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Credibility Dynamics and Disinflation Plans¹**Prepared by Rumen Kostadinov and Francisco Roldán**

Authorized for distribution by Daniel Leigh

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Abstract

We study the optimal design of a disinflation plan by a planner who lacks commitment. Having announced a plan, the Central banker faces a tradeoff between surprise inflation and building reputation, defined as the private sector's belief that the Central bank is committed to the plan. Some plans are harder to sustain: the planner recognizes that paving out future grounds with temptation leads the way for a negative drift of reputation in equilibrium. Plans that successfully create low inflationary expectations balance promises of lower inflation with dynamic incentives that make them more credible. When announcing the disinflation plan, the planner takes into account these anticipated interactions. We find that, even in the zero reputation limit, a gradual disinflation is preferred despite the absence of inflation inertia in the private economy.

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Author's E-Mail Address: froldan@imf.org

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INTRODUCTION

Macroeconomic models give expectations about future policy a large role in the determination of current outcomes. Policy is then generally set under one of two assumptions: commitment to future actions or discretion. Attempts to model policy departing from these extreme cases have found limited success.

However, governments actively attempt to influence beliefs about future policy. Examples include forward guidance and inflation targets but also fiscal rules and the timing of introduction of policies. Such promises rarely constrain future choices, yet they can shift expectations substantially. Standard macroeconomic models cannot capture this idea directly, as expectations of the public are fully determined by the policy chosen with commitment, or with discretion as part of an equilibrium. In either of these cases the public understands that announcements do not bind the government in any way. Announcements do not grant any additional credibility to the policy maker, as the public is convinced of her course of action.

In this paper we develop a rational-expectations theory of government credibility and apply it to policy design questions. Our notion of credibility is based on the concept of reputation in game theory (Kreps and Wilson, 1982; Milgrom and Roberts, 1982). In our model, the government (or planner, or long-lived player) could be rational and strategic, or one of many possible ‘behavioral’ types described by a policy that they stubbornly follow. The public is uninformed about the government’s type and makes statistical inference about it after observing the government’s announcements and actions. This inference is central to our analysis because it turns out to be in the best interest of the rational type to pretend to be one of the behavioral types.

We consider a stylized environment. In the initial period, the government makes an announcement of its policy ‘targets’ and is then free to choose policy. However, the private sector knows that if the government is behavioral it announced exactly what it will implement. As a consequence, the rational type has an ex-post incentive to stay close to any announced targets, which might earn it a reputation for being ‘committed’ to them. The incentive exists at any positive level of reputation, though its strength depends on the announced targets. In anticipation of these interactions, the rational type chooses carefully which targets to announce. Our main question concerns the optimal policy announcement in the presence of these reputational concerns.

We set our model of reputation in a modern version of the classic environment of Barro (1986) and Backus and Driffill (1985), where a monetary authority sets inflation subject to an expectations-augmented Phillips curve. The monetary authority dislikes inflation but constantly faces an opportunity to engineer surprise inflation, which would deliver output closer to potential. We model these features through the standard, cashless-limit New Keynesian setup for the private economy. To focus on incentives and reputation dynamics, we abstract from an IS curve and let the monetary authority control inflation directly.

A natural definition of the government’s reputation is the private sector’s belief that the government is indeed the behavioral type whose plan was announced. The credibility of a plan is a measure of closeness between expected inflation under the plan and what the plan actually calls for. We refer to the total, ex-ante probability of

the behavioral types as the government's initial reputation. While credibility generally increases with reputation, the insights of the reputation literature mean in our case that credibility need not converge to zero as reputation vanishes.

A key assumption we introduce is that the government exerts imperfect control over inflation, perhaps due to underlying shocks to money demand. Imperfect control masks the government's choice of policy: the private sector understands that realized inflation is only an imperfect signal of intended inflation. We consider additive and normally distributed noise which implies that the public can never be certain of the government's action. This assumption distinguishes us in technical terms from the early studies of reputation in monetary policy referenced above, where the public perfectly observes the inflation chosen by the government. But, crucially, it also creates a smooth tradeoff for the government: overshooting the target by more creates, in expectation, a larger boom but also larger reputational losses.

When designing policy, the planner takes into account its own future behavior, which it can influence but not control. 'Future' governments have complete freedom and will only respect promises made at time 0 to the extent that it suits them. Preserving reputation turns out to be a powerful disciplining force for the planner's future self. Crucially, the value of reputation depends on the plan in place. Plans differ in the outcomes they intend to deliver and in how closely they are expected to be followed in the future, their remaining credibility. Both features contribute to current outcomes through the private sector's expectations. These forces lead the planner to weigh a plan's intended outcomes against the reputation dynamics it generates.

Our main result is that the government chooses a policy under which inflation starts high and diminishes gradually (except maybe when initial reputation is very high). Plans with gradual disinflation are more credible: having a higher target for today than tomorrow boosts the gains from sticking to the plan. This slows down the pace of reputational losses sufficiently to offset the negative effect of higher announcements on expected inflation. In contrast, the reputation literature typically considers the limit as the long-lived player becomes arbitrarily patient ([Fudenberg and Levine, 1989](#)). In that case, the government can obtain a payoff arbitrarily close to its commitment payoff by announcing a static plan with zero inflation in every period.

The gradualism of our optimal policy might lead an outside observer to conclude that there is substantial inflation inertia in the economy and that the government avoids a costly recession when bringing inflation down. However, in our model past inflation does not enter the Phillips curve. Rather, as it turns out, a plan that promises decreasing inflation is easier to keep.

A second result concerns the limit as initial reputation becomes arbitrarily small. At zero initial reputation, the only Markov equilibrium is a repetition of the static Nash with high inflation and output at the natural level. However, as is usual in the reputation literature, even a small amount of reputation creates a large departure from the Nash outcome. To make this point, we focus on the case of vanishingly small initial reputation. A discontinuity at zero reputation allows the credibility dynamics we emphasize to have an impact on the government's plan even in the limit. While the solution of the model forces us to consider all levels of reputation, we view the limiting

case as the most interesting and as a sensible refinement in the broader game played by the government and the private sector. Along the zero-reputation limit, optimal plans retain their gradualist property.

Discussion of the Literature We contribute to a long literature dealing with commitment, imperfect credibility, and reputation. The time-inconsistency of optimal policy (Kydland and Prescott, 1977) has long been recognized by researchers, who have set out to ask whether reputation can be a substitute for commitment.

We build on models such as Barro (1986), Backus and Driffill (1985), and more recently Sleet and Yeltekin (2007) and Dovis and Kirpalani (2019) who introduce reputation and behavioral types in models of monetary policy. The key departure from that literature is our assumption of imperfect control of inflation. With perfect control, all deviations by the government are detected by the private sector: on the equilibrium path, any deviation completely destroys the reputation. Imperfect control complicates the private sector's inference and enables the tradeoffs that shape our optimal plans. In these models of reputation with perfect control, gradualism is never a feature of the optimal plan.

A related literature looks at subgame perfect equilibria in games between the government and the private sector applying the tools of Abreu, Pearce, and Stacchetti (1990). However, this notion of sustainable plans (Chari and Kehoe, 1990; Phelan and Stacchetti, 2001) generally generates a large set of equilibria, which limits the theory's predictions.

There is also a large literature that makes use of imperfect control in the same way we do, along with uncertainty about the preferences of the planner. Examples include Phelan (2006), Cukierman and Meltzer (1986), Faust and Svensson (2001), among many others. We view our model with behavioral types as more directly suited to address the issue of announcements about future policy that motivates us.

Even though the announcements in our model do not constrain the actions of the rational government, they are not cheap talk, as they can be sent by only one of the behavioral types. This distinguishes us from cheap talk models of monetary policy such as Turdaliev (2010).

The recent work of King, Lu, and Pastén (2008, 2016), and Lu (2013) shares some of our ingredients. They also consider a model with imperfect control of inflation and behavioral types, but in these papers the planner has commitment power. The tradeoff becomes that the planner wants to make it clear that it is the rational type but also deliver good outcomes. If the behavioral type resembles the Ramsey plan at any point in time, these objectives create a tension. Finally, in the rational limit that we are interested in, the tension between delivering the Ramsey plan and separating from a behavioral type dissipates. These plans then simply converge to the Ramsey outcome.

Faingold and Sannikov (2011) study a general model of reputation in continuous time which applies to our framework of monetary policy with imperfect control, with the important exception that behavioral types are limited to static plans. Hence, their model cannot address the gradualism of announcements we are interested in. Faingold and Sannikov (2011) find conditions for a unique equilibrium which is Markovian in reputation, which informs our restriction to Markovian equilibria.

Layout The rest of the paper is structured as follows. Section 2 introduces our model of reputation. Notions of equilibrium are defined and discussed in Section 3. Section 4 lays out our main results and Section 5 discusses how optimal plans depend on parameters. Section 6 relates our results to other salient models. Finally, Section 7 concludes.

2. MODEL

We consider a government which dislikes inflation and deviations of output from a target y^* according to a loss function

$$L_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t [(y^* - y_t)^2 + \gamma \pi_t^2] \right] \quad (1)$$

where y_t, π_t denote output and inflation at time t , $\gamma \geq 0$ is the relative weight on inflation, and $\beta \in (0, 1)$ is a discount factor. A Phillips curve relates current output to current and expected inflation

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t [\pi_{t+1}] \quad (2)$$

where $\mathbb{E}_t$ represents the expectations operator based on information up to time t and $\kappa \geq 0$ is the slope of the Phillips curve. We assume that the government has imperfect control over inflation so that

$$\pi_t = g_t + \sigma \epsilon_t. \quad (3)$$

The government controls g_t at time t and $\epsilon \stackrel{iid}{\sim} \mathcal{N}(0, 1)$.

2.1 Reputation

We introduce reputation by considering the possibility of behavioral types for the government indexed by a set $\mathcal{C}$. A government of behavioral type $c \in \mathcal{C}$ is committed to an *inflation plan* $(a_t^c)_{t=0}^{\infty}$. An inflation plan consists of inflation announcements for each t . For convenience, we identify each behavioral type by the strategy it follows and write c and $(a_t^c)_{t=0}^{\infty}$ interchangeably.

We assume that the government is rational with probability $1 - z$. A probability distribution defined over $\mathcal{C}$ with density ν describes the distribution of possible behavioral types, which have total probability z .

2.2 Timing of play

At time 0 an announcement $a = (a_t)_{t=0}^{\infty}$ of *inflation targets* takes place. The announcement includes targets a_t for all time periods into the future.

If the government happens to be behavioral of type $c \in \mathcal{C}$, it announces $(a_t^c)_t$ for sure. The rational type of the government chooses an announcement r , possibly $r \in \mathcal{C}$. The government understands that announcing

$r \notin \mathcal{C}$ reveals rationality. After observing an announcement $c \in \mathcal{C}$, the private sector attaches probability 1 to the government being either rational or behavioral of type c .

