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VAT Notches, Voluntary Registration, and Bunching: Theory and UK Evidence

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Abstract

Using administrative tax records for UK businesses, we document both bunching in annual turnover below the VAT registration threshold and persistent voluntary registration by almost half of the firms below the threshold. We develop a conceptual framework that can simultaneously explain these two apparently conflicting facts. The framework also predicts that higher intermediate input shares, lower product-market competition and a lower share of business to consumer (B2C) sales lead to voluntary registration. The predictions are exactly the opposite for bunching. We test the theory using linked VAT and corporation tax records from 2004-2014, finding empirical support for these predictions.

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I. Introduction

Most countries around the world use the value-added tax (VAT) as their primary indirect tax.[1] It is standard to set a minimum registration threshold, usually based on annual turnover, below which businesses do not need to register for VAT. In the EU, a large majority of countries currently have a registration threshold, with the UK threshold being the highest at £85,000 ($110,000).[2] As VAT rates are often quite high (in excess of 20% in many EU countries), this may create a large and salient tax notch for businesses whose turnover is around the threshold, depending on firm characteristics as we explain below.[3] The effects of these VAT notches on firm behavior have received little attention in the existing literature.

In this paper, we study the behavior of firms around the VAT registration threshold theoretically and also empirically, using administrative data on UK corporations. We begin by documenting two stylized facts. First, we find strong evidence that some firms bunch below the registration threshold by restricting their reported turnover to avoid having to register for the VAT.[4] Second, we observe that in any year, a large fraction of firms with turnover below the current-year threshold were registered for VAT (on average 43%, over the period 2004-2014). This behavior seems to be deliberate, rather than accidental. For example, of the firms initially registered with turnover below the threshold on 2004/5, about half of them are still reg-

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[2]There is a positive registration threshold in all but five EU countries: Greece, Hungary, Malta, Spain and Sweden. For details, see www.vatlive.com.

[3]A notch arises when the tax liability changes discontinuously.

[4]Bunching is generally inefficient for two reasons. First, it might artificially reduce some firms' size and growth. Moreover, unregistered firms face so-called embedded VAT on inputs they acquire from VAT-registered firms, which distorts their input choices and may cascade through the production chain (Ebrill and others, 2001).

istered and below the threshold three years later. We call these firms *voluntarily registered*.[5] This is noteworthy because voluntary payment of any tax by businesses, when it is explicitly optional, is uncommon.[6]

How can we explain the coexistence of voluntary VAT registration and bunching at the threshold? We develop a simple general equilibrium model that can explain both these phenomena in a unified way. Our first observation is that for both behaviors to occur simultaneously within a given sector, firms in that sector must make *both* sales to final consumers (B2C sales) and sales to other VAT-registered businesses (B2B sales). Moreover, these firms must themselves use intermediate inputs in production. To see this, suppose that firms make only B2C sales. Then, it is easily shown that irrespective of the degree of competition between firms, the cost of voluntary registration exceeds the benefit, because the burden of VAT paid on output when registered exceeds the burden of VAT paid on inputs when not registered.[7] Conversely, with only B2B sales, voluntary registration is always weakly optimal, absent compliance costs, because the burden of output VAT can be passed on to the buyer. However, it is only strictly optimal if the firm uses inputs on which it can then claim back input VAT.

It follows that the most parsimonious model that can explain both voluntary registration and bunching must have three stages of production, because the small firms must sell some of their output to other VAT-registered firms and buy inputs that bear VAT.[8]

In this paper, we present such a model, and derive three main predictions. First, we show that voluntary registration by a firm is more likely when (i) the cost of inputs relative to sales is high; (ii) when the proportion of B2C sales by the firm is low; (iii) when markets are less

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[5]The UK fiscal year starts in early April.

[6]Of course, this behavior may be due to the fact that, in a VAT system, firms play the role of fiscal intermediaries. As such, they do not necessarily bear the burden of the tax which they must legally remit to the government.

[7]This is proved very generally in Appendix A below.

[8]This implies that the two best-known models of VAT thresholds, Keen and Mintz (2004) and de Paula and Scheinkman (2010), are not suitable for our purposes because both models only have two stages of production. Indeed, as explained in more detail in Section V, neither of these models can explain the coexistence of voluntary registration and bunching.

competitive i.e. firms have higher mark-ups. [9] The intuition for (i) is that, when input costs are important, registration allows the firm to claim back a considerable amount of input VAT. The intuition for (ii) is simply that, if most customers are VAT-registered, the burden of an increase in VAT can easily be passed on in the form of a higher price, because the customer itself can claim back the increase. The intuition for (iii) is also simple; in a more competitive market, it is more difficult to pass on the burden of output VAT.

Second, we show that the determinants of bunching at the registration threshold are the same as for voluntary registration, with the signs of the effects reversed. Specifically, bunching is more likely when (i) the cost of inputs relative to sales is low; (ii) when the proportion of B2C sales is high, or (iii) when markets are more competitive [10]

In addition to the three main predictions, note that extending the model to allow for evasion responses does not affect any of our qualitative predictions. We discuss this extended model in online Appendix A.

The third contribution of the paper is to test the main predictions of our model using an administrative dataset created by linking the population of VAT and corporation tax records in the UK for the period 2004-2014. One advantage of studying the UK's VAT system is that businesses below the registration threshold are not subject to any other tax to replace the VAT, as is the case in many other countries.[11] So, the UK gives us quite a clean test of our theoretical predictions.

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[9]Both these characteristics clearly differ widely across small firms that are close to the registration threshold. For example, a small sole trader such as a plumber or electrician may typically have mostly B2C sales of his services to householders, and make relatively light use of intermediate inputs. So, they would face a low effective VAT rate when not registered, but a high rate when registered. Conversely, a small specialist engineering firm, such as a car component firm, may make mostly B2B sales with heavy use of intermediate inputs, and so will be in the reverse position.

[10]Predictions (i) and (ii) concerning bunching have been made informally (e.g. Brashares and others (2014)) but to our knowledge, this is the first formal derivation of them. Predictions concerning the effect of market competition on bunching, and on any determinants of voluntary registration, are to our knowledge, new.

[11]These businesses may still be liable for corporation tax (if they are incorporated) or income tax (if they are sole-proprietorships or other kinds of businesses).

We first show that the pattern of voluntary registration in the data is consistent with the theory. In particular, voluntary registration is more likely with a low share of B2C sales, a high share of input costs, or when the industry is less competitive. Quantitatively, the probability that a firm voluntarily registers for VAT is increased by 4 percentage points for a one-standard deviation (s.d.) decrease in the share of B2C sales, by 4.9 percentage points for a one-s.d. *increase* in the input-cost ratio, and by 3.9 percentage points for a one-s.d. decrease in the Lerner index of competition. The results are robust to the use of either a linear probability model or fixed-effects logit model, and to the inclusion of additional firm-level control variables such as the distance to the VAT threshold.

We then look at bunching. In the aggregate, there is clear evidence of bunching at the VAT threshold. To test the predictions from the model, we partition the sample into two groups of firms based on their predicted likelihood of registering voluntarily, regardless of whether their turnover is above or below the threshold. Focusing on the subset of firms that are less likely to register voluntarily, we find that the bunching response is larger when (i) the proportion of B2C sales is low, (ii) the cost of inputs relative to sales is high, or (iii) when the Lerner index of their industry is low. In contrast, the bunching patterns for firms that are highly likely to register voluntarily are unrelated to those three characteristics, as expected. Thus, we conclude that the heterogeneous bunching patterns are consistent with the theoretical predictions.

We further investigate some of the mechanisms underlying the registration decision. There is some suggestive evidence that part of the bunching response is driven by evasion, in the form of under-reporting of sales.[12] We also analyze changes in registration status in a dynamic regression setting. Specifically, we address the possibility that voluntary registration may not be an optimizing choice of firms, but simply a failure to deregister due to inertia (or high deregistration costs) when sales fall below the threshold. Our empirical findings suggest that, while there is some persistence in firm behavior, the decision is not entirely driven by inertia.[13]

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[12]The details of this exercise are reported on online Appendices B and D.

[13]See online Appendix E for details.

The rest of the paper proceeds as follows. The next section reviews related literature. Section III establishes the two stylized facts referred to in the introduction. Section IV develops the conceptual framework to analyze VAT bunching and voluntary registration. Section V derives the main empirical predictions. Section VI provides an overview of the VAT system in the UK and describes the data. Sections VII.A and VII.B present the empirical analysis for voluntary registration and bunching, respectively. Finally, Section IX concludes.

II. RELATED LITERATURE

This paper contributes to several strands of literature. First, our work relates to the literature on the effect of tax and regulatory thresholds, and in particular, the effect of VAT thresholds on small business behavior. In an important paper, Keen and Mintz (2004) were the first to set up a model of VAT including a threshold, and they show that there will be bunching below the threshold. However, there are a number of differences between their approach and ours. First, their model only allows for final consumer sales (B2C sales), which cannot by itself explain voluntary registration, as argued above. Second, their main focus is on the optimal registration threshold, whereas our focus is on the coexistence and determinants of voluntary registration and bunching. Kanbur and Keen (2014) extend the Keen and Mintz (2004) framework to allow for VAT evasion and avoidance. A simplified version of our model without B2B is closely-related to the framework in Keen and Mintz (2004), as explained in more detail in Section IV.A below. In a related paper, Brashares and others (2014) use a calibrated formula from Keen and Mintz (2004) to infer that for a 10 percent VAT rate, the optimal level for the threshold in the United States is $200,000.

Second, there is a related literature on the relationship between the VAT and business informality in developing countries (de Paula and Scheinkman, 2010; Emran and Stiglitz, 2005). In particular, de Paula and Scheinkman (2010) present a model where firms can choose between formal and informal production, where the distinction is whether they must register for VAT. Firms can also choose to buy inputs from formal or informal suppliers. In their model, informality can be interpreted as producing below a VAT threshold. Moreover, because they

model two stages in the chain of production (upstream and downstream firms), they allow for both B2C and B2B sales. Nevertheless, their model is not really suited to our task because voluntary registration cannot occur in equilibrium, as there are only two stages of production. The formal argument for this is further discussed in Section IV.A and in the online appendix.

Third, there is a small empirical literature on the effects of VAT registration thresholds in different countries. Onji (2009) documents the effects of the introduction of a VAT threshold in Japan, focusing on the incentives for a large firm to split in order to stay below the threshold. More recent papers, appearing after the first version of this paper was completed, document bunching of small firms at the VAT registration threshold in Finland (Harju, Matikka, and Rauhanen, 2016), as well as lack of bunching in response to the VAT threshold in Armenia (Asatryan and Peichl, 2016).[14] In particular, Harju, Matikka, and Rauhanen (2016) provide strong evidence of bunching below the VAT threshold in Finland, and argue that compliance costs are the main driver.[15] However, neither of these papers studies the determinants of voluntary registration in detail, nor do they develop a theoretical framework specific to the VAT.[16]

Our work also relates to the broader literature on tax notches (Almunia and Lopez-Rodriguez, 2018; Best and Kleven, 2018; Kleven, 2016; Kleven and Waseem, 2013; Kopczuk and Munroe, 2015; Slemrod, 2010). As shown below, the "bunching equation" in this paper, which relates the amount of bunching to the elasticity of the tax base and parameters of the tax system, is

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[14]The first version of the current study was published as a working paper in 2015 (Liu and Lockwood, 2015).

[15]There are several reasons that compliance costs may be more important in Finland than for our study. First, the VAT threshold in Finland is very low, at €8,500 ($ 9,800), compared to well over $ 100,000 in the UK over our study period. Other things equal, larger firms find compliance less costly. Second, in the UK, all active companies are required to file company accounts and corporation tax returns, so they already have the information required for the VAT return, and the VAT return itself is short and simple.

[16]Harju, Matikka, and Rauhanen (2016) present formulae for the reduced-form elasticity implied by observed bunching which are taken directly from Kleven and Waseem (2013). However, these formulae are originally developed for a labour supply model, and are not directly applicable to the VAT due to the influence of input costs on the incentives for registration. In an earlier version of our paper, (Liu and Lockwood, 2015), we show that to apply the Kleven-Waseem formulae directly, one has to assume that all sales are B2C, firms are price-takers, and the production function is fixed-coefficients. Moreover, the elasticity estimated is an output supply elasticity, taking the price of output as given, not the elasticity of the tax base, because the latter will also be determined by the elasticity of demand.

mathematically very similar to the equation of Kleven and Waseem (2013). However, for reasons discussed below, it is much more difficult to back out credible elasticity estimates in the VAT case, and so we do not attempt this in the paper.

III. TWO STYLIZED FACTS

To motivate the theoretical analysis, we present two key stylized facts from our UK administrative dataset. The construction of the dataset is described in detail in Section VI.B. Here, we just note that we have more than 3.4 million turnover observations above and below the VAT threshold, for almost one million unique companies between April 1, 2004 and April 5, 2015.

The first fact is bunching of firm turnover just below the VAT registration threshold. We normalize turnover by subtracting the threshold value so that the threshold is located at zero in any year. The histogram of normalized turnover is shown in Figure 1, where there is clear evidence of excess mass to the left of the registration threshold in an otherwise smooth distribution. The figure indicates that the VAT registration threshold is binding for at least a subset of British firms. We discuss the bunching estimates later, in section VII.B.

It is worth noting that the bunching spike is not as sharp as in other bunching studies, in particular those studying firms' responses to notches (e.g., Almunia and Lopez-Rodriguez, 2018; Best and others, 2015; Harju, Matikka, and Rauhanen, 2016). One possible explanation is that firms that benefit from voluntary registration, do not respond in any way to the location of the threshold. Another potential reason is measurement error, because the registration threshold is set in terms of VAT taxable turnover, but in the corporation tax records we cannot distinguish domestic sales from exports. The latter suggests that our measures of bunching should be interpreted as a lower bound of the true behavioral response. We explore these hypotheses in more detail later in the paper.

The second stylized fact is that, in any given year, a significant number of firms are registered for VAT even though their turnover is below the threshold in the current year. On average, over our sample period, 43% of firms below the threshold are in this position. Possibly, part

of this may be due to rules of registration. In the UK, a business must register for VAT if its taxable turnover is likely to go over the threshold in the next 30 days, or if its taxable turnover in the previous 12 months was above the threshold. So, for example, a firm may register on the basis of previous year's turnover, and then its turnover may fall below the threshold in the current year.

However, our data show that there is considerable persistence in registration below the threshold. This is shown in Figure 2 which shows what happens to firms initially registered and below the threshold during in 2004/5. Almost one-half are still registered three years later, and over one-third are still registered five years later. So, it is likely that registration below the threshold is a conscious decision by firms, rather than just due to inability to forecast turnover one year in advance, or inertia. So, we refer to this stylized fact as *voluntary registration*.

IV. CONCEPTUAL FRAMEWORK

Key Features of the Model. We aim to model the behavior of "small" firms selling to both final consumers and to businesses, where both voluntary registration and bunching can be equilibrium outcomes. As already explained in the introduction, the coexistence of these two behaviors requires that the "small" firms make *both* B2B and B2C sales, and that they buy produced inputs. So, as already remarked, the model must have (at least) three stages of production.[17]

Second, we wish to study the effect of the input cost ratio, share of B2C sales, and the level of industry competition on voluntary registration and bunching, so the model must incorporate parameters measuring these.

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[17]Suppose that firms make only B2C sales. Then, it is easily shown that irrespective of the degree of competition between firms, the cost of voluntarily registering exceeds the benefit, because the burden of VAT paid on output when registered exceeds the burden of VAT paid on inputs when not registered. Conversely, with only B2B sales, the voluntary registration is always optimal, because the burden of output VAT can be passed on to the buyer. Both these points are made formally below.

We construct the simplest general equilibrium model that has the required features. There is a single representative consumer that supplies labour and buys two kinds of goods; a differentiated good sold by the small firms, and a single good produced by a large downstream firm. The large firm also buys inputs from the small firms, generating a B2B demand. We assume that the large firm is operating at a scale where non-registration for VAT (i.e. operating so that the value of sales are below the VAT threshold) is never profitable. Finally, a homogeneous input to the small firm is produced by a third sector of upstream competitive firms from a labour input via a constant returns technology. The behavior of this last sector is summarized by a zero-profit condition that says that the price of the small-firm input is equal to the wage. The structure of this economy is illustrated in Figure 3. Note that there are three stages of production.

Consumers. There is a representative household that has preferences over the homogeneous good, consumed at level Y, a set of differentiated goods $a \in [\underline{a}, \overline{a}]$, consumed at levels $x(a)$, $a \in [\underline{a}, \overline{a}]$, and leisure l. These preferences are of the following form:

$$U(X) + V(Y) + l, \ X = \left[\int_{\underline{a}}^{\overline{a}} (x(a))^{(e_C - 1)/e_C} \, da \right]^{e_C/(e_C - 1)}, \ e_C > 1 \tag{1}$$

where X is a CES index of differentiated products, and

$$U(X) = \lambda^{1/\phi} \frac{X^{1 - 1/\phi}}{1 - 1/\phi}, \ V(Y) = (1 - \lambda)^{1/\gamma} \frac{Y^{1 - 1/\gamma}}{1 - 1/\gamma}, \ \phi > 0, \ \gamma > 1$$

Here, λ is a measure of the final demand by households for the goods produced by the small firms relative to demand for the good produced by the large downstream firm; as such, it will be the parameter that measures B2C demand in what follows.

Each differentiated good a is produced by a single small firm a, which can be either registered for VAT or not. For reasons further discussed below, we allow the homogeneous good Y to be subject to VAT or not. We also allow the firm to price-discriminate between final and intermediate consumers, so let $p_C(a)$, $p_B(a)$ be the prices charged to final consumers and the large firm respectively for good a, excluding VAT.

So, the household faces a budget constraint

$$P(1+t)Y + \int_{\underline{a}}^{\overline{a}} [p_C(a)(1+I(a)t)]x(a)da = w(1-l) + \Pi \tag{2}$$

where $1-l$ is labour supply, w is the wage, P is the price of the homogeneous good produced by the large firm, and $I(a) \in \{0,1\}$ is an indicator recording whether the firm registers for VAT or not, with a "1" indicating registration. So, if the firm is registered, the consumer price is grossed up by VAT i.e. $p(a)(1+t)$. Finally, Π is aggregate profit, which can be defined formally using ((11)) below; because utility is linear in leisure, there are no income effects in demand for $x(a), Y$ and so this term plays no further role.

So, by standard arguments, maximization of ((1)) subject to ((2)) gives household demand for the homogeneous and differentiated goods:

$$Y = (1-\lambda)(1+t)^{-\gamma}P^{-\gamma} \tag{3}$$

$$x(a) = \lambda \left(\frac{p_C(a)(1+I(a)t)}{Q} \right)^{-e_C} Q^{-\phi} \tag{4}$$

where Q is the CES price index corresponding to the quantity index X i.e.

$$Q = \left[\int_{\underline{a}}^{\overline{a}} (p_C(a)(1+I(a)t))^{1-e_C} da \right]^{1/(1-e_C)} \tag{5}$$

We assume that in equilibrium, positive leisure is consumed, so that from ((1)), the wage is fixed at the marginal utility of leisure, unity.

The Large Firm. The large firm combines inputs $y(a)$, $a \in [\underline{a}, \overline{a}]$ bought from the small firms via a constant returns CES production technology to produce output Y. This production technology is characterized by a CES cost function per unit of output of

$$C = \left[\int_{\underline{a}}^{\overline{a}} (p_B(a))^{1-e_B} da \right]^{1/(1-e_B)}, \ e_B > 1 \tag{6}$$

where $p_B(a)$ is the price of the input net of tax (as the large firm is VAT-registered, it can claim back any tax on inputs). So, the large firm chooses P to maximize $(1-\lambda)P^{-\gamma}(P -$

C). This gives the usual mark-up equation for price i.e.

$$P = \frac{\gamma}{\gamma - 1} C \tag{7}$$

and thus, combining ((7)),((3)), ultimately, output is

$$Y = (1 - \lambda)(1 + t)^{-\gamma} \left(\frac{\gamma}{\gamma - 1} C \right)^{-\gamma} \tag{8}$$

Finally, input demand for variety a is, by Shephard's Lemma and ((8)):

$$y(a) = \frac{\partial C}{\partial p_B(a)} Y = (1 - \lambda)(1 + t)^{-\gamma} \left(\frac{\gamma}{\gamma - 1} \right)^{-\gamma} C^{-\gamma} \left(\frac{p_B(a)}{C} \right)^{-e_B} \tag{9}$$

The Small Firms. To keep things simple, we assume, following Keen and Mintz (2004), that the production technology is fixed coefficients, with one unit of output for an a type firm requiring ω/a units of the input, and $(1 - \omega)/a$ units of labour.[18] The input costs are w, r, where r is the cost of the input. By assumption, $w = 1$, and we have also assumed, without loss of generality that one unit of the intermediate good requires one unit of labor, so $r = 1$ also.

