intermediation frictions. Therefore, some shocks which can be ignored by countries with deep FX markets cannot be ignored by countries with shallow FX markets. Figure 4 illustrates how different financial shocks affect the net demand curve for bonds in the domestic currency debt market.

5.3.1 World Interest Rate Shocks

We consider a shock to the value of the world interest rate, $(1 + i_1^*)$.

Under PCP, the planner optimally accommodates the shock using all policy instruments in a complementary fashion. To begin, consider the allocation with only monetary policy (MP) and capital controls (CC), represented by the dashed lines in figure 21. When the world interest rate declines, the households' Euler condition suggests that it is optimal for imports to increase in period 1 and decline in period 2. However, an increase in imports requires external borrowing, which increases

Figure 4: Financial Shocks under Shallow FX Markets



the UIP wedges that foreign-owned intermediaries earn at the expense of the domestic economy. Recognizing this problem, the planner does not stimulate a large increase in debt: it reduces the domestic policy rate only slightly. Capital controls reflect a combination of both the new financial TOT externality and the old intertemporal TOT externality.

Next, consider the solid lines in figure 21, which plot the allocation when FX intervention is also available. The use of FX intervention allows more accommodation of the shock, and more rather than less exchange rate volatility. The planner can now reduce the premium accruing to intermediaries by conducting ex post FX sales and purchasing domestic currency bonds. Since the premium is now lower, and indeed $(B_2 + FXI_1)$ is pushed to zero, the planner can reduce the ex post policy rate more and stimulate more ex post debt. At these higher levels of debt, capital controls remain only insofar as the intertemporal TOT externality is relevant.

Therefore, the planner uses a combination of looser monetary policy and expansionary FX intervention in order to take advantage of the lower world interest rate, both of these policies accommodating the shock rather than leaning against the wind. The aim is to boost aggregate demand while minimizing the UIP wedge.

Under DCP, FX intervention is again used to accommodate the low realization of the shock, increasing debt and aggregate demand while minimizing the UIP wedge (figure 22). However, there are two main differences from the PCP case. Firstly, the exchange rate is more volatile under DCP than PCP, as in the deep FX markets case. Secondly, the introduction of FX intervention reduces rather than increases the ex post movement in the policy rate. The reason is that FX intervention makes the ex post policy rate get closer to its deep-FX-market equivalent, and under deep FX markets, the policy rate is kept constant in response to the shock.

Result 9. *Countries facing world interest rate shocks should use ex post FX intervention to accommodate the shocks, rather than lean against them, under both PCP and DCP.*

Remark 10. *After world interest rate shocks, exchange rates are more volatile under DCP than PCP.*

5.3.2 Foreign Appetite Shocks

A foreign appetite shock is a shock to the value of S_1. These shocks are changes to the foreign demand for domestic currency bonds that are motivated by factors other than returns. Potential triggers includes booms/busts in foreigners' "animal spirits," changes in perceived creditworthiness, and/or the abrupt entry of large institutional investors into the thin market for a borrower country's debt.

Under PCP, the domestic policy rate appears to be a poor substitute for capital controls and FX intervention. Figures 23 and 24 trace out the allocations as we add instruments one by one to the planner's toolkit.

The dashed lines in figure 23 plot the allocation with only monetary policy. When foreigners' appetite improves, there is a decline in the UIP wedge, because the entry of the new category of foreigners reduces the debt held, and the premium demanded, by the Gamma intermediaries. Domestic households increase their borrowing to finance an increase in imports and aggregate demand. Their borrowing is excessive, because each household does not internalize that their increase in debt is increasing the interest rate for all other households. As the shock is insensitive to the premium, monetary policy is not an effective instrument to insulate the economy from the shock: raising the policy rate to deter overborrowing generates the cost of larger UIP wedges for the intermediaries. For a fixed policy rate, the exchange rate appreciation hurts export volumes and destabilizes employment, so the planner reduces the policy rate to limit the appreciation, stimulating export demand and stabilizing employment.

The solid lines in figure 23 plot the allocation with both monetary policy and capital controls. When foreigners' appetite improves, positive ex post capital controls are more effective than monetary policy. Capital controls cannot absorb the full impact of all the new foreigners, but they can address the financial TOT externality. Ex post overborrowing is significantly reduced. Since the shock is being tackled closer to its source, there are fewer remaining symptoms, and the ex post policy rate needs to move less to stabilize employment.

The dashed lines in figure 24 show the allocation with monetary policy and FX intervention. They show that ex post FX accumulation is also more effective than monetary policy. FX intervention directly limits the impact of foreigners' entry. By selling domestic currency bonds and purchasing dollar bonds, the planner absorbs some of the favorable foreign appetite shock and earns carry profits on the flow, instead of letting it be absorbed in an inefficient manner by domestic households. Again, the ex post policy rate needs to move less to stabilize employment.

FX intervention partially leans against the wind but does allow the appetite shock to move the UIP wedges: $(B_2 + FXI_1 - S_1) \in (-|S_1|, 0) < 0$ after the high realization of the shock and $(B_2 + FXI_1 - S_1) \in (0, |S_1|) > 0$ after the low realization. The planner internalizes that it is not only stabilizing macro allocations but also opportunistically exploiting the inflows and outflows to earn carry profits, and those profits would diminish if all of the shock is absorbed.

The solid lines in figure 24 plot the allocation with monetary policy, capital controls, and FX intervention. FX intervention partially leans against the wind for the appetite shock and capital controls move in the direction of the remaining financial TOT externality. When both are used together, macro allocations are perfectly stabilized across states and over time. Capital controls and FX intervention appear to be mild substitutes with each other under PCP, as both are used a little less together than when they were used alone. They are strong substitutes for the domestic policy rate: if both capital controls and FX intervention are available, they insulate the economy from the volatility of the foreign appetite shocks, and the domestic policy rate need not move at all.

The use of capital controls and FX intervention increases monetary autonomy by tackling the appetite shock close to its source, and freeing up monetary policy to be used to address the domestic sources of price pressures.

Under DCP, the the main results from PCP on the optimal use of instruments and their complementarity/substitutability continue to hold (figures 25 and 26). However, there are some key differences: firstly, exchange rates are more volatile under DCP than PCP; secondly, employment is destabilized under DCP and not PCP; thirdly, the policy rate increases rather than decreases after the high appetite shock because it is no longer used to stabilize exports; and fourthly, consumption of the home tradable good decreases rather than increases after the high appetite shock.

Result 10. *Countries facing foreign appetite shocks should use ex post capital controls and ex post FX intervention to insulate the domestic economy and increase ex post monetary autonomy under both PCP and DCP.*

Remark 11. *After foreign appetite shocks, capital controls and FX intervention are substitutes for the domestic policy rate.*

5.3.3 Bank Debt Limit Shocks

We consider a shock to the value of banks' pledgability parameter, κ_{H1}.

Under both PCP and DCP, for B_0 high enough for banks' external constraints to bind after the low realization of the shock, figures 27 and 28 show that the optimal policies are identical between the deep and shallow FX markets cases except the steady-state level of FX intervention. The insights from the analysis of the commodity price shock carries over to this case. Under shallow FX markets, a positive domestic currency debt level means that there are steady-state financial TOT externalities, and FX intervention should focus on absorbing these premia. In both period-1 states, $(B_2 + FXI_1)$

is kept at zero, establishing that FX intervention is used solely to minimize external premia, and not to otherwise manage the level of exchange rate. In fact, managing premia is consistent with the exchange rate having the same degree of flexibility as in the deep FX markets case.

The similarity between the cases for deep and shallow FX markets may be puzzling, as it would seem intuitive that using FX intervention to defend the exchange rate ex post, i.e., $(B_2 + FXI_1) < 0$, should help relax borrowing constraints while letting the domestic policy rate address aggregate demand. However, there is a flaw in this intuition. When the banks' debt limit binds, the interest rate ρ_1 in the households' Euler condition between periods 1 and 2 is determined by the overall quantity of available external funds and is no longer equal to the policy rate i_1. Therefore, the policy rate has no impact on aggregate demand through the Euler condition; it only affects demand through the exchange rate term in the borrowing constraint.

In the case of deep FX markets, the policy rate is assigned solely to managing the level of the exchange rate (trading off import/export substitution against the relaxation of the borrowing constraint) after low realizations of the shock, as it has no other use. In the case of shallow FX markets, both the policy rate and FX intervention could potentially manage the level of the exchange rate when the debt limit binds. The optimal instrument to manage the exchange rate is actually the policy rate, because the policy rate is not needed for any other purpose, while FX intervention has a carry cost. Therefore, it is not optimal to use ex post FX intervention except to minimize external premia, i.e., to set $(B_2 + FXI_1)$ to zero.

Result 11. *Countries with high initial external FX debt that are subject to debt limit shocks should impose positive ex ante capital controls. FX intervention should minimize premia while allowing the same degree of exchange rate flexibility as in the deep FX markets case.*

Remark 12. *After debt limit shocks, FX intervention is used to absorb external premia but not otherwise to influence the exchange rate.*

Remark 13. *For countries with high initial external FX debt and vulnerability to debt limit shocks, ex ante capital controls are larger under DCP than PCP.*

5.4 Extension: Ban on Open FX Positions

In section 4, we showed that currency mismatches generate vulnerability to bank external FX debt limit shocks. In the current section, we showed that shallow FX markets generate vulnerability to shocks to the foreign appetite for domestic currency debt. In practice, countries may suffer from both currency mismatches and shallow FX markets and may therefore be vulnerable to both shocks. In this case, it is important that policy actions to address one kind of shock do not inadvertently increase the vulnerability of the country to the other kind of shock.

Next, we illustrate this conundrum by considering the impact of a particular macroprudential regulation: a ban on open FX positions for those intermediaries which are domestically owned. Since the representative household acquires currency mismatch through its ownership of intermediaries who have dollar liabilities, such a ban may be seen as a way to reduce the economy's vulnerability to FX debt limit shocks.

The mechanical impact of such a ban is to remove the fraction of intermediaries which are domestically owned, λ, from participation in the FX market. As a result, domestic currency debt can only be absorbed by those intermediaries who are foreign-owned, and the Gamma equations (14) and (15) need to be replaced with the following equations:

$$\frac{\Gamma}{(1-\lambda)}\left(\frac{D_1}{E_0}+FXI_0-S_0\right)=\mathbb{E}_0\left[\eta_1-(1+i_0^*)\right] \tag{32}$$

$$\frac{\Gamma}{(1-\lambda)}\left(\frac{D_2}{E_1}+FXI_1-S_1\right)=\eta_2-(1+i_1^*)\,. \tag{33}$$

The economy-wide resource constraint (18) should be replaced as well:

$$\begin{aligned} D_{t+1} \geq & -E_t P_{Ft}^*\left[\omega C_t^* - C_{Ft}\right] - E_t P_{Zt}^* Z_t \\ & - O_t\left[\left(1+\widehat{i}_{t-1}\right)-\left(1+i_{t-1}^*\right)\frac{E_t}{E_{t-1}}\right] + \left(1+\widehat{i}_{t-1}\right)D_t. \end{aligned} \tag{34}$$

Relative to equation (18), there are two main differences. Firstly, there is no fraction λ of debt with currency mismatch, so an exchange rate depreciation does not increase the domestic currency value of debt repayments. Secondly, the effective carry cost of FX intervention is larger, because all carry profits accrue to foreign-owned intermediaries and none to domestically-owned intermediaries. The entire amended system of constraints and FOCs is summarized in Appendix A.4.

This macroprudential regulation has both benefits and costs, which we will discuss next with reference to two figures: figure 29, which shows the impact of the regulation on the allocations after debt limit shocks; and figure 30, which shows the impact of the regulation on the allocations after foreign appetite shocks. In both figures, the dashed lines represent allocations without the regulation and the solid lines represent allocations with the regulation. Both figures are constructed for the DCP case, and FX intervention is removed for illustrative purposes, as we impose the following constraint for $t \in \{0, 1\}$:

$$FXI_t = 0. \tag{35}$$

The first key result is that the regulation reduces the economy's vulnerability to debt limit shocks and removes the need for ex ante capital controls to address pecuniary AD externalities. As the country transitions from high to zero currency mismatch, depreciations can freely be used to reduce the FX value of repayments and relax external debt limits in period 1. Correspondingly, there is no pecuniary externality associated with the debt limit which needs to be addressed by distorting the exchange rate from the value that stabilizes price pressures. Therefore, there is no pecuniary-AD-externality rationale for ex ante capital controls.

If FX markets are deep, i.e., $\Gamma = 0$, the country simply transitions from high to zero currency mismatch with no side effects, as the UIP conditions in equation (21) continue to hold. If FX markets are shallow, i.e., $\Gamma > 0$, however, the regulation effectively makes the FX markets even more shallow, i.e., the effective Gamma becomes $\frac{\Gamma}{(1-\lambda)}$, which is higher than Γ.

The second key result is that the regulation increases the steady-state cost of debt financing if the planner cannot simply use FX intervention to bypass all the intermediaries. Figure 29, which illustrates debt limit shocks, shows that the regulation generates significant deleveraging when FX intervention is set to zero. The reason is as follows. Open FX positions only arise in equilibrium for countries who need intermediaries to finance domestic currency debt. If markets are shallow, this financing generates positive UIP wedges. If the FX market becomes shallower, UIP wedges increase and debt decreases in equilibrium. Even though ex ante capital controls are not needed for pecuniary AD externalities (indeed, the external debt limit no longer binds after the regulation), there are large steady-state capital controls to mitigate financial TOT externalities.

The third key result is that the regulation increases the vulnerability of the economy to foreign appetite shocks and increases the marginal value of ex post FX intervention. Figure 30, which illustrates foreign appetite shocks, plots the allocations with monetary policy (MP) and capital controls (CC) only, and then we can infer the marginal value of ex post FX intervention. Since FX markets are shallower owing to the ban, allocations become more volatile in response to foreign appetite shocks. Both the ex post policy rate and ex post capital controls also become more volatile.

The marginal value of ex post FX intervention can be assessed from the values of the period-1 multipliers, $\left\{\Theta_1^H, \Theta_1^L\right\}$, on constraint (35). Moving from the allocations with the ban to the allocations with the ban, the values of the multipliers increase from $\left\{\Theta_1^H = -0.03, \Theta_1^L = 0.02\right\}$ to $\left\{\Theta_1^H = -0.04, \Theta_1^L = 0.12\right\}$, establishing that the value of ex post FX intervention increases. The country becomes more reliant on FX intervention after foreign appetite shocks.

Result 12. *Banning open FX exposures reduces the country's vulnerability to banks' debt limit shocks and removes the need for ex ante capital controls to address AD externalities. However, it has side effects: (i) it makes debt harder to finance in steady state, and it alters steady-state capital controls; (ii) it increases the country's vulnerability to foreign appetite shocks and its reliance on FX intervention.*

5.5 Summary

Countries with shallow FX markets have more reasons to deviate from the traditional Mundell-Fleming prescription, and to use capital controls and FX intervention, than countries with deep FX markets. Our model suggests there is a case for ex post capital controls and ex post FX intervention after shocks which generate changes in external debt and UIP wedges. Countries worried about whether to respond to capital inflows and outflows should not focus on stabilizing the level or volatility of the exchange rate, but they should instead monitor the level of UIP wedges which arise from imperfect arbitrage by financial intermediaries in the FX market.

Shocks which generate exchange rate fluctuations without generating changes in external debt, such as permanent productivity and commodity price shocks, do not rationalize ex post capital controls. In such cases, the only use of FX intervention is to minimize external premia and thereby to facilitate the same degree of exchange rate volatility as in the deep FX markets case. Shocks which generate changes in external debt, such as foreign appetite shocks, may warrant the joint use of capital controls and FX intervention to lean against the wind, limit some exchange rate movements, and improve monetary autonomy, without fully eliminating the premia that the shocks generate.

Countries which have both currency mismatches and shallow FX markets should carefully design any macroprudential regulations to reduce external FX debt without harmful spillovers onto FX market depth. Regulations which reduce currency mismatches while limiting participation in the FX market may reduce the vulnerability of the economy to banks' debt limit shocks only at the cost of increasing the vulnerability of the economy to foreign appetite shocks. The country may then find itself increasingly dependent on FX intervention to manage volatility in external premia.

Mapping the global financial cycle to our model

Consider a retrenchment of the global financial cycle which encompasses three component shocks: a rise in world interest rates, a decline in foreign appetite for domestic currency debt, and a contraction in debt limits for borrower countries. Whether the planner wishes to accommodate, ignore, or fight each component shock has parallels to the case of deep FX markets, but the policy implementation is different under shallow FX markets.

The planner still wishes to accommodate the world interest rate shock, and under shallow FX markets, it optimally uses ex post FX intervention to reinforce the shock rather than lean against it.

The planner wishes to ignore the foreign appetite shock and set monetary policy based on domestic considerations only. Unfortunately, in the case of shallow FX markets, these shocks compromises monetary autonomy, and the policy rate is forced to respond to external factors. Since the foreign appetite shock is not return-sensitive, monetary policy is ineffective at addressing the shock at its source, and the policy rate is assigned to address the employment destabilization generated by the shock. The use of ex post capital controls and ex post FX intervention restores monetary autonomy, so the economy is insulated from the global financial cycle while the policy rate can be set based only on domestic considerations.

The planner's response to the debt limit shock is similar under deep and shallow FX markets, because FX intervention is unable to enhance monetary autonomy in the state when the economy is most stressed. When the debt limit binds, there is no benefit from using FX intervention to target the exchange rate, beyond the role of FX intervention in reducing the premia on external debt. By contrast, access to FX resources from swap lines in order to directly relax banks' external debt limits remains useful as in the deep FX markets case, and would reduce the necessary ex ante capital controls and macroprudential taxes.

6 Housing Sector

Finally, we consider the additional friction of housing sector borrowing constraints and focus on the optimal integrated use of the policy rate, capital controls, FX intervention, and domestic macroprudential taxes. Whether a country as a whole has high or low external debt, there may be substantial stocks of debt contracted between different domestic agents, and domestic borrowing constraints are likely to be related to the domestic currency value of nontraded assets. For the housing sector, an appropriate collateral would be land. Leveraged domestic borrowing is relevant for most advanced economies and a growing number of emerging markets which have gradually developed domestic credit markets over time.

Housing frictions have usually been analyzed in closed economy models where fire sales of land are triggered by domestic shocks, and the possibility of crisis-time fire sales rationalizes taxes or quantity restrictions on domestic housing sector debt in normal times. In this section, we nest such housing sector frictions in an open economy model where fire sales of land may be triggered by both domestic and external shocks, and may rationalize a combination of domestic (policy rate and macroprudential debt taxes) and external adjustment tools (capital controls and FX intervention).

6.1 Policy Instruments and Wedges

The case with housing frictions draws on the full set of constraints and FOCs summarized in Appendix A.2. For housing frictions to matter for the equilibrium allocations, we require two conditions: firstly, that the planner can only impose macroprudential taxes on the linear housing subsector, i.e., $\theta_{Rt}^{Linear} \in \mathbb{R}$, while the concave subsector is unregulated, i.e., $\theta_{Rt}^{Concave} \equiv 0$; and secondly, that housing sector borrowing capacity is limited, i.e., κ_{L1} is sufficiently low. Housing frictions may rationalize a combination of domestic and external adjustment tools. In this subsection, we first explain the rationale for macroprudential taxes on housing debt. Then we turn to additional policy instruments, including those only available in open economies.

Relative to the previous sections, there is now a meaningful decision for the planner to make regarding the quantity of land held by the linear and concave housing subsectors. Let us begin with describing period-1 crisis-time outcomes and then derive the optimal prudential policies in period 0. The following trade-off determines the constrained-efficient quantity of land held by the linear subsector in the period-1 state s:[20]

$$\begin{aligned}\underbrace{\frac{\beta \alpha_R \tau_{R2}}{Y_{R2}}}_{\text{Minimize housing distortion}} &= \underbrace{\Psi_{R1} \widehat{q}_1 \left(1-\kappa_{L1}\right)}_{\text{Housing constraint}} + \underbrace{\Psi_{R1} \frac{\partial \widehat{q}_1}{\partial L_1^{Linear}} \left(\left(1-\kappa_{L1}\right) L_1^{Linear} - L_0^{Linear}\right)}_{\text{Tightening of constraint}} \\ &\quad + \frac{1}{\pi_1} \mathbb{E}_0 \left[\Psi_{R1} \underbrace{\frac{\partial \left(\chi_1 \widehat{q}_0\right)}{\partial L_1^{Linear,s}} \left(L_0^{Linear} - L_{-1}^{Linear}\right)}_{\text{Effect on period-0 land price}} \right], \end{aligned} \tag{36}$$

[20]In our notation, a derivative of the form $\mathbb{E}_0 \left[\Psi_{R1} \frac{\partial X_1}{\partial Y_1^s}\right]$ indicates the marginal impact of changing the variable Y_1 in a particular period-1 state s on the expected value of the variable X_1 across states, weighted by the housing multiplier Ψ_{R1} in each state. All derivatives depend on whether the planner has access to capital controls or consumer macroprudential taxes, and they are documented in Appendix A.2.

where Ψ_{R1} is the multiplier on the housing constraint in period 1, a hat over a variable indicates the FX value of that variable, and the FX gross return related to domestic interest repayments between periods t and $t+1$ is given by:

$$\chi_{t+1} = (1+i_t)\,\frac{E_t}{E_{t+1}} > 0.$$

The FX gross return covers both the borrowing rate and the exchange rate movements between the two periods. Unlike the gross return η_{t+1} of international financial intermediaries, the gross return χ_{t+1} is not subject to capital controls because it covers purely domestic transactions.

If the housing constraint is not binding, i.e., $\Psi_{R1} = 0$, then there is no distortion to housing output, i.e., $\tau_{R2} = 0$, and all land is held by the linear subsector, i.e., $L_1^{Linear} = 1$. If the housing constraint is binding, i.e., $\Psi_{R1} > 0$, the first term on the right hand side shows that the housing wedge τ_{R2} exceeds 0, indicating that L_1^{Linear} decreases below 1. The decrease in L_1^{Linear} reduces the period-1 land price $\widehat{q}_1$, i.e., $\frac{\partial \widehat{q}_1}{\partial L_1^{Linear}} > 0$, which tightens the constraint via the second term. The associated decrease in the period-0 land price affects the inherited debt of the linear subsector, represented by the third term.

The following trade-off determines the constrained-efficient quantity of land held by the linear subsector in period 0:

$$\underbrace{\frac{\beta\alpha_R\tau_{R1}}{Y_{R1}}}_{\text{Minimize housing distortion}} = \mathbb{E}_0\left[\Psi_{R1}\left\{\begin{array}{c}\underbrace{\left(\chi_1\widehat{q}_0 - \widehat{P}_{R1} - \widehat{q}_1\right)}_{\text{Hedging motive}} \\ +\underbrace{\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial L_0^{Linear}}\left(L_0^{Linear} - L_{-1}^{Linear}\right)}_{\text{Effect on period-0 land price}} - \underbrace{\frac{\partial \widehat{P}_{R1}}{\partial L_0^{Linear}}L_0^{Linear}}_{\text{Effect on rent}}\end{array}\right\}\right]. \quad (37)$$

If the housing constraint is not binding in any of the period-1 states, there is no distortion to land usage by the linear subsector in period 0. Let us now consider an allocation when the housing constraint is binding in one of the period-1 states.

The first term on the right hand side is the hedging motive. The term is positive if the linear subsector's net profit from land (i.e., the value of rents $\widehat{P}_{R1}$ and the land price $\widehat{q}_1$ minus interest payments $\chi_1\widehat{q}_0$) is negative in the period-1 state where the housing constraint is binding. If so, the linear subsector's constraint in that state could be relaxed if the subsector were holding less land and less inherited debt from the previous period. It is indeed optimal for the planner to relax the constraint in this manner because there is a pecuniary production externality: individual firms in the linear housing subsector do not internalize that their period-0 debt decisions affect the period-1 land price and thereby the tightness of the period-1 constraint. The planner relaxes the constraint by reducing L_0^{Linear} below 1. The second and third terms on the right hand side capture the side-effects of the reduction in L_0^{Linear} on the period-0 land price and on period-1 rents.

The planner can reduce L_0^{Linear} below 1 using a period-0 domestic macroprudential tax on the debt of the linear housing subsector. The housing macroprudential tax in period t follows the expression:

$$\left(1+\theta_{Rt}^{Linear}\right) = \begin{cases} \frac{\mathbb{E}_t\left[\frac{1}{L_t^{Linear}+G\left(1-L_t^{Linear}\right)}\frac{\alpha_R}{\alpha_F}\frac{E_{t+1}}{E_t}P^*_{Ft+1}C_{Ft+1}\right]+\mathbb{E}_t\left[\frac{E_{t+1}}{E_t}\widehat{q}_{t+1}\right]}{(1+i_t)\widehat{q}_t} & \text{if } \Psi_{Rt}=0 \\ 0 & \text{if } \Psi_{Rt}>0 \end{cases}, \tag{38}$$

$$\text{where } \widehat{q}_t = \begin{cases} \frac{1}{(1+i_0)}\frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)}\frac{\alpha_R}{\alpha_F}\mathbb{E}_0\left[\frac{E_1}{E_0}P^*_{F1}C_{F1}\right] \\ \quad +\frac{1}{(1+i_0)}\mathbb{E}_0\left[\frac{1}{\chi_2}\frac{E_1}{E_0}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\right] & \text{if } t=0 \\ \frac{1}{\chi_2}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right) & \text{if } t=1 \\ \widehat{q}_2 & \text{if } t=2, \end{cases}$$

where $\widehat{q}_2$ is exogenously fixed.[21] The desired reduction in L_0^{Linear} causes a decrease in the period-0 land price, $\widehat{q}_0$, as the concave subsector is forced to hold some land. To prevent the linear subsector purchasing all the land at this lower price, the planner imposes a positive ex ante tax on that subsector, i.e., $\theta_{R0}^{Linear}>0$.

In the absence of other instruments, the hedging motive is positive, because fire sales decrease the net payoffs from land. Therefore, the housing macroprudential tax imposed in period 0 is also

[21] Fixing $\widehat{q}_2$, the FX value of the land price in period 2, captures the assumption that short-term policy actions cannot alter the long-term relative price of land to foreign tradable goods. The corollary of this assumption is that depreciations increase the period-2 land price in domestic currency.

positive.

However, the planner possesses additional policy tools to help relax the housing constraint (20), and these tools may alter the hedging motive and thereby the rationale for the housing macroprudential tax. We turn next to the use of these other instruments, both domestic and external. The planner can relax the housing constraint by reducing the policy rate and domestic borrowing rate, which raises the land price via the no-arbitrage condition of the concave housing subsector. The planner can also use a combination of policy tools to depreciate the exchange rate, which relaxes the housing constraint via two channels: firstly, it generates substitution in consumption from imports to home goods including housing, thus boosting rents and house prices; and secondly, it increases the domestic currency price of land in period 2, which filters back to a higher domestic currency price in period 1 as well.

The use of these additional instruments to stabilize the housing sector generates distortions for the non-housing sectors of the economy, which must be balanced against the relaxation of the housing constraint. Exactly which distortions are generated in the rest of the economy depends on the set of available instruments, and in particular whether the planner has access to capital controls or consumer macroprudential taxes. The formula for the FX value of the housing sector's gross borrowing rate depends on which of the two instruments is available:

$$\chi_{t+1} = \begin{cases} \dfrac{\frac{\alpha_F}{P^*_{Ft} C_{Ft}}}{\beta E_{t+1} \mathbb{E}_t \left\{ \frac{1}{E_{t+1}} \frac{\alpha_F}{P^*_{Ft+1} C_{Ft+1}} \right\}} & \text{if } \varphi_t \in \mathbb{R} \text{ but } \theta_{HHt} \equiv 0 \\ \eta_{t+1} & \text{if } \theta_{HHt} \in \mathbb{R} \text{ but } \varphi_t \equiv 0. \end{cases}$$

If capital controls are allowed but consumer macroprudential taxes are not, i.e., $\varphi_t \in \mathbb{R}$ but $\theta_{HHt} \equiv 0$, the housing sector's FX borrowing rate is identical to the borrowing rate of domestic households, and altering the borrowing rate must be balanced against distorting the domestic consumption path. If consumer macroprudential taxes are allowed but capital controls are not, i.e., $\theta_{HHt} \in \mathbb{R}$ but $\varphi_t \equiv 0$, the housing sector's FX borrowing rate is identical to the return received by international intermediaries, and altering the borrowing rate rate must be balanced against altering the UIP wedges paid by the domestic economy to foreigners.

In other words, the occasionally-binding constraint of the housing sector breaks the result of substitutability between capital controls and consumer macroprudential taxes (assuming perfect coverage of both instruments) from sections 4 and 5. The two instruments are in principle substitutable in period 0, but they are not substitutable in period-1 states when the housing constraint binds, and the divergence in allocations in those period-1 states causes a divergence in the optimal period-0 levels of the instruments as well.

We can catalogue the constrained efficient FOCs depending on whether the planner has access to capital controls or consumer macroprudential taxes. First, if capital controls are allowed as in section 5, the expressions (30) and (31) remain unchanged, as do the exchange rate FOCs in periods 0 and 2 represented by equation (29), but the FOC for the exchange rate in period 1 changes to the following:

$$\underbrace{\alpha_H \tau_{H1}}_{\text{Stabilize demand for home goods}} = \underbrace{\begin{array}{c} -\mathbb{I}^{PCP} \cdot \left\{ \omega C_1^* \frac{\alpha_F}{C_{F1}} \tau_{X1}^{PCP} \right\} \\ -\mathbb{I}^{DCP} \cdot \left\{ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t \omega C_t^* \frac{\alpha_F}{C_{Ft}} \tau_{Xt}^{DCP} \right] \frac{1}{\beta \pi_1} \left(-\frac{E_1}{P_X} \frac{\partial P_X}{\partial E_1} \right) \right\} \end{array}}_{\text{Optimize TOT on export goods}}$$
$$+ \underbrace{\frac{\Psi_{B1}}{\beta \left[(1+i_0^*) + (1-\lambda) \tau_{\Gamma 1} \right]} \kappa_{H1} \frac{P_H}{E_1}}_{\text{Relax bank constraint}} \pm \underbrace{\frac{1}{\beta \pi_1} \Lambda E_1 \left[(1+i_0^*) + \tau_{\Gamma 1} \right]}_{\text{Prevent excessive contingency of exchange rate}}$$
$$+ \mathbb{E}_0 \left[\Psi_{R1} \left\{ \underbrace{E_1^s \frac{\partial \chi_1}{\partial E_1^s} \left[\left(1+i_{-1}^*\right) B_{R0}^{Linear} - \widehat{P}_{R0} L_{-1} \right]}_{\text{Inherited housing debt relative to rent and land price}} + \underbrace{E_1^s \frac{\partial \left(\chi_1 \widehat{q}_0 \right)}{\partial E_1^s} \left(L_0 - L_{-1} \right)}_{\text{Effect on period-0 land price}} \right\} \right]. \tag{39}$$

The last term is new and, since $\frac{\partial \chi_1^s}{\partial E_1^s} < 0$ and $\frac{\partial \chi_1^s}{\partial E_1^{-s}} > 0$, indicates that the planner finds it optimal to depreciate the exchange rate in period-1 states in which the housing constraint binds, relative to those period-1 states in which it does not bind. We explained above that such a depreciation raises rents and the land price. Another way to view the same mechanism is that there is a reduction in the ratio of inherited debt to period-1 rents and the land price, and this view is captured in the above expression. The distortion from using the exchange rate to support the housing sector is a reduction in τ_{H1}, indicating positive price and AD pressures.

The new Euler condition between periods 0 and 1 is:

$$
\begin{aligned}
&\frac{\alpha_F}{P^*_{F0}C_{F0}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H0}\right]-\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{P^*_{F0}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F0}}\right\}\\
&=\beta\mathbb{E}_0\left[\frac{\alpha_F\left[(1+i^*_0)+(1-\lambda)\,\tau_{\Gamma 1}\right]}{P^*_{F1}C_{F1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H1}\right]\right]\\
&-\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\,\tau_{\Gamma 1}}{\pi_1P^*_{F1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F1}}\right]\\
&+\Gamma\Omega_0+\mathbb{E}_0\left[\Psi_{R1}\left\{\begin{array}{c}\left(\frac{1}{P^*_{F0}}\frac{\partial\chi_1}{\partial C_{F0}}-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{P^*_{F1}\pi_1}\right]\frac{\partial\chi_1}{\partial C^s_{F1}}\right)\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0}-\widehat{P}_{R0}L_{-1}\right]\\
-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{P^*_{F1}\pi_1}\right]\frac{\partial(\chi_1\widehat{q}_0)}{\partial C^s_{F1}}\left(L_0-L_{-1}\right)-\chi_1\frac{1}{P^*_{F0}}\frac{\partial\widehat{P}_{R0}}{\partial C_{F0}}L_{-1}\\
-\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{P^*_{F1}\pi_1}\left\{-\frac{\partial\widehat{P}_{R1}}{\partial C_{F1}}L_0+\frac{\partial\widehat{q}_1}{\partial C_{F1}}\left(\left(1-\kappa_{L1}\right)L_1-L_0\right)\right\}\end{array}\right\}\right],
\end{aligned}
\tag{40}
$$

and the new formula for the ex ante capital control tax, φ_0, is:

$$
\varphi_0=1-\frac{\frac{(1+i^*_0)+\mathbb{E}_0\tau_{\Gamma1}}{\mathbb{E}_0\left[\frac{E_0}{E_1}\right]}\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right]\left\{\begin{array}{c}\frac{\alpha_F}{P^*_{F0}C_{F0}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H0}\right]\\-\mathbb{I}^{DCP}\cdot\left\{\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\frac{1}{P^*_{F0}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F0}}\right\}\end{array}\right\}}{\frac{\alpha_F}{P^*_{F0}C_{F0}}\left\{\begin{array}{c}\mathbb{E}_0\left\{\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{(1+i^*_0)}\frac{\alpha_F}{P^*_{F1}C_{F1}}\left[1+\frac{\alpha_H}{\alpha_F}\tau_{H1}\right]\right\}+\Gamma\Omega_0\\-\mathbb{I}^{DCP}\cdot\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\omega C^*_t\frac{\alpha_F}{C_{Ft}}\tau^{DCP}_{Xt}\right]\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{\pi_1P^*_{F1}}\frac{1}{P_X}\frac{\partial P_X}{\partial C_{F1}}\right]\\+\mathbb{E}_0\left[\Psi_{R1}\left\{\begin{array}{c}\left(\frac{1}{P^*_{F0}}\frac{\partial\chi_1}{\partial C_{F0}}-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{P^*_{F1}\pi_1}\right]\frac{\partial\chi_1}{\partial C^s_{F1}}\right)\\\times\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0}-\widehat{P}_{R0}L_{-1}\right]\\-\mathbb{E}_0\left[\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{P^*_{F1}\pi_1}\right]\frac{\partial(\chi_1\widehat{q}_0)}{\partial C^s_{F1}}\left(L_0-L_{-1}\right)-\chi_1\frac{1}{P^*_{F0}}\frac{\partial\widehat{P}_{R0}}{\partial C_{F0}}L_{-1}\\-\frac{(1+i^*_0)+(1-\lambda)\tau_{\Gamma1}}{P^*_{F1}\pi_1}\left\{-\frac{\partial\widehat{P}_{R1}}{\partial C_{F1}}L_0+\frac{\partial\widehat{q}_1}{\partial C_{F1}}\left(\left(1-\kappa_{L1}\right)L_1-L_0\right)\right\}\end{array}\right\}\right]\end{array}\right\}}.
\tag{41}
$$

This expression captures how the inclusion of the housing sector alters the optimal capital controls. First, the formula reveals how capital controls interact with the policy rate. We know that reducing the policy rate in period 0 (via increasing C_{F0}, with $\frac{\partial\chi_1}{\partial C_{F0}}<0$) and in the period-1 state when the constraint binds (via increasing C_{F1} in that state, with $\frac{\partial \chi_1}{\partial C^s_{F1}} > 0$) help relax the housing constraint. The formula shows that for given UIP wedges, such policy rate reductions reduce the necessary ex ante capital controls. Second, the formula includes terms related to $\frac{\partial \widehat{P}_{Rt}}{\partial C_{Ft}}$ and $\frac{\partial \widehat{q}_1}{\partial C_{F1}}$, which establish that capital controls should shift consumption intertemporally in order to bolster rents and house prices in period-1 states when the housing constraint is binding.

Next, we consider the allocations if consumer macroprudential taxes are allowed but capital controls are not. The FOCs for the exchange rate and FX intervention, given by equations (29) and (31), remain unchanged relative to section 5. However, the expression (30) summarizing the trade-off for the optimal UIP wedge between periods 0 and 1, $\tau_{\Gamma 1}$, is altered to the following:

$$\underbrace{\Omega_0}_{\text{Ability to borrow more today}} = \underbrace{(1-\lambda)\left(\Phi_1 + \Psi_{B1}\right)\frac{B_1 + FXI_0}{I_0}}_{\text{Higher repayments tomorrow}} \pm \underbrace{\frac{1}{\pi_1}\Lambda E_1}_{\text{Prevent excessive contingency of premium}}$$
$$+ \underbrace{(1-\lambda)\,\Omega_1 \Gamma\left(B_1 + FXI_0\right)}_{\text{Higher premium tomorrow owing to rollover needs}} + \underbrace{\Psi_{R1}\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0} - \widehat{P}_{R0}L_{-1}\right]}_{\text{Higher inherited housing debt for linear subsector}}. \tag{42}$$

The last term is new and indicates that the planner finds it optimal to reduce the UIP wedge between period 0 and period-1 states in which the housing constraint binds, i.e., to depreciate the exchange rate in those period-1 states. The distortion from using the exchange rate to support the housing sector is an increase in Ω_0, indicating that international financial intermediaries are less willing to finance domestic currency debt in period 0. To restore the attractiveness of the debt, the planner can commit to appreciate the exchange rate in period-1 states in which the housing constraint does not bind, distorting allocations in those states as well.

The trade-off for the optimal UIP wedge between periods 1 and 2, $\tau_{\Gamma 2}$, is altered to the following:

$$\underbrace{\Omega_1}_{\text{Ability to borrow more today}} = \underbrace{(1-\lambda)\,\Phi_1 \frac{B_2 + FXI_1}{I_0 I_1}}_{\text{Higher repayments tomorrow}}$$

$$+\underbrace{\frac{1}{\pi_1}\mathbb{E}_0\left[\Psi_{R1}\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial\eta_2^s}\left(L_0-L_{-1}\right)\right]}_{\text{Reduction in period-0 land price}}+\underbrace{\Psi_{R1}\frac{\partial\widehat{q}_1}{\partial\eta_2}\left(\left(1-\kappa_{L1}\right)L_1-L_0\right)}_{\text{Lower period-1 land price, tighter constraint}}. \quad (43)$$

The last two terms are again new. The terms capture the impact of the UIP wedge on the constraint via the period-0 and period-1 land prices.