At time $t \geq 0$, the government sets inflation. If the government happens to be behavioral of type c , it sets $g_t = a_t^c$. The rational type may instead choose g_t strategically. As described above, inflation also has a random component and so the private sector applies Bayes' rule to update beliefs about the government's type.

2.3 Beliefs

After the initial announcement, the private sector applies Bayes' rule to update beliefs about the government's type. By our discussion above, if an announcement $c \in \mathcal{C}$ has been made, the government can only be rational or of type c .

Suppose that in equilibrium the rational type announces c with density $\mu(c)$. By Bayes' rule, the posterior probability of the government being behavioral of type c , the reputation with which it will start implementing its announcement is

$$p_0(c; z, \mu) = \frac{z\nu(c)}{z\nu(c) + (1-z)\mu(c)} \quad (4)$$

At time t , the private sector's posterior of the government being of behavioral type c is formed by applying Bayes' rule to the private sector's information. Suppose that inflation π_t is realized at time t . If the government is behavioral of type c , then it must have chosen $g_t = a_t^c$ and the current shock must have been $\epsilon_t = \pi_t - a_t^c$, which has density $f_\epsilon(\pi_t - a_t^c)$. Let g_t^* denote the rational type's strategy. Then if the government is rational, it must have chosen $g_t = g_t^*$ so the shock must have been $\epsilon_t = \pi_t - g_t^*$. Therefore, updating from a prior belief of p_t , we have that

$$p_{t+1} = \frac{p_t \cdot f_\epsilon(\pi_t - a_t)}{p_t \cdot f_\epsilon(\pi_t - a_t) + (1 - p_t) \cdot f_\epsilon(\pi_t - g_t^*)}$$

It is useful to rewrite this condition as

$$p_{t+1} = p_t + p_t(1 - p_t) \frac{f_\epsilon(\pi_t - a_t) - f_\epsilon(\pi_t - g_t^*)}{p_t f_\epsilon(\pi_t - a_t) + (1 - p_t) f_\epsilon(\pi_t - g_t^*)} \quad (5)$$

which makes it evident that reputation moves when (i) it started far away from 0 and 1 and (ii) when realized inflation is closer to either the target or the rational type's strategy. Only if a_t is far away from g_t^* can actual inflation fall closer to one than to the other. Some of this intuition is leveraged heavily later on.

2.4 The set of behavioral types

We parametrize the set $\mathcal{C}$ of possible behavioral types. We assume that behavioral type c 's inflation plan is defined by three parameters (a_0, ω, χ) so that

$$a_t^c = (a_0 - \chi) e^{-\omega t} + \chi$$

This parametrization makes $\mathcal{C}$ finitely-dimensional but also allows us to write each plan recursively. For each $c \in \mathcal{C}$, $a_{t+1}^c = \chi + e^{-\omega} (a_t^c - \chi) = \phi_c(a_t)$. We also assume that all plans are bounded above by the inflation in the static Nash equilibrium: $a_0, \chi \in \mathcal{A} = [0, \pi^N]$ for any (a_0, ω, χ) .

Figure 1 illustrates some possible paths. Paths start at a_0 and converge towards χ with an exponential decay rate of ω . The set $\mathcal{C}$ contains constant, decreasing, and increasing paths, which obtain by appropriately setting a_0 and χ .

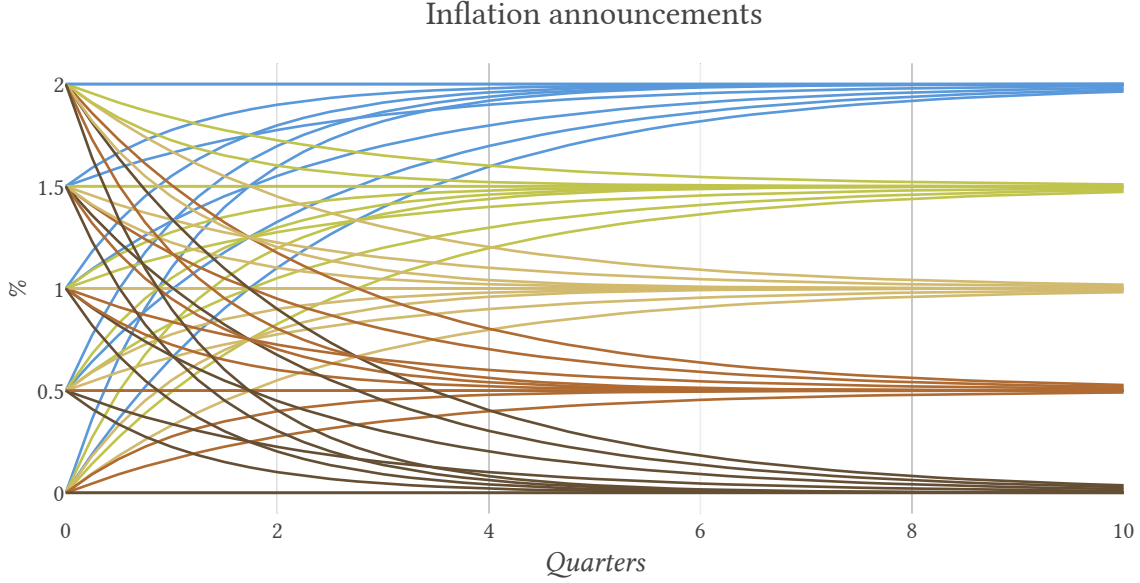


FIGURE 1: POSSIBLE BEHAVIORAL TYPES' ANNOUNCEMENTS

2.5 Bellman equations after an announcement

Given an announcement c , the problem of the rational type is to choose mean inflation g_t in period t to maximize (1) subject to (2), (3), and (5). The time- t government chooses, taking as given its reputation p_t , its future strategy, and the private sector's expectations about the behavioral and rational types' choices.

At time t , the private sector expects the behavioral type to choose $g_t = a_t^c$. Let g_t^* denote the private sector's expectations of the rational type's choice. We focus on Markovian strategies with $g_t^* = g^*(p_t, a_t^c)$. This allows us to write the rational government's problem recursively as

$$\mathcal{L}^c(p, a) = \min_g \mathbb{E} [(y^* - y)^2 + \gamma \pi^2 + \beta \mathcal{L}^c(p', \phi_c(a))] \quad (6)$$

$$\text{subject to } \pi = g + \epsilon$$

$$\pi = \kappa y + \beta [p' \phi_c(a) + (1 - p') g^*(p', \phi_c(a))]$$

$$p' = p + p(1 - p) \frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^*(p, a))}{p f_\epsilon(\pi - a) + (1 - p) f_\epsilon(\pi - g^*(p, a))}$$

Problem (6) illustrates how the government best-responds to the public's beliefs $g^*(p, a)$. In the Phillips curve, expected inflation is a weighted average between $\phi_c(a)$, the action of the behavioral type (in the next period), and $g^*(p', \phi_c(a))$, the conjectured choice of the rational type. This gives the government a degree of freedom: it can influence expected inflation by affecting its reputation.

By choosing different levels of inflation today, the rational type can, on average, affect the evolution of reputation. Problem (6) shows that changes in p' affect the government in three ways: by altering the weighting in the computation of expected inflation, by altering the expected action of the behavioral type $g^*(p', \phi_c(a))$, and by altering the continuation value $\mathcal{L}^c(p', \phi_c(a))$.

At low levels of p , the Phillips curve puts most of the weight on the expected choice of the rational type. Therefore, at low levels of p , the government affects inflationary expectations mostly through $g^*(p', \phi_c(a))$. If future governments are expected to value their reputation and choose $g^*(p, a)$ close to a when $p > 0$, the current government has an incentive not to let its reputation go to zero and, therefore, choose the current g^* close to a .

3. EXPECTATIONS AND EQUILIBRIUM

A solution to (6) describes the government's choices $g(p, a)$ as a function of the private sector's expectations g^* . Our equilibrium definition makes it clear that rational expectations requires finding a fixed point of that function.

Definition Given an announcement $c \in \mathcal{C}$, a *continuation equilibrium* consists of a loss function $\mathcal{L}^c : [0, 1] \times \mathcal{A} \rightarrow \mathbb{R}$ and a policy function $g_c^* : [0, 1] \times \mathcal{A} \rightarrow \mathbb{R}$ such that

1. The loss function $\mathcal{L}^c$ solves the government's Bellman equation (6) taking as given expectations g_c^*
2. g_c^* is the policy function that corresponds to the solution of (6)

A useful property of continuation equilibria follows from close observation of problem (6): given the decay and asymptote parameters, it is equivalent to start the plan at a different initial announcement a or to just have arrived at a current announcement a as the continuation equilibrium unfolded.

Observation Suppose $(\mathcal{L}, g^*)$ is a continuation equilibrium for announcement $c = (a_0, \chi, \omega) \in \mathcal{C}$. Then for any b_0 , the same pair $(\mathcal{L}, g^*)$ is a continuation equilibrium for plan $c' = (b_0, \chi, \omega)$.

Lemma 1. *In any continuation equilibrium, the rational type's reputation is a supermartingale:*

$$\mathbb{E}[p_{t+1} \mid \text{rational}, \mathcal{F}_t] \leq p_t$$

That is, the planner cannot design a policy that generates expected reputational gains.

Conditional on a rational government, either $g^*(p, a) = a$, in which case $p' = p$ a.s., or $g^*(p, a) \neq a$, in which case π is a signal centered away from a , which is revealing on average.

One reason why one might think a planner prefers a gradual disinflation is that gradualism allows the planner to accumulate credibility by delivering on ‘easy’ promises at first in order to be able to grow more ambitious. Lemma 1 says that the planner cannot strategize in this way. It cannot design its plan in a way that makes it expect to increase reputation over time. What the planner can do is to design its plan in a way that provides incentives to deliver on it.

Definition Given an initial reputation z , a *reputational equilibrium* is a distribution μ_z over $\mathcal{C}$ along with continuation equilibria $\{\mathcal{L}^c, g_c^*\}_{c \in \mathcal{C}}$ and a posterior reputation $p_0 : \mathcal{C} \rightarrow [0, 1]$ such that

1. Posterior reputation is set according to Bayes’ rule (4), given the distribution μ_z .
2. The distribution of mimicked types μ_z minimizes the starting reputation-adjusted loss function

$$\mathcal{L}_r^*(\mu_z, z) = \int_{\mathcal{C}} \mathcal{L}^c(p_0(c), a_0(c)) d\mu_z(c)$$

taking as given the starting reputation function p_0 .