Let the unit cost function be denoted $c(I(a); a)$, where $I(a)$ is the variable recording registration status. Then, under the assumptions just stated, the unit cost function is:

$$c(1; a) = \frac{1}{a}, c(0; a) = \frac{1 + \omega t}{a} \tag{10}$$

So, cost of the input is grossed up by the tax t if the firm is not registered, as the firm cannot claim the input tax back. Note that a is a measure of productivity, and ω is a measure of the firm's use of intermediate inputs relative to labour, independently of productivity.

Suppressing the dependence of p_C, p_B, I and etc. on a to lighten notation, the firm's profit is

$$\pi(p_C, p_B, I; a) = (p_C - c(I; a))x + (p_B - c(I; a))y \tag{11}$$

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[18]All the results below generalize straightforwardly if the small firm production function is assumed as a constant returns CES function of labor and the intermediate input.

where from ((4)), ((9)):

$$x = \lambda A_C \left(p_C (1 + I.t) \right)^{-e_C}, \ y = (1 - \lambda) A_B (p_B)^{-e_B} \tag{12}$$

where

$$A_C = Q^{e_C - \phi}, \ A_B = (1 + t)^{-\gamma} \left(\frac{\gamma}{\gamma - 1} \right)^{-\gamma} C^{e_B - \gamma} \tag{13}$$

are parameters that the small firms take as given, but are determined in equilibrium.

The small firm chooses $p_C, p_B \in [0, \infty)$, $I \in \{0, 1\}$ to maximize $\pi(p_C, p_B, I; a)$ subject to the registration constraint, which says that if the firm chooses not to register ($I = 0$), the value of sales $s \equiv p_C x + p_B y$ must be less than the VAT sales threshold s^*. This allows of course, for *voluntary registration*, which is defined by a choice $I = 1$ when $s < s^*$. The costs and benefits of registration are clear from equations ((11)) and ((12)). The benefit is that registration, $I = 1$, lowers the unit cost of production. The cost is that at a fixed price, registration lowers B2C sales, because demand by the household is reduced by the tax.

(a) Equilibrium.   An *equilibrium* is (i) a price P for the homogeneous product given by ((7)); (ii) for each $a \in [\underline{a}, \overline{a}]$, prices $p_C(a)$, $p_B(a)$ and a registration decision $I(a)$ that maximises ((11)) subject to ((12)) and A_C, A_B fixed; (iii) demand shifts A_C, A_B in ((13)) that are functions of $p_C(a), p_B(a), I(a)$ via ((6)) and ((5)). It should be noted that this describes a general equilibrium for the whole economy. Note that the equilibrium is conditional on fixed values of the share of B2C sales, λ, the intensity of input use, ω, and demand elasticities e_C, e_B and we will vary these parameters in what follows.

A. Discussion

Here, we discuss some modelling choices. First, as already remarked, we allow the small firms to price-discriminate between B2C and B2B customers. The reason for this is the following. As long as the elasticities of demand from the household and the large firm are different, i.e. $e_C \neq e_B$, the small firm prefers to price discriminate if it can. Moreover, ruling out price discrimination is not very tractable, as the profit-maximising price cannot then be solved

for in closed form, unless it is assumed that $e_C = e_B$. However, for some results (Proposition V.B, parts (c) and (d), Propositions V.C-B) we do assume $e_C = e_B$, so that given this, the assumption of price-discrimination is without loss of generality.

Second, in practice, there are compliance costs to VAT registration. These can be introduced at the cost of some additional complexity - see Section V.D below. However, it should be noted that these costs are relatively small for the UK: for example, a recent literature review found that at the registration threshold, these costs were around 1.5% of turnover, declining to 0.1% or less for large companies (Federation of Small Businesses, 2010), and so we do not feel it necessary to include these in our analysis.[19]

Third, it has been argued that the amount of output exported is a determinant of registration, because in practice, exports are exempt from VAT, and so firms exporting more of their output are more likely to register (Brashares and others, 2014). Our model could be interpreted to cover this case. This is because in the case of exports, the exporter does not bear any of the burden of the output VAT, and so from the supplier's point of view, domestic B2B sales and exports are equivalent in this respect. So, we could interpret good Y as being purchased by foreigners, rather than domestic consumers.[20] Finally, in our baseline model, we do not allow for evasion, to avoid overloading the analysis. We introduce evasion in Section B.

Note that when $\lambda = 1$, our model is closely related to the Keen and Mintz model. Then, both models have a set of firms of different productivity producing a good from labor and an intermediate input, which they sell only to final consumers. The main differences are that we have constant returns to scale and monopolistic competition, whereas in their model, firms are perfectly competitive, facing a world producer price for the good, but have decreasing returns to scale. But under both sets of assumptions, when $\lambda = 1$, the firms bear the burden of output VAT to a point where voluntary registration is not desirable.

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[19]Also, empirically, there are no major changes in compliance requirements during our sample period.

[20]In that case, technically, we would have to replace the term $1 + t$ in ((3)) by $1 + t^*$ where t^* is the foreign rate of VAT.

Note finally that our model has three stages of production; upstream, an input is produced, which is then sold to the small firms, who produce goods, some of which are then sold as inputs to the downstream large firm. This is no accident; as argued in the introduction, it is clear that *at least three stages are required* to generate voluntary registration in equilibrium. This observation explains why the model of de Paula and Scheinkman (2010), which is elegant in many ways, is not suitable for our investigation, as it only has two stages of production.[21]

V. ANALYSIS

A. Necessary Conditions for Voluntary Registration and Bunching

As already argued above, the coexistence of both voluntary registration and bunching requires that the "small" firms make *both* B2B and B2C sales. This is relatively easy to show formally in our framework.[22]

If all sales are B2C i.e. $\lambda = 1$, there is no voluntary registration. If all sales are B2B, i.e. $\lambda = 0$, all firms register voluntarily, whatever their turnover, so there can be no bunching. So, to observe both bunching and voluntary registration simultaneously, we require that there are both B2C and B2B sales i.e. $0 < \lambda < 1$.

Proposition 1 is in fact completely general; as shown in the online appendix, it does not rely on our technical assumptions such as isoelastic demand or constant marginal cost. For the remainder of the theoretical section of the paper, we thus focus on the general case where $0 < \lambda < 1$.

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[21]It may be asked why we did not develop a theoretical framework by adding a third stage to the de Paula and Scheinkman model. We did not do this, for two reasons. First, because with this addition, it is not clear that the algebra would be less burdensome than what we have. In particular, although their model is very simple to write down, establishing comparative statics results is surprisingly difficult, as prices are endogenous. In our setting with constant returns production technologies and prices being a constant mark-up on marginal cost, comparative statics results are much more easily established. Second, their model assume perfect competition, and so cannot be used for generating predictions about the effect of changing market power.

[22]All proofs are in online Appendix A.

Here, we face a problem; generally, A_C, A_B depend on equilibrium prices and this considerably complicates the analysis. So, for the remainder of the theoretical Section, we assume that $e_C = \phi$, $e_B = \gamma$. This ensures that the demand parameters are exogenous i.e. from ((13)), $A_C = 1$, $A_B = (1+t)^{-e_B} \left(\frac{e_B - 1}{e_B} \right)^{e_B}$.

B. Voluntary Registration

We now turn to the registration decision, which is a choice $I \in \{0, 1\}$, with the firm then maximizing profit given its registration decision. We begin by defining two crucial cost and demand changes. First, from ((10)), the percentage increase in the firm's unit costs due to non-registration, because of input VAT, can be defined independently of firm productivity a as

$$\Delta_c = \frac{c(0; a)}{c(1; a)} - 1 = \omega t. \tag{14}$$

Call this the *input VAT effect* on cost.

Second, if $e_C = e_B = e$, we can define a similar kind of *output VAT effect* on demand. Specifically, it is easy to calculate that any fixed price $p_C = p_B = p$, the percentage reduction in overall demand for the firm's product due to the charging of output VAT on B2C sales is:[23]

$$\Delta_d = \frac{\lambda (1 - (1+t)^{-e})}{\lambda + (1-\lambda)(1+t)^{-e}(\frac{e-1}{e})^e} > 0 \tag{15}$$

This is because when output VAT is charged, at a fixed price p, all B2C sales (which count for λ of the total) are reduced by a factor $(1+t)^{-e}$; call this the *output VAT effect*. We can then show:

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[23]To see this, note that from ((12)), at any fixed price p, the ratio of total demand with VAT registration to without is

$$\frac{\lambda (p(1+t))^{-e} + (1-\lambda)A_B p^{-e}}{\lambda p^{-e} + (1-\lambda)A_B p^{-e}} = \frac{\lambda(1+t)^{-e} + (1-\lambda)A_B}{\lambda + (1-\lambda)A_B}$$

Using the fact that $A_B = (1+t)^{-e} \left(\frac{e-1}{e} \right)^e$, we see that this expression is equal to $1 - \Delta_d$.

(a) A firm of type a will register voluntarily iff

$$a^{e_B - e_C} \geq \frac{\lambda B_C}{(1-\lambda)B_B} \frac{(1+\Delta_c)^{1-e_C} - (1+t)^{-e_C}}{1 - (1+\Delta_c)^{1-e_B}} \tag{16}$$

where B_C, B_B are exogenous demand shift parameters:

$$B_C = \left(\frac{e_C - 1}{e_C}\right)^{e_C} \frac{1}{e_C - 1}, \; B_B = (1+t)^{-e_B} \left(\frac{e_B - 1}{e_B}\right)^{2e_B} \frac{1}{e_B - 1} \tag{17}$$

(b) If the voluntary registration condition ((16)) holds at λ, ω, it also holds at $\omega' \geq \omega, \lambda' \leq \lambda$; that is, voluntary registration is more likely, the higher the input cost ratio, ω, and the lower the share of B2C sales, λ.

(c) If $e_B = e_C = e$, ((16)) holds independently of a iff

$$T = (1 - \Delta_d)(1 + \Delta_c)^{e-1} \geq 1 \tag{18}$$

The voluntary registration condition is most easily interpreted in the form ((18)). There, it says that if the input VAT effect on cost due to non-registration, Δ_c, is large relative to the output VAT effect, Δ_d, there will be voluntary registration. Note also when $e_C = e_B$, T is a sufficient statistic that captures the entire effect of the VAT system on voluntary registration. We will see later that it is also a sufficient statistic for the degree of bunching. Note finally that via ((14)), ((15)), T depends on the rate of VAT, t, and our three key parameters, the share of B2C sales λ, the input cost ratio ω, and the elasticity of demand e.

The intuition for these results is the following. Generally, voluntary registration occurs when output effect Δ_d is small, and/or when the input VAT effect Δ_c is large. The first observation is that other things equal, the larger λ, the bigger is the output VAT effect Δ_d; this explains the fact that T falls with λ. Second, other things equal, the larger ω, the bigger is the input VAT effect Δ_c; this explains why T rises with ω.

It is also of interest to study how the level of competition, measured by e, affects voluntary registration. Here, we assume that $e_B = e_C = e$. Then, we see from ((14)), ((15)), ((18)) that there are two effects of a higher e, working in opposite directions.

First, the input effect, $(1 + \omega t)^{e-1}$, is increasing in e, which captures the fact that the input VAT burden from non-registration rises with e, because the higher cost (due to embedded VAT) is harder to pass on to both B2C and B2B consumers when demand becomes more elastic. *Second*, the output effect in ((18)) is decreasing in e, and captures the fact that the output VAT burden from registration rises with e, because the tax on output (due to embedded VAT) is harder to pass on to B2C consumers when demand becomes more elastic.

As e becomes large, the output VAT effect becomes very strong. Specifically, as $e \to \infty$, $1 - \Delta_d$ is proportional to $1/(1+t)^e$, which dominates the input effect $(1 + \omega t)^{e-1}$. Hence, eventually $T \to 0$. In fact, we can prove:[24]

In the competitive limit, as $e \to \infty$, voluntary registration is never optimal.

To get some further insights on the effect of e on the sufficient statistic, T, we report some numerical simulations. Table 1 shows the value of T for as e increases, for different combinations of λ and ω.

In the first row, the input cost ratio is very high, and the share of B2C sales is very low. These are the conditions under which we expect voluntary registration ($T > 1$) to occur, and indeed this is the case. Note here that T is not monotonic in e; it first rises and then falls. This non-monotonicity is due to the fact that for low values of e, the effect of increasing competition on the input VAT component of T can dominate the effect on the output component of T. The second row is where both the input cost ratio and the share of B2C sales are very high. Here, these two opposing forces always lead to a T below 1 that is monotonically decreasing in T. The same qualitative picture as in the second row also emerges when the input cost ratio is low, for both a low and high B2C ratio.

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[24]The proof is simple. From ((14)), ((15)), ((18)) we see that for e large, T behaves like $(\frac{1+\omega t}{1+t})^e$. But as $\omega < 1$, this term goes to zero as $e \to \infty$, and so $T \to 0$ as $e \to \infty$.

C. Bunching

Now consider what happens if $T < 1$. In this case, registration leads to a drop in profit for any fixed value of sales. This implies that there must be an interval of firms, $a \in [a^*, a^* + \Delta a^*]$, who *bunch* by restricting the value of sales in order to stay at the VAT threshold. As demand is elastic by assumption, i.e. $e > 1$, they do this by cutting price to keep sales low. The firm at the bottom of this bunching interval, a^*, is the one that has a profit-maximising total value of sales of exactly s^* when not registered.

The firm at the top of the interval, $a^* + \Delta a^*$ is indifferent between restricting the value of sales to s^* and not registering, and registering and choosing price and thus sales without any restriction. If $\pi(I; a)$ denotes optimized profit, conditional on the registration decision $I \in \{0, 1\}$, this indifference condition can be written as

$$\pi(1; a^* + \Delta a^*) = \pi(0; a^* + \Delta a^*) \tag{19}$$

So, the amount of bunching in the space of firms is measured by Δa^*.

Now, we do not observe a but we do observe firm sales. Following Saez (2010) and Kleven and Waseem (2013), we reason as follows. First, let $s^* + \Delta s^*$ be the value of sales of the firm $a^* + \Delta a^*$, assuming that this firm does not have to register for VAT. So, Δs^* is the difference in sales between the VAT threshold and what the value of sales for the firm at the top of the bunching interval, $a^* + \Delta a^*$, *would have been* had it been unconstrained by the threshold. So, following Saez (2010), the fraction of firms bunching, B, in the space of sales is given by

$$B = \int_{s^*}^{s^* + \Delta s^*} h(s) ds \tag{20}$$

where $h(s)$ is the distribution of firms in the space of sales, assuming that firms do not have to register. Moreover, because each variety a is produced by a single firm, the distribution of firms on the space of varieties is uniform on $[0, 1]$, and so $h(s) = 1/\bar{s}$, where $\bar{s}$ is the sales of the highest-productivity firm, $a = 1$. So, from ((20)), we see $B = \frac{\Delta s^*}{\bar{s}}$.

As *B* is empirically observable, this means that our empirical predictions need to be about the determinants of Δs^*. In the online appendix, assuming $e_C = e_B = e$, we show [25]:

The amount of bunching at the VAT threshold Δs^* is given by the implicit relationship

$$\frac{e}{(1 + \Delta s^*/s^*)} - (e - 1) \left[\frac{1}{1 + \Delta s^*/s^*} \right]^{e/(e-1)} - T = 0 \tag{21}$$

where $T < 1$ is the VAT sufficient statistic.

Note that the entire effect of VAT on bunching is captured by the sufficient statistic T. Note also that $T < 1$, because otherwise there would be voluntary registration. We can now use ((21)) to look at some of the determinants of bunching. We have:

The amount of bunching Δs^* rises (i) as the fraction of B2C sales, λ, increases, and (ii) as the share of inputs in total cost, ω, falls. Moreover, if the sufficient statistic T is decreasing in e, the amount of bunching Δs^* increases as e rises.

The intuition for (i) and (ii) is very similar to the case of voluntary registration. That is, factors that make voluntary registration less attractive also provide incentives for staying under the VAT threshold by bunching. Specifically, this will be the case when most customers are not VAT-registered, so that the burden of an increase VAT can not easily be passed on to the buyer, and/or when input costs are relatively unimportant relative to labour costs. We will bring these predictions to the data below.

Finally, increased competition increases bunching as long as T is decreasing in e. While we cannot establish analytically that T is decreasing in e, the simulation results reported above indicate this is the case for a wide range of parameter values. The intuition here is again re-

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[25]Note that ((21)) is closely related to the Kleven and Waseem (2013) formula relating bunching at a notch of the personal income tax schedule to the elasticity of the labor supply. In particular, in their formula, the tax notch is measured by the term $\Delta t/(1 - t)$ where t is the lower rate of income tax, and Δt is the increase in the tax rate at the notch. In fact, it is easily verified if we take ((21)) and substitute $e_L = e - 1$, where e_L is th elasticity of labor supply, replace $\Delta s^*/s^*$ by $\Delta z^*/z^*$, and replace $T^{1/e}$ by $1 - \frac{\Delta t}{1-t}$, we get equation (5) in their paper.

lated to the intuition with voluntary registration; increased product market competition makes it harder for a firm to pass on output market VAT and thus increases the incentive to bunch.

D. Evasion and Compliance Costs

Here, we briefly explain how our theoretical results extend to evasion and compliance costs. First, VAT in the UK has been susceptible to fraud and avoidance, as in many other countries. According to HMRC estimates, the VAT tax gap, which is defined as the difference between theoretical VAT liabilities and total VAT receipts on a timely basis, is currently around 10% of theoretical VAT liability (HM Revenue and Customs, 2015). Some of this gap can be accounted for by VAT debt owed by firms, and by sophisticated fraud schemes, but most of the VAT gap is probably due to sales under-reporting and cost over-reporting.[26]

In online Appendix B, we model the simplest and most common form of VAT evasion, under-reporting of sales. This has two aspects. First, as is widely recognized, a non-registered firm can hide sales, for example by using cash transactions. However, it is also the case that registered sellers can use this strategy, albeit at a higher cost of detection. We model both these evasion strategies.

With evasion, it turns out that the qualitative effects of λ, ω, and e on T do not change, and so our predictions about the determinants of voluntary registration and bunching do not change. Therefore, our key empirical predictions are robust to the presence of evasion.

However, as already mentioned in Section I, we do not measure evasion directly. Nor do we have any obvious way of decomposing the total bunching effect into an evasion response and a real response, although this can be done plausibly for business taxes in some other countries, using special features of national tax systems.[27] Our empirical strategy therefore fo-

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[26]For example, HM Revenue and Customs (2015) estimates that the total VAT gap in 2014-15 was 13.1 billion. Of this, at most £1.0 billion is due to Missing Trader Intra-Community fraud, £1.2 billion is VAT debt, and finally £0.2 billion due to VAT avoidance. This means that over 80% of the gap is due to other factors.

[27]For example, Best and others (2015) use a minimum tax scheme for corporations in Pakistan which has a kink point where the real incentive for bunching is small, but the evasion incentive is large, and they find large bunch-

cuses primarily on identifying the effects of changes in the B2C ratio, input-cost ratio and level of competition as predicted by the theory, without taking a view on how much of this effect works through evasion. However, in online Appendix D, we do present some suggestive evidence that firms under-report turnover to stay below the threshold.

Now suppose that there is a fixed compliance cost $\Gamma > 0$ for the firm if it registers. What will the effect be on the equilibrium? If the voluntary registration condition condition ((18)) above does *not* hold, so we are in the bunching regime, then firms continue to bunch, but the upper end of the bunching interval is now described by

$$\pi(1;a^* + \Delta a^*) - \Gamma = \pi(0;a^* + \Delta a^*) \tag{22}$$

Clearly, the larger Γ, the higher is the upper end of the bunching interval in ((22)), as firms now have an incentive not only to avoid charging the output VAT but also the compliance cost. If the voluntary registration condition condition ((18)) above *does* hold, then there are two possibilities. If Γ is small enough, then even the least productive $\underline{a}$-firm will choose to register voluntarily and so part (c) of Proposition V.B continues to hold. If Γ is larger, then low-productivity firms start to bunch.

VI. CONTEXT AND DATA

A. The Value-Added Tax System in the UK

Approximately two million registered businesses remit value-added tax (VAT) in the UK every fiscal year. The VAT is the third largest source of government revenue following income tax and national insurance contributions. In 2017/18, the VAT raised £125.1 billion, accounting for 21.1% of total tax revenue and 6.1% of GDP in the UK.[28]

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ing around the minimum tax kink. Waseem (2016), again for Pakistan, shows very large responses when reforms cut the rate of income tax to zero for the self-employed, which he interprets as being largely an evasion response.

[28] See www.gov.uk/government/statistics/hmrc-tax-and-nics-receipts-for-the-uk.

