

**WP/19/35**

# **IMF Working Paper**

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## **Multivariate Filter Estimation of Potential Output for the United States: An Extension with Labor Market Hysteresis**

by Ali Alich, Hayk Avetisyan, Douglas Laxton, Shalva Mkhatrihvili, Armen Nurbekyan, Lusine Torosyan, and Hou Wang

I N T E R N A T I O N A L M O N E T A R Y F U N D

**IMF Working Paper**

Research Department

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An Extension with Labor Market Hysteresis**

Prepared by Ali Alich, Hayk Avetisyan, Douglas Laxton, Shalva Mkhatriashvili, Armen Nurbekyan, Lusine Torosyan, and Hou Wang<sup>1</sup>

Authorized for distribution by Benjamin Hunt

February 2019

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**Abstract**

This paper extends the multivariate filter approach of estimating potential output developed by Alich and others (2018) to incorporate labor market hysteresis. This extension captures the idea that long and deep recessions (expansions) cause persistent damage (improvement) to the labor market, thereby reducing (increasing) potential output. Applying the model to U.S. data results in significantly smaller estimates of output gaps, and higher estimates of the NAIRU, after the global financial crisis, compared to estimates without hysteresis. The smaller output gaps partly explain the absence of persistent deflation despite the slow recovery during 2010-2017. Going forward, if strong growth performance continues well beyond 2018, hysteresis is expected to result in a structural improvement in growth and employment.

JEL Classification Numbers: C51, E31, E52

Keywords: Macroeconomic Modeling, Potential Output

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<sup>1</sup> The estimates of potential output and the output gap presented in this paper are not official IMF estimates. The programs and potential output estimates can be downloaded from [www.douglaslaxton.org](http://www.douglaslaxton.org). The authors would like to thank Rania Al-Mashat, Jaromir Benes, Aram Butavyan, Robert Ford, Narek Ghazaryan, Vahagn Grigoryan, Mane Harutyunyan, Anahit Hovhannisyan, Edgar Hovhannisyan, Mariam Kharaishvili, Akaki Liqokeli, Karolina Matikyan, Gevorg Minasyan, Andrei Orlov, Babken Pashinyan, Garik Petrosyan, Yekaterina Rezepina, Aleksandr Shirkhanyan, Tamta Sopromadze, Erik Vardanyan, and Jiaxiong Yao for helpful comments and suggestions. All errors and omissions are our own.

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## I. INTRODUCTION

The Great Recession (GR) in the United States that began in 2009 not only resulted in one of the most severe losses of output and financial wealth in a century, but has also been associated with a marked increase in uncertainty about both the level and the growth rate of U.S. potential output. This uncertainty stems, at least in part, from the following stylized facts: an uncommonly weak recovery despite an aggressive policy response to the crisis; repeated downward revisions to growth forecasts and to estimates in potential growth since the onset of the crisis through 2017; and only small declines in inflation notwithstanding large increases in unemployment and drops in output.

There is little agreement on the sources of the apparent slowdown in potential growth. For example, Fernald (2014) argues that it started to decelerate before the crisis as broad-based productivity gains from information technology ran their course. Reifschneider, Wascher, and Wilcox (2015), on the other hand, argue that a significant part of the slowdown reflects long-term damage to the labor market, weak business capital accumulation, and a sharp decline of spending on research and development. Yellen (2016) also cites many of these factors.

The uncertainty surrounding potential output and its growth has important implications for policy, both now and in the future. For example:

- For the Federal Reserve, which struggled during 2010-2017 to bring inflation back up to its target despite near-zero interest rates and a massive increase in its balance sheet, uncertainty about where the economy stands relative to its potential increases the risk of policy missteps that could tip the U.S. into another recession or increase inflation more than desired.
- U.S. fiscal policy currently needs prudent estimates of the economy's cyclical position and its underlying potential growth rate, given that U.S. public debt has reached post-war highs and concerns about sustainability of public health and social security programs.<sup>2</sup>

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<sup>2</sup> For example, the IMF's 2018 Article IV Staff Report projects that federal debt held by the public will rise in the baseline from around 75 percent of GDP in 2017 to over 95 percent of GDP by 2027, but notes that these estimates are highly sensitive to assumptions regarding economic growth. See: <https://www.imf.org/en/Publications/CR/Issues/2018/07/03/United-States-2018-Article-IV-Consultation-Press-Release-Staff-Report-and-Statement-by-the-46048>

- Fears of secular stagnation have led to growing calls for structural reforms, in the United States and elsewhere, to kick start sustained growth and employment.

Building on previous work, this paper introduces labor market hysteresis. Laxton and Tetlow (1992) and Alichí and others (2018) construct a multivariate filter for jointly estimating potential output and the NAIRU — i.e., the levels of output and unemployment that can be achieved without triggering inflationary pressures. This approach has the advantages of considering the relationship between these two concepts and actual inflation, and of exploiting a rich array of both temporary and permanent shocks. Like these earlier authors, this paper relies on Bayesian estimation techniques. Alichí and others (2019) discuss the advantages of using Bayesian estimation approach.

Hysteresis in the labor market can help explain the distinct features of the Great Recession in relation to other cyclical downturns, particularly the so-called Volcker recession of the early 1980s. In this paper's model, we assume that the NAIRU changes when output persistently deviates from its potential, although it eventually goes back to its long-run steady state when the output gap closes. We refer to this as partial hysteresis.

In particular, the model is extended to admit the possibility that the GR—owing to its severity and especially its persistence—left long-lasting scars on labor markets. This type of damage can arise, for example, from: long-term shifts in the sectoral composition of domestic production that render existing workers without the skills needed to meet new demands; extended periods of unemployment that cause a gradual deterioration in skills and weakening labor market attachment; and an impairment of the ability of the labor market to direct the unemployed to vacancies that suit their skills.

These sorts of effects feed into potential output. The effect could be direct, notably by lowering the available supply of effective labor, and indirect, including the possibility that the same change of labor supply also results in an adjustment of the capital stock. The Appendix provides the formal details of micro-foundations for these types of effects.

The possibility that temporary shocks produce persistent output losses due to labor market hysteresis dates back to at least Blanchard and Summers (1986). Duval, Eris, and Furceri (2011) demonstrate, in a sample of 30 countries, that large recessions can have significant and persistent impacts on labor participation rates. Blanchard and others (2015) find that as many as two-thirds of recessions have been followed by a fall in potential output levels and growth. Cerra and Saxena (2017) show that recessions, on average, lead to permanent output losses and poor real-time output gap estimates. Bluedorn and Leigh (2018) find, for a wide range of advanced and emerging market economies, that a 1 percent shock to current-period output typically changes the 10-year-ahead forecast of output made by professional forecasters by about 2 percent, and argue that hysteresis effects contribute to this strong perceived persistence of output.

Hysteresis is a very important factor to consider in the current juncture of the U.S. economy as well. Growth performance in 2018 was very strong. If this continues well beyond 2018, the opposite of what happened during the GR could ensue with a structural improvement in growth performance. In fact, the previous Fed chair, Janet Yellen, and the current chair, Jerome Powell have both alluded to the possibility of positive gains from hysteresis:

Janet Yellen (2016): “A tight labor market might draw in potential workers who would otherwise sit on the sidelines and encourage job-to-job transitions that could also lead to more efficient and, hence, more productive job matches”.

Jerome Powell (2018): “While persistently strong economic conditions can pose risks to inflation and perhaps financial stability, we can ask whether there maybe lasting benefits”.

While in this paper much of the discussion is around deep recessions, the model incorporates hysteresis in a symmetrical way, making it equally suitable for strong expansions too.

The paper is organized in the following way. The next two sections provide some stylized facts about the differences between the two largest post-WWII recessions in the U.S.: the Volcker recession of the early 1980s and the GR that began in 2009. Section IV describes how hysteresis is incorporated into the model. Section V provides estimates of potential output and the NAIRU for the U.S. and discusses how the results resemble the distinct features of the GR. The final section concludes. The Appendix provides technical details about the model.

## **II. BACKGROUND: A TALE OF TWO RECESSIONS**

The United States has suffered two large cyclical downturns in the past 40 years: the recession in the early 1980 and the GR that began in 2009, comparison of which provides an opportunity to examine the lasting effects of large macroeconomic shocks:

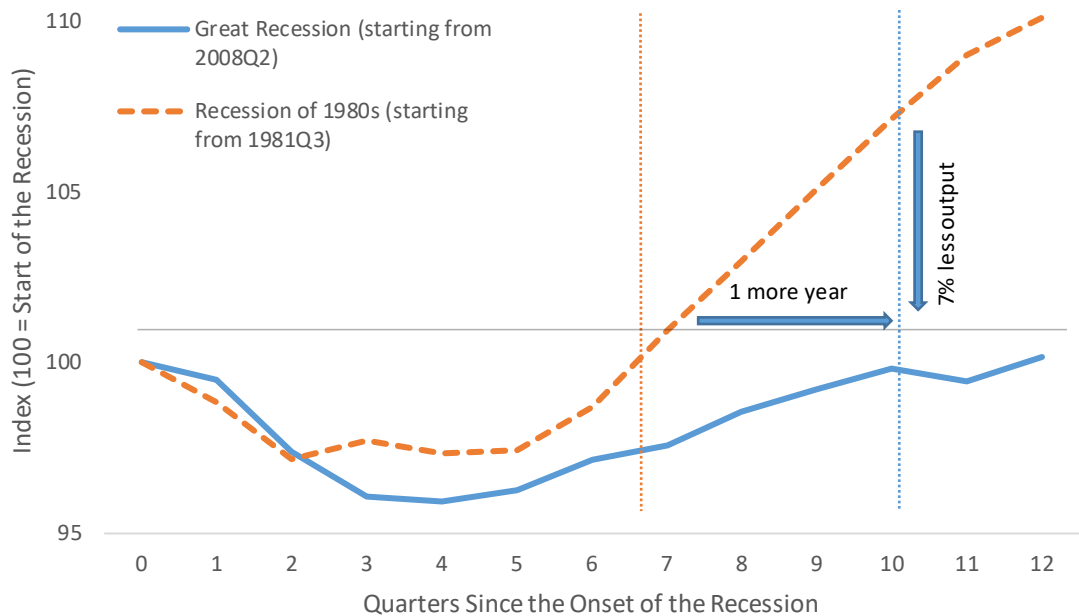
- The Volcker recession of the early 1980s was largely due to a significant tightening of monetary policy in response to an overheated economy and rising inflation, which peaked around 14 percent in 1980. The resulting recession was significant, with the output dropping by almost 3 percent from peak to trough and the unemployment rate ratcheting up to 10.9 percent in 1982, from around 6 percent at the end of 1979. The Volcker policy was successful, in the sense that inflation fell sharply and has been well contained ever since.
- The GR has often been assessed as the worst cyclical downturn since the Great Depression. Still, in some respects it was not hugely different from the recession in early 1980s. The decline in output and the rise in unemployment were only slightly

worse than during the Volcker recession. Output declined by around 4 percent from peak to trough and the unemployment rate peaked at 10 percent in 2009, compared to about 5 percent at the end of 2007.

But the two episodes were remarkably different in other respects:

- After the onset of the GR, it took 4 more quarters for GDP to return to its pre-recession peak than in the Volcker recession. Put differently, while real GDP needed 10 quarters to return to its pre-crisis level after the GR, 10 quarters after the Volcker recession real GDP was already 7 percent above its pre-crisis level (Figure 1).