Notice that, as a consequence of 2, in a reputational equilibrium the planner is indifferent among plans in the support of μ_z and prefers them to plans outside the support

$$\begin{aligned} \mathcal{L}^c(p_0(c), a_0(c)) &= \mathcal{L}^{c'}(p_0(c'), a_0(c')) && \text{for } c, c' \in \text{supp}(\mu_z) \\ \mathcal{L}^c(p_0(c), a_0(c)) &\leq \mathcal{L}^{c'}(1, a_0(c')) && \text{for } c \in \text{supp}(\mu_z), c' \notin \text{supp}(\mu_z) \end{aligned}$$

where we highlight the fact that announcements which are not made in equilibrium grant full reputation: $p_0(c) = 1$ for all $c \notin \text{supp}(\mu_z)$.

Definition An *equilibrium with vanishingly small reputation* is the limit of reputational equilibria as $z \rightarrow 0$.

$$\mu^* = \lim_{z \rightarrow 0} \mu_z$$

An important part of finding a reputational equilibrium is determining which plans are announced and which ones are not. A plan c can only be outside the support if its expected loss (at full reputation) is still greater than what the plans that are played deliver.

An alternative definition of equilibrium follows Kambe (1999) and does away with the initial inference by the private sector. Instead, it corresponds to the case where the government first announces a plan c and subsequently becomes committed to following it with some exogenous probability p_0 , independent of what c is.

Definition For a given $p_0 \in [0, 1]$, a *K-equilibrium* is an announcement c and a continuation equilibrium $\{\mathcal{L}^c, g_c^*\}$ such that c minimizes the loss function

$$c_K^*(p_0) = \arg \min_c \mathcal{L}^c(p_0, a_0(c))$$

Once more, in our application we are especially interested in $\lim_{p_0 \rightarrow 0} c_K^*(p_0)$.

Finding a *K-equilibrium* is simple. For given plan c and reputation p_0 , the value of starting plan c at reputation p_0 , $\mathcal{L}^c(p_0, a_0(c))$ can be found applying our previous discussion. We then optimize over this function by choosing c in $\mathcal{C}$, keeping p_0 fixed.

To find a reputational equilibrium, we proceed as follows: given $k \in \mathbb{R}$, we partition the space of plans according to whether

$$\mathcal{L}(1, c) \leq k$$

Plans that have a loss greater than k are assigned probability zero of being played, $\mu(c) = 0$. For the remainder of plans, we find a probability $p_0(c)$ that delivers k by requiring

$$\mathcal{L}(p_0(c), c) = k$$

For plan c to start with a reputation of $p_0(c)$, the initial application of Bayes' rule (4) tells us that c must be played with a probability $\mu(c)$ such that

$$p_0(c) = \frac{z\nu(c)}{z\mu(c) + (1 - z)\mu(c)}$$

Finally, the planner's strategy is required to be a probability distribution. Therefore, we pin down k by requiring that at the end of this process the non-negative function $\mu(c)$ integrates to 1 over the set of possible plans $\mathcal{C}$.

3.1 Reputation-building incentives

First-order conditions in the government's problem involve critically the marginal effect of inflation on output and on future reputation. Solving for output in the Phillips curve yields that output is affected by inflation according to

$$\frac{\partial y}{\partial \pi} = \frac{1}{\kappa} \left[1 - \beta \frac{\partial p'}{\partial \pi} \left(\phi_c(a) - g^*(p', \phi_c(a)) + (1 - p') \frac{\partial g^*(p', \phi_c(a))}{\partial p'} \right) \right] \quad (7)$$

Inflation affects current output through three different channels, corresponding to three terms in equation (7). The first term, $\frac{1}{\kappa} \cdot 1$, describes the standard, direct effect of inflation on output.

The second term, $\beta \frac{1}{\kappa} \left(-\frac{\partial p'}{\partial \pi} \right) (\phi_c(a) - g^*(p', \phi_c(a)))$, describes an expectation-shifting effect by which more inflation reduces the posterior p' and therefore moves expectations of future inflation away from the target $\phi_c(a)$ and toward the expected choice of the rational type $g^*(p', \phi_c(a))$.

Finally, the third effect is given by $\beta \frac{1}{\kappa} \left(-\frac{\partial p'}{\partial \pi} \right) (1 - p') \frac{\partial g^*(p', \phi_c(a))}{\partial p'}$. It describes how more inflation today moves the expected choice of future rational governments through its effect on their reputation.

3.2 Reputation and credibility

Let π^N be the (mean) level of inflation in the Nash equilibrium of the stage game or, equivalently, in a Markov equilibrium when $p = 0$. First-order conditions of the government's choice in this case imply that

$$\pi^N = y^* \frac{\kappa}{1 - \beta + \kappa^2 \gamma}$$

Lemma 2. *In any continuation equilibrium, the rational type's choice of inflation is bounded above by the choice in the Nash equilibrium of the stage game:*

$$\forall c \in \mathcal{C} : \quad g_c^*(p, a) \leq \pi^N$$

We define the credibility of a plan as the ratio of announced and (expected) realized inflation, normalized by their distance from Nash inflation.

Definition Given a plan c , its *remaining credibility* in state (p, a) is

$$\begin{aligned} C(p, a; c) &= \mathbb{E} \left[(1 - \beta) \frac{\pi^N - \pi}{\pi^N - a} + \beta C(p'_c(p, a), \phi_c(a)) \right] \\ &= (1 - \beta) \frac{\pi^N - [pa + (1 - p)g_c^*(p, a)]}{\pi^N - a} + \beta \mathbb{E} [C(p'_c(p, a), \phi_c(a))] \end{aligned} \quad (8)$$

where π^N is Nash inflation. The *credibility* of a plan in a K -equilibrium is then given by

$$C^K(c) = \lim_{p \rightarrow 0} C(p, a_0(c); c)$$

while in a reputational equilibrium it is

$$C^*(c) = \lim_{z \rightarrow 0} \int C(p_0(c), a_0(c); c) d\mu_z(c)$$

4. ANALYSIS AND NUMERICAL RESULTS

We solve the model numerically for different announcements $c \in \mathcal{C}$.

4.1 Parametrization

We parametrize our model following [King, Lu, and Pastén \(2016\)](#) and pick our preference and technology parameters γ, κ, y^* consistently with the planner's objective function and Phillips curve in a standard New Keynesian economy calibrated to US data ([Galí, 2015](#); [Galí and Gertler, 1999](#)). Table 1 summarizes our parameter choices.

Parameter	Value	Definition	Source / Target
β	0.995	Discount factor	2% real interest rate
γ	60	Inflation weight	King, Lu, and Pastén (2016)
σ	1%	Std of control shock	King, Lu, and Pastén (2016)
κ	0.17	Slope of Phillips curve	King, Lu, and Pastén (2016)
y^*	5%	Output target	King, Lu, and Pastén (2016)

TABLE 1: BENCHMARK CALIBRATION

4.2 Continuation equilibrium after announcement c

Figure 2 shows a typical value function $\mathcal{L}^c(p, a)$ for an arbitrary plan c . All plots have current reputation p in the x -axis. Darker lines correspond to lower current announcements a .

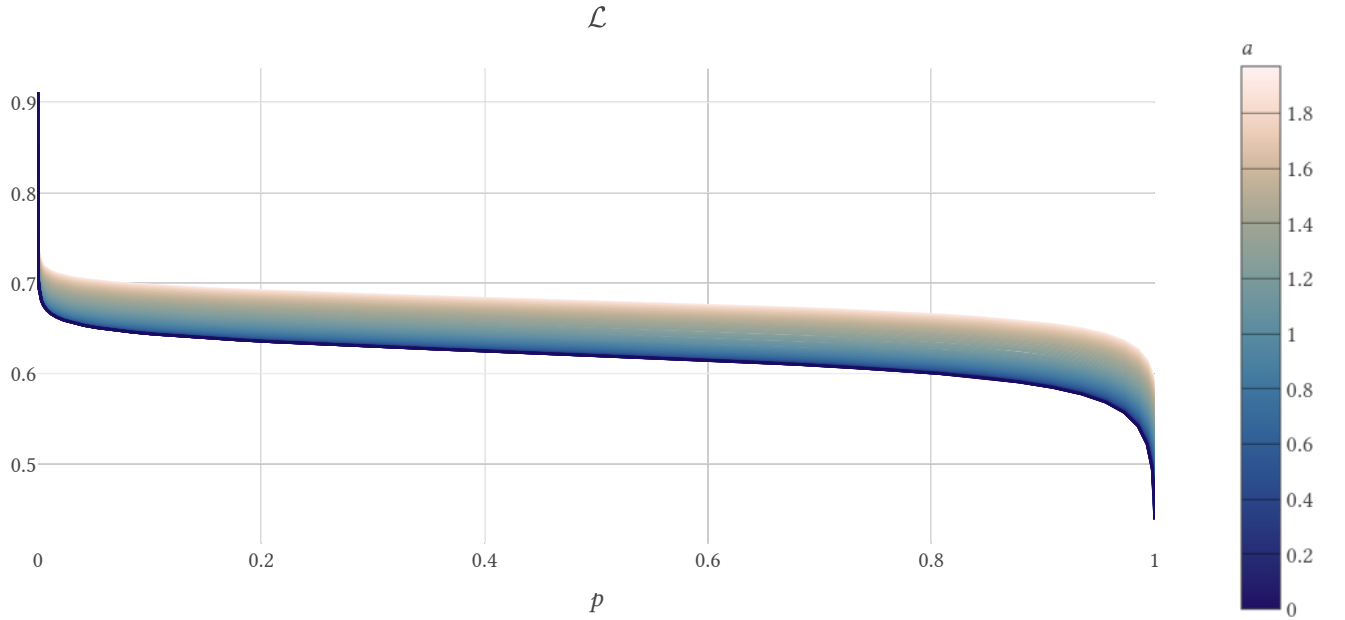


FIGURE 2: LOSS FUNCTION AFTER ANNOUNCEMENT c

There are three observations to make. First, $\mathcal{L}$ is decreasing in p . An increase in reputation generally decreases expected inflation leading to higher current output.

Second, the loss function has a convex-concave shape, which reflects the dynamics of reputation. A value close to either 0 or 1 for reputation means that the public is confident in its assessment of who it faces. As reputation approaches the bounds, more evidence is required to make it move. An increase (decrease) of the same magnitude is hard to revert when reputation is close to 1 (0), which makes the government attach a larger value to it.

Finally, at high levels of reputation, a lower current target a is unambiguously good. When p is high, lower a mostly means lower inflation expectations. However, when p is small a lower a also means a larger expected loss of reputation (as a more ambitious target fosters a larger deviation), which makes more modest targets preferable.