VAT is levied on most goods and services provided by registered businesses in the UK, goods and some services imported from countries outside the European Union (EU), and brought into the UK from other EU countries. Key information about rates and thresholds for VAT in the UK is given in Table 2. As can be seen from that Table, the standard VAT rate in the UK was constant at 17.5% between April 2004 and January 2011, except for a temporary reduction to 15% during the worst months of financial crisis (between December 2008 and January 2010). The standard rate was further raised to 20% in January 2011 and has not been modified since then. A small number of goods and services have VAT levied at a reduced rate of 5% and there are also goods and services that are zero-rated or exempt from VAT altogether.[29]

All businesses must register for VAT if their taxable turnover is above a given threshold.[30] In the case of the UK, the registration threshold is regularly updated to keep up with inflation, as shown in Table 2.[31] The UK has the highest registration threshold in the EU, which is perceived as a way for the government to reduce the compliance costs of small businesses not wishing to register for VAT.[32]

As is standard in most VAT systems, VAT-registered businesses pay VAT on their purchases (input tax), and charge VAT on the full sale price of their taxable supplies (output tax). Businesses with a turnover below the threshold may choose to register voluntarily in order to recover the VAT paid on their intermediate inputs. Businesses cannot charge output VAT on

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[29]The 5% reduced rate is charged on a small number of supplies under schedule 7A of the *Value Added Tax Act of 1994 (VATA)*. This mainly includes the supply of domestic fuel and power, the installation of energy saving materials, women's sanitary products, children's car seats and certain types of construction work.

[30]Taxable turnover includes the value of any goods or services a business supplies *within* the UK, unless they are exempt from VAT. All zero-rated sales, except exports, are included as part of the taxable turnover.

[31]Specifically, under Article 24(2)(c) of the sixth EC VAT directive (77/388/EEC 17 May 1977). These provisions are now consolidated in the principal VAT directive (2006/112/EC); article 287 allows for States to increase the registration threshold in line with inflation. As far as we are aware, there is no interaction between the VAT registration threshold with other major taxes in the UK, not does it change the probability of being audited for firms around the threshold.

[32]Small firms with annual taxable turnover above the registration threshold but below £150,000 can use a simplified flat-rate VAT scheme. In 2007, only 16% of eligible firms were registered under the flat-rate scheme (Vesal, 2013).

their sales of zero-rated or exempt goods. In the case of zero-rated supplies, the firm can claim back the VAT paid on inputs, whereas the same is not true for exempt supplies.

In the specific case of the UK, there are two rules governing registration: a forward-looking rule and a backward-looking one. Under the forward-looking rule, a business must register for VAT if its taxable turnover is likely to go over the threshold in the next 30 days. Under the backward-looking rule, a business must register if its taxable turnover in the previous 12 months was above the threshold. Strictly speaking, our theoretical model is static and applies only to the forward-looking decision; that is, the business must register if turnover in the current year is expected to exceed the threshold. In our sample, around 67.4% of first-time registers have a previous-year turnover lower than the VAT threshold. This suggests that the forward-looking decision is the most relevant in practice.

B. Data

We construct our data by linking two administrative datasets: (i) the universe of VAT returns and (ii) the universe of corporation tax records in the UK (called CT600). The first dataset provides detailed information on VAT-registered businesses, which may take a variety of legal forms including sole proprietorships, partnerships, and companies. To obtain information on business not registered for the VAT, we link the VAT records to the population of corporation tax records based on a common anonymized taxpayer reference number. The linked dataset allows us to identify whether *companies* are registered in the VAT or not, and contains rich information on VAT and corporation tax for each business and year.[33]

We further merge the linked tax dataset with two additional data sources: (i) the Financial Analysis Made Easy (FAME) annual company account database, which contains additional firm characteristics and accounting information and (ii) the annual sector-level statistics on

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[33]Note that this linked dataset does not include sole proprietorships or partnerships that are below the VAT threshold and have chosen to not register for VAT. These types of business are taxed through the individual income tax. The data on the universe of self-assessment taxpayers is available (it has been used recently by Almunia, Lockwood, and Scharf (2017). However, this dataset does not have an identifier to link individual taxpayers to the VAT records.

the share of sales to final consumers based on the Office of National Statistics' (ONS) Input-Output tables. The latter gives us an empirical proxy for λ, the share of sales that are B2C at the 2-digit SIC industry level.

The final dataset contains 3,461,247 observations for 968,353 unique companies between April 1, 2004 and April 5, 2015.[34] For each company-year observation, we have information on the VAT-exclusive turnover taken from the corporate tax records, and whether the company is registered for VAT.[35] We examine several key factors that drive firms' decisions about voluntary registration, including the share of input costs relative to total turnover (input-cost ratio), the share of sales to final consumers (B2C sales ratio), firm-specific history of registration, and the degree of industry-level competitiveness.

The CT600 data contains an aggregate measure of input costs that includes both salaries and other inputs. Therefore, the input cost ratio derived from this dataset is higher than the magnitude relevant in our setting. The FAME dataset does report salaries and other inputs separately, but only 7 percent of the firms in our study sample have non-missing salaries in FAME, severely limiting our sample size. To obtain a measure of the input cost ratio closer to the theory, we extrapolate from the subset of firms in the FAME dataset. Specifically, we rescale the input-cost ratio reported in the CT600 data to match the mean and standard deviations observed for each industry in the FAME data.

For the industry competitiveness measure, we use the Lerner index of competition. This is defined as one minus the average ratio of trading profit to value of sales for firms in a given

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[34]We take several steps to refine the sample to study the VAT registration decisions of individual companies. First, we eliminate companies which are part of a larger VAT group and focus only on standalone independent companies. This is because companies under common control–for example, subsidiaries of a parent company–can register as a VAT group and submit only one VAT return for all companies in a VAT group. Second, we drop all observations with partial-year tax or accounting records because the registration decision can be based on turnover in the previous 12 months. We further eliminate companies that mainly engage in overseas activities based on the HMRC trade classification. This is because the taxable VAT turnover excludes exports. We exclude companies/organizations that are non-profit making because they may have specific objectives that are different from other companies. Online Appendix F provides details on how we construct our sample.

[35]Our empirical analysis is based on turnover reported in the CT600 for two reasons. The first is data availability, as we only observe VAT-liable turnover for firms that are registered for VAT. The second is related to salience, given that firms that are not registered for VAT are more likely to base their registration decision on the overall amount of turnover, instead of computing a separate measure of turnover that is subject to VAT.

industry. It is easily checked that if demand is iso-elastic at e, for all firms in an industry, as it is in our theoretical model, the Lerner index is simply $(e-1)/e$. This means that the Lerner index is an ideal measure for testing our predictions about the effect of competition as measured by e.[36]

Notice that the input cost ratio varies at the firm level, while the share of B2C sales and the Lerner index vary at the 2-digit and 4-digit industry level respectively. All three variables have annual variation, allowing us to include them in the panel regressions with fixed effects that we present in the next section.

We focus on two different subsamples to test hypotheses developed in Section V. When studying the voluntary registration decision, we include all firms with turnover below the current-year VAT registration threshold to examine the choice of voluntary registration. We define a firm as voluntarily registered if it is registered in the current year, and (1) has never registered before and has a turnover below the VAT threshold, or (2) if it was registered in the previous year and has a turnover below the VAT deregistration threshold. For the bunching analysis, we include all firms with turnover in the range between £50,000 below the current-year registration threshold and £100,000 above. In this larger sample, 69.5% of firms have a turnover below the VAT threshold, of which 42.9% are registered for VAT. So, overall, 29.8% of firms in the main sample of companies are voluntarily registered.

C. Summary Statistics

Table 3 reports summary statistics for companies in the neighborhood of current-year VAT notch, i.e. those with nominal turnover of between £10,000 and £200,000. We report the mean, standard deviation, various percentiles (10th, 50th and 90th) and the number of non-missing observations for the key variables used in empirical analysis. Firms in the final dataset have £74,690 of average turnover and £21,880 of trading profit. The average salary-*inclusive*

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[36]Other commonly used measures of competition are the four-firm concentration ratio, or the Herfindahl index, but they measure the relative importance of the largest firms in an industry, and are not closely related to the demand elasticity faced by small firms close to the VAT registration threshold.

input cost ratio (using data from CT600) is 71% of total turnover, while the average salary-*exclusive* input cost ratio (using data from FAME for a subsample of firms) is 38%. The input cost ratio calculated with the extrapolation procedure explained above yields an average of 48%, which is in between but closer to the FAME subsample, as expected. The average share of B2C sales is 55%, and the average Lerner index is 0.75.[37]

VII. RESULTS

We present two sets of empirical results. For voluntary registration, we estimate a linear probability model with firm and year fixed effects focusing on firms with turnover below the VAT registration threshold. The regression equation includes the share of business-to-consumer sales (B2C), the input cost ratio (ICR), the Lerner index as a proxy for the competitiveness of the industry and the distance from the registration threshold. In the bunching analysis, we use graphical evidence and standard nonparametric techniques to estimate the excess bunching mass just below the threshold. We then investigate whether the amount of bunching varies with the three key variables mentioned above in the way predicted by the theory.

A. Voluntary Registration

We examine whether the decision to voluntarily register for the VAT is consistent with the three theoretical predictions stated in Propositions V.B and V.B: a firm is more likely to register voluntarily for VAT if it sells mostly to other VAT-registered businesses (as opposed to final consumers), has a larger share of intermediate input costs (relative to total costs), or operates in a more competitive industry.

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[37]Recall that the share of B2C sales and the Lerner index vary at the industry-year level, but here we report the firm-year level averages.

We evaluate these relationships more formally using a panel regression framework. We model the decision of voluntary registration as a binary choice model of the following form:

$$R_{it} = \alpha_i + \alpha_t + \gamma_1 B2C_{it}^j + \gamma_2 ICR_{it} + \gamma_3 L_{it}^j + \gamma_4 D_{it} + \upsilon_{it}, \tag{23}$$

where R_{it} is a dummy indicator that takes value 1 if the firm is voluntarily registered and zero otherwise. $B2C_{it}^j$ denotes the share of B2C sales in industry j where firm i operates in year t, ICR_{it} denotes the input-cost ratio for firm i in year t, and L_{it}^j is the Lerner index for competitiveness for industry j in year t. Additionally, we control for the distance to the VAT threshold, D_{it}, defined as the difference between total turnover and the registration threshold in year t. The time-invariant firm fixed effects and year dummies are denoted by α_i and α_t, respectively, and υ_{it} is a random error term.

We estimate equation ((23)) using a linear probability model, which allows us to include firm fixed effects without a bias due to the incidental parameters problem.[38] The estimation sample includes all firms with turnover below the current-year VAT registration threshold. According to Proposition V.B, we expect to obtain $\gamma_1 < 0$, $\gamma_2 > 0$ and $\gamma_3 < 0$.

Table 4 reports the estimation results from the linear probability model.[39] The first four columns include year dummies but not firm fixed effects, which allows us to examine the total effect of the industry-level variation in the B2C sales ratio and the Lerner index on the probability of voluntary registration. We first include each of the three key variables one at a time (columns 1-3), and then include them all together in column 4. The coefficients in the letter column are -0.17 for B2C sales, 0.20 for the input cost ratio and -0.36 for the Lerner index, all statistically significant at the one percent level. These coefficients are consistent with the predictions from our theoretical framework, and similar to those in columns (1)-(3).

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[38]The predicted values are all within the $(0,1)$ interval, indicating that the main drawback of the linear probability model is not a concern in this setting.

[39]The estimation results from a conditional logit model with fixed effects are very similar and are available from the authors upon request.

In columns (5)-(8), we include firm fixed effects and follow the same progression as before. The fixed effects absorb a substantial part of the cross-sectional variation in the industry-level variables and reduce the size of their coefficient estimates. While all coefficients are statistically significant and have the expected signs in columns (5)-(7), the coefficient on the share of B2C sales becomes essentially zero in column (8). In that last specification, the coefficient on the input cost ratio is 0.064, and the one on the Lerner index is -0.214, both statistically significant.

One advantage of including firm fixed effects is that they partially control for inertia in registration status, by controlling for whether a firm has previously been above the registration threshold. However, including firm fixed effects also absorbs part of the variation underlying the predictions in our theoretical framework. This is because some of the characteristics that affect the incentives to register voluntarily, in particular the share of B2C sales and the input cost ratio, are fairly stable over short periods of time. Thus, it is not surprising that the coefficients decrease in size in the fixed-effects specification. While neither specification (with or without fixed effects) is flawless, we think the regression without fixed effects represents the best possible test of our theoretical predictions.

Therefore, we evaluate the size of the effects focusing on column (4), our preferred specification. Given these results, the likelihood of being registered voluntarily is on average 4.0 percentage points higher as the B2C ratio *decreases* by one standard deviation (s.d.), 4.9 percentage points higher as the input-cost ratio increases by one s.d., and 3.9 percentage points higher as the Lerner index *decreases* by one s.d. These are sizable effects that confirm the importance of these three variables in the firms' decision to register voluntarily for VAT.

Table 5 reports similar specifications using the two alternative measures of the input cost ratio. Columns (1)-(4) use the measure from the CT600 dataset, and columns (5)-(8) use the measure from the FAME dataset, for the subsample of firms observed. All the coefficient estimates are qualitatively similar to those in Table 4, and they are all statistically significant except for the coefficient on the share of B2C sales in the fixed-effects specifications (4 and

8). We conclude that the results are robust to the use of alternative measures of the input cost ratio.

Dynamic Behavior   One potential limitation of the above analysis is that we do not explicitly consider the dynamic behavior of firms. A change in the registration status involves some costs to firms, raising the possibility that firms who are initially above the registration threshold and later fall below may stay registered simply to avoid the cost of deregistration. Hence, some of the firms who seem to be voluntarily registered may just be behaving in this way because of inertia. As noted above, the firm fixed effects partially control for this type of behavior. As a further robustness check, we conduct additional regressions taking into account these dynamic effects. Specifically, we estimate a probit model with random effects where we include a lag of the dependent variable (i.e., whether the firm was registered the previous year), the initial registration status, and the averages of the key explanatory variables. A more detailed explanation of these regressions and the results can be found in the online appendix.

B. Bunching Evidence

1. Estimation Method

As explained in Section IV, the VAT registration threshold at the cutoff turnover value s^* will induce excess bunching at the threshold by companies for which voluntary registration is not optimal. From ((20)), we can write bunching as $B = \Delta s^* h(s^*)$, where $h(s^*)$ is the counterfactual density of firms over the bunching interval, assuming that this is constant. Following Kleven and Waseem (2013), We will express this as a fraction of the counterfactual density of firms at the notch, so our empirical measure of bunching is

$$ b = \frac{\sum\limits_{j=s^*_-}^{s^*} (c_j - \widehat{c}_j)}{\frac{1}{N} \sum\limits_{j=s^*_-}^{s^*} \widehat{c}_j}. \tag{24} $$

Here, c_j is the actual number of firms in each £1,000 turnover bin, and $\widehat{c}_j$ is the counterfactual bin counts without the notch. The range $\left(s_-^*, s_+^*\right)$ specifies turnover bins around the notch where bunching occurs and are therefore excluded from predicting the counterfactual distribution. Specifically, the lower bound of the excluded turnover region, s_-^*, is set at the point where excess bunching starts. The upper bound of the excluded region, s_+^*, is estimated in an iteration procedure to ensure that the excess mass below the VAT notch is equal to the missing mass above.[40] Finally, N is the number of bins in the excluded range $\left(s_-^*, s_+^*\right)$.

So, to summarize, equation ((24)) says that the excess mass is empirically measured by the difference between the predicted and actual mass of firms in the excluded range, divided by the average counterfactual density of firms in that range.

2. Graphical Evidence

This section presents evidence of bunching below the VAT notch using the main sample of companies with turnover in a range between £45,000 below and £100,000 above the registration threshold. Figure 1 shows the distribution of turnover for all companies in that range, pooling together data from fiscal year 2004/05 through 2014/15. Following Chetty and others (2011) and Kleven and Waseem (2013), we estimate the counterfactual distribution by fitting a flexible polynomial of order 5 to the empirical distribution, excluding a range around to the VAT notch.[41]

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[40]We follow the standard procedure to estimate the counterfactual distribution. By grouping companies into bins of £1,000, we estimate the counterfactual distribution of turnover around the VAT notch s^* in the following regression:

$$c_j = \sum_{l=0}^{q} \beta_i \left(s_j\right)^l + \sum_{i=s_-^*}^{s_+^*} \gamma_i I\{j=i\} + \varepsilon_j, \tag{25}$$

where c_j is the number of companies in turnover bin j, s_j is the distance between turnover bin j and the VAT notch s^*, q is the order of the polynomial, and $I\{\cdot\}$ is an indicator function. The error term ε_j reflects misspecification of the density equation.

[41]As a robustness check, we have tried values between 4 and 7 for the order of the polynomial and our results are not significantly changed (available on request). The excluded range goes from -£14,000 to £24,000, which ensures that the excess bunching mass to the left of the notch is almost identical to the missing mass to the right. As in Kleven and Waseem (2013), the upper bound of the excluded region is estimated using an iterative procedure to ensure that the area under the estimated counterfactual density is equal the area under the observed density.

Two points are worth noting in Figure 1. First, the VAT notch creates evident bunching below the threshold. The bunching estimate is 1.361 (std. error: 0.202), meaning that the total excess bunching mass is almost 1.4 times as large as the average height of the counterfactual over the excluded range.[42] Second, in contrast with the large spike at the threshold, there is only a small hole in the distribution above the VAT notch. We do not attempt to estimate the magnitude of optimization frictions implied by the missing mass to the right of the notch for the various reason discussed in section IV.

Kleven (2016) argues that, in the context of the personal income tax, bunching is much more likely to be due to evasion, rather than to real earnings responses. Here, as already noted, both evasion and real responses may be driving observed bunching. We do not attempt to decompose observed bunching into real and evasion responses because there is no variation that allows us to do that. In the online appendix, we show some suggestive evidence that the bunching behavior may partly be due to misreporting of turnover.

3. Heterogeneity in Bunching

We now explore potential heterogeneity in bunching to see whether the empirical patterns are consistent with the predictions set out in Proposition V.C. Implementing this analysis is challenging because some firms have incentives to voluntarily register for VAT, and therefore are indifferent to the existence of the VAT threshold. To address this issue, we leverage the fact that we observe which firms choose to register voluntarily among those below the threshold. Specifically, we partition the sample into two groups of firms based on their predicted likelihood of registering voluntarily (regardless of their turnover) following three steps. First, we regress voluntary registration status on the three key variables (share of B2C sales, input cost ratio and Lerner index), including only firms below the turnover threshold. Second, we use the estimated coefficients to obtain a predicted probability of being voluntarily registered for

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[42]Bunching is sharp and significant every year, as shown in earlier versions of this paper (e.g., Liu and Lockwood, 2015). Unlike to studies analyzing bunching in the taxable income of individuals (Kleven and Waseem, 2013) and corporations (Devereux, Liu, and Loretz, 2014), we do not find any evident bunching at round numbers.

each firm i in year t. Third, we divide firms into two groups depending on whether their predicted probability is above or below the median.

First, we explore how companies with different shares of B2C sales respond to the same VAT notch. We divide companies into four quartiles of the B2C share distribution and estimate bunching at the VAT registration threshold for the subsamples of firms more and less likely to register voluntarily. The top panel of Figure 4 shows the bunching estimates and 95-percent confidence intervals for each quartile of the B2C share distribution, for firms predicted to not register voluntarily (i.e., the subgroup for which the VAT notch is binding). The bunching estimate is positively correlated with the share of B2C sales, taking a value of 0.5 for the first quartile (Q1) and about 1.4 for the fourth quartile (Q4). The bottom panel of Figure 4 shows the equivalent estimates for the subgroup of firms predicted to register voluntarily, for whom the VAT threshold is not binding. In this case, the bunching estimates are consistently low, between 0.3 and 0.6, and they do not follow any clear pattern across quartiles.

Second, we examine the extent of bunching depending on the degree of competition in the product market, measured by the Lerner index at the 4-digit industry level. Since this index is defined as one minus the average profit margin in the industry, higher values of the index indicate that the industry is more competitive. As in the previous cases, we examine how bunching varies across quartiles of the Lerner index distribution for firms predicted to register vs. not. The top panel of Figure 5 shows a strong positive correlation between the bunching estimates and the degree of competition for firms predicted to not register, with an estimate of 1.7 for firms in the top quartile (Q4). When studying firms predicted to register voluntarily in the bottom panel of Figure 5, we observe consistently low bunching estimates at all four quartiles without any specific pattern.