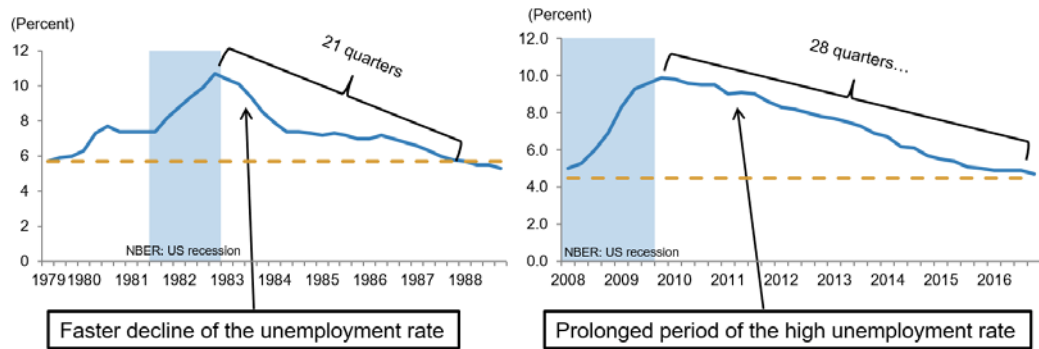
**Figure 1. Real GDP Dynamics after the Two Recessions**



Sources: Federal Reserve Bank of St. Louis, and authors' estimates.

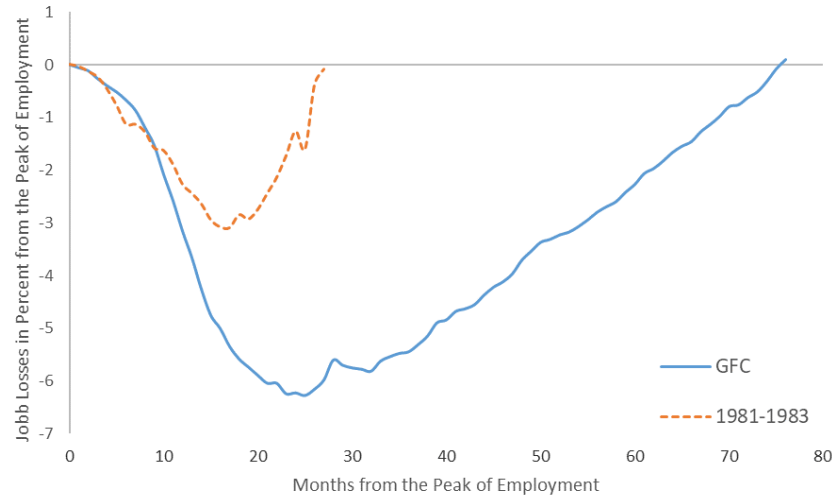
- Similarly, apart from the unemployment rate, the labor market was affected much more severely during the GR. After the unemployment rate peaked at 10 percent, it took 2 years longer for it to return to the pre-crisis level, after the GR, compared to the recession in 1980s (Figure 2). The impact of the GR on employment was even greater than suggested by the rise in the unemployment rate, given the marked decline in the participation rate that occurred after 2008; the participation rate did not decline during the 1980-85 period.

**Figure 2. Unemployment Rate in the Two Recessions**



Source: Federal Reserve Bank of St. Louis.

**Figure 3. Employment in the Two Recessions**

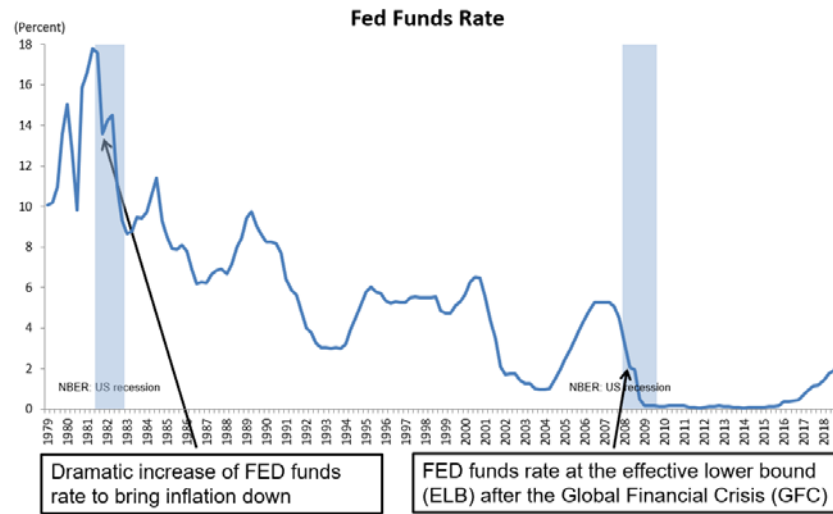


Sources: Federal Reserve Bank of St. Louis, Plotnikov (2014), and authors' estimates.

- Monetary policy was also sharply different. In the early 1980s, the Fed deliberately and successfully tightened its policy stance to cut inflation. By contrast, in the GR the Fed reduced its policy rate to zero and hugely expanded its balance sheet. Fiscal policy was also different, as the federal budget deficit rose substantially more during the GR (Figure 5).

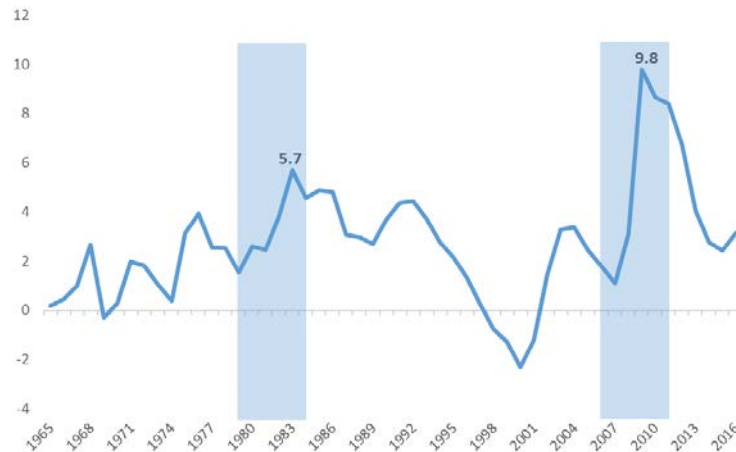


**Figure 4. Monetary Policy in the Two Recessions**



Source: Federal Reserve Bank of St. Louis.

**Figure 5. Federal Budget Deficit in the Two Recessions**

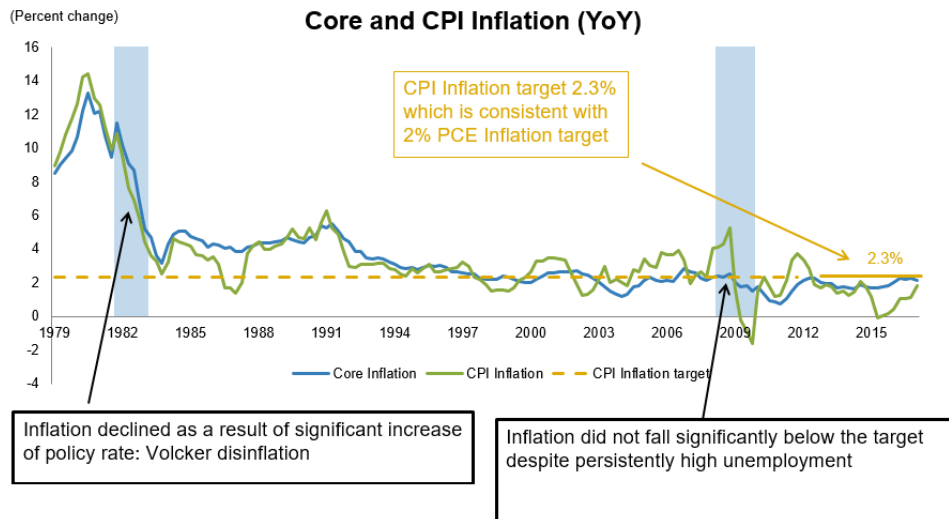


Source: Federal Reserve Bank of St. Louis.

- In the wake of the Volker recession, inflation had fallen 10 percentage points by the mid-1980s. However, notwithstanding persistent deflation fears, the drop in inflation during the GR was quite small (Figure 6), even though the rise in the unemployment rate and the fall in output were more persistent. Judging from the historical relationship between inflation and unemployment, such “missing deflation” (during the recession) and “missing inflation” (in the recovery) seem to be a puzzle. Possible explanations include hysteresis effects, which are examined in this paper (the unemployment and output gaps were not as large as they appeared); the Phillips Curve may have flattened (see Chapter 3 of the April 2013 World Economic

Outlook); the Phillips Curve is very convex (larger gaps are needed to reduce inflation when it is already low) (Laxton and others, 1999); and significantly lower inflation expectations in 1980s contributed to lower inflation during the recession (for a given output gap), while expectations were already low and, more importantly, broadly stable during the GR (Alichi and others, 2018).

**Figure 6. Inflation in the Two Recessions**

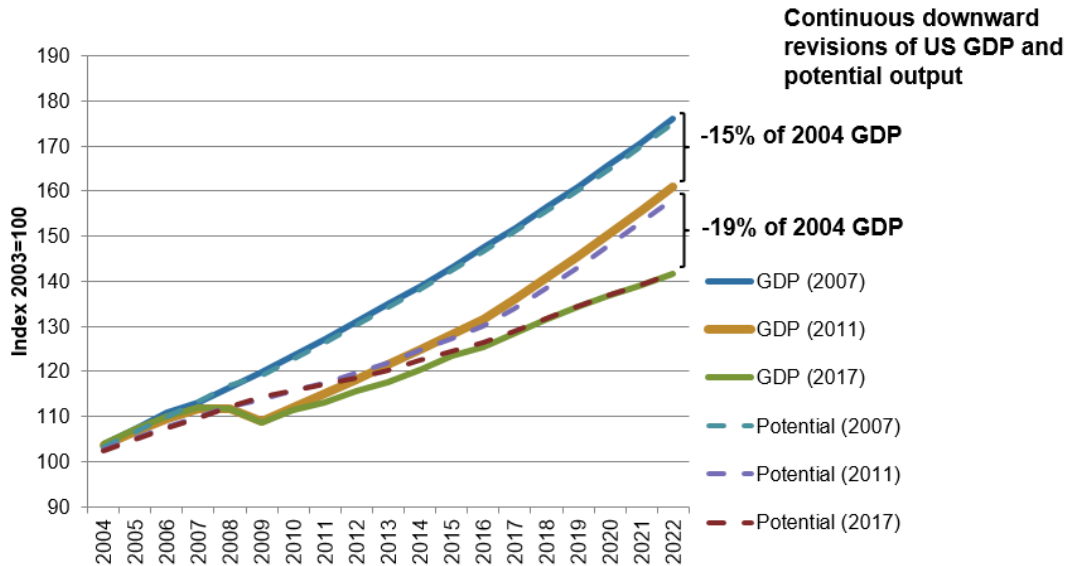


Source: Federal Reserve Bank of St. Louis.

### III. SYMPTOMS OF LABOR MARKET HYSTERESIS AFTER THE GR

The long-lasting demand deficiency during the GR plausibly resulted in labor market hysteresis, given the persistence of high unemployment and weak growth. This persistence was something of a surprise, in the sense that forecasters have had to repeatedly revise down projections of both actual and potential output. For example, the level of output in 2022 as expected in 2017 is below that as expected in 2007 by a whopping 34 percent of 2004 GDP (Figure 7). The same difference regarding potential output estimates is also evident.

