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Monetary Policy and Models of Currency Demand

by Mariam El Hamiani Khatat

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Monetary Policy and Models of Currency Demand

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Abstract

Two types of currency in circulation models are identified: (1) a first generation derived from the theory of money demand and (2) a second generation aimed at producing daily forecasts of currency in circulation. In this paper, we transform the currency demand function into a VAR to capture the dynamic link between interest rates and the demand for cash. We also apply ARIMA modeling to forecast the daily currency in circulation for Brazil, Kazakhstan, Morocco, New Zealand, and Sudan. Our empirical work shows that some of the conclusions in the economic literature on the impact of interest rates on the demand for currency do not necessarily hold, and that central banks would benefit from running both generations of currency in circulation models. The fundamental longer-run determinants of the demand for cash are distinct from its short-run determinants.

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Glossary

AR	Auto-Regressive
ARIMA	Auto-Regressive Integrated Moving Average
ATM	Automated Teller Machine
BAM	Bank Al-Maghrib
BCB	Banco Central do Brasil
BoE	Bank of England
CIC	Currency in Circulation
CNB	Czech National Bank
COPOM	Banco Central do Brasil Monetary Policy Committee
CPI	Consumer Price Index
ECB	European Central Bank
EUR	Euro
FX	Foreign Exchange
GDP	Gross Domestic Product
IFS	International Financial Statistics
IMF	International Monetary Fund
IT	Inflation Targeting
KASE	Kazakhstan Stock Exchange
MA	Moving Average
NBK	National Bank of Kazakhstan
NCB	National Central Bank
NFA	Net Foreign Assets
NZD	New Zealand Dollar
OCR	Official Cash Rate
OLS	Ordinary Least Squares
OMO	Open Market Operation
R	Bank Reserves
RBNZ	Reserve Bank of New Zealand
REPO	Repurchase Agreement Operation
RR	Reserve Requirement
SELIC	Sistema Especial de Liquidação e de Custódia
SLPB	Structural Liquidity Position of the Banking System
STS	Structural Time Series
TONIA	Tenge Overnight Index Average
USD	US Dollar
VAR	Vector Autoregression
VECM	Vector Error Correction Model
YOY	Year-on-Year

I. INTRODUCTION

A number of central banks of advanced, emerging, and developing countries forecast the daily change of their currency in circulation (CIC) to calibrate the volume of their monetary operations. Yet not all central banks use models to forecast the daily CIC, and many of them still rely on expert judgment. Modeling the daily CIC improves the quality of central banks' liquidity forecasts. At the same time, central banks use several CIC models for the purpose of monetary policy, but also for their currency issuance activities. CIC forecasts over the short run rely on daily data, and on the longer run on lower frequency data (for example, monthly, or quarterly). Models based on daily frequency time series are typically used to calibrate the volume of central banks' Open Market Operations (OMOs), conducted on a regular basis. Accurate short-term (daily) forecasts help properly anticipate short-term liquidity shocks and stabilize money market rates, thereby fostering the development of money markets and strengthening monetary transmission. However, even when moving toward a more active liquidity management, it is important to retain lower frequency data models to project the central bank balance sheet over horizons beyond the short-term (from a few months to few years).

The fundamental long-run determinants of the demand for cash are distinct from its very short-term determinants. In the short run (that is, on a daily basis), the demand for currency appears to be mostly affected by recurring seasonal factors such as weekends, payroll dates, and holidays. Over the longer run, the main determinants of the demand for cash include—among others—economic activity, inflation, the interest rate, financial crises, innovations in the payment systems as well as the exchange rate. While drivers of banking system liquidity may differ from one economy to another, the short-run determinants of the demand for cash display significant similarities across countries.

The drivers of banking system liquidity depend on: (1) the country's monetary and foreign exchange (FX) policy; (2) the magnitude of capital flows and current account shocks; (3) the capacity of the central bank to efficiently manage banking system liquidity; (4) the nature of the government budget financing and effectiveness of the government cash management; (5) the size of other items of central banks' balance sheets; (6) the structure of the financial system; and (7) the magnitude of the lender-of-last-resort operations. Yet during specific periods of time, in particular the post-global financial crisis period, many emerging and developing countries have displayed similar patterns, characterized by reversals of overall liquidity conditions, in response to capital flow reversals or current account shocks.

In such context, distinguishing liquidity shocks stemming from short-term fluctuations of autonomous factors[2] from those induced by large and persistent exogenous shocks or internal

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[2] Autonomous factors (or autonomous liquidity factors) are components of the central bank balance sheet other than its monetary policy operations and banks reserves. They are called autonomous because they are outside the control of the central bank (Cabrero, Camba-Mendez, Hirsh, and Nieto (2002)), and the main role of the liquidity management is often to offset the liquidity shocks induced by autonomous factors fluctuations.

idiosyncrasies can help improve the conduct of monetary policy, regardless of the monetary policy and exchange rate regime in place.

The outstanding amount of currency issued by the central bank—that is, the CIC—is typically one of the main determinants of the structural liquidity position of the banking system (SLPB),[3] together with the central bank's net foreign assets (NFA) and the net deposits of the government at the central bank. While central banks have the exclusive right to issue banknotes and coins, changes in the CIC are usually driven by the demand of economic agents and are therefore usually considered an autonomous liquidity factor. Nonetheless, in a few cases, some central banks have attempted to control or restrict the supply of cash to the economy. CIC is usually, on trend, a liquidity-absorbing factor since the issuance of CIC is mirrored by a reduction in bank reserves.[4] However, changes in the trend of the CIC may occur in times of crisis. In cash-based economies, CIC swings are expected to have a stronger liquidity effect. Nonetheless, CIC can also become the autonomous factor with the most important liquidity effect in economies with flexible exchange rates, low levels of NFA, and low and stable government deposits at the central bank.

This paper provides a framework for modeling and forecasting the daily CIC. For this, it reviews the CIC models used by central banks and identifies two types: (1) a first generation based on the theory of transaction and portfolio demand for money, and (2) a second generation developed by central banks to produce daily CIC forecasts in the context of the management of banking system liquidity. Though the focus of the paper is more on the second generation of models, it emphasizes the need to continue to use the first generation of models. In this paper, we transform the currency demand function in a vector autoregression (VAR) to capture the dynamic link between interest rates (that is, the policy rate) and the demand for cash in Brazil, Kazakhstan, Morocco, and New Zealand. We also apply auto-regressive integrated moving average (ARIMA) modeling to daily CIC for the same countries as well as Sudan. The remainder of the paper is structured as follows: section II discusses the main determinants of the demand for cash; section III introduces the models for CIC forecasts; section IV provides an empirical assessment of the reaction of the CIC to monetary policy shocks; section V presents an application of ARIMA models for Brazil, Kazakhstan, Morocco, New Zealand, and Sudan; and section VI concludes.

II. The Determination of the Demand for Cash

A number of factors can potentially affect the demand for cash,[5] not only economic and financial, but also political, cultural, and technological. Income, prices, and interest rates—

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[3] The SLPB is defined in Appendix I.

[4] Cabrero, Camba-Mendez, Hirsh, and Nieto (2002).

[5] In this paper, "currency demand" or "demand for cash" generally refers to the stock of the CIC in the liability side of the central bank's balance sheet. In some cases, banknotes in circulation are used as a proxy for CIC.

that proxy the opportunity cost of holding cash—have been traditionally and early identified as the main drivers of the demand for cash. Lower interest rates reduce the opportunity cost of holding currency, therefore increasing the relative attractiveness of cash. Economic growth tends to boost the transaction demand for currency. In many countries, the CIC usually displays a growing trend reflecting the expansion of the economic activity. Recessions and crises affect this trend, especially when depositors are concerned by the liquidity or solvency of some depository institutions. The public perception of the banking system's overall soundness influences the demand for cash, and a history of banking crises can durably undermine the confidence in banks and the perception of the relative safety of banks' accounts in comparison to cash.[6]

During the early phase of the global financial crisis, a slowdown in the US dollar CIC growth rate was observed, followed by a spike in 2009 and again in 2011. A surge in euro banknotes in circulation also followed the bankruptcy of Lehman Brothers and the worsening of the crisis. The insolvency of Lehman Brothers in autumn 2008 and the subsequent loss of confidence in banks' soundness led to sizeable cash withdrawals: euro banknotes served as a safe haven then. In October 2008 about half of the net withdrawals of EUR 35 billion went to regions outside the euro area. The opposite occurred during the debt crises in Greece and Cyprus: mistrust in the stability of the euro led to a decrease in international demand for euro currency between April 2010 and March 2013 (Figure 1).[7]

Technological change in the area of payment systems also impacts CIC trends in several, and not always linear, ways. The intuition that an increase of automated teller machines (ATMs) occurring simultaneously with innovation in payment systems would result in the reduction in currency demand is not always valid. For instance, a rise in the number of bank branches or ATMs tends to reduce the need for holding cash, but also makes cash more easily accessible,[8] and an increase in the number of ATMs does not always alter consumers' preference for holding cash.[9] Hence, factors such as the availability or popularity of alternative stores-of-value or payment methods, as well as consumers' preference for holding cash instead of payment alternatives, influence the CIC.

The way the central bank decides on how it breaks down the overall CIC can, in turn, affect the demand for cash, both domestically and externally. In May 2010 currency exchange offices in the UK were banned from selling EUR 500 banknotes because of their alleged use

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[6] Della Valle and others, forthcoming.

[7] Mersch (2014), https://www.ecb.europa.eu/press/key/date/2014/html/sp140519.en.html.

[8] Gerst and Wilson (2011).

[9] In New Zealand, for example, the increase of ATMs in the 1990s had made cash more accessible to the public, and it is more likely that the overall demand for cash should have increased as a result of the technological innovation in the payment system rather than the contrary (Cassino, Misich, and Barry, 1997).

for money laundering.[10] On May 4, 2016, the Governing Council of the European Central Bank (ECB) decided to permanently stop producing the EUR 500 banknote and to exclude it from the Europa series, taking into account concerns that this banknote could facilitate illicit activities. The issuance of the EUR 500 will be stopped around the end of 2018, when the introduction of the EUR 100 and EUR 200 banknotes is planned. In view of the international role of the euro and the widespread trust in its banknotes, the EUR 500 will remain legal tender and can therefore continue to be used as a means of payment and store of value.[11]

When modeling and forecasting the overall CIC in the context of monetary policy activities, the central bank aims to assess the total demand for cash, but the way it then chooses the relevant denominations and coin-note boundary can also affect the public's subsequent preference. Therefore, central banks often maintain CIC models for different types of banknotes in their cash-issuance-related activities. However, from a monetary policy implementation perspective, it is the overall demand for cash that is considered as an autonomous factor, and that constitutes the focus of this paper.

Currency substitution can increase the volatility of currency demand as a result of shifts between domestic and foreign currency. These shifts can be driven by political instabilities or exchange rate expectations, as revealed by the experiences of countries such as Kazakhstan, Nigeria, Surinam, and Ukraine. In Kazakhstan, devaluation expectation episodes led to shifts from domestic to FX currency, inverting the positive trend of the CIC (Figure 1). In Nigeria, CIC is also sensitive to the exchange rate due to currency substitution in times of high inflation and inflation expectations.[12] More generally, political instability, corruption, and weak confidence in countries' institutions and banking sector, as well as a sizeable informal economy, can induce a significant increase in the demand for cash. Currency substitution can also affect the country that issues the foreign currency. However, the effect on the country that issues the parallel currency is generally smaller than the effect on the country that uses the parallel currency, since only a small fraction of its currency is affected.[13]

In most countries, the CIC displays relatively stable and strong seasonal patterns. The intra-weekly, intra-monthly, and intra-yearly seasonalities reflect regularities in payments and receipts (payrolls, pensions, and so on), as well as patterns in the consumption behavior associated with holidays. Usually, the CIC tends to increase just before the weekend and decreases after (intra-weekly seasonality). It increases as a result of salary payments (intra-monthly seasonality). It also rises during holidays and toward the end of the year (intra-annual seasonality). Common calendar effects associated with holidays include those related

[10] Hellerstein and Ryan (2011).