Finally, we examine how companies with different input cost ratios respond to the VAT notch. Again, we divide the sample into quartiles of the distribution of this variable, and look separately at firms predicted to register voluntarily vs. those not predicted to register. For this test, we use the input cost ratio constructed using information from the FAME subsample. The top panel of Figure 6 shows that the degree of bunching generally decreases with the input

cost ratio for firms predicted to not register voluntarily, although the relationship is not fully monotonic because the estimate for the first quartile is relatively low. In the bottom panel of Figure 6, we observe that the pattern of bunching estimates is flat for firms predicted to register voluntarily, again confirming that, as expected, the model's predictions do not apply to that group of firms.

VIII. THE VAT THRESHOLD

We conclude by discussing some of the implications of our work for the setting of the VAT threshold. The well-known work of Keen and Mintz (2004) makes clear that the basic trade-off in choosing the threshold is between minimizing administration costs for the revenue authority and compliance costs for businesses (implying a high threshold) and raising VAT revenue (implying a low threshold). On top of this, they show that behavioral responses to the threshold also affect threshold design.

In the online appendix, we develop a formula for the optimal threshold in our model which refines their basic insight in two ways. First, it allows for a behavioral response that is specific to our model. Second, it also allows for B2B sales, a feature not present in Keen and Mintz (2004). We find that the optimal threshold in the presence of B2B sales ($\lambda < 1$) is higher than in the absence of B2B sales.[43]

The intuition for this result is that only B2C sales by small firms above the threshold are taxed in our model, whereas the value of B2B sales to the large firm is eventually taxed, as all of the large firm's sales are taxed. This implies that raising the threshold is less costly in terms of forgone tax revenue when more of the sales of the small firm are B2B. This intuition is further developed in online Appendix C.

While our formula is obviously specific to our model, the mechanism at work is likely to be more general, i.e. for a variety of different market structures, B2B sales are more likely to

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[43]A fuller analysis of the optimal threshold is beyond the scope of this paper, as it would require calibration of the model. We leave this as a topic for future work.

eventually be taxed than B2C sales.[44] In order to test this, we have compiled some cross-country empirical evidence. Figure 7 shows the cross-country relationship between the average ratio of B2C sales in a country and the ratio of the VAT threshold in year 2017 over GDP per capita.[45] We focus on this ratio to adjust for the relative size of turnover by small businesses in countries at different levels of development. Moreover, the size of the informal sector might also influence the choice of threshold. We proxy the latter by the share of the agricultural sector in GDP, following Keen and Lockwood (2010).

The empirical pattern in Figure 7 is quite intriguing and consistent with the model prediction in the online Appendix C. There is a positive correlation between the average B2C ratio and the VAT threshold as a fraction of per-capita GDP (Panel A). The estimated slope coefficient is 0.06 and is significant at the 10 percent level (with a $p-$value of 0.089). However, when controlling for the size of informal sector, the relation between the average B2C ratio and the VAT threshold normalized by GDP per capita becomes negative, with the slope coefficient estimated to be -0.03 and highly significant (with a $p-$value of 0.005). As a large informal sector (proxied by the share of agriculture sector) is typically associated with higher compliance and administration costs, the patterns suggest that the VAT threshold tends to be higher with higher compliance and administration costs and less direct selling to consumers.

IX. CONCLUSIONS

In this paper, we first developed a conceptual framework which can explain the co-existence of voluntary VAT registration and bunching at the registration threshold. We showed that this required (at least) three stages of production, with firms at the intermediate stage selling both to final consumers and to other firms. This framework predicts that voluntary registration is

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[44]There is also a second, more indirect mechanism at work in our model with B2B sales. An increase in the threshold increases the prices that the small firms who are bunching charge for inputs to the large firm, and this is passed on to prices by the large firm, increasing the VAT base. See online Appendix C for details.

[45]Specifically, the B2C ratio is calculated as the final sales to consumers, including sales to households, non-profit institutions serving households, and governments, relative to total sales to industries and consumers in each of the 103 countries. The cross-country input/output data is from the multi-region input-output table (MRIO) of the Eora global supply chain database, available at https://worldmrio.com/eora26/.

more likely, and bunching is less likely, when (i) the cost of inputs relative to sales is high, (ii) when the proportion of B2C sales is low, or (iii) when the level of product market competition is low. Additionally, evasion opportunities will generally make voluntary registration more likely, while having an ambiguous effect on bunching.

We then brought these predictions to an administrative dataset that was created by linking the population of corporate income and value-added tax records in the UK. We found that voluntary registration is more likely with a low share of B2C sales, a high input-cost ratio, and more competition in the industry. Moreover, there is clear evidence of bunching at the VAT threshold. Consistently with the theory, there is a clear pattern of heterogeneity in bunching; the amount of bunching is increasing in the B2C sales ratio, and decreasing in the input-cost ratio and the level of competition in the industry. In the last section of the paper, we provided a discussion and some cross-country evidence of the implications of our results for the optimal design of the VAT threshold.

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FIGURES

Figure 1. Turnover Distribution around the Registration Threshold
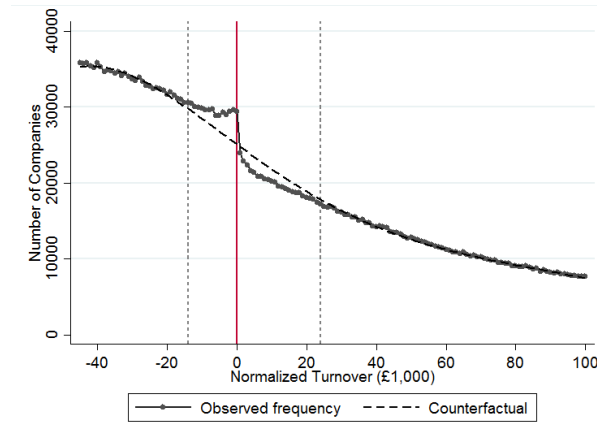
Notes: This figure shows the histogram of companies' turnover net of current-year VAT registration threshold (normalized VAT notch) by pooling data between 2004/05-2014/15. The bin width is £1,000 and the red line denotes the normalized VAT notch. The dashed line is counterfactual density fitted by excluding bins around the VAT notch.

Figure 2. Persistence of Registration Below the Threshold
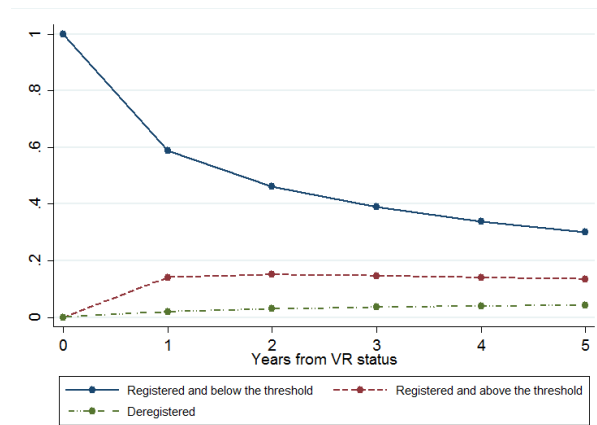
Notes: This figure plots the transition probability for firms voluntarily registered during 2004/05-2009/10, to remain registered and below the threshold, registered and above the threshold or deregistered, in the following years. Firms leave the sample when they are dissolved or become part of a larger VAT group in following years.

Figure 3. Structure of the Economy
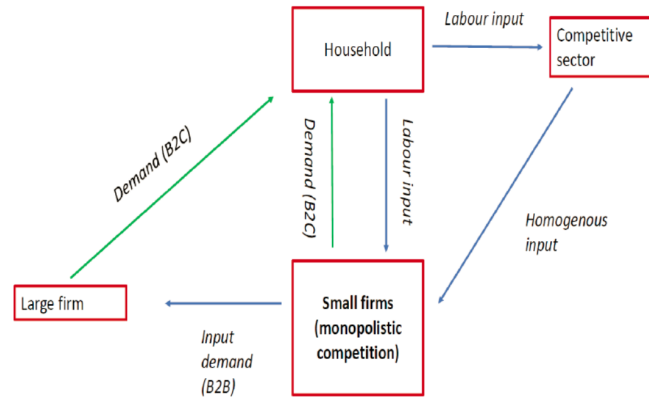
Notes: this figure shows the key features of the economy in which we model behavioral responses of firms with respect to the VAT threshold, including voluntary registration and bunching.

Figure 4. Bunching across Quartiles of the B2C Share Distribution
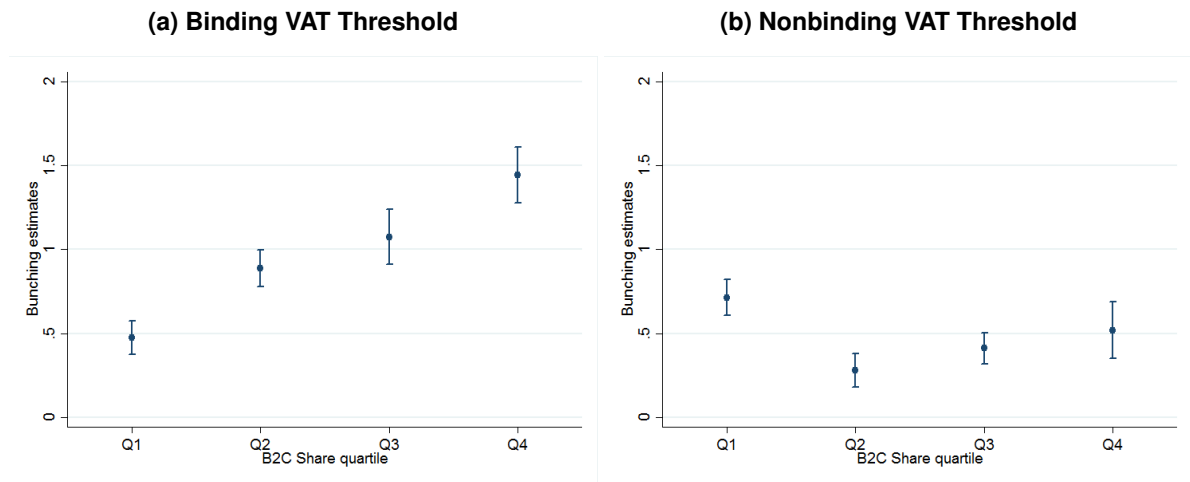
(a) Binding VAT Threshold **(b) Nonbinding VAT Threshold**

Notes: the figure shows the bunching estimates around the VAT notch across four different quartiles of the distribution of the share of B2C sales for the pooled dataset for period 2004/05-2014/15. Panel (a) shows the point estimates and 95 percent confidence intervals for the subset of firms not predicted to register voluntarily. Panel (b) shows the estimates for the subset of firms predicted to register voluntarily, for whom the threshold is nonbinding.

Figure 5. Bunching across Quartiles of the Lerner Index Distribution
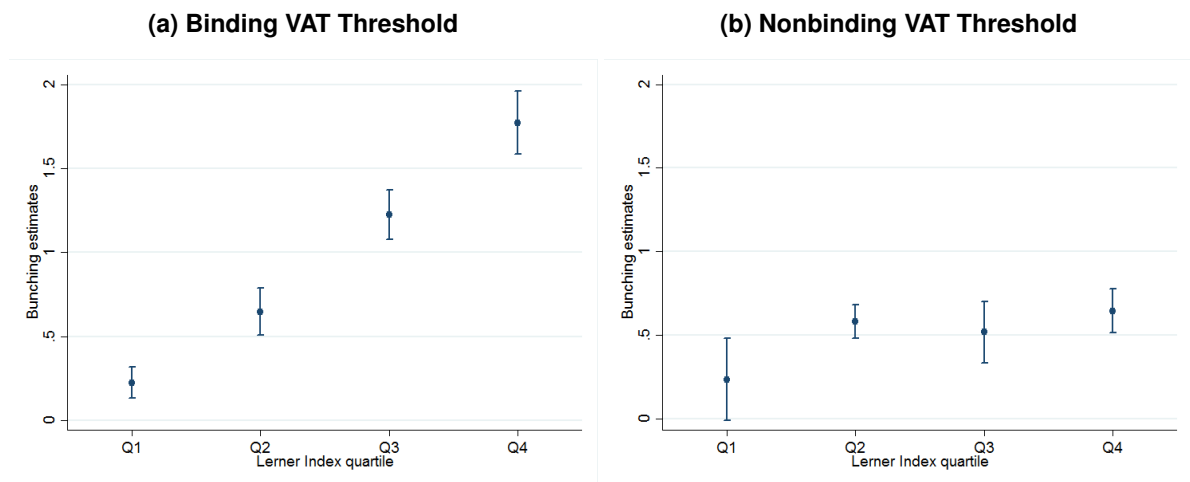
(a) Binding VAT Threshold **(b) Nonbinding VAT Threshold**

Notes: the figure shows the bunching estimates around the VAT notch across four different quartiles of the distribution of Lerner Index for the pooled dataset for period 2004/05-2014/15. Panel (a) shows the point estimates and 95 percent confidence intervals for the subset of firms not predicted to register voluntarily. Panel (b) shows the estimates for the subset of firms predicted to register voluntarily, for whom the threshold is nonbinding.

Figure 6. Bunching across Quartiles of the Input-Cost Ratio Distribution

(a) Bunching Evidence

(b) Bunching Estimates

Notes: the figure shows the bunching estimates around the VAT notch across four different quartiles of the distribution of Input-cost Ratio for the pooled dataset for period 2004/05-2014/15. Panel (a) shows the point estimates and 95 percent confidence intervals for 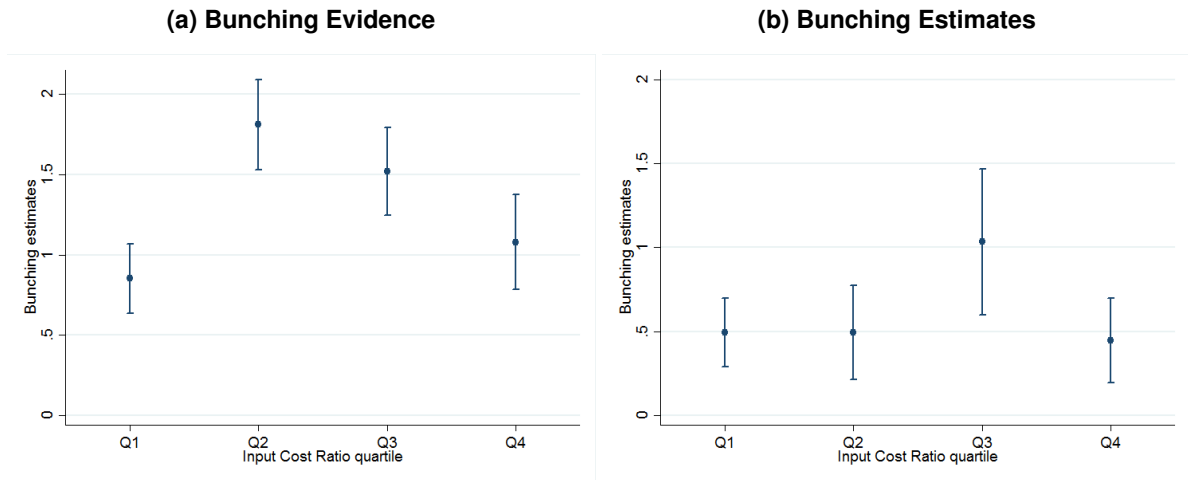the subset of firms not predicted to register voluntarily. Panel (b) shows the estimates for the subset of firms predicted to register voluntarily, for whom the threshold is nonbinding.

Figure 7. VAT Threshold and B2C Ratio: Cross-Country Evidence

(a) Raw Correlation

(b) Controlling for the Size of Informal Sector
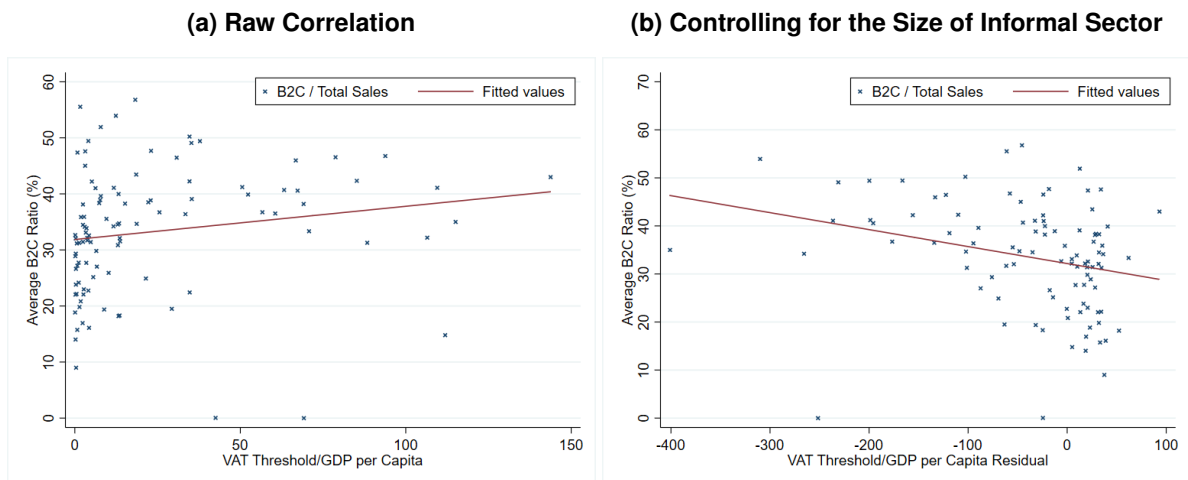
Notes: the figure shows the observed correlation between the B2C sales ratio and the VAT threshold across 103 countries in 2017. Panel A controls for the influence of GDP per capita, expressing the VAT threshold as a fraction of GDP per capita in each country. Panel B further controls for the size of the informal sector, by regressing the VAT threshold as a fraction of GDP per capita on the share of agriculture sector in each country and plotting the residual against the B2C sales ratio.

TABLES

Table 1. The Effect of Competition on the Sufficient Statistic T

e	2	5	10	20	50	100
$\omega = 0.9, \lambda = 0.1$	1.039	1.408	1.967	2.361	1.564	0.678
$\omega = 0.9, \lambda = 0.9$	0.826	0.796	0.739	0.629	0.381	0.164
$\omega = 0.1, \lambda = 0.1$	0.898	0.786	0.530	0.148	0.001	0.000
$\omega = 0.1, \lambda = 0.9$	0.714	0.444	0.199	0.039	0.000	0.000

Notes: the table shows how the value of the sufficient statistic T varies with the elasticity of demand (e), the share of B2C sales (λ) and the input cost ratio (ω).

Table 2. Value-Added Tax System in the UK

Fiscal Year	Registration Threshold (£)	Deregistration Threshold (£)	Standard Rate (%)	Flat-Rate Scheme Threshold (£)
2004-05	58,000	56,000	17.5	150,000
2005-06	60,000	58,000	17.5	150,000
2006-07	61,000	59,000	17.5	150,000
2007-08	64,000	62,000	17.5	150,000
Apr 1, 2008-Nov 30, 2008	67,000	65,000	17.5	150,000
Dec 1, 2008-Mar 30, 2009	67,000	65,000	15.0	150,000
Apr 1, 2009-Dec 31, 2009	68,000	66,000	15.0	150,000
Jan 1, 2010-Mar 30, 2010	68,000	66,000	17.5	150,000
Apr 1, 2010-Jan 3, 2011	70,000	68,000	17.5	150,000
Jan 4, 2011-Mar 31, 2011	70,000	68,000	20.0	150,000
2011-2012	73,000	71,000	20.0	150,000
2012-2013	77,000	75,000	20.0	150,000
2013-2014	79,000	77,000	20.0	150,000
2014-2015	81,000	79,000	20.0	150,000

Notes: the table shows changes in the registration threshold, deregistration threshold, Flat-Rate scheme threshold, and VAT standard rate over recent fiscal years. For more information on the UK VAT tax system, see http://www.hmrc.gov.uk/vat/forms-rates/rates/rates-thresholds.htm. For the values of past registration thresholds, see https://www.gov.uk/government/publications/vat-notice-7001-should-i-be-registered-for-vat/vat-notice-7001-supplement–2.