**Figure 7. WEO Revisions of Actual and Potential Output**

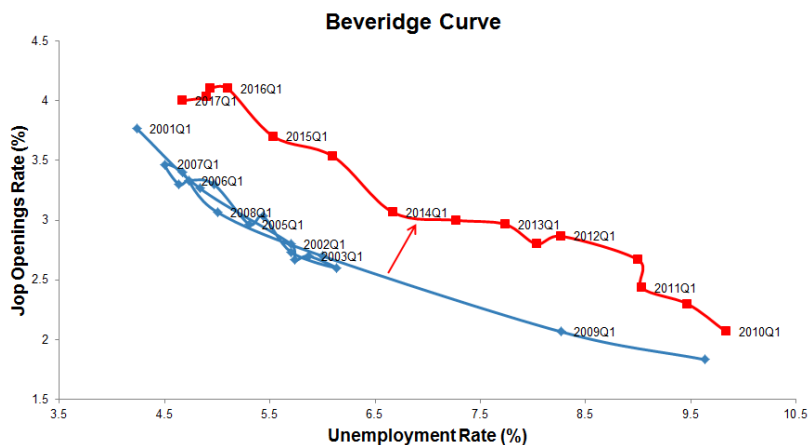


Downward revisions of US GDP and potential output.

Source: IMF WEO database: April 2007, April 2011, and April 2017.

The Beveridge Curve (the relationship between unemployment and job openings) implies a deterioration in labor market performance consistent with hysteresis effects taking hold during the GR. After 2010, it shifted outward: the same job opening rate was associated with around a 1.5 to 2.0 percentage point higher unemployment rate (Figure 8). In the last two years, which have been marked by a return to low unemployment, there have been signs that the Beveridge Curve is shifting back in.

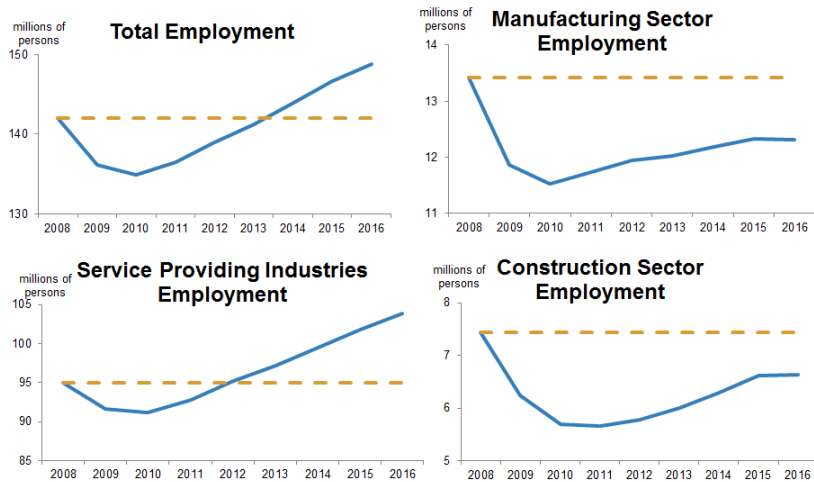
**Figure 8. Efficiency of Matching between Vacancies and Workers Before and After 2010**



Source: Federal Reserve Bank of St. Louis.

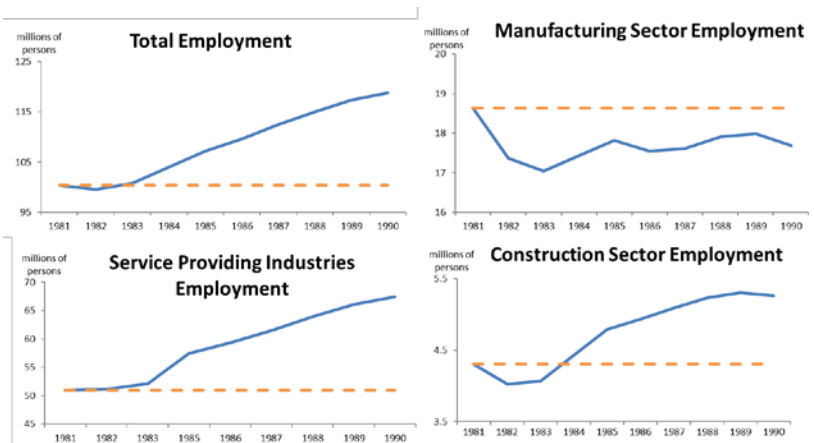
One explanation for the higher job mismatch implicit in the Beveridge Curve shift is a shift in the sectoral pattern of output that has made some workers' skills obsolete. At a fairly aggregate level, this appears to have happened in the GR. Total employment exceeded its pre-crisis level by 2013 and services employment a year before that, while manufacturing and construction employment have yet to recover (Figure 9). However, these trends need to be judged against the background of the longer-term trend decline in manufacturing employment and a corresponding increase in services employment. Comparison with the 1980s recession suggests that something of the same employment shifts were at work then, too, except for construction which made a strong recovery (Figure 10).

**Figure 9. Sectoral Employment after the Great Recession**



Source: Bureau of Economic Analysis.

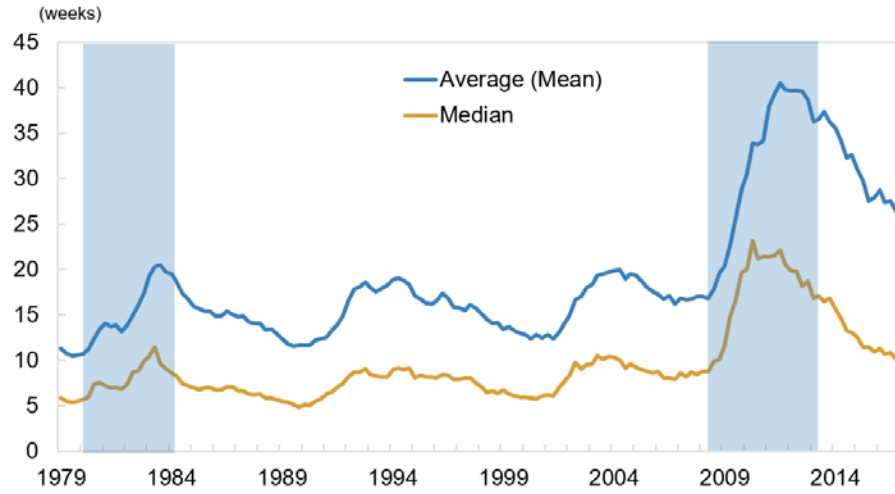
**Figure 10. Sectoral Employment after the Volcker Disinflation**



Source: Bureau of Economic Analysis.

The GR resulted in the highest recorded unemployment duration. The average unemployment duration was double that in the recession of early 1980s (Figure 11). Such a high level of unemployment duration plausibly contributed to the erosion of skills and weaker labor market attachment, and thus a reduction in the effective labor supply.

**Figure 11. Unemployment Duration**



Source: Federal Reserve Bank of St. Louis.

## IV. THE MODEL

This paper extends the model developed by Alichì and others (2018), which is described in full in the Appendix. In this section, we explain in detail only the modifications to that model. In Section IV.2 we compare the simulation results for both models to make clear the differences of output gap shocks on unemployment, NAIRU and inflation. In Section IV.3 we discuss Bayesian estimation procedure. Parameter values and variances of shocks, estimated with Bayesian estimation techniques, are provided in Table 1.

### IV.1. Adding Unemployment Hysteresis to the Model

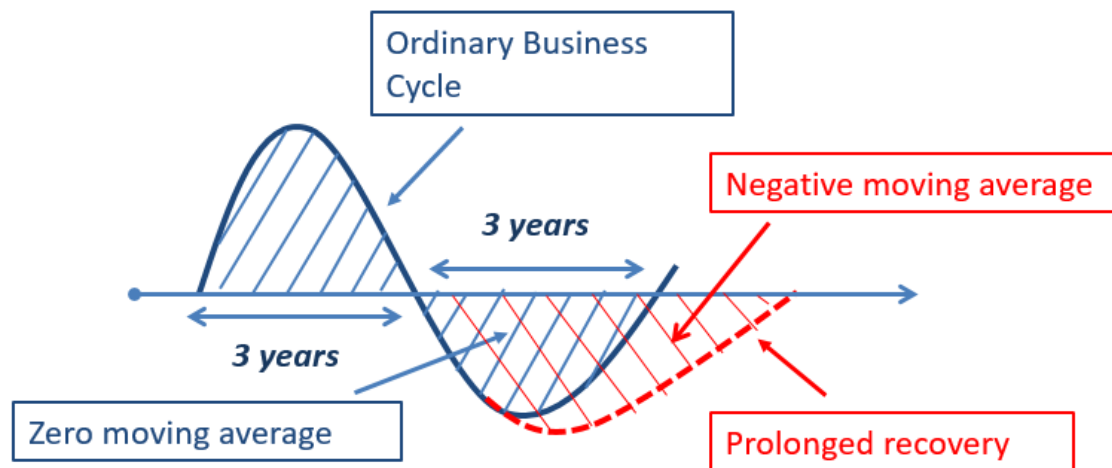
Following the discussion in Benes and others (2010), we generate the partial hysteresis effect by including the output gap in the NAIRU equation. Additions to the model of Alichì and others (2018) are in bold.

$$(1) \quad \bar{u}_t = \rho_{\bar{u}} \bar{u}_{t-1} + (1 - \rho_{\bar{u}}) u^{ss} + g_{\bar{u},t} - \xi * \mathbf{movavg}(\hat{y}_{t+3}, -6) + \epsilon_{\bar{u},t}$$

Here,  $\bar{u}_t$  is the equilibrium value of the unemployment rate (the NAIRU), which is time varying. It is subject to shocks ( $\epsilon_{\bar{u},t}$ ) as well as to a temporary variation in the trend ( $g_{\bar{u},t}$ ), which is itself also subject to shocks ( $\epsilon_{g_{\bar{u},t}}$ ).

Importantly, and unlike in Benes and others (2010), the NAIRU responds to the moving average ( $movavg(\hat{y}_{t+3}, -6)$ ) of demand conditions over the past three years and the expectation over the following three years. This specification allows only persistent output gaps to affect the NAIRU. Six years is the average duration of a U.S. business cycle, as judged by the NBER. Ordinary business cycle variations therefore do not change the NAIRU, as the moving average equals zero. However, if the recovery is unusually long (i.e. business cycle lasts more than 6 years) the moving average becomes negative and, hence, increases the NAIRU. For the sake of simplicity, we model hysteresis symmetrically, which means that if the business cycle is shorter and the moving average term is positive, it has positive effects on potential. The question of symmetry of hysteresis is still open in the literature; Blanchard (2017) provides a recent overview of arguments both in favor and against. Technically it is possible to build in an asymmetric hysteresis effects by making the function non-linear, however, we do not complicate the analysis, since we concentrate on the GFC. This is illustrated in Figure 12.

**Figure 12. Illustrative Effects of Business Cycles on the NAIRU**



Source: Authors' construction.

We include both lag and lead terms in the moving average. Past deficiencies in aggregate demand result in higher unemployment, which, if it lasts long enough, erodes the skills of unemployed. However, the NAIRU is also affected if firms expect low demand for their products in the coming years. These expectations may result in significant changes to business models, perhaps requiring new skills, and thus pose skills mismatch problems. Consequently, persistent deviations of the NAIRU from its steady-state value are permitted, both directly as a result of shocks to the level and growth of the NAIRU, but also indirectly, as a result of large and relatively long-lasting shocks to GDP.