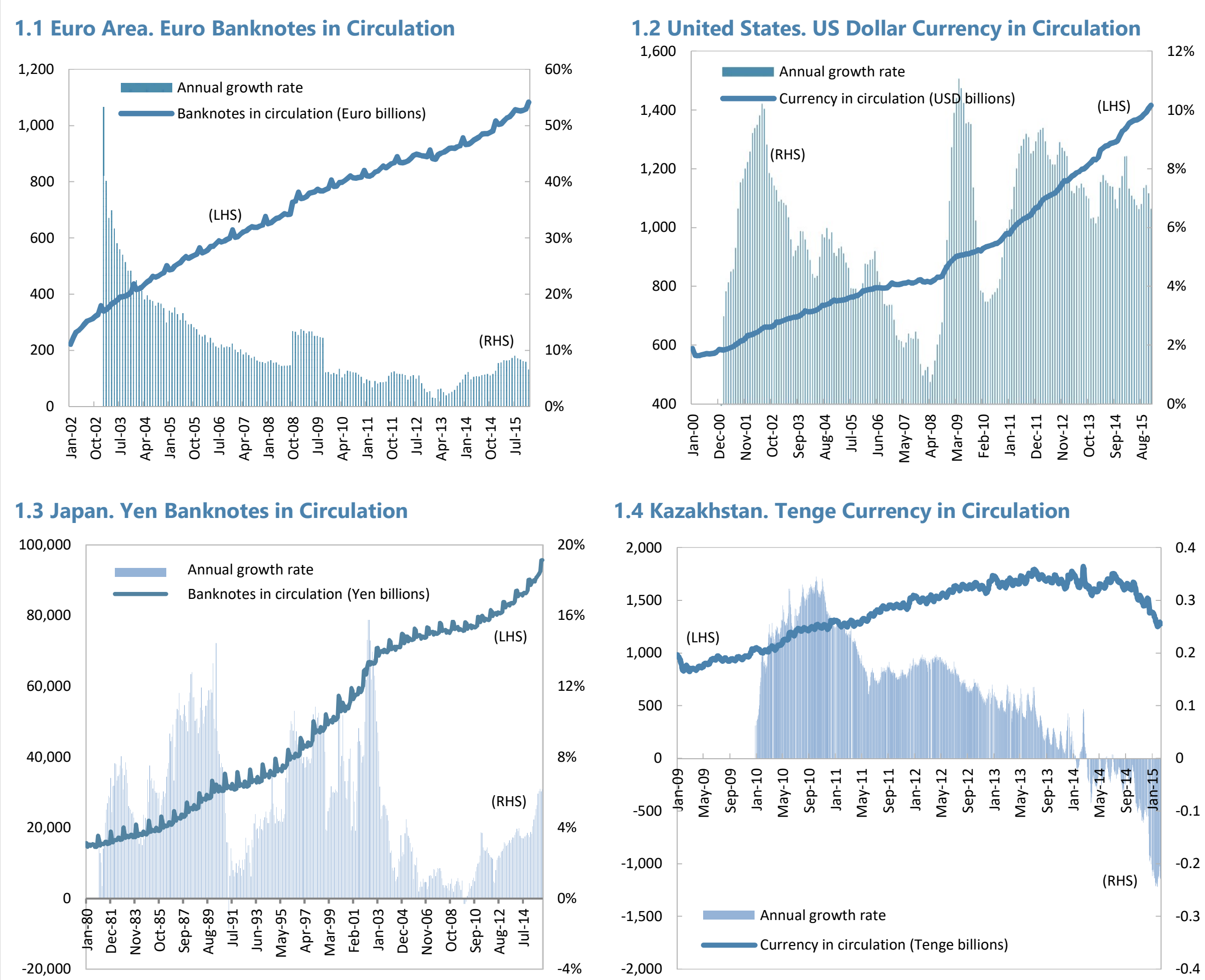
[11] https://www.ecb.europa.eu/press/pr/date/2016/html/pr160504.en.html.

[12] Ikoku (2014).

[13] Schaechter (2000).

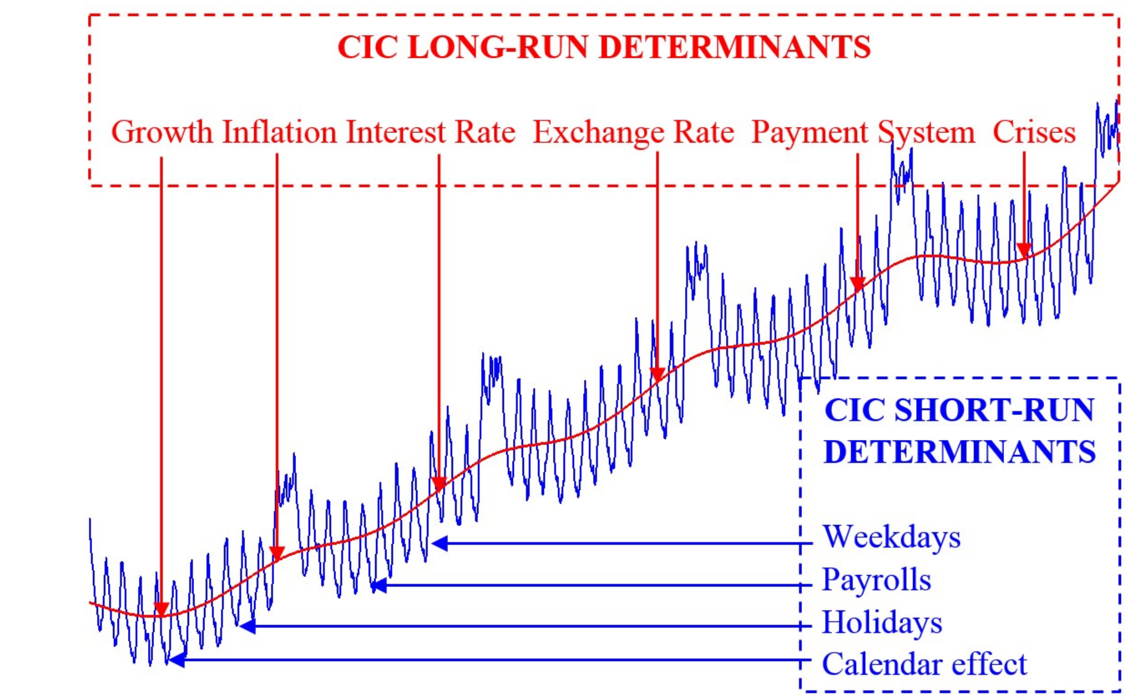
to Christmas and the Gregorian New Year. However, other specific calendar effects exist in countries that do not strictly follow the Gregorian calendar, or where CIC is affected by specific religious holidays attached to other calendars such as the Muslim, Hindu, and Buddhist calendars. Specific calendar effects include, for example, those of the two main Muslim religious holidays: Eid Al-Adha and Eid Al-Fitr (Figure 15). In countries where tourism accounts for an important share of the economic activity a particularly significant increase in the CIC occurs during the summer holidays. Considering the different seasonal patterns and multiple calendar effects, specific challenges arise when attempting to model the CIC on a daily basis. These challenges have not constrained central banks from developing sophisticated models to forecast the daily changes of the CIC. Figure 2 summarizes the main determinants of the demand for cash.

Figure 1. Currency in circulation in Normal and Crisis Times



Source: Central banks, central banks' websites, and author's calculations.

Figure 2. Stylized Determinants of the Currency in Circulation



Source: The author.

III. The Models for Currency in Circulation Forecasts

A. First Generation Models and the Theory of Demand for Money

First generation models are based on low frequency data; they rely on the theory of transaction and portfolio demand for money. According to the theory, economic agents tend to hold both cash and fixed income assets, mainly due to their desynchronized expenditures and receipts. The demand for cash departs from the need of transaction, but also depends on the opportunity cost of holding cash, that is, the opportunity cost of not storing the currency in interest-bearing assets. Higher interest rates incentivize economic agents to hold less cash. Therefore, economic agents tend to hold a mixed portfolio of cash and assets.[14] Two broad approaches have been used by central banks to model the CIC based on low frequency data:

- In the first approach, the demand for money or cash M^d in period t is a function of prices P, output Y, and the opportunity cost of holding cash R (for example, the interest rate):

$$M_t^d = f(P_t, Y_t, R_t) \qquad (1)$$

[14] Tobin (1956).

- In the second approach, the ARIMA technique is used. ARIMA models can be independent of any particular economic theory and combine two regression processes: an auto-regressive (AR) process, which assumes that the dependent variable is a function of its own past values, and a moving average (MA) process, allowing the inclusion of persistent random shocks.

A comparison of the two approaches for New Zealand indicates that the first approach outperforms the second one in sample, while the second outperforms the first out of sample.[15]

More sophisticated first generation models expand traditional models of currency demand to include other variables such as dummy variables that account for financial crises and other specific, isolated events, as well as payment system variables (number of ATMs, bank branches, and so on). However, this type of model usually aims to identify the main determinants of the currency demand rather than produce forecasts for the purpose of monetary policy implementation. Cusbert and Rohling (2013) estimated such a model for Australia. The authors found that the increase in currency holdings in late 2008 was substantially larger than what can be attributed to the normal response to the interest rate cuts and the fiscal stimulus: around 20 percent of the increase can be attributed to the normal response of currency holdings to the decrease of interest rates and the increase in income following the fiscal stimulus; the remaining 80 percent was potentially for precautionary holdings in response to the financial turmoil. Gerst and Wilson (2011) also provided a detailed analytical framework for the demand for Federal Reserve cash services; they focused on the long-term outlook for cash demand that entails different empirical approaches than those used for short-term forecasting.

A central bank's reliance on OMOs when moving toward more active liquidity management requires daily monitoring and forecasting of autonomous factors, hence the CIC, based on high-frequency (daily) time series, which are typically not available for macroeconomic variables such as output and inflation. Even when the central bank conducts its main OMOs or manages liquidity on weekly frequency, the daily modeling allows a closer monitoring of the liquidity situation, as the central bank may need to use fine-tuning operations if the daily monitoring indicates a significant forecasting error or warns of an extraordinary liquidity shock.[16] When attempting to model the daily CIC, other specific challenges can arise, such as the intra-monthly and intra-weekly seasonal patterns. Yet, when moving toward a more interest-rate-based monetary policy, it is important to retain CIC models based on lower frequencies time series since daily frequency models may fail to provide reliable forecasts over horizons beyond a few days. Therefore, forecasting CIC over quarters or years may require models based on similar frequencies. Because central banks can absorb and provide

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[15] Cassino, Misich, and Barry (1997).

[16] Cabrero, Camba-Mendez, Hirsh, and Nieto (2002).

liquidity on the short run and the longer run, it is important to retain models based on lower frequency, as they help forecast the SLPB and project the central bank balance sheet over longer maturities.

The most recent context of negative interest rates raises additional questions about reliance on high-frequency data models only, and may revive the interest in the first generation type of models. To this end, we transform the currency demand function in a VAR form including the same variables. Simple macroeconomic models that take the form of an ordinary least square (OLS) regression can be based on assumptions that do not capture all the explanatory variables or do not take into account the significance of variables lags or consider certain variables as exogenous. Therefore, the dynamic interactions between variables is usually insufficiently represented in simple regression models and the conclusions from such models incomplete. The general form of the VAR estimated in this paper is presented in Box 1. All the variables included are considered endogenous. For the estimations, we follow similar order of variables as suggested in Christiano, Eichenbaum, and Charles (2005): gross domestic product (GDP), inflation, interest rate, and the CIC. We use the cases of Brazil, Kazakhstan, Morocco, and New Zealand to empirically explore the interest rate-CIC relation. VAR estimations and results are presented in Section IV.[17]

Box 1. VAR Structure of the Currency Demand Function

Matrix Structure:

$$\begin{bmatrix} y_t \\ \pi_t \\ i_t \\ m_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\pi \\ c_i \\ c_m \end{bmatrix} + \begin{bmatrix} a^1_{y,1} a^1_{y,2} a^1_{y,3} a^1_{y,4} \\ a^1_{\pi,1} a^1_{\pi,2} a^1_{\pi,3} a^1_{\pi,4} \\ a^1_{i,1} a^1_{i,2} a^1_{i,3} a^1_{i,4} \\ a^1_{m,1} a^1_{m,2} a^1_{m,3} a^1_{m,4} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \\ m_{t-1} \end{bmatrix} + \ldots + \begin{bmatrix} a^p_{y,1} a^p_{y,2} a^p_{y,3} a^p_{y,4} \\ a^p_{\pi,1} a^p_{\pi,2} a^p_{\pi,3} a^p_{\pi,4} \\ a^p_{i,1} a^p_{i,2} a^p_{i,3} a^p_{i,4} \\ a^p_{m,1} a^p_{m,2} a^p_{m,3} a^p_{m,4} \end{bmatrix} \begin{bmatrix} y_{t-p} \\ \pi_{t-p} \\ i_{t-p} \\ m_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{\pi,t} \\ \varepsilon_{i,t} \\ \varepsilon_{m,t} \end{bmatrix}$$

(2)

Where:

y_t	is the year-on-year (yoy) change in nominal GDP at time *t*
π_t	is the yoy inflation at time *t*
i_t	is the central bank main policy rate at time *t*
m_t	is the yoy change of the CIC at time *t*
c_v	is the constant of the regression of the dependent variable *v*
$a^l_{v,j}$	are the regression coefficients
$\varepsilon_{v,t}$	is the error term of the dependent variable *v* at time *t*

[17] Section IV only discusses the CIC response to monetary policy shocks and not inflation response to an increase in the policy rate.

B. Second Generation Models in the Context of Liquidity Forecasting

Models based on daily frequency time series of CIC are used to forecast the daily fluctuations of bank reserves over the maturity of the central bank standard OMOs as well as over the reserve maintenance period. Time series models have been widely used by central banks, with many of them having similar characteristics. To the best of our knowledge, ARIMA models including dummy variables to account for the different seasonal patterns and calendar effects seem to be one of the most common.[18] The ECB and the Bank of England (BoE) also use Structural Time Series (STS) models. Forecasts of the CIC can be obtained by combining the forecasts of different types of statistical models—for example, ARIMA and STS—as well as with expert judgment.

The ARIMA models developed by central banks combine a dummy variable regression with an ARIMA process, consistent with the view that the CIC is a random variable following a compound process with seasonal and stochastic components.[19] The deterministic component of the model describes the different seasonal patterns using several dummy variables, while the stochastic component is modeled using AR and MA processes. In addition, the use of trigonometric functions can help capturing some of the patterns, especially the intra-monthly deterministic pattern, and may help reduce the number of dummies used.[20] An example of the general structure of such ARIMA models is presented in Box 2. Rather than decomposing the time series in its deterministic and stochastic components, STS modeling consists of separating the stochastic trend, cycle, and seasonal components of a time series and specifying a structural equation for each component. The general structure of the STS model is provided in Appendix II.[21]

ARIMA models of CIC usually include a wide range of dummies as key components, but not all of the explanatory variables are kept for the forecast; only statistically significant variables are maintained. CIC ARIMA models should fulfill standard criteria of stationarity of the dependent variable, minimal serial correlation of residuals, and good in-sample

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[18] Among others, the following central banks rely on such models and/or have published papers using them: Bank Al-Maghrib (BAM), Banque de France, Central Bank of Nigeria, Central Bank of Sri Lanka, Croatian National Bank, Czech National Bank (CNB), European Central Bank (ECB), National Bank of Kazakhstan (NBK), National Bank of Poland, and Qatar Central Bank.