Table 3. Summary Statistics

Variable	Mean	St.Dev.	p10	p50	p90	Observations
Total Turnover	74.69	48.88	19.88	62.70	151.80	3461247
Trading profit	21.88	27.25	0.00	11.82	59.71	3461247
Total Input Costs (CT600)	52.81	44.27	12.04	36.55	123.46	3461247
Intermediate Input Costs (FAME)	31.34	33.79	2.00	18.00	82.00	238838
Input-Cost Ratio (CT600)	0.71	0.28	0.28	0.78	1.00	3461247
Input-Cost Ratio (FAME)	0.38	0.25	0.04	0.37	0.73	238838
Input-Cost Ratio (Adj)	0.48	0.24	0.11	0.53	0.77	3024673
Share of B2C Sales	0.55	0.24	0.29	0.45	0.91	3461247
Lerner Index	0.75	0.11	0.58	0.77	0.90	3461247
VAT Registered	0.630	0.483	0	1	1	3461247
VAT Registered (below threshold)	0.429	0.495	0	0	1	2405144

Notes: this table shows the mean, standard deviation, the 10th, 50th (median) and 90th percentiles, and the number of nonmissing observations of the key variables used in the empirical analysis. The top four variables are expressed in thousands of pounds (GBP), where 1 GBP = 1.29 USD as of September 2018. The rest of variables are defined to be in the interval $[0, 1]$. Note that we only have data on salary-exclusive input costs for a subset of companies from the FAME dataset. The input-cost ratio (Adj) is constructed by normalizing input-cost ratio (CT600) to match the mean and standard deviation of input-cost ratio (FAME) at industry level. The share of B2C sales denotes the proportion of turnover that comes from sales to final consumers, as opposed to sales to other VAT-registered businesses. VAT Registered is indicator for companies registered for VAT, and VAT Registered (below threshold) is that defined for companies below registration threshold.

Table 4. Determinants of Voluntary VAT Registration

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Share of B2C Sales	-0.233***			-0.167***	-0.025**			0.008
	(0.003)			(0.003)	(0.010)			(0.011)
Input-Cost Ratio		0.153***		0.204***		0.064***		0.064***
		(0.002)		(0.002)		(0.001)		(0.001)
Lerner Index			-0.417***	-0.356***			-0.195***	-0.214***
			(0.006)	(0.007)			(0.015)	(0.016)
Observations	2405144	2143833	2405144	2143833	2405144	2143833	2405144	2143833
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm FE	No	No	No	No	Yes	Yes	Yes	Yes

Notes: this table presents estimation results from the binary choice model of VAT registration based on equation ((23)). The dependent variable is the binary indicator of VAT registration status that takes on the value 1 if a firm is voluntarily registered for VAT and zero otherwise. Columns (1)-(4) present results from the linear probability model without firm-fixed effects, and columns (5)-(8) present results by adding firm-fixed. The input-cost ratio is the adjusted measure - input-cost ratio (CT600) normalized to match the mean and standard deviation of input-cost ratio (FAME) at industry level. Additional firm-level control variables include distance to the registration threshold. *,**,*** denotes significance at 10%, 5% and 1%, respectively. Standard errors are clustered at firm level.

Table 5. Determinants of Voluntary VAT Registration - other Input cost measures

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Share of B2C Sales		-0.192***		-0.004		-0.158***		0.028
		(0.003)		(0.010)		(0.010)		(0.033)
Input-Cost Ratio	0.070***	0.139***	0.067***	0.068***	0.220***	0.287***	0.032***	0.032***
	(0.002)	(0.002)	(0.001)	(0.001)	(0.008)	(0.008)	(0.005)	(0.005)
Lerner Index		-0.341***		-0.226***		-0.696***		-0.163***
		(0.007)		(0.015)		(0.026)		(0.056)
Observations	2405144	2405144	2405144	2405144	156571	156571	156571	156571
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-FE	No	No	Yes	Yes	No	No	Yes	Yes
Cost-Ratio Source	CT600	CT600	CT600	CT600	FAME	FAME	FAME	FAME

Notes: this table presents estimation results from the binary choice model of VAT registration based on equation ((23)). The dependent variable is the binary indicator of VAT registration status that takes on the value 1 if a firm is voluntarily registered for VAT and zero otherwise. Columns (1)-(2) present results from the linear probability model without firm-fixed effects with input-cost ratio (CT600), and columns (3)-(4) present results by adding firm-fixed effects. Column (5)-(6) present results without firm-fixed effects with input-cost ratio (FAME), and columns (7)-(8) present results by adding firm-fixed effects. Additional firm-level control variables include distance to the registration threshold. *,**,*** denotes significance at 10%, 5% and 1%, respectively. Standard errors are clustered at firm level.

APPENDIX A.   THEORETICAL APPENDIX

(A.0.0.1) ***Proof of Proposition V.A.***   First, assume only B2C sales i.e. $\lambda = 1$. In this case, the firm chooses p_C, p_B to maximize ((11)) subject to ((12)),((13)). This is easily solved to give price as a mark-up over cost i.e. $p_C = \frac{e_C}{e_C - 1} c(I; a)$ and thus profit for non-registered and registered firms respectively is

$$\pi(0; a) = B_C(c(0, a))^{1 - e_C}, \; \pi(1; a) = B_C(c(1, a))^{1 - e_C} \frac{1}{(1 + t)^{e_C}} \qquad \text{A.1}$$

where $B_C = A_C \left(\frac{e_C - 1}{e_C} \right)^{e_C} \frac{1}{e_C - 1}$. So, we only need show that $\pi(0; a) > \pi(1; a)$. But, from (A.1), ((10)), this holds iff

$$\left(\frac{c(0, a)}{c(1, a)} \right)^{1 - e_C} = (1 + \omega t)^{1 - e_C} > \frac{1}{(1 + t)^{e_C}}$$

Now note that as $e_C > 1$,

$$(1 + \omega t)^{1 - e_C} \geq (1 + t)^{1 - e_C} > (1 + t)^{-e_C}$$

as required.

Next, assume only B2B sales i.e. $\lambda = 0$. Then, it is easily checked profit is $(p - c(I; a)) A_B p^{-e_B}$. So, here, the argument is even simpler; cost is decreasing, and therefore profit is increasing, if $I = 1$. $\square$

(A.0.0.2) ***Proof of Proposition V.B.***   (a) If registered, the firm chooses p_C, p_B to maximize ((11)) subject to ((12)),((13)) when $I = 1$. This is easily solved to give prices as a mark-up over cost i.e.

$$p_C = \frac{e_C}{e_C - 1} c(1; a), \; p_B(a) = \frac{e_B}{e_B - 1} c(1; a). \qquad \text{A.2}$$

Substituting A.2 back into the profit function ((11)), and using $A_C = 1, A_B = (1 + t)^{-e_B} \left(\frac{e - 1}{e} \right)^e$, we can derive the following formula for maximized profit:

$$\pi(1; a) = \lambda B_C (1 + t)^{-e_C} c(1; a)^{1 - e_C} + (1 - \lambda) B_B c(1; a)^{1 - e_B} \qquad \text{A.3}$$

Next, if not registered, the firm chooses p_C, p_B to maximize ((11)) subject to ((12)),((13)) when $I = 0$, subject also to the constraint $p_C x + p_B y \leq s^*$ that the total value of sales is below the threshold. Now consider voluntary registration. A necessary and sufficient condition for this is that profit with registration is greater than profit without, ignoring the constraint that sales be below the threshold. We will assume that a firm will register if indifferent, to break the tie. Solving this problem ignoring the threshold, following the steps for the registered firm, we get maximized profit of

$$\pi(0;a) = \lambda B_C c(0;a)^{1-e_C} + (1-\lambda)B_B c(0;a)^{1-e_B} \qquad \text{A.4}$$

Then, the voluntary registration condition is $\pi(1;a) \geq \pi(0;a)$. After some simple rearrangement, using (A.3),(A.4), this reduces to ((16)).

(b) We only need show that the RHS of ((16)) is increasing in λ and decreasing in ω. The statement for λ is obvious by inspection. For ω, note first that $1 + \Delta_c \equiv 1 + \omega t$ is increasing in ω. So, both $(1+\Delta_c)^{1-e_C}$, $(1+\Delta_c)^{1-e_B}$ are decreasing in ω as $e_C, e_B > 1$. Finally, note that as $\Delta_c > 0$, $e_B > 1$, $1 - (1+\Delta_c)^{1-e_B} > 0$. So, the RHS of ((16)) is decreasing in ω as required.

(c) If $e_B = e_C = e$, ((16)) rearranges to

$$\frac{\lambda(1+t)^{-e} + (1-\lambda)B}{\lambda + (1-\lambda)B}(1+\Delta_c)^{e-1} > 1, A = \frac{B_B}{B_C} = (1+t)^{-e}\left(\frac{e-1}{e}\right)^e \qquad \text{A.5}$$

But then from ((15)), it is clear that the first term on the left-hand side of (A.5) is $1 - \Delta_d$, so we obtain ((18)), as required. $\square$

(A.0.0.3) **Proof of Proposition V.C.** (i) First, we characterize the relationship between firm type and sales, under the assumption that the firm does not have to register for VAT. From ((12)), the optimal price and total sales (B2C and B2B) of a non-registered firm are;

$$p(a) = \frac{e}{e-1}c(0;a) = \frac{1+\Delta_c}{a}, \; s(a) = (\lambda + (1-\lambda)A_B)(p(a))^{1-e} \qquad \text{A.6}$$

Combining these two expressions to substitute out $p(a)$, we see that the relationship we need between firms type and sales is:

$$s = (\lambda + (1-\lambda)A_B)\left(\frac{e(1+\Delta_c)}{e-1}\right)^{1-e} a^{e-1} \qquad \text{A.7}$$

(ii) We now write the indifference condition ((19)) that determines $a^* + \Delta a^*$ explicitly in the space of sales. To To lighten notation, and using $e_C = e_B = e$, define

$$A_0 = \lambda + (1-\lambda)A_B, \; A_1 = \lambda(1+t)^{-e} + (1-\lambda)A_B$$

First, from (A.3), and using the restriction that $e_C = e_B = e$, a firm that registers has maximized profit:

$$\pi(1;a) = A_1 \left(\frac{e}{1-e}\right)^{-e} \frac{a^{e-1}}{1-e} \qquad\qquad \text{A.8}$$

Next, the profit from being just at the VAT threshold for an $a-$type when constrained is

$$\pi(0;a) = s^* - \frac{1+\Delta_c}{a}\frac{s^*}{p} \qquad\qquad \text{A.9}$$

But solving for p from the constraint $A_0 p^{1-e} = s^*$, we get $p = \left(\frac{s^*}{A_0}\right)^{-1/(e-1)}$. Substituting this back into (A.9), and using $a = a^* + \Delta a^*$, we get

$$\pi(0;a^*+\Delta a^*) = s^* - \frac{1+\Delta_c}{A_0^{1/(e-1)}(a^*+\Delta a^*)}(s^*)^{e/(e-1)} \qquad\qquad \text{A.10}$$

Then combining (A.7), evaluated at $a = a^* + \Delta a^*$, and (A.10), we get:

$$\begin{aligned}
\pi(0;a^*+\Delta a^*) &= s^* - \frac{1+\Delta_c}{A_0^{1/(e-1)}(a^*+\Delta a^*)}(s^*)^{e/(e-1)} \qquad\qquad \text{A.11}\\
&= s^* - (s^*)^{e/(e-1)}\left(\frac{e-1}{e}\right)(s^*+\Delta s^*)^{1/(1-e)}
\end{aligned}$$

Again using (A.7), evaluated at $a = a^* + \Delta a^*$, in (A.8), we get:

$$\begin{aligned}
\pi(1;a^*+\Delta a^*) &= A_1\left(\frac{e}{1-e}\right)^{-e}\frac{1}{1-e}(a^*+\Delta a^*)^{e-1} \qquad\qquad \text{A.12}\\
&= \frac{A_1}{A_0}\left(\frac{e}{1-e}\right)^{-e}\frac{1}{1-e}\left(\frac{e(1+\Delta_c)}{e-1}\right)^{e-1}(s^*+\Delta s^*)\\
&= \frac{T}{e}(s^*+\Delta s^*)
\end{aligned}$$

So, using (A.12),(A.11), the indifference condition $\pi(1;a^*+\Delta a^*) = \pi_0(0;a^*+\Delta a^*)$ becomes

$$s^* - (s^*)^{e/(e-1)}\left(\frac{e-1}{e}\right)(s^*+\Delta s^*)^{1/(1-e)} - \frac{T}{e}(s^*+\Delta s^*) = 0 \qquad\qquad \text{A.13}$$

After some simplification of (A.13) (divide through by s^*, then $1 + \frac{\Delta s^*}{s^*}$, and multiply by e) we get ((21)) as required. $\square$

(A.0.0.4) **Proof of Proposition V.C.** First, ((21)) can be rewritten as

$$f(x,e) - T(\lambda, \omega, e) = 0, f(x) \equiv ex - (e-1)x^{e/(e-1)}, x = \frac{1}{(1 + \Delta s * / s*)} \qquad \text{A.14}$$

So, from (A.14):

$$\frac{dx}{d\lambda} = \frac{T_\lambda}{f_x}, \frac{dx}{d\omega} = \frac{T_\omega}{f_x}, \frac{dx}{de} = \frac{T_e - f_e}{f_x} \qquad \text{A.15}$$

Moreover, note that

$$f_x = e(1 - x^{1/(e-1)}) > 0 \qquad \text{A.16}$$

because $x < 1$, and $e > 1$, so $x^{1/(e-1)} < 1$. Also, we know that $T_\lambda < 0$, $T_\omega > 0$ and so from (A.15), (A.16), we conclude that $\frac{dx}{d\lambda} < 0$, $\frac{dx}{d\omega} > 0$. As x is an inverse measure of bunching, it follows that as λ increases, Δs^* rises, and ω rises, Δs^* falls.

Finally, from (A.15),

$$f_e = x - x^{e/(e-1)} - (e-1)\frac{d(x^{e/(e-1)})}{de} = x - x^{e/(e-1)} - \frac{\ln x}{e-1}x^{e/(e-1)}$$

So, for $f_e > 0$, we require

$$x^{-1/(e-1)} > 1 + \frac{\ln x}{e-1}$$

But as $x < 1$, $\ln x < 0$, so we require $x^{1/(e-1)} < 1$, which certainly holds as $x < 1$ and $1/(e-1) > 0$. So, we conclude that $f_e > 0$. It then follows from (A.15) ,(A.16), that $\frac{dx}{de} < 0$ as long as $T_e < 0$. But then bunching increases if $T_e < 0$, as claimed. $\square$

Proof that Voluntary Registration is Not Possible in the de Paula and Scheinkman (2010) Model

We follow the notation of their paper. There are two kinds of firms, upstream firms, and downstream firms. An informal downstream firm is by definition, not registered for VAT, and get gets profit $\pi_N = \max_{x \leq \bar{x}}\{\theta^d x^\alpha - p_i x\}$, where x is sales, $\bar{x}$ is the registration threshold, and p_i is the price of the input of bought from an upstream informal firm. If this firm registers, it gets profit $\pi_R = \max_x\{(1-\tau)(\theta^d x^\alpha - p_f x)\}$, where τ is the rate of VAT, and $p_f > p_i$ is the cost

of buying from the formal sector. Voluntary registration involves registering at a turnover (say $x_0 < \bar{x}$), giving profit $(1-\tau)(\theta^d x_0^\alpha - p_f x_0)$. But then, as $\tau > 0$, $p_f > p_i$;

$$\pi_R = (1-\tau)(\theta^d x_0^\alpha - p_f x_0) < \theta^d x_0^\alpha - p_f x_0 < \theta^d x_0^\alpha - p_i x_0 \leq \pi_N$$

so that the downstream firm can do better by not registering. Now consider an upstream firm in their model. This firm gets $\pi_N = \min \theta_u, \bar{x}$ if not registered, and $\pi_R = (1-\tau)\theta_u$ if registered. So, such a firm registers voluntarily iff $\theta_u < \bar{x}$. But, for such a firm, $\pi_N = \theta_u > (1-\tau)\theta_u = \pi_R$; again, the downstream firm can do better by not registering.

General Proof that Voluntary Registration is not Possible with Only B2C Sales

Consider a firm facing a residual demand function from final consumers of $x(q)$, where q is the consumer price. This covers both the cases of monopoly, where $x(.)$ is also the actual demand curve, and monopolistic competition, where $x(.)$ is demand for that firm's product, taking the prices of all other firms as fixed. Assume all sales are to final consumers i.e. B2C. If a firm is registered for VAT, profit is then

$$\pi_R(p) = px(p(1+t)) - c(x(p(1+t)), w, r)$$

where p is the producer price, and $c(x, w, r)$ is the cost function given output x, and prices of labour and the intermediate input w, r. This is completely general cost function that clearly includes the cost function in the paper as a special case. If the firm is not registered for VAT, profit is

$$\pi_N(p) = px(p) - c(x(p), w, r(1+t))$$

Then, we have

$$
\begin{aligned}
\pi_R &= \max_p \left\{ px(p(1+t)) - c(x(p(1+t)), w, r) \right\} \\
&= \max_q \left\{ \frac{q}{1+t} x(q) - c(x(q), w, r) \right\} \\
&= \frac{1}{1+t} \max_q \left\{ qx(q) - (1+t)c(x(q), w, r) \right\} \\
&< \max_q \left\{ (q - c(x(q), w, r(1+t))) \right\} \\
&= \pi_N
\end{aligned}
$$

So, with only B2C sales, no firm would every wish to register voluntarily.

APPENDIX B. EXTENDED MODEL WITH VAT EVASION

Here, we model the simplest and most common form of VAT evasion, under-reporting of sales. This has two aspects. First, a non-registered firm can hide a share of sales, for example by using cash transactions. It is widely believed that the VAT chain makes it more difficult to conceal sales to other registered businesses, so we will assume that only that a share v_N of B2C sales can be hidden. Then, sales can be s high as $s^* + v_N px$ without registering for VAT.

We also allow registered sellers can use this strategy, albeit at a higher cost of detection. So, suppose that such a seller does not charge VAT on some proportion v_R of B2C sales px. The total cost of x units of the good to the household, if purchased from a registered seller, will be

$$v_R xp + (1 - v_R)xp(1 + t) = xp(1 + (1 - v_R)t).$$

That is, the household faces an average price of $p(1 + (1 - v_R)t)$, and the firm continues to get revenue p on every unit sold to the household.

We will assume that v_N, v_R are exogenously fixed, both for simplicity, and also because there are some analytical issues in endogenizing them.[46] The main qualitative points will extend to the endogenous case.[47] We will assume that $0 \le v_R \le v_N < 1$, reflecting the fact that non-registered firms are less likely to be audited than registered ones.

It is then easily verified that for voluntary registration, the analysis proceeds much as before except that the VAT sufficient statistic becomes

$$T(v) = (1 - \Delta_d(v_R))(1 + \Delta_c)^{e-1}, \ \Delta_d(v_R) = \frac{\lambda(1 - (1 + (1 - v_R)t)^{-e})}{\lambda + (1 - \lambda)A_B} \qquad \text{B.17}$$

---

[46]For example, suppose that the registered firm chooses v_R to maximize profit minus evasion cost $g(v_R)$. It is easily verified that optimized profit is convex in v_R, as it only depends on v_R via the term $(1 + t(1 - v_R))^{-e}$, which is a convex function of v_R. So, to have an interior solution, $g(.)$ also has to be sufficiently convex in v_R. But then, a closed-form solution for v_R cannot be found.

[47]For example, suppose the cost of evasion is linear in v, up to a limit $\overline{v} < 1$. Then, as profit is convex in v, as explained in the previous footnote, and the evasion cost is small, the firm will always choose $\overline{v}$, so that it is effectively exogenous.

Thus, with evasion, the output VAT effect depends on v_R and is smaller than without evasion i.e. $\Delta_d(v_R) < \Delta_d(0)$. This is intuitive; with some VAT evaded on sales, output VAT becomes less of a burden. It then follows that T is increasing in v_R i.e. voluntary registration is more likely, the greater the opportunities for evasion, as measured by v_R.

As regards bunching, evasion has two opposing effects. First, evasion relaxes the constraint imposed by the VAT threshold, as the tax authority only observes $1 - v_N$ of B2C sales, and so the firm can in fact produce over the threshold without registering. Second, as just discussed, if $v_R > 0$, evasion makes registration less costly, because output VAT becomes less of a burden.

Both of these effects appear formally as follows. With evasion, we show below that the term T in the bunching equation ((21)) is replaced by

$$\hat{T}(v_N, v_R) = \frac{(1-v_N)\lambda + (1-\lambda)A_B}{\lambda + (1-\lambda)A_B}T(v_R) \qquad \text{B.18}$$

and in particular, positive bunching will occur when $\hat{T}(v_N, v_R)$ is less than 1.[48] An increase in v_N has the expected effect of lowering $\hat{T}$ and thus increasing bunching, as a higher v_N makes it less costly for the firm to hold its observed sales under the threshold. An increase in v_R has the opposite effect of reducing bunching, via the fact that $T(v_R)$ rises; this captures the effect that evasion reduces the burden of output VAT.