Constrained efficient allocations with consumer macroprudential taxes instead of capital controls follow the Euler condition (40), but some of the derivatives inside the expression take different values.[22] In particular, the borrowing rate for households and the housing sector are no longer connected, so we need to impose that $\frac{\partial\chi_1}{\partial C_{F0}}=\frac{\partial\chi_1}{\partial C_{F1}^s}=0$. Nevertheless, we preserve the recommendation that consumption levels should be altered (now via consumer macroprudential taxes instead of capital controls) in order to bolster rents and house prices in period-1 states when the housing constraint is binding. The value of the ex ante consumer macroprudential tax is obtained by setting $\varphi_0=0$ in the period-0 version of equation (26).

For the remainder of this section, we show constrained efficient policy responses as a function of different shocks and country characteristics. In subsections 6.2-6.3, we show whether a range of real and financial shocks can cause the housing constraint to bind. In subsection 6.4, we illustrate interactions between the two occasionally-binding constraints in our model: the banks' external debt limit and the linear housing subsector's domestic debt limit.

Although we have shown graphs for both PCP and DCP cases in sections 4 and 5, we focus on illustrating the DCP case in this section, while documenting any differences with the PCP case in the main text. One figure for the PCP case at the end of the section encapsulates the key qualitative difference relative to the DCP case.

6.2 Real Shocks

In this subsection, we consider whether housing sector constraints can become binding as a result of domestically- and externally-generated shocks which affect the real income of domestic households.

[22] For more details on the derivatives, please see Appendix A.2.

6.2.1 Productivity Shocks

We consider permanent shocks to the value of productivity, $A_1 = A_2$.

Under DCP, the introduction of the housing constraint alters the finding that exchange rate flexibility should be the main tool to absorb the shock. Figure 31 shows that after the low realization of the productivity shock, the exchange rate appreciates, reducing the consumption of home goods and thereby the domestic currency value of rents and the land price. As a result, the housing constraint becomes binding. In light of this problem, the planner imposes a housing macroprudential tax in period 0 and lowers the policy rate in the period-1 state in which the constraint binds.

The ex post reduction in the policy rate helps reduce the size of the necessary housing macroprudential tax, which reduces the distortion in the land market. However, the reduction in the policy rate generates distortions elsewhere: there is a destabilization of imports across states and periods, and the lower AD wedge indicates positive price and AD pressures.

Ex ante capital controls are negative because the planner tries to induce higher consumption in period 0, which can raise rents so that the linear housing subsector can repay inherited debt more easily before the shock hits. Ex post capital control subsidies are provided when the constraint binds. These subsidies indicate that for the specific parameterization in the simulation, the policy rate reduction to support the housing sector causes an excessive exchange rate depreciation: while some depreciation is helpful in supporting the domestic currency value of rents and the land price, a large depreciation can move the exchange rate far from the level which optimally balances the import/export substitution margin against housing sector support. Capital control subsidies are needed to attract inflows and limit the depreciation. If capital controls are available, $(B_2 + FXI_1)$ is kept at zero in both period-1 states, establishing that FX intervention is used solely to minimize external premia and not otherwise to influence the exchange rate.

Figure 32 shows the allocations if consumer macroprudential taxes are allowed but capital controls are not. Relative to capital controls, consumer macroprudential taxes have advantages and disadvantages. The advantage is that the planner can set a low policy rate in period 0 to reduce the interest burden on inherited debt for the housing sector before the shock hits, while applying positive consumer macroprudential taxes on households so that their consumption levels are not destabilized. The disadvantage is that in the absence of capital controls, inflow subsidies cannot be provided to limit the depreciation after the period-1 policy rate reduction. Instead, there is a role for FX sales, i.e., $(B_2 + FXI_1) < 0$, to limit the depreciation, but this intervention comes at a carry cost.

Under PCP, the main results from DCP continue to hold.

Result 13. *Countries with high housing debt may experience fire sales in the housing sector after adverse productivity shocks. These countries should impose an ex ante housing macroprudential tax and they should lower the policy rate when the housing constraint binds. They may also need to undertake capital controls and/or FX intervention when the constraint binds.*

Remark 14. *FX intervention is less efficient than capital controls at limiting depreciations after policy rate reductions which are aimed at relaxing the housing constraint.*

6.2.2 Commodity Price Shocks

We consider permanent shocks to the dollar price of commodities, $P^*_{Z1} = P^*_{Z2}$. Since our focus here is on domestic rather than external debt, we show figures only for a low initial level of unhedged external FX debt, $B_0 = 0$.

Under DCP, the housing constraint does not necessarily bind after low realizations of commodity prices. The reason is that although the shock makes the domestic economy poorer relative to the rest of the world, which tends to reduce the level of imports and rents, the large exchange rate depreciation associated with DCP substantially increases the domestic currency value of rents and the land price. In figure 33, rents and the land price are actually higher after low commodity price realizations than after high ones, so the housing constraint does not bind after low realizations.

In commodity exporters, exchange rate flexibility not only benefits the country by optimizing the import/export substitution margin for consumption and stabilizing non-commodity-sector employment, but it also has an additional benefit in helping domestic credit markets avoid binding constraints.

Under PCP, the same qualitative mechanisms apply as in the DCP case, but the exchange rate depreciation is smaller after low realizations of commodity prices. There is still some depreciation, which increases the period-2 land price and filters back to a higher period-1 land price as well. However, rents may end up being lower owing to the economy being poorer relative to the rest of the world. As a result, there is more chance of a binding housing constraint under PCP.

Result 14. *Countries with large commodity sectors and DCP may not necessarily suffer from binding housing sector constraints after low realizations of commodity prices.*

Remark 15. *Low commodity price realizations are more likely to generate binding housing sector constraints under PCP than DCP.*

6.3 Financial Shocks

In this subsection, we consider both domestically- and externally-generated shocks which affect borrowing/lending transactions on two dimensions: between domestic banks and the rest of the world; and between different domestic agents.

6.3.1 World Interest Rate Shocks

We consider a shock to the value of the world interest rate, $(1 + i_1^*)$.

Under DCP, the housing constraint does not necessarily bind after high realizations of the world interest rate. The reason is that in response to the shock, the exchange rate depreciates substantially in period 1 and then appreciates substantially in period 2, as shown in figure 34. The period-2 appreciation reduces the domestic currency value of the period-2 land price, which filters through to a low period-1 land price as well and tends to tighten the constraint. However, the period-1 depreciation bolsters the domestic currency value of rents, which tends to relax the housing constraint. Again, as in the case of the commodity price shock, exchange rate flexibility not only addresses price and AD pressures, but also helps stabilize domestic credit markets.

Under PCP, the same qualitative mechanisms apply as in the DCP case, but the exchange rate depreciation is smaller after high realizations of the world interest rate. As a result, there is a smaller boost to the domestic currency value of rents, which means that the housing constraint does occasionally bind in our simulations when the world interest rate increases.

Result 15. *Countries with DCP may not necessarily suffer from binding housing sector constraints after high realizations of the world interest rate.*

Remark 16. *High world interest rate realizations are more likely to generate binding housing sector constraints under PCP than DCP.*

6.3.2 Foreign Appetite Shocks

A foreign appetite shock is a shock to the value of S_1. Countries with deep FX markets are not affected by this shock, so there is no risk of the housing constraint becoming binding. However, countries with shallow FX markets are affected.

Under DCP, capital controls are more effective than consumer macroprudential taxes in stabilizing the housing sector after foreign appetite shocks. Figure 35 shows the relevant comparison. The dashed lines show the allocations with capital controls but without consumer macroprudential taxes. They confirm the result from subsection 5.3.2 that countries with shallow FX markets can fully stabilize allocations using a combination of capital controls and FX intervention. Capital inflow subsidies after negative foreign appetite shocks raise returns to foreigners without requiring any change in the policy rate. If there is no change in the policy rate, there is no variation of rents and the land price across period-1 states, so the housing sector is fully stabilized with no risk of binding constraints. Therefore, there is no need for an ex ante housing macroprudential tax.

After foreign appetite shocks in countries with shallow FX markets, capital controls and FX intervention together prevent transmission of the shocks not only into AD pressures but also into the housing sector. This result extends our previous finding on the benefits of these instruments for monetary autonomy. The policy rate can focus on domestic sources of price pressures, ignoring not only the direct effect of the external shocks onto households' borrowing and demand, but also any indirect effect through the housing market.

The solid lines show the allocations if consumer macroprudential taxes are allowed but capital controls are not. To replicate the loosening of capital controls after negative foreign appetite shocks, the planner needs to raise the policy rate for intermediaries while loosening consumer macroprudential taxes for households. However, the higher policy rate causes a crash in house prices and as a result may make the housing constraint bind. To remedy this problem, the planner finds it optimal to support the housing sector by raising the policy rate by less than would be necessary to fully stabilize the exchange rate. As a result, macro allocations are partially destabilized by the foreign appetite shock. In addition, the planner optimally imposes an ex ante housing macroprudential tax.

Under PCP, the main results from DCP continue to hold.

Result 16. *Countries with shallow FX markets and high housing debt, but without access to capital controls, should impose higher ex ante housing macroprudential taxes if there is a possibility of foreign appetite shocks. Macro allocations are not stabilized after such shocks.*

Remark 17. *Countries with deep FX markets, and countries with shallow FX markets and the ability to use both capital controls and FX intervention, can set ex ante housing macroprudential taxes independently of the possibility of foreign appetite shocks.*

6.3.3 Housing Debt Limit Shocks

A housing sector debt limit shock is a shock to the value of κ_{L1}, the pledgability parameter in the linear housing subsector's borrowing constraint when they borrow from domestic banks. A decline in κ_{L1} reflects either a downgrade in the perceived creditworthiness of the housing sector, or a reversal in the ability of domestic banks to extend credit to anyone to purchase land.

Under DCP, external adjustment turns out to be necessary to handle this domestically-generated shock. Indeed, countries with open economies possess policy instruments to be used in addition to, and perhaps instead of, the domestic policy instruments usually assigned to handle domestic housing sector shocks in closed-economy contexts.

The dashed line in figure 36 shows the allocation when the exchange rate is pegged and no policy instruments are available beyond the housing macroprudential tax. In this case, there is a positive hedging motive after the low realization of the debt limit shock, because the shock reduces the land price as it tightens the constraint. As expected, the planner optimally imposes an ex ante housing macroprudential tax to address the pecuniary production externality.

The solid line in the figure shows the allocation when the exchange rate is flexible. In this case, the planner allows the exchange rate to depreciate after the low realization of the shock in order to relax the constraint: the decline in κ_{L1} is partially offset by an increase in the domestic currency value of rents and the land price. This ex post exchange rate depreciation may result in the domestic currency rents and land price being similar across period-1 states, or even being *higher* after the low shock. If so, the hedging motive disappears, and we may obtain the counterintuitive result that an ex ante housing macroprudential tax is not necessary to address a domestic housing sector shock. In the simulation plotted, the optimal value of this tax actually hits its lower bound of zero.[23]

Figure 37 provides the allocations when more policy instruments are available. The dashed lines in the figure show the allocations when all instruments are available except the consumer macroprudential tax. In addition to the exchange rate depreciation, the policy rate is reduced after the debt limit shock, while the ex ante housing macroprudential tax continues to be zero. As we observed for the productivity shock, this monetary loosening comes at the cost of a lower AD wedge, indicating positive price and AD pressures. The sign of the ex ante capital inflow tax is similar to that in the case of the productivity shock, and for the same reason. For the specific parameterization in the simulation, the policy rate reduction to support the housing sector again causes an excessive exchange rate depreciation, so capital control subsidies are needed to attract inflows and limit the depreciation. If capital controls are available, $(B_2 + FXI_1)$ is kept at zero in both period-1 states, establishing that FX intervention is used solely to minimize external premia and not otherwise to

[23]The upper bound on land use in the linear subsector, i.e., $L_0^{Linear} \leq 1$, corresponds to a lower bound on the ex ante housing macroprudential tax, i.e., $\theta_{R0}^{Linear} \geq 0$.

influence the exchange rate.

The solid lines show the allocations if consumer macroprudential taxes are allowed but capital controls are not. Consumer macroprudential taxes allow a lower ex ante policy rate, which reduces the interest burden and relaxes the housing constraint so much that the ex post policy rate increases substantially. As a result, for the specific parameterization in the simulation, there is no longer an excessive depreciation in the period-1 state when the constraint binds. Instead, the depreciation is insufficient relative to the level which optimally balances the import/export substitution margin against housing sector support. Therefore, the planner accumulates FX, i.e., $(B_2 + FXI_1) > 0$ to further depreciate the exchange rate. The more limited ex post support for the housing market means that the ex ante housing macroprudential tax does actually rise above zero.

Under PCP, the main results from DCP continue to hold.

Result 17. *Countries with high housing debt that are subject to housing debt limit shocks should lower the policy rate and depreciate the exchange rate after such shocks, and may or may not need to impose an ex ante housing macroprudential tax. They may also need to undertake capital controls and/or FX intervention when the constraint binds.*

Remark 18. *FX intervention is less efficient than capital controls at limiting depreciations after policy rate reductions which are aimed at relaxing the housing constraint.*

6.4 Domestic and External Borrowing Constraints

Many countries, especially emerging markets, may find themselves vulnerable to two kinds of occasionally-binding borrowing constraints: domestic and external. This topic has been the subject of both theoretical and empirical work (e.g., Caballero and Krishnamurthy, 2001, Kaminsky and Reinhart, 1999). Domestic borrowing constraints typically feature domestic-currency-denominated debt which may be collateralized using domestic nontradable assets such as housing, so the constraints become relaxed as the policy rate is reduced and the exchange rate depreciates. External borrowing constraints typically feature dollar-denominated debt which may be collateralized using some element of domestic production, so the constraints become tighter as the exchange rate depreciates and the dollar value of domestic collateral declines.

In this subsection, we explore whether and how these borrowing constraints may interact with each other within our model framework.

6.4.1 Housing Debt Limit Shock to Bank Constraint

In subsection 6.3.3, we established that the exchange rate is optimally depreciated to relax the housing sector constraint after a low realization of the value of κ_{L1}, the parameter summarizing the linear housing subsector's ability to pledge land as collateral. Such a depreciation poses no problems for countries with no initial unhedged external FX debt, i.e., $B_0 = 0$, but it may cause external constraints to bind for countries with high initial unhedged external FX debt, i.e., $B_0 > 0$.

Under DCP, the depreciation may indeed cause the external constraint to bind. Figure 38 shows the allocations with both domestic and external constraints binding in the same period-1 state. The planner relaxes the banks' external constraint by limiting the depreciation in that state, even at the expense of tightening the housing constraint. Ex post capital controls do not work in the binding state. Ex post FX intervention should be used to absorb external premia but not otherwise to defend the exchange rate, for a reason similar to that in subsection 5.3.3: once the banks' debt limit binds, the interest rate ρ_1 in the households' Euler condition and the housing sector's no-arbitrage condition becomes disconnected from the policy rate i_1, so the latter can be used to manage the exchange rate. Therefore, the planner sets $(B_2 + FXI_1)$ to zero.

The limited room for manuever ex post enhances the case for ex ante policy actions. The planner sets a low policy rate in period 0 to reduce the interest burden on inherited debt for the housing sector before the shock hits, thereby mitigating the pecuniary production externality, and sets positive ex ante capital controls to limit external FX debt, thereby mitigating the pecuniary AD externality.

Under PCP, the main results from DCP continue to hold, but the ex ante capital controls are lower for the same reason as in subsection 4.3.3.

Result 18. *Countries with high housing debt and external FX debt should use the policy rate to set the ex post exchange rate to the level that balances the tightness of domestic and external constraints after housing debt limit shocks which cause all external constraints to bind. They may or may not need to impose an ex ante housing macroprudential tax, but they should impose ex ante capital controls and help reduce the housing sector's interest burden.*

Remark 19. *After debt limit shocks, FX intervention is used to absorb external premia but not otherwise to influence the exchange rate.*

Remark 20. *For countries with high housing debt and external FX debt, ex ante capital controls are larger under DCP than PCP.*

6.4.2 Extension: Housing Debt Limit Shock to Household Constraint

In this subsection, we extend the model to generate an interesting role for FX intervention when both domestic and external constraints are binding. In the previous subsection, the only role for ex post FX intervention is to absorb external premia, never to influence the exchange rate at the cost of generating external premia. The reason is that once the external constraint binds, the policy rate i_1 becomes available to manage the exchange rate costlessly, as it is no longer inside the optimization conditions of domestic agents.

We extend the model by assuming that domestic households and domestic housing firms use different banks, so it is possible for some of those banks to be externally constrained while others are not. In particular, we trace out the implications for allocations if a low realization of the value of κ_{L1} causes external debt limits to bind only for those banks which are lending to households. In the set of constraints and FOCs summarized in Appendix A.2, the external constraint of all domestic banks is replaced by the external constraint of only those banks which lend to households, which in equilibrium can be rearranged as follows:

$$\left(1+i_{-1}^{*}\right) B_0 \leq P_{F0}^{*}\left[\omega C_0^{*}-C_{F0}\right]+P_{Z0}^{*} Z_0$$

$$+\frac{P^*_{F1}\left[\omega C^*_1 - C_{F1}\right] + P^*_{Z1}Z_1 - (1-\lambda)\,FXI_0\left[\eta_1 - (1+i^*_0)\right]}{\lambda\,(1+i^*_0) + (1-\lambda)\,\eta_1}$$

$$+\frac{\kappa_{H1}\frac{P_H}{E_1}}{\lambda\,(1+i^*_0) + (1-\lambda)\,\eta_1} - \frac{\frac{\alpha_R}{\alpha_F}P^*_{F1}C_{F1} + \chi_1\frac{\alpha_R}{\alpha_F}P^*_{F0}C_{F0}}{\lambda\,(1+i^*_0) + (1-\lambda)\,\eta_1}, \tag{44}$$

one equation per period-1 state s_1 $[\Psi_{HH}]$

where the last term represents the housing rents which are no longer cancelled out by any aggregation over households and banks.

The policy rate i_1 is no longer inside the optimization conditions of domestic households, but it still remains inside the optimization conditions of the domestic housing sector. We also assume that capital controls are not available, as we have seen above that capital controls may be more efficient than FX intervention for the purpose of inserting a gap between the policy rate and external premia. Consumer macroprudential taxes are available instead.

Under DCP, the depreciation may indeed cause the external constraint to bind. Figure 39 also shows that there is also an interesting role for FX intervention when both domestic and external constraints are binding. The planner knows that the policy rate does affect some domestic activity, and should be reduced to relax the domestic housing constraint. On the other hand, lowering the policy rate for international financial intermediaries causes a depreciation which tightens the external constraint. FX sales can help limit the depreciation, and the planner accordingly sets $(B_2 + FXI_1) < 0$.

Under PCP, the main results from DCP continue to hold.

Result 19. *Countries with high housing debt and external FX debt, but without access to capital controls, should use the policy rate and FX sales to set the ex post exchange rate to the level that balances the tightness of domestic and external constraints if housing debt limit shocks cause some but not all external constraints to bind.*

6.4.3 Bank Debt Limit Shock to Housing Constraint

Returning to the baseline framework, we consider as our final experiment the possibility that a shock to banks' external debt limits, i.e., to the value of κ_{H1}, may cause domestic housing constraints to bind. This time we begin with PCP and then proceed to DCP to highlight how the difference in exchange rate volatility under PCP and DCP that we identified in subsection 4.3.3 affects the transmission channel from the external to the domestic constraint.

Under PCP, the banks' debt limit shock may indeed cause the domestic housing constraint to bind. Figure 40 illustrates the mechanism. The binding external constraint is associated with a large decrease in the policy rate i_1 and an exchange rate depreciation which tends to increase the domestic currency value of rents and the land price. However, it is also associated with an increase in the borrowing rate ρ_1 for domestic households and the housing sector, and a decrease in household consumption. These factors tend to reduce rents and the land price. If the latter effects outweigh the former ones, as they do in our simulations, the housing constraint may bind.

The planner relaxes the housing constraint by reducing the policy rate and allowing more depreciation in that state, even at the expense of tightening the banks' external constraint. Ex post capital controls do not work in the binding state. Ex post FX intervention should be used to absorb external premia but not otherwise to defend the exchange rate. The limited room for manuever ex post enhances the case for ex ante policy actions, and the planner imposes an ex ante housing macroprudential tax.

Under DCP, we know from subsection 4.3.3 that after the banks' debt limit shock, the exchange rate is more depreciated and yet the external constraint is also more relaxed. Figure 41 illustrates that both of these factors alter the likelihood that the domestic housing constraint binds. The larger depreciation means that there is a larger boost to the domestic currency value of rents and the land price. The more relaxed external constraint means that the borrowing rate ρ_1 for domestic households and the housing sector is lower under DCP than PCP, which also supports rents and the land price.

As a result, it is less likely under DCP than PCP that the tightening of external constraints causes the domestic housing constraint to bind. Moreover, even when the housing constraint does bind, it is less severe. Therefore, while ex ante capital controls are larger under DCP than PCP, ex ante housing macroprudential taxes are lower under DCP than PCP.

Result 20. *Countries with high housing debt and external FX debt whose banks are vulnerable to external debt limit shocks should impose positive ex ante capital controls and ex ante housing macroprudential taxes.*

Remark 21. *For countries with high housing debt and external FX debt and vulnerability to external debt limit shocks, ex ante capital controls are larger under DCP than PCP, but ex ante housing macroprudential taxes are lower under DCP than PCP.*

6.5 Extension: Limits to Regulation

The above subsections assumed that all linear housing subsector firms borrow only from domestic banks, and these banks are the sole domestic counterparties of the international financial intermediaries. If so, then housing macroprudential taxes are able to achieve full coverage of the linear subsector's debt transactions. In this subsection, we consider the joint use of housing macroprudential taxes and housing-sector-specific capital controls when there is imperfect coverage of that subsector. As in subsection 4.4, we consider two forms of imperfect coverage, but this time applied to housing taxes and the linear subsector.

First, housing macroprudential taxes may cover a subset of linear housing subsector firms while the remainder of them borrow directly from international financial intermediaries instead of borrowing from domestic banks. For simplicity, we assume that the form of the borrowing constraint is identical for all the linear subsector firms. In this case, the housing macroprudential taxes are an imperfect substitute for housing-sector-specific capital controls. The planner should set housing macroprudential taxes, θ_{Rt}^{Linear}, for domestically-regulated firms in the linear subsector of a magnitude pinned down by (38), and should set capital controls, φ_{Rt}^{Linear}, on intermediaries lending to the other firms in the linear subsector of the following magnitude:

$$\varphi_{Rt}^{Linear} = \frac{\theta_{Rt}^{Linear}}{1 + \theta_{Rt}^{Linear}}. \tag{45}$$

Second, all linear housing subsector firms may officially be regulated by housing macroprudential taxes, but they can circumvent the taxes and issue debt directly to international financial intermediaries. For simplicity, we again assume that the form of the borrowing constraint is identical for all the linear subsector firms. In this case, housing macroprudential taxes and housing-sector-specific capital controls are perfect complements. For housing macroprudential taxes, θ_{Rt}^{Linear}, to be effective at all, these taxes need to be complemented by housing-sector-specific capital controls, φ_{Rt}^{Linear}, following the above expression.

More generally, imperfect coverage by domestic macroprudential taxes on a specific sector should be remedied by the use of sector-specific capital controls, and vice versa.

Result 21. *Capital controls and housing macroprudential taxes are imperfect substitutes if housing macroprudential taxes do not cover all housing sector firms who are vulnerable to borrowing constraints, and they are perfect complements if any housing sector firms who are vulnerable to borrowing constraints can circumvent the macroprudential taxes via cross-border transactions which are not intermediated through the domestic banking system.*

6.6 Summary

Most advanced economies and a growing number of emerging markets have high levels of domestic debt which may trigger binding domestic borrowing constraints after domestic and external shocks. Housing debt is often a large component of this domestic debt. Housing frictions are relevant for macro allocations if the housing sector cannot be perfectly regulated (i.e., some housing firms such as our concave subsector are beyond the regulatory perimeter) and if the housing sector borrowing capacity is limited (e.g., with an upper limit given by the price of collateralized land).

Analyzing a small open economy with a housing sector allows us to identify how and whether external adjustment tools should be used to complement the domestic policy tools which have typically been recommended for housing sector stabilization in closed-economy contexts. The use of these external adjustment tools may justify a deviation from the traditional Mundell-Fleming prescription of relying solely on the domestic policy rate and exchange rate flexibility for some shocks and country characteristics, while bolstering the prescription in other cases.

The following themes emerge from the above analysis on constrained efficient policy responses.

Firstly, for some shocks, exchange rate flexibility may mitigate or prevent housing constraints from binding. After adverse commodity price shocks, increases in world interest rates, and bank debt limit shocks, for example, exchange rate depreciations bolster rents and the land price and help relax housing constraints if this housing sector support is not offset by a reduction in rents. This result represents an extension of the benefits of exchange rate flexibility: it not only benefits the country by optimizing the import/export substitution margin for consumption and stabilizing non-commodity-sector employment, but it also has an additional benefit in helping domestic credit markets avoid binding constraints. For countries with vulnerability to housing sector debt limit shocks, contrary to the intuition of the closed-economy literature, ex post exchange rate flexibility may even remove the need for housing macroprudential taxes in normal times, even though it does not eliminate the housing constraint.

Secondly, for other shocks, exchange rate flexibility may not be sufficient or may even tighten housing constraints, e.g., after productivity and foreign appetite shocks. In these circumstances, countries should use not just domestic instruments but also external adjustment tools to handle housing constraints, deviating from the Mundell-Fleming prescription even if the origin of the shocks is purely domestic. Countries with high housing debt may optimally use a combination of housing macroprudential taxes in normal times coupled with policy rate reductions and exchange rate depreciations when housing constraints bind. If the policy rate reduction causes a depreciation that is too large relative to the level that optimally balances the import/export substitution margin against housing sector support, then capital inflow subsidies or FX sales (with the latter instrument being less efficient than the former) should be used to limit the depreciation. If the depreciation is too small, then capital inflow taxes or FX purchases are the appropriate response.

Thirdly, the set of available policy instruments matters for the stabilization of the housing sector. After foreign appetite shocks, the enhancement of monetary autonomy through the joint use of capital controls and FX intervention is further strengthened in a model with a housing market: the policy rate can focus on domestic sources of price pressures, ignoring not only the direct effect of these shocks onto households' borrowing and demand, but also any indirect effect through the housing market. In the absence of capital controls, the planner can try to replicate the loosening of capital controls after negative foreign appetite shocks by raising the policy rate for intermediaries while loosening consumer macroprudential taxes for households. However, the higher policy rate causes a crash in house prices and as a result may make the housing constraint bind. Therefore, if a country has shallow FX markets and no access to capital controls, the planner needs to impose higher ex ante housing macroprudential taxes because of unavoidable instability from the global financial cycle.

Fourthly, domestic and external constraints may interact with each other if countries have both high housing debt and high external FX debt. After domestic housing sector debt limit shocks, the planner optimally depreciates the exchange rate to support the domestic currency value of rents and the land price, but the exchange rate depreciation may make external constraints bind. If so, there is limited room for manuever ex post. Countries should use the policy rate to set the ex post exchange rate to the level that balances the tightness of domestic and external constraints. They may or may not need to impose an ex ante housing macroprudential tax, but they should impose ex ante capital controls and help reduce the housing sector's interest burden. After bank external debt limit shocks, there is optimally a large exchange rate depreciation which tends to bolster the housing sector, but an increase in the borrowing rate and a decrease in household consumption which tend to reduce rents and the land price, and the latter effects may outweigh the former ones to make the housing constraint bind.

Fifthly, the pricing paradigm matters for both the likelihood of housing constraints after individual shocks and for the interaction between domestic and external constraints. Since exchange rate depreciations are optimally larger under DCP than PCP after adverse commodity price shocks and after increases in world interest rates, and since these depreciations support the housing sector, it is less likely that the housing constraint binds under DCP. After the bank external debt limit shock, the exchange rate is more depreciated (so there is a larger boost to the domestic currency value of rents and the land price under DCP than PCP) and yet the external constraint is also more relaxed (so the borrowing rate for domestic agents is lower under DCP than PCP). Therefore, while ex ante capital controls are larger under DCP than PCP, ex ante housing macroprudential taxes are lower under DCP than PCP.

Finally, housing macroprudential taxes and housing-sector-specific capital controls may appear at first glance to be substitutes for each other, but in practice, they should be used in a complementary fashion to minimize gaps in coverage. Countries should seek to influence the debt levels of any leveraged agents in the housing sector who may be vulnerable to domestic borrowing constraints in the future.

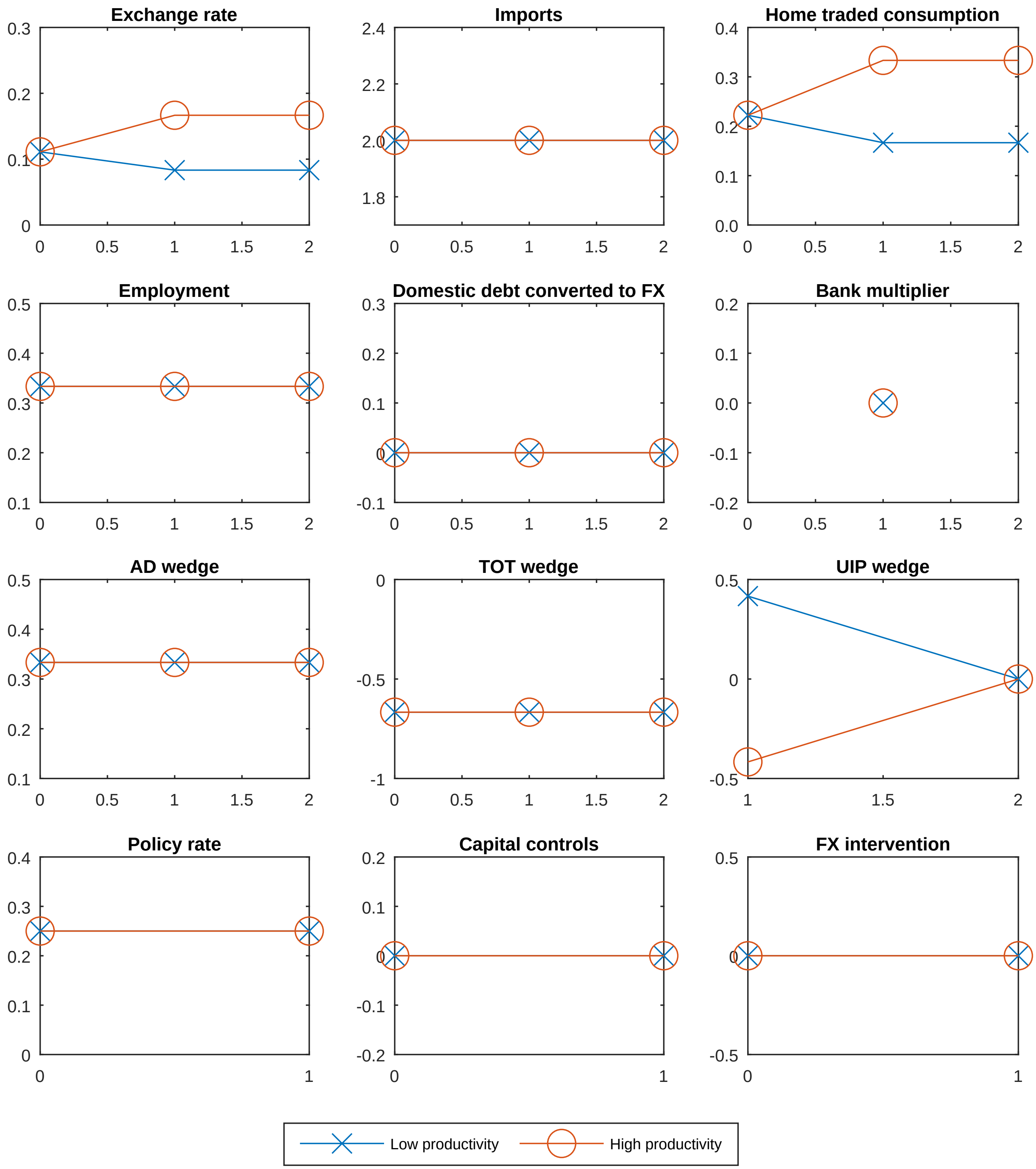
7 Conclusion

In this paper we have built a model of a small open economy that features real and nominal frictions. We have used this framework to characterize the optimal integrated use of monetary policy, capital controls, FX intervention, and macroprudential policy for different shocks and country characteristics. We allowed countries to differ in terms of their currency of trade invoicing, external debt levels, degree of currency mismatches, external and domestic borrowing constraints, and the depth of their FX markets. As a general principle, we have established that not just the number but the workings of individual policy instruments matter. In addition, instruments interact with each other in complex, sometimes unexpected, ways, making it essential that they are considered jointly. Finally, we have found that there is no strict assignment of domestic policies (policy rate and macroprudential debt taxes) to domestic shocks and domestic frictions, or external policies (capital controls and FX intervention) to external shocks and external frictions.

We have built a comprehensive framework that integrates several frictions and policy tools, but as is true for any model, it does not feature all aspects of the real world. First, country characteristics can be endogenous to policy actions, particularly in the long term. For example, it is possible that the private sector tends to take on more currency mismatches in countries with high stocks of FX reserves, if there is an expectation that these reserves will be used in a manner to excessively stabilize the exchange rate after all future shocks. It is also possible that the development of FX markets is hindered through the use of capital controls or FX intervention, if the instruments affect not just the optimal level of debt but also the possibility of market entry. Second, considerations of imperfect policy credibility may call for using fewer rather than more instruments until the country builds credibility, and the policy trade-offs may be different during this transition. Finally, we have analyzed the optimal policy problem from the perspective of a small open economy, but spillovers and spillbacks should be considered when assessing the cost and benefits of policies from a global perspective.

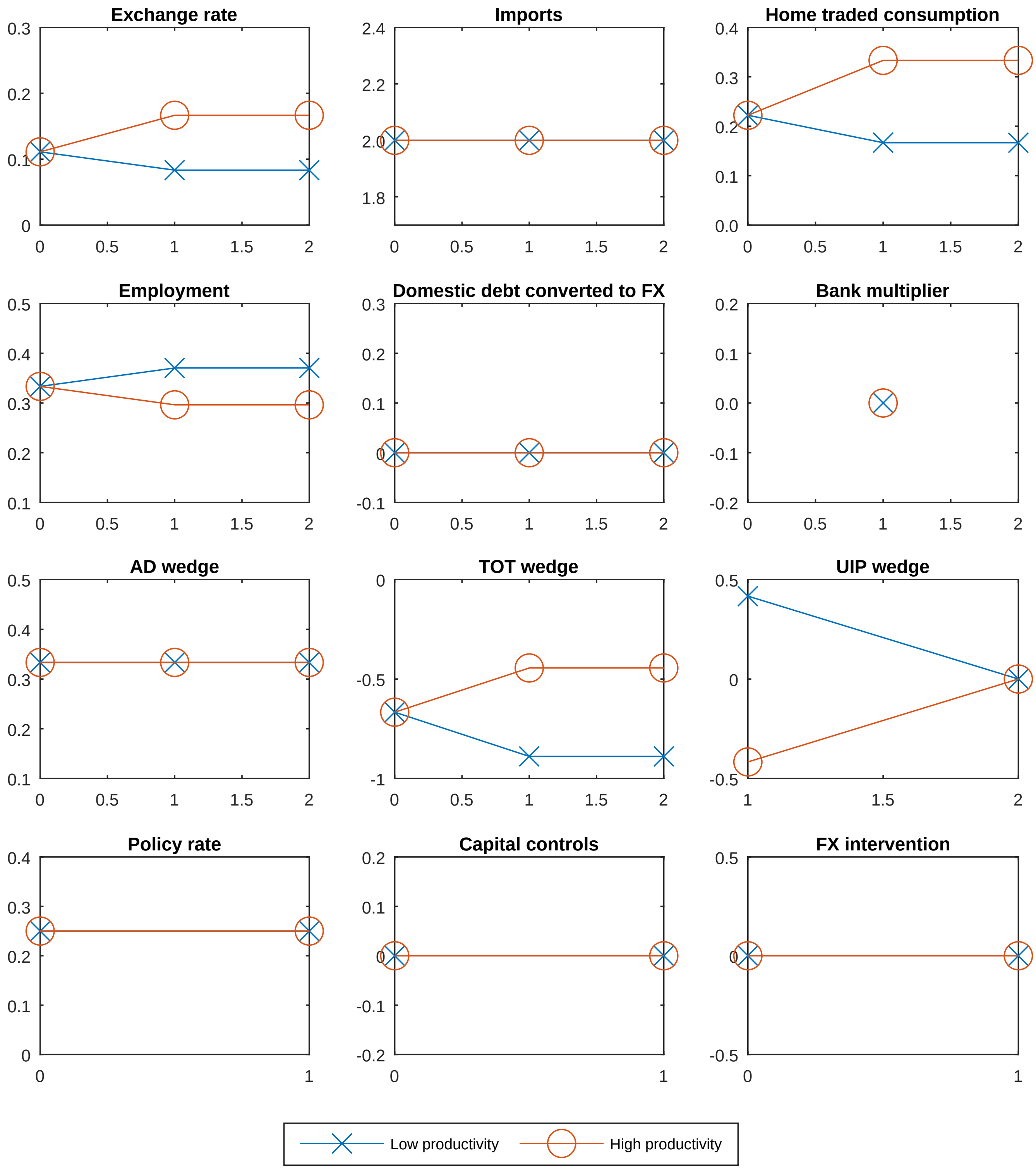
Several practical challenges may arise when it comes to implementing policies in an integrated fashion. First, the policies considered in this paper may be assigned to different agencies in some countries, and coordination between them may be imperfect. Second, the identification of shocks in real time may be difficult. Third, while each policy tool could have its own merit in certain circumstances as outlined in this paper, in practice central banks will need to consider how to carefully incorporate multiple objectives and tools into their policy and communication strategy, as well as their operational framework. Central banks affect the economy both through their immediate policy actions and the impact of their announcements on the public's expectations about future policy actions. The stabilization of expectations requires clear and sustained communication of how policy tools will be used in different states of nature in the future, and how to verify that the central bank is honoring its previous promises. With multiple instruments, this communication problem becomes more complex, and transitional arrangements may be necessary.