Another amendment to the system is the relationship between the potential growth and NAIRU. As in the core version in Alichì and others (2018), the level of potential output ( $\bar{y}_t$ ) evolves according to the underlying trend growth rate of potential ( $g_{\bar{y},t}$ ) and is subject to level- ( $\epsilon_{\bar{y},t}$ ) and growth- ( $\epsilon_{g_{\bar{y},t}}$ ) shocks. However, here the level of potential output is assumed to be also affected by the changes in the NAIRU.

$$(2) \quad \bar{y}_t = \bar{y}_{t-1} + g_{\bar{y},t} - \eta (\bar{u}_t - \bar{u}_{t-1}) - (1 - \eta) \frac{(\bar{u}_{t-1} - \bar{u}_{t-5})}{4} + \epsilon_{\bar{y},t}$$

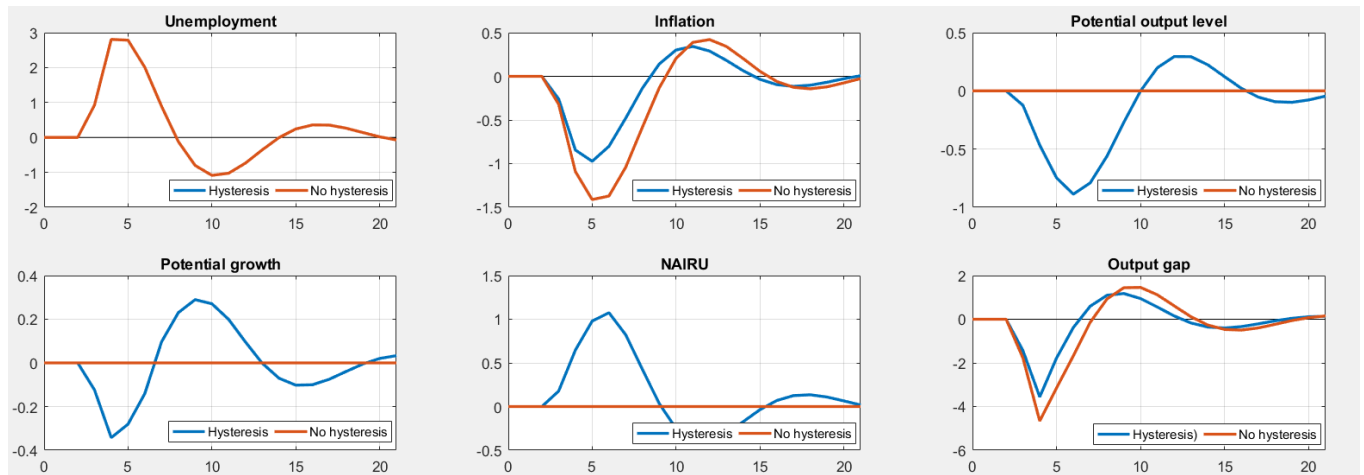
The term  $(\eta (\bar{u}_t - \bar{u}_{t-1}))$  captures the contemporaneous impact of a change in the NAIRU on potential output, which is proportional to an estimate of labor's share in a Cobb-Douglas production function. In addition, a higher NAIRU, implying a permanent drop of labor available for production, triggers a gradual downward capital stock adjustment captured by the term  $\left( (1 - \eta) \frac{(\bar{u}_{t-1} - \bar{u}_{t-5})}{4} \right)$ . This extends the negative effect by another 4 years reflecting adjustment costs on capital accumulation. The final total impact on potential output is proportional to the initial increase in NAIRU. The formal justification of this specification is provided in the Appendix, which derives an expression similar to the one shown here.

## IV.2. Simulations Matching the Actual Data

To illustrate the effects arising from hysteresis, we simulate the model using shocks to the output gap that result in an output drop comparable to the one observed in 2008-09 (Figure 13). Results are reported for two versions of the model: one without hysteresis (the Alichì and others (2018) model) and the one developed in this paper. The simulations are conducted in the following way. First, each model is separately estimated using Bayesian techniques (see the next section for details of the estimation of the hysteresis model). After that, the output gap in the with-hysteresis model is shocked for two consecutive years (meant to replicate 2008 and 2009) such that the output drop is similar in magnitude to the actual 2008-09 output decline. Then the resulting unemployment rate path is imposed on the non-hysteresis model by exogenizing the unemployment rate and endogenizing the output gap shock. This yields the same path for the unemployment rate in both models, but different paths for the output gap and, hence, inflation. The results are reported in Figure 13.

Hysteresis means that, for the same initial shock, the NAIRU rises and potential output falls in response to persistently deficient demand. Thus, the estimated output gap is smaller for the same increase in unemployment. Put another way, had the shocks to the output gap been the same for the two models, the one with hysteresis would have predicted less disinflation. Thus, hysteresis goes some way to explaining persistent unemployment and less disinflationary pressure.

**Figure 13. Model Simulations of Impact of Negative Demand Shocks (Percent)**



Source: Authors' estimates.

### IV.3. Bayesian Estimation

As in the case of Alichì and others (2018), the model is estimated with Bayesian techniques, using annual U.S. data from 1980 to 2018 on real GDP, CPI inflation, unemployment rate, capacity utilization rate in the manufacturing sector, and interest rates, as well as Consensus Economics multi-year-ahead forecasts for CPI inflation and GDP growth. The Consensus series help to avoid the usual end-point problems associated with filtering techniques and to better identify shocks. The incorporation of Consensus forecasts can be thought of as a heuristic approach to blending forecasts from different sources and methods. As for the Bayesian estimation approach, Alichì and others (2019) discuss how it can help overcome the problem of short samples and structural shifts which can render pure maximum likelihood estimation unreliable and often counterintuitive. The Bayesian approach is a practical way to incorporate expert judgment and narrow down the search perimeter.

Table 1 summarizes the Bayesian priors and the posterior estimates for the model's parameters. Somewhat tight priors are imposed on most of the parameters. Exceptions are the coefficient of potential output shock in the Phillips curve, the coefficient linking capacity utilization and the output gap, and the coefficient of the inflation target shock on the short-term interest rate. The prior for steady state GDP growth (2.1 percent) is calibrated to the value in the January 2017 Consensus Economics long-term survey of GDP growth in 2027. The prior for the steady-state NAIRU is taken from the latest median of FOMC participants'



estimates of the longer-run natural rate of unemployment, which was 4.8 percent.<sup>3</sup> Both of these act as attractors in the system and determine where GDP growth and unemployment converge to over the long term.

**Table 1. Model Parameter Priors and Posterior Estimates**

	Mode		Standard Error	
	Prior	Posterior	Prior	Posterior
$\eta$	0.700	0.683	0.210	0.029
$\xi$	0.450	0.433	0.180	0.027
$\rho_{g\gamma}$	0.291	0.299	0.009	0.008
$\Phi_1$	0.671	0.655	0.020	0.014
$\Phi_2$	0.400	0.399	0.120	0.009
$\Phi_3$	0.400	0.385	0.120	0.011
$\Phi_4$	0.300	0.319	0.090	0.021
$\Phi_5$	0.800	0.812	0.240	0.020
$\lambda_1$	0.410	0.409	0.001	0.001
$\lambda_3$	0.082	0.083	0.002	0.002
$\lambda_4$	0.100	0.099	0.600	0.017
$\beta_1$	0.400	0.399	0.001	0.001
$\alpha_1$	0.500	0.499	0.002	0.001
$\alpha_2$	1.500	1.504	0.005	0.004
$\alpha_3$	0.050	0.057	0.015	0.010
$\alpha_4$	2.000	2.030	0.060	0.027
$\rho^{\pi\pi^{1Y}}$	0.900	0.883	0.270	0.018
$\rho^{\sigma^{Term}}$	0.700	0.697	0.021	0.014
$\tau$	0.521	0.503	0.156	0.028
$\rho_{\hat{u}}$	0.407	0.416	0.122	0.026
$\rho_{\bar{u}}$	0.120	0.125	0.036	0.016
$\rho_{g\pi}$	0.880	0.880	0.004	0.004
$\delta_1$	0.100	0.104	0.030	0.016
$\delta_2$	0.200	0.198	0.060	0.017
$\kappa$	2.167	2.148	0.585	0.047

Source: Authors' estimates.

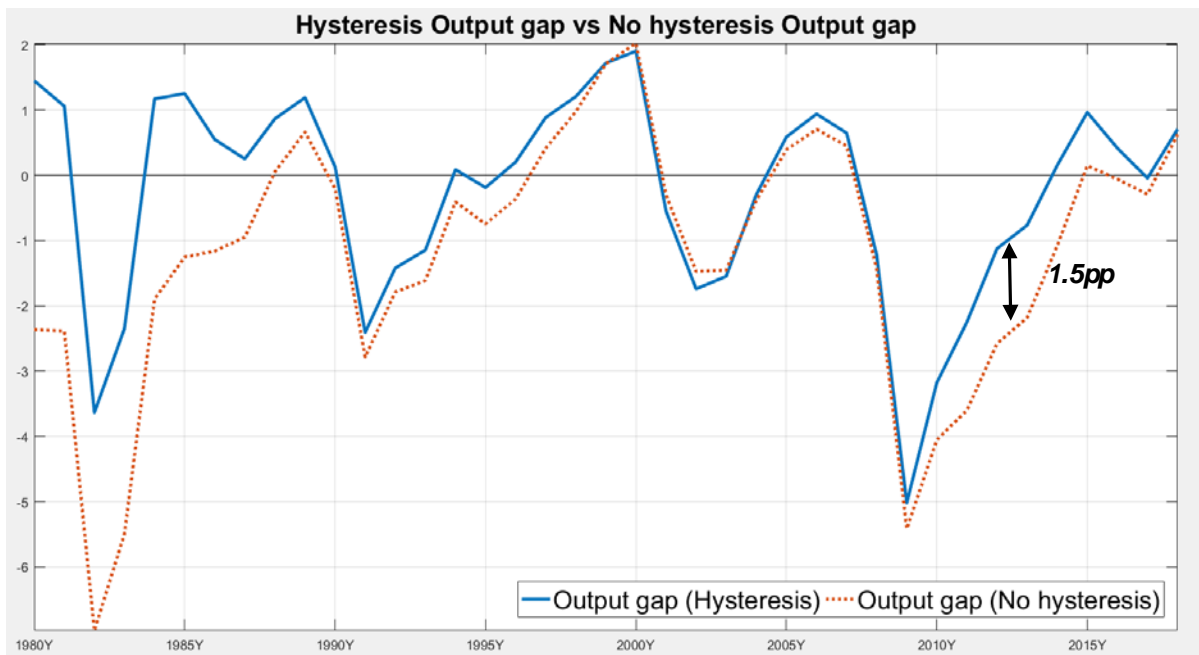
The hysteresis parameter priors are calibrated to match the discussion of the shift in the Beveridge Curve observed since 2010. The outward shift of the Beveridge Curve is 2.0-2.5 percentage points of the unemployment rate. However, as discussed in Daly and others (2012), the magnitude of the implied NAIRU change depends on the sources of the shift in the Beveridge curve. A widely-preferred estimate in the literature implies an increase of the NAIRU from its pre-recession level of about half the shift in the Curve, or close to 1 percentage point (Daly and others, 2012). The model was calibrated to match this.