Note that, with evasion, the qualitative effects of λ and ω on T do not change, and so our predictions about the determinants of voluntary registration do not change; this is clear by inspection from (B.17). This is also true of bunching; it is seen by inspection that $\hat{T}$ is decreasing in λ, and increasing in ω, as is T. So, our key empirical predictions are robust to the presence of evasion. We can summarize as follows:

An increase in evasion by non-registered firms, v_N, increases bunching. An increase in evasion by registered firms, v_R, raises the likelihood of voluntary registration and reduces bunching. Moreover, evasion does not affect our qualitative predictions about the effects of λ, the fraction of B2C sales, and input-cost ratio, ω, on voluntary registration and bunching.

---

[48]For a formal proof, see below.

Derivation of the Bunching Equation with Evasion

The proof follows the proof of Proposition 4 in the paper, with the following changes. First, we define A_0, A_1 as

$$A_0(v) = \lambda(1 - v_N) + (1 - \lambda)A_B, \; A_1(v) = \lambda(1 + (1 - v_R)t)^{-e} + (1 - \lambda)A_B$$

As in (A.3) in the paper, any firm that registers has maximized profit:

$$\pi(1;a) = A_1(v_R)\left(\frac{e}{1-e}\right)^{-e}\frac{a^{e-1}}{1-e} \qquad \text{B.19}$$

Next, the payoff from being on the VAT threshold for an $a-$type when constrained is now

$$\pi(0;a) = (s^* + vpx) - \frac{1 + \Delta_c}{a}\frac{(s^* + vpx)}{p} \qquad \text{B.20}$$

This is because the firm can actually produce and sell $s^* + vpx$ with a threshold s^* because sales vpx are "cash" and thus not observable by the tax authority. Solving for p from the definition that non-concealed sales must be equal to s^* i.e. $((1 - v_N)\lambda + (1 - \lambda)A_B)p^{1-e} = s^*$, we get:

$$p = \left(\frac{s^*}{(1 - v_N)\lambda + (1 - \lambda)A_B}\right)^{-1/(e-1)}$$

Combining this with the fact that $x = \lambda p^{1-e}$, we get

$$s^* + vpx = s^*\frac{\lambda + (1 - \lambda)A_B}{(1 - v_N)\lambda + (1 - \lambda)A_B} \equiv \mu s^*$$

Substituting this back into (B.20), and setting $a = a^* + \Delta a^*$, we get

$$\pi(0;a^* + \Delta a^*) = \mu\left(s^* - \frac{1 + \Delta_c}{(A_0(v_N))^{1/(e-1)}(a^* + \Delta a^*)}(s^*)^{e/(e-1)}\right) \qquad \text{B.21}$$

Also observed non-cash sales map into type by

$$s^* + \Delta s^* = A_0(v_R)\left(\frac{e(1 + \Delta_c)}{e - 1}\right)^{1-e}(a^* + \Delta a^*)^{e-1} \qquad \text{B.22}$$

Combining (B.21) and (B.22), we get:

$$
\begin{aligned}
\pi(0; a^* + \Delta a^*) &= \mu \left(s^* - \frac{1 + \Delta_c}{(A_0(v_N))^{1/(e-1)}(a^* + \Delta a^*)} (s^*)^{e/(e-1)} \right) && \text{B.23} \\
&= \mu \left(s^* - (1 + \Delta_c)(s^*)^{e/(e-1)} \left(\frac{e-1}{e(1+\Delta_c)} \right) (s^* + \Delta s^*)^{1/(1-e)} \right) \\
&= \mu \left(s^* - (s^*)^{e/(e-1)} \left(\frac{A_0(v_R)}{A_0(v_N)} \right)^{1/(e-1)} \left(\frac{e-1}{e} \right) (s^* + \Delta s^*)^{1/(1-e)} \right)
\end{aligned}
$$

Now using (B.22) in (B.19), we get:

$$
\begin{aligned}
\pi(1; a^* + \Delta a^*) &= A_1(v_R) \left(\frac{e}{1-e} \right)^{-e} \frac{1}{1-e} (a^* + \Delta a^*)^{e-1} && \text{B.24} \\
&= \frac{A_1(v_R)}{A_0(v_R)} \left(\frac{e}{1-e} \right)^{-e} \frac{1}{1-e} \left(\frac{e(1+\Delta_c)}{e-1} \right)^{e-1} (s^* + \Delta s^*) \\
&= \frac{A_1(v_R)(1+\Delta_c)^{e-1}}{A_0(v_R)} \frac{(s^* + \Delta s^*)}{e} \\
&= T(v_R) \frac{(s^* + \Delta s^*)}{e} && \text{B.25}
\end{aligned}
$$

So, using (B.24),(B.25), the indifference condition $\pi(1; a^* + \Delta a^*) = \pi_0(0; a^* + \Delta a^*)$ becomes

$$
s^* - (s^*)^{e/(e-1)} \left(\frac{e-1}{e} \right) (s^* + \Delta s^*)^{1/(1-e)} - \frac{T(v_R)}{\mu} \frac{(s^* + \Delta s^*)}{e} = 0 \qquad \text{B.26}
$$

After some simplification of (B.26) (divide through by s^*, then $1 + \frac{\Delta s^*}{s^*}$, and multiply by e) we get ((21)) in the paper with the tax term $\frac{T(v_R)}{\mu}$ as required. $\square$

APPENDIX C. THE OPTIMAL VAT THRESHOLD

In this section, we show how a formula for the optimal VAT threshold can be derived for our model. Following Keen and Mintz (2004), we assume that any firm that is registered has to pay a compliance cost $\Gamma > 0$ and the revenue authority incurs an administration cost $A > 0$. To keep things, simple, we will also assume that $e_C = e_B = e$, so $p_C(a) = p_B(a) = p(a)$.

As a first step, it is helpful to rule out parameter values under which all firms choose to register voluntarily. As discussed in Section V.D, this situation arises when Γ is small enough, and condition ((18)) in the paper holds. Then, even the least productive $\underline{a}$-firm will choose to register voluntarily, so the location of the registration threshold s^* becomes irrelevant to firm

behavior, government revenue and welfare. Thus, we assume that either; (i) ((18)) does not hold, or (ii) ((18)) does hold, and Γ is large enough to cause some firms to bunch.

Let the firm at the top of the bunching interval be $a^* + \Delta a^* \equiv \hat{a}$ for convenience. Then $\hat{a}$ is defined by condition ((22)) in the paper. Note for future reference that $\hat{a}$ is strictly increasing in s^*, as the higher the threshold, the high the productivity of the firm that is just willing to cut output to stay below the threshold. We do not need to solve for the details of this relationship in what follows, we just need to recall that $\frac{\partial \hat{a}}{\partial s^*} > 0$.

We focus on the choice of the VAT threshold to maximize tax revenue, to bring out the basic issues. The VAT base, which is taxed at rate t, is

$$B = PY + \int_{\hat{a}}^{\overline{a}} p(a)x(a)da + \omega \int_{\hat{a}}^{\overline{a}} \frac{x(a)+y(a)}{a}da \qquad \text{C.27}$$

This is composed of three terms; the value of final sales to the household of the large firm, the value of final sales of the registered small firms, and the value of inputs purchased by non-registered small firms. Revenue is therefore

$$R = tB - (\overline{a} - \hat{a})A \qquad \text{C.28}$$

where A is the administration cost per firm of implementing the VAT, and $(\overline{a} - \hat{a})$ is the measure of firms that are registered.

To further study this expression, we note that the price $p(a)$ charged by the firms in equilibrium is as follows:

$$p(a) = \begin{cases} \frac{e}{e-1}\frac{1}{a}, & a > \hat{a} \\ \left(\frac{\Omega}{s^*}\right)^{1/(e-1)}, & a^* \leq a \leq \hat{a} \\ \frac{e}{e-1}\frac{1+\omega t}{a}, & a < a^* \end{cases} \qquad \text{C.29}$$

where $\Omega = \lambda + (1-\lambda)A_B$. The explanation is as follows. For firms $a > \hat{a}$, the price is a fixed mark-up over marginal cost of $1/a$. The same is true for firms $a < a^*$, except that now marginal cost is $(1+\omega t)/a$, due to embedded VAT. For firms in the bunching interval, the price is set to just satisfy demand, while making the total value of sales equal to s^* i.e. $p(a)$ must satisfy $p(a)(x(a)+y(a)) = s^*$; using ((12)), ((13)), and solving for p gives the formula (C.29). Finally, in what follows, define $p(s^*) \equiv \left(\frac{\Omega}{s^*}\right)^{1/(e-1)}$.

Next, we note that from ((6)), ((8)) in the paper, and using $e = \gamma$, gives

$$PY = (1-\lambda)(1+t)^{-e} \left(\frac{e}{e-1} C \right)^{1-e} = (1-\lambda)A_B \int_{\underline{a}}^{\overline{a}} (p(a))^{1-e} da \qquad \text{C.30}$$

So, combining (C.29), (C.30):

$$PY = (1-\lambda)A_B \left[(p(s^*))^{1-e}(\hat{a} - a^*) + \int_{\hat{a}}^{\overline{a}} (p(a))^{1-e} da + \int_{\underline{a}}^{a^*} (p(a))^{1-e} da \right] \qquad \text{C.31}$$

Also, using the fact that for $a^* \le a \le \hat{a}$, $x(a) + y(a) = s^*/p(s^*)$, we can write:

$$\int_{\underline{a}}^{\hat{a}} \frac{x(a)+y(a)}{a} da = \frac{s^*}{p(s^*)} \int_{a^*}^{\hat{a}} \frac{1}{a} da + \int_{\underline{a}}^{a^*} \frac{x(a)+y(a)}{a} da \qquad \text{C.32}$$

So, combining (C.31),(C.32) and (C.27), we get:

$$\begin{aligned} B &= (1-\lambda)A_B \left[(p(s^*))^{1-e}(\hat{a} - a^*) + \int_{\hat{a}}^{\overline{a}} (p(a))^{1-e} da + \int_{\underline{a}}^{a^*} (p(a))^{1-e} da \right] \\ &+ \int_{\hat{a}}^{\overline{a}} p(a)(x(a)) da + \omega \frac{s^*}{p(s^*)} \int_{a^*}^{\hat{a}} \frac{1}{a} da + \omega \int_{\underline{a}}^{a^*} \frac{x(a)+y(a)}{a} da \end{aligned} \qquad \text{C.33}$$

Now, consider the effect of a change in the threshold on the tax base B. When s^* changes, it changes the endpoints of the bunching interval $a^*, \hat{a}$. It also changes the prices via (C.29). Some of these effects will be zero, because the price function $p(a)$ is continuous in a, and also B is continuous in $a^*, \hat{a}$ with one exception; at $\hat{a}$, the terms in the integrands in the second and third terms of (C.33) are generally not equal. In fact, the first is the value of final sales for the $\hat{a}$ firm, and the second is the overall cost of production for the $\hat{a}$ firm, which will be smaller. So, differentiating (C.33), and integrating the term $\int_{a^*}^{\hat{a}} \frac{1}{a} da$ explicitly, we get

$$\begin{aligned} \frac{\partial B}{\partial s^*} &= \left(\frac{\omega}{\hat{a}} \frac{s^*}{p(s^*)} - p(\hat{a})x(\hat{a}) \right) \frac{\partial \hat{a}}{\partial s^*} \\ &+ p'(s^*) \left[(1-\lambda)A_B(1-e)(p(s^*))^{-e}(\hat{a} - a^*) - \omega \frac{s^*}{(p(s^*))^2} ln \left(\frac{\hat{a}}{a^*} \right) \right] + \frac{\omega}{p(s^*)} ln \left(\frac{\hat{a}}{a^*} \right) \end{aligned} \qquad \text{C.34}$$

Now from (C.29) and ((12)), it is easy to establish the following:

$$p'(s^*) = \frac{p(s^*)}{s^*(1-e)}, \quad p(\hat{a})x(\hat{a}) = \lambda p(\hat{a})^{1-e} = \frac{\lambda s^*}{\Omega} \qquad \text{C.35}$$

Combining (C.34), (C.35), after some simplification, we get

$$\begin{aligned} \frac{\partial B}{\partial s^*} &= \left(\frac{\omega}{\hat{a}} \frac{s^*}{p(s^*)} - (1-\beta)s^* \right) \frac{\partial \hat{a}}{\partial s^*} + \beta(\hat{a} - a^*) \\ &+ \frac{e}{(e-1)} \frac{\omega}{p(s^*)} ln \left(\frac{\hat{a}}{a^*} \right) \end{aligned} \qquad \text{C.36}$$

where $\beta \equiv \frac{(1-\lambda)A_B}{\lambda + (1-\lambda)A_B}$ is the share of demand for the bunching firms that is B2B.

Now using (C.28), we see that the optimal threshold for maximizing revenue is characterized by

$$\frac{\partial R}{\partial s^*} = t\frac{\partial B}{\partial s^*} + A\frac{\partial \hat{a}}{\partial s^*} = 0 \qquad \text{C.37}$$

Combining (C.36) and (C.37), we get the FOC that defines the optimal threshold:

$$\frac{\partial R}{\partial s^*} = \left(t\frac{\omega}{\hat{a}}\frac{s^*}{p(s^*)} - t(1-\beta)s^* + A\right)\frac{\partial \hat{a}}{\partial s^*} + t\beta(\hat{a} - a^*) + t\frac{e}{(e-1)}\frac{\omega}{p(s^*)}ln\left(\frac{\hat{a}}{a^*}\right) = 0 \quad \text{C.38}$$

To interpret this, and make the link to Keen and Mintz (2004), we proceed as follows. In deriving their simple analytical formula, equation (2) in their paper, Keen and Mintz assume no behavioral responses i.e. no bunching (which can be captured here by $e = 0$), and they also assume no B2B sales i.e. $\beta = 0$. Making these two simplifications in (C.38), we can write

$$\frac{\partial R}{\partial s^*} = (-ts^*v + A)\frac{\partial \hat{a}}{\partial s^*} = 0, \ v = 1 - \frac{\omega}{\hat{a}p(s^*)} \qquad \text{C.39}$$

where v is the share of value added in sales for the $\hat{a}$ firm. Solving (C.39) for s^* gives

$$s^* = \frac{A}{tv} \qquad \text{C.40}$$

This is *exactly* the formula in Keen and Mintz (2004) for the threshold in the special case where the government maximizes revenue[49] This formula is very intuitive; the higher are administrative costs A, the higher the threshold should be, and the higher is the tax rate t or the share of value-added (the tax base) for the firm at the threshold, the lower the threshold will be, as reducing it will increase revenue proportionately to tv.

More generally, we can solve (C.38) for s^* to get a generalization of the Keen and Mintz formula:

$$s^* = \frac{A + t\left(\beta(\hat{a} - a^*) + \frac{e}{(e-1)}\frac{\omega}{p(s^*)}ln\left(\frac{\hat{a}}{a^*}\right)\right) / \frac{\partial \hat{a}}{\partial s^*}}{t(v - \beta)} \qquad \text{C.41}$$

---

[49]This is their equation (2), in the limit as δ, the weight on revenue, goes to infinity, which is $s^* = A/tv$ in our notation.

This formula tells us that introducing either B2B sales tends to raise the threshold. [50] This is straightforward to see mathematically from (C.41): if not all sales are B2C i.e. $\beta > 0$, then first, a positive term in β appears in the numerator of (C.41), and second, a negative term in β appears in the denominator. Both of these make s^* higher.

The intuition is as follows. First, an increase in s^* increases PY by increasing the prices that the small firms who are bunching charge for inputs to the large firm, and this raises B. This is captured by the term in β in the numerator of (C.41), which is proportional to the mass of bunching small firms, $\beta(\hat{a} - a^*)$.

Second, an increase in s^* raises a^* and therefore the number of firms who bunch. This in turn decreases the net VAT paid by bunching firms, as more firms now below the threshold. But, the size of this effect depends on B2B sales. Specifically, this reduction is proportional to the value of B2C sales minus the value of inputs used to produce *all* sales. So, the higher are B2B sales, the smaller this effect is, and thus the lower the revenue cost of raising s^*. This explains the term in β in the denominator in (C.41).

APPENDIX D.   EVIDENCE OF BUNCHING VIA TURNOVER MISREPORTING

In this section, we provide some suggestive evidence on the extent of bunching due to turnover misreporting. When bunching is due to a decrease in real output, we expect companies to reduce their input costs in proportion, so that the distribution of input-cost ratio for non-registered companies should be smooth around the VAT notch. When bunching is due to turnover misreporting, we conjecture that the non-registered companies are less likely to under-report their input costs and wage expenses. Both costs are deductible for corporation taxes and the latter is subject to third-party reporting. In other words, the gain from under-reporting the deductible costs is considerably smaller than the gain from under reporting the turnover to avoid VAT registration. If the majority of companies bunch via turnover misre-porting, we would expect to see a higher average input-cost ratio for the non-registered group just below the VAT notch, relative to that for the registered group.

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[50]It can also be seen that the effect of a behavioral response i.e. $e > 0$ on the threshold is positive. This is most easily seen by setting $\beta = 0$ in (C.41) to get the special case of only B2C sales. Then, the term in $e/(e-1)$ is positive. This is a somewhat different finding from Keen and Mintz (2004), who find an ambiguous effect of the behavioral response on the threshold

Figure A.9 pools all observations in the sample period and plots the distribution of average input-cost ratio for registered and non-registered companies in £1,000 turnover bins, respectively. In Panel A, the input-cost ratio is salary exclusive and represents the share of direct cost of sales relative to total turnover. The solid blue line shows the average input cost relative to sales for registered companies within each turnover bin of £1,000 normalized by the current-year VAT notch, and the dashed blue line shows the average input cost ratio for the unregistered companies. Consistent with the theory, voluntary registers incur a much larger input cost as indicated by their average input-cost ratio which is consistently larger than that for the non-registered companies below the VAT notch. On the other hand, there is no evident increase in the average input-cost ratio just below the VAT notch for the non-registered group. The distribution is relatively smooth and continues to increase with turnover above the VAT notch.

In comparison, Panel B plots the distribution of average input-cost ratio *inclusive* of salary, for registered and non-registered companies, respectively. There is striking difference between the two input-cost ratio series just below the VAT notch. The two series move in parallel directions until the average input-cost ratio for the non-registered companies starts to increase drastically just below the VAT notch. The sharp increase in the salary-inclusive cost ratio can be partly attributed to the fixed nature of salary cost which takes longer to adjust than variable costs of input. On the other hand, the sharp increase is also consistent with the fact that salary is subject to third-party reporting and thus it is more costly/difficult for small businesses to underreport salary expenses. Overall, Panel A and B in Figure A.9 provide suggestive yet not conclusive evidence that part of bunching is due to turnover misreporting.

APPENDIX E. DYNAMIC REGRESSIONS OF VOLUNTARY REGISTRATION

In this section, we investigate the importance of inertia in driving VAT registration by analyzing the dynamic behavior of firms when they cross the registration and deregistration thresholds. First, we compute a transition probability matrix for firms changing their registration states in Table A.1, which shows the probability of being registered or not registered *t* years after initially being in a given state.[51] For example, the entry in the first cell of the matrix, in-

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[51]Changes in the transition probability could also driven by attrition, therefore we focus on a balanced sample of firms that we observe throughout the sample period.

dicates that of all the firms that where initially registered in year 2004/05, 82.2% remained registered a year later.

Table A.1 shows that there is considerable persistence in registration status. On the other hand, comparing to the registered firms, this persistence does decline substantially over time for non-registered firms. For example, 81.4% of initially non-registered firms remain unregistered after 5 years, whereas 64% of registered firms remain registered after 5 years. The difference in persistence is due to the fact that the majority of firms are growing over our sample, and so will tend to stay above the registration threshold once they cross it.

Next, we investigate to what extent the registration decision is driven by persistence in turnover versus the costs of changing registration status. To answer this question, we augment equation ((23)) as follows:

$$
\begin{aligned}
R_{it} = {}& \gamma_0 + \gamma_1 R_{i,t-1} + \gamma_2 (1 - R_{i,t-1}) IR_{it} + \gamma_3 R_{it-1} ID_{it} \\
& + \gamma_4 B2C_{it}^{j} + \gamma_5 ICR_{it} + \gamma_6 L_{it}^{j} + \gamma_7 D_{it} + \rho_t + \phi_i + \upsilon_{it}
\end{aligned}
\qquad \text{E.42}
$$

where R_{it} is a dummy indicator that takes value 1 if the firm is currently registered and zero otherwise, as defined previously in Section VII.A. In addition,

$$
IR_{it} = \left\{ \begin{array}{ll} 1, & Y_{it} \geq Z_t \\ 0, & Y_{it} < Z_t \end{array} \right. , \quad ID_{it} = \left\{ \begin{array}{ll} 1, & Y_{it} \geq Z'_t \\ 0, & Y_{it} < Z'_t \end{array} \right. ,
$$

where Z_t, Z'_t are the registration and deregistration thresholds at time t, Y_{it} denotes the current-period turnover, so IR_{it} and ID_{it} are dummy indicators recording whether the firm is above the registration and deregistration thresholds respectively at time t. All the other variables are defined as before, and υ_{it} is the error term. We estimate equation (E.42) in a fixed-effect Probit model, and augment the estimation equation with the initial registration status R_{i0} and the mean characteristics of all the time-varying regressors (Wooldridge, 2005).