Figure 5: Productivity Shock under PCP with Deep FX Markets



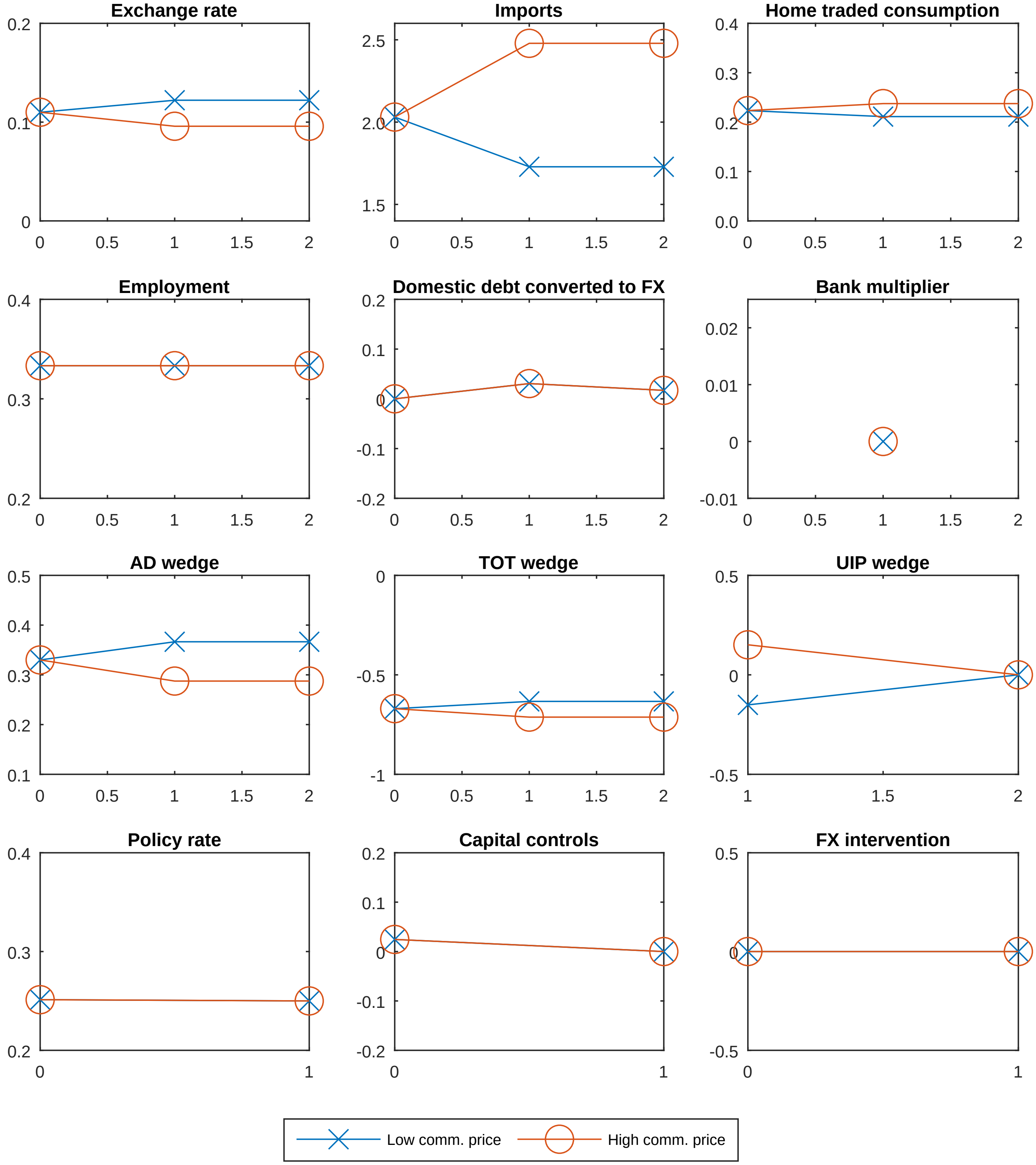
Notes: This figure plots the responses of key variables to a permanent productivity shock under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.75, 1.5]$. This calibration of the shock neutralizes the intertemporal TOT externality.

Figure 6: Productivity Shock under DCP with Deep FX Markets



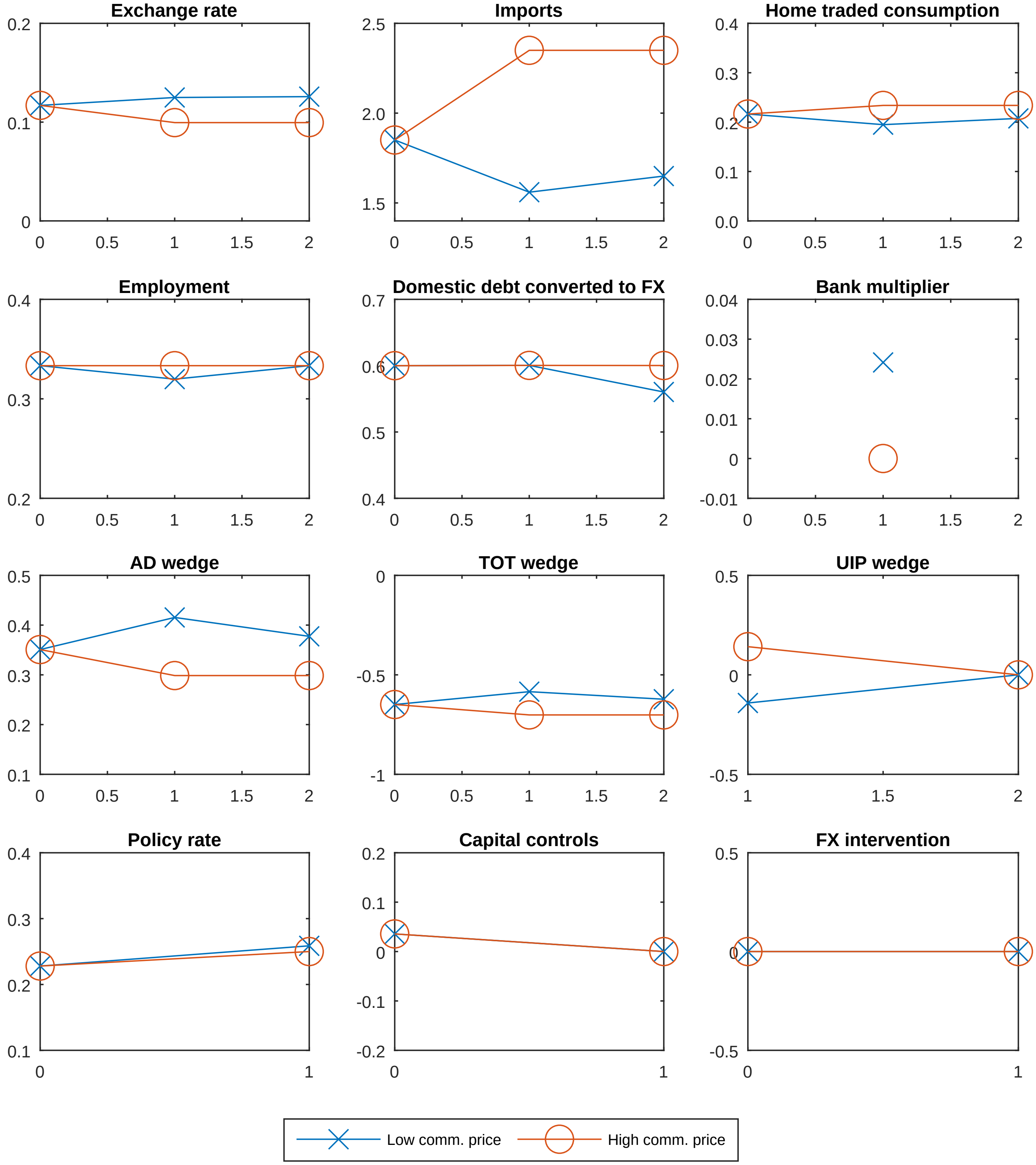
Notes: This figure plots the responses of key variables to a permanent productivity shock under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.75, 1.5]$. This calibration of the shock neutralizes the intertemporal TOT externality.

Figure 7: Commodity Price Shock under PCP with Deep FX Markets



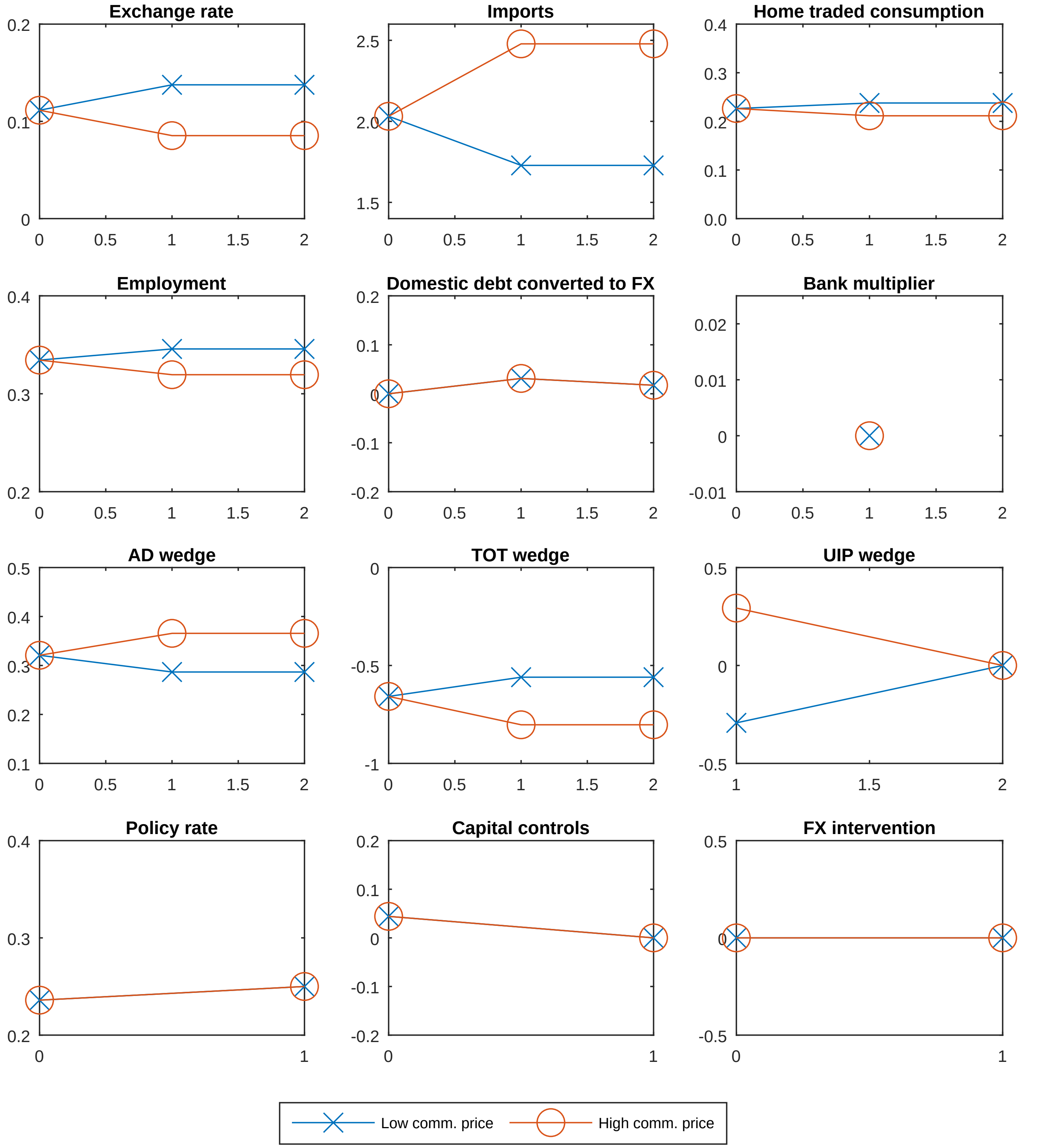
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 8: Commodity Price Shock under PCP with Deep FX Markets and High External Debt



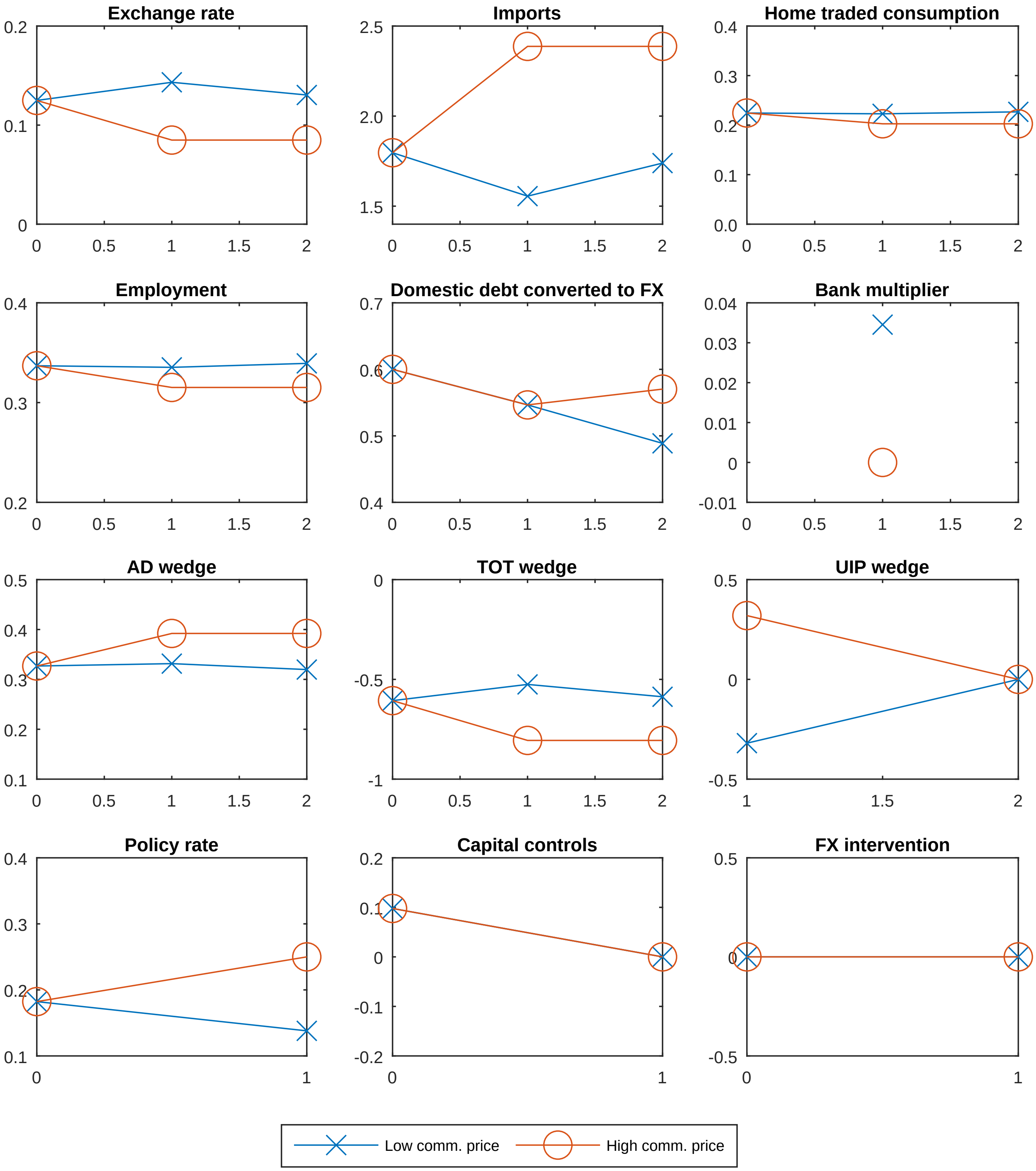
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. Initial external debt is set as $B_0 = 0.6$

Figure 9: Commodity Price Shock under DCP with Deep FX Markets



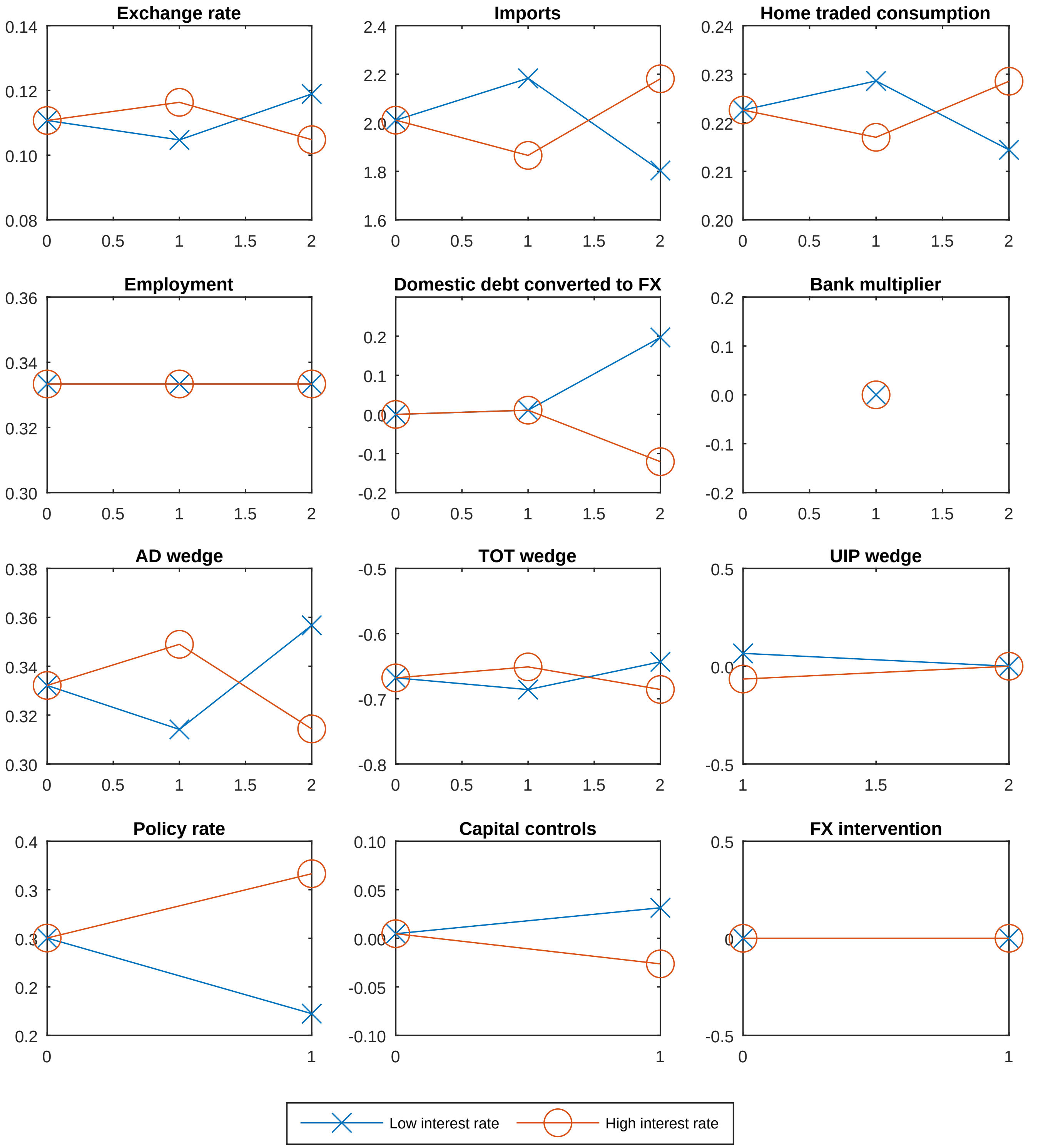
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with deep FX markets in presence of borrowing constraints. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 10: Commodity Price Shock under DCP with Deep FX Markets and High External Debt



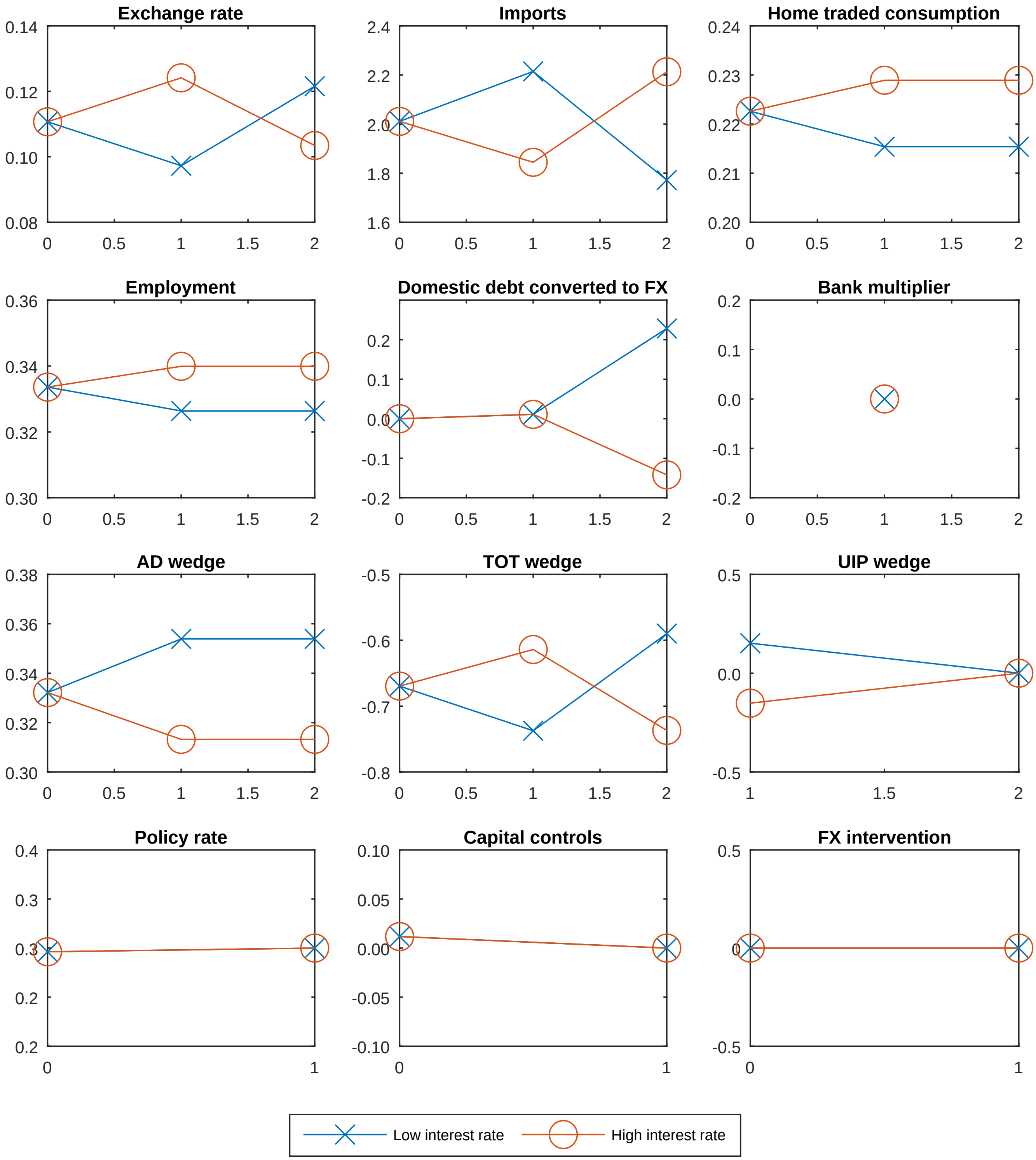
Notes: This figure plots the responses of key variables to a permanent commodity price shock under DCP with deep FX markets in presence of borrowing constraints. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. Initial external debt is set as $B_0 = 0.6$

Figure 11: Interest Rate Shock under PCP with Deep FX Markets



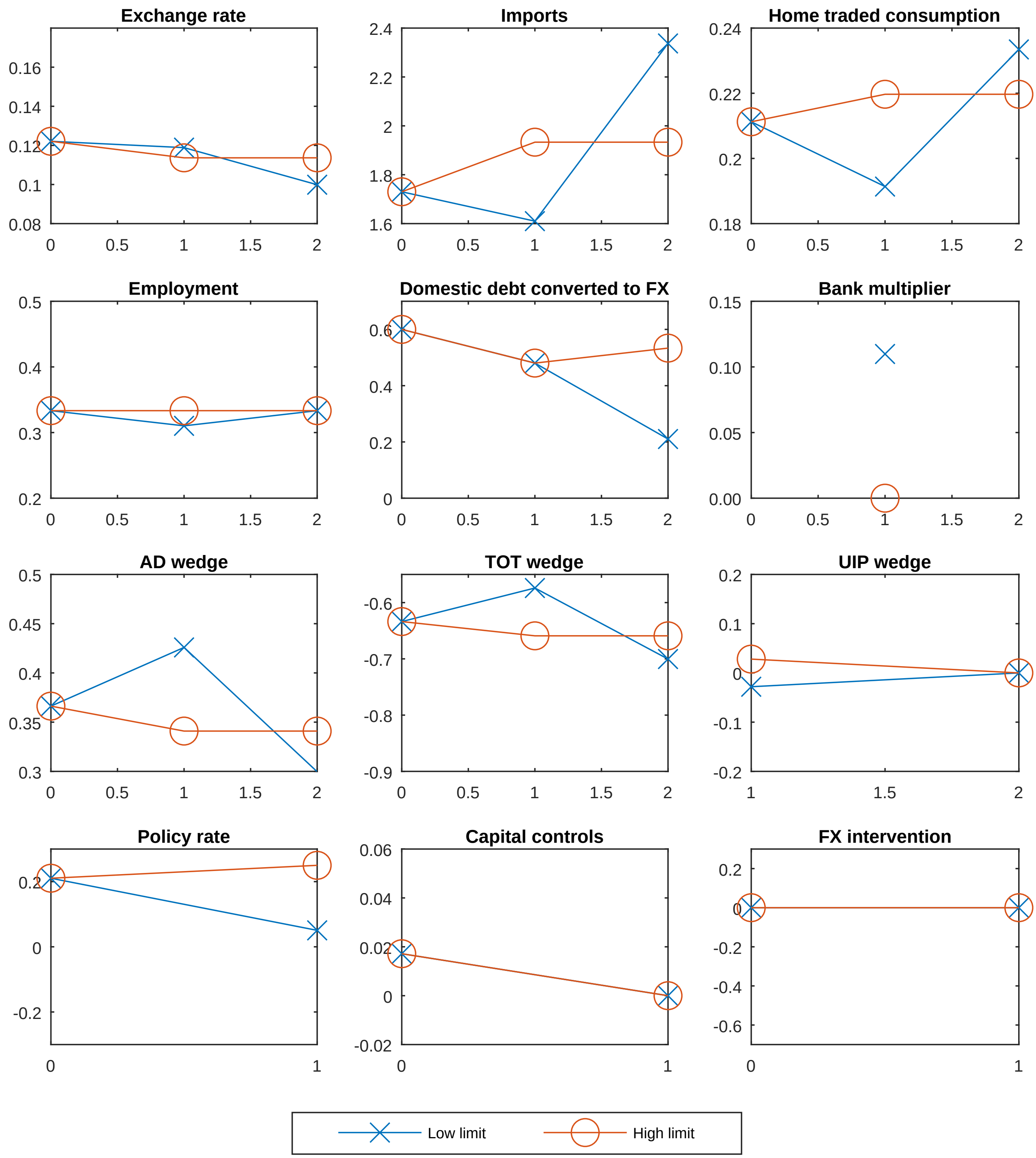
Notes: This figure plots the responses of key variables to a world interest rate shock under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 12: Interest Rate Shock under DCP with Deep FX Markets



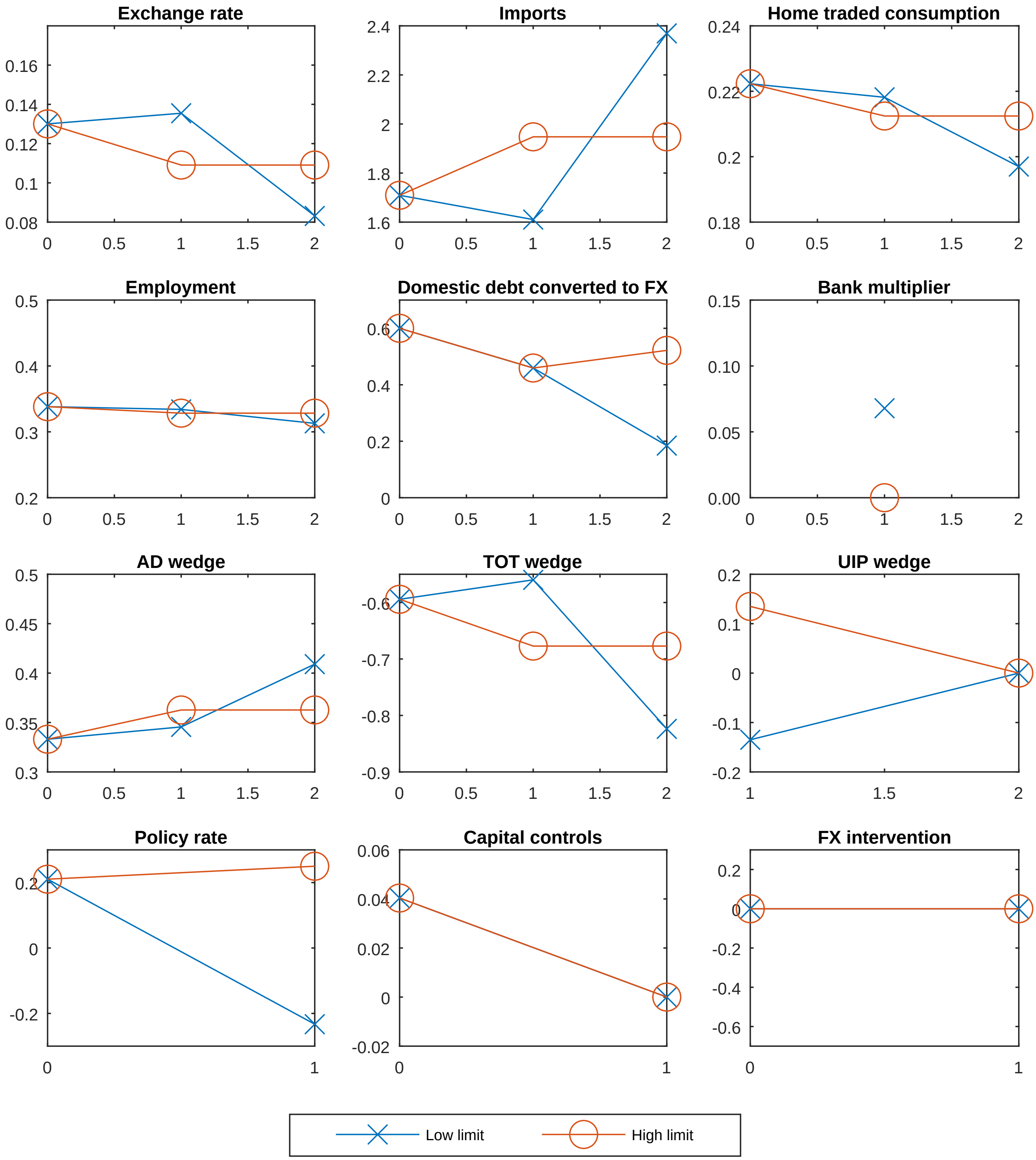
Notes: This figure plots the responses of key variables to a world interest rate shock under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 13: Debt Limit Shock under PCP with Deep FX Markets



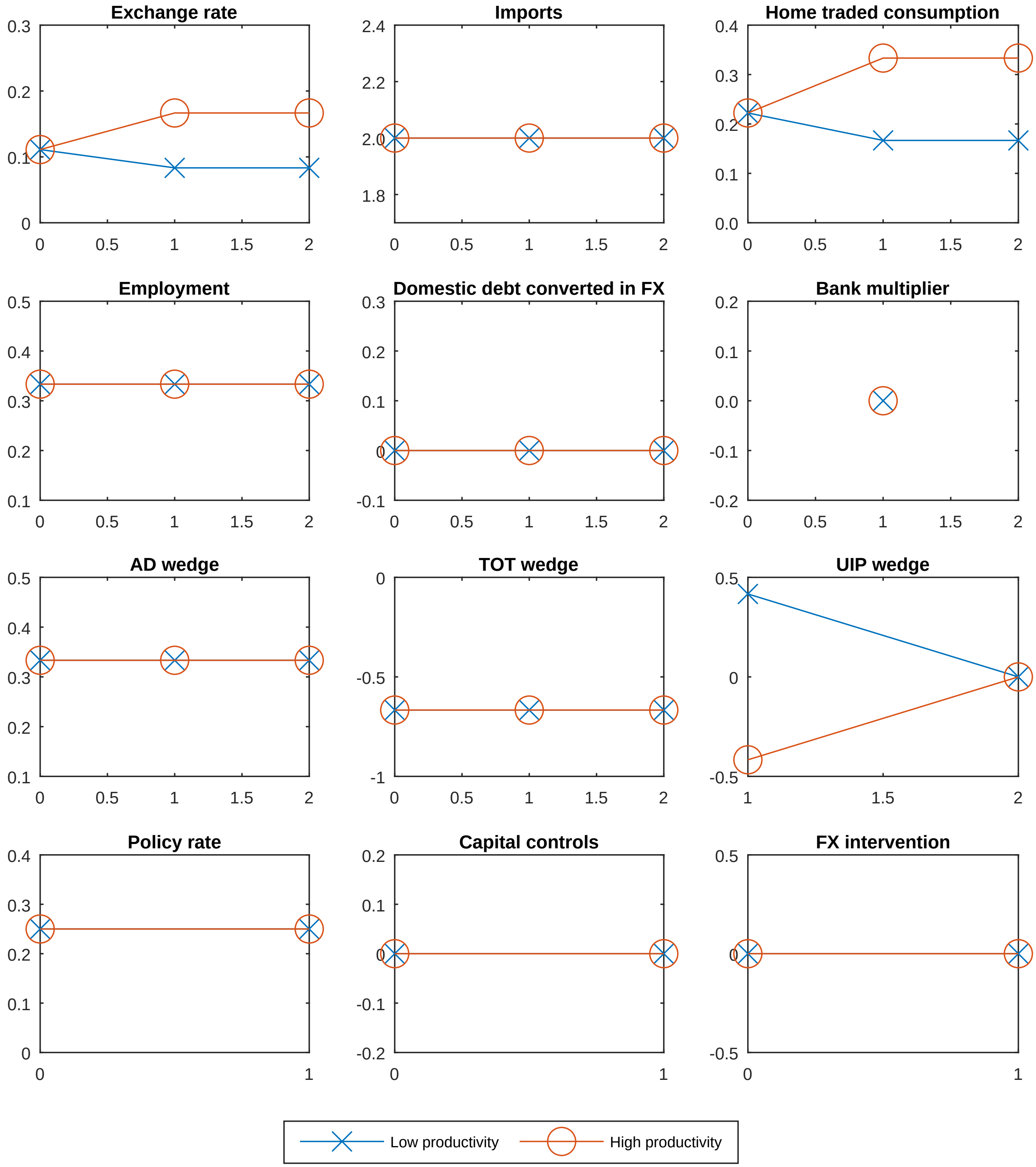
Notes: This figure plots the responses of key variables to a debt limit shock under under PCP with deep FX markets. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 14: Debt Limit Shock under DCP with Deep FX Markets



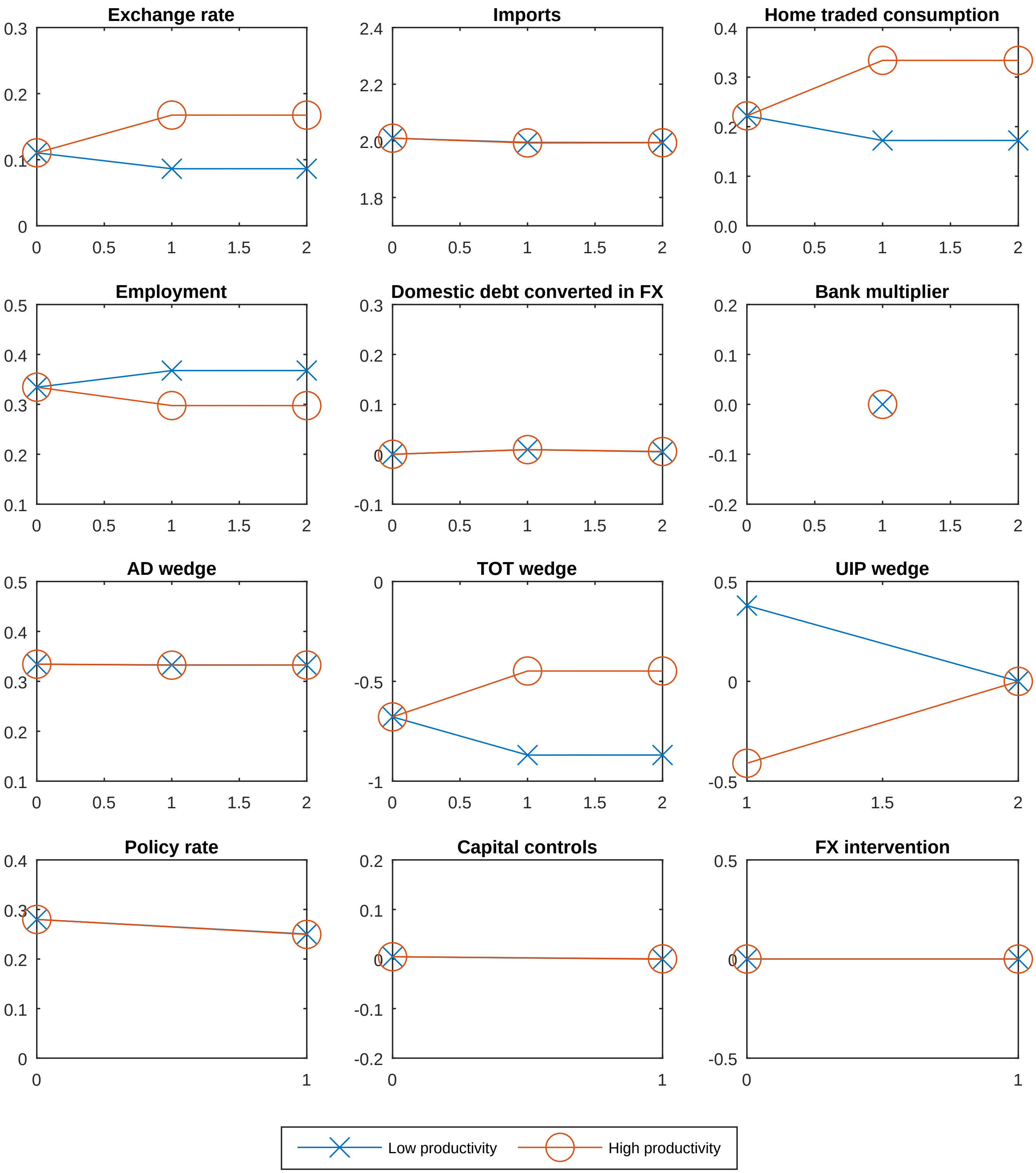
Notes: This figure plots the responses of key variables to a debt limit shock under under DCP with deep FX markets. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 15: Productivity Shock under PCP with Shallow FX Markets



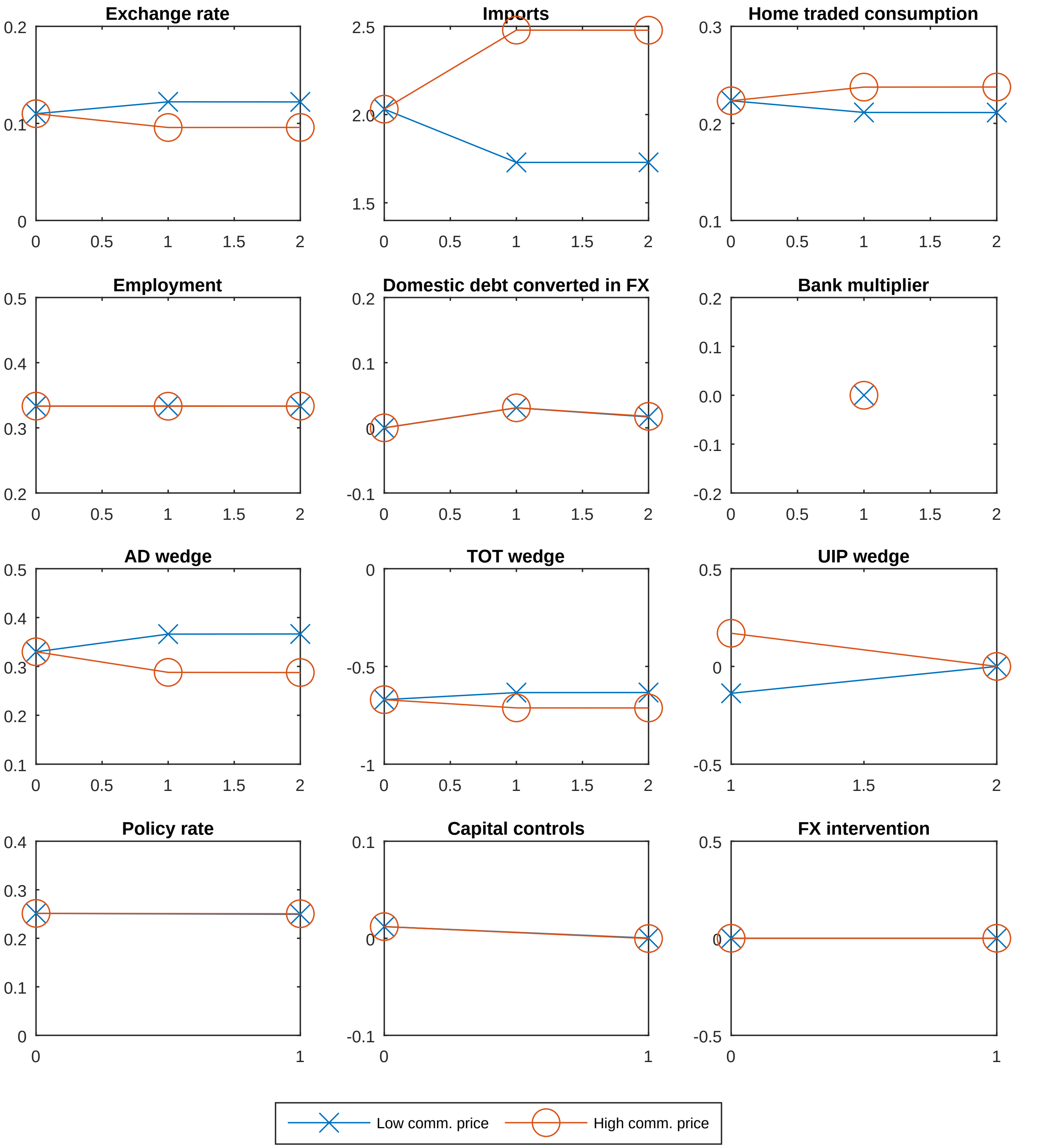
Notes: This figure plots the responses of key variables to a permanent productivity shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.8, 1.5]$. This calibration of the shock neutralizes the intertemporal TOT externality.