[19] Hlaváček and others (2005).

[20] An alternative specification of the intra-monthly effect takes the form of a trigonometric function which fits the deterministic seasonal patterns: $d_t = \sum_{j=1}^{p} (a_j \sin \frac{2\pi j m_t}{M_t} + b_j \cos \frac{2\pi j m_t}{M_t})$ where m_t stands for the day of the month and M_t is the total number of days of a given month. p defines the number of different frequencies used in modeling the intra-monthly effect and should be large enough for this variable to account for all the seasonality.

[21] For more details on STS models, see Cabrero and others (2002), Norat (2008), and Appendix II.

characteristics and forecasting properties. First order difference of the stock of the CIC is commonly used to stationarize the series.

A specification of ARIMA models with dummy variables was produced for the euro area and published in 2002. Cabrero and others (2002) applied two major statistical approaches for modeling the daily volume of banknotes in circulation in the euro area: the ARIMA approach of Bell and Hillmer (1982) and the STS model proposed by Harvey, Koopman, and Riani (1997).[22] The authors compared the performance of the two models as well as the aggregated forecasts computed by the National Central Banks (NCBs). Combinations of forecasts derived from the different models were also built. The authors found that overall the combination of statistical model forecasts outperforms the aggregated forecasts obtained from the NCBs. The ARIMA model has the best forecasting performance over horizons of five days, while the STS model performs better over one- to four-day horizons. Since 2006 the BoE also has used a STS model similar to one of the ECB to forecast the demand of banknotes in the UK (Norat, 2008).

Hlaváček and others (2005) applied both ARIMA and a new neural network model—the feedforward structured neural network, which has not been applied to time series analysis and forecasting before—to the CIC in the Czech Republic. The out-of-sample performance of the models developed by the CNB shows that both models outperform the previous forecasts based on expert knowledge. According to the authors, the neural network model is slightly more accurate than the ARIMA model, mainly due to its nonlinear characteristic that is supposed to fit the seasonal patterns of the CIC more accurately. However, comparison of predictive accuracy with the use of a Diebold-Mariano test[23] didn't show a statistically significant difference between the forecasts produced by these two models.

In 2006, Dheerasinghe modeled the CIC for Sri Lanka for daily, weekly, and monthly data using an ARIMA model combined with a regression on time in order to model the trend as well as the use of dummy variables. Lang and others (2008) applied ARIMA modeling with dummies to the daily currency outside banks in Croatia and found that the model outperforms the preexisting forecasts based on expert knowledge. Balli and Elsamadisy (2010) also used the ARIMA technique to model the CIC issued by Qatar Central Bank and found persistent significant effects of major religious events related to the Muslim calendar, that is, Eid Al-Adha and Eid Al-Fitr. Koziński and Świst (2015) applied the ARIMA model with dummy variables to forecast the CIC in Poland.

While forecasts obtained from ARIMA models are usually found to outperform those based on pure expert knowledge, the combination of ARIMA forecasts with expert judgment is warranted, especially around the periods of significant and unexpected change in the CIC.

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[22] Cabrero, Camba-Mendez, Hirsh, and Nieto (2002).

[23] Diebold and Mariano (1995).

Certain seasonal patterns or exceptional events may not be perfectly captured by the structure of the model. ARIMA models in particular fail to accurately predict exceptional events. No matter how good the forecasting performance of a model is, models may not substitute expert knowledge perfectly. They are intended to provide the forecaster with a baseline that needs to be assessed and complemented or corrected to ensure that the baseline forecast fits the forecaster's own knowledge of the CIC behavior at a given point in time. By incorporating forward-looking information into the forecasts, the forecaster's intervention has a value added.[24] Hence, changes in the CIC need to be adequately monitored and corrected ex-ante by expert judgment whenever deemed necessary and ex-post by re-estimations and adjustments of the CIC model. The statistical significance of the variables changes over time according to changes affecting the structure of the CIC, and a significant variable at a given point in time, can run out of explanatory power later. It is thus important to periodically re-estimate the model (at least once a year). Further, an update of the CIC data and forecast on a regular basis is necessary at least before the meeting of each committee that makes the decision on the volume of the central bank's main OMOs.

Box 2. Example of the Structure of the ARIMA Currency in Circulation Model

$$\Delta y_t = \sum_{i=1}^{5} \alpha_i D_{it} + \sum_{i=1}^{k} \beta_i W_{it} + \sum_{i=1}^{4} \delta_i P_{it} + \sum_{i=1}^{12} \gamma_i M_{it} + \sum_{i=1}^{l} \Theta_i(B) H_{it} + \Phi(B) S_t + \sum_{i=1}^{m} \eta_i O_{it} + \sum_{i=1}^{p} \mu_i \Delta y_{t-i} + \sum_{i=0}^{q} \theta_i \varepsilon_{t-i} \quad (3)$$

Where:

y_t	is the CIC at time *t*
D_{it}	is a dummy variable that takes the value of 1 if the day at time *t* is *i* (*i*=Monday,…,Friday) and 0 otherwise
W_{it}	is a dummy variable that takes the value of 1 during week *i* of the year and 0 otherwise
k	is the number of weeks in the year
P_{it}	is the dummy variable that reflects the position of the week within a specific month; it takes the value of 1 when the position of the week is *i* (*i*=1,…,4) in a given month and 0 otherwise
M_{it}	is the dummy variable that takes the value of 1 during month *i* of the year and 0 otherwise
B	is the standard backshift operator ($By_t=y_{t-1}$)
$\Theta_i(B)$	is a polynomial in variable B. The term $\Theta_i(B)H_{it}$ captures the change of the CIC around the holiday *i*
H_{it}	is the dummy variable that takes the value of 1 when it is a public holiday at time t and 0 otherwise
l	is the number of calendar variation effects
S_t	is the dummy variable that takes the value of 1 if salaries are paid at time *t* and 0 otherwise
$\Phi(B)$	is a polynomial in variable B. The term $\Phi(B)S_t$ captures the change of the CIC around the salary's payment day
O_{it}	are dummy variables controlling for the effect of outliers identified
ε_t	is an independent and identically distributed (*iid*) stochastic process with zero mean and a variance of σ^2

[24] Norat (2008).

IV. Currency Demand Responses to Monetary Policy Shocks: Country Experiences

According to the economic theory, the currency demand is expected to have an inverse relationship with the interest rate: it is expected to decrease/increase with the increase/decrease of interest rates. To explore this relation empirically, we estimate VARs for Brazil, Kazakhstan, Morocco, and New Zealand, including the same variables as the currency demand function. The VARs and results are presented in the following subsection. Due to the fully fledged Islamic banking system and unavailability of quarterly GDP data, VARs investigating CIC response to monetary policy shocks are not estimated for Sudan in this working paper.[25]

A. Brazil

Banco Central do Brasil (BCB) adopted an inflation-targeting (IT) framework in June 1999. The annual inflation targets are established by the National Monetary Council, composed of the Minister of Finance (chairman), the Governor of the BCB, and the Minister of Planning and Budget. The inflation target for 2016 was 4.5 percent with a tolerance range of 2 percentage points above and below the target. This tolerance range was reduced to 1.5 percentage points for 2017 and 2018.

The operational target of the BCB's monetary policy framework is the overnight secured interest rate, that is, the Selic rate. This is the weighted average of the interest rate charged on overnight repurchase agreement operations (repo) collateralized by public debt securities registered at *Sistema Especial de Liquidação e de Custódia* (Selic).[26] The policy rate—that is, the target for the Selic rate—is set by the BCB's Monetary Policy Committee, the COPOM (Figure 3). The COPOM holds eight regular meetings per year, each of which lasts two days (Tuesday and Wednesday). The operational framework of the BCB includes: (1) short-term repos and reverse repos (in general, overnight) for daily liquidity management; (2) medium-term reverse repos with maturities up to 45 days, between COPOM meetings; (3) long-term reverse repos (three and six months); and (4) outright purchase/sale of domestic government debt securities.

To explore the currency demand-interest rate relation in Brazil, we transform the currency demand function in a VAR including four endogenous variables: the yoy GDP growth (GDP_YOY), the yoy inflation (INFLATION_YOY), the policy rate (POLICY_RATE), and the yoy growth rate of the CIC (CIC_YOY). The VAR is estimated using quarterly data over the period 2000:Q1-2014:Q3. The impulse response of the CIC_YOY to a POLICY_RATE

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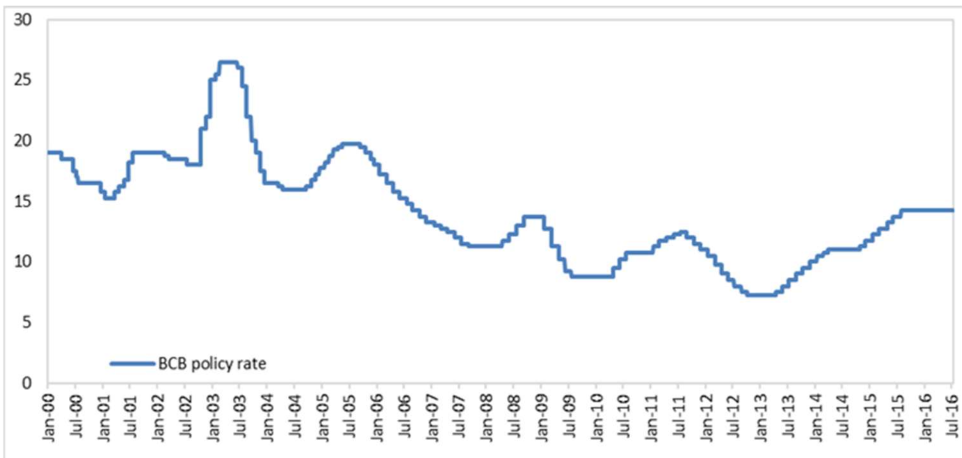
[25] Future researches for the case of Sudan could consider the Murabahah profit margin as a proxy for the opportunity cost of holding cash. In addition, substitution between cash and assets may operate differently in Sudan due to the limited availability of interest-bearing financial assets. Future researches could also discuss the best proxy for quarterly GDP in the absence of industrial production index.

[26] This weighted average includes the operations of the BCB in the repo market.

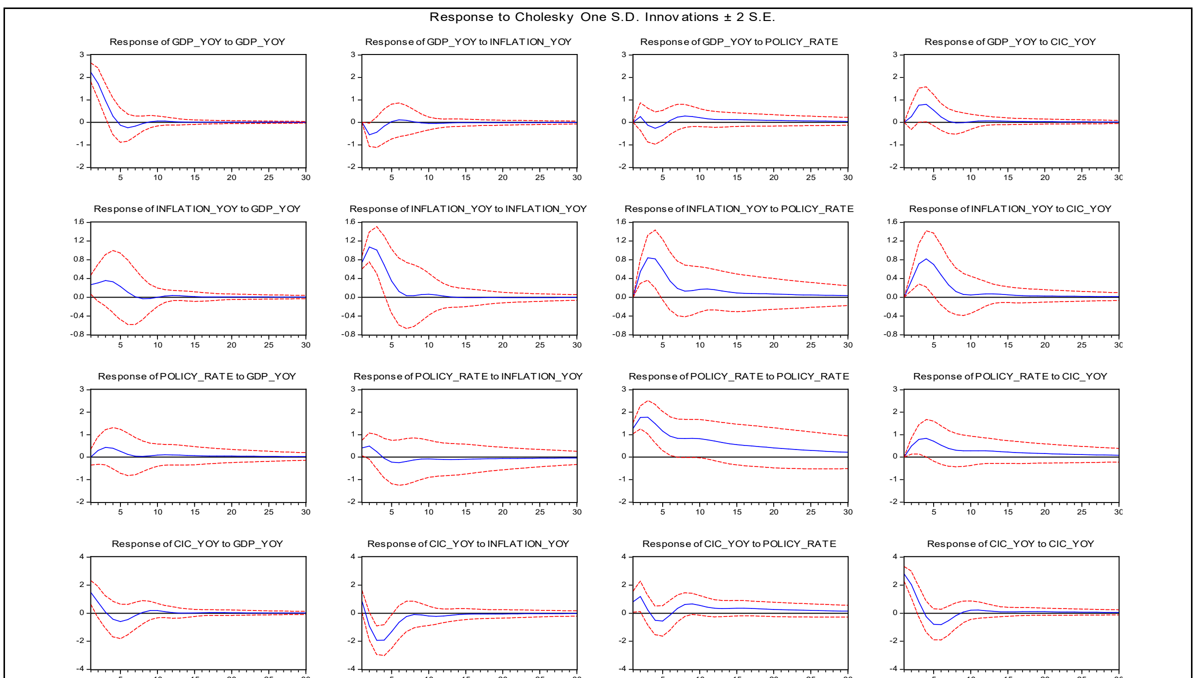
shows an increase of the growth rate of the CIC before it starts decreasing. The impulse responses also suggest that the growth rate of the CIC increases with GDP growth (Figure 4). However, the coefficients associated with the policy rate lags in the equation where the CIC_YOY is the dependent variable are not statistically significant.

Figure 3. Banco Central do Brasil Policy Rate (2000–16)



Source: BCB website and author's calculation.