<sup>3</sup> See the Fed's Statement on Longer-Run Goals and Monetary Policy Strategy, amended effective January 31, 2017. This is an example of expert judgments that can and should be applied to this simple model when using it in practice.

## V. RESULTS: NAIRU AND POTENTIAL GROWTH FOR THE UNITED STATES

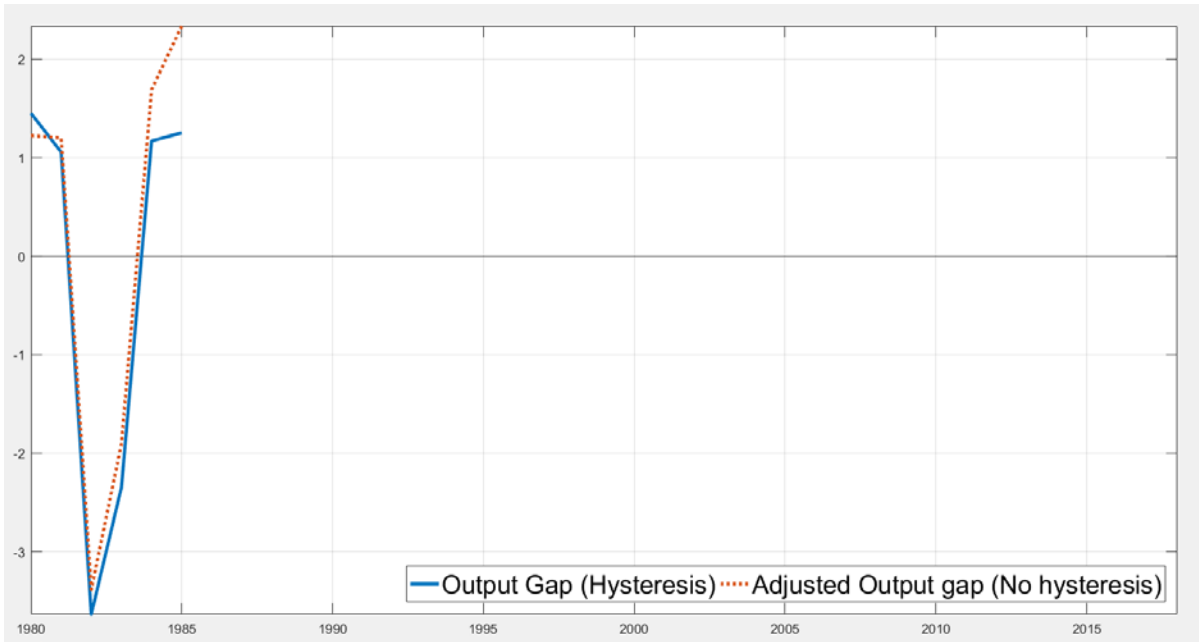
Figure 14 shows output gap estimates derived from the non-hysteresis and hysteresis models. First, the output gap estimates have similar magnitudes of around -5 percent at the height of the crisis in 2009. The output gap is less negative at the height of 1980 recession in the model with hysteresis. However, this does not reflect hysteresis during Volcker disinflation, but rather a different output gap at the beginning of the estimation period, which was inherited, as it were, from the previous boom that had led to the 1980s inflation. Mechanically adjusting for this difference by a parallel shift, to have the same output gaps in the trough, shows that there is little hysteresis effect due to the recession itself during Volcker disinflation period (Figure 15). Indeed, adjusting for the level, dynamics of the output gap is very similar during that period (Figure 16). Given the high inflation in the 1970s, the 1980 gap in the hysteresis model looks more plausible, and suggests a benefit of adding hysteresis.

**Figure 14. Model-Based Estimates of the Output Gap (Percent)**



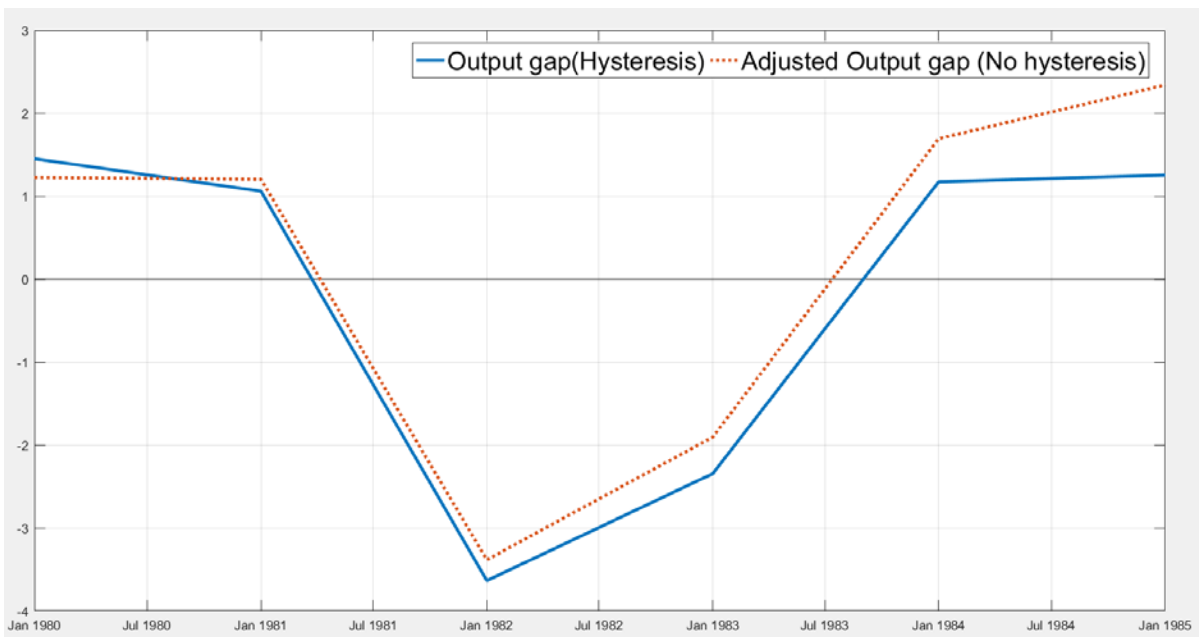
Source: Authors' estimates.

**Figure 15. Adjusted Estimates of the Output Gap for the Volcker Disinflation Period (Percent)**



Source: Authors' estimates.

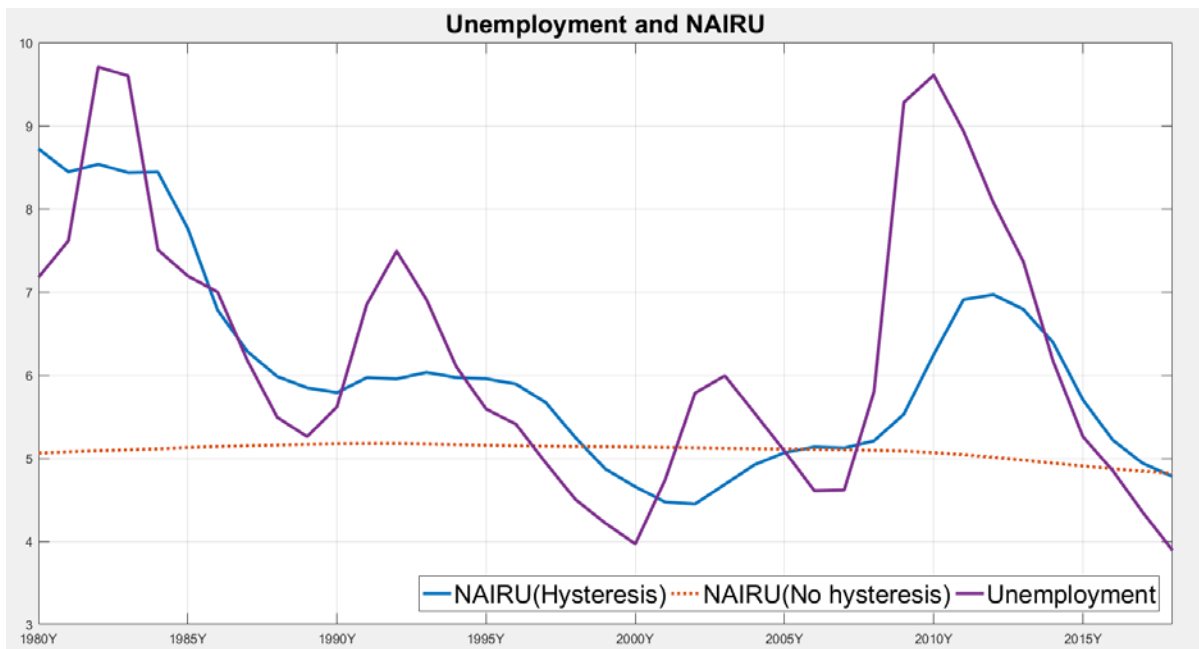
**Figure 16. Adjusted Estimates of the Output Gap for the Volcker Disinflation Period (Percent)**



Source: Authors' estimates.

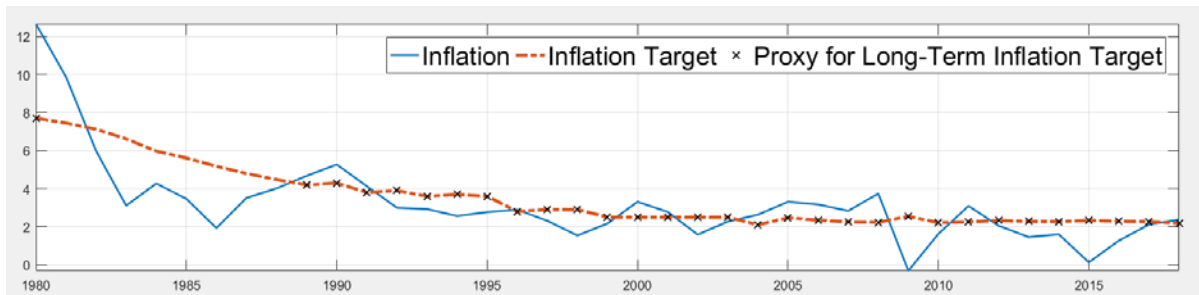
The model with hysteresis suggests significantly less open output gap in the Great Recession; that is, it estimates lower potential output than the non-hysteresis model. The difference builds up gradually, reaching 1.5 percentage points in 2012, as it takes time for the cyclical downturn to affect potential. The less open output gap can explain some, but not all, of the absence of strong disinflation during the GR. In the model with hysteresis the NAIURU is, not surprisingly, much more volatile (Figure 17). It increases by 2 percentage points after the GFC reaching 7 percent in 2012. It then decreases to its pre-crisis level by 2016. Again, we don't estimate sizeable hysteresis effects in 1980s; the NAIURU doesn't increase significantly during the Volcker disinflation.