So, if firm registration decision was entirely backward-looking and ignores its current turnover relative to Z_t, Z'_t, we would expect the coefficients γ_2 to γ_5 to be insignificant. However, we expect most firms to comply with the VAT law. Specifically, we expect firms to register when they are initially not registered and their turnover passes above the threshold. Such a firm has a value 1 for the term $(1 - R_{it-1}) IR_{it}$ and so we expect to find a positive γ_2.

The VAT legislation also requires firms to stay registered if they are registered in the previous year and their current turnover is above the deregistration threshold. Such a firm will have a value of 1 for $R_{it-1}ID_{it}$. So, we also expect to find that $\gamma_3 > 0$. On the other hand, if the firm remains registered simply due to the cost of deregistration such that whether crossing the deregistration threshold plays no role in the registration behavior, we would expect to find that $\gamma_3 = 0$.

Finally, we already know from our analysis of voluntary registration that the registration decision is significantly affected by the industry B2C ratio $B2C_{it}^j$, the firm input cost ratio ICR_{it}, and the industry-level Lerner index L_{it}^j, so we expect γ_4, γ_5 and γ_6 to be positive.

Table A.2 reports the full results from estimating equation (E.42) using a fixed-effects Probit model, following Wooldridge (2005). For ease of interpretation, Table A.3 reports the relevant average partial effects, which refer to the effect on the mean probability of registration after averaging the unobserved heterogeneity across all firms in the sample. For example, to calculate the APE of a discrete change of *ID* from 0 to 1, we first compute the average predicted probability of registration at fixed values of $R_{t-1} = 1, ID_t = 0$ and $R_{t-1} = 1, ID_t = 1$, respectively, across all firms in the sample. We then take the difference between the two average probabilities to obtain the average partial effect of *ID*. We use a similar procedure to compute the average partial effect of a one-standard-deviation increase in $B2C_{it}^j$, ICR_{it}, and L_{it}^j, noting that the one-standard-deviation increase applies to their mean characteristics in addition to the time-varying values. Column (1) shows the mean predicted probability of VAT registration at fixed value of R_{t-1}, ID_t, IR_t, the mean predicted probability across all firms in the sample and that for one-standard-deviation increase in $B2C_{it}^j$, ICR_{it}, and L_{it}^j. Column (2) shows the average partial effects of these variables by taking the difference in the mean predicted probabilities given the change in their value. For example, for firms that are registered in the previous year, falling below the deregistration threshold lowers the probability of being currently registered by 5 percentage points. Alternatively, for firms that are not registered in the previous year, going above the registration threshold increases their probability of registration by 70.4 percentage points. These findings suggest that the registration decision is not entirely driven by the cost of deregistration or inertia.

Finally, the short-run partial effects of the share of B2C sales and the input cost ratio in the dynamic model are considerably smaller than the static estimates in section VII.A. A one standard deviation increase in the B2C ratio and the Lerner index reduces the probability of

registration by 0.1 and 0.25 percentage points, respectively, and there is no significant change in the probability of registration for one standard deviation increase in the input cost ratio.

Overall, we see that while there is a considerable amount of persistence in firm behavior, the registration decision is not entirely driven by inertia due to fixed cost of deregistration. Firms respect the legal registration requirement, and at the same time change their registration decisions in a way that is consistent with profit maximization behavior depicted in Section IV. The probability of registration is also affected significantly by the more fundamental determinants of VAT registration. The positive coefficient estimates and partial effects of $R_{it-1}ID_{it}$, the B2C ratio and the input-cost ratio provides supportive evidence that the VAT registration decision is rational and relates to the fundamental determinants of VAT registration as predicted by the theory in Section V.

APPENDIX F. DATA CONSTRUCTION

Our data combined two administrative datasets âĂŞ Corporation Tax returns (CT600) and VAT returns, with two additional data sources - Financial Analysis Made Easy (FAME), and annual sector-level statistics on the share of sales to final consumers (B2C ratio) based on the Office for National Statistics (ONS) Input-Output tables. This section describes how we construct our sample.

We first link CT600 corporate tax return and FAME company account using company identifier (93% of records in CT600 were merged with FAME with 12 months accounting period), resulting in 7,914,902 company-year records covering April 1, 2004-April 4, 2015. This includes company that has (i) non-missing turnover, (ii) single CT600 account in one accounting year with full 12-months period, (iii) non-negative fixed assets (measured in FAME), (iv) do not engaged mainly in overseas activities based on HMRC trade classification, or (v) is not part of company group. Our sample exclude non-profit organizations/companies âĂŞ that includes charitable organization, industrial and provident society and company limited by guarantee; we also exclude LLP, public investment trust; we exclude companies in financial services industry under SIC.

We then merge the CT600-FAME data with the VAT returns, excluding company in their VAT record that has (i) registered as a group, (ii) the same registration and deregistration date, (iii)

non-corporation status, or (iv) measurement error âĂŞ when input VAT is greater than the input value, or output VAT tax rate is greater than the statutory rate.

We further link the CT600-FAME-VAT data with annual B2C ratio from ONS at 2-digit SIC industry level, for industries which the B2C ratios are available from the Input-Output tables - that consists of 6,536,170 company-year observations. Our sample for analysis includes observations with yearly turnover between Âč10,000 to Âč200,000, consists of 3,461,247 company-year observations with 968,353 unique companies between 2004/2005 to 2014/2015.

APPENDIX FIGURES

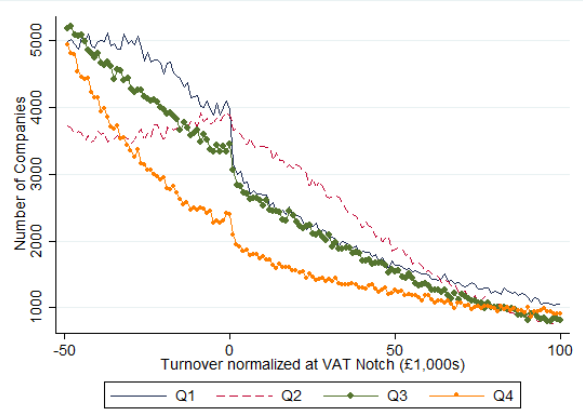**Figure A.1. Registration status of firms below threshold by year**
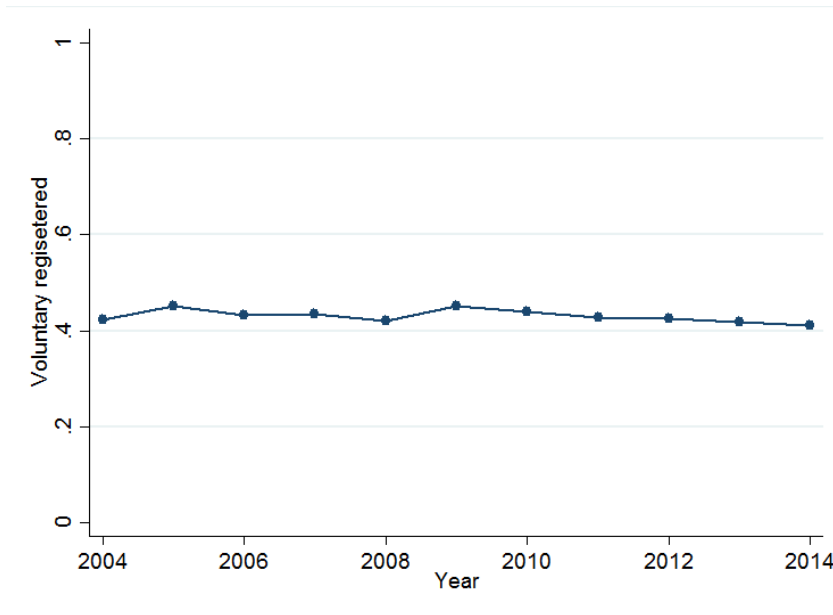
Notes: The figure plots the probability of firms voluntarily registered during 2004/05-2009/10, for firms that are below VAT threshold.

Figure A.2. Annual Turnover Distribution around the Registration threshold, 2004/05-2014/15

(a) 2004

(b) 2005

(c) 2006

(d) 2007

(e) 2008

(f) 2009

(g) 2010

(h) 2011

(i) 2012

(j) 2013

(k) 2014
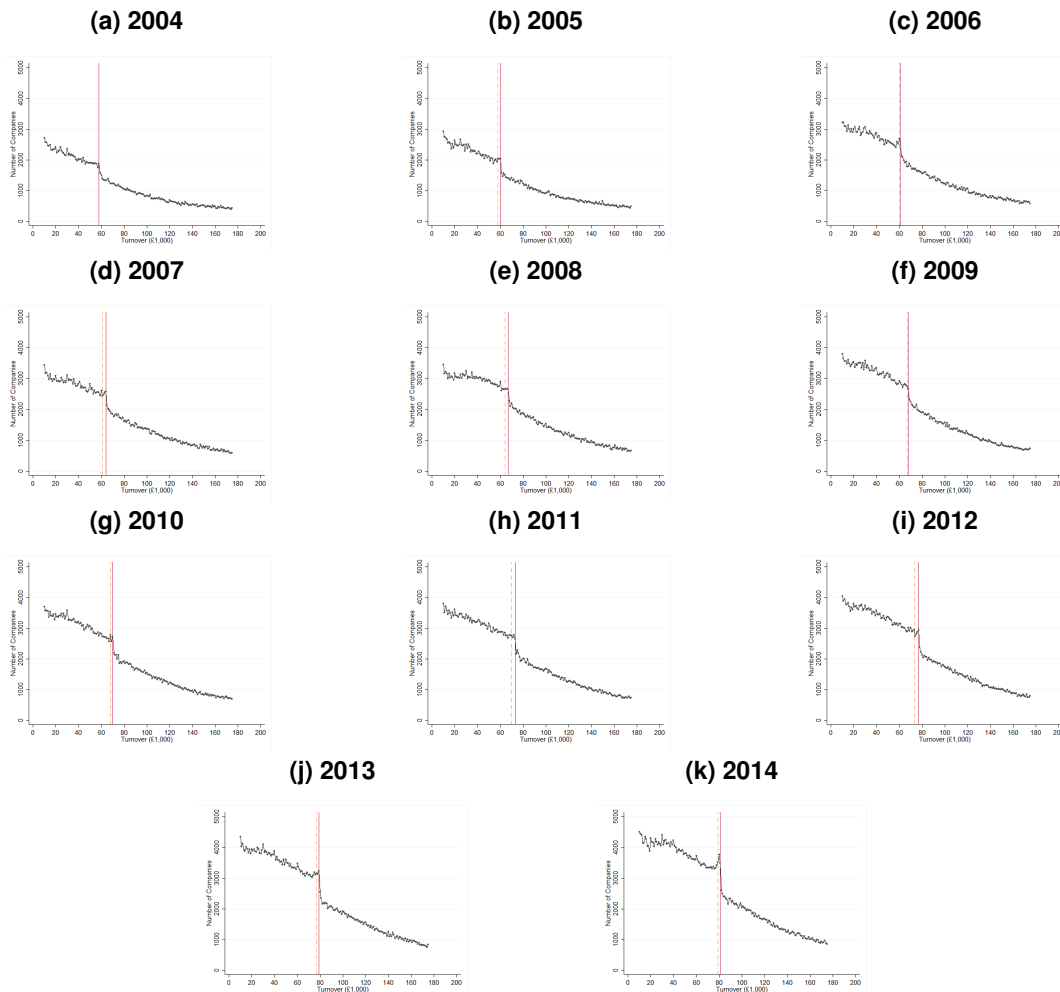
Notes: This figure shows the histogram of companies within the neighborhood of turnover for each year between 2004/05-2014/15. The bin width is £1,000 and the red line denotes the VAT notch. The dash line denotes the VAT notch in the previous year.

Figure A.3. Bunching across Quartiles of the B2C Share Distribution
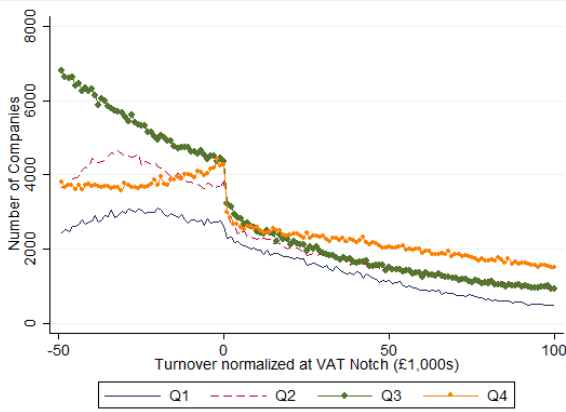
(a) Binding VAT Threshold **(b) Nonbinding VAT Threshold**

Notes: this figure shows the raw distribution of companies' turnover around the normalized VAT notch across four different quartiles of the distribution of the share of B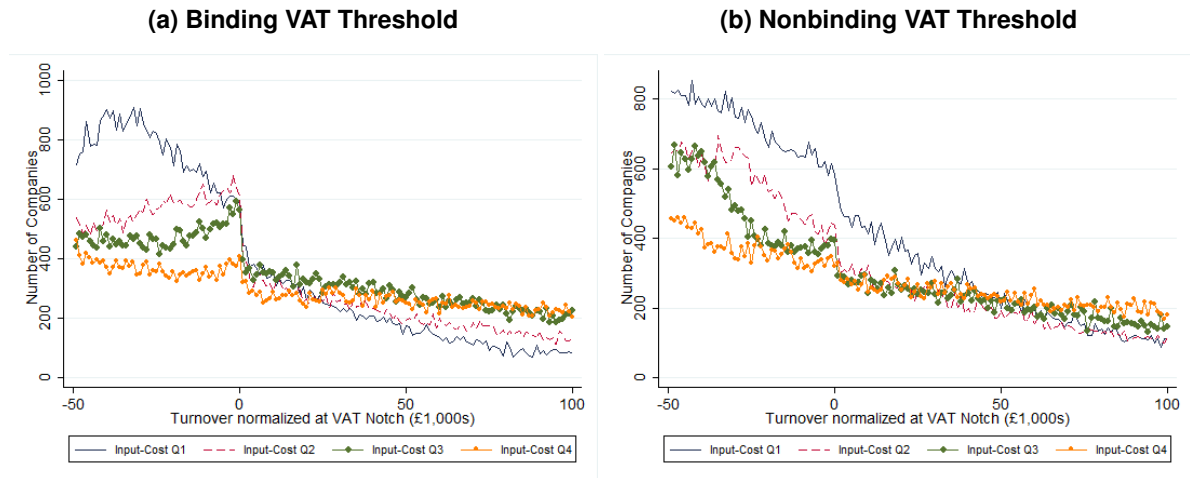2C sales for the pooled dataset for period 2004/05-2014/15. Panel (a) shows the distributions for the subset of firms not predicted to register voluntarily. Panel (b) shows the distributions for the subset of firms predicted to register voluntarily, for whom the threshold is nonbinding. The corresponding bunching estimates are reported in Figure 4 in the main text.

Figure A.4. Bunching across Quartiles of the Lerner Index Distribution
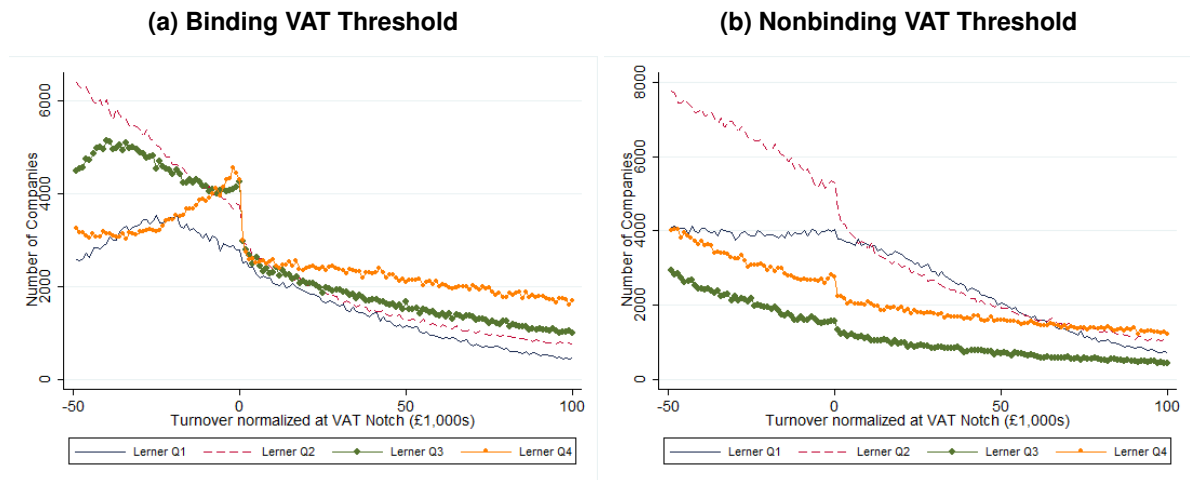
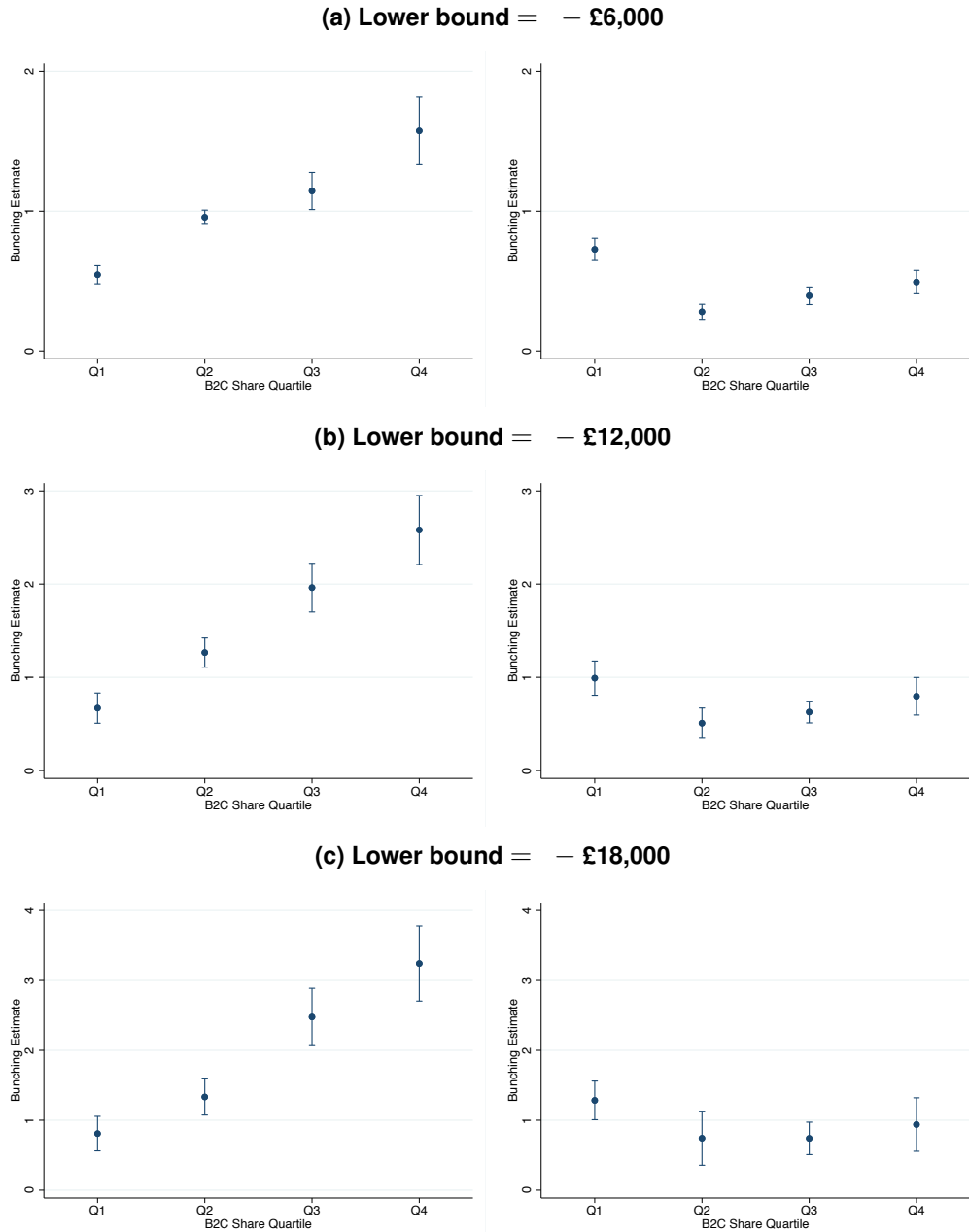**(a) Binding VAT Threshold**   **(b) Nonbinding VAT Threshold**

Notes: this figure shows the raw distribution of companies' turnover around the normalized VAT notch across four different quartiles of the distribution of the Lerner Index for the pooled dataset for period 2004/05-2014/15. Panel (a) shows the distributions for the subset of firms not predicted to register voluntarily. Panel (b) shows the distributions for the subset of firms predicted to register voluntarily, for whom the threshold is nonbinding. The corresponding bunching estimates are reported in Figure 5 in the main text.