Figure 4. Brazil—Reaction of the Currency in Circulation to the Policy Rate



Sources: IFS, BCB, and author's calculations.

B. Kazakhstan

The National Bank of Kazakhstan (NBK) has maintained a stabilized exchange rate against the US dollar during several years. However, following several exogenous shocks,[27] the NBK decided that an adjustment of the exchange rate regime and its overall monetary policy framework was desirable. As a first step, on July 15, 2015, the NBK widened the exchange rate band and, on August 20, 2015, announced the adoption of a floating exchange rate regime. Shortly after, in early September 2015, the NBK introduced a new operational framework composed of a new policy rate, the base rate. Before September 2015 the main policy rate of the NBK was the refinance rate, that is, the rate of its main refinancing operations. In February 2016, the NBK resumed the practice of using the base rate—that was suspended earlier—as the main tool of monetary policy. In order to meet the balance of risks, the base rate was set at 17 percent. When financial markets conditions started stabilizing and forecasts of macroeconomic indicators improving, the NBK started gradually lowering the base rate. In 2017, the base rate was decreased from 12 to 10.25 percent within a +/-100 basis point corridor width.

In order to maintain the Tenge Overnight Index Average (TONIA)[28] within the newly established corridor system, the NBK uses several liquidity injection and absorption instruments. The NBK operational framework comprises a standing credit facility (the Kazakhstan Stock Exchange [KASE] reverse repo from the NBK perspective), and a liquidity-providing OMO auction with a seven-day maturity (purchase of securities with reverse sale). In terms of liquidity-absorbing instruments, the NBK issues its own notes as main sterilization instruments in times of excess liquidity, but also makes use of seven-day deposits and overnight uncollateralized deposits as well as the KASE direct repo (Figure 5).

To explore the currency demand-interest rate relation in Kazakhstan, we estimated the same VAR including four endogenous variables: the yoy GDP growth (GDP_YOY), the yoy inflation (INFLATION_YOY), the policy rate (POLICY_RATE), and the yoy growth rate of the CIC (CIC_YOY). Over the estimation period, 2004:Q4–2015:Q1, the NBK refinance rate is used as the main policy rate. The introduction of the base rate in September 2015 does not provide sufficient data to assess its impact on the CIC. The impulse responses show no significant response of the growth rate of the CIC to an increase in the POLICY_RATE: the result is not unexpected since the refinance rate was targeted to other objectives than monetary policy.[29] The impulse responses also suggest that the growth rate of the CIC increases with GDP growth and decreases with inflation (Figure 6). Since the Engel-Granger tests indicate that the series included in the VAR are rather co-integrated, we also estimate a

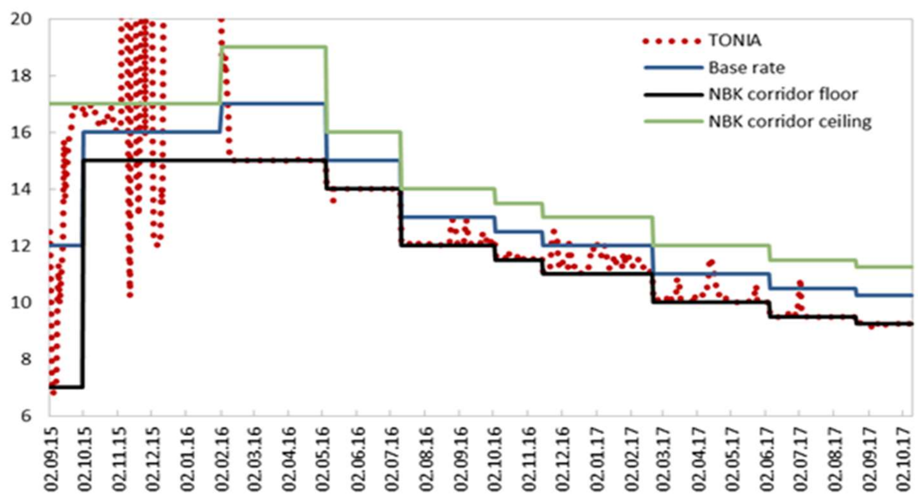
[27] These shocks included the oil price slump, the depreciation of the Russian ruble, and the slowdown in China.

[28] TONIA is the weighted average interest rate on one business day repo opening deals concluded on KASE.

[29] Since the NBK refinance rate was not reflective of monetary conditions during the period considered, other indicators, such as the nominal exchange rate could be tested in future researches.

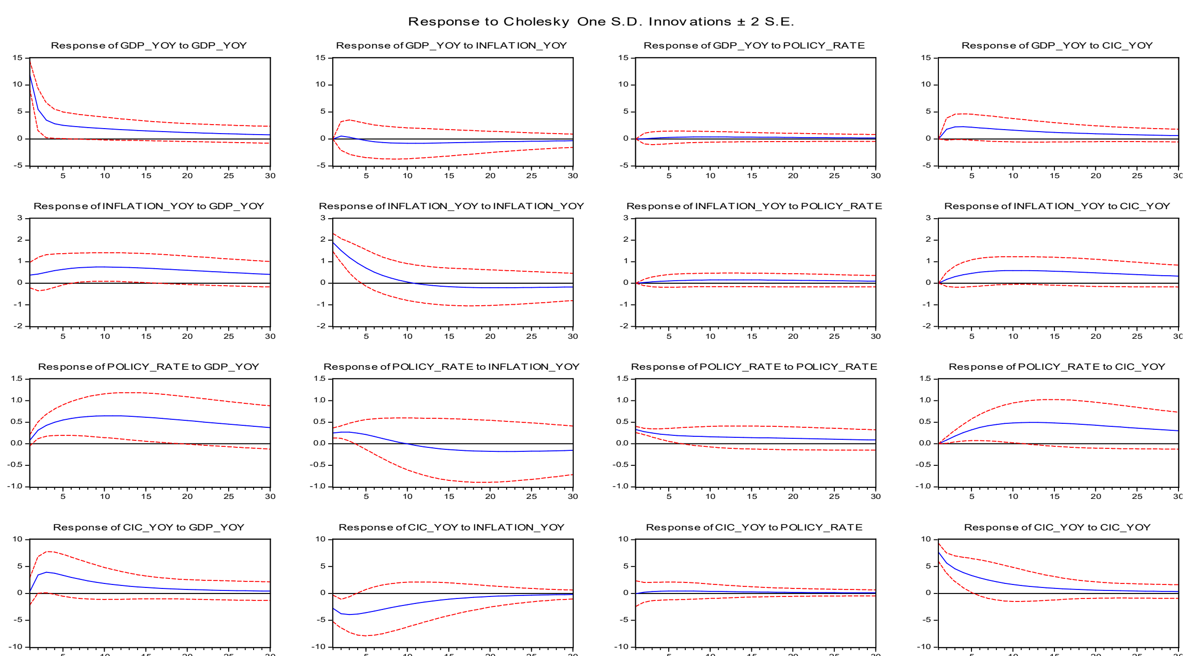
Vector Error Correction Model (VECM) with the same variables: the impulse responses of the VECM display similar results than the VAR, that is, no significant response of the growth rate of the CIC to an increase in the POLICY_RATE.

Figure 5. National Bank of Kazakhstan Introduction of Corridor System (2015–17)



Sources: NBK, KASE, and author's calculations.

Figure 6. Kazakhstan—Reaction of the Currency in Circulation to the Policy Rate



Sources: IFS, NBK, and author's calculations.

C. Morocco

Since 1973 Morocco has maintained a fixed exchange rate against a basket of currencies, reflecting the structure of Morocco's foreign trade. Since April 2015 the foreign currency weights of the Moroccan dirham currency basket have been 60 percent EUR and 40 percent USD. The composition and weights of the basket as well as the bands around the central parity are transparent and disclosed by Bank Al-Maghrib's (BAM). The currency basket allowed Morocco to enjoy several years of low and stable inflation and BAM to fulfill its price stability mandate while preparing for a move to further exchange rate flexibility.

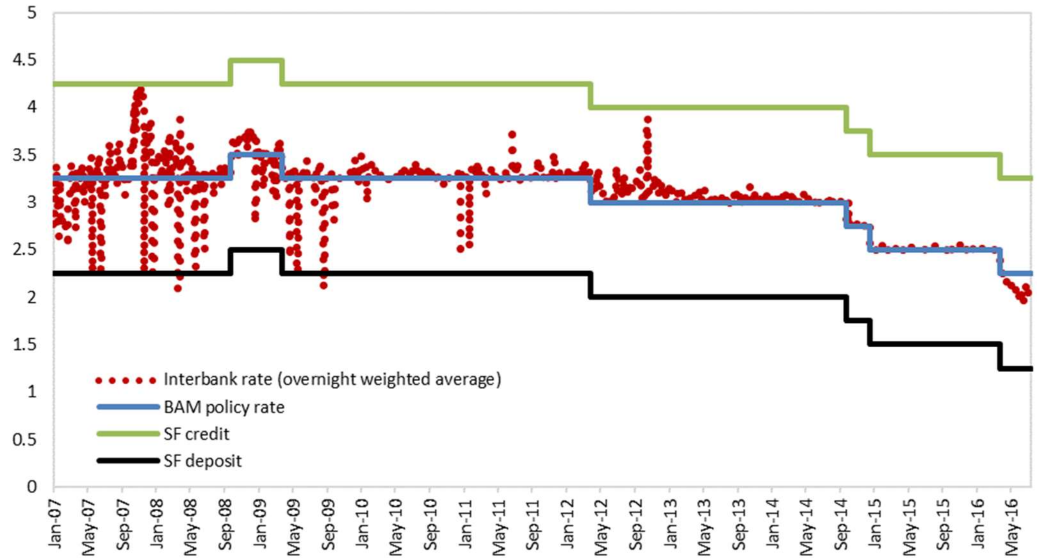
According to Article 6 of its Law, BAM's monetary policy objective is to ensure price stability. The Board of BAM meets quarterly to make monetary policy decisions. The Board sets BAM's main policy rate as well as the reserve requirement (RR) ratio and remuneration. Monetary policy decisions made by the Board are publicly communicated in a quarterly statement released after each meeting, and explained in a quarterly Monetary Policy Report. BAM's Board is composed of the Governor (chairman), the general manager of the central bank, the head of the Treasury Department, and six members appointed by the Prime Minister, three on the Governor's proposal. The head of the Treasury Department does not take part in monetary policy votes.[30]

In this context, BAM has successfully operated a corridor system since 2007 (Figure 7). BAM's corridor system aims to maintain the overnight transaction-based uncollateralized interbank market rate (weighted average) within a +/-100 basis points interest rate corridor. To achieve this objective, BAM uses mainly its seven-day liquidity-providing OMOs conducted as repo operations, as well as overnight standing credit and deposit facilities. BAM's operational framework also includes a full range of structural, long-term, and fine-tuning operations that are occasionally used when needed.

To explore the currency demand-interest rate relation in Morocco, we estimated the VAR including four endogenous variables: the yoy GDP growth (GDP_YOY), the yoy inflation (INFLATION_YOY), the policy rate (POLICY_RATE), and the yoy growth rate of the CIC (CIC_YOY). The estimation period is 2002:Q4–2015:Q1. The impulse responses show no significant reaction of the growth rate of the CIC to a policy rate increase (Figure 8).

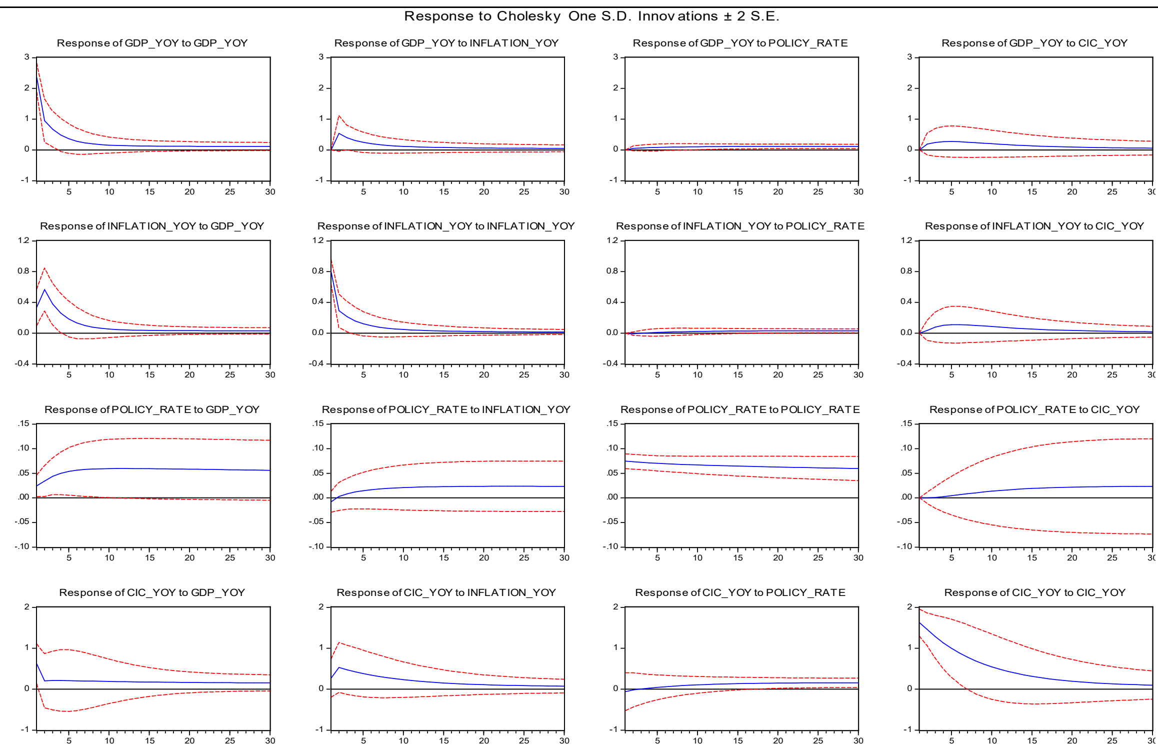
[30] BAM website.