Figure 16: Productivity Shock under DCP with Shallow FX Markets



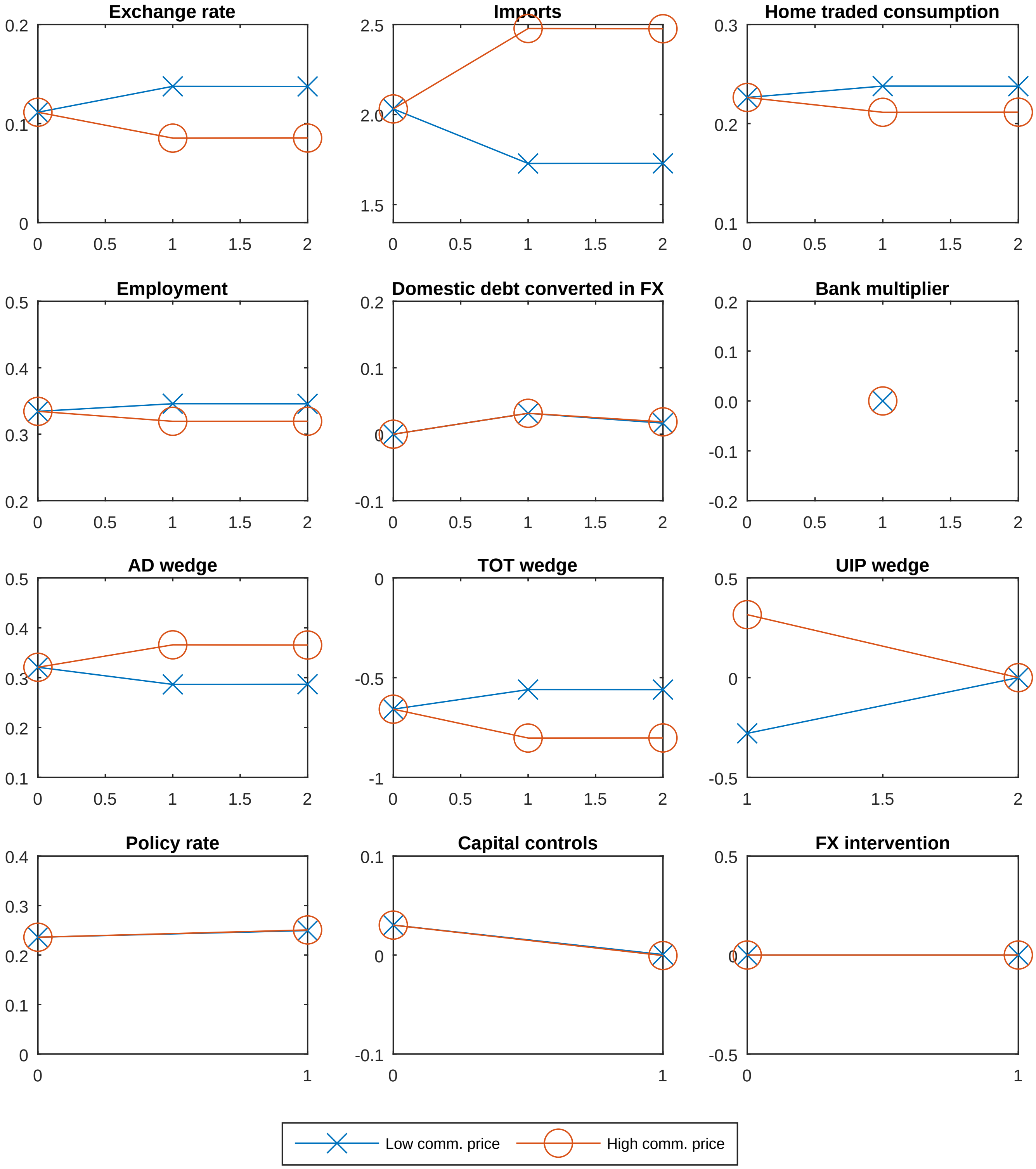
Notes: This figure plots the responses of key variables to a permanent productivity shock under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $A_1 \in [0.8, 1.5]$. This calibration of the shock neutralizes the intertemporal TOT externality.

Figure 17: Commodity Price Shock under PCP with Shallow FX Markets



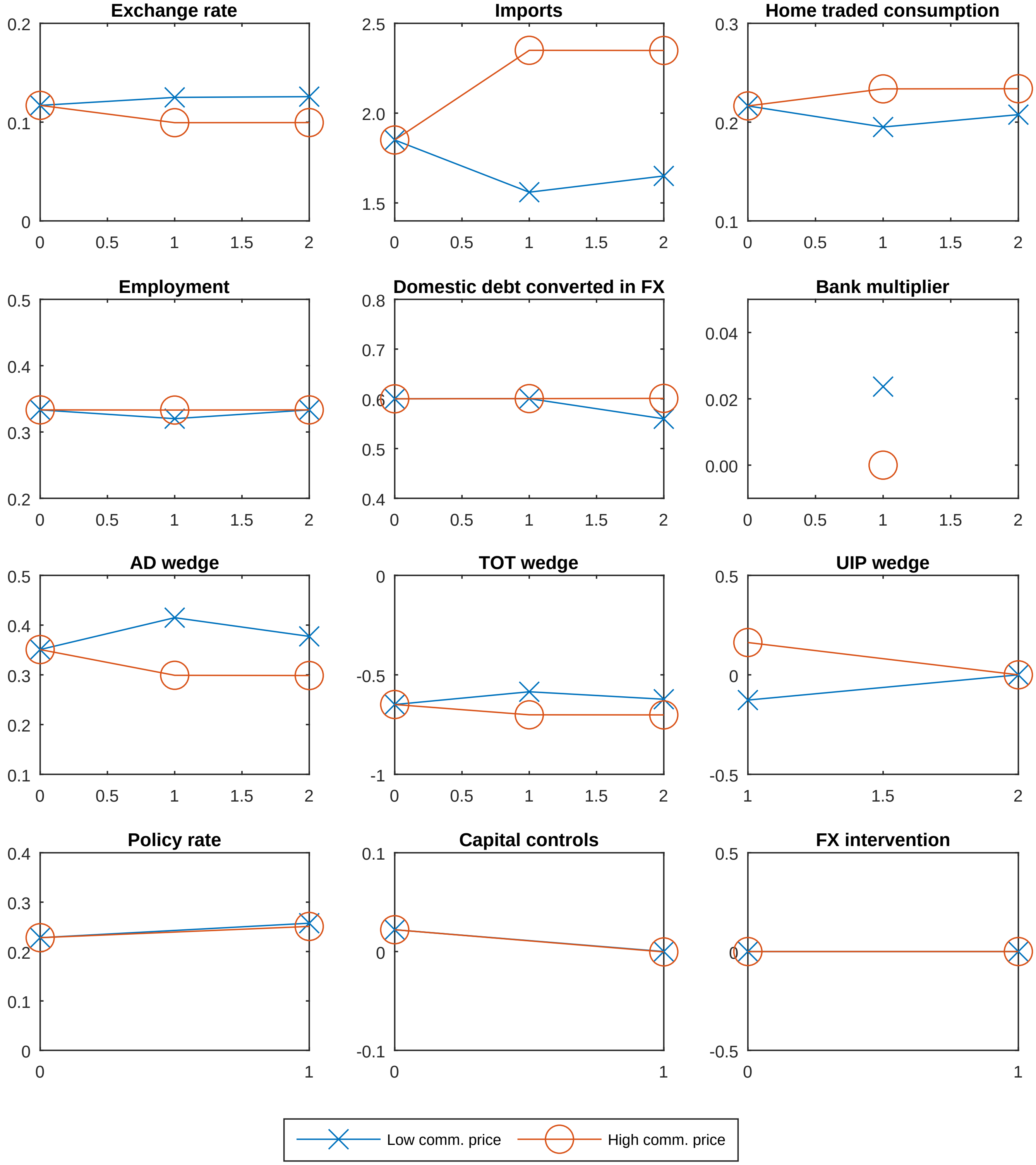
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 18: Commodity Price Shock under DCP with Shallow FX Markets



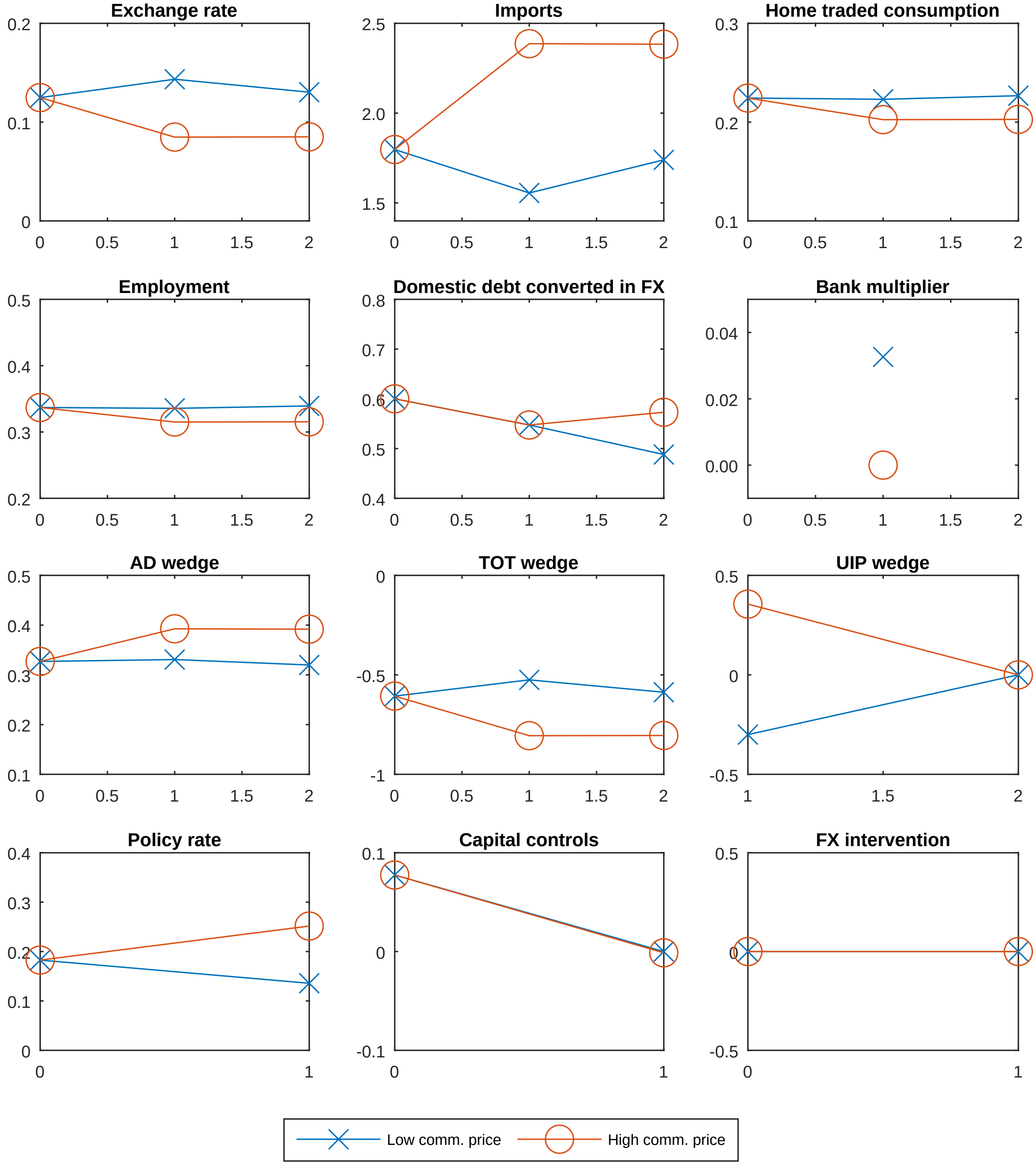
Notes: This figure plots the responses of key variables to a permanent commodity price shock under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 19: Commodity Price Shock under PCP with Shallow FX Markets and High External Debt



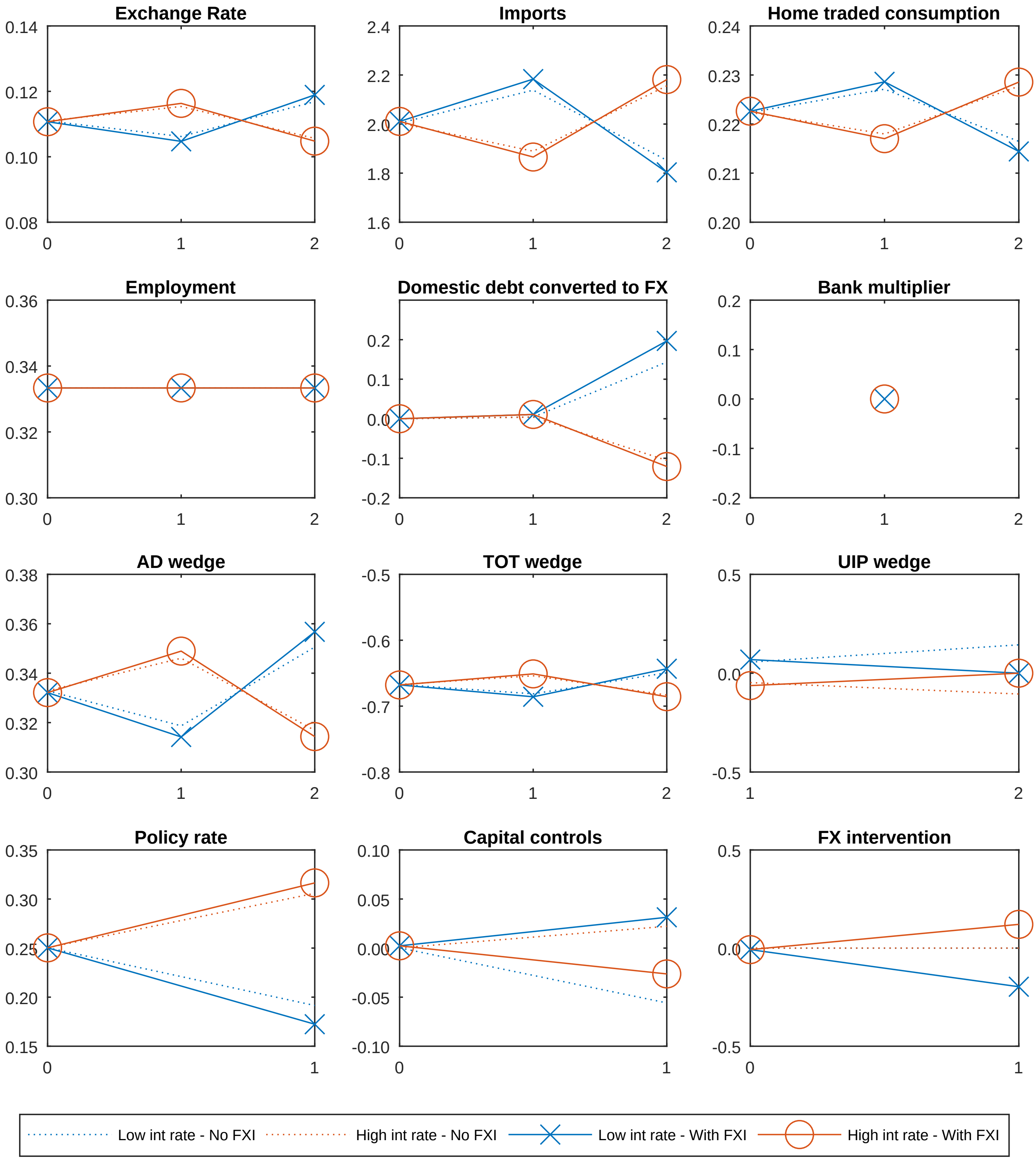
Notes: This figure plots the responses of key variables to a permanent commodity price shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. Initial external debt is set as $B_0 = 0.6$

Figure 20: Commodity Price Shock under DCP with Shallow FX Markets and High External Debt



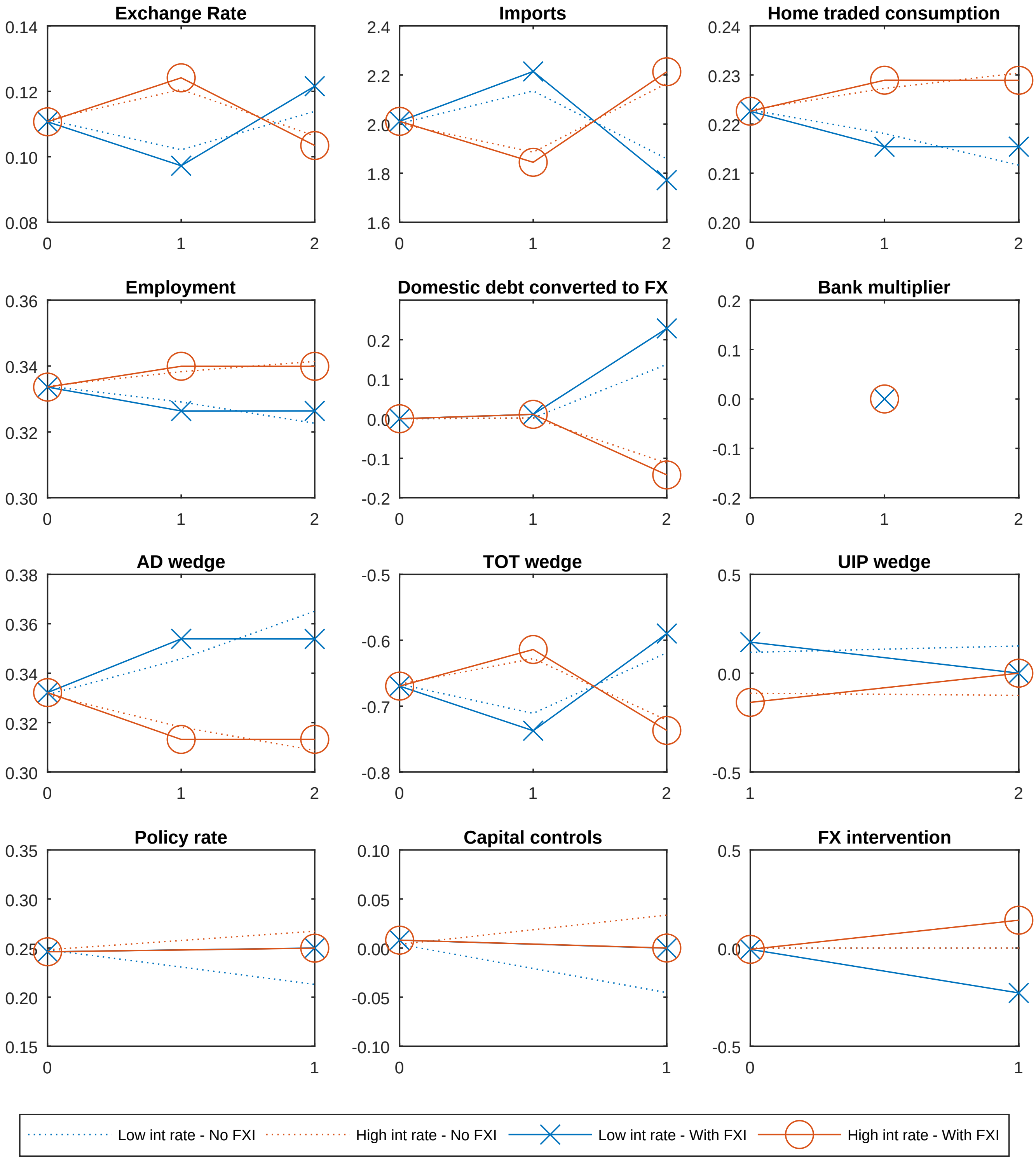
Notes: This figure plots the responses of key variables to a permanent commodity price shock under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$. Initial external debt is set as $B_0 = 0.6$

Figure 21: Interest Rate Shock under PCP with Shallow FX Markets



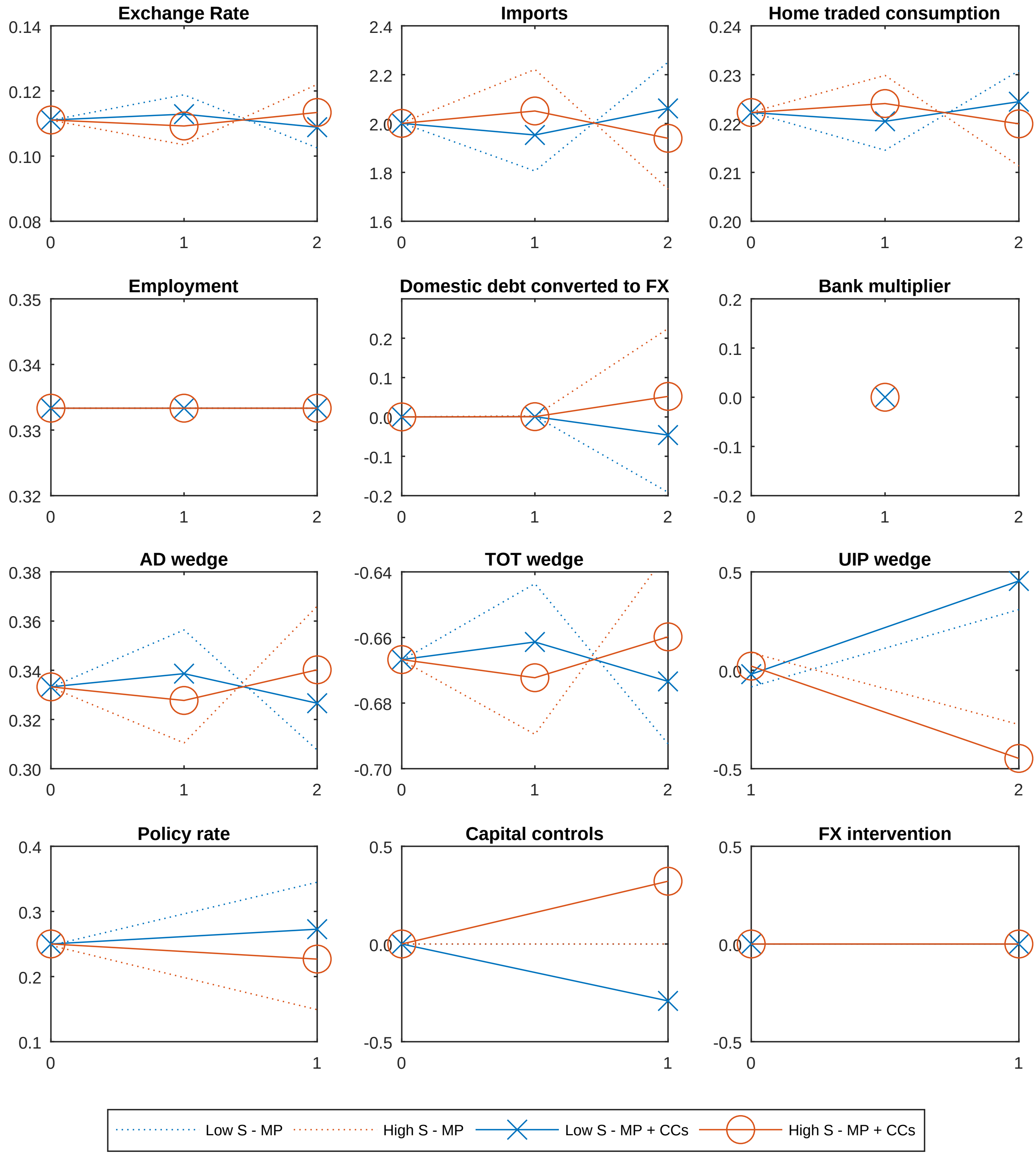
Notes: This figure plots the responses of key variables to a shock to the world interest rate under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 22: Interest Rate Shock under DCP with Shallow FX Markets



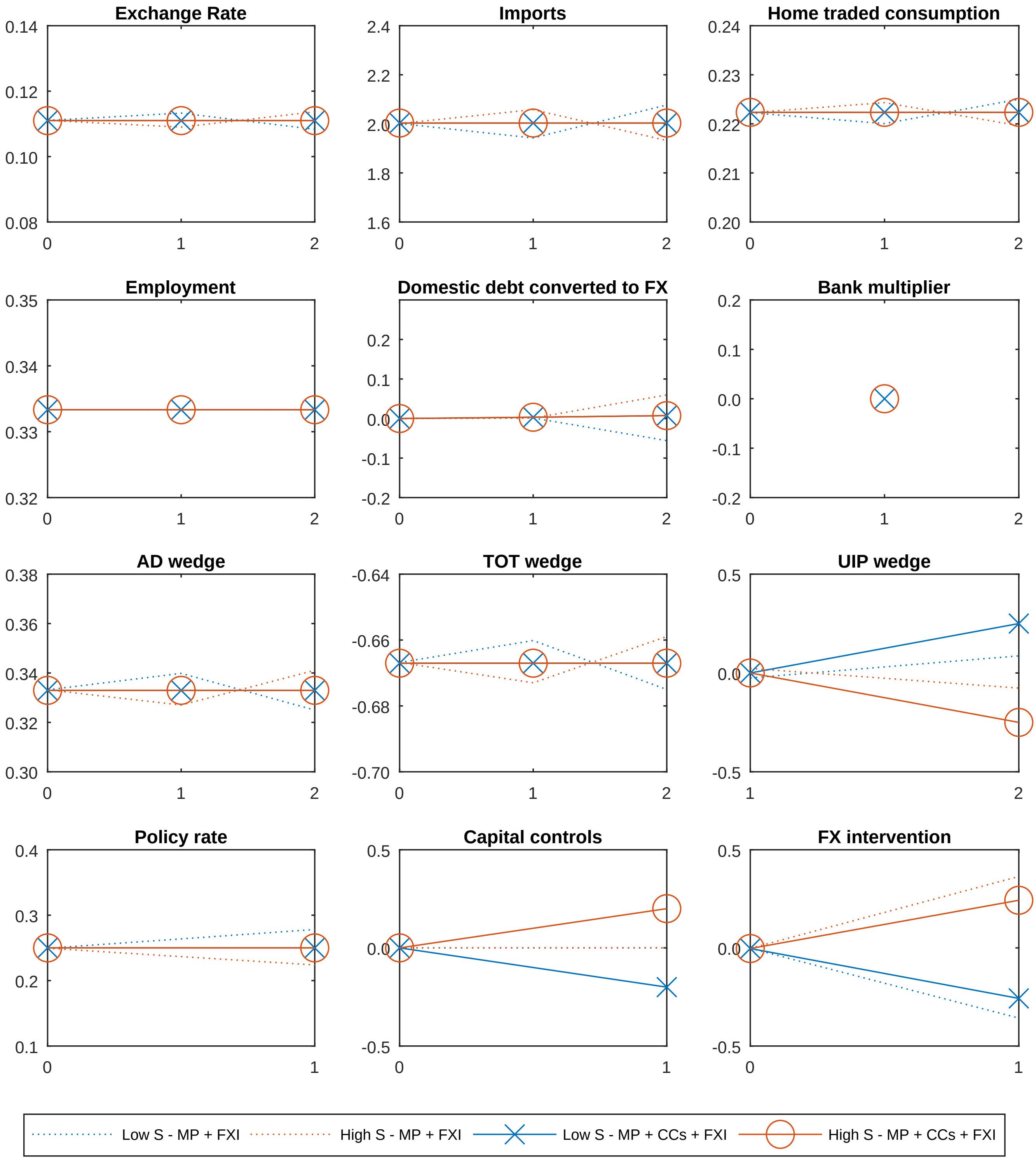
Notes: This figure plots the responses of key variables to a shock to the world interest rate under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 23: Foreign Appetite Shock under PCP with MP and CC



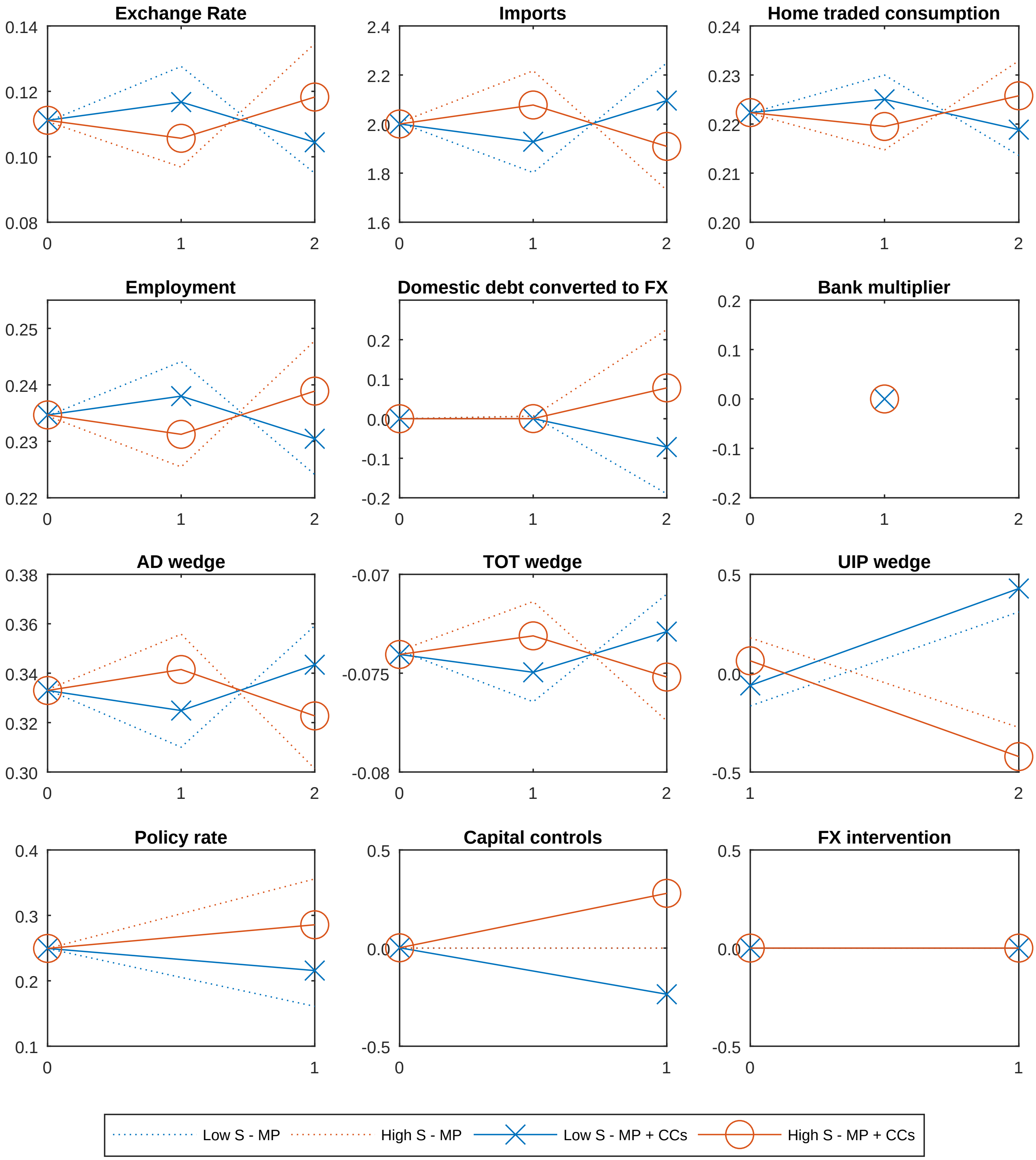
Notes: This figure plots the responses of key variables to a foreign appetite shock under PCP with shallow FX markets, monetary policy (MP) and capital controls (CC). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 24: Foreign Appetite Shock under PCP with MP, CC, and FXI



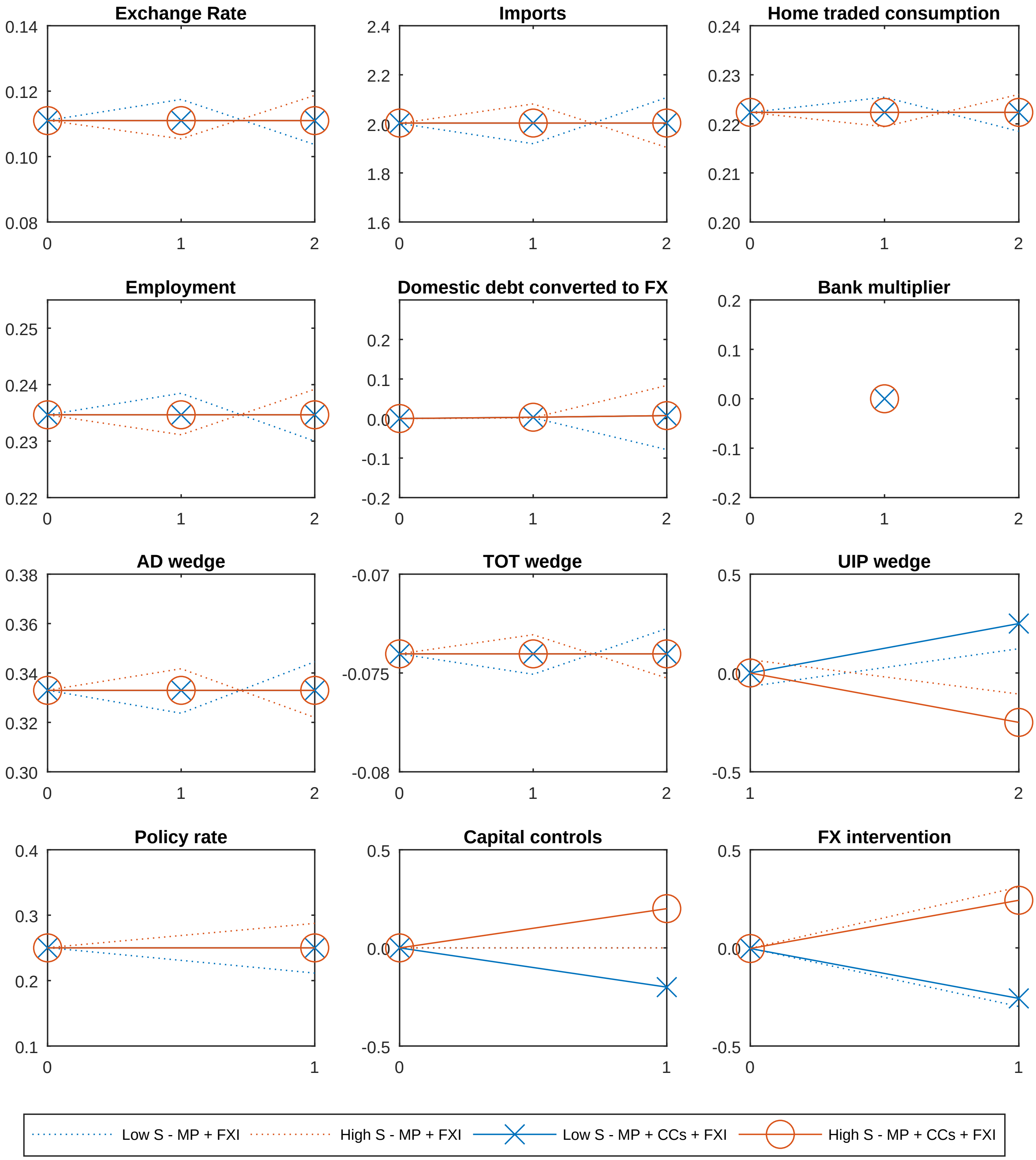
Notes: This figure plots the responses of key variables to a foreign appetite shock under PCP with shallow FX markets, monetary policy (MP), capital controls (CC), and FX intervention (FXI). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 25: Foreign Appetite Shock under DCP with MP and CC