On the other hand, as reputation decreases, the gap between a -lines shrinks as two effects arise. The first effect is that with lower reputation the current announcement becomes less relevant as its weight in expected inflation decreases. The second effect concerns the government's choice and how close to a it chooses to set mean inflation. If at low reputation the government chooses to deviate more from lower announcements, this second force might make it prefer a higher announcement today.

Figure 3 shows deviations from the current target as a function of current reputation and target. It confirms that as reputation approaches zero governments tend to deviate more from their target. The figure also reveals a discontinuity at zero reputation, where the government reverts to Nash inflation regardless of announcements. When reputation is exactly zero, Bayes' rule prevents it from moving. This makes the government entirely disregard the plan. By the same logic, a planner with zero reputation is indifferent across all plans, as they all yield the stage Nash payoff at $p = 0$.

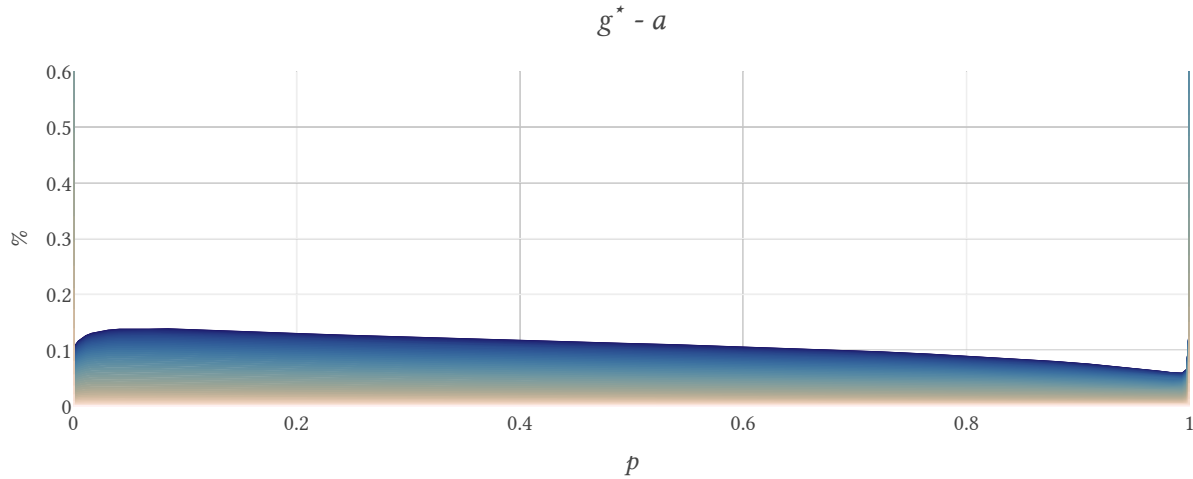


FIGURE 3: INFLATION DEVIATIONS

The effect of reputation p on the deviation $g^*(p, a) - a$ is complicated and arises from the sum of many forces. On the one hand, a larger stock of reputation makes the planner more inclined to spend it. Moreover, at higher levels of reputation Bayes' rule implies that reputation is more difficult to lose, which increases incentives to gamble. But on the other hand, at higher reputation delivering on the announcement is less costly, especially when the current announcement is also high.

A higher current announcement a has a more clear effect on the deviation: the lower a , the further away from it will the rational type set inflation. The reason is simple: getting inflation close to target rewards the government in roughly the same way, but it is more costly to set inflation close to target when the target is lower.

Figure 4 shows average reputation p' as function of current reputation p and announcement a . First, $\mathbb{E}[p']$ is always below p , as predicted by Lemma 1. At the highest announcement we consider, which coincides with the Nash equilibrium of the stage game, by definition the government has no incentives to deviate so it chooses $g = a$ for all levels of p . As a result, reputation does not move. For all announcements lower than Nash inflation, reputation falls on average.

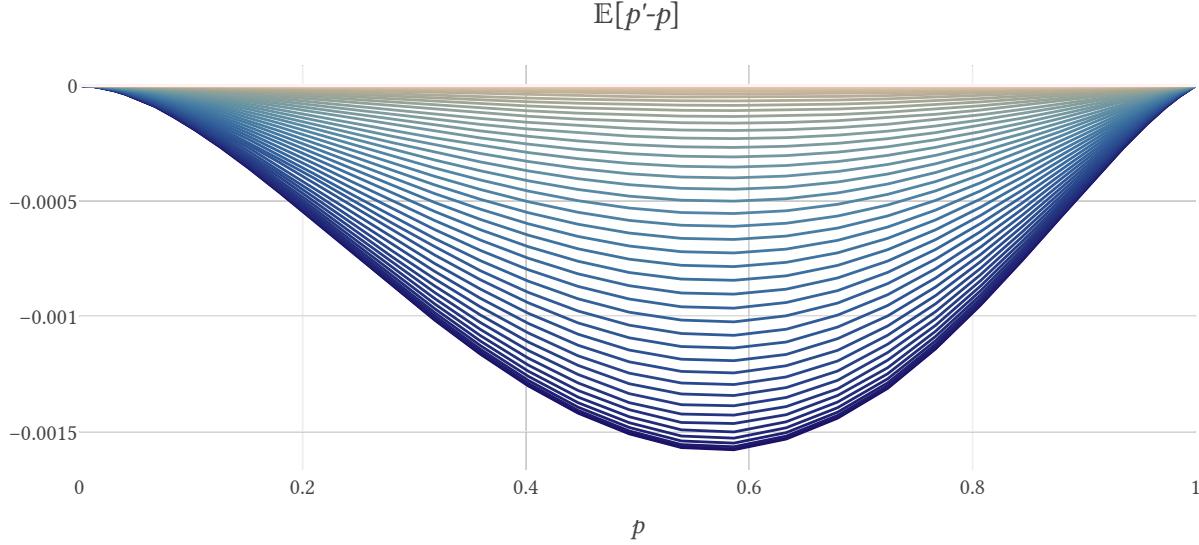


FIGURE 4: EXPECTED REPUTATION LOSSES

Second, lower announcements are associated with a larger expected reputation loss. Lower current announcements generate weaker incentives to deliver target inflation: as the temptation to inflate grows larger, the government prefers to spend more of its reputation to achieve more output.

Third, Bayes' rule forces p' to be close to p when p is close to either 0 or 1. However, the picture looks skewed to the right, which means that the government 'spends' more reputation when it has more of it. This is especially true at high levels of p , consistent with Figure 3. At low levels of reputation, the government expects to lose more reputation when its current target a is lower.

4.3 Announcements in the K-equilibrium

Figure 5 shows the K-equilibrium as a function of p_0 , which is simply the loss-minimizing plan conditional on starting with reputation p_0 . The top panel shows the decay rate $1 - e^{-\omega}$ (in percent terms) while the bottom panel shows the choice of initial inflation a_0 and asymptote χ .

At $p_0 = 1$, any announcement is believed by the private sector, regardless of expectations about the behavior of the rational type. The planner sets expectations at their most advantageous level by promising zero inflation throughout. The rational type intends to break this promise, given that at full reputation the private sector never

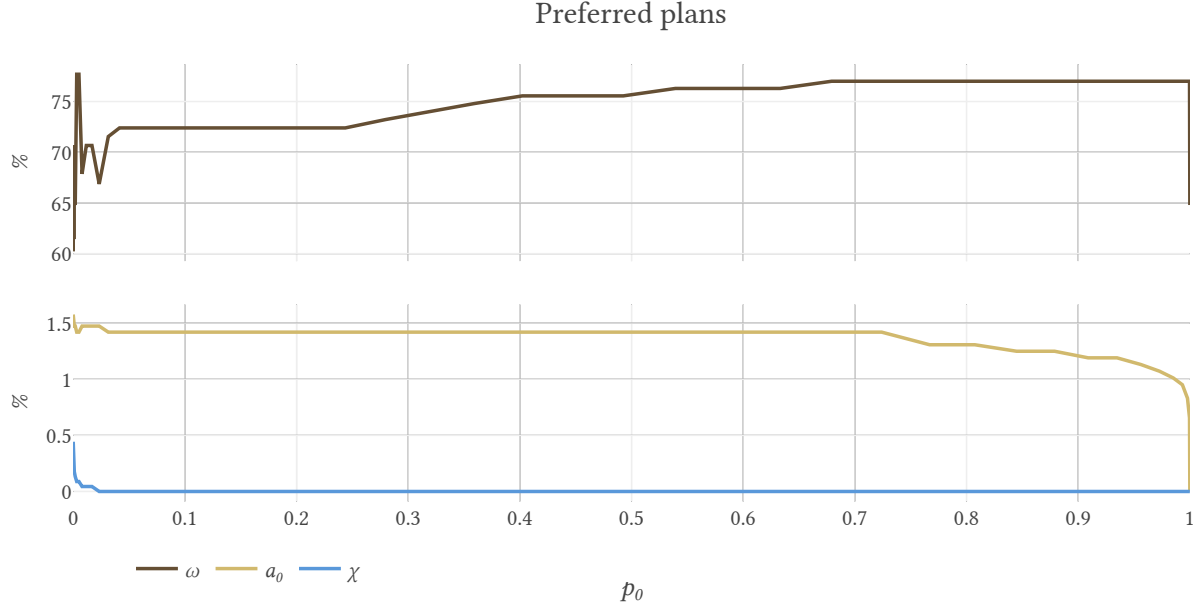


FIGURE 5: PREFERRED PLANS WITH DIFFERENT p_0

learns. As soon as the initial posterior p_0 is less than one, the planner starts caring about incentivizing future governments to behave and conserve reputation. This leads the planner to prefer plans that have a higher initial inflation a_0 . The planner also chooses plans that make inflation decrease over time by setting $a_0 > \chi$, meaning that the planner attempts a gradual disinflation. This property holds even as p_0 approaches zero.

Figure 6 shows the determination of the K -equilibrium when p_0 is small. For each decay ω and asymptote χ we plot the minimized loss function $\min_{a_0} \mathcal{L}(p_0, (a_0, \omega, \chi))$. The x -axis moves ω while different curves plot different values of χ .

Some patterns are evident from Figure 6. The overall minimum is achieved at a point with both $\omega, \chi > 0$: the K -equilibrium has the initial planner promise a gradual disinflation that does not converge to the first best level of zero inflation.

When χ is small, plans eventually imply very low levels of inflation which makes them more difficult to sustain: reputation is lost quickly when a approaches zero. This gives rise to unfavorable continuation values as the government is revealed to be rational and reverts back to the high-inflation stage Nash. For this reason, at low χ the planner prefers to make the decay rate slow by choosing ω as low as possible. This way, the plan only promises very low inflation in the far future. When χ is higher, the planner uses a decay rate that provides incentives even in the short run. These values of χ turn out to be preferred.