Figure 7. Bank Al-Maghrib Corridor System and Interbank Rate (2007–16)



Source: BAM website and author's calculations

Figure 8. Morocco—Reaction of the Currency in Circulation to the Policy Rate



Sources: IFS, BAM website, and author's calculations.

D. New Zealand

By targeting a specific band for inflation, in 1989–90 New Zealand pioneered a new monetary policy regime: inflation targeting (IT). Since then, a number of central banks have adopted IT including those of Brazil, Canada, the UK, Norway, Poland, South Africa, Sweden, and Australia. The Reverse Bank of New Zealand (RBNZ) was given statutory authority to control inflation, provided for in Section 8 of its Act of 1989. The specifics were set out in a contract between the Governor of the RBNZ and the Minister of Finance, signed in 1990. This Policy Targets Agreement (PTA) initially called for a reduction of inflation to a 0–2 percent increase in the consumer price index (CPI) by 1992. A new PTA must be signed each time a Governor is appointed or reappointed, but a new PTA can also be written at other times. Since 1990 there have been a number of PTAs, and the target band has been revised several times as circumstances have changed. According to the RBNZ Act of 1989, the government has the power to override the PTA for a 12-month period. However, any override must be done publicly and transparently.[31] The current PTA, signed in September 2012, defines price stability as annual increases in the CPI of between 1–3 percent on average over the medium term, with a focus on keeping future average inflation near the 2 percent target midpoint.

Since March 1999 the RBNZ has implemented monetary policy by setting the official cash rate (OCR). The OCR is reviewed seven times a year by the RBNZ. Unscheduled adjustments to the OCR may occur at other times in response to unexpected developments. By setting the OCR, the RBNZ is able to influence the wholesale price of money and, via the linkages to the banking system and financial markets, influence a range of economic factors that help keep inflation under control.

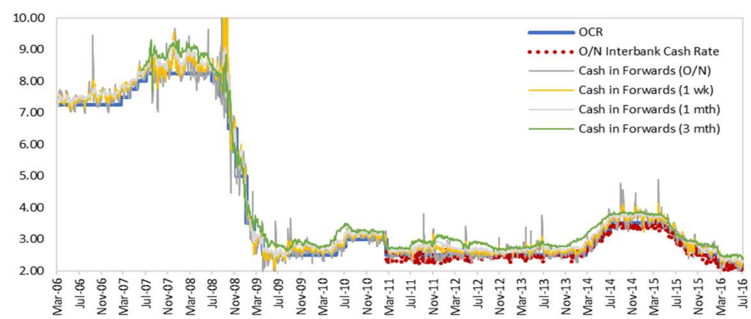
Currently, the RBNZ operates a floor system where the OCR, the key policy rate, is the rate associated to the overnight standing deposit facility (compared to OCR-25 basis points previously). In 2006, the RBNZ moved to a fully cashed-up payment system where the settlement cash level is set by the central bank from time to time. The level is driven by the desire to have short-term interest rates trading close to the OCR and the medium-term demand revealed by payment system participants. This new system was introduced with a target for bank reserves of New Zealand dollar (NZD) 7 billion, and currently reserves sit around NZD 7.6 billion. At the same time as the 2006 introduction of the new operational framework, the RBNZ increased the rate of its overnight standing credit facilities to the OCR+50 basis points (compared to OCR+25 basis points previously). The RBNZ also discontinued its intra-day liquidity facility at that time. The RBNZ predominantly uses FX swaps to manage the level of reserves in the banking system and also holds OMOs, as and

[31] RBNZ (2007).

when required, using RB bills and repurchase transactions. Since 2008, the RBNZ has accepted a wide range of acceptable securities for its repurchase transactions (Figure 9).[32]

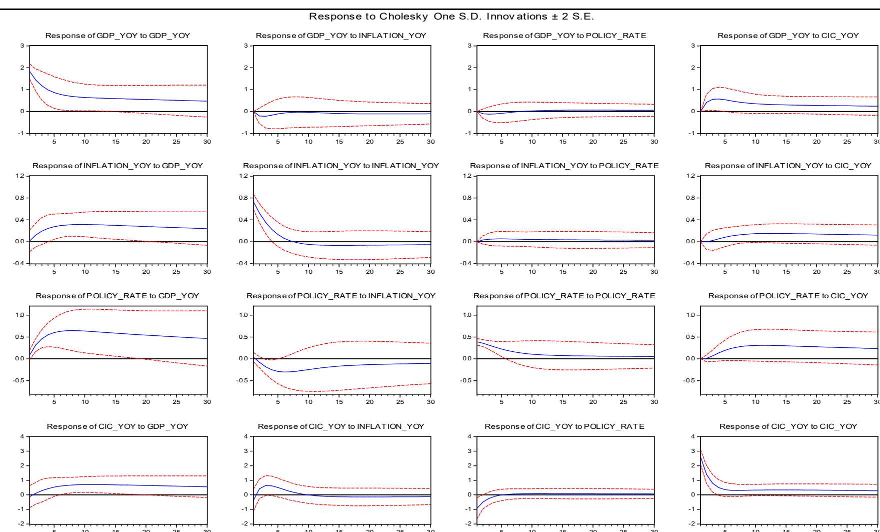
VAR estimations for New Zealand over the period 2000:Q2–2013:Q4 and impulse responses show a decrease of the growth rate of the CIC over five quarters following a monetary policy shock. However, the coefficient associated with the policy rate lag in the equation where the CIC_YOY is the dependent variable is not statistically significant. The impulse response also suggests that the growth rate of the CIC increases with GDP growth (Figure 10).

Figure 9. Reserve Bank of New Zealand Official Cash Rate and Selected Wholesale Funding Rates (2006–16)



Source: RBNZ.

Figure 10. New Zealand—Reaction of the Currency in Circulation to the Policy Rate



Sources: IFS, RBNZ website, and author's calculations.

[32] Nield (2006).

V. Modeling the Daily Currency in Circulation: Country Cases

Central banks conduct short-term liquidity forecasts whether their operating target is an interest rate, an exchange rate, or a quantity of money. In particular, accurate liquidity forecasting is a cornerstone of the process of monetary policy transition to an interest rate-based monetary policy, because it replaces quantitative targets by an active calibration of central bank monetary operations based on autonomous factor forecasts. Moving to a calibration of central bank monetary operations based on liquidity forecasts usually means targeting an aggregated banking system excess liquidity—above the reserve requirement (RR)—as low as possible on average over the reserve maintenance period on a forward-looking basis.

Upgrading central bank liquidity forecasting and operational frameworks should start earlier than the official adoption of an interest rate-based monetary policy. Stabilizing short-term fluctuations of money market rates is an operational fine-tuning process that is usually separated from setting or changing the level of the policy rate. In countries with fixed exchange rate regimes, money is primarily endogenous, and the central bank can lose part of its control over the size of its balance sheet and the growth of the monetary base—but it can still control the size of excess liquidity left in the banking system above the RRs. It can also decide to fully sterilize the surplus liquidity and offset the short-term fluctuations of autonomous factors. Money market rates move close to the policy rate when the market for bank reserves is balanced over the reserve maintenance period. The demand for banks' reserves is primarily linked to banks' need to meet their RR; but banks may also want to hold additional precautionary reserves over the minimum requirement. Large excess reserves held beyond the RR can put downward pressure on money market rates, while an insufficient level of reserves causes interest rates to rise.

Changes in bank reserves are directly derived from autonomous factors' forecasts, using the balance sheet of the central bank. Trends and forecasts of autonomous factors are country specific, in particular with regard to the extent to which NFA fluctuations and government operations affect bank reserves. In economies with flexible exchange rates, central bank FX operations usually do not cause important changes in bank reserves; however, large NFA fluctuations and FX interventions may cause important fluctuations in the SLPB. Government cash flows can also result in large liquidity shocks and induce higher funding cost volatility whenever government accounts at the central bank fluctuate significantly. Other specific components of the central bank balance sheet can also have a significant liquidity effect, such as stabilization funds, sovereign funds, and oil funds deposits.

Liquidity management requires daily forecasts, typically over the reserve maintenance period. However, liquidity forecasting does not rely only on models and also involves a well-designed framework including processes, organization, and coordination with the Department of Treasury and market operators. Liquidity forecasting features strong coordination, expert judgment, and market data monitoring components, and necessitates centralizing all the necessary information as well as the knowledge and the capacity in a dedicated unit of the central bank. Forecasting errors are common; addressing them needs not only the improvement of the central bank liquidity forecasting framework, but also a well-designed operational framework including the necessary set of monetary operations.[33]

In this section, we apply the same type of ARIMA modeling with dummy variables as the one used in Cabrero and others (2002) to the daily change of the CIC in five countries: Brazil, Kazakhstan, Morocco, New Zealand, and Sudan. The key findings are the following:

- In the five countries, the CIC displays marked seasonal patterns.

- There is a clear *trading day* effect on Fridays for Brazil, Kazakhstan, and Morocco, and on Thursdays for New Zealand. This reflects that the CIC increases just before the weekend and decreases after the weekend, at the beginning through the middle of the week (Figures 11 and 12).

- The CIC also increases with salary payments, which occur twice a month in Kazakhstan and New Zealand, and once a month in Brazil, Morocco, and Sudan (intra-monthly seasonality).

- Finally, the CIC rises during holidays periods and toward the end of the year (intra-annual seasonality).

To model the effect of the trading day, the intra-monthly behavior of the CIC related to payroll dates, as well as the holiday effects, several dummy variables are constructed and tested for each country following the ARIMA model structure presented in Box 2. These dummy variables are the following:

- *Variables related to the intra-weekly seasonality*. Five variables have been created to simulate the intra-weekly behavior of the CIC. These variables are *Monday, Tuesday, Wednesday, Thursday,* and *Friday.* Each of these variables takes the value of 1 on the correspondent day and 0 otherwise.

- *Variables related to the intra-monthly seasonality*. Four variables, that is, *Week1, Week2, Week3,* and *Week4,* are generated to take into account the patterns of the CIC, which usually increases during the first two weeks of the month and decreases in the second half of the month.

- *Variables related to the intra-yearly seasonality*. Two sets of dummy variables are used to simulate the intra-yearly behavior of the CIC. The first set includes *k*

[33] See Appendix I for further details on liquidity forecasting.

variables, *W1,…, Wk, k* being the number of weeks in the year. The second set of variables are *M1,…, M12,* each one taking the value of 1 during the corresponding month of the year and 0 otherwise.

- *Holiday dummy variables*. The dummy variable *H* takes the value of 1 on the dates corresponding to the country's holidays and 0 otherwise. Further, to take into account the fact that a holiday has a different effect on the CIC when it is a Monday, a Tuesday, a Wednesday, a Thursday, or a Friday, several other dummies have been generated. These variables are: *M1,…, Mi* when the holiday is a Monday; *T1,…, Tj* when the holiday is a Tuesday; *Wed1,…, Wedk* when the holiday is a Wednesday; *Th1,…, Thl* when the holiday is a Thursday; and *F1,…, Fm* when the holiday is a Friday.

- *Salary variables*. The dummy variable *S* as well as its lags and future values simulate the effect of payroll dates on the CIC. *S* takes the value 1 when salaries are paid at the corresponding day and 0 otherwise.

In addition to these dummies, specific variables have been created for each country to model significant calendar effects such as those related to Christmas for Brazil and New Zealand, as well as Easter for New Zealand, and Eid Al-Adha and Eid Al-Fitr for Morocco and Sudan. These dummy variables are detailed hereafter in each country's subsection.