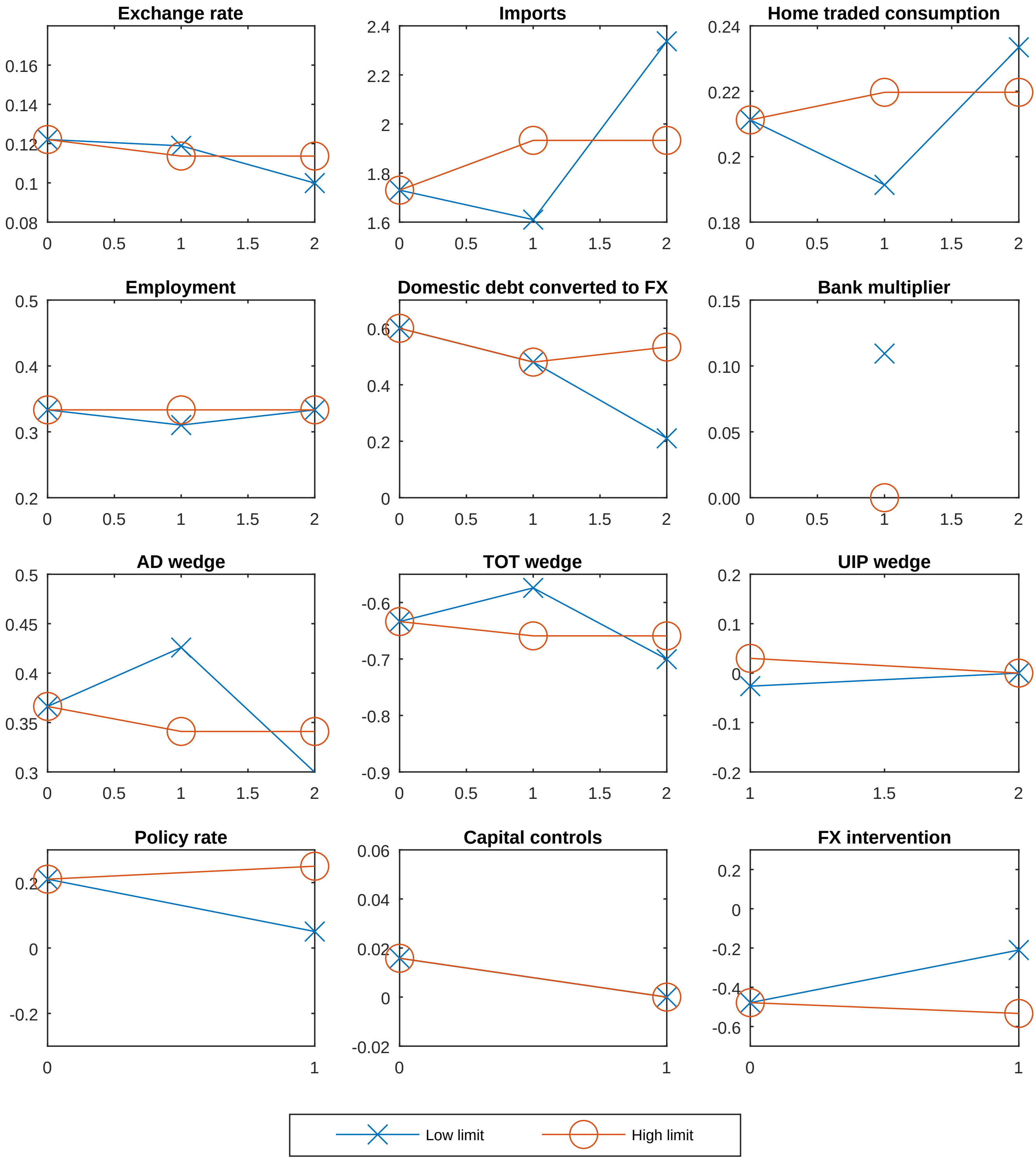
Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets, monetary policy (MP) and capital controls (CC). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 26: Foreign Appetite Shock under DCP with MP, CC, and FXI



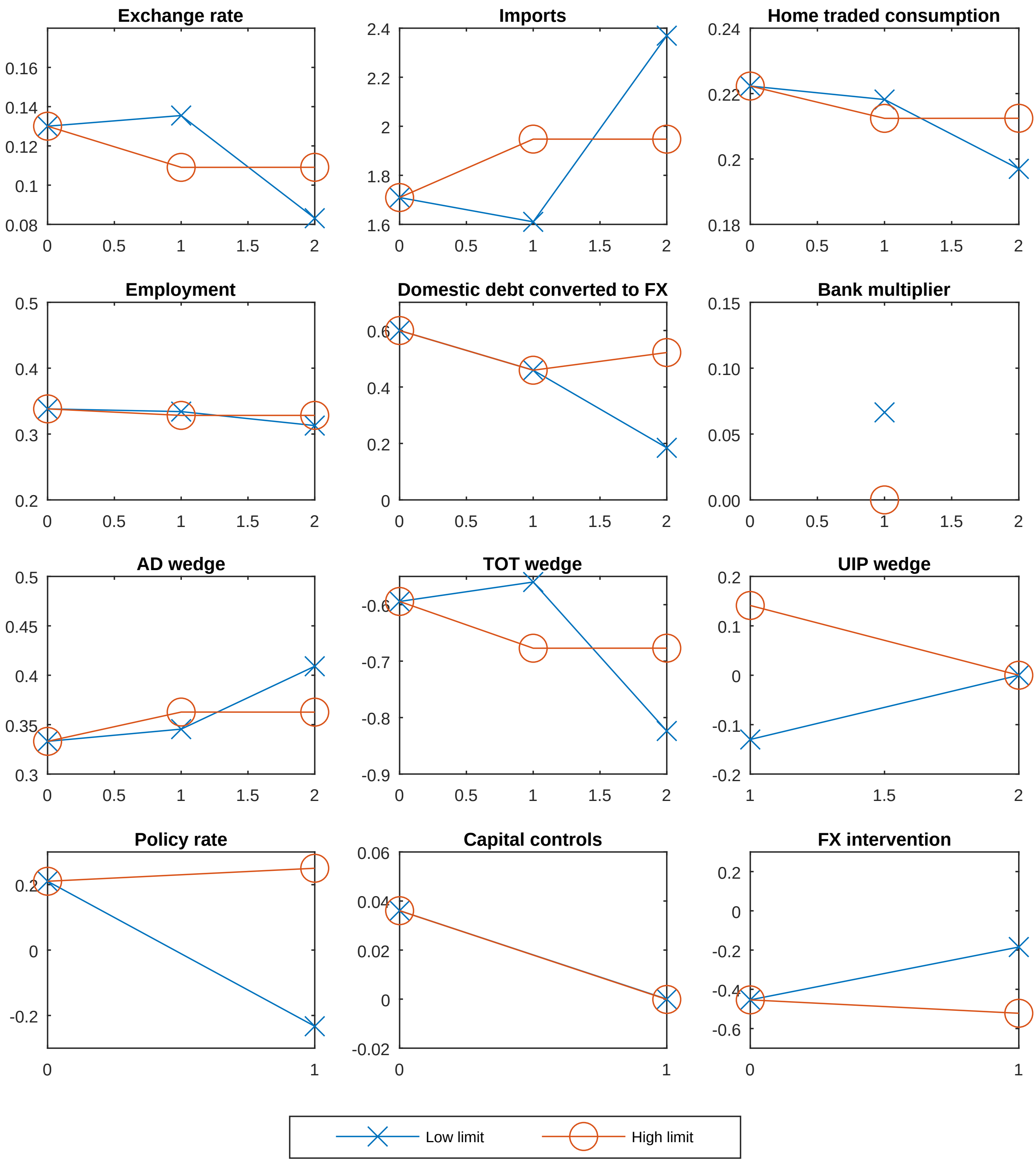
Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets, monetary policy (MP), capital controls (CC), and FX intervention (FXI). The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 27: Debt Limit Shock under PCP with Shallow FX Markets



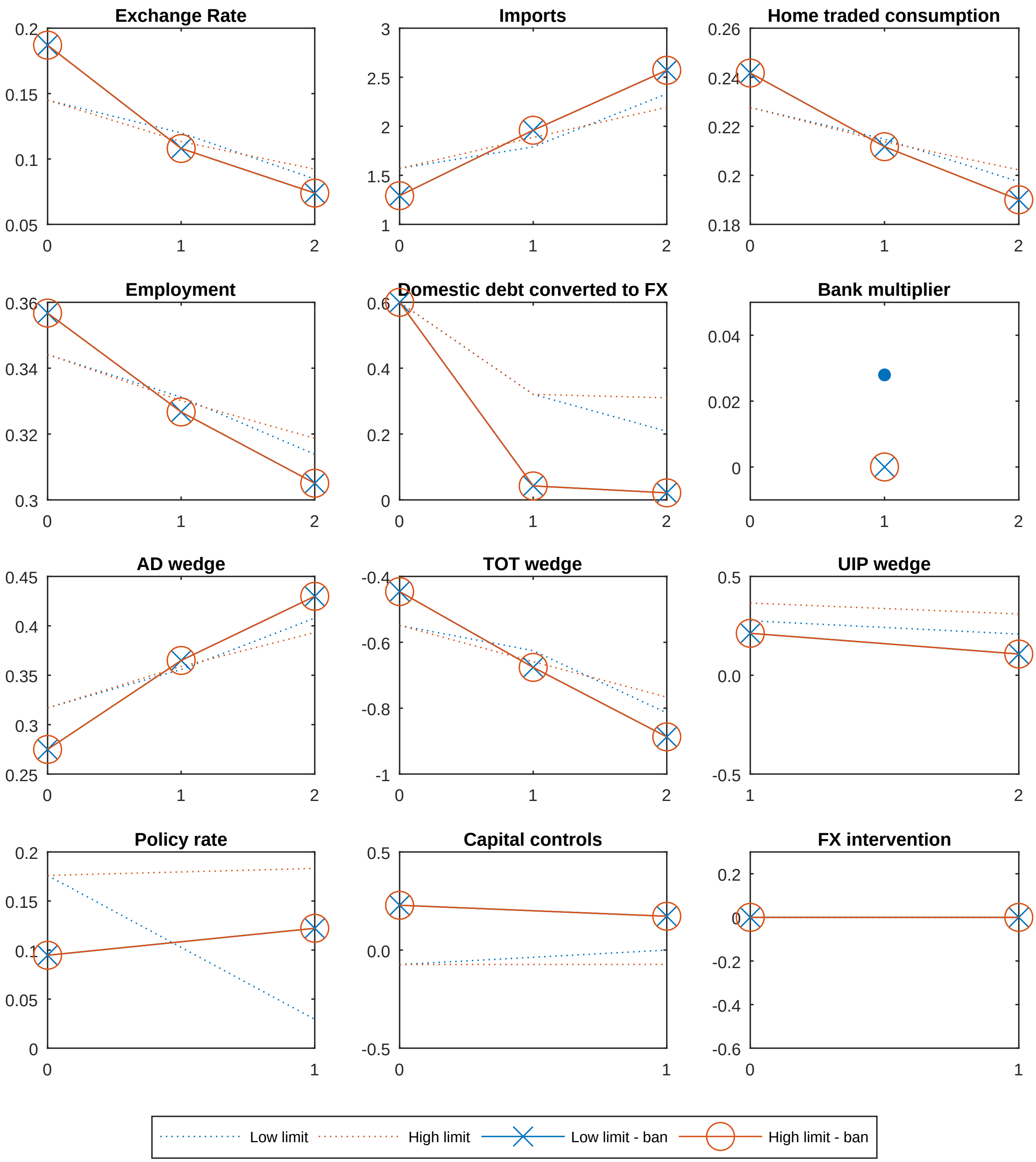
Notes: This figure plots the responses of key variables to a debt limit shock under PCP with shallow FX markets. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 28: Debt Limit Shock under DCP with Shallow FX Markets



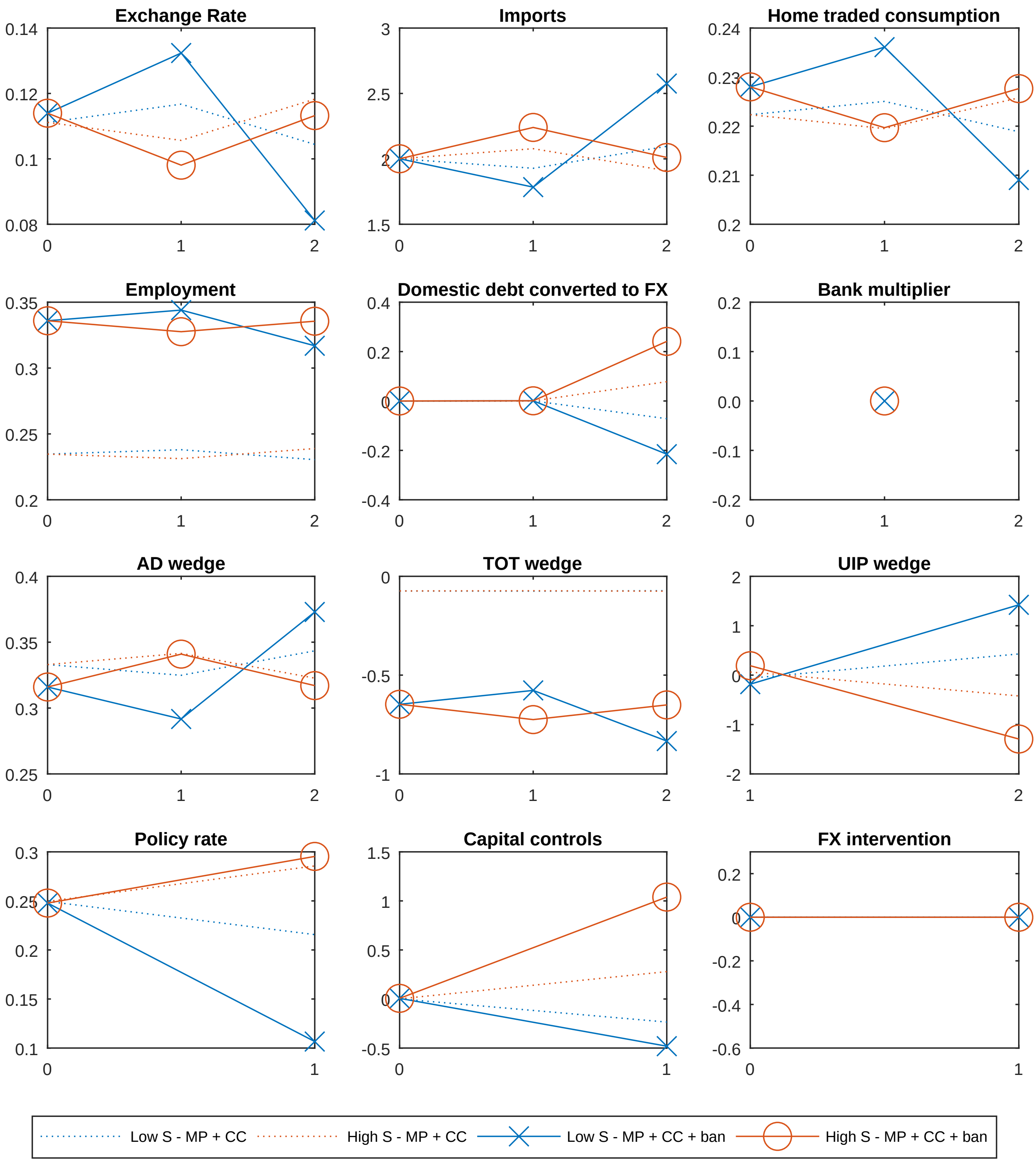
Notes: This figure plots the responses of key variables to a debt limit shock under DCP with shallow FX markets. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 29: Debt Limit Shock under DCP and Banning FX Exposures



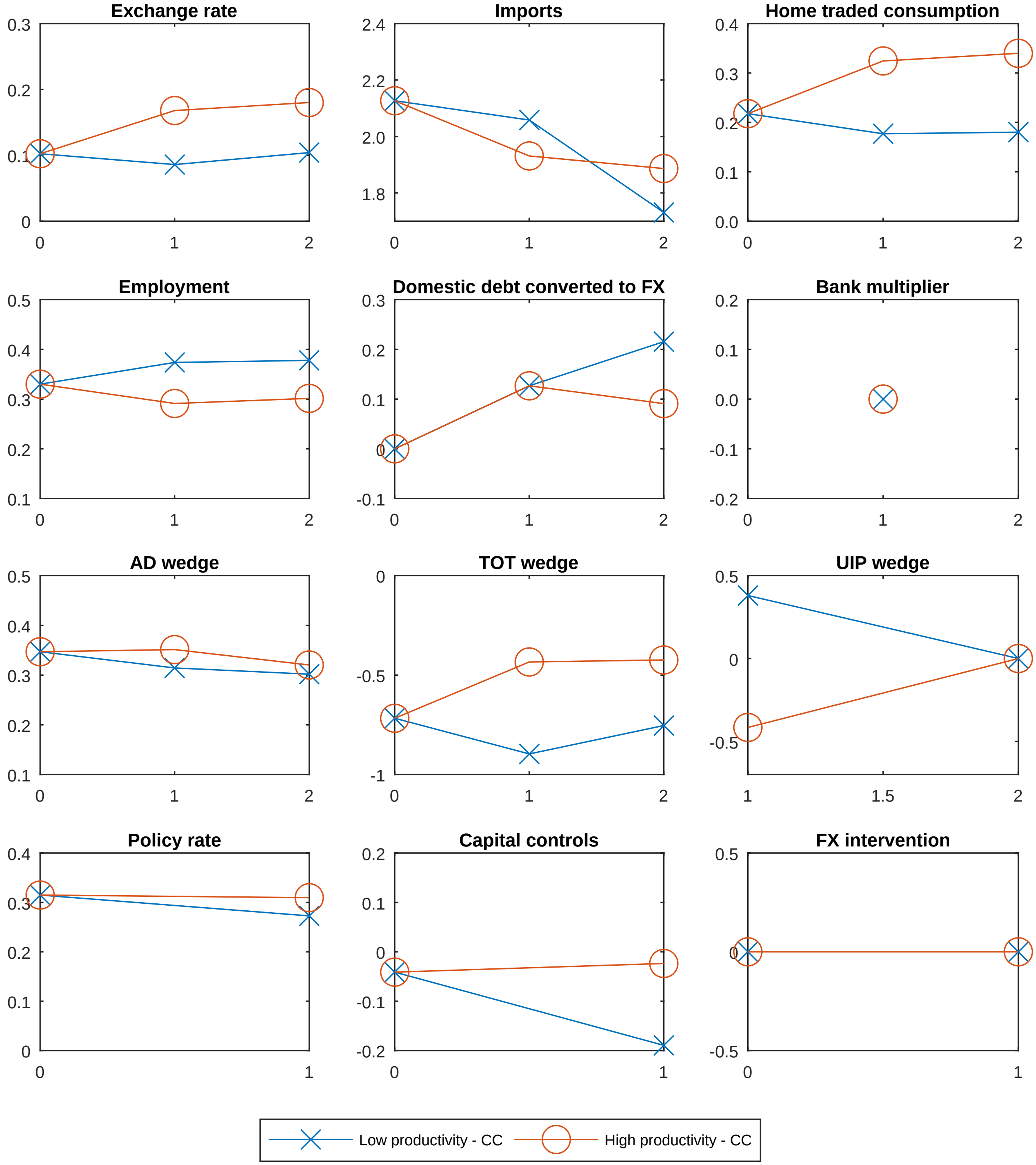
Notes: This figure plots the responses of key variables to a debt limit shock under DCP with shallow FX markets, banning of open FX positions. The shock hits at date-1 and is calibrated as $\kappa \in [0.025, 10]$ such that the constraint binds in the case of a bad realization of the shock but not after a good realization.

Figure 30: Foreign Appetite Shock under DCP with MP, CC, and Banning FX Exposures



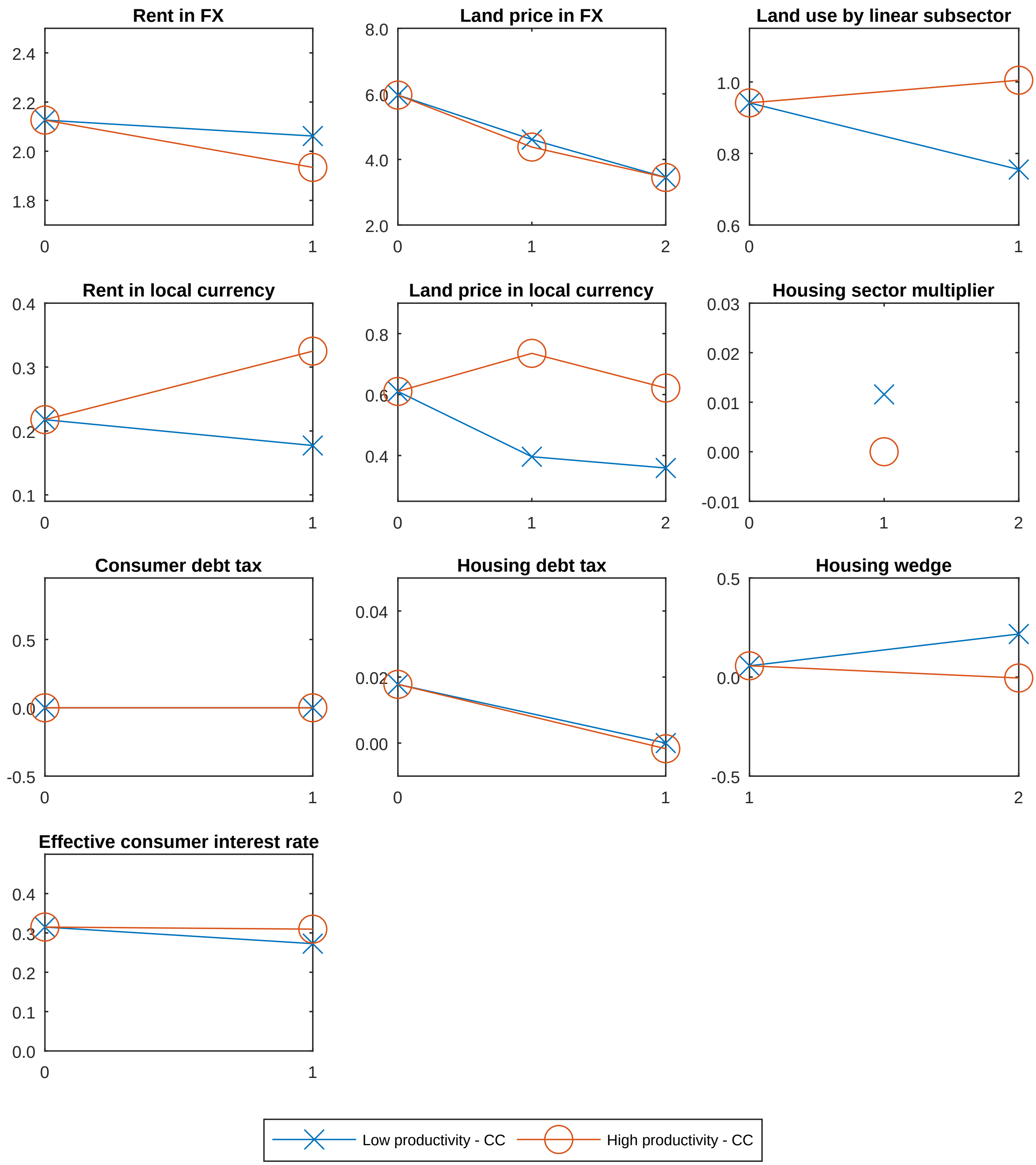
Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets, monetary policy (MP), capital controls (CC) and banning of open FX positions. The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 31: Productivity Shock under DCP with Housing Sector Frictions (With Capital Controls)



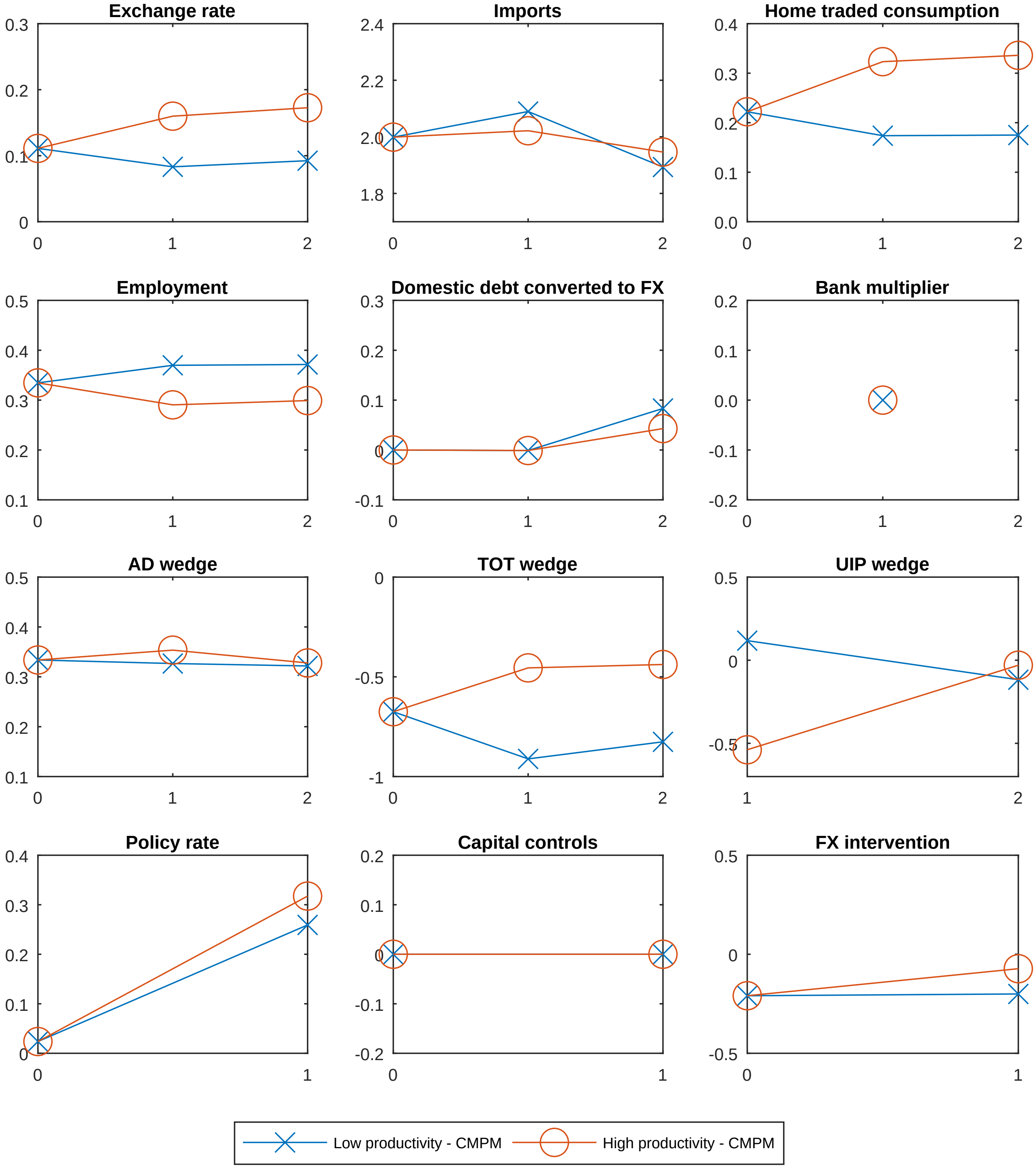
Notes: This figure plots the responses of key variables to a productivity shock under DCP with shallow FX markets and housing debt taxes. The shock hits at date-1 and is calibrated as $A_1 \in [0.8, 1.5]$.

Figure 31: Productivity Shock under DCP with Housing Sector Frictions (With Capital Controls) cont.



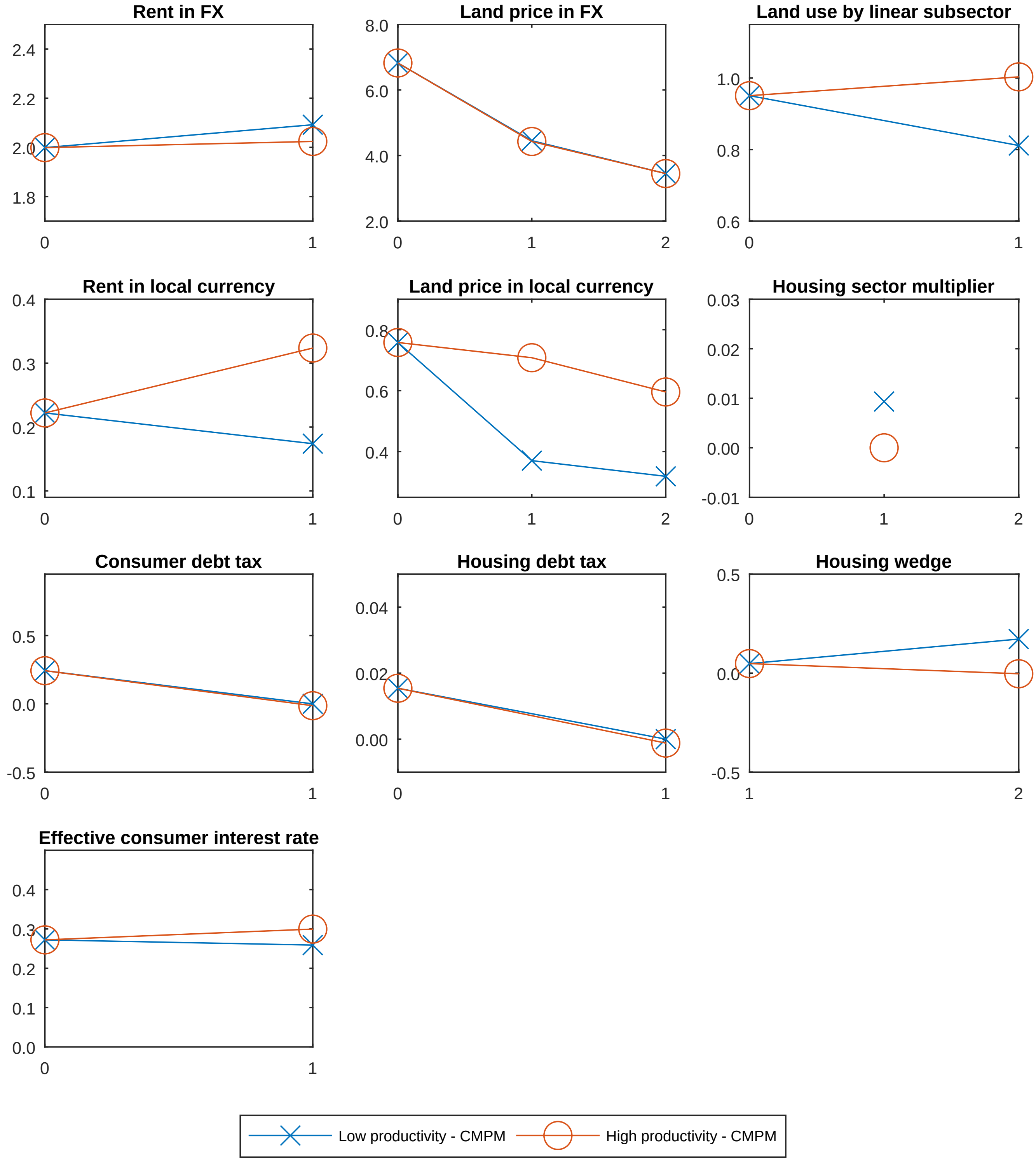
Notes: This figure plots the responses of key variables to a productivity shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $A_1 \in [0.8, 1.5]$.

Figure 32: Productivity Shock under DCP with Housing Sector Frictions (With Consumer Debt Taxes)



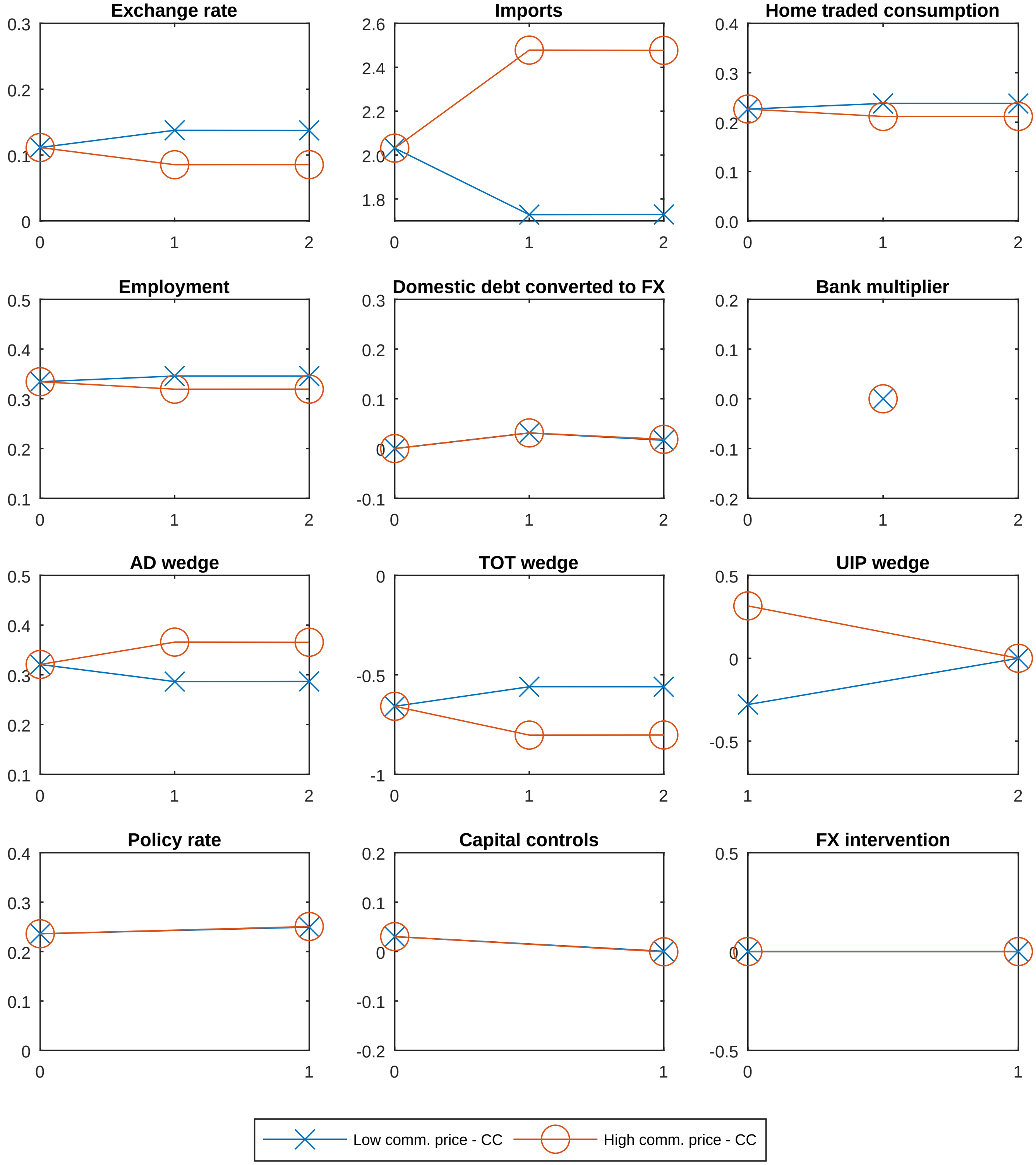
Notes: This figure plots the responses of key variables to a productivity shock under DCP with shallow FX markets and housing sector frictions. Capital controls have been turned off. The shock hits at date-1 and is calibrated as $A_1 \in [0.75, 1.5]$.

Figure 32: Productivity Shock under DCP with Housing Sector Frictions (With Consumer Debt Taxes) cont.