Figure A.5. Bunching across Quartiles of the Input-Cost Ratio Distribution

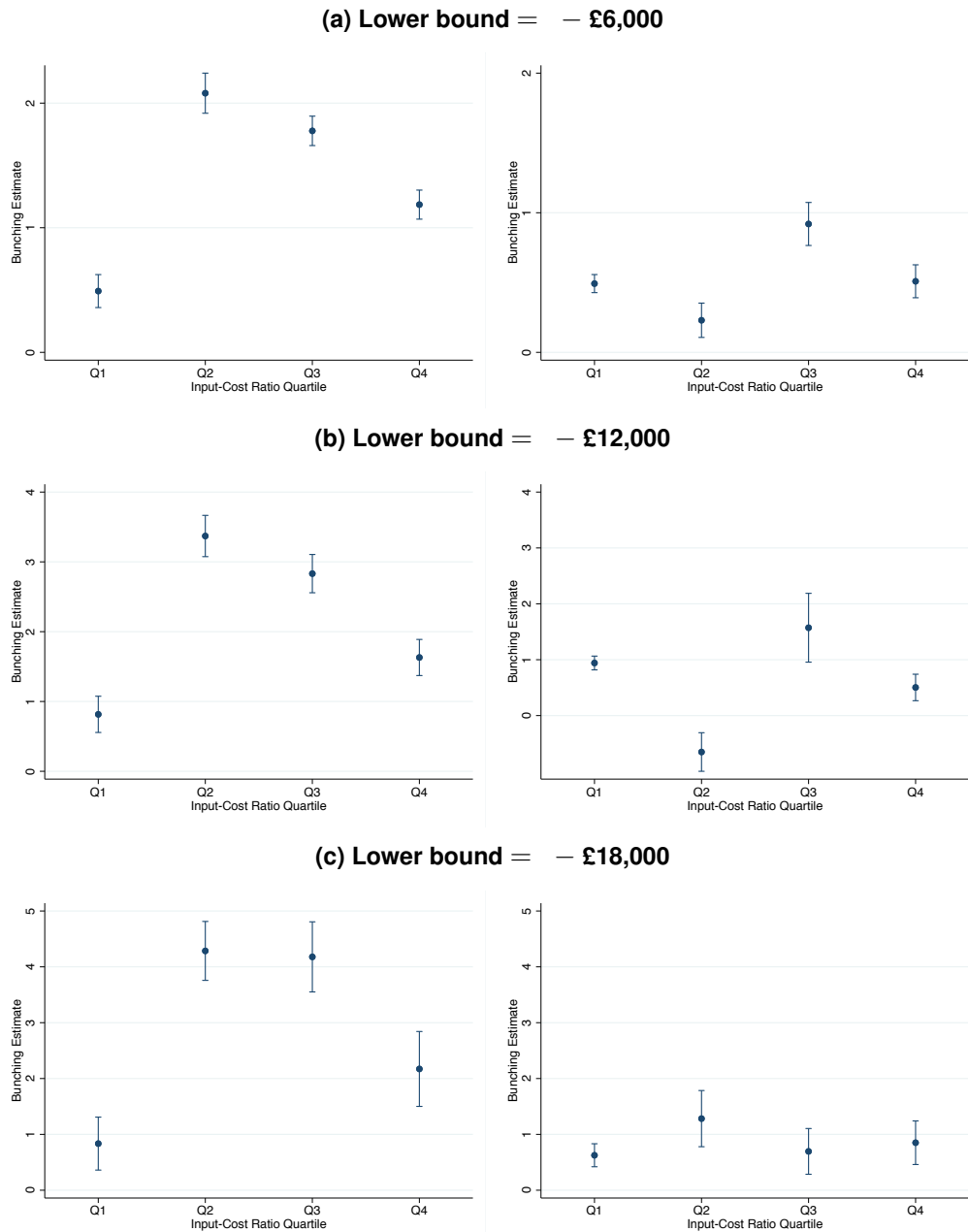**(a) Binding VAT Threshold**　　　　　　　　**(b) Nonbinding VAT Threshold**

Notes: this figure shows the raw distribution of companies' turnover around the normalized VAT notch across four different quartiles of the distribution of the input-cost ratio for the pooled dataset for period 2004/05-2014/15. Panel (a) shows the distributions for the subset of firms not predicted to register voluntarily. Panel (b) shows the distributions for the subset of firms predicted to register voluntarily, for whom the threshold is nonbinding. The corresponding bunching estimates are reported in Figure 6 in the main text.

Figure A.6. Bunching by Quartiles of B2C, by Lower Bound of Excluded Region

(a) Lower bound $= -$ £6,000

(b) Lower bound $= -$ £12,000

(c) Lower bound $= -$ £18,000

Notes: This figure presents the bunching estimates by quartiles of the distribution of share of B2C sales. The left figure in each panel is constructed for firms predicted not to register voluntarily (i.e., for which the VAT threshold is binding), while the right figure is constructed using only firms predicted to register voluntarily, for which the VAT threshold is non-binding. In panels A (top), B (middle) and C (bottom), we report bunching estimates under the assumption that the excluded range begins £6,000, £12,000 and £18,000 below the VAT threshold, respectively.

Figure A.7. Bunching by Quartiles of Input-Cost Ratio, by Lower Bound of Excluded Region

(a) Lower bound $= -£6,000$

(b) Lower bound $= -£12,000$

(c) Lower bound $= -£18,000$

Notes: This figure presents the bunching estimates by quartiles of the distribution of input-cost ratio. The left figure in each panel is constructed for firms predicted not to register voluntarily (i.e., for which the VAT threshold is binding), while the right figure is constructed using only firms predicted to register voluntarily, for which the VAT threshold is non-binding. In panels A (top), B (middle) and C (bottom), we report bunching estimates under the assumption that the excluded range begins £6,000, £12,000 and £18,000 below the VAT threshold, respectively.

Figure A.8. Bunching by Quartiles of Lerner Index, by Lower Bound of Excluded Region
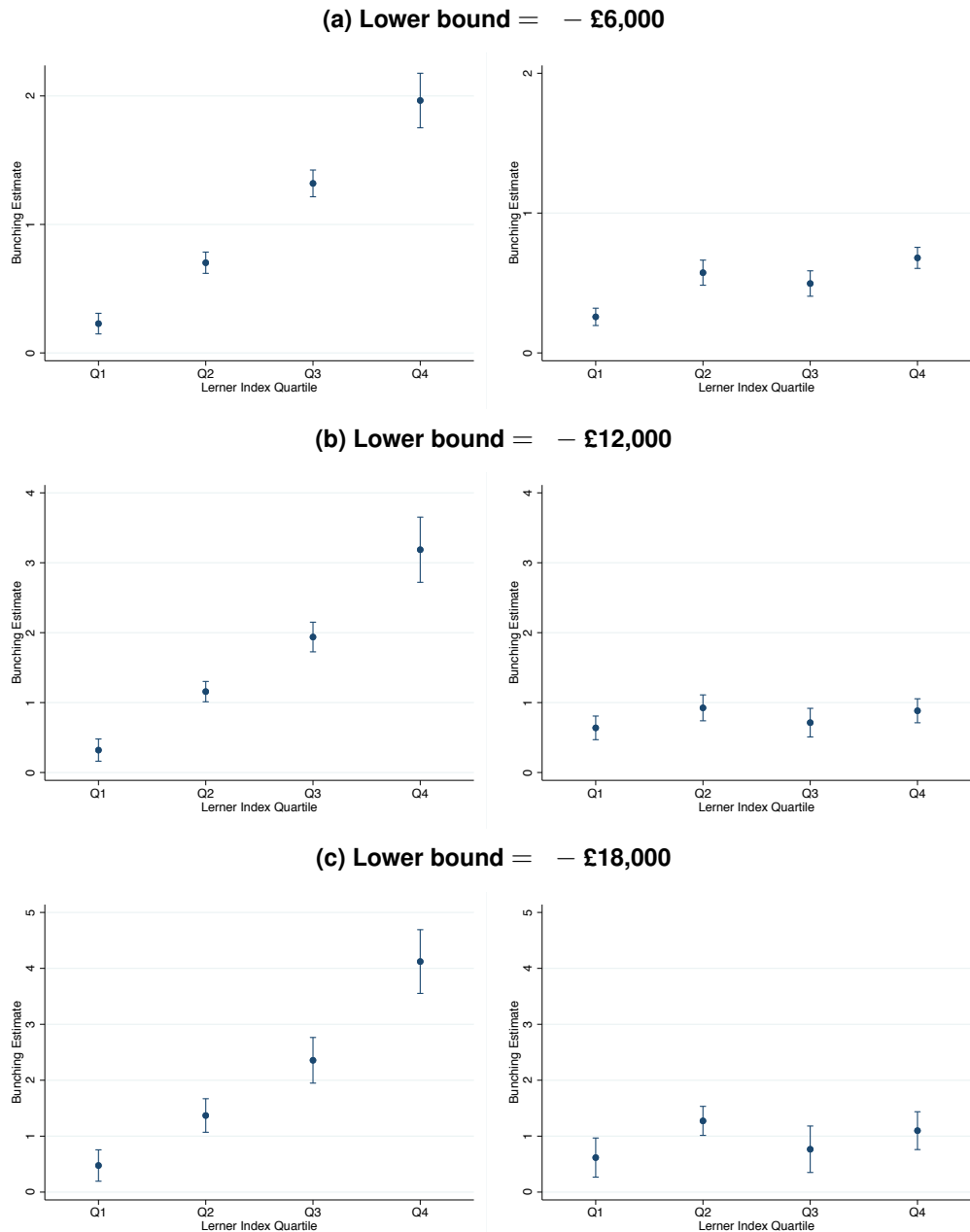
(a) Lower bound $= -$ £6,000

(b) Lower bound $= -$ £12,000


**(c) Lower bound $= -$ £18,000**

Notes: This figure presents the bunching estimates by quartiles of the distribution of share of B2C sales. The left figure in each panel is constructed for firms predicted not to register voluntarily (i.e., for which the VAT threshold is binding), while the right figure is constructed using only firms predicted to register voluntarily, for which the VAT threshold is non-binding. In panels A (top), B (middle) and C (bottom), we report bunching estimates under the assumption that the excluded range begins £6,000, £12,000 and £18,000 below the VAT threshold, respectively.

Figure A.9. Bunching via Turnover Misreporting

(a) Distribution of Direct Input-Cost Ratio
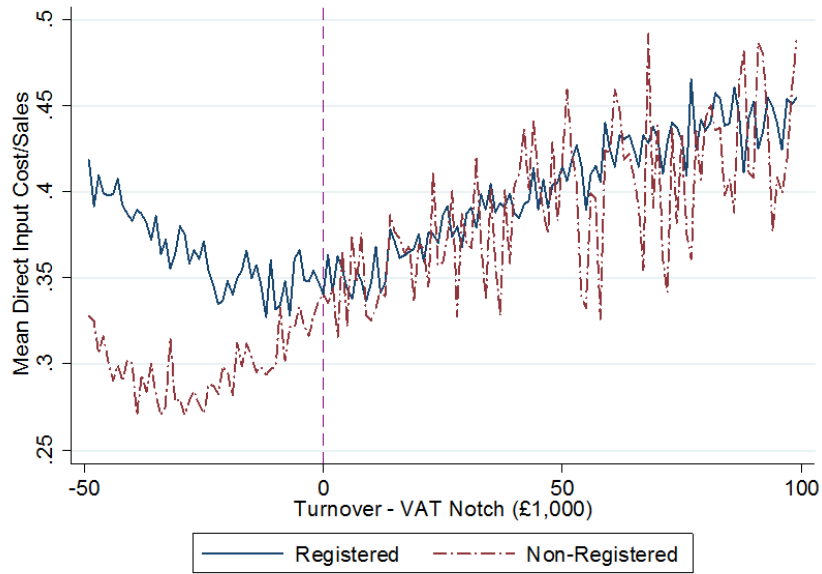
(b) Distribution of Salary-Inclusive Cost Ratio
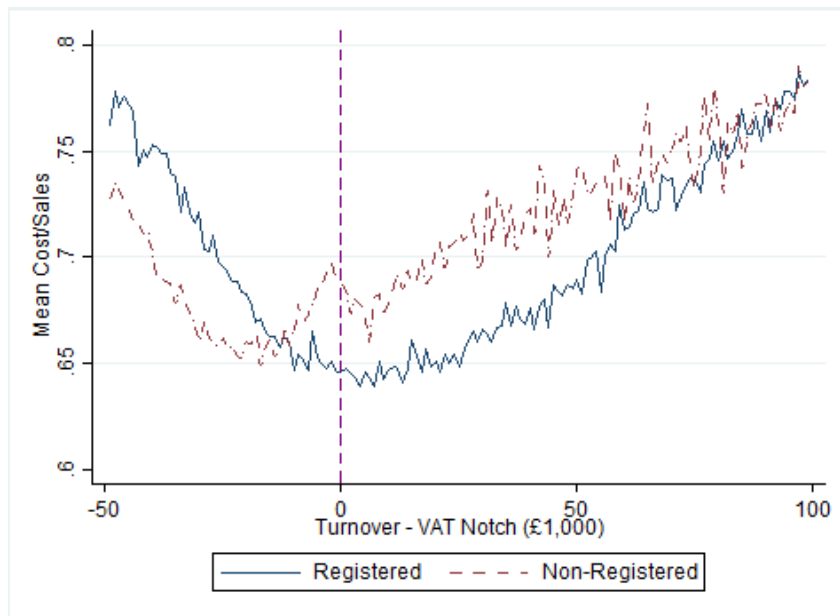
Notes: the figure plots separately the average input cost ratio for registered and non-registered firms with a turnover in the neighborhood of normalized VAT notch during 2004/05-2009/10. Panel A uses the input cost ratio calculated from FAME and exclude the salary expenses. Panel B uses the input cost ratio calculated from the corporation tax records and includes salary expenses in the overall cost.

APPENDIX TABLES

Table A.1. Transition Matrix of VAT Registration Status

	$R_t = 1$ $ID_t = 1$ (1)	$R_t = 1$ $ID_t = 0$ (2)	$R_t = 0$ $IR_t = 1$ (3)	$R_t = 0$ $IR_t = 0$ (4)
$t = 1$				
$R_0 = 1, ID_0 = 1$	82.23%	16.79%	0.30%	0.68%
$R_0 = 1, ID_0 = 0$	19.03%	78.06%	0.16%	2.75%
$R_0 = 0, ID_0 = 1$	9.69%	2.97%	61.57%	25.77%
$R_0 = 0, ID_0 = 0$	2.23%	3.28%	5.60%	88.90%
$t = 2$				
$R_0 = 1, ID_0 = 1$	76.93%	21.13%	0.38%	1.55%
$R_0 = 1, ID_0 = 0$	22.66%	72.31%	0.29%	4.74%
$R_0 = 0, ID_0 = 1$	11.18%	4.97%	53.29%	30.56%
$R_0 = 0, ID_0 = 0$	4.14%	4.99%	5.87%	85.00%
$t = 3$				
$R_0 = 1, ID_0 = 1$	72.67%	24.46%	0.45%	2.42%
$R_0 = 1, ID_0 = 0$	23.53%	69.96%	0.33%	6.19%
$R_0 = 0, ID_0 = 1$	12.29%	5.99%	48.45%	33.26%
$R_0 = 0, ID_0 = 0$	5.24%	6.25%	5.56%	82.94%
$t = 4$				
$R_0 = 1, ID_0 = 1$	68.63%	27.06%	0.63%	3.69%
$R_0 = 1, ID_0 = 0$	23.19%	68.47%	0.40%	7.94%
$R_0 = 0, ID_0 = 1$	12.35%	6.87%	45.13%	35.65%
$R_0 = 0, ID_0 = 0$	6.00%	7.19%	4.88%	81.93%
$t = 5$				
$R_0 = 1, ID_0 = 1$	64.35%	30.68%	0.52%	4.45%
$R_0 = 1, ID_0 = 0$	21.88%	69.23%	0.33%	8.56%
$R_0 = 0, ID_0 = 1$	12.57%	8.01%	41.40%	38.02%
$R_0 = 0, ID_0 = 0$	6.07%	7.96%	4.53%	81.44%

Notes: this table shows in each cell the probability of changing from registration status in year t to year $t + 1$.

Table A.2. Determinants of VAT Voluntary Registration: Probit Model

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Lagged Registration (Rt-1)	3.700***	3.420***	3.973***	3.693***	3.689***	3.681***	3.716***	3.679***	3.702***
	(0.011)	(0.011)	(0.010)	(0.010)	(0.011)	(0.011)	(0.012)	(0.011)	(0.012)
Initial Reg Status (R0)	0.937***	0.869***	0.375***	0.511***	0.756***	0.758***	0.669***	0.756***	0.665***
	(0.023)	(0.022)	(0.016)	(0.018)	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)
R*ID		1.838***		1.677***	0.961***	0.971***	0.958***	0.967***	0.957***
		(0.020)		(0.019)	(0.021)	(0.021)	(0.021)	(0.021)	(0.021)
(1-R)*IR			4.937***	5.200***	4.730***	4.729***	4.790***	4.715***	4.764***
			(0.062)	(0.066)	(0.072)	(0.072)	(0.084)	(0.072)	(0.084)
Distance to Threshold					-0.015***	-0.015***	-0.014***	-0.015***	-0.015***
					(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Average Distance to Threshold					0.003***	0.003***	0.004***	0.003***	0.003***
					(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Share of B2C Sales						0.094			0.150
						(0.129)			(0.136)
Average Share of B2C Sales						-0.598***			-0.410***
						(0.130)			(0.136)
Input-Cost Ratio							0.229***		0.203***
							(0.020)		(0.020)
Average Input-Cost Ratio							-0.396***		-0.136***
							(0.024)		(0.025)
Lerner Index								0.719***	0.464**
								(0.182)	(0.191)
Average Lerner Index								-2.121***	-1.573***
								(0.187)	(0.197)
Constant	-2.163***	-2.168***	-2.146***	-2.240***	-1.807***	-1.528***	-1.758***	-0.746***	-0.861***
	(0.011)	(0.011)	(0.010)	(0.011)	(0.011)	(0.012)	(0.012)	(0.022)	(0.023)
Insig2u									
Constant	-0.743***	-0.686***	-2.163***	-1.419***	-0.920***	-0.927***	-1.058***	-0.950***	-1.079***
	(0.030)	(0.029)	(0.077)	(0.044)	(0.032)	(0.032)	(0.036)	(0.033)	(0.036)
Observations	2016377	2016377	2016377	2016377	2016377	2016377	1773902	2016377	1773902
Chi-sq stat B2C						1734.538			373.728
Chi-sq p-val B2C						0			0
Chi-sq stat CostRatio							126.958		18.274
Chi-sq p-val CostRatio							0		0
Chi-sq stat Lerner								2623.417	1201.127
Chi-sq p-val Lerner								0	0
Sigma(u)	0.690	0.710	0.339	0.492	0.631	0.629	0.589	0.622	0.583
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-FE	No	No	No	No	No	No	No	No	No
Cost-Ratio Source							CT600 (new)		CT600 (new)

Notes: this table presents the coefficient estimates from the dynamic estimation of VAT registration in equation (E.42) in a fixed-effect Probit model.

Table A.3. Determinants of VAT Voluntary Registration: Average Partial Effects

Evaluated at:		Mean $\widehat{\Pr(R_t = 1)}$ (1)	Average Partial Effect (2)
$R_{t-1} = 1$	$ID_t = 1$	0.987	
$R_{t-1} = 1$	$ID_t = 0$	0.931	0.0557***
			(0.0010)
$R_{t-1} = 0$	$IR_t = 1$	0.989	
$R_{t-1} = 0$	$IR_t = 0$	0.206	0.782***
			(0.0034)
Average in the sample:		0.602	
$B2C + \sigma_{B2C}$		0.600	-0.0028***
			(0.00015)
$ICR + \sigma_{ICR}$		0.603	0.0073***
			(0.00017)
$Lerner + \sigma_{Lerner}$		0.597	-0.0058***
			(0.00017)

Notes: this table presents the partial effects of the key variables of interest from the dynamic estimation of VAT registration in equation (E.42). The partial effects are based on the coefficient estimates reported in column 9 of Table A.2.