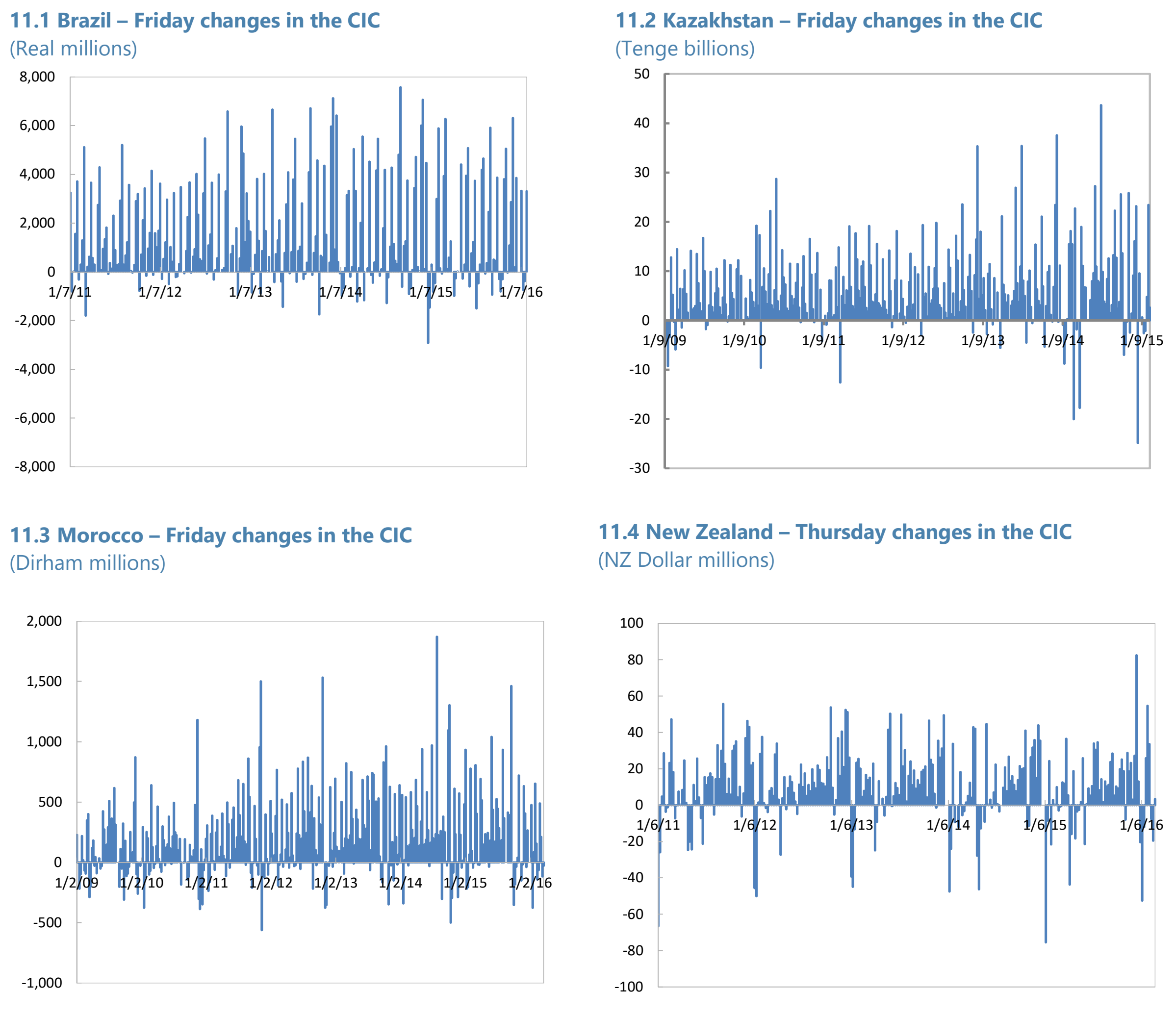
The ARIMA model developed for each country is estimated using OLS regression. The dependent variable is the daily change of the CIC that is stationary for the five countries.[34] CIC is not stationary and displays a trend for the five countries (Appendix III). The identification of the models is based on the significance of the parameters, the autocorrelation function and partial autocorrelation function, as well as the residuals diagnosis. Estimation results display adjusted R-squared that range from a maximum of 0.92 for Brazil and 0.71 for New Zealand, as shown in Table 1. However, the CIC model for New Zealand does not include the salary payment dummy, while in Brazil, the salary dummy and its past and future values are statistically significant.

Table 1. Currency in Circulation Models—R-Squared and Adjusted R-Squared

	R-squared	Adjusted R-squared
Brazil	0.927939	0.919730
Sudan	0.874755	0.862072
Morocco	0.805276	0.799527
Kazakhstan	0.748668	0.741218
New Zealand	0.726685	0.713336

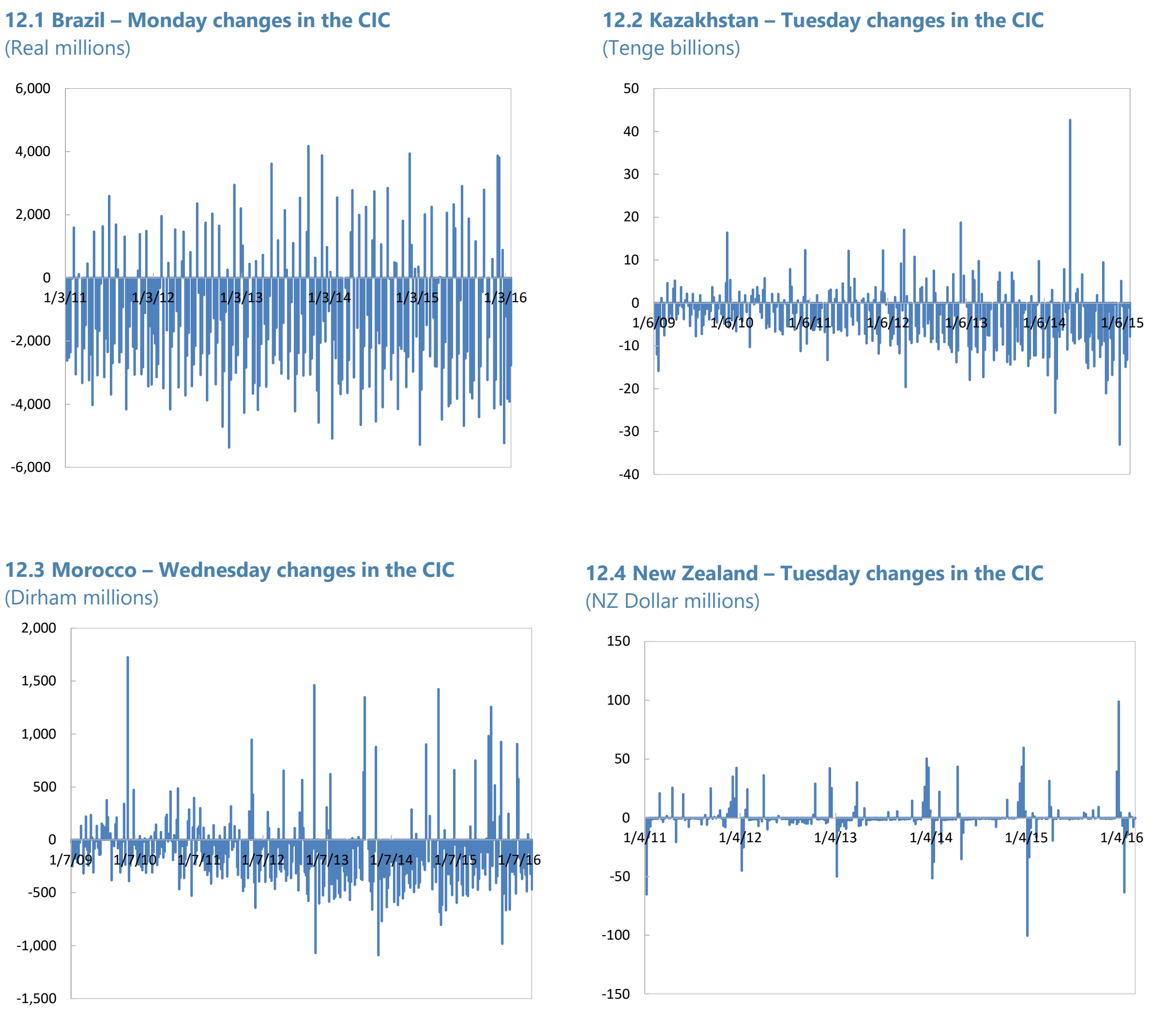
[34] Augmented Dickey-Fuller tests have been applied to test for the stationarity of the daily change of the CIC.

Figure 11. Trading Day Effect



Sources: Central banks and author's calculations.

Figure 12. Beginning to Midweek Decrease in the Currency in Circulation



Sources: Central banks and author's calculations.

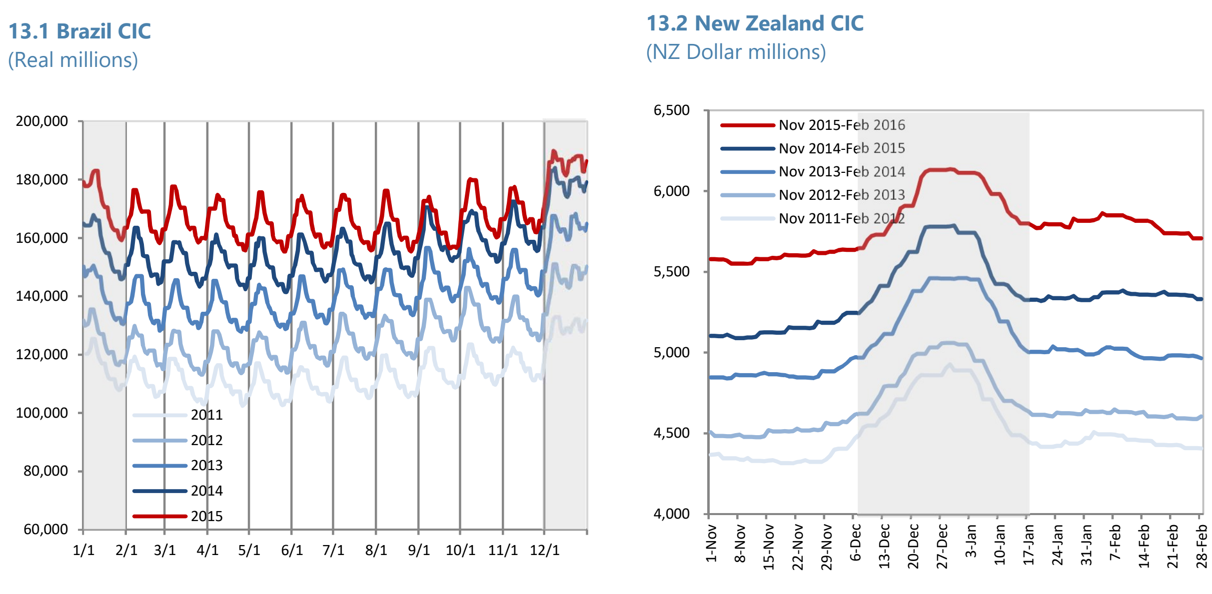
A. Brazil

In Brazil, the CIC displays a similar profile each year, as shown in Figure 13, with marked increases at the beginning of each month. This replicable profile over months and years, as well as the persistent effect of payrolls, resulted in the highest adjusted R-squared obtained among the five countries considered. In Brazil, salaries are usually paid in the first days of each month, up to the fifth day of each month. An important seasonal effect is related to the thirteen salary payment, usually paid in December.

The CIC model for Brazil has been estimated over the period 1/03/2011 to 1/29/2016. The stochastic structure of the model includes 22 AR terms and one MA term, as shown in the regression results (Appendix IV). The deterministic structure of the model includes intra-yearly dummy variables, holiday variables, and all the days of the week. Specific dummy variables have also been introduced for Christmas: these variables are *C_1, …, C_15* that take into account the increase in the CIC 15 days before Christmas, and *C1, …, C15* that model the decrease of the CIC after Christmas. However, only four of these variables turned out to be significant. In addition, several lags and future values of the salaries variable *S* have been introduced in the model to account for the persistence of the payroll effect.

Brazil's main national holidays include the Gregorian New Year (January 1), Carnival Day and day after (February), Good Friday (March), Tiradentes Day (April), Labor Day (May 1), Corpus Christi (May–June), Independence Day (September 7), Our Lady Aparecida (October 12), All Souls' Day (November 2), Republic Proclamation Day (November 15), and Christmas Day (December 25).The most important calendar effect is the cumulated effect of Christmas and New Year, which coincides with the payment of the thirteen salary. During this period, the CIC increases substantially (Figure 13).

Figure 13. Currency in Circulation in Brazil and New Zealand during Christmas and New Year



Sources: Central banks and author's calculations.

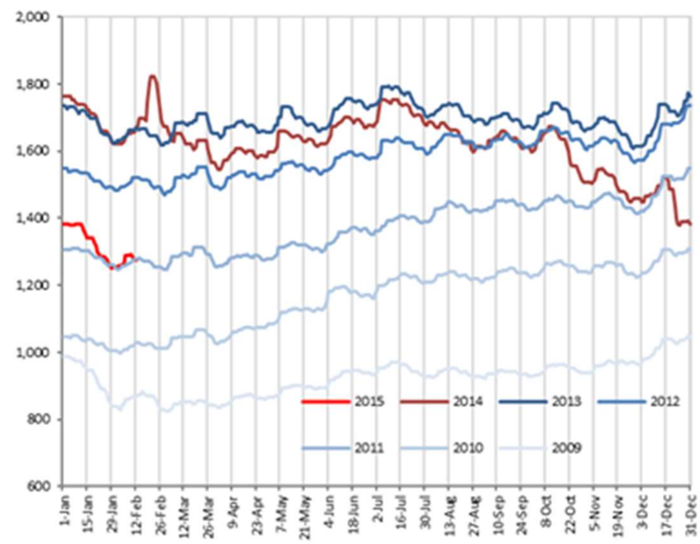
B. Kazakhstan

The ARIMA model for Kazakhstan has been estimated over the period 1/03/2011 to 12/31/2014. The stochastic structure of the CIC model includes 13 AR terms and one MA term, as shown in the regression results (Appendix V). The deterministic structure of the model includes intra-yearly dummy variables, holiday variables, and the trading day effect, FRIDAY, that turned out to be statistically significant, as well as the salary dummy variable.

Kazakhstan's main holidays include the Gregorian New Year (January 1), the Eastern Orthodox Christmas (January 7), Nowruz, the Persian New Year (March), Kazakhstan People's Unity Day (May 1), Defender of the Fatherland Day (May 7), Great Patriotic War Against Fascism Victory Day (May 9), Capital City Day (July 6), Constitution Day (August 30), Eid Al-Adha, First President Day (December 1), and Independence Day (December 16). Eid Al-Adha occurs the tenth day of Dul-Hijja month following the Muslim calendar; on the Gregorian calendar, the dates vary from year to year, drifting approximately 13 days earlier each year (Figure 14).

Figure 14. Kazakhstan Currency in Circulation Intra-Yearly Seasonality

(Tenge billions)



Sources: NBK and author's calculations.