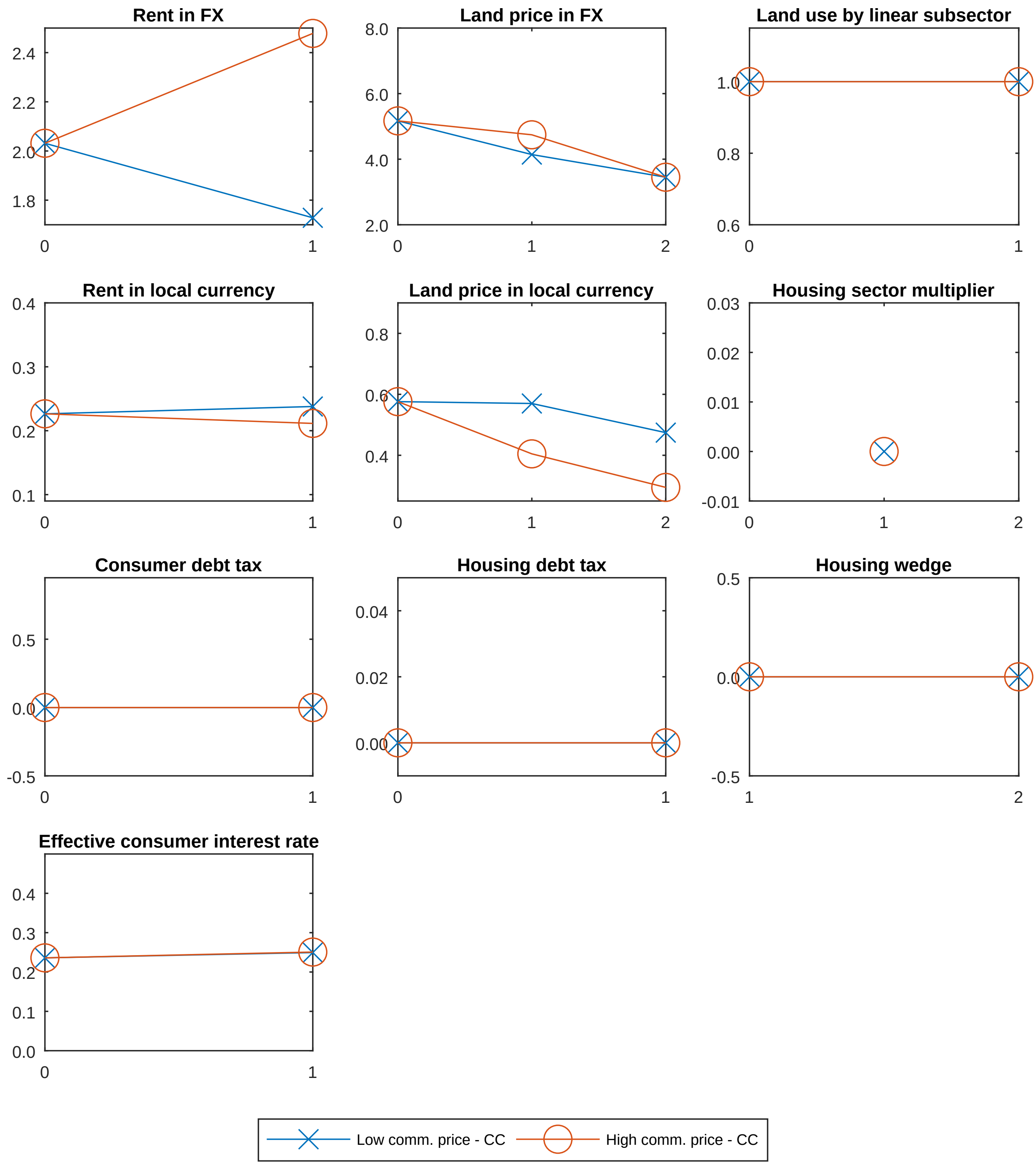
Notes: This figure plots the responses of key variables to a productivity shock under DCP with shallow FX markets and housing sector frictions. Capital controls have been turned off. The shock hits at date-1 and is calibrated as $A_1 \in [0.75, 1.5]$.

Figure 33: Commodity Price Shock under DCP with Housing Sector Frictions (With Capital Controls)



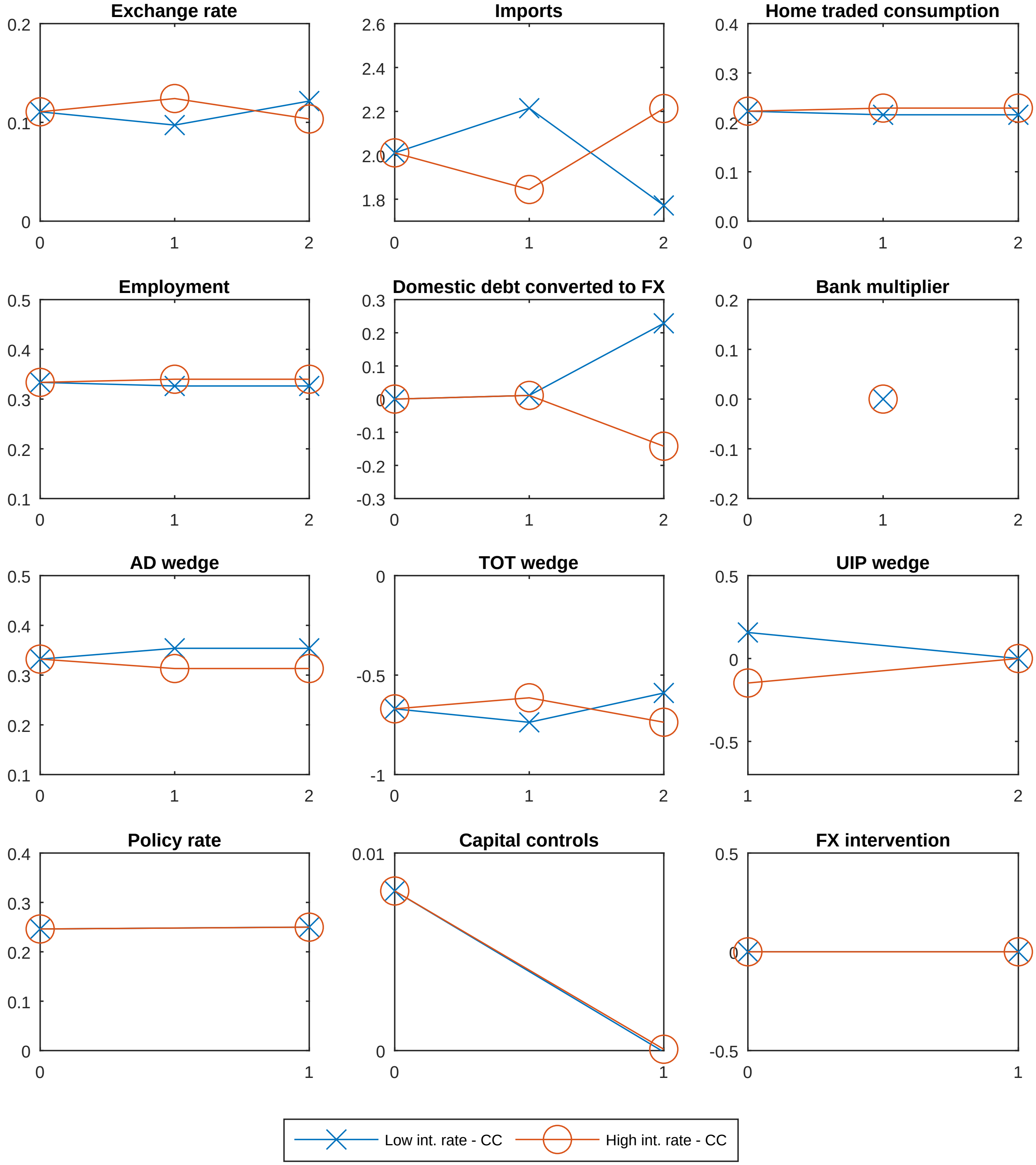
Notes: This figure plots the responses of key variables to a commodity price shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 33: Commodity Price Shock under DCP with Housing Sector Frictions (With Capital Controls) cont.



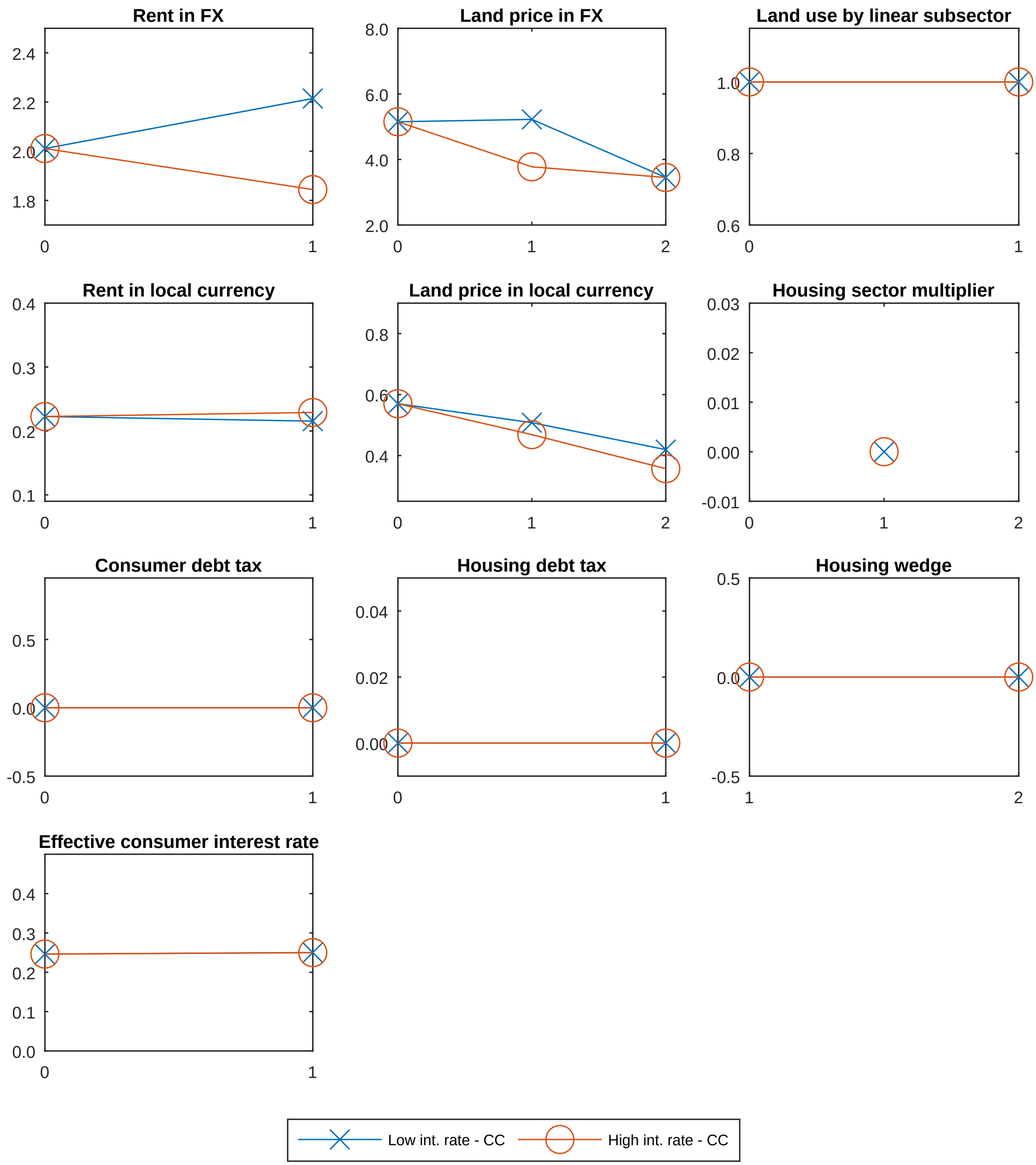
Notes: This figure plots the responses of key variables to a commodity price shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $P^*_{Z1} \in [0.75, 1.5]$.

Figure 34: Interest Rate Shock under DCP with Housing Sector Frictions (With Capital Controls)



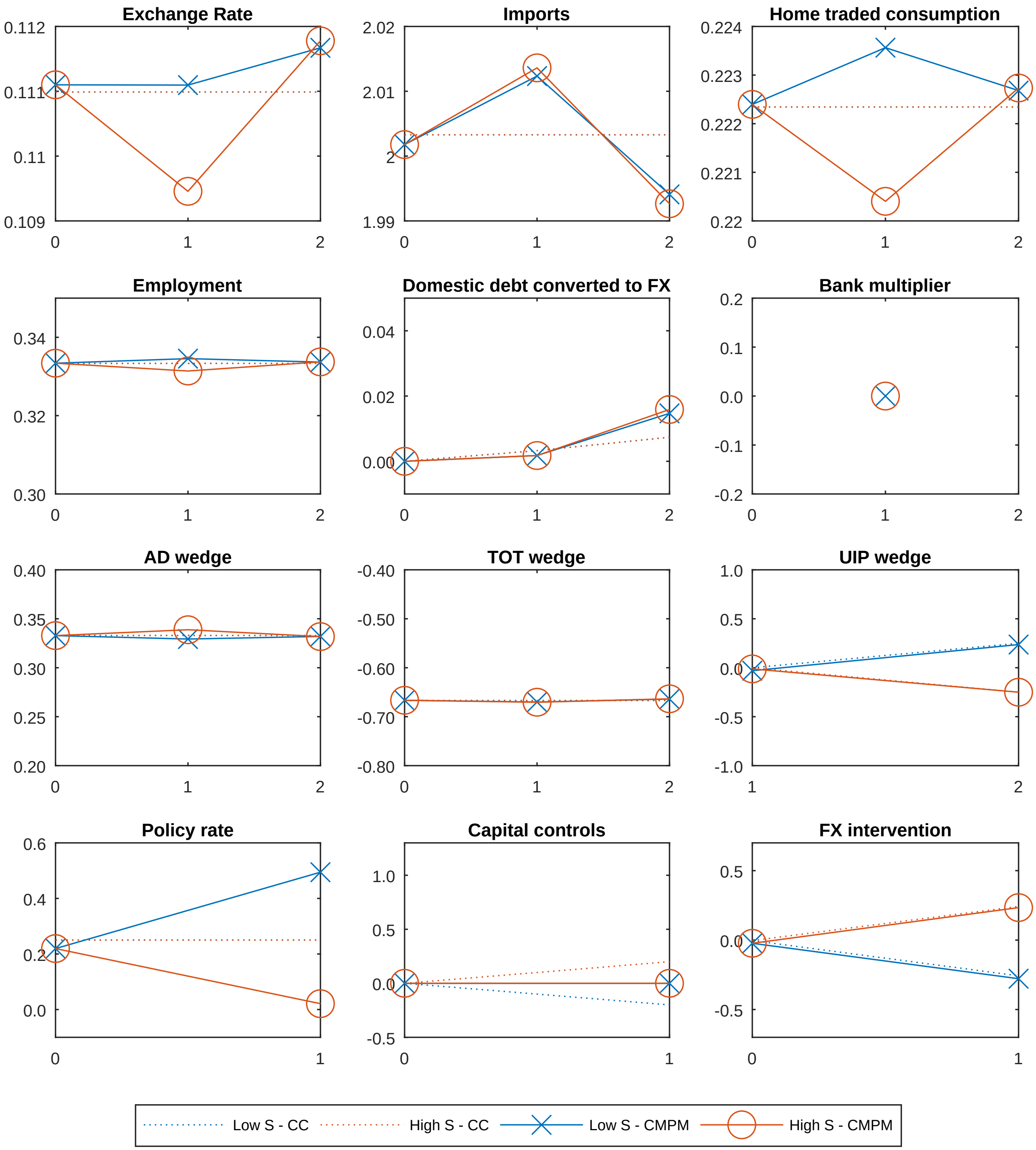
Notes: This figure plots the responses of key variables to an interest rate shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $(1 + i^*_1) \in [1, 1.5]$.

Figure 34: Interest Rate Shock under DCP with Housing Sector Frictions (With Capital Controls) cont.



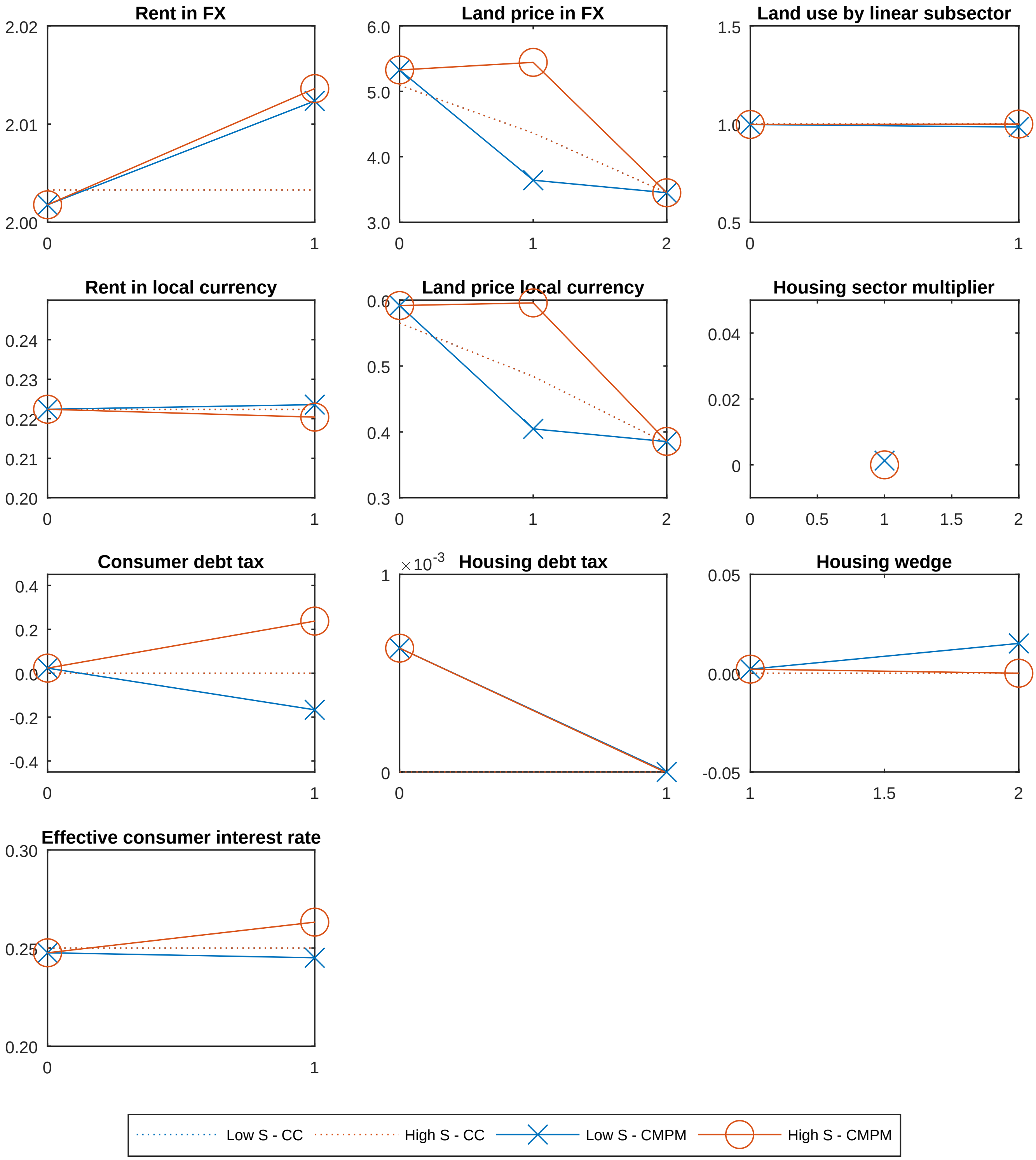
Notes: This figure plots the responses of key variables to an interest rate shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $(1 + i_1^*) \in [1, 1.5]$.

Figure 35: Foreign Appetite Shock under DCP with Housing Sector Frictions



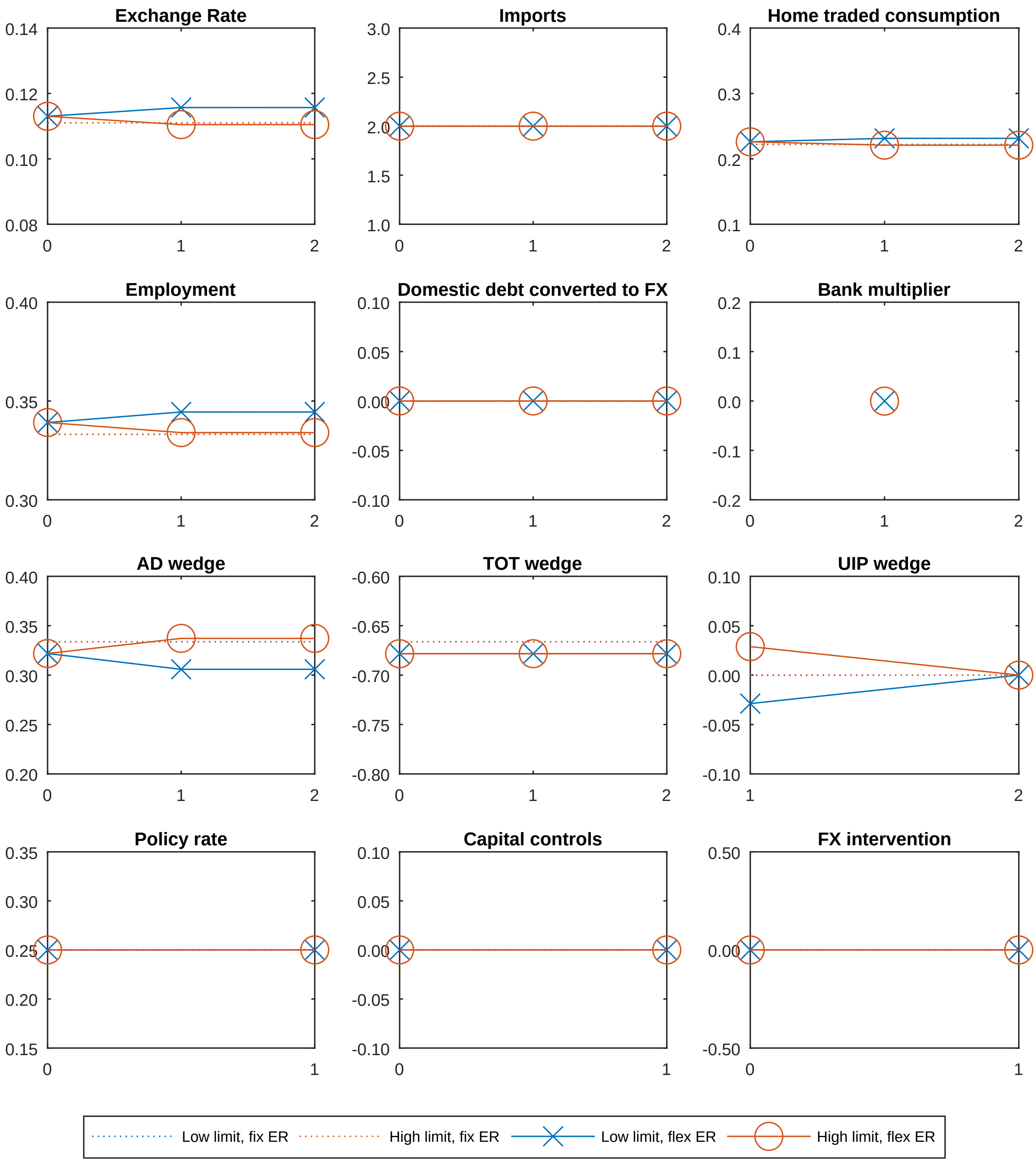
Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 35: Foreign Appetite Shock under DCP with Housing Sector Frictions cont.



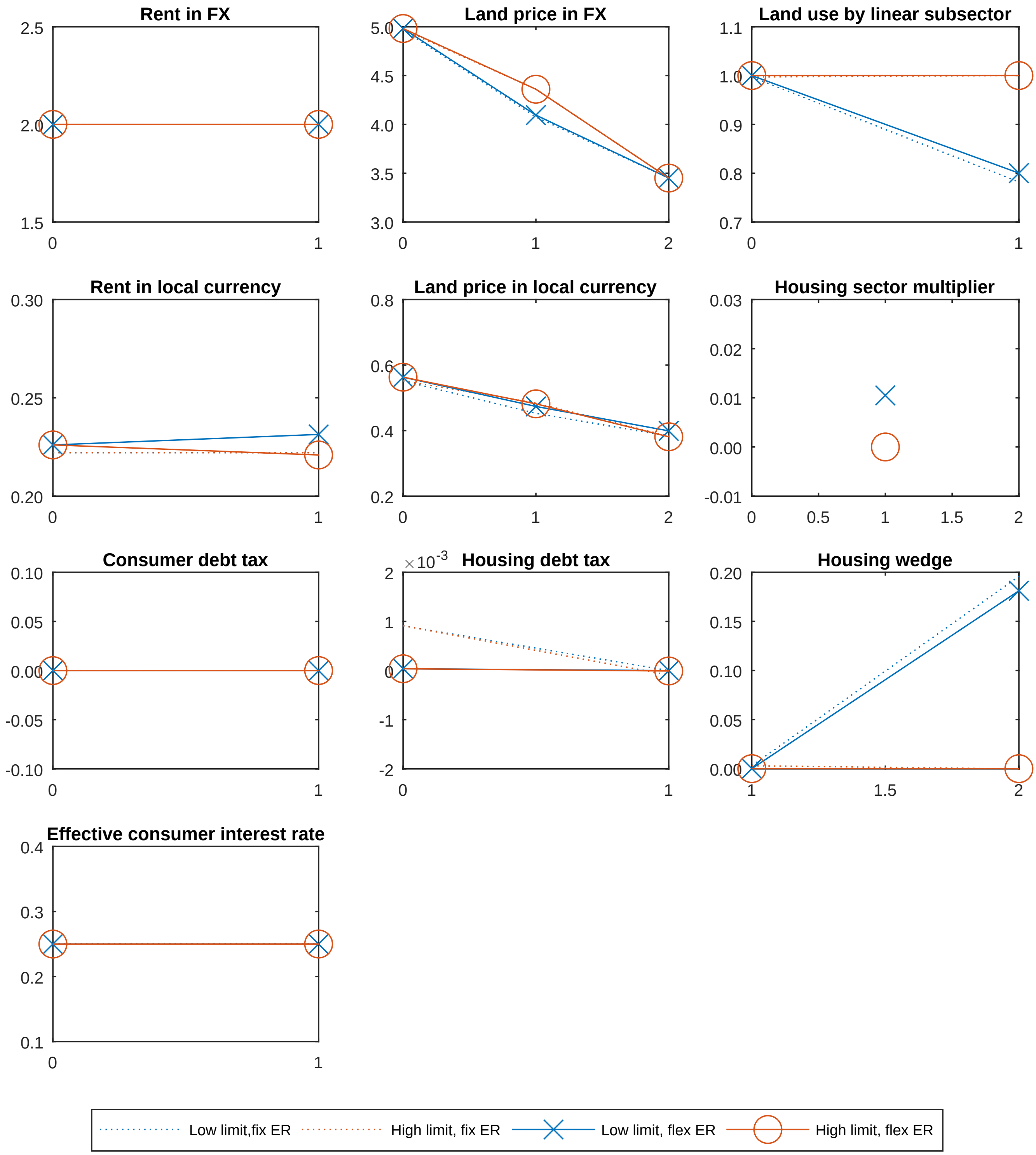
Notes: This figure plots the responses of key variables to a foreign appetite shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $S_1 \in [-0.5, 0.5]$.

Figure 36: Housing Debt Limit Shock under DCP with Housing Sector Frictions



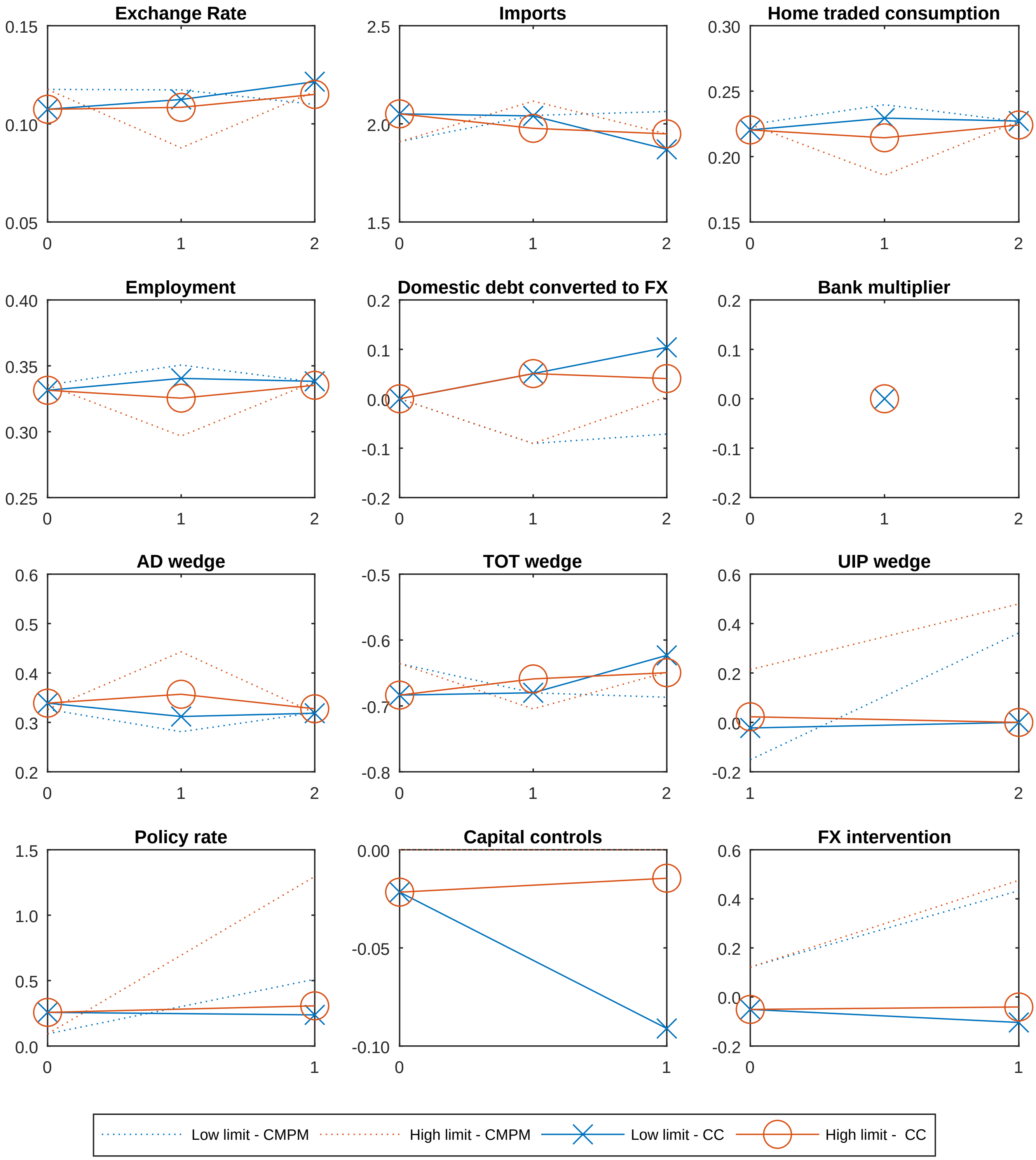
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: housing debt taxes and flexible exchange rates; dashed lines: housing debt taxes and fixed exchange rates. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 36: Housing Debt Limit Shock under DCP with Housing Sector Frictions cont.



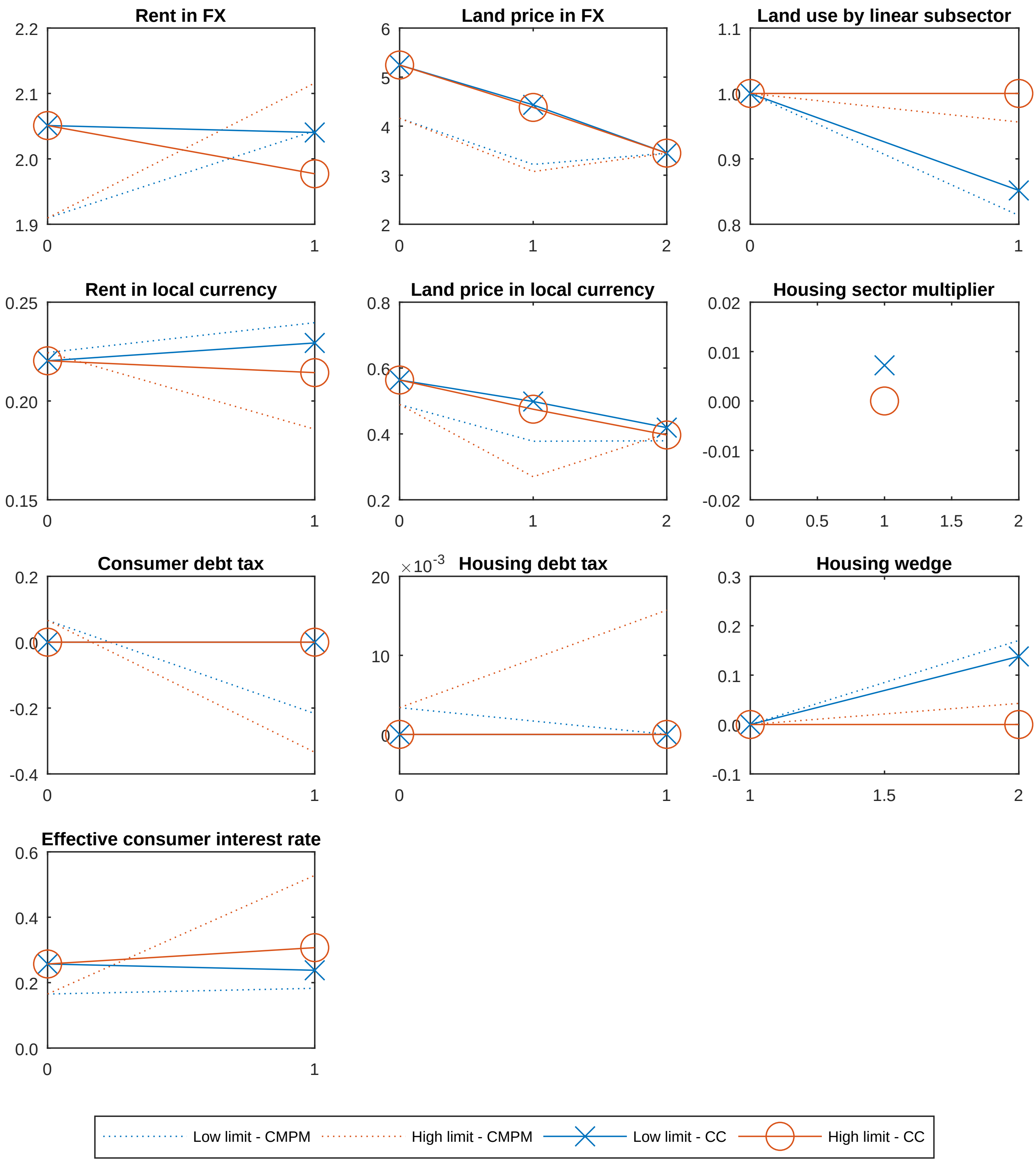
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: housing debt taxes and flexible exchange rates; dashed lines: housing debt taxes and fixed exchange rates. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 37: Housing Debt Limit Shock under DCP with Housing Sector Frictions



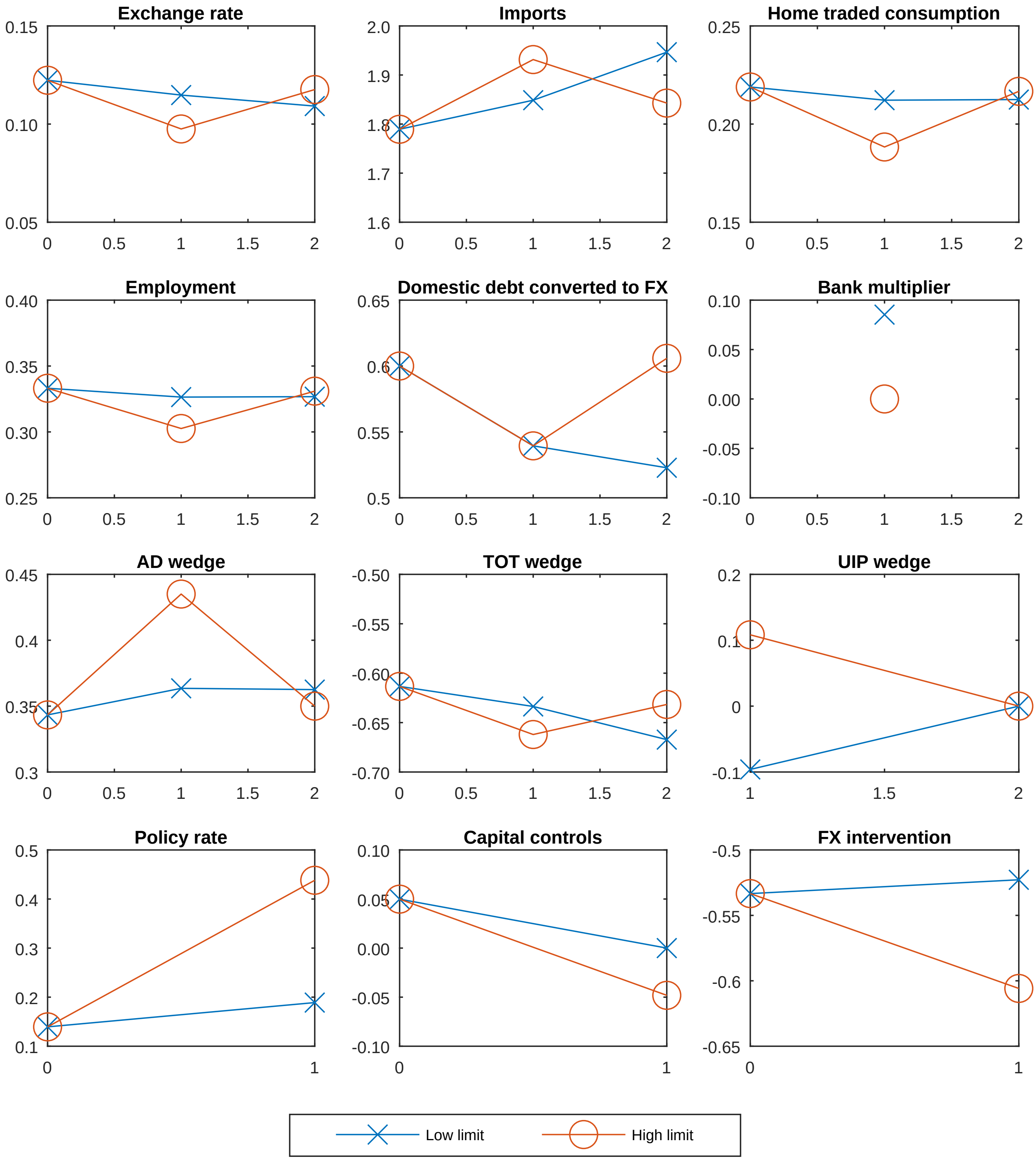
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: capital controls; dashed lines: consumer debt taxes. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 37: Housing Debt Limit Shock under DCP with Housing Sector Frictions cont.