Finally, χ cannot grow too much either. As χ approaches Nash inflation, the plan becomes arbitrarily easy to keep, but provides very small gains.

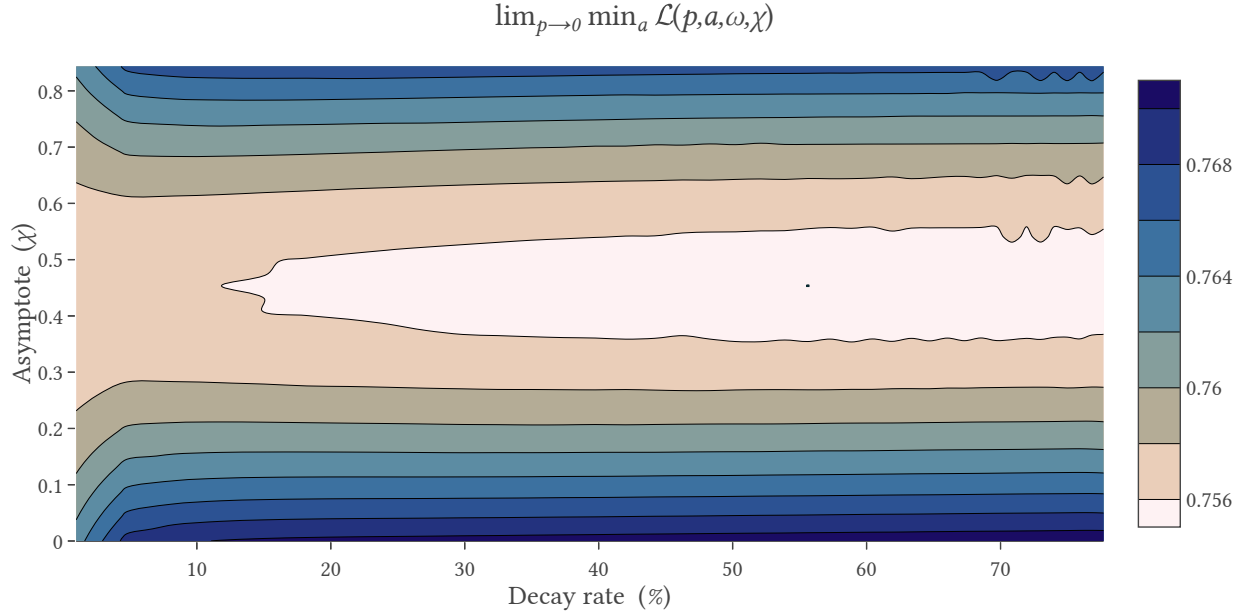


FIGURE 6: LOSS FUNCTION ACROSS ANNOUNCEMENTS

4.4 Credibility

Our setup distinguishes reputation p , the posterior that the government is the behavioral type that was announced, from credibility $C(p, a; c)$, the expected discounted deviations from plan c at reputation p and current announcement a , as defined in (8). Figure 7 plots the credibility of different plans at vanishingly small reputation, as a function of the decay rate ω and the asymptote χ , for the loss-minimizing initial inflation a_0 at those parameters.

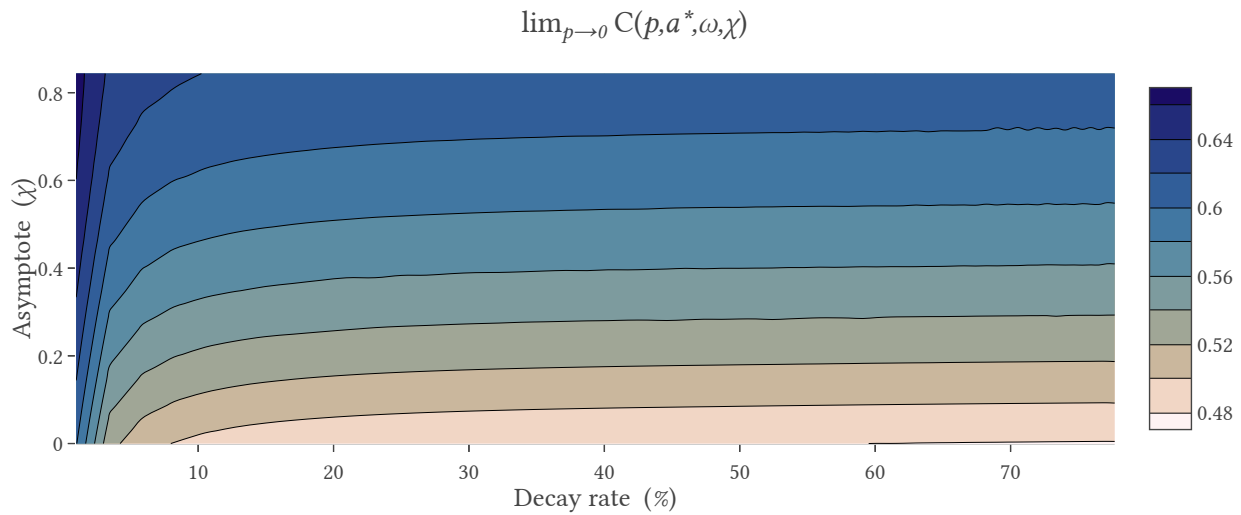


FIGURE 7: CREDIBILITY

Plans with a lower asymptote are less credible, as are plans with a steeper promised descent of inflation. One should be careful about this result, as it mostly reflects the fact that plans with fast decay reach a phase in which the targets are almost constant more quickly.

4.5 Distribution of announcements in the reputational equilibrium

We now turn to a description of the reputational equilibrium and its distributions μ_z . Figure 8 plots the average plan as a function of initial reputation z .

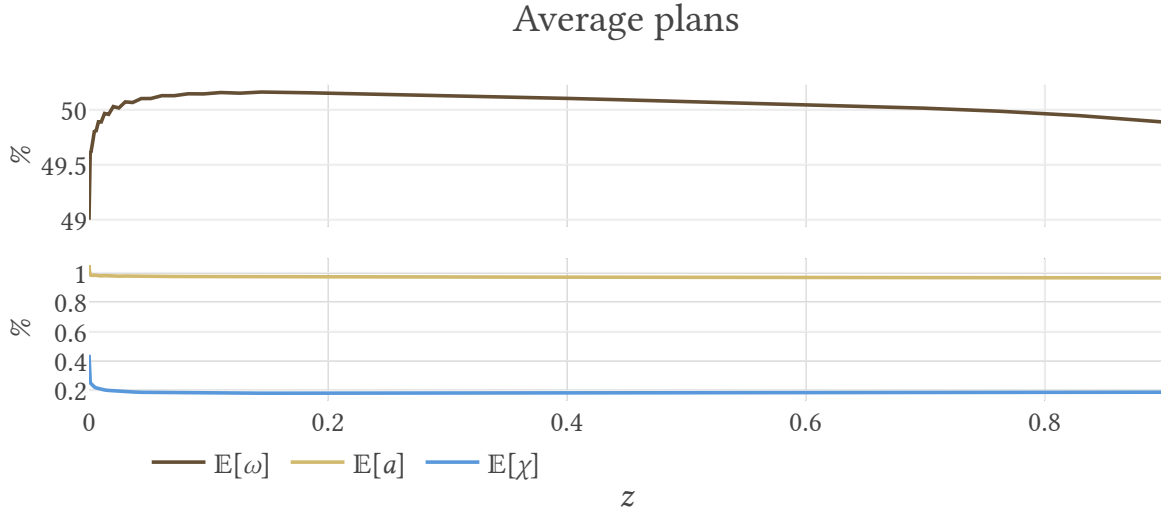


FIGURE 8: REPUTATIONAL EQUILIBRIUM ANNOUNCEMENTS

For intermediate values of initial reputation z , the planner chooses (on average) a disinflation path that starts from about half Nash inflation and converges towards a tenth of it, by about half the distance each period. As initial reputation becomes very small, as before, the planner starts to put more weight on plans that converge toward a higher asymptote χ .

For extremely low initial reputation, Figure 9 shows the limiting distribution $\mu^* = \lim_{z \rightarrow 0} \mu_z$. The left panel shows the distribution of types as a function of the asymptote χ and initial inflation a_0 , integrating over the decay rate ω , while the right panel integrates over initial inflation. Figure 9a on the left shows that the planner tends to choose gradual plans with higher initial inflation a_0 than asymptote inflation χ . This probability is $\mathbb{P}(a_0 > \chi) = 71.3\%$. Moreover, the planner chooses plans whose initial inflation is at least five times the asymptote about half the time, $\mathbb{P}(a_0 > 5\chi) = 18\%$. While the level of initial inflation varies from plan to plan, the asymptote seems more precisely set: the density of a_0 and χ in the reputational equilibrium announcement falls sharply for χ away from its maximum, while it stays flat over many more values of initial inflation a_0 .

On the right, Figure 9b bears a close resemblance to Figure 6 which plotted the K -equilibrium loss function at low p_0 . Plans with a lower loss in the K -equilibrium are good for the planner, so they are announced more often.

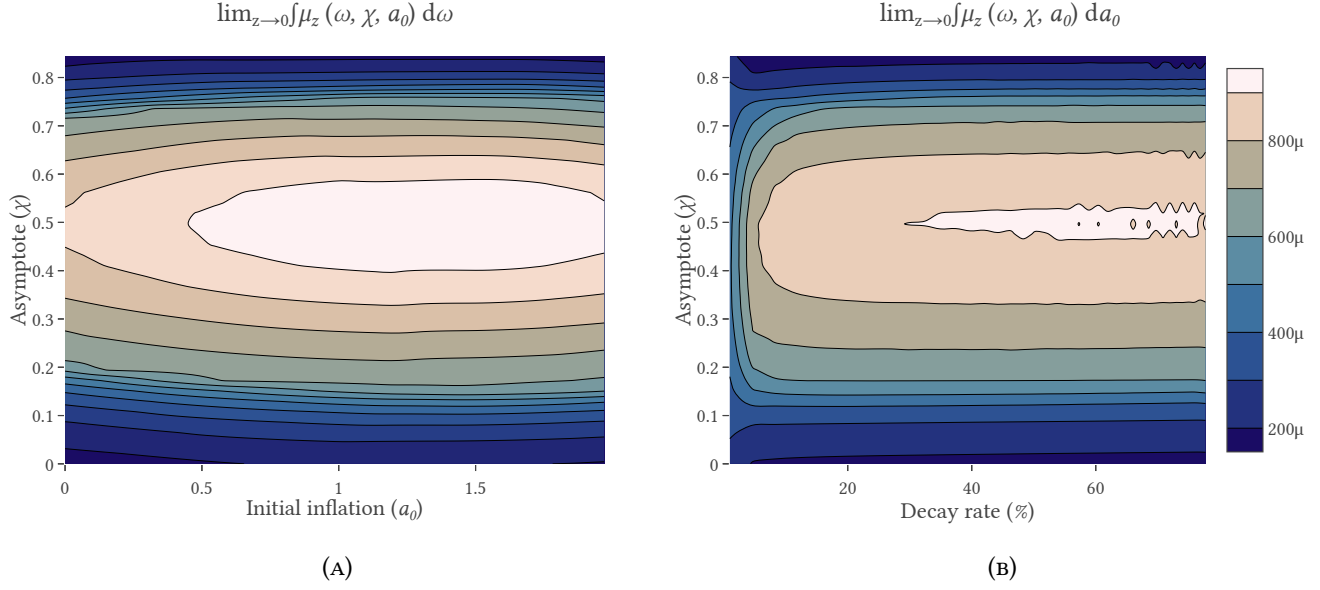


FIGURE 9: DISTRIBUTION OF TYPES

Hence, the planner starts with lower reputation in those plans, which lowers their value in the equilibrium. This initial update of reputation (from z to p_0) makes the planner indifferent across all plans (announced in equilibrium), which ultimately justifies the mixed strategy of announcement. There are however some differences between Figures 6 and 9b: the planner does not appear to play plans with a low decay rate, as the density μ^* falls sharply as ω becomes small. Other than this, the density of announcement also seems flat along the decay rate dimension, as happened with the initial level of inflation.