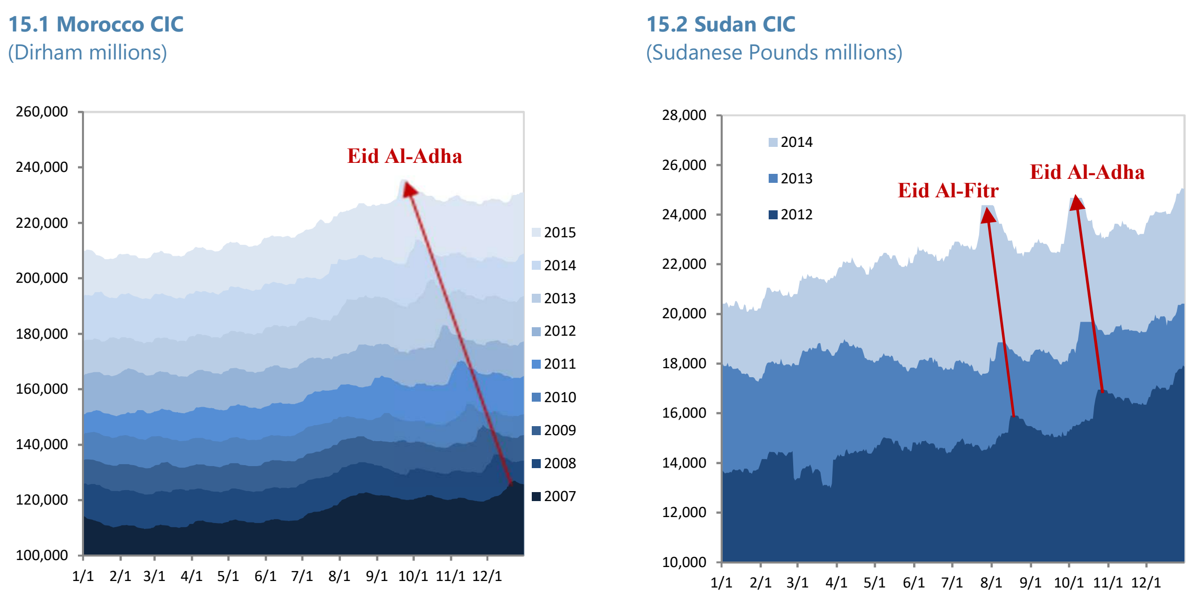
C. Morocco

Since 2007 Morocco has used an ARIMA model with dummy variables to forecast the daily changes in the CIC in the context of BAM liquidity management. For the purpose of homogeneity with the other country cases considered in this paper, the ARIMA model for Morocco is estimated over the period 1/03/2011 to 3/21/2016. The stochastic structure of the CIC model includes four AR terms and one MA term, as shown in the regression results. The deterministic structure of the model includes the holiday dummy variable *H,* the salary payment dummies *S* and *S1,* the intra-yearly dummy variables, the trading day FRIDAY, as well as the effects of MONDAY and WEDNESDAY, which are statistically significant. Specific dummy variables have also been introduced to account for the effects of the two major Muslim religious holidays, Eid Al-Adha and Eid Al-Fitr. The dummy variables related to Eid Al-Adha are: *AA_1, …, AA_10* that take into account the increase in the CIC 10 days before Eid Al-Adha, and *AA1, …, AA10* that model the decrease of the CIC after Eid Al-

Adha. Four dummies, *AF1,…, AF4,* have been included to simulate the CIC increase during Eid Al-Fitr (Appendix VI).

Morocco's main holidays include the Gregorian New Year (January 1), Anniversary of the Independence Manifesto (January 11), Labor Day (May 1), Feast of Throne (July 30), Recovery Oued Ed-Dahab (August 14), Revolution of the King and the People (August 20), Youth Day (August 21), Green March (November 6), and Independence Day (November 18). The holidays specific to the Muslim calendar are the Hijra New Year (Muharram, 1 on the Muslim calendar), Eid Al-Mawlid (Rabi'I, 12), Eid Al-Adha (Dul-Hijja, 10), and Eid Al-Fitr (Shawwal, 1). In the Gregorian calendar, the dates vary from year to year, drifting approximately 13 days earlier each year (Figure 15).

Figure 15. Morocco and Sudan Specific Calendar Effects



Sources: Central banks and author's calculations.

D. New Zealand

The CIC model for New Zealand has been estimated over the period 1/03/2011 to 2/29/2016. The stochastic structure of the model includes six AR terms and 10 MA terms, as shown in the regression results (Appendix VII). The deterministic structure of the model includes intra-yearly dummy variables, holiday variables, and all the days of the week except THURSDAY. As for Brazil, specific dummy variables have been introduced for Christmas (*C_1, …, C_15* and *C1, …, C15).* In addition, specific dummy variables have been also added to model the CIC increase during Easter (E_1, ..., E_10 and E1, …, E10).

New Zealand's main national holidays include the Gregorian New Year and the day after (January 1 and 2), Waitangi Day, Good Friday, Easter Monday, ANZAC Day, Queen's Birthday, Labour Day, Christmas, and Boxing Day. The most important calendar effect in New Zealand is the cumulated effect of Christmas and New Year, during which the CIC increases substantially (Figure 13.2 and appendix III).

E. Sudan

Sudan is an interesting case in the modeling and forecasting of the CIC, as it differs from the other countries both in terms of calendar and behavior of the CIC during holidays. Even though Morocco is a Muslim country where the CIC increases during Eid Al-Fitr and Eil Al-Adha, weeks in Morocco start on Monday with weekends on Saturday and Sunday, whereas in Sudan, the week starts on Sunday. In addition, during some Sudan official national holidays, a change in the CIC can be observed. That is not the case in the other four countries considered, where a zero-change in the CIC is usually the standard on official holiday dates. These specific features have not constrained the adaptation of the same type of ARIMA modeling to the CIC in Sudan, the adjusted R-squared of the model for Sudan being 0.86, as shown in Appendix VIII.

The CIC model for Sudan has been estimated over the period 1/01/2012 to 12/31/2014. The stochastic structure of the model includes 22 AR terms and one MA term. The deterministic structure of the model includes intra-yearly dummy variables and holiday variables as well as SUNDAY and WEDNESDAY. Specific dummy variables have also been introduced to account for the effects of the two major Muslim religious holidays, Eid Al-Adha and Eid Al-Fit. The dummy variable related to Eid Al-Adha are: *ADHA_1, …, ADHA_10* that take into account the increase of the CIC 10 days before Eid Al-Adha, and *ADHA1, …, ADHA10* that model the decrease of the CIC after Eid Al-Adha. Four dummies, *FITR_1,…, FITR_4,* have been included to simulate the CIC increase before Eid Al-Fitr (Appendix VIII).[35]

Sudan's main holidays include Independence Day (January 1), the Coptic Christmas (January 8), the Coptic Easter (May 1), and Revolution Day (June 30). The holidays specific to the Muslim calendar are the Hijra New Year (Muharram 1 on the Muslim calendar), Eid Al-Mawlid (Rabi'I, 12), Eid Al-Adha (Dul-Hijja, 10), and Eid Al-Fitr (Shawwal, 1). On the Gregorian calendar, the dates vary from year to year, drifting approximately 13 days earlier each year (Figure 10).

[35] Further research for Sudan could investigate how to better capture the effects of the two agricultural seasons (June-July and October-November) as well as the riot period of September 2013. The first riot period (July 2005) is outside the estimation period (1/01/2012 to 12/31/2014). Currency Change Overs in Sudan (April–August 2007 and July–October 2011) are also outside the estimation period.

VI. Conclusion

The following conclusions can be drawn from our work:

- *Long-term versus short-term determinants of the CIC*. When considering the determinants of the CIC, a clear distinction between the macroeconomic long-term determinants of the demand for cash and its short-term determinants is needed. Consequently, modeling the CIC over the short run and the longer run necessitates different frameworks and modeling techniques. On the macroeconomic level, income, prices, and interest rates have been identified by the economic theory as the main drivers of the demand for cash. However, our empirical work shows that higher interest rates do not necessarily decrease the relative attractiveness of cash.

- *Transmission of monetary policy shocks to CIC demand*. Our empirical work shows that the transmission of monetary policy shocks to the currency demand may be different from one country to another, as monetary transmission, more generally, differs from one country to another. Substitution between cash and interest-bearing assets depends on the availability of fixed income assets in the economy, the depth of bonds markets, and more generally the structure of the financial system.

- *Impact of non-macroeconomic variables*. When analyzing the transmission of central banks policy rate changes to the currency demand, it is also important to acknowledge that other non-macroeconomic variables such as the denomination and banknote boundary and the existence of secure ways of storage for cash, among other factors, may also affect the transmission of policy rate changes to the demand for cash.

- *CIC behavior over the short term.* While the transmission of monetary policy shocks to the CIC may differ from one country to another, CIC behavior over the short run displays relatively similar patterns under IT or prohibited interest rate environments. The main short-term determinants of currency demand include weekdays, payroll dates, holidays, and calendar effects. Central banks have mainly relied on ARIMA and STS techniques to model and forecast the daily CIC. An application of ARIMA models to the CIC in five countries—Brazil, Kazakhstan, Morocco, New Zealand, and Sudan—provided good performances, suggesting that they can be adapted to the daily change of the CIC under different monetary policy frameworks and macro-financial environments.

- *Benefits of modeling versus expert knowledge*. Forecasts derived from ARIMA models are usually found to outperform those based on pure expert knowledge. This suggests that central banks will gain in terms of accuracy of their liquidity forecasting by modeling the CIC. However, statistical models cannot perfectly substitute experts' judgment.

- *Forecasting the CIC over longer horizons*. No matter how sophisticated they are, ARIMA models using daily time series are generally not well suited to producing reliable forecasts over horizons longer than a few days; they are typically intended to calibrate central banks' main OMOs. Therefore, since central banks' monetary operations can be undertaken beyond short-term horizons, it is important to retain lower frequency first generation models, as they help not only forecast the SLPB over the longer run, but also quantify the effect of monetary policy shocks and other macroeconomic variables on the currency demand.

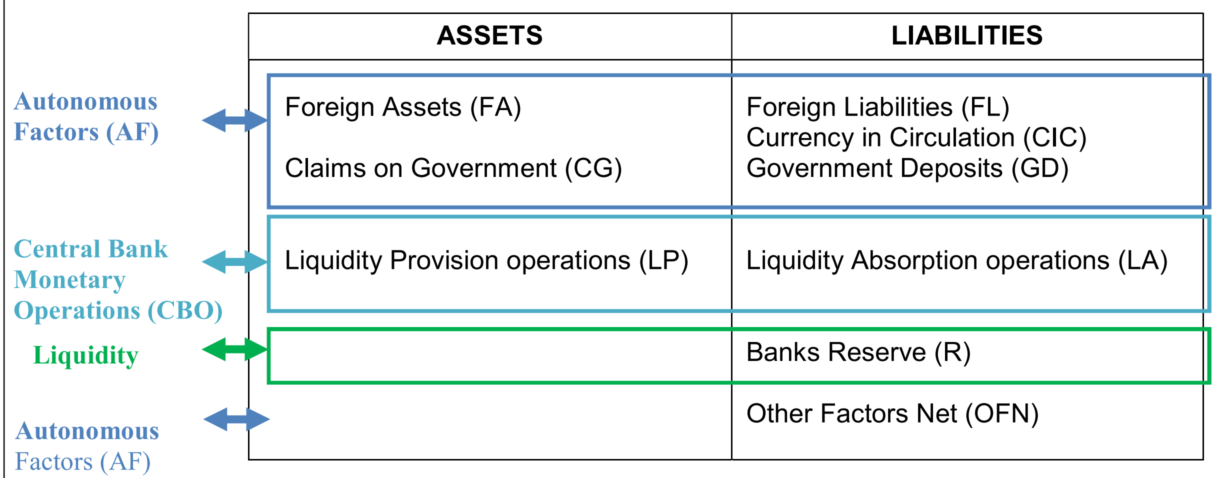
Appendix I. Liquidity Forecasting

Liquidity forecasting is the process of centralizing all relevant information that determines the future stance of the overall banking system liquidity, in order to calibrate the volume of the central bank liquidity injection or absorption operations. Hence, the main purpose of short-term liquidity forecasts is to create an information set of the projected evolution of autonomous factors that allows the central bank to smooth the future changes in liquidity conditions. Smoothing liquidity fluctuations creates stable liquidity conditions and helps the central bank to steer a benchmark short-term money market rate. These are preconditions of an effective conduct of monetary policy, since they help market participants to clearly distinguish between changes in the monetary policy stance and temporary "noises" in the liquidity conditions.[1]

The SLPB represents the amount of aggregated banking system liquidity (required and free reserves) before any liquidity injection or absorption has been undertaken by the central bank. Liquidity can be derived and forecasted, over the short run or the long run, using the other components of the central bank balance sheet. At a certain point in time, the SLPB is simply equivalent to bank reserves—RRs and excess reserves—minus central bank net liquidity injections. Once the SLPB has been observed or forecasted, the other component used to assess the level of bank liquidity are the RRs that create an additional need for central bank money (Figure 16).

[1] Schaechter (2000).