**Figure 17. Model-based Estimates of the NAIURU (Percent)**



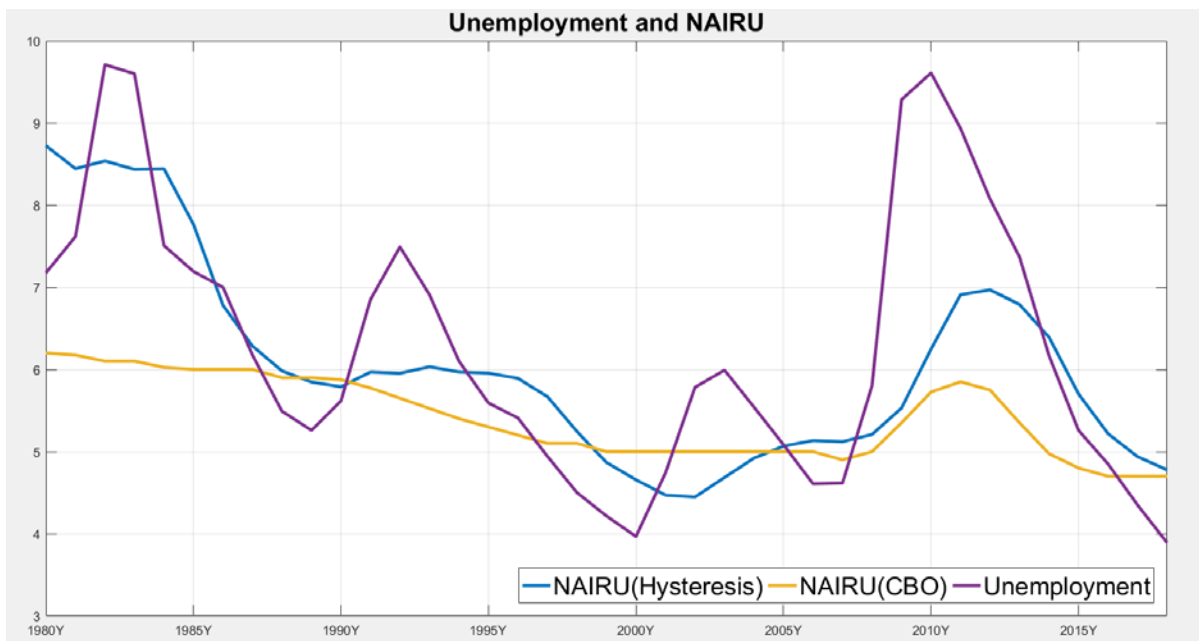
Source: Authors' estimates.

Hysteresis alone cannot explain the lack of disinflation during the GR in the face of high and persistent output and unemployment gaps. Nevertheless, the model can shed some light on the issue. It attributes a big role to shocks to the perceived inflation target in 1980s (Figure 18). That is, following the credible Volcker recession, people became more convinced that the Fed would control inflation and revised down their view of the inflation target significantly. As a result, expectations became more anchored, stabilizing inflation itself. This anchoring was reinforced more recently when the Fed adopted an explicit inflation target in 2012.

**Figure 18. Perceived Inflation Target (Percent)**

Source: Authors' estimates.

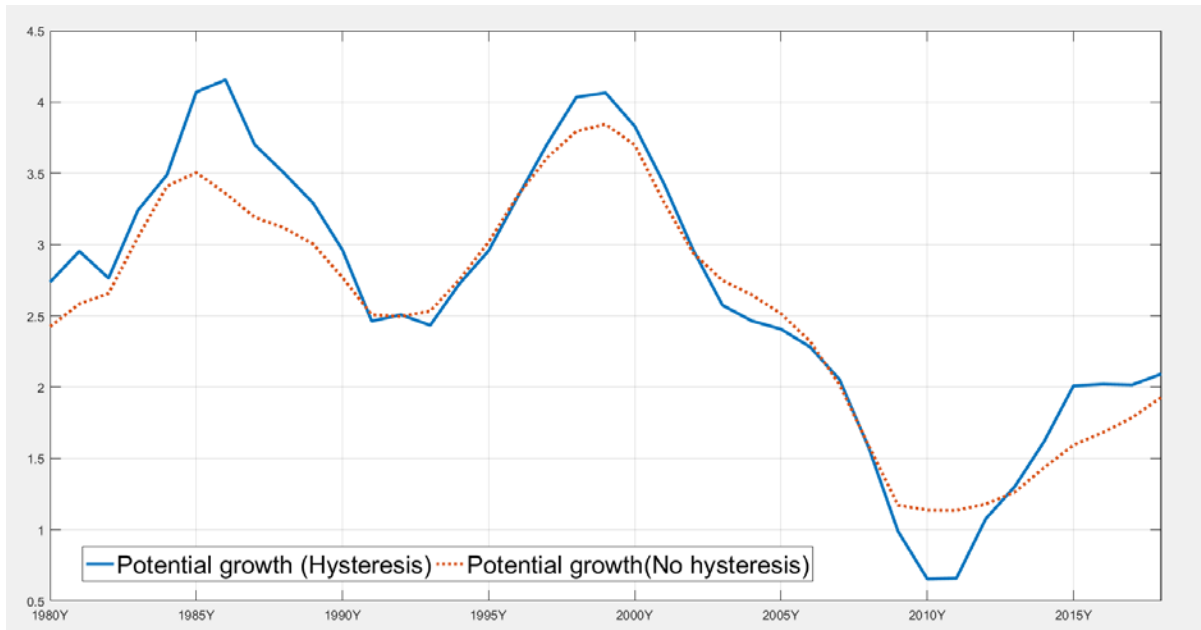
To assess whether the model's NAIRU estimates are sensible, we compare them with CBO estimates (Figure 19). In both cases the NAIRU rises during the GR. The CBO has a temporary increase in the NAIRU to capture structural problems in the labor market, which is consistent with our approach. Indeed, our model calls for an even larger increase in NAIRU. Second, in contrast with CBO estimates, we estimate a much higher NAIRU in 1980, suggesting a smaller unemployment gap, which is easier to square with the high inflation at the time.

**Figure 19. Model-based and CBO Estimates of the NAIRU (Percent)**

Sources: U.S. Congressional Budget Office, and authors' estimates.

Figure 20 compares hysteresis and non-hysteresis estimates of potential growth. The movements of potential growth are, again unsurprisingly, more pronounced in the model with hysteresis. The persistent decline of the potential growth helps to reconcile the continuous revisions to the actual and potential growth rates after the great recession.

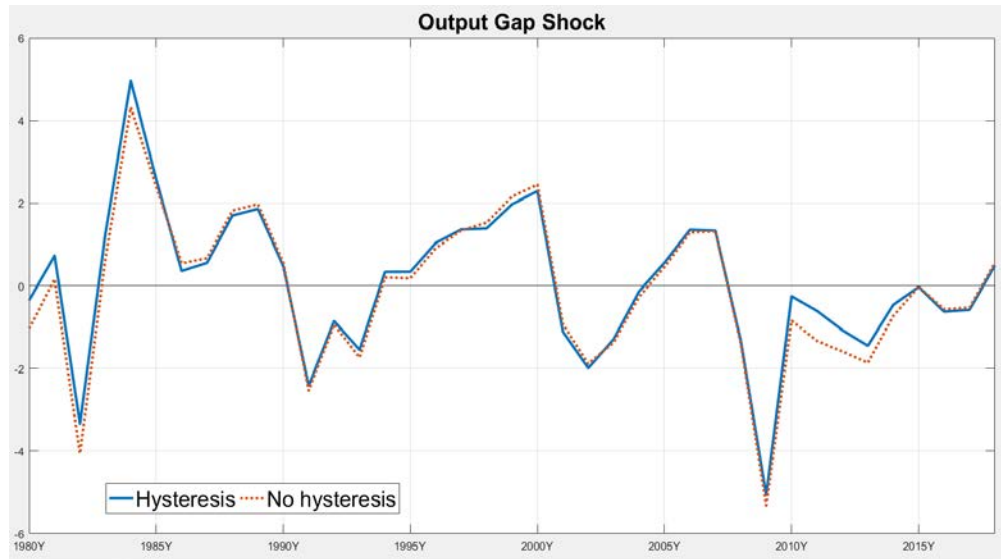
**Figure 20. Model-based Estimates of the Potential Growth (Percent)**



Source: Authors' estimates.

Figure 21 compares the output gap shocks needed to explain the data from the models with and without hysteresis. After the GFC, the model with hysteresis needs much smaller shocks to demand to explain the slow recovery. Indeed, the model without hysteresis, after 2009, generates around 2 percentage points more negative output gap shocks cumulatively to match the downturn. The model with hysteresis doesn't need to generate these shocks, as the structural problems which build up gradually help to match the slow recovery.

**Figure 21. Model-based Estimates of Output Gap Shocks (Percent)**



Source: Authors' estimates.

## VI. CONCLUSION

The aim of this paper is to incorporate the hysteresis effects in a multivariate filter that could be used to estimate potential output. Many indicators point to the existence of hysteresis in the labor market, which can have substantial effect on production possibilities. The hysteresis model is developed and then estimated on U.S. data. The results of this exercise are as expected. Hysteresis generates much larger movements in the NAIRU and potential output. Hysteresis is fairly strong in the protracted GR, certainly when compared to the recession in the early 1980s, which was in some respects as deep, but which was also significantly shorter. As a result, in the GR, taking account of hysteresis results in significantly smaller estimates of output gaps.

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## APPENDIX

### A. The Complete Model

The core data for the model are: GDP, CPI, unemployment rate, capacity utilization rate in the manufacturing sector, and short- and long-term interest rates. We measure the data at annual frequency to reduce the noise in the quarterly data. In addition, we use data on consensus forecasts of annual CPI inflation and real GDP growth (from Consensus Economics) to help better identify supply and demand shocks and deal with end-point problems. In this section, we present all the equations of the model. Parameter values and the standard errors of shock terms for these equations are estimated using Bayesian estimation techniques and are provided in the main text.

The output gap is defined as the deviation of real GDP, in log terms ( $y_t$ ), from its potential level ( $\bar{y}_t$ ):

$$(A1) \quad \hat{y}_t = y_t - \bar{y}_t$$

The stochastic process for output (real GDP) is defined by three equations, (2)-(4), and three types of shocks:

$$(A2) \quad \bar{y}_t = \bar{y}_{t-1} + g_{\bar{y},t} - \eta(\bar{u}_t - \bar{u}_{t-1}) - (1 - \eta) \frac{(\bar{u}_{t-1} - \bar{u}_{t-5})}{4} + \epsilon_{\bar{y},t}$$

$$(A3) \quad g_{\bar{y},t} = (1 - \rho_{g_{\bar{y}}})g_{\bar{y},t-1} + \rho_{g_{\bar{y}}}g_{\bar{y}}^{SS} + \epsilon_{g_{\bar{y},t}}$$

$$(A4) \quad \hat{y}_t = \phi_1 \hat{y}_{t-1} - \phi_2 \widehat{r}_t^{1Y} - \phi_3 \widehat{r}_{t-1}^{1Y} + \phi_4 \epsilon_{g_{\bar{y},t}} - \phi_5 \epsilon_{\bar{y},t} + \epsilon_{\hat{y},t}$$