5. COMPARATIVE STATICS

Figure 10 shows the average plan announced in the reputational equilibrium, as a function of the variance of the control shock σ . Broadly speaking, more noise in the control makes deviations less observable. Therefore, the level of adherence to plans decreases with the noise. This makes the planner choose less ambitious plans when the control over inflation is less tight: as σ increases, the average plan has a higher asymptote χ , a slightly higher starting point a_0 , and a slower rate of decay ω .

Figure 11 repeats the exercise varying the discount factor β and the slope of the Phillips curve κ . It reveals some subtleties in the manipulation of the three parameters that describe our plans.

Figure 11a shows the average plan as a function of the discount rate $1/\beta - 1$ (whose benchmark value is 2% in annual terms). As the planner becomes more impatient, average plans start higher and converge to lower inflation, with a faster decay rate. With more impatience, the public expects a larger inflation bias. For this reason, the planner tends to choose plans that are more ‘resilient.’ Increasing initial inflation makes the plan easier to

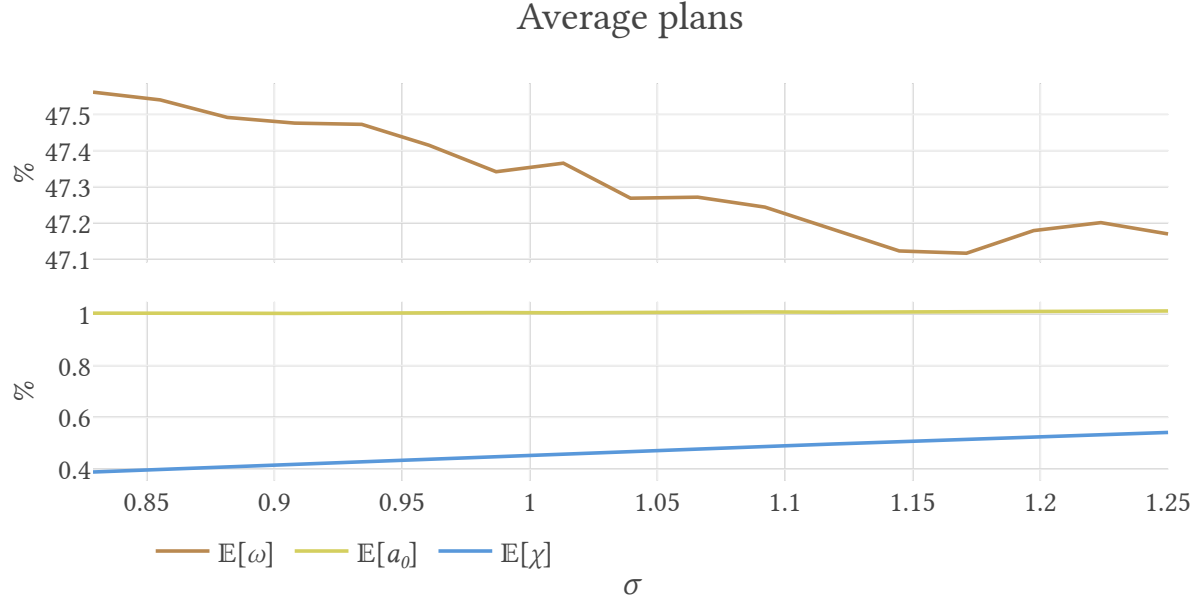


FIGURE 10: AVERAGE PLANS AND THE CONTROL SHOCK VARIANCE

keep, while decreasing asymptotic inflation makes it more costly to deviate early on. Having a steeper descent of inflation contributes to both objectives.

When we vary the slope of the Phillips curve, the planner chooses to announce lower inflation throughout. Figure 11b shows that when the Phillips curve is steeper (meaning that the same increase in current inflation produces a smaller output boom), the planner chooses to lower targets for inflation. Here the logic is that with a steep Phillips curve there are weaker incentives to create surprise inflation, which allows the planner to announce less inflation throughout.

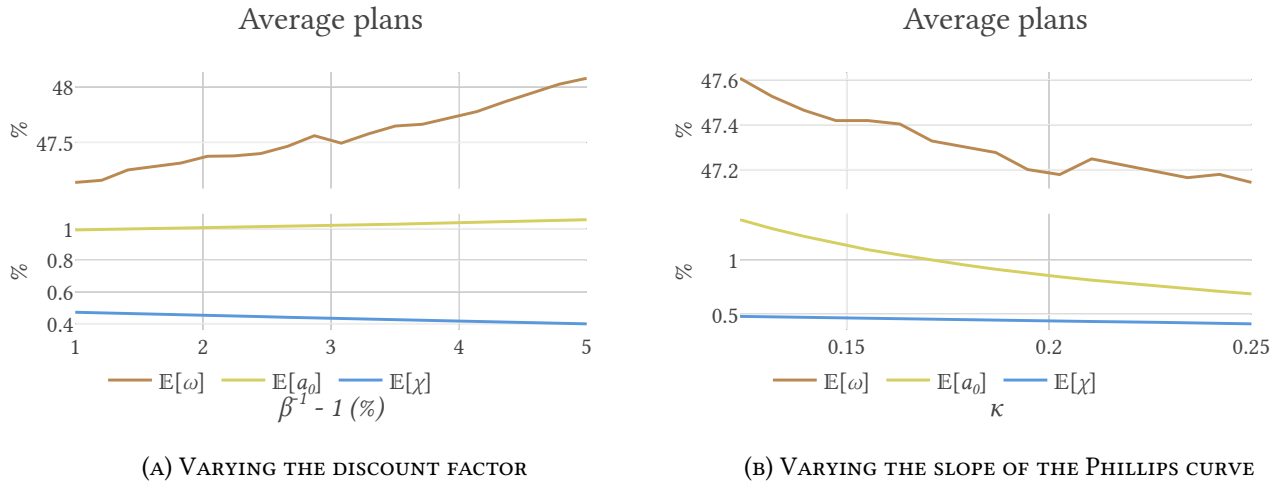


FIGURE 11: AVERAGE PLANS

6. OTHER MODELS

Our benchmark model of reputation with imperfect control yields gradual disinflation plans. We dissect this result by comparing our model to salient models in the literature on sustainable plans. We start from the Ramsey plan and slowly enrich the structure of the model to see which mathematical features create the incentive for gradualism.

6.1 The Ramsey plan

We refer to choosing an entire path $\{g_t\}_t$ with commitment to minimize (1) subject to (2) and (3) as the *Ramsey plan*. The linear-quadratic structure of this problem allows us to disregard the issue of imperfect control: the presence of the shocks ϵ_t only affects the planner through a variance term independent of policy.

Standard techniques allow us to use Lagrange multipliers of each time- t constraint to write the problem recursively for $t > 0$ as

$$v^{FB}(\theta) = \max_{\theta'} \min_{y, \pi} (y - y^*)^2 + \gamma \pi^2 + \theta'(\pi - \kappa y) - \theta \pi + \beta v^{FB}(\theta') \quad (9)$$

Problem (9) produces policy functions $g_\pi^{FB}(\theta)$, $g_y^{FB}(\theta)$, $g_\theta^{FB}(\theta)$ that describe the planner's actions for each period. At time 0, the planner does not carry any multipliers from the past and attains a value

$$f^{FB} = v^{FB}(0) \quad (10)$$

To reconstruct the plan, we recursively apply the policy functions g^{FB} , starting from the solution θ^{FB} to (10)

$$\theta_t = \begin{cases} g_\theta^{FB}(\theta_{t-1}) & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases} \quad \text{and} \quad \pi_t = g_\pi^{FB}(\theta_t) \quad (11)$$

Figure 12 plots the Ramsey plan against the average announcement in the reputational equilibrium, as well as the announcement in the K -equilibrium. The power of commitment enables a complete disinflation: Ramsey inflation is basically zero about a year and a half after the announcement of the plan. The Ramsey plan also starts from a high level, about three quarters of the way to Nash inflation. This is because the planner understands that inflation in the first period of the plan does not affect past inflation, which is sunk at the moment of the announcement. Inflation then comes down as the planner smooths the benefits of initial inflation on output over a few of the initial periods.

In contrast, the reputational equilibrium announcements do not converge to zero (Figure 9a shows a low probability that $\chi = 0$). The K -equilibrium announcement, which minimizes the loss conditional on starting from a very low (but constant) level of reputation, starts above the Ramsey plan. It needs high initial inflation to create the descent that fuels incentives. Turning to the average reputational equilibrium announcement, it starts

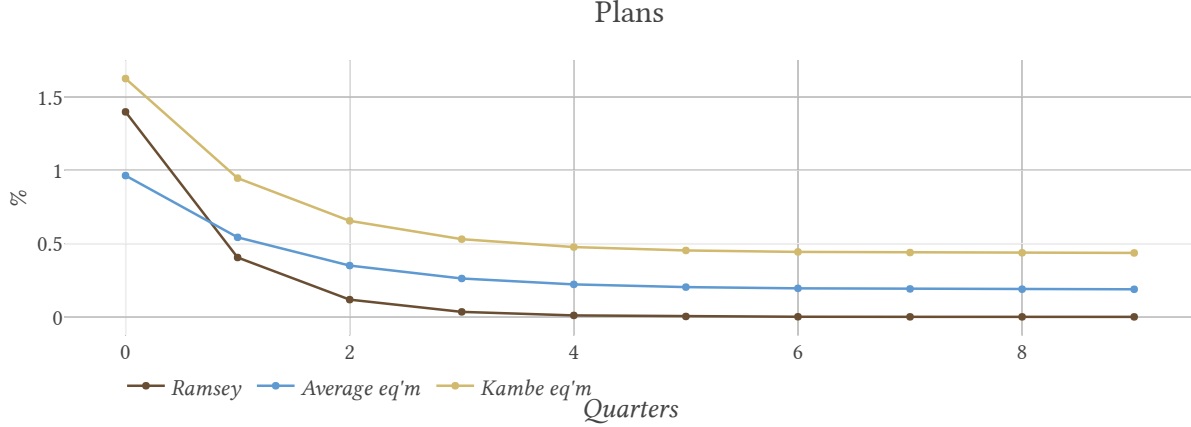


FIGURE 12: THE RAMSEY PLAN AND EQUILIBRIUM ANNOUNCEMENTS

below the K -equilibrium plan. This is because some of the plans in the support of the announcement distribution do not resort to such high initial inflation: as they are announced less often, they start with higher reputation. This makes them attractive even as their credibility would be lower at the same p .