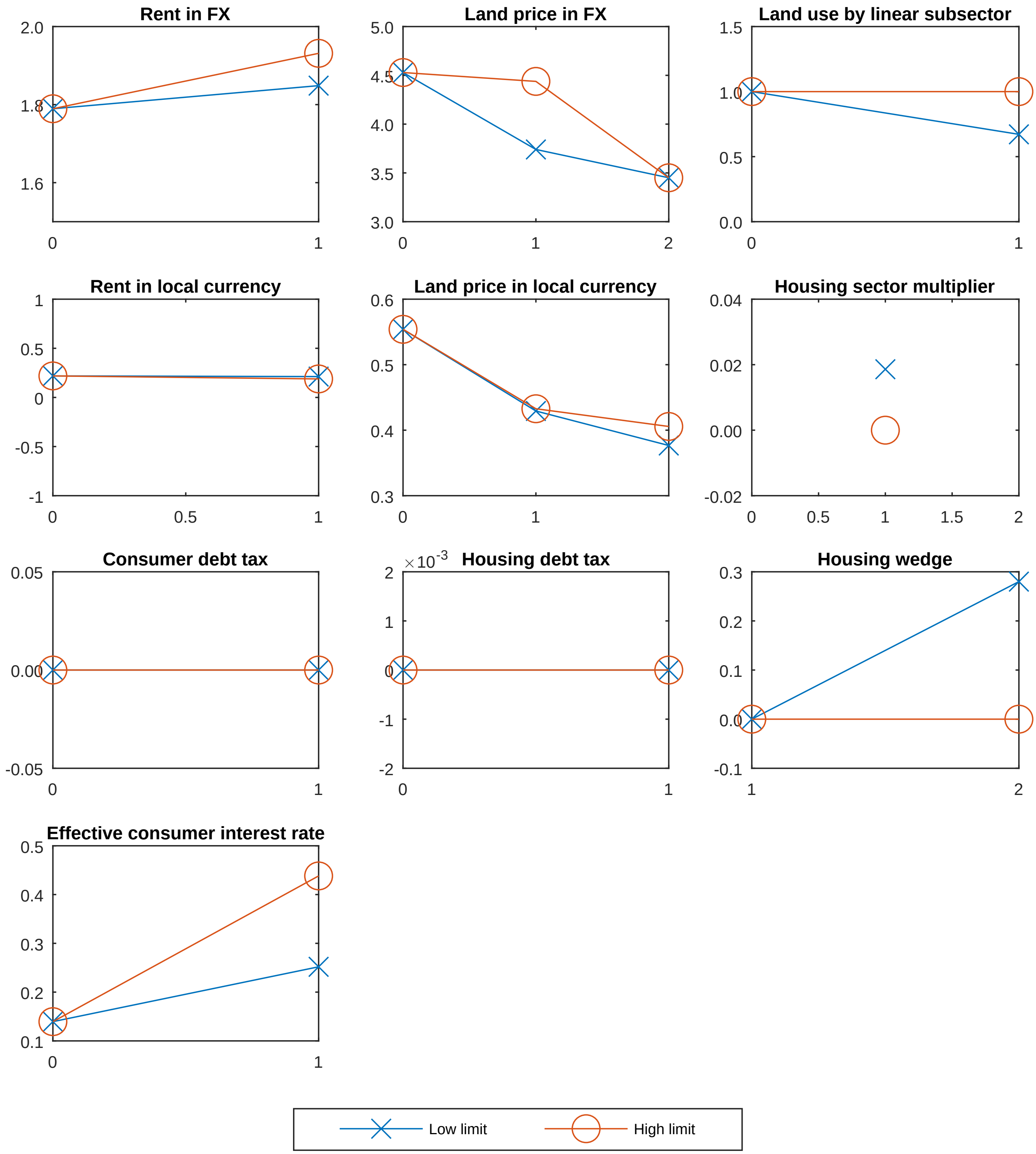
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. Solid lines: capital controls; dashed lines: consumer debt taxes. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 38: Housing Constraint Spilling Over to Banks' Constraint under DCP



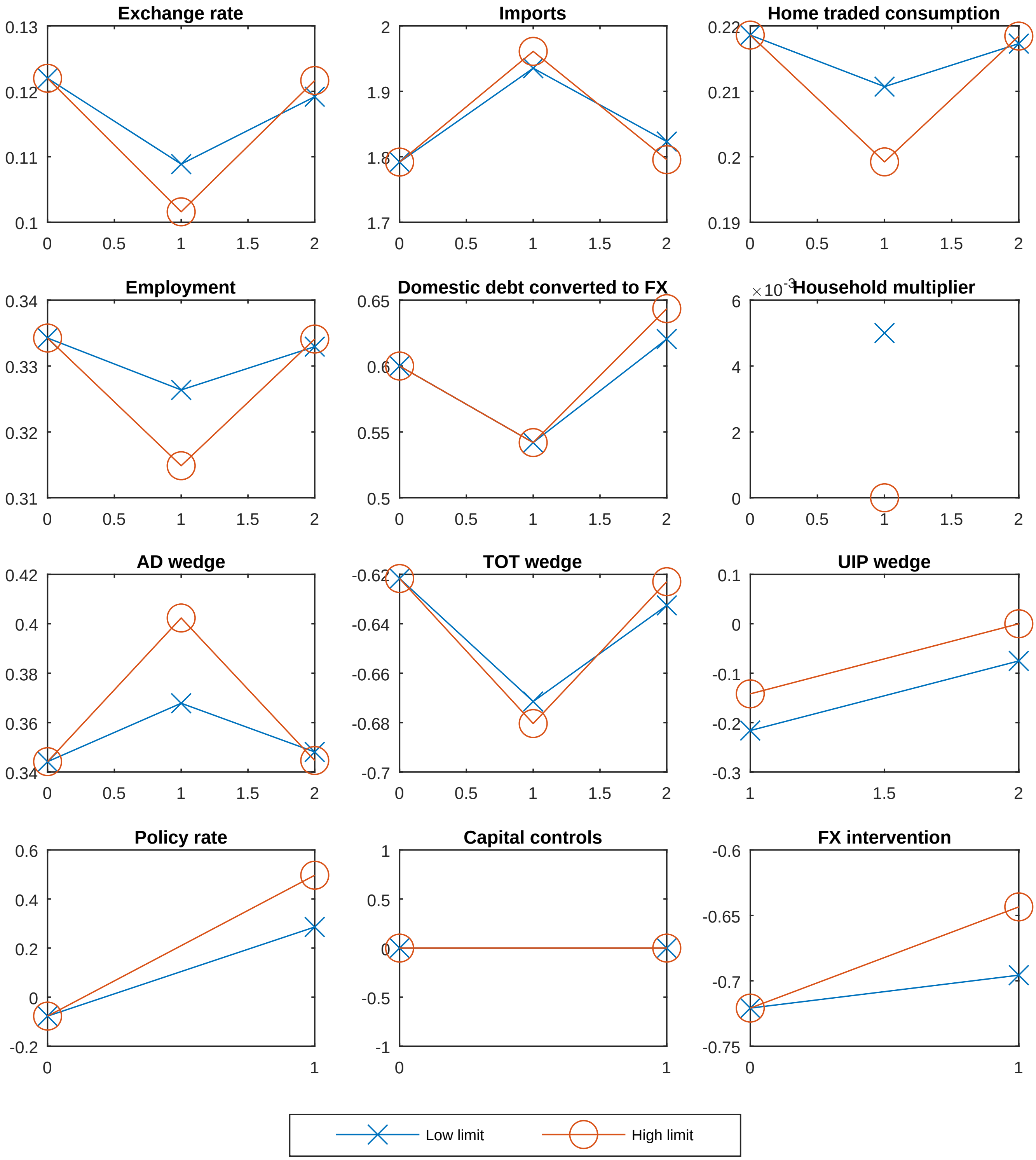
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 38: Housing Constraint Spilling Over to Banks' Constraint under DCP cont.



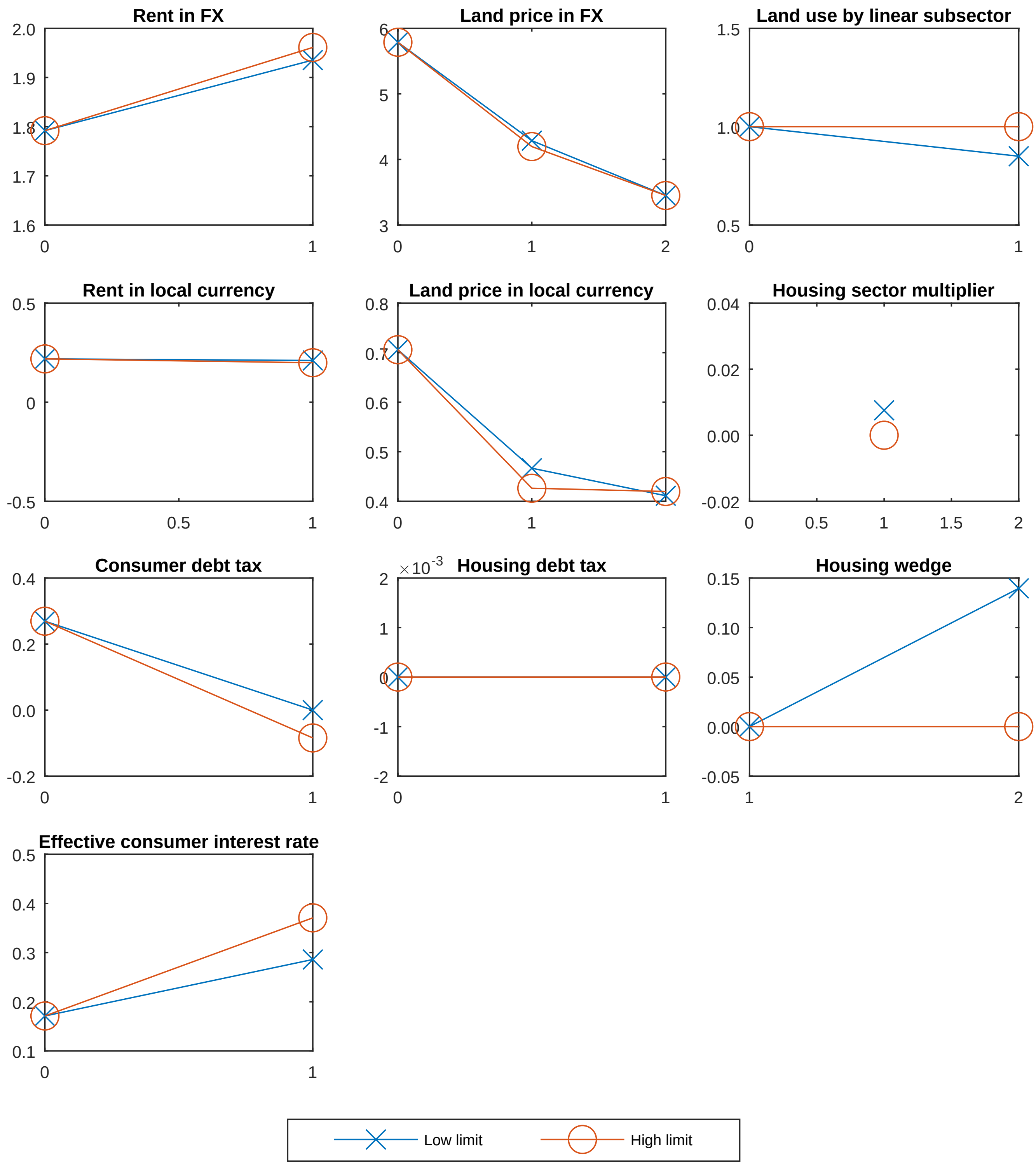
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 39: Housing Constraint Spilling Over to Households' Constraint under DCP



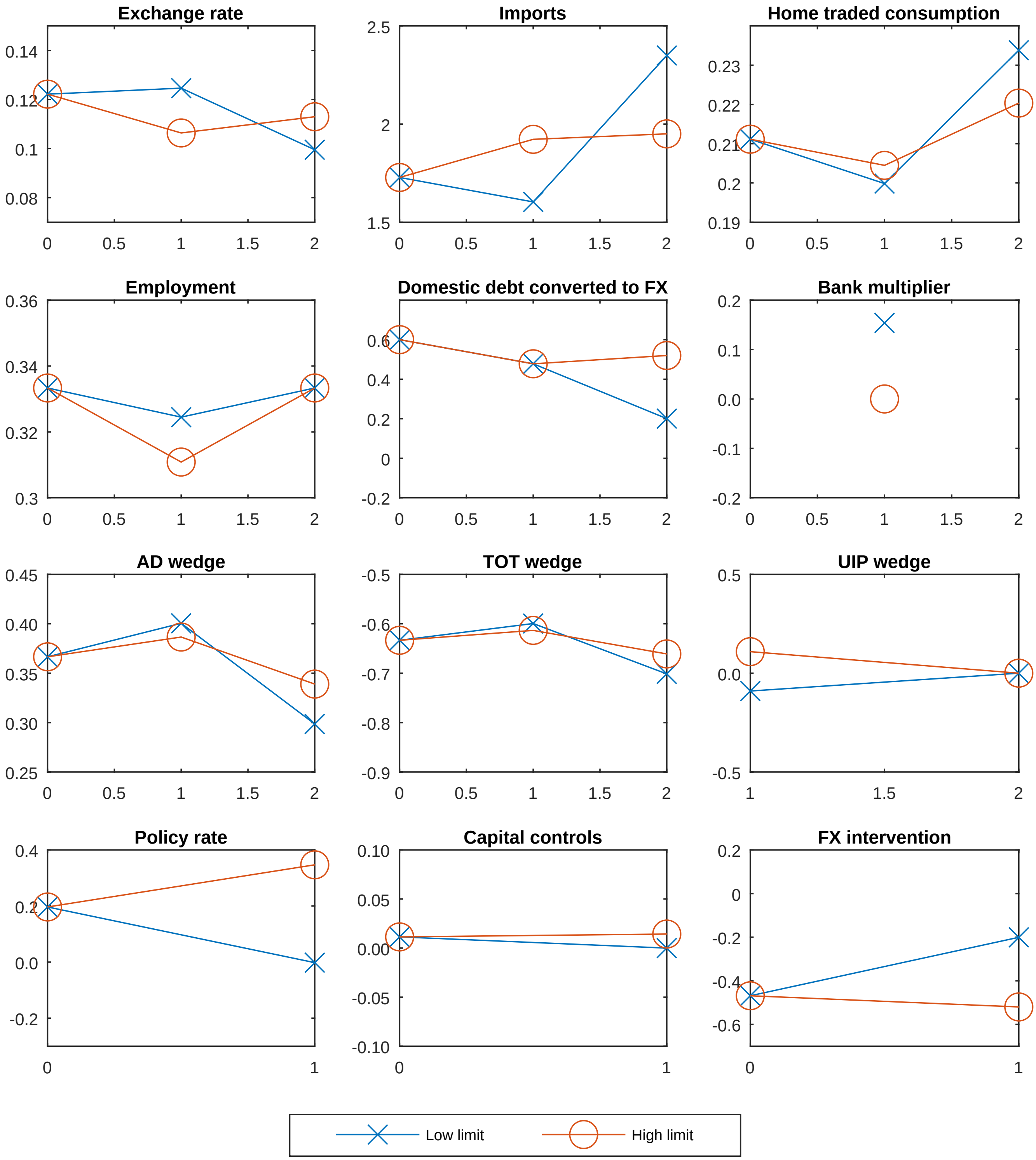
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 39: Housing Constraint Spilling Over to Households' Constraint under DCP cont.



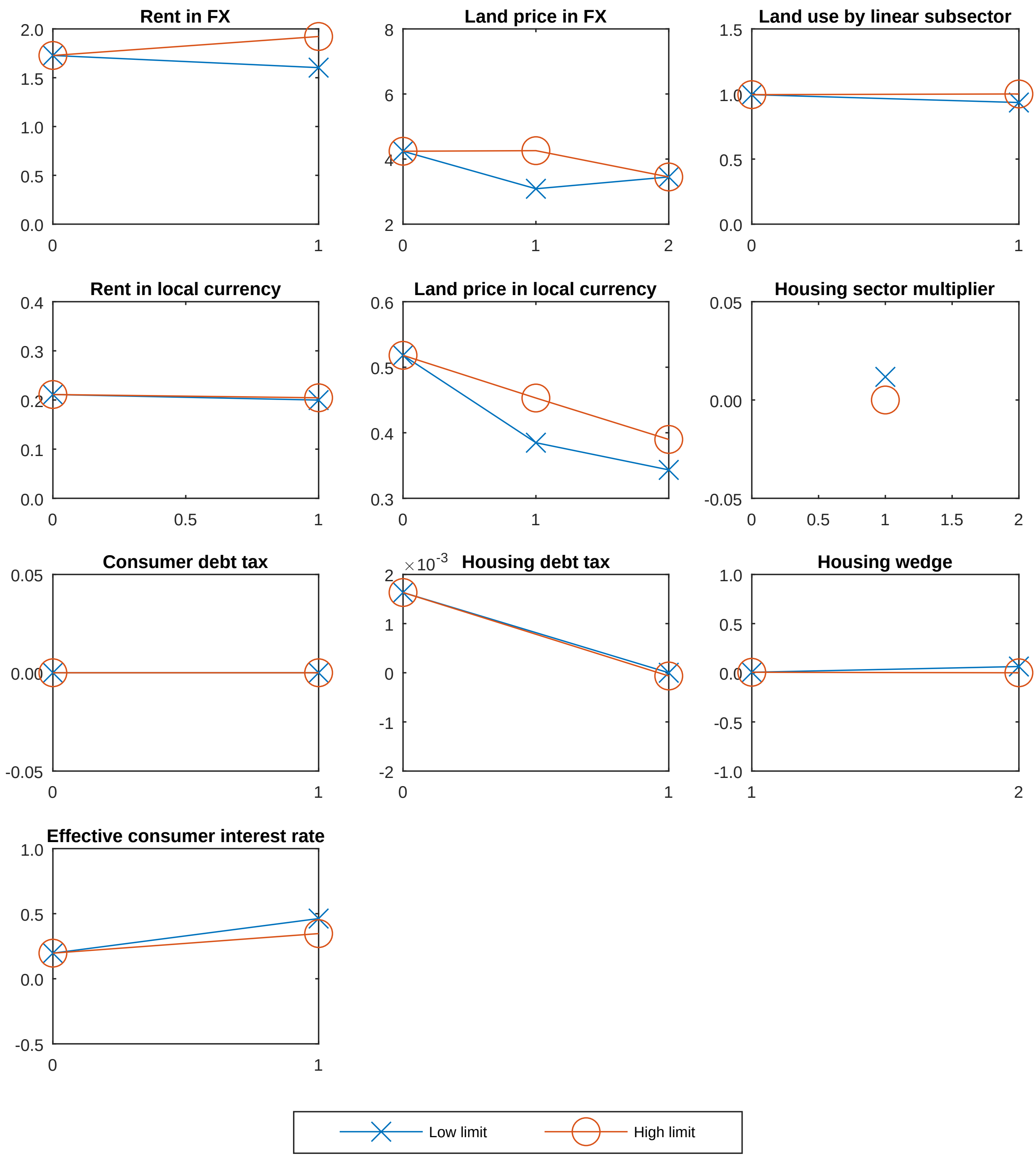
Notes: This figure plots the responses of key variables to a housing debt limit shock under DCP with shallow FX markets and and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{L1} \in [0.025, 10]$.

Figure 40: Banks' Constraint Spilling Over to Housing Constraint under PCP



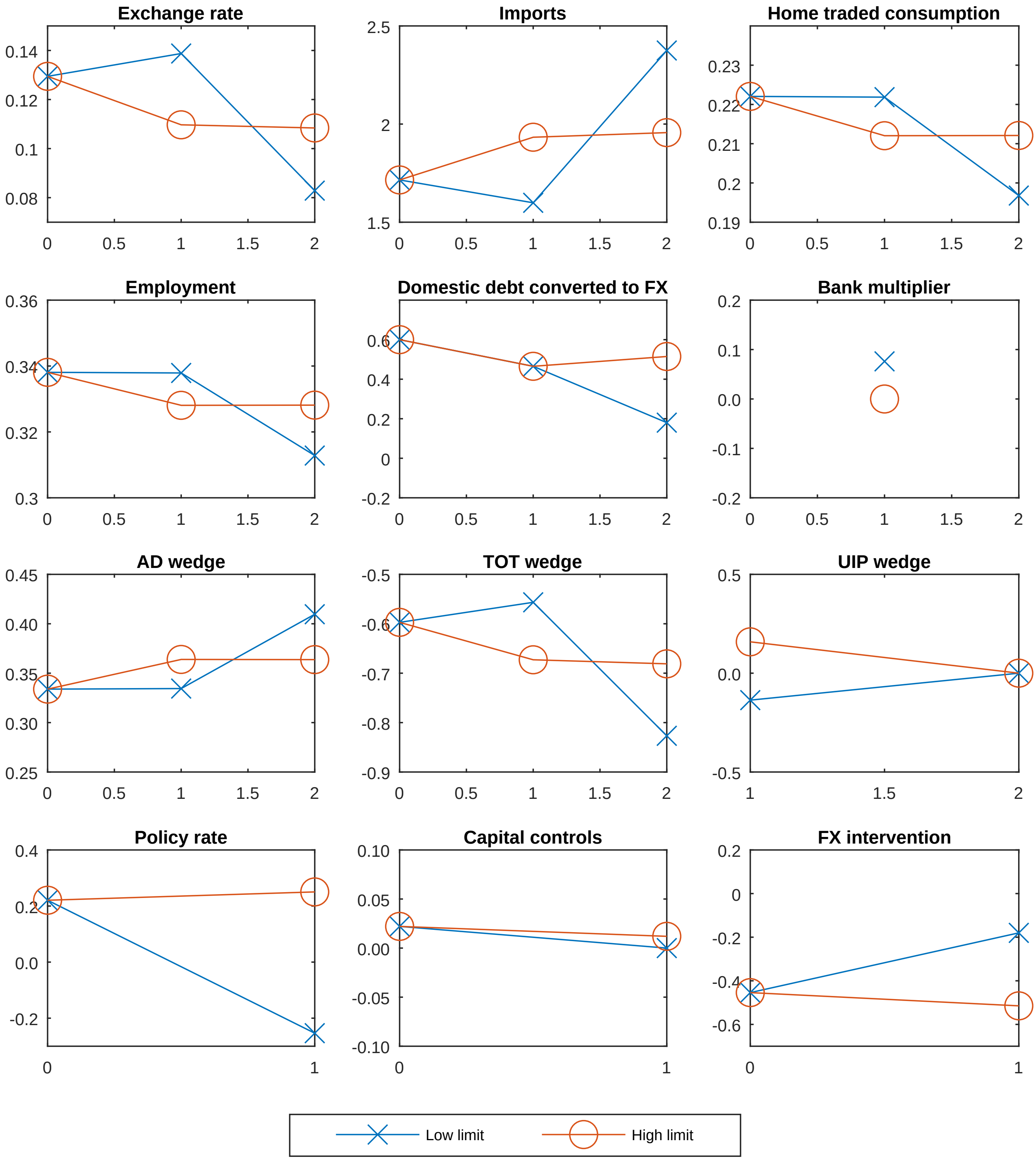
Notes: This figure plots the responses of key variables to a bank debt limit shock under PCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

Figure 40: Banks' Constraint Spilling Over to Housing Constraint under PCP cont.



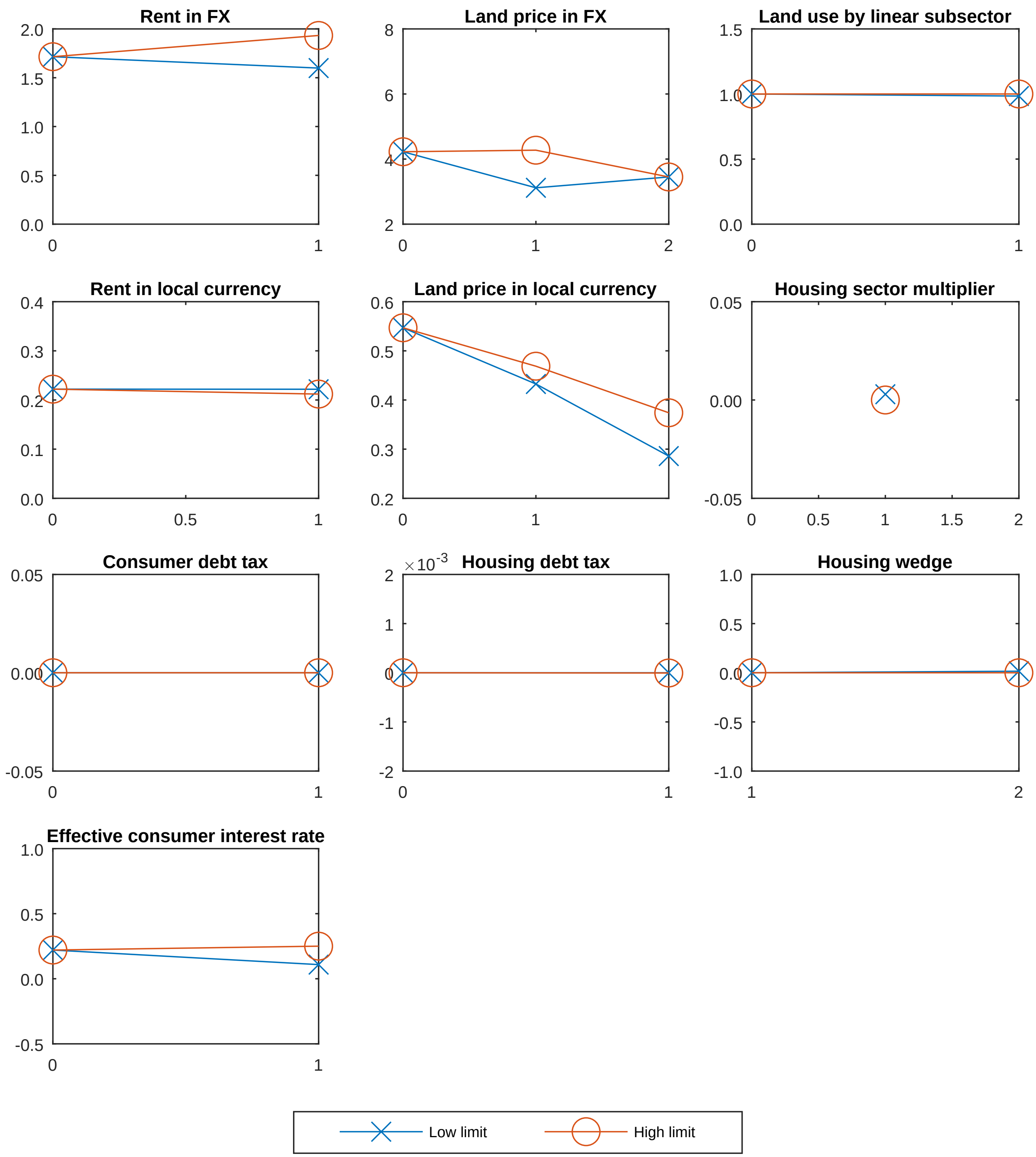
Notes: This figure plots the responses of key variables to a bank debt limit shock under PCP with shallow FX markets and and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

Figure 41: Banks' Constraint Spilling Over to Housing Constraint under DCP



Notes: This figure plots the responses of key variables to a bank debt limit shock under DCP with shallow FX markets and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

Figure 41: Banks' Constraint Spilling Over to Housing Constraint under DCP cont.



Notes: This figure plots the responses of key variables to a bank debt limit shock under DCP with shallow FX markets and and housing sector frictions. The shock hits at date-1 and is calibrated as $\kappa_{H1} \in [0.025, 10]$.

Table 1: Parameter Values

Parameter	Description	Value		
		Deep FX	Shallow FX	Shallow FX + housing
α_H	Expenditure share of tradable goods	1/3	1/3	1/3
α_F	Expenditure share of imports	1/3	1/3	1/3
α_R	Expenditure share of housing services	1/3	1/3	1/3
β	Discount factor	0.8	0.8	0.8
ω	Elasticity of export demand	1	1	1
P_F^*	Dollar price of imports	1	1	1
C^*	World demand level	1	1	1
Y_{NT}	Endowment of nontradable goods	1	1	1
Z	Endowment of commodities	1	1	1
P_{Z0}^*	Initial dollar price of commodity exports	1	1	1
i_0^*	Initial world interest rate	$1/\beta$-1	$1/\beta$-1	$1/\beta$-1
A_0	Initial level of productivity	1	1	1
B_0	Initial debt level	[0, 0.6]	[0, 0.6]	[0, 0.6]
B_{R0}	Initial housing sector debt level	NA	NA	3.5
L_0	Initial land	NA	NA	1
λ	Domestic share of intermediaries	1	0.8	0.8
Γ	Balance sheet friction	0	1	1
Shocks	Description	Value		
π	Probability of good/bad shock	0.5	0.5	0.5
A	Productivity	[0.75, 1.5]	[0.8, 1.5]	[0.8, 1.5]
P_{Z1}^*	Commodity price	[0.75, 1.5]	[0.75, 1.5]	[0.75, 1.5]
i_1^*	World interest rate	[0, 0.5]	[0, 0.5]	[0, 0.5]
κ_{H1}	Bank Debt limit	[0.025, 10]	[0.025, 10]	[0.025, 10]
κ_{L1}	Housing Sector Debt limit	NA	NA	[0.025, 10]
S_1	Foreign risk appetite	NA	[-0.5, 0.5]	[-0.5, 0.5]

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A APPENDIX

A.1 Price Setting Condition under DCP

The functional form for $P_X = P_X\left(P_H, C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}\right)$ is as follows:

$$P_X = P_H \frac{X_1}{X_2}\frac{X_3}{X_4}$$

where $X_1 = \frac{1}{A_0}\left(P_{F0}^*\right)^2 C_0^* C_{F0} + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[\frac{1}{A_1}\left(P_{F1}^*\right)^2 C_1^* C_{F1}\right] + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[\frac{1}{(1+i_1^*)}\frac{1}{A_2}\left(P_{F2}^*\right)^2 C_2^* C_{F2}\right]$

$$X_2 = P_{F0}^* C_0^* + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[P_{F1}^* C_1^*\right] + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[\frac{1}{(1+i_1^*)} P_{F2}^* C_2^*\right]$$

$$X_3 = P_{F0}^* C_{F0} + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[P_{F1}^* C_{F1}\right] + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[\frac{1}{(1+i_1^*)} P_{F2}^* C_{F2}\right]$$

$$X_4 = \frac{1}{A_0} E_0\left(P_{F0}^* C_{F0}\right)^2 + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[\frac{1}{A_1} E_1\left(P_{F1}^* C_{F1}\right)^2\right] + \frac{1}{(1+i_0^*)}\mathbb{E}_0\left[\frac{1}{(1+i_1^*)}\frac{1}{A_2} E_2\left(P_{F2}^* C_{F2}\right)^2\right]$$

This price-setting constraint on the planner captures the fact that when setting the export price at the beginning of period 0, firms take into account the planner's anticipated actions in all future periods. The solution of the constrained efficient allocation will require the following derivatives:

$$\frac{\partial P_X}{\partial P_H} = \frac{P_X}{P_H}$$

$$\begin{aligned}\frac{\partial P_X}{\partial C_{F0}} &= P_H \frac{\frac{1}{A_0}\left(P_{F0}^*\right)^2 C_0^*}{X_2}\frac{X_3}{X_4} + P_H \frac{X_1}{X_2}\frac{X_4 P_{F0}^* - X_3 \frac{2}{A_0} E_0\left(P_{F0}^*\right)^2 C_{F0}}{\left(X_4\right)^2} \\ &= P_X\left[\frac{\frac{1}{A_0}\left(P_{F0}^*\right)^2 C_0^*}{X_1} + \frac{X_4 P_{F0}^* - X_3 \frac{2}{A_0} E_0\left(P_{F0}^*\right)^2 C_{F0}}{X_3 X_4}\right], \text{ a single equation}\end{aligned}$$

$$\begin{aligned}\frac{\partial P_X}{\partial C_{F1}} &= \pi_1 P_H \frac{\frac{1}{(1+i_0^*)}\frac{1}{A_1}\left(P_{F1}^*\right)^2 C_1^*}{X_2}\frac{X_3}{X_4} \\ &\quad + \pi_1 P_H \frac{X_1}{X_2}\frac{X_4 \frac{1}{(1+i_0^*)} P_{F1}^* - X_3 \frac{1}{(1+i_0^*)}\frac{2}{A_1} E_1\left(P_{F1}^*\right)^2 C_{F1}}{\left(X_4\right)^2} \\ &= \frac{\pi_1}{(1+i_0^*)} P_X\left[\frac{\frac{1}{A_1}\left(P_{F1}^*\right)^2 C_1^*}{X_1} + \frac{X_4 P_{F1}^* - X_3 \frac{2}{A_1} E_1\left(P_{F1}^*\right)^2 C_{F1}}{X_3 X_4}\right],\end{aligned}$$

one equation per period-1 state s_1

$$\begin{aligned}\frac{\partial P_X}{\partial C_{F2}} &= \pi_1 P_H \frac{\frac{1}{(1+i_0^*)(1+i_1^*)}\frac{1}{A_2}\left(P_{F2}^*\right)^2 C_2^*}{X_2}\frac{X_3}{X_4} \\ &\quad + \pi_1 P_H \frac{X_1}{X_2}\frac{X_4 \frac{1}{(1+i_0^*)(1+i_1^*)} P_{F2}^* - X_3 \frac{1}{(1+i_0^*)(1+i_1^*)}\frac{2}{A_2} E_2\left(P_{F2}^*\right)^2 C_{F2}}{\left(X_4\right)^2}\end{aligned}$$

$$= \frac{\pi_1}{(1+i_0^*)(1+i_1^*)} P_X \left[\frac{\frac{1}{A_2}(P_{F2}^*)^2 C_2^*}{X_1} + \frac{X_4 P_{F2}^* - X_3 \frac{2}{A_2} E_2 (P_{F2}^*)^2 C_{F2}}{X_3 X_4} \right],$$

one equation per period-1 state s_1

$$\frac{\partial P_X}{\partial E_0} = -P_H \frac{X_1}{X_2} \frac{X_3}{(X_4)^2} \frac{1}{A_0} (P_{F0}^* C_{F0})^2 = -P_X \frac{\frac{1}{A_0}(P_{F0}^* C_{F0})^2}{X_4}, \text{ a single equation}$$

$$\frac{\partial P_X}{\partial E_1} = -\pi_1 P_H \frac{X_1}{X_2} \frac{X_3}{(X_4)^2} \frac{1}{(1+i_0^*)} \frac{1}{A_1} (P_{F1}^* C_{F1})^2$$
$$= -\frac{\pi_1}{(1+i_0^*)} P_X \frac{\frac{1}{A_1}(P_{F1}^* C_{F1})^2}{X_4},$$

one equation per period-1 state s_1

$$\frac{\partial P_X}{\partial E_2} = -\pi_1 P_H \frac{X_1}{X_2} \frac{X_3}{(X_4)^2} \frac{1}{(1+i_0^*)(1+i_1^*)} \frac{1}{A_2} (P_{F2}^* C_{F2})^2$$
$$= -\frac{\pi_1}{(1+i_0^*)(1+i_1^*)} P_X \frac{\frac{1}{A_2}(P_{F2}^* C_{F2})^2}{X_4},$$

one equation per period-1 state s_1.

A.2 FOCs for Constrained Efficient Allocations

The constrained efficient allocation under full commitment is:

$$\max_{\left\{C_{Ft}, P_H, E_t, \eta_{t+1}, FXI_t, L_{t-1}^{Linear}\right\}} \left\{ \begin{array}{lr} \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t V \left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{E_t P_{Ft}^*}{P_H}, L_{t-1}^{Linear} \right) \right] & \text{if PCP} \\ \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t V \left(C_{Ft}, \frac{E_t P_{Ft}^*}{P_H}, \frac{P_{Ft}^*}{P_X}, L_{t-1}^{Linear} \right) \right] & \text{if DCP,} \\ \text{with } P_X = P_X \left(C_{F0}, \{C_{F1}\}, \{C_{F2}\}, E_0, \{E_1\}, \{E_2\}, P_H \right) & \end{array} \right\}$$

subject to the following constraints:

$$\begin{aligned} \left(1+i_{-1}^*\right) B_0 \leq\ & P_{F0}^* \left[\omega C_0^* - C_{F0}\right] + P_{Z0}^* Z_0 \\ & + \frac{P_{F1}^* \left[\omega C_1^* - C_{F1}\right] + P_{Z1}^* Z_1 - (1-\lambda) FXI_0 \left[\eta_1 - (1+i_0^*)\right]}{\lambda (1+i_0^*) + (1-\lambda)\eta_1} \\ & + \frac{P_{F2}^* \left[\omega C_2^* - C_{F2}\right] + P_{Z2}^* Z_2 - (1-\lambda) FXI_1 \left[\eta_2 - (1+i_1^*)\right] + B_3}{\left[\lambda (1+i_0^*) + (1-\lambda)\eta_1\right]\left[\lambda (1+i_1^*) + (1-\lambda)\eta_2\right]}, \end{aligned}$$

one equation per period-1 state s_1 $[\Phi]$

$$\begin{aligned} \left(1+i_{-1}^*\right) B_0 \leq\ & P_{F0}^* \left[\omega C_0^* - C_{F0}\right] + P_{Z0}^* Z_0 \\ & + \frac{P_{F1}^* \left[\omega C_1^* - C_{F1}\right] + P_{Z1}^* Z_1 - (1-\lambda) FXI_0 \left[\eta_1 - (1+i_0^*)\right]}{\lambda (1+i_0^*) + (1-\lambda)\eta_1} \\ & + \frac{\kappa_{H1} \frac{P_H}{E_1}}{\lambda (1+i_0^*) + (1-\lambda)\eta_1}, \text{ one equation per period-1 state } s_1 \ [\Psi_B] \end{aligned}$$

$$\Gamma\left(\begin{array}{c}\left(1+i_{-1}^{*}\right)B_0+FXI_0-S_0\\ -P_{F0}^{*}\left[\omega C_0^{*}-C_{F0}\right]-P_{Z0}^{*}Z_0\end{array}\right)=\mathbb{E}_0\left[\eta_1-\left(1+i_0^{*}\right)\right]\text{, a single equation }\left[\Omega_0\right]$$

$$\Gamma\left(\begin{array}{c}\left[\begin{array}{c}\left(1+i_{-1}^{*}\right)B_0\\ -P_{F0}^{*}\left[\omega C_0^{*}-C_{F0}\right]-P_{Z0}^{*}Z_0\end{array}\right]\left[\lambda\left(1+i_0^{*}\right)+\left(1-\lambda\right)\eta_1\right]+FXI_1-S_1\\ -P_{F1}^{*}\left[\omega C_1^{*}-C_{F1}\right]-P_{Z1}^{*}Z_1+\left(1-\lambda\right)FXI_0\left[\eta_1-\left(1+i_0^{*}\right)\right]\end{array}\right)$$
$$=\eta_2-\left(1+i_1^{*}\right)\text{, one equation per period-1 state } s_1\ \left[\Omega_1\right]$$

$$E_1^H\eta_1^H=E_1^L\eta_1^L\text{, a single equation }\left[\Lambda\right]$$

$$0\geq B_{R2}^{Linear,s}=\chi_1^s\left[\left(1+i_{-1}^{*}\right)B_{R0}^{Linear}-\frac{\alpha_R}{\alpha_F}\frac{P_{F0}^{*}C_{F0}}{L_{-1}^{Linear}+G\left(1-L_{-1}^{Linear}\right)}L_{-1}^{Linear}\right]$$
$$+\left\{\begin{array}{c}\frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)}\frac{\alpha_R}{\alpha_F}\mathbb{E}_0\left[\frac{E_1}{E_1^s}P_{F1}^{*}C_{F1}\right]\\ +\mathbb{E}_0\left[\frac{1}{\chi_2}\frac{E_1}{E_1^s}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P_{F2}^{*}C_{F2}+\widehat{q}_2\right)\right]\end{array}\right\}\left(L_0^{Linear}-L_{-1}^{Linear}\right)$$
$$-\frac{P_{F1}^{*}C_{F1}}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)}\frac{\alpha_R}{\alpha_F}L_0^{Linear}$$
$$+\frac{1}{\chi_2}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P_{F2}^{*}C_{F2}+\widehat{q}_2\right)\left(\left(1-\kappa_{L1}\right)L_1^{Linear}-L_0^{Linear}\right),$$
one equation per period-1 state s_1 $\left[\Psi_R\right]$

$$\chi_{t+1}^s=\left\{\begin{array}{ll}\eta_{t+1}^s & \text{if capital controls are not permitted}\\ \dfrac{\frac{\alpha_F}{P_{Ft}^{*}C_{Ft}}}{\beta\mathbb{E}_t\left\{\frac{E_{t+1}^s}{E_{t+1}}\frac{\alpha_F}{P_{Ft+1}^{*}C_{Ft+1}}\right\}} & \text{if household macroprudential controls are not permitted}\end{array}\right.$$

where we define all the constraints in dollar terms, we use the superscript s to refer to the state of nature, and we indicate the multipliers in capital Greek letters in square brackets after each constraint. We fix the dollar value of initial debt repayments for the economy as a whole at $\left(1+i_{-1}^{*}\right)B_0$ in order to avoid the artefact depreciating away domestic-currency debt repayments at time 0, and we fix the dollar value of final debt at $B_3=B_0$ in order to normalize the debt path. We fix the dollar value of initial debt repayments for linear subsector housing firms at $\left(1+i_{-1}^{*}\right)B_{R0}^{Linear}=-\left(1+i_{-1}^{*}\right)B_{R0}^{Concave}$, and the dollar value of the final house price at $\widehat{q}_2$, in order to avoid the artefact depreciating away the value of domestic currency in all periods as a means of circumventing this subsector's borrowing constraint.