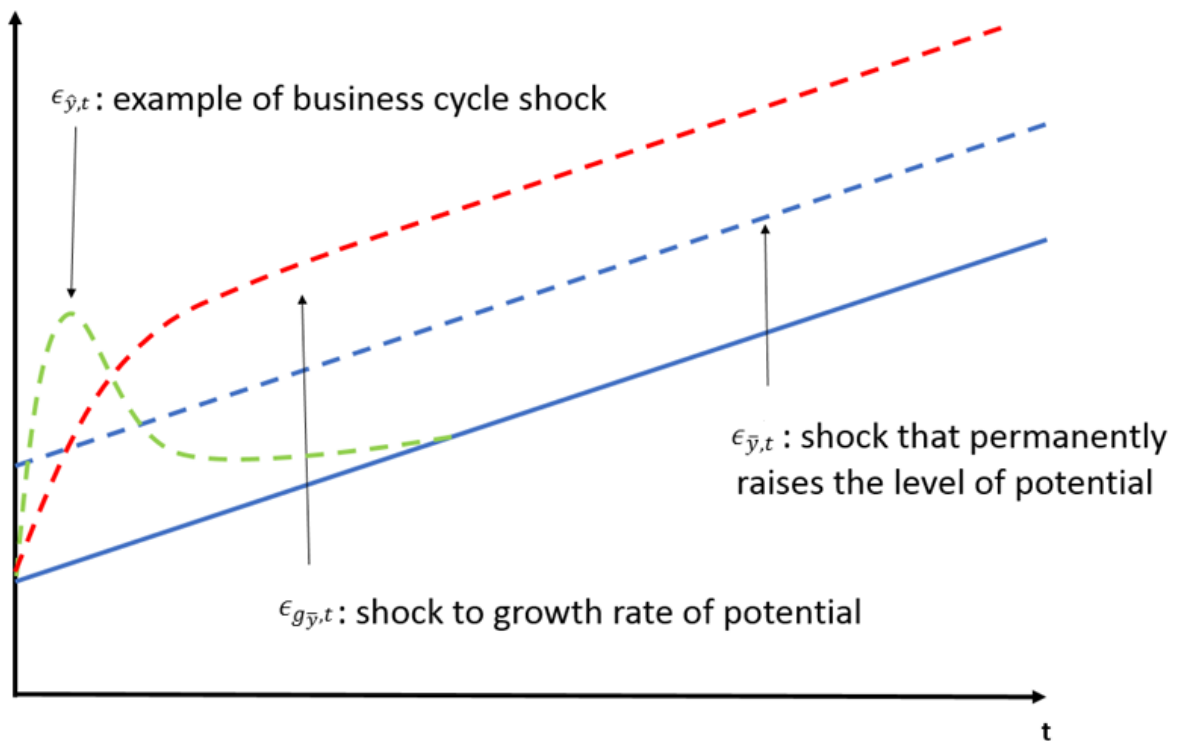
$$(A5) \quad g_t = y_t - y_{t-1}$$

$$(A6) \quad g_{\bar{y},t} = \bar{y}_t - \bar{y}_{t-1}$$

The level of potential output ( $\bar{y}_t$ ) evolves according to trend potential growth ( $g_{\bar{y},t}$ ) and a level-shock term ( $\epsilon_{\bar{y},t}$ ). Potential growth is also subject to shocks ( $\epsilon_{g_{\bar{y},t}}$ ), whose impact fades gradually according to the parameter  $\rho_{g_{\bar{y}}}$  (a lower value means a slower adjustment back to the steady-state growth rate following a shock). In addition, as discussed in the main text this

model also incorporates hysteresis effects as captured by the part of the equation (A2) that is in bold. Finally, the output gap ( $\hat{y}_t$ ) is function of contemporaneous and lagged values of the one-year real interest rate gap ( $\widehat{r}_t^{1Y}$ ) which is the deviation of short-term interest rate from its equilibrium level. The output gap equation also incorporates potential growth ( $\epsilon_{g_{\bar{y},t}}$ ) and level effects ( $\epsilon_{\bar{y},t}$ ). It is also subject to shocks ( $\epsilon_{\hat{y},t}$ ), which are – interpreted as demand shocks in that they raise demand above supply. A stylized representation of how GDP responds to each shock term is expressed graphically in Figure A1:

**Figure A1: Shocks to the Level and Growth Rate of Potential Output, and the Output Gap**



Source: Authors' Construction.

In the absence of a shock, output follows its steady-state path, which is shown above by the solid blue line (which has a slope of  $g_{\bar{y}}^{SS}$ ). However, any of the three shocks causes output to deviate from this path. A shock to the level ( $\epsilon_{\bar{y},t}$ ) will raise (or, if negative, lower) potential output once and for all (as in the dashed blue line). A shock to the growth rate of potential ( $\epsilon_{g_{\bar{y},t}}$ ), illustrated by the dashed red line, raises the growth rate of potential; however the growth rate ultimately returns to the steady state, resulting in a rise in the level of potential output that depends on the size of the shock and the speed with which its effect decays. A shock to the output gap ( $\epsilon_{\hat{y},t}$ ) causes a temporary deviation of the level of output from the level of potential, as shown by the dashed green line.

In order to help identify the three output shock terms, a Phillips Curve equation for inflation ( $\pi_t$ ) is added, which links the evolution of the output gap (an unobservable variable) to observable data on inflation, according to the process:<sup>4</sup>

$$(A7) \quad \pi_t = \lambda_1 E_t \pi_{t+1} + (1 - \lambda_1) \pi_{t-1} + \lambda_3 \hat{y}_t + \epsilon_{\pi,t} - \lambda_4 \epsilon_{\bar{y},t}$$

The last term allows the model to mimic the effects of shocks to productivity which lower marginal cost and therefore reduce inflation.

The inflation target, which can be time-varying, is modeled as a random walk:

$$(A8) \quad \pi_t^{Tar} = \pi_{t-1}^{Tar} + \epsilon_{\pi^{Tar},t}$$

The measure of inflation expectation that is used to calculate the real return on financial instruments is modeled as a linear combination of model-consistent expected inflation and lagged inflation:

$$(A9) \quad \pi_t^e = \beta_1 E_t \pi_{t+1} + (1 - \beta_1) \pi_{t-1}$$

The real one-year interest rate is defined as the difference between the nominal one-year interest rate and expected inflation:

$$(A10) \quad rr_t^{1Y} = rs_t^{1Y} - \pi_t^e$$

To close the model, we introduce a policy interest rate reaction function, where the one-year nominal interest rate responds to the deviation of inflation from target and the output gap:

$$(A11) \quad rs_t^{1Y} = \alpha_1 rs_{t-1}^{1Y} + (1 - \alpha_1) [\bar{r}r_t^{1Y} + \pi_t^e + \alpha_2 (\pi_t - \pi_t^{Tar}) + \alpha_3 \hat{y}_t] + \epsilon_{rs^{1Y},t} - \alpha_4 \epsilon_{\pi^{Tar},t}$$

The equilibrium real interest rate is modeled as a slow-moving autoregressive process that reverts to its long-run steady-state level of  $\bar{r}r^{SS}$ .

$$(A12) \quad rr_t^{1Y} = \bar{r}r_t^{1Y} + \hat{r}r_t^{1Y}$$

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<sup>4</sup> There has been much work suggesting that the slope of the Phillips curve relationship ( $\beta$ ) has flattened; see for example Chapter 3 of the April 2013 World Economic Outlook). However, other studies suggest that it may have steepened in some countries in recent years (Riggi and Venditti, 2014). Although the methodology in this paper does not allow for time variation in parameter estimates, modest changes in the estimated value of the parameter  $\beta$ , on its own, do not materially change the estimates of potential output and the output gap.

$$(A13) \quad \bar{r}r_t^{1Y} = \rho^{\bar{r}r^{1Y}} \bar{r}r_{t-1}^{1Y} + (1 - \rho^{\bar{r}r^{1Y}}) \bar{r}r^{SS} + \epsilon_{\bar{r}r^{1Y},t}$$

The model allows for longer-term bond yields to shed light on the estimates of the equilibrium real interest rate. Based on the expectations theory of the term structure, the interest rate on 10-year government bonds is modeled as the expected sum of future short-term interest rates 10 years into the future, plus a term premium.

$$(A14) \quad rS_t^{10Y} = \frac{\sum_{i=t}^{t+9} rS_i^{1Y}}{10} + \sigma_t^{Term} + \epsilon_{rS^{10Y}}$$

$$(A15) \quad \sigma_t^{Term} = \rho^{\sigma^{Term}} \sigma_{t-1}^{Term} + (1 - \rho^{\sigma^{Term}}) \sigma^{Term,SS} + \epsilon_{\sigma^{term},t}$$

Equations describing the evolution of unemployment provide further identifying information for the estimation of the output gap:

$$(A16) \quad \hat{u}_t = \bar{u}_t - u_t$$

$$(A17) \quad \bar{u}_t = (1 - \rho_{\bar{u}}) \bar{u}_{t-1} + \rho_{\bar{u}} u^{SS} + g_{\bar{u},t} - \xi * \mathit{movavg}(\hat{y}_{t+3}, -6) + \epsilon_{\bar{u},t}$$

$$(A18) \quad g_{\bar{u},t} = \rho_{g_{\bar{u}}} g_{\bar{u},t-1} + \epsilon_{g_{\bar{u},t}}$$

$$(A19) \quad \hat{u}_t = \rho_{\hat{u}} \hat{u}_{t-1} + \tau \hat{y} + \epsilon_{\hat{u},t}$$

Here,  $\bar{u}_t$  is the equilibrium value of the unemployment rate (the NAIRU), which is time varying, and subject to shocks ( $\epsilon_{\bar{u},t}$ ) and to variation in its trend ( $g_{\bar{u},t}$ ), which is itself also subject to shocks ( $\epsilon_{g_{\bar{u},t}}$ ) — this specification allows for long-lasting deviations of the NAIRU from its steady-state value. Finally, the NAIRU also depends on moving average of output gap, as discussed in the paper, which is meant to capture the labor market hysteresis effects of demand shocks.

Most importantly, equation (19) specifies an Okun's law relationship wherein the gap between actual unemployment and its equilibrium rate (given by  $\hat{u}_t$ ) is a function of the output gap ( $\hat{y}_t$ ).

Finally, we incorporate information from measures of capacity utilization rates in the manufacturing sector to help shed some light on the overall slack in the entire economy at a given point in time.

$$(A20) \quad \hat{c}_t = c_t - \bar{c}_t$$

$$(A21) \quad \bar{c}_t = (1 - \delta_2) \bar{c}_{t-1} + \delta_2 \bar{c}^{SS} + g_{\bar{c},t} + \epsilon_{\bar{c},t}$$

$$(A22) \quad g_{\bar{c},t} = (1 - \delta_1)g_{\bar{c},t-1} + \epsilon_{g_{\bar{c},t}}$$

$$(A23) \quad \hat{c}_t = \kappa \hat{y}_t + \epsilon_{\hat{c},t}$$

In the above,  $\bar{c}_t$  is the equilibrium value of the capacity utilization rate, which changes over time, and is subject to shocks ( $\epsilon_{\bar{c},t}$ ). The equilibrium capacity utilization rate grows at  $g_{\bar{c},t}$ , which is itself also subject to shocks ( $\epsilon_{g_{\bar{c},t}}$ ), with their impact fading gradually according to the parameter  $\delta_2$ . This specification allows for permanent movements in the equilibrium capacity utilization rate. The capacity utilization gap, which is meant to capture the economic slack in the manufacturing sector, should be correlated with the measure of the overall economic slack in the economy ( $\hat{y}_t$ ).

Equations (A1)-(A23) comprise the core of the model for the output gap and potential output. In addition, data on growth and inflation expectations are added to help identify shocks, and to improve the accuracy of the estimates at the end of the sample:

$$(A24) \quad \pi_{t+j}^c = \pi_{t+j} + \epsilon_{\pi^c,t+j}, j = 5$$

$$(A25) \quad g_{t+j}^c = g_{t+j} + \epsilon_{g^c,t}, j = 1, \dots, 5$$

For real GDP growth ( $g$ ), the model is augmented with forecasts from Consensus Economics for five years following the end of any particular sample of historical observations. For inflation, expectations data are added for 5-year-ahead inflation whenever such survey data is available. These equations relate the model-consistent forward expectation for growth and inflation ( $\pi_{t+j}$  and  $g_{t+j}$ ) to observable data on how various forecasts expect these variables to evolve over various horizons (one to five years ahead) at any given time ( $g_{t+j}^c$  and  $\pi_{t+j}^c$ ). The ‘strength’ of the relationship between the survey data and the model’s forward expectation is determined by the standard deviation of the error terms ( $\epsilon_{\pi^c,t+j}$  and  $\epsilon_{g^c,t}$ ). In practice, setting non-zero variance of these terms allows consensus data to influence, but not completely override, the model’s expectations, particularly at the end of the sample period. In a way, the incorporation of survey data can be thought as a heuristic approach to blending forecasts from different sources and methods. The resulting impact of this information on the historical estimates of potential and the output gap can be significant.