6.2 Sustainable plans with expectations as threats

The Ramsey plan can be decentralized by choosing appropriately high inflation expectations ξ in case of deviation. We start by considering a world in which control over inflation is perfect. Letting $p \in \{0, 1\}$ be the indicator of whether the planner is keeping past promises,

$$\begin{aligned}
 v^\xi(p, a) &= \min_{y, \pi, a'} (y - y^*)^2 + \gamma \pi^2 + \beta v^\xi(p', a') \\
 \text{subject to } \pi &= \kappa y + \beta (p' g_\pi^\xi(1, a') + (1 - p') \xi) \\
 p' &= \begin{cases} 1 & \text{if } \pi = a \\ 0 & \text{otherwise} \end{cases}
 \end{aligned} \tag{12}$$

where as our main analysis $g_\pi^\xi(a)$ is the equilibrium policy function (when the punishment is ξ) at state a , which the planner takes as given but coincides with its v^ξ -minimizing choice. In the first period, the planner is also allowed to choose the initial a_0 , attaining a value

$$\mathcal{F} = \min_a v^\xi(1, a) \tag{13}$$

We let the planner start with ‘full reputation’ ($p = 1$) as in this interpretation p stands for whether or not the planner has deviated from the path. Figure 13 plots sustainable plans for different values of ξ , along with the Ramsey plan. We recover a well-known result from the literature: when the punishment is harsh enough, (12) recovers the Ramsey plan. However, when the Ramsey plan is not sustainable, the planner deviates from

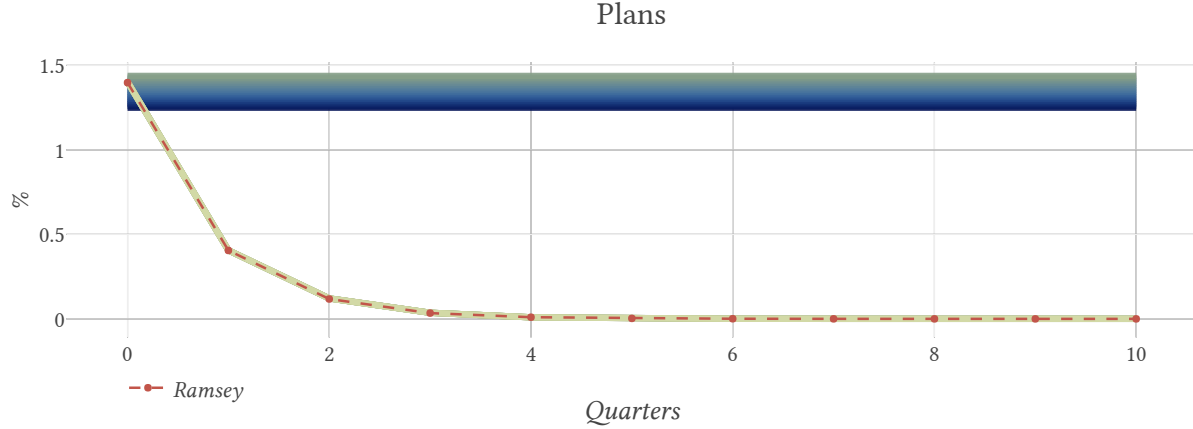


FIGURE 13: SUSTAINABLE PLANS AT DIFFERENT PUNISHMENTS

any plan. ξ then only affects the level of inflation expectations that the planner, who deviates immediately, is best-responding to.

6.3 Sustainable plans with reverting triggers (inspired by [Green and Porter, 1984](#))

The Ramsey plan is built on the fact that all deviations are detected. In this case, the private sector can threaten to, in our language, send the reputation to zero. With imperfect control, this is no longer the case. The private sector understands that any threat might be triggered on-path without actual deviations having taken place. Assessing punishment strategies needs to weigh deterrent against the costs from false positives. This is why we consider a version in which, like in [Green and Porter \(1984\)](#), the private sector shifts to a ‘punishment regime’ whenever realized inflation deviates from the target by more than some threshold, and reverts stochastically from it.

An equilibrium with reverting triggers is parametrized by a distance D between realized and announced inflation that triggers the punishment, a probability of return θ to the normal regime, and ‘punishing’ expectations ξ . We have

$$\begin{aligned}
 v^G(a) &= \min_{g, a'} \mathbb{E} [(y - y^*)^2 + \gamma \pi^2 + \beta (p' v^G(a) + (1 - p') v^P)] \\
 \text{subject to } \pi &= g + \epsilon \\
 \pi &= \kappa y + \beta (p' g^G(a') + (1 - p') \xi) \\
 p' &= \begin{cases} 1 & \text{if } \frac{|\pi - a|}{a} < D \\ 0 & \text{otherwise} \end{cases}
 \end{aligned} \tag{14}$$

with, as the control shock only leaves a variance term like before,

$$v^p = \min_{\pi, a'} (y - y^*)^2 + \gamma \pi^2 + \beta (\theta v^G(a) + (1 - \theta)v^p) + \sigma_\epsilon^2 \left(\gamma + \frac{1}{\kappa^2} \right) \quad (15)$$

subject to $\pi = \kappa y + \beta \xi$

where, once more, in the first period the planner can choose the very first announcement in the G regime ($p = 1$) attaining a value

$$f^G = \min_a v^G(a) \quad (16)$$

Figure 14 shows the path of mean inflation $g^G(a_t)$ assuming that the punishment regime is never triggered, for different levels of ξ .

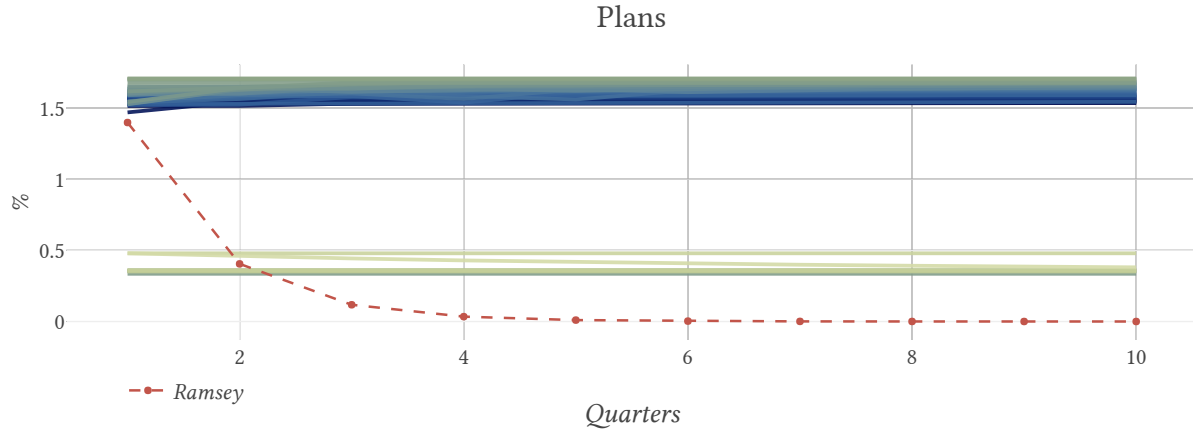


FIGURE 14: SUSTAINABLE PLANS WITH REVERTING TRIGGERS AT DIFFERENT PUNISHMENTS

As before, when ξ is small the planner understands that promises will be broken and simply best responds to these expectations. When ξ is larger, the planner is able to target lower levels of inflation. In this case with imperfect control, something interesting happens: while the planner is able to announce lower levels of inflation, it can promise neither zero inflation nor the Ramsey plan.

6.4 Recursive plans with reputation (inspired by [Dovis and Kirpalani, 2019](#))

Notice that in all these previous versions, the roles of $g(a)$ and ξ are reversed in comparison to our baseline notions of reputational and K -equilibrium. In the Ramsey model and our version of the Green-Porter model, the private sector expects the government's strategy when 'reputation' is high and the exogenous parameter ξ (the private sector's threat) when it is low. In both reputational and K -equilibria, the private sector expects the behavioral government to follow the announcement and the rational government to follow its strategy. While this may appear as an innocuous relabelling of types, we will now show that this shift makes the optimal plan look very different.

We now consider a version in which, like in [Dovis and Kirpalani \(2019\)](#), the government is made up of two agents: a ‘morning’ planner and an ‘afternoon’ policy maker. In the morning of period t , the planner announces a policy recommendation for the afternoon of $t + 1$ (given an announcement for the afternoon of t made earlier). The action in each period happens in the afternoon, when the policy maker chooses inflation. The policy maker can be either a commitment type who stubbornly follows the recommendation, or a rational type who chooses whether to follow it. The planner and the private sector do not know which type of policy maker they face but make statistical inference based on past realizations of inflation. In contrast to [Dovis and Kirpalani \(2019\)](#), we retain the feature of imperfect control and the new Keynesian Phillips curve that we use in our baseline. We refer to the equilibrium announcements in this game as *recursive plans (with reputation)*.