The above planner problem assumes that all instruments (i.e., the policy rate, capital controls, and FX intervention) are available. For determinacy of the instruments, we need to assume that only one of capital controls and consumer macroprudential taxes are available. The optimal allocations from the problem can be used to produce the implied optimal domestic policy rates and capital controls:

$$\left(1+\widehat{i}_t\right)=\left(1-\varphi_t\right)\left(1+i_t\right)=\eta_{t+1}\frac{E_{t+1}}{E_t}$$

$$\frac{\left(1-\varphi_0\right)}{\left(1+\theta_{HH0}\right)}=\frac{\eta_1\frac{E_1}{E_0}\beta\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^{*}C_{F1}}\right]}{\frac{\alpha_F}{P_{F0}^{*}C_{F0}}}\text{ and }\left(1+i_0\right)=\frac{\eta_1\frac{E_1}{E_0}}{\left(1-\varphi_0\right)}$$

$$\begin{array}{lr} \frac{(1-\varphi_1)}{(1+\theta_{HH1})}=\frac{\eta_2\beta P^*_{F1}C_{F1}}{P^*_{F2}C_{F2}} \text{ and } (1+i_1)=\frac{\eta_2\frac{E_2}{E_1}}{(1-\varphi_1)} & \text{if } \Psi_{Bt}=0 \\ (1+i_1)=\left(1+\widehat{i}_1\right) \text{ and } \varphi_1=\theta_{HH1}=0 & \text{if } \Psi_{Bt}>0 \end{array}$$

$$\chi_{t+1}=\frac{\eta_{t+1}}{(1-\varphi_t)}$$

$$(1+\theta_{Rt})=\begin{cases} \frac{\mathbb{E}_t\left[\frac{1}{L_t^{Linear}+G\left(1-L_t^{Linear}\right)}\frac{\alpha_R}{\alpha_F}\frac{E_{t+1}}{E_t}P^*_{Ft+1}C_{Ft+1}\right]+\mathbb{E}_t\left[\frac{E_{t+1}}{E_t}\widehat{q}_{t+1}\right]}{(1+i_t)\widehat{q}_t} & \text{if } \Psi_{Rt}=0 \\ 0 & \text{if } \Psi_{Rt}>0 \end{cases},$$

$$\text{where } \widehat{q}_t=\begin{cases} \frac{1}{(1+i_0)}\frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)}\frac{\alpha_R}{\alpha_F}\mathbb{E}_0\left[\frac{E_1}{E_0}P^*_{F1}C_{F1}\right] & \\ +\frac{1}{(1+i_0)}\mathbb{E}_0\left[\frac{1}{\chi_2}\frac{E_1}{E_0}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\right] & \text{if } t=0 \\ \frac{1}{\chi_2}\left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right) & \text{if } t=1 \\ \widehat{q}_2 & \text{if } t=2, \end{cases}$$

If FX intervention is not permitted, then we need to set:

$$FXI_0=FXI_1=0,$$

and remove the FOCs with respect to FXI_t.

If both capital controls and consumer macroprudential controls are not permitted, then the household Euler conditions need to be added as constraints:

$$\frac{\alpha_F}{P^*_{F0}C_{F0}}=\beta E_1^H\eta_1^H\mathbb{E}_0\left[\frac{1}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right], \text{ a single equation } [\Upsilon_0]$$

$$\frac{\alpha_F}{P^*_{F1}C_{F1}}\geq\beta\eta_2\frac{\alpha_F}{P^*_{F2}C_{F2}}=\beta\chi_2\frac{\alpha_F}{P^*_{F2}C_{F2}}, \text{ one equation per period-1 state } s_1\ [\Upsilon_1]$$

If mortgage MPMs are set to zero, i.e., $\theta_{Rt}\equiv 0$, then we need to add the following period-0 constraint:

$$L_0=1, \text{ a single equation } [\Delta_0]$$

If capital controls are not permitted and the domestic policy rate cannot be used, then the additional constraints are:

$$E_1^H\eta_1^H=\frac{1}{\beta}E_0, \text{ a single equation } [\Xi_0]$$

$$\eta_2E_2=\frac{1}{\beta}E_1, \text{ one equation per period-1 state } s_1\ [\Xi_1]$$

If consumer macroprudential controls are not permitted and the domestic policy rate cannot be used, then the additional constraints are:

$$\frac{\alpha_F}{P^*_{F0}C_{F0}}=\mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P^*_{F1}C_{F1}}\right], \text{ a single equation } [\Sigma_0]$$

$$\frac{\alpha_F}{P^*_{F1}C_{F1}}=\frac{E_1}{E_2}\frac{\alpha_F}{P^*_{F2}C_{F2}}, \text{ one equation per period-1 state } s_1\ [\Sigma_1]$$

Finally, if the exchange rate regime is a peg, the four additional constraints are:

$$\left\{\begin{array}{l} E_0 = E_1^s \\ E_0 = E_2^s \end{array}\right\}, \text{ one equation per state } s_1 \text{ in each of periods 1 and 2 } [\Pi_1^s \text{ and } \Pi_2^s]$$

The FOCs for the constrained efficient allocation are:

$$\begin{aligned} C_{F0} &: \frac{\alpha_F}{P_{F0}^* C_{F0}} \left[1 + \frac{\alpha_H}{\alpha_F}\left(1 - \frac{1}{A_0}\frac{C_{H0}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP} \cdot \left\{\frac{1}{P_{F0}^*}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F0}}\right\} \\ &= \mathbb{E}_0\left[\Phi + \Psi_B\right] + \Gamma\Omega_0 + \Gamma\mathbb{E}_0\left[I_0\Omega_1\right] + \frac{\alpha_F}{\left(P_{F0}^* C_{F0}\right)^2}\left(\Upsilon_0 + \Sigma_0\right) \\ &+ \mathbb{E}_0\left[\Psi_R\left\{\frac{1}{P_{F0}^*}\frac{\partial \chi_1}{\partial C_{F0}}\left[\left(1 + i_{-1}^*\right)B_{R0}^{Linear} - \widehat{P}_{R0} L_{-1}^{Linear}\right] - \chi_1 \frac{1}{P_{F0}^*}\frac{\partial \widehat{P}_{R0}}{\partial C_{F0}} L_{-1}^{Linear}\right\}\right], \text{ a single equation} \end{aligned}$$

$$\begin{aligned} C_{F1} &: \beta I_0 \frac{\alpha_F}{P_{F1}^* C_{F1}}\left[1 + \frac{\alpha_H}{\alpha_F}\left(1 - \frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP} \cdot \left\{\frac{I_0}{\pi_1 P_{F1}^*}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F1}}\right\} \\ &= \Phi + \Psi_B + \Gamma I_0 \Omega_1 \\ &+ \frac{I_0}{P_{F1}^* \pi_1}\mathbb{E}_0\left[\Psi_R\left\{\frac{\partial \chi_1}{\partial C_{F1}^s}\left[\left(1 + i_{-1}^*\right)B_{R0}^{Linear} - \widehat{P}_{R0} L_{-1}^{Linear}\right] + \frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial C_{F1}^s}\left(L_0^{Linear} - L_{-1}^{Linear}\right)\right\}\right] \\ &+ \frac{I_0}{P_{F1}^* \pi_1}\Psi_R\left\{-\frac{\partial \widehat{P}_{R1}}{\partial C_{F1}} L_0^{Linear} + \frac{\partial \widehat{q}_1}{\partial C_{F1}}\left(\left(1 - \kappa_{L1}\right)L_1^{Linear} - L_0^{Linear}\right)\right\} \\ &+ I_0 \frac{\alpha_F}{\left(P_{F1}^* C_{F1}\right)^2}\left[\Upsilon_1 - \Upsilon_0 \beta \eta_1\right] + I_0 \Upsilon_1 \beta \frac{\alpha_F}{P_{F2}^* C_{F2}}\frac{1}{P_{F1}^*}\frac{\partial \chi_2}{\partial C_{F1}} + I_0 \frac{\alpha_F}{\left(P_{F1}^* C_{F1}\right)^2}\left[\Sigma_1 - \frac{E_0}{E_1}\Sigma_0\right], \\ &\text{one equation per period-1 state } s_1 \end{aligned}$$

$$\begin{aligned} C_{F2} &: \beta^2 I_0 I_1 \frac{\alpha_F}{P_{F2}^* C_{F2}}\left[1 + \frac{\alpha_H}{\alpha_F}\left(1 - \frac{1}{A_2}\frac{C_{H2}}{\alpha_H}\right)\right] + \mathbb{I}^{DCP} \cdot \left\{\frac{I_0 I_1}{\pi_1 P_{F2}^*}\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2}\right]\frac{\partial P_X}{\partial C_{F2}}\right\} \\ &= \Phi + \frac{I_0 I_1}{\pi_1 P_{F2}^*}\mathbb{E}_0\left[\Psi_R \frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial C_{F2}^s}\left(L_0^{Linear} - L_{-1}^{Linear}\right)\right] + \frac{I_0 I_1}{P_{F2}^*}\Psi_R \frac{\partial \widehat{q}_1}{\partial C_{F2}}\left(\left(1 - \kappa_{L1}\right)L_1^{Linear} - L_0^{Linear}\right) \\ &+ \Upsilon_1 I_0 I_1\left\{\beta \chi_2 \frac{\alpha_F}{P_{F2}^* C_{F2}}\frac{1}{P_{F2}^*}\frac{\partial \chi_2}{\partial C_{F2}} - \beta \chi_2 \frac{\alpha_F}{\left(P_{F2}^* C_{F2}\right)^2}\right\} - \Sigma_1 I_0 I_1 \frac{E_1}{E_2}\frac{\alpha_F}{\left(P_{F2}^* C_{F2}\right)^2}, \\ &\text{one equation per period-1 state } s_1 \end{aligned}$$

$$\begin{aligned} E_0 &: \alpha_H\left(1 - \frac{1}{A_0}\frac{C_{H0}}{\alpha_H}\right) = \mathbb{I}^{PCP} \cdot \left\{\frac{E_0 P_{F0}^*}{P_H}\frac{\omega}{A_0} C_0^*\right\} + \mathbb{I}^{DCP} \cdot \left\{E_0 \mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t \frac{\omega}{A_t} C_t^* \frac{P_{Ft}^*}{(P_X)^2}\right]\left(-\frac{\partial P_X}{\partial E_0}\right)\right\} \\ &+ \Xi_0 \frac{E_0}{\beta} + \Sigma_0 \mathbb{E}_0\left[\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^* C_{F1}}\right] - E_0\left(\Pi_1^H + \Pi_1^L + \Pi_2^H + \Pi_2^L\right), \text{ a single equation} \end{aligned}$$

$$
\begin{aligned}
E_1^H &: \beta\alpha_H\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\pi_1-\frac{\Psi_B}{I_0}\kappa_{H1}\frac{P_H}{E_1}\pi_1-E_1\Pi_1\\
&+\Lambda E_1\eta_1-\Upsilon_0\beta\eta_1^L\frac{\alpha_F}{P_{F1}^*C_{F1}^L}\pi_1^L+\Xi_0E_1\eta_1-\Xi_1\frac{E_1}{\beta}\pi_1+\Sigma_0\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\pi_1-\Sigma_1\frac{E_1}{E_2}\frac{\alpha_F}{P_{F2}^*C_{F2}}\pi_1\\
&=\mathbb{I}^{PCP}\cdot\left\{\beta\frac{E_1P_{F1}^*}{P_H}\frac{\omega}{A_1}C_1^*\pi_1\right\}+\mathbb{I}^{DCP}\cdot\left\{E_1\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{\left(P_X\right)^2}\right]\left(-\frac{\partial P_X}{\partial E_1}\right)\right\}\\
&+\mathbb{E}_0\left[\Psi_R\left\{E_1^H\frac{\partial\chi_1}{\partial E_1^H}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear}-\widehat{P}_{R0}L_{-1}^{Linear}\right]+E_1^H\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial E_1^H}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right\}\right],
\end{aligned}
$$

a single equation for the H-state

$$
\begin{aligned}
E_1^L &: \beta\alpha_H\left(1-\frac{1}{A_1}\frac{C_{H1}}{\alpha_H}\right)\pi_1-\frac{\Psi_B}{I_0}\kappa_{H1}\frac{P_H}{E_1}\pi_1-E_1\Pi_1\\
&-\Lambda E_1\eta_1+\Upsilon_0\beta\eta_1\frac{\alpha_F}{P_{F1}^*C_{F1}}\pi_1-\Xi_1\frac{E_1}{\beta}\pi_1+\Sigma_0\frac{E_0}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\pi_1-\Sigma_1\frac{E_1}{E_2}\frac{\alpha_F}{P_{F2}^*C_{F2}}\pi_1\\
&=\mathbb{I}^{PCP}\cdot\left\{\beta\frac{E_1P_{F1}^*}{P_H}\frac{\omega}{A_1}C_1^*\pi_1\right\}+\mathbb{I}^{DCP}\cdot\left\{E_1\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{\left(P_X\right)^2}\right]\left(-\frac{\partial P_X}{\partial E_1}\right)\right\}\\
&+\mathbb{E}_0\left[\Psi_R\left\{E_1^L\frac{\partial\chi_1}{\partial E_1^L}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear}-\widehat{P}_{R0}L_{-1}^{Linear}\right]+E_1^L\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial E_1^L}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right\}\right],
\end{aligned}
$$

a single equation for the L-state

$$
\begin{aligned}
E_2 &: \beta^2\alpha_H\left(1-\frac{1}{A_2}\frac{C_{H2}}{\alpha_H}\right)\pi_1-E_2\Upsilon_1\beta\frac{\alpha_F}{P_{F2}^*C_{F2}}\frac{\partial\chi_2}{\partial E_2}\pi_1+\Xi_1E_2\eta_2\pi_1+\Sigma_1\frac{E_1}{E_2}\frac{\alpha_F}{P_{F2}^*C_{F2}}\pi_1-E_2\Pi_2\\
&+\mathbb{E}_0\left[\Psi_RE_2^s\frac{\partial\left(\chi_1\widehat{q}_0\right)}{\partial E_2^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right]+\Psi_RE_2\frac{\partial\widehat{q}_1}{\partial E_2}\left(\left(1-\kappa_{L1}\right)L_1^{Linear}-L_0^{Linear}\right)\pi_1\\
&=\mathbb{I}^{PCP}\cdot\left\{\beta^2\frac{E_2P_{F2}^*}{P_H}\frac{\omega}{A_2}C_2^*\pi_1\right\}+\mathbb{I}^{DCP}\cdot\left\{E_2\mathbb{E}_0\left[\sum_{t=0}^{2}\beta^t\frac{\omega}{A_t}C_t^*\frac{P_{Ft}^*}{\left(P_X\right)^2}\right]\left(-\frac{\partial P_X}{\partial E_2}\right)\right\},
\end{aligned}
$$

one equation per period-1 state s_1

$$
\begin{aligned}
\eta_1^H &: \left(\Phi+\Psi_B\right)\left(1-\lambda\right)\frac{B_1+FXI_0}{I_0}+\Psi_B\left(1-\lambda\right)\frac{\kappa_{H1}\frac{P_H}{E_1}-B_2}{\left(I_0\right)^2}\\
&+\Psi_R\frac{\partial\chi_1}{\partial\eta_1}\left[\left(1+i_{-1}^*\right)B_{R0}^{Linear}-\widehat{P}_{R0}L_{-1}^{Linear}\right]\\
&=\Omega_0-\Omega_1\Gamma\left(1-\lambda\right)\left(B_1+FXI_0\right)+\frac{1}{\pi_1}\Lambda E_1-\frac{1}{\pi_1}\Upsilon_0\beta E_1\mathbb{E}_0\left\{\frac{1}{E_1}\frac{\alpha_F}{P_{F1}^*C_{F1}}\right\}+\frac{1}{\pi_1}\Xi_0E_1,
\end{aligned}
$$

a single equation for the H-state

$$
\eta_1^L : \left(\Phi+\Psi_B\right)\left(1-\lambda\right)\frac{B_1+FXI_0}{I_0}+\Psi_B\left(1-\lambda\right)\frac{\kappa_{H1}\frac{P_H}{E_1}-B_2}{\left(I_0\right)^2}
$$

$$
\begin{aligned}
&+\Psi_R \frac{\partial \chi_1}{\partial \eta_1}\left[\left(1+i_{-1}^*\right) B_{R0}^{Linear}-\widehat{P}_{R0} L_{-1}^{Linear}\right] \\
&=\Omega_0-\Omega_1 \Gamma(1-\lambda)\left(B_1+FXI_0\right)-\frac{1}{\pi_1} \Lambda E_1, \text{ a single equation for the } L\text{-state}
\end{aligned}
$$

$$
\begin{aligned}
\eta_2 \quad : \quad & \Phi(1-\lambda) \frac{B_2+FXI_1}{I_0 I_1}+\frac{1}{\pi_1} \mathbb{E}_0\left[\Psi_R \frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial \eta_2^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right] \\
& +\Psi_R \frac{\partial \widehat{q}_1}{\partial \eta_2}\left(\left(1-\kappa_{L1}\right) L_1^{Linear}-L_0^{Linear}\right) \\
= \quad & \Omega_1-\Upsilon_1 \beta \frac{\alpha_F}{P_{F2}^* C_{F2}}+\Xi_1 E_2, \text{ one equation per period-1 state } s_1
\end{aligned}
$$

$$
\begin{aligned}
FXI_0: 0= & -(1-\lambda) \mathbb{E}_0\left\{\left(\Phi+\Psi_B\right) \frac{\left[\eta_1-\left(1+i_0^*\right)\right]}{I_0}\right\} \\
& -\Gamma \Omega_0-(1-\lambda) \Gamma \mathbb{E}_0\left\{\Omega_1\left[\eta_1-\left(1+i_0^*\right)\right]\right\}, \text{ a single equation}
\end{aligned}
$$

$$
FXI_1:-\Phi \frac{(1-\lambda)\left[\eta_2-\left(1+i_1^*\right)\right]}{I_0 I_1}-\Gamma \Omega_1=0, \text{ one equation per period-1 state } s_1,
$$

$$
\begin{aligned}
& L_0: \beta \alpha_R \frac{1-G^{\prime}\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \\
& =\mathbb{E}_0\left[\Psi_R\left\{\left(\chi_1 \widehat{q}_0-\widehat{P}_{R1}-\widehat{q}_1\right)+\frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial L_0}\left(L_0^{Linear}-L_{-1}^{Linear}\right)-\frac{\partial \widehat{P}_{R1}}{\partial L_0} L_0\right\}\right], \\
& \text{a single equation}
\end{aligned}
$$

$$
\begin{aligned}
L_1: \beta^2 \alpha_R \frac{1-G^{\prime}\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)}= & \frac{1}{\pi_1} \mathbb{E}_0\left[\Psi_R \frac{\partial\left(\chi_1 \widehat{q}_0\right)}{\partial L_1^s}\left(L_0^{Linear}-L_{-1}^{Linear}\right)\right] \\
+\Psi_R\left\{\widehat{q}_1\left(1-\kappa_{L1}\right)+\frac{\partial \widehat{q}_1}{\partial L_1}\left(\left(1-\kappa_{L1}\right) L_1^{Linear}-L_0^{Linear}\right)\right\}&, \text{ one equation per period-1 state } s_1
\end{aligned}
$$

where $\Theta_t \geq 0$ and $\sum_{s=0}^t FXI_t \geq 0$ with complementary slackness, and the FOCs with respect to P_H are redundant, so we normalize $P_H=1$. We define:

$$
I_t=\lambda\left(1+i_t^*\right)+(1-\lambda) \eta_{t+1}
$$

$$
\begin{aligned}
B_1 \equiv \frac{D_1}{E_0} & =\left(1+i_{-1}^*\right) B_0-\left(P_{F0}^*\left[\omega C_0^*-C_{F0}\right]+P_{Z0}^* Z_0\right) \\
B_2 \equiv \frac{D_2}{E_1} & =B_1 I_0-\left(P_{F1}^*\left[\omega C_1^*-C_{F1}\right]-(1-\lambda) FXI_0\left[\eta_1-\left(1+i_0^*\right)\right]+P_{Z1}^* Z_1\right) \\
B_3 \equiv \frac{D_3}{E_2} & =B_2 I_1-\left(P_{F2}^*\left[\omega C_2^*-C_{F2}\right]-(1-\lambda) FXI_1\left[\eta_2-\left(1+i_1^*\right)\right]+P_{Z2}^* Z_2\right),
\end{aligned}
$$

We define the following derivatives for the case when capital controls are not permitted:

$$\frac{\partial \chi_1^s}{\partial C_{F0}} = \frac{\partial \chi_1^s}{\partial C_{F1}^s} = \frac{\partial \chi_1^s}{\partial C_{F1}^{-s}} = \frac{\partial \chi_1^s}{\partial E_1^s} = \frac{\partial \chi_1^s}{\partial E_1^{-s}} = \frac{\partial \chi_2}{\partial C_{F1}} = \frac{\partial \chi_2}{\partial C_{F2}} = \frac{\partial \chi_2}{\partial E_2} = 0$$

$$\frac{\partial \chi_1^s}{\partial \eta_1^s} = \frac{\partial \chi_2}{\partial \eta_2} = 1,$$

and the following derivatives for the case when consumer macroprudential controls are not permitted:

$$\frac{\partial \chi_1^s}{\partial C_{F0}} = -\frac{\frac{\alpha_F}{P_{F0}^*(C_{F0})^2}}{\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}}, \frac{\partial \chi_1^s}{\partial C_{F1}^s} = \frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{\alpha_F}{P_{F1}^* \left(C_{F1}^s\right)^2} \pi_1^s}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}, \frac{\partial \chi_1^s}{\partial C_{F1}^{-s}} = \frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{E_1^s}{E_1^{-s}} \frac{\alpha_F}{P_{F1}^* \left(C_{F1}^{-s}\right)^2} \pi_1^{-s}}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}$$

$$\frac{\partial \chi_1^s}{\partial E_1^s} = -\frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{1}{E_1^{-s}} \frac{\alpha_F}{P_{F1}^* C_{F1}^{-s}} \pi_1^{-s}}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}, \frac{\partial \chi_1^s}{\partial E_1^{-s}} = \frac{\frac{\alpha_F}{P_{F0}^* C_{F0}} \beta \frac{E_1^s}{\left(E_1^{-s}\right)^2} \frac{\alpha_F}{P_{F1}^* C_{F1}^{-s}} \pi_1^{-s}}{\left[\beta \mathbb{E}_0 \left\{ \frac{E_1^s}{E_1} \frac{\alpha_F}{P_{F1}^* C_{F1}} \right\}\right]^2}$$

$$\frac{\partial \chi_2}{\partial C_{F1}} = -\frac{P_{F2}^* C_{F2}}{\beta P_{F1}^* \left(C_{F1}\right)^2}, \frac{\partial \chi_2}{\partial C_{F2}} = \frac{P_{F2}^*}{\beta P_{F1}^* C_{F1}}, \frac{\partial \chi_2}{\partial E_2} = 0 \text{ and } \frac{\partial \chi_1^s}{\partial \eta_1^s} = \frac{\partial \chi_2}{\partial \eta_2} = 0$$

We define the following derivatives:

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F1}^s} = \frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F1}^* \pi_1^s + \frac{\partial \widehat{q}_1^s}{\partial C_{F1}^s} \pi_1^s \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F2}^s} = \frac{\partial \widehat{q}_1^s}{\partial C_{F2}^s} \pi_1^s$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F1}^{-s}} = \frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \frac{E_1^{-s}}{E_1^s} P_{F1}^* \pi_1^{-s} + \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial C_{F1}^{-s}} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial C_{F2}^{-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial C_{F2}^{-s}} \pi_1^{-s}$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_1^s} = -\frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \frac{E_1^{-s}}{(E_1^s)^2} P_{F1}^* C_{F1}^{-s} \pi_1^{-s} - \frac{E_1^{-s}}{(E_1^s)^2} \widehat{q}_1^{-s} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_2^s} = \frac{\partial \widehat{q}_1^s}{\partial E_2^s} \pi_1^s$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_1^{-s}} = \frac{G'\left(1-L_0^{Linear}\right)}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \frac{\alpha_R}{\alpha_F} \frac{1}{E_1^s} P_{F1}^* C_{F1}^{-s} \pi_1^{-s} + \frac{1}{E_1^s} \widehat{q}_1^{-s} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial E_2^{-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial E_2^{-s}} \pi_1^{-s}$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial L_0} = \left\{ \frac{\left[-G''\left(1-L_0^{Linear}\right)\right]}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} - \frac{G'\left(1-L_0^{Linear}\right)\left[1-G'\left(1-L_0^{Linear}\right)\right]}{\left[L_0^{Linear}+G\left(1-L_0^{Linear}\right)\right]^2} \right\} \frac{\alpha_R}{\alpha_F} \mathbb{E}_0 \left[\frac{E_1}{E_1^s} P_{F1}^* C_{F1} \right] \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial \eta_2^s} = \frac{\partial \widehat{q}_1^s}{\partial \eta_2^s} \pi_1^s$$

$$\frac{\partial(\chi_1^s \widehat{q}_0)}{\partial L_1^{Linear,s}} = \frac{\partial \widehat{q}_1^s}{\partial L_1^{Linear,s}} \pi_1^s, \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial L_1^{Linear,-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial L_1^{Linear,-s}} \pi_1^{-s} \qquad \frac{\partial(\chi_1^s \widehat{q}_0)}{\partial \eta_2^{-s}} = \frac{E_1^{-s}}{E_1^s} \frac{\partial \widehat{q}_1^{-s}}{\partial \eta_2^{-s}} \pi_1^{-s}$$

$$\frac{\partial \widehat{q}_1}{\partial C_{F1}} = -\frac{1}{\left(\chi_2\right)^2} \left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial C_{F1}}$$

$$\frac{\partial \widehat{q}_1}{\partial C_{F2}} = -\frac{1}{\left(\chi_2\right)^2} \left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial C_{F2}} + \frac{1}{\chi_2} \frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^*$$

$$\frac{\partial \widehat{q}_1}{\partial E_2} = -\frac{1}{\left(\chi_2\right)^2} \left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial E_2}$$

$$\frac{\partial \widehat{q}_1}{\partial \eta_2} = -\frac{1}{\left(\chi_2\right)^2} \left(\frac{G'\left(1-L_1^{Linear}\right)}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} \frac{\alpha_R}{\alpha_F} P_{F2}^* C_{F2} + \widehat{q}_2 \right) \frac{\partial \chi_2}{\partial \eta_2}$$

$$\frac{\partial \widehat{q}_1^s}{\partial L_1^s} = \frac{1}{\chi_2}\left\{\frac{\left[-G''\left(1-L_1^{Linear}\right)\right]}{L_1^{Linear}+G\left(1-L_1^{Linear}\right)} - \frac{G'\left(1-L_1^{Linear}\right)\left[1-G'\left(1-L_1^{Linear}\right)\right]}{\left[L_1^{Linear}+G\left(1-L_1^{Linear}\right)\right]^2}\right\}\frac{\alpha_R}{\alpha_F}P_{F2}^*C_{F2}$$

$$\frac{\partial \widehat{P}_{R0}}{\partial C_{F0}} = \frac{\alpha_R}{\alpha_F}\frac{P_{F0}^*}{L_{-1}^{Linear}+G\left(1-L_{-1}^{Linear}\right)} \quad \frac{\partial \widehat{P}_{R1}}{\partial C_{F1}} = \frac{\alpha_R}{\alpha_F}\frac{P_{F1}^*}{L_0^{Linear}+G\left(1-L_0^{Linear}\right)} \quad \frac{\partial \widehat{P}_{R1}}{\partial L_0} = -\frac{\alpha_R}{\alpha_F}\frac{P_{F1}^*C_{F1}\left[1-G'\left(1-L_0^{Linear}\right)\right]}{\left[L_0^{Linear}+G\left(1-L_0^{Linear}\right)\right]^2}$$

A.3 Numerical Solution

We start from the relevant set of planner constraints and FOCs from the preceding subsections. First, we select the set of policy instruments available to the planner:

- If all policy instruments (i.e., the policy rate, capital controls, FX intervention, and macroprudential controls) are available to the planner, then use all the FOCs above but set $\Upsilon_0 = \Upsilon_1 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = 0$.
- If FX intervention is not permitted, then set $FXI_0 = FXI_1 = 0$, and remove the FOCs with respect to FXI_t.
- If capital controls and consumer macroprudential controls are not permitted, then use all the FOCs above but set $\Delta_0 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = \Pi_1 = \Pi_2 = 0$.
- If housing sector macroprudential controls are not permitted, then use all the FOCs above but set $\Upsilon_0 = \Upsilon_1 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = \Pi_1 = \Pi_2 = 0$.
- If capital controls and the domestic policy rate are not permitted, then use all the FOCs above but set $\Sigma_0 = \Sigma_1 = \Pi_1 = \Pi_2 = 0$.
- If consumer macroprudential controls and the domestic policy rate are not permitted, then use all the FOCs above but set $\Xi_0 = \Xi_1 = \Pi_1 = \Pi_2 = 0$.
- If the exchange rate is pegged, then use all the FOCs above but set $\Delta_0 = \Xi_0 = \Xi_1 = \Sigma_0 = \Sigma_1 = 0$.

Next, we characterize the solution numerically by running the following iterative process to convergence.

1. Fix guess on whether the banks' external borrowing constraint (19) is slack or binding in every period-1 state. For states where the constraint is slack, fix $\Psi_B = 0$ and remove the borrowing constraint. For states where the constraint is binding, set the borrowing constraint to be satisfied with equality and allow $\Psi_B \neq 0$. Run the following iterative process to convergence.
   - Fix guess on whether the housing sector borrowing constraint (20) is slack or binding in every period-1 state. For states where the constraint is slack, fix $\Psi_R = 0$ and remove the borrowing constraint. For states where the constraint is binding, set the borrowing constraint to be satisfied with equality and allow $\Psi_R \neq 0$. Run the following iterative process to convergence.
   - Verify that the housing sector borrowing constraint is slack for states where the constraint was guessed to be slack; otherwise, change the guess. Verify that $\Psi_R \geq 0$ for states where the constraint was guessed to be binding; otherwise, change the guess.
2. Verify that the household borrowing constraint is slack for states where the constraint was guessed to be slack; otherwise, change the guess. Verify that $\Psi_B \geq 0$ for states where the constraint was guessed to be binding; otherwise, change the guess.

A.4 Ban on FX Positions

If domestically-owned intermediaries are prohibited from taking open FX positions, then the constrained planner problem changes:

$$\max_{\left\{C_{Ft},P_H,E_t,\eta_{t+1},FXI_t,L^{Linear}_{t-1}\right\}}\left\{\begin{array}{cc} \mathbb{E}_0\left[\sum\limits_{t=0}^{2}\beta^t V\left(C_{Ft},\frac{E_tP^*_{Ft}}{P_H},\frac{E_tP^*_{Ft}}{P_H},L^{Linear}_{t-1}\right)\right] & \text{if PCP} \\ \mathbb{E}_0\left[\sum\limits_{t=0}^{2}\beta^t V\left(C_{Ft},\frac{E_tP^*_{Ft}}{P_H},\frac{P^*_{Ft}}{P_X},L^{Linear}_{t-1}\right)\right] & \text{if DCP,} \\ \text{with } P_X=P_X\left(C_{F0},\{C_{F1}\},\{C_{F2}\},E_0,\{E_1\},\{E_2\},P_H\right) & \end{array}\right\}$$

$$\begin{aligned}\left(1+i^*_{-1}\right)B_0 \le\; & P^*_{F0}\left[\omega C^*_0-C_{F0}\right]+P^*_{Z0}Z_0\\ &+\frac{P^*_{F1}\left[\omega C^*_1-C_{F1}\right]+P^*_{Z1}Z_1-FXI_0\left[\eta_1-\left(1+i^*_0\right)\right]}{\eta_1}\\ &+\frac{P^*_{F2}\left[\omega C^*_2-C_{F2}\right]+P^*_{Z2}Z_2-FXI_1\left[\eta_2-\left(1+i^*_1\right)\right]+B_3}{\eta_1\eta_2},\\ &\text{one equation per period-1 state } s_1\ \ [\Phi]\end{aligned}$$

$$\begin{aligned}\left(1+i^*_{-1}\right)B_0 \le\; & P^*_{F0}\left[\omega C^*_0-C_{F0}\right]+P^*_{Z0}Z_0\\ &+\frac{P^*_{F1}\left[\omega C^*_1-C_{F1}\right]+P^*_{Z1}Z_1-FXI_0\left[\eta_1-\left(1+i^*_0\right)\right]}{\eta_1}+\frac{\kappa_{H1}\frac{P_H}{E_1}}{\eta_1},\\ &\text{one equation per period-1 state } s_1\ \ [\Psi_B]\end{aligned}$$

$$\frac{\Gamma}{(1-\lambda)}\left(\begin{array}{c}\left(1+i^*_{-1}\right)B_0+FXI_0-S_0\\ -P^*_{F0}\left[\omega C^*_0-C_{F0}\right]-P^*_{Z0}Z_0\end{array}\right)=\mathbb{E}_0\left[\eta_1-\left(1+i^*_0\right)\right],\text{ a single equation }\ [\Omega_0]$$

$$\begin{aligned}&\frac{\Gamma}{(1-\lambda)}\left(\begin{array}{c}\left[\begin{array}{c}\left(1+i^*_{-1}\right)B_0\\ -P^*_{F0}\left[\omega C^*_0-C_{F0}\right]-P^*_{Z0}Z_0\end{array}\right]\eta_1+FXI_1-S_1\\ -P^*_{F1}\left[\omega C^*_1-C_{F1}\right]-P^*_{Z1}Z_1+FXI_0\left[\eta_1-\left(1+i^*_0\right)\right]\end{array}\right)\\ &=\eta_2-\left(1+i^*_1\right),\text{ one equation per period-1 state } s_1\ \ [\Omega_1]\end{aligned}$$

$$E^H_1\eta^H_1=E^L_1\eta^L_1,\text{ a single equation }\ [\Lambda]$$

$$\begin{aligned}0\ge B^{Linear,s}_{R2}=\chi^s_1&\left[\left(1+i^*_{-1}\right)B^{Linear}_{R0}-\frac{\alpha_R}{\alpha_F}\frac{P^*_{F0}C_{F0}}{L^{Linear}_{-1}+G\left(1-L^{Linear}_{-1}\right)}L^{Linear}_{-1}\right]\\ &+\left\{\begin{array}{c}\frac{G'\left(1-L^{Linear}_0\right)}{L^{Linear}_0+G\left(1-L^{Linear}_0\right)}\frac{\alpha_R}{\alpha_F}\mathbb{E}_0\left[\frac{E_1}{E^s_1}P^*_{F1}C_{F1}\right]\\ +\mathbb{E}_0\left[\frac{1}{\chi_2}\frac{E_1}{E^s_1}\left(\frac{G'\left(1-L^{Linear}_1\right)}{L^{Linear}_1+G\left(1-L^{Linear}_1\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\right]\end{array}\right\}\left(L^{Linear}_0-L^{Linear}_{-1}\right)\\ &-\frac{P^*_{F1}C_{F1}}{L^{Linear}_0+G\left(1-L^{Linear}_0\right)}\frac{\alpha_R}{\alpha_F}L^{Linear}_0\\ &+\frac{1}{\chi_2}\left(\frac{G'\left(1-L^{Linear}_1\right)}{L^{Linear}_1+G\left(1-L^{Linear}_1\right)}\frac{\alpha_R}{\alpha_F}P^*_{F2}C_{F2}+\widehat{q}_2\right)\left(\left(1-\kappa_{L1}\right)L^{Linear}_1-L^{Linear}_0\right),\end{aligned}$$

one equation per period-1 state s_1 $[\Psi_R]$

$$\chi_{t+1}^s = \begin{cases} \eta_{t+1}^s & \text{if capital controls are not permitted} \\ \dfrac{\frac{\alpha_F}{P_{Ft}^* C_{Ft}}}{\beta \mathbb{E}_t \left\{ \frac{E_{t+1}^s}{E_{t+1}} \frac{\alpha_F}{P_{Ft+1}^* C_{Ft+1}} \right\}} & \begin{array}{c}\text{if household macroprudential controls} \\ \text{are not permitted}\end{array} \end{cases}$$

where we define all the constraints in dollar terms, we use the superscript s to refer to the state of nature, we fix the dollar value of initial debt repayments at $\left(1 + i_{-1}^*\right) B_0$, and we set $B_3 = B_0$.