### B. Micro-foundations for the Effects of NAIRU on Potential Output

The part of the equation linking the NAIRU (which is the opposite of potential level of employment) and potential output can be derived from a firm’s cost minimization problem. For this purpose we assume that firms produce output using a constant returns to scale Cobb-Douglas production function and face adjustment costs when changing the level of capital used in production. The production function takes the form:

$$(A26) \quad \bar{Y}_t = A_t K_t^\alpha L_t^{1-\alpha}$$

where  $\bar{Y}_t$  is potential output,  $K_t$  is the stock of capital level and  $L_t$  is the potential level of employment; while  $\alpha$  is the parameter that governs the share of capital in production.

Given this, firms minimize costs, which include labor wage costs  $W_t L_t$ , capital rental costs  $r_t^k K_t$ , and capital adjustment costs that are assumed to be proportional to rental costs. The adjustment costs, following the literature using quadratic adjustment costs (e.g. Benes and others, 2014), take the form:

$$(A27) \quad r_t^k K_t \frac{1}{2} c \left( \frac{K_t}{K_{t-1}} - 1 \right)^2$$

where  $c$  is the adjustment cost parameter.

Finally, we assume that labor and capital markets are competitive, so firms take wages and rental rate of capital as given. In this setting, a firm's cost minimization problem is:

$$\min_{L_{t+i}, K_{t+i}} E_t \sum_{i=0}^{\infty} \beta^i \left( W_{t+i} L_{t+i} + r_{t+i}^k K_{t+i} \left[ 1 + \frac{1}{2} c \left( \frac{K_{t+i}}{K_{t+i-1}} - 1 \right)^2 \right] + \lambda_{t+i} (\bar{Y}_{t+i} - A_{t+i} K_{t+i}^\alpha L_{t+i}^{1-\alpha}) \right)$$

where  $E_t$  is the expectation operator (with the time- $t$  information set),  $\beta$  is the discount factor and  $\lambda_t$  is the Lagrange multiplier.

The first order conditions (FOCs) of the optimization problem are:

$$(A28) \quad \mathbf{L}_t: W_t - \lambda_t (1 - \alpha) \frac{\bar{Y}_t}{L_t} = 0$$

$$(A29) \quad \mathbf{K}_t: r_t^k \left[ 1 + \frac{1}{2} c \left( \frac{K_t}{K_{t-1}} - 1 \right)^2 \right] + r_t^k K_t c \left( \frac{K_t}{K_{t-1}} - 1 \right) \frac{1}{K_{t-1}} + \beta c E_t \left[ r_{t+1}^k K_{t+1} \left( \frac{K_{t+1}}{K_t} - 1 \right) \left( -\frac{K_{t+1}}{K_t^2} \right) \right] - \lambda_t \alpha \frac{\bar{Y}_t}{K_t} = 0$$

After little rearrangement and substitution, these two FOCs give the following equation:



$$(A30) \quad \frac{r_t^k K_t \left[ 1 + \frac{1}{2} c \left( \frac{K_t}{K_{t-1}} - 1 \right)^2 \right] + r_t^k K_t c \left( \frac{K_t}{K_{t-1}} - 1 \right) \frac{K_t}{K_{t-1}} - \beta c E_t \left[ r_{t+1}^k K_{t+1} \left( \frac{K_{t+1}}{K_t} - 1 \right) \frac{K_{t+1}}{K_t} \right]}{\alpha} - \frac{W_t L_t}{1-\alpha} = 0$$

This equation shows how capital evolves given the level of employment. If there were no adjustment costs ( $c = 0$ ) then this equation would boil down to the familiar expression:

$$(A31) \quad \frac{r_t^k K_t}{W_t L_t} = \frac{\alpha}{1-\alpha}$$

which says that capital-to-labor ratio is constant and proportional to the production function parameter  $\alpha$ . However, when there are capital adjustment costs, it takes some time before capital approaches the level given in equation (A31).

In order to see how much time capital would need to go to the new steady state, given by (A31), after a persistent change in the potential level of employment, we can linearize the equation (A30) and solve the differential equation. The linearized version of this equation is:

$$(A32) \quad \hat{k}_t = \frac{c}{c+\beta c+1} \hat{k}_{t-1} + \frac{\beta c}{c+\beta c+1} E_t \hat{k}_{t+1} + \frac{1}{c+\beta c+1} \hat{l}_t$$

where  $\hat{k}_t$  and  $\hat{l}_t$  are the deviations of capital and labor from their initial steady-states.

Note that without adjustment costs  $\hat{k}_t = \hat{l}_t$  at all times; that is, the change in capital is instantaneous and the same size as the change in employment, again reflecting equation (A31).

To solve equation (A32) using the method of undetermined coefficients (Christiano, 2002), posit the solution:

$$(A33) \quad \hat{k}_t = \rho \hat{k}_{t-1} + \eta \hat{l}_t$$

Substituting this into (A32) and restricting the combination of coefficients in front of  $\hat{k}_{t-1}$  to zero yields the following quadratic equation:

$$(A34) \quad \beta c \rho^2 + (c + \beta c + 1) \rho + c = 0$$

for which the solution is:

$$(A35) \quad \rho = \frac{c+\beta c+1 \pm \sqrt{(c+\beta c+1)^2-4\beta c^2}}{2\beta c}$$

After ruling out the explosive solution, it is easy to check that  $\rho$  is strictly increasing in  $c$  (for positive values of the discount factor  $\beta$ ). In other words, higher adjustment costs imply that capital adjusts to the new steady state more slowly, as expected. It is also easy the check that the coefficient in front of labor in the solved equation is  $\eta = 1 - \rho$ . This is so, because the sum of all the coefficients in equation (A32), equals one.

Finally, in order to see how output reacts to a persistent change in employment, both through the direct effect of labor (which would be contemporaneous) and indirect effect of capital (which would be spread over time due to capital adjustment costs), we can express capital in the following way:

$$(A36) \quad K_t = \rho K_{t-1} + (1 - \rho) \frac{K}{L} L_t = \sum_{i=0}^{\infty} \rho^i (1 - \rho) \frac{K}{L} L_{t-i}$$

and output (using the production function) in the following way:

$$(A37) \quad \bar{Y}_t = A_t [K(L_t, L_{t-1}, \dots)]^\alpha L_t^{1-\alpha}$$

and differentiate the latter with respect to  $L_t$ .

Then the first-year effect of a persistent change in employment on output is:

$$(A38) \quad \frac{\partial \bar{Y}_t}{\partial L_t} = A_t [K(L_t, L_{t-1}, \dots)]^\alpha (1 - \alpha) L_t^{-\alpha} + A_t \alpha [K(L_t, L_{t-1}, \dots)]^{\alpha-1} (1 - \rho) \frac{K}{L} L_t^{1-\alpha} = (1 - \alpha) \frac{\bar{Y}_t}{L_t} + \alpha (1 - \rho) \frac{K}{L} \frac{\bar{Y}_t}{K_t}$$

while the indirect effects from second year on ( $i = 1, 2, \dots$ ) are:

$$(A39) \quad \frac{\partial \bar{Y}_{t+i}}{\partial L_t} = A_{t+i} \alpha [K(L_{t+i}, L_{t+i-1}, \dots)]^{\alpha-1} \rho^i (1 - \rho) \frac{K}{L} L_{t+i}^{1-\alpha} = \alpha \rho^i (1 - \rho) \frac{K}{L} \frac{\bar{Y}_{t+i}}{K_{t+i}}$$

From equation (A36), for sufficiently large  $i$   $\frac{K}{L} \frac{1}{K_{t+i}} \approx \frac{1}{L_t}$  and by multiplying (A38) and (A39) by  $\frac{\bar{Y}_t}{L_t}$  we arrive at the final results, which show the elasticity of output with respect to employment, including both direct (current year) and indirect (second year on) effects:

$$(A40) \quad \frac{\partial \bar{Y}_t / \bar{Y}_t}{\partial L_t / L_t} = (1 - \alpha) + \alpha (1 - \rho)$$

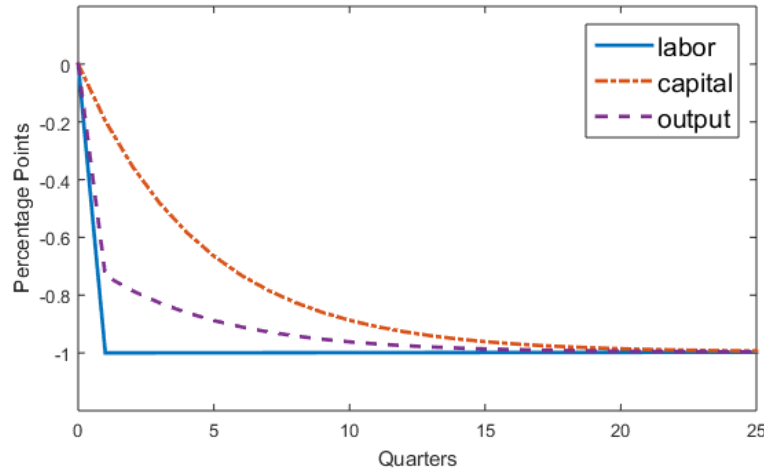
$$(A41) \quad \frac{\partial \bar{Y}_{t+i} / \bar{Y}_{t+i}}{\partial L_t / L_t} = \alpha \rho^i (1 - \rho) \quad \text{for } i = 1, 2, \dots$$

The sum of these two, direct and indirect, effects converge to one as  $i$  increases. Hence, in the long run any change in NAIRU affects potential output one-for-one.

We calibrate the discount factor at  $\beta = 0.99$ ,<sup>5</sup> the labor share parameter at  $\alpha = 2/3$  and capital adjustment cost parameter at  $c = 20$ . The value of the adjustment cost parameter seems to be broadly in line with the literature on models that use this kind of adjustment cost (e.g. Benes and others, 2014). Setting  $c = 20$  implies that a 1 percent increase in the capital stock would result in additional 0.1 percent in the cost of capital services due to adjustment costs (hence the total cost of the investment would be roughly 1.1 percent). Reasonable changes to this calibration do not alter the results significantly.

We simulate a 1 percentage point permanent decrease in potential employment (i.e. an increase in NAIRU) to see how fast capital and potential output react. The result is shown in Figure B2. The bulk of output adjustment (roughly 70 percent) occurs in the first year, and much of the rest happens during the next 4 years.

**Figure B2. Effect of Permanent Change in Employment on Output and Capital**



Source: Authors' estimates.

<sup>5</sup> This implies a 4 percent real interest rate in the steady-state. While some recent estimates show that natural real interest rate have declined (e.g. Holston, Laubach and Williams, 2017), calibrating the value used here has been standard for DSGE models (e.g. Smets and Wouters, 2003). Small changes to the calibration of discount factor have only a negligible effect on final results.