Our timing assumption is done for simplicity and to improve comparability to our benchmark. The target at $t + 1$ can depend on reputation at t but not on realized inflation at t , or reputation at $t + 1$. Thus, when faced with a target a and its current reputation is p , the policy maker attains a value of

$$v^R(p, a) = \min_{g, a'} \mathbb{E} [(y - y^*)^2 + \gamma \pi^2 + \beta v^R(p', a')] \quad (17)$$

$$\text{subject to } \pi = g + \epsilon$$

$$\pi = \kappa y + \beta (p' a' + (1 - p') g^R(p', a'))$$

$$p' = p + p(1 - p) \frac{f_\epsilon(\pi - a) - f_\epsilon(\pi - g^R(p, a))}{p f_\epsilon(\pi - a) + (1 - p) f_\epsilon(\pi - g^R(p, a))}$$

Like in our benchmark analysis, we are interested in the low-reputation case. The planner then attains an initial value of

$$\mathcal{J}^R = \lim_{p \rightarrow 0} \min_a v^R(p, a)$$

Figure 15 plots the recursive plan with reputation for different levels of initial reputation. The planner with

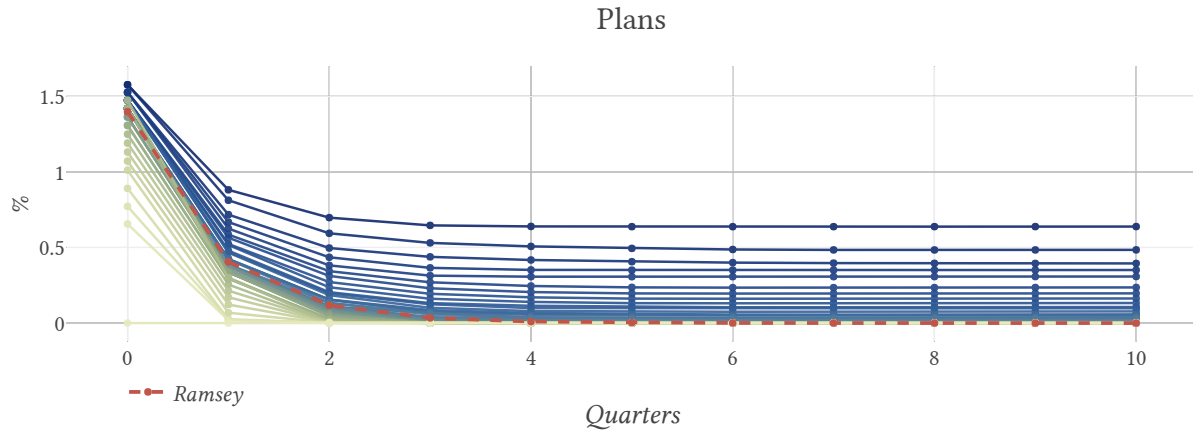


FIGURE 15: RECURSIVE PLANS WITH REPUTATION

full reputation enjoys complete credibility for all plans and hence announces a plan of zero inflation throughout,

which it intends to break, as this will not alter expectations in any way. With moderate initial reputation the planner announces a positive level of initial inflation, which converges to zero inflation. When initial reputation is low, the plan stops converging to zero, in a similar pattern to the one delivered by the K -equilibrium.

6.5 A discussion of imperfect control, reputation, and the gradualism of optimal plans

In this section, we explored the role of the various assumptions that describe our model in relation to the literature. Our model differs from the Ramsey plan (and its decentralization as a sustainable plan) along two key aspects: reputation and imperfect control.

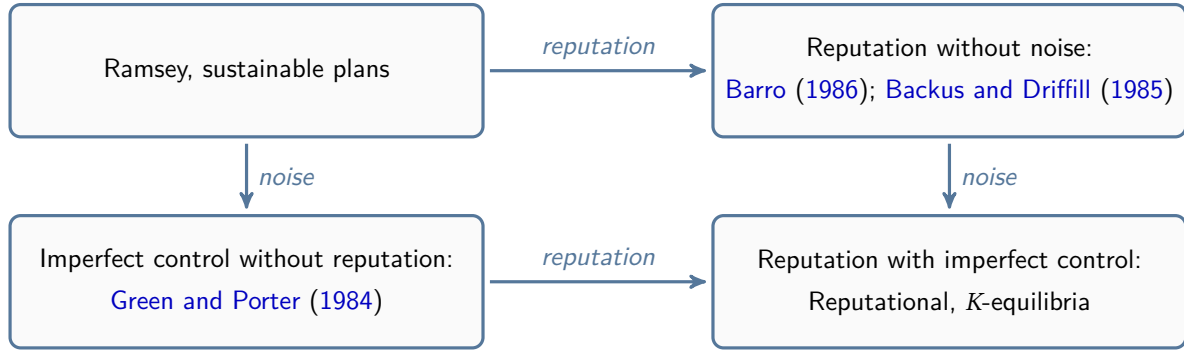


FIGURE 16: MODELS

Figure 16 illustrates this relation. Early papers like Barro (1986) and Backus and Driffill (1985) departed from the sustainable plan by introducing reputation without noise. Their results were negative in the sense that they recovered the Ramsey plan (zero inflation throughout in their context of a ‘simultaneous’ Phillips curve). Our reformulation of the Green and Porter (1984) model introduces noise but leaves out reputation. This switch produced the result that, at high enough threat, the planner chooses a plan with inflation below Nash. However, the threat structure under which ‘reputation’ can only jump to zero induced the planner to choose inflation targets that are constant over time.

Our model of reputational equilibria mixes both of these assumptions by considering reputation combined with noise. In this case, the planner is able to take advantage of the dynamics of targets as they impart dynamics to its reputation. Our reformulation in terms of recursive plans with reputation shows that the result does not depend on the planner being able to choose all targets at once.

While recursive plans with reputation are inspired on Dovis and Kirpalani (2019), we point out that the models are very different. Dovis and Kirpalani (2019) differs from our recursive plans with reputation by considering perfect control of inflation as well as an old-style Phillips curve in which what matters is the difference between realized and expected inflation for the current period. Both differences turn out to be important: both intermediate combinations (no noise plus forward-looking Phillips curve and noise plus old-style Phillips curve) yield a flat optimal plan.

6.6 Gains from preannouncement and flexibility

Figure 17 plots the recursive plan with reputation against some of the other plans we have considered so far.

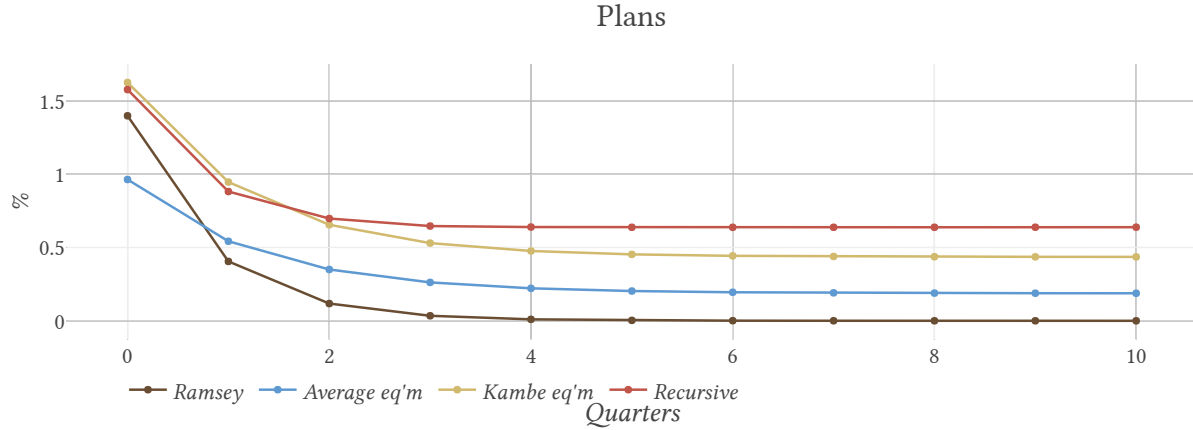


FIGURE 17: PLANS WITH REPUTATION

Inspecting problems (6) and (17) reveals that, while very similar, the recursive plan differs from optimal plans in reputational and K -equilibria in two important ways. In the latter, inflation targets for all periods are chosen at the beginning of time, while in the recursive plan they are chosen sequentially. Secondly, as a consequence of this, the recursive plan allows for feedback between the evolution of reputation and future targets.

Because of these differences, comparing the recursive plans to our baseline notions of equilibrium will mix two distinct effects. On the one hand, because the recursive plan can respond to the evolution of reputation, it benefits from the *flexibility* of tailoring future announcements to the current assessment of the credibility of different plans. On the other hand, reputational and K -equilibrium plans benefit from *pre-announcing* the targets. This allows the planner to use the partial credibility of plans in the future to induce movements in expectations which provide incentives in the present.

To disentangle both effects, we consider an ‘average’ recursive plan, as follows. After solving for the recursive plan, we take the expected path of announcements which takes into account the fact that reputation will drift down over time, as the planner and the private sector understand. We then project this path onto the (ω, χ, a_0) space of our announcements, and solve for the continuation equilibrium after the announcement of this projected plan.

Table 2 provides a summarized comparison of our plans. In this parametrization, the gains from pre-announcing seem to be of the same magnitude as the gains from flexibility.

The recursive plan has a higher asymptote than reputational and K -equilibrium plans. This reveals a second form of time-inconsistency in the Ramsey plan. At time 0, the planner wants to promise to deliver low inflation in the future because this decreases the level of expected inflation in earlier periods. When the announcement is

Model	Ramsey	K -equilibrium	‘Average’ recursive plan	Recursive plan
Initial inflation	1.40%	1.63%	1.58%	1.58%
Long-run inflation	0%	0.44%	0.65%	0.65%
Value of loss function	0.3364	0.7552	0.7589	0.7554

TABLE 2: INFLATION PLANS

made sequentially this gain evaporates and the planner attains a higher long-run level for inflation.

7. CONCLUDING REMARKS

This paper addresses an old question: can reputation be a substitute for commitment? We find that a simple model of reputation combined with imperfect control on the part of the government creates incentives for staying close to announced targets. The central bank’s optimal policy after a plan was announced trades off the benefits of surprise inflation against the possibility that a deviation becomes known to the public. In this way, the monetary authority’s reputation becomes an important state variable in the optimal policy problem under discretion.

Various characteristics of announced plans come to bear when determining the value of reputation. We find that a pervasive feature of optimal plans is gradualism. In anticipation of the continuation equilibrium, the planner finds it desirable to set itself up in situations where keeping its reputation is both easy and valuable. These are situations in which current announced inflation is higher now than in the future. In our model, gradualism is therefore an artifact of incentives and not the reflection of inflation inertia. Understanding how the presence of sources of true inertia might interact with our results is one of our goals going forward.

The gradualist property of optimal plans holds at positive levels of reputation and also in the limit as initial reputation vanishes to zero. We interpret this limit case as a sensible refinement of the game between a rational government and the private sector.

As an important aside, we provide a comparison to models that stand between our reputational equilibria and more standard sustainable plans, loosely based on some contributions to the literature. We argue that our model of reputation effectively operates by modifying the incentive constraint in the recursive version of a planning problem. When reputation is low, large option values of sticking to the plan are created, which become even greater when the plan is backloaded.

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