Figure 1. Banking System Liquidity and the Central Bank Balance Sheet



ASSETS = LIABILITIES

$FA + CG + LP = FL + CIC + GD + LA + R + OFN$
$NFA = FA – FL \Rightarrow NFA + CG + LP = CIC + GD + LA + R + OFN$
$R = NFA + CG + LP – CIC – GD – LA – OFN$
$R = (NFA + CG – CIC – GD – OFN) + (LP – LA)$
$R = AF + CBO$
$R – CBO = AF = SLPB$

Appendix II. General Structure of the Structural Time Series Model

$$y_t = \mu_t + \gamma_t + \psi_t + \sum_{i=1}^{m} \eta_i O_{it} + \varepsilon_t$$

$$\mu_t = \beta_{t-1} + \mu_{t-1} + \eta_t$$

$$\beta_t = \beta_{t-1} + \upsilon_t$$

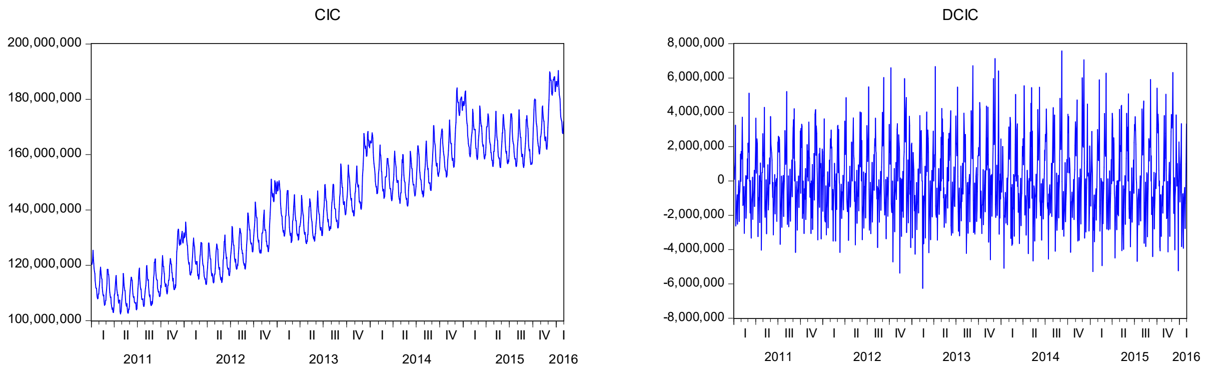
Where:

y_t	is the dependent variable
μ_t	is the stochastic trend component of the dependent variable
β_t	is the slope of the trend component
γ_t	is the stochastic seasonal component of the dependent variable. The seasonal component is usually set up in terms of stochastic trigonometric function or in terms of dummy-variable formulation
ψ_t	is the cycle component
O_{it}	are the dummy variables controlling for the effect of the largest outliers identified
ε_t, η_t, υ_t	are *iid* stochastic processes with zero mean and a variance of σ_ε^2, σ_η^2 and σ_υ^2 respectively[1]

[1] For further details on STS models, see Cabrero and others (2002), Norat (2008), and Harvey, Koopman, and Riani (1997).

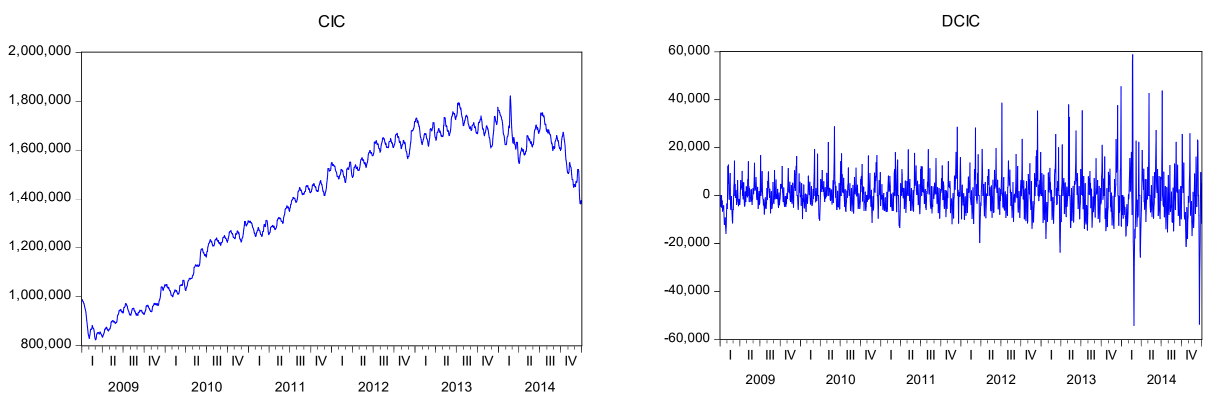
Appendix III. Currency in Circulation—Levels and Daily Changes

Brazil



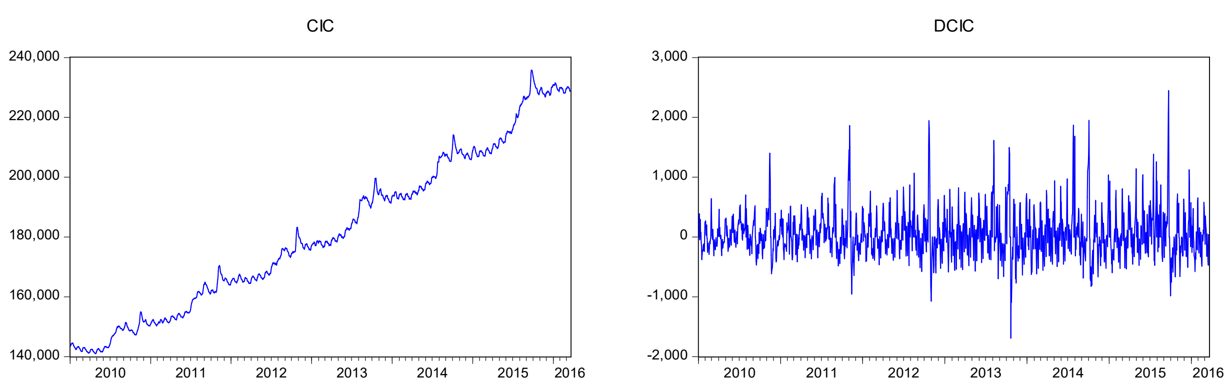
Sources: BCB and author's calculations.

Kazakhstan



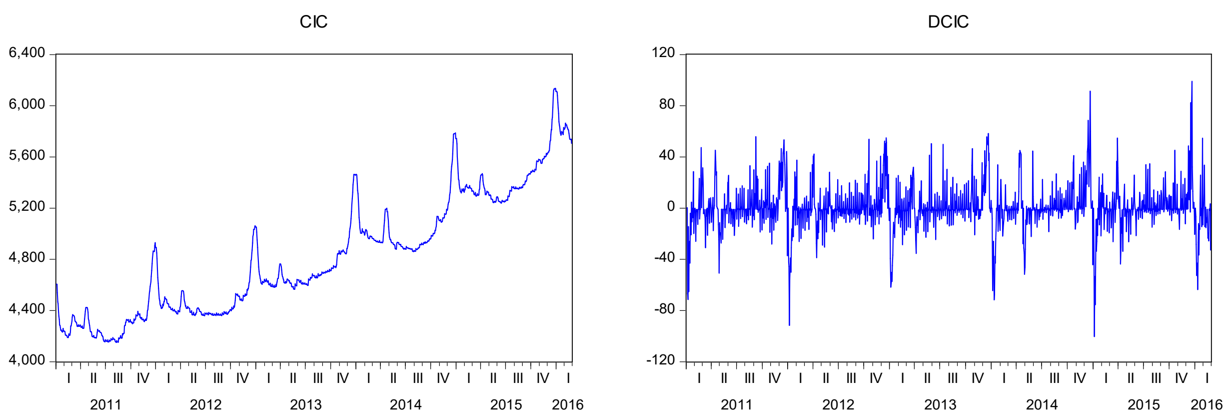
Sources: NBK and author's calculations.

Morocco



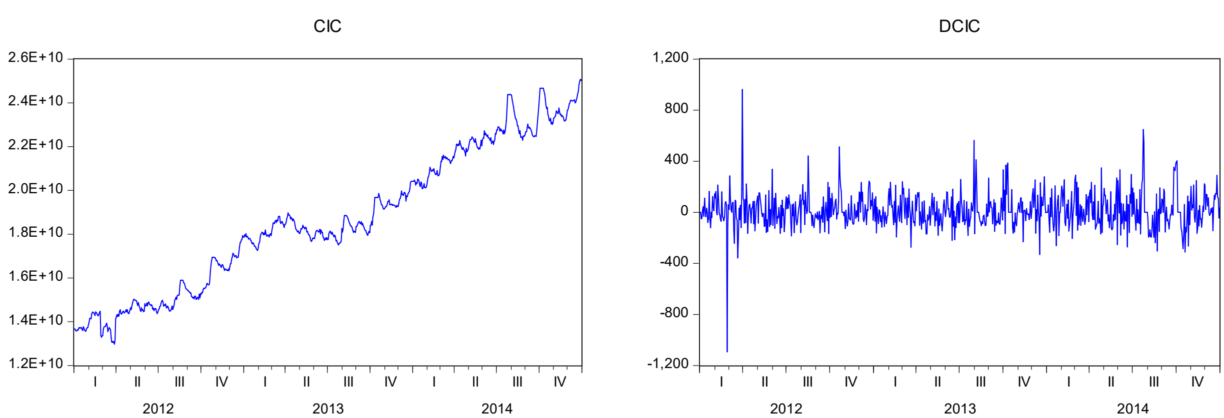
Sources: BAM and author's calculations.

New Zealand



Sources: RBNZ and author's calculations.

Sudan



Sources: Bank of Sudan and author's calculations.

Appendix IV. Brazil ARIMA Model of Currency in Circulation—Regression Results

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DAY	34923.64	8071.438	4.326818	0.0000
MONTH	-952166.1	102847.2	-9.258067	0.0000
S(-13)	415343.1	135112.7	3.074050	0.0022
S(-9)	-1583162.	154766.6	-10.22935	0.0000
S(-8)	-1660587.	154494.9	-10.74849	0.0000
S(-7)	-1677334.	151936.8	-11.03968	0.0000
S(-6)	-1411373.	154962.3	-9.107846	0.0000
S(-1)	-596799.6	148256.4	-4.025456	0.0001
S(11)	-796436.6	131244.6	-6.068337	0.0000
S(12)	-894093.4	146211.9	-6.115050	0.0000
S(16)	691795.8	157118.5	4.403020	0.0000
S(17)	1466382.	157078.7	9.335333	0.0000
S(18)	2033679.	150086.7	13.55003	0.0000
S(19)	2120215.	159901.4	13.25952	0.0000
S(20)	1785731.	157252.1	11.35585	0.0000
S(21)	1081496.	173378.9	6.237761	0.0000
S(22)	2631228.	153971.9	17.08902	0.0000
S(23)	2640102.	141413.4	18.66940	0.0000
S(24)	1715511.	146110.9	11.74115	0.0000
S(25)	346289.2	138089.1	2.507723	0.0123
S(27)	-528189.9	135069.1	-3.910515	0.0001
S(34)	547975.2	155507.0	3.523797	0.0004
H	-1256794.	244066.8	-5.149384	0.0000
MONDAY	9117550.	1111823.	8.200541	0.0000
TUESDAY	9885197.	1113144.	8.880432	0.0000
WEDNESDAY	10317636	1108574.	9.307126	0.0000
THURSDAY	11211913	1101992.	10.17422	0.0000
FRIDAY	12366833	1100794.	11.23447	0.0000
M2	709774.9	261034.4	2.719086	0.0067
M3	2779286.	357651.8	7.770927	0.0000
M5	-1515198.	293876.3	-5.155903	0.0000
T3	2306737.	407138.6	5.665730	0.0000
T4	-1380320.	363890.3	-3.793231	0.0002
F2	1344729.	284093.2	4.733409	0.0000
F3	-1709068.	375741.5	-4.548522	0.0000
W1	-11397458	994705.1	-11.45813	0.0000
W2	-11093698	1077530.	-10.29548	0.0000
W3	-11862310	1084411.	-10.93894	0.0000
W4	-11873108	1088763.	-10.90514	0.0000
W5	-7816951.	930654.9	-8.399409	0.0000
W6	-9382312.	990568.4	-9.471645	0.0000
W7	-10667086	970707.8	-10.98898	0.0000
W8	-10422072	981470.8	-10.61883	0.0000
W9	-7175944.	785982.5	-9.129903	0.0000
W10	-8294541.	890015.3	-9.319548	0.0000

W11	-9639147.	893301.2	-10.79048	0.0000
W12	-8849751.	888545.1	-9.959821	0.0000
W13	-6401306.	701060.8	-9.130886	0.0000
W14	-7784671.	777336.1	-10.01455	0.0000
W15	-8545299.	785253.1	-10.88222	0.0000
W16	-8175028.	778130.5	-10.50599	0.0000
W17	-4857302.	589954.8	-8.233347	0.0000
W18	-6802630.	696613.5	-9.765287	0.0000
W19	-7692416.	678189.4	-11.34258	0.0000
W20	-7445134.	683530.1	-10.89218	0.0000
W21	-3848539.	508892.5	-7.562577	0.0000
W22	-5500320.	589595.6	-9.328971	0.0000
W23	-6639484.	584119.7	-11.36665	0.0000
W24	-5554610.	567054.9	-9.795542	0.0000
W25	-2850158.	385107.2	-7.400948	0.0000
W26	-5543846.	483863.1	-11.45747	0.0000
W27	-5666348.	480679.0	-11.78821	0.0000
W28	-5536462.	498257.6	-11.11165	0.0000
W29	-1932287.	313782.6	-6.158045	0.0000
W30	-4030266.	382332.7	-10.54125	0.0000
W31	-4599429.	377168.0	-12.19464	0.0000
W32	-4215678.	380241.0	-11.08686	0.0000
W33	-1065674.	245269.6	-4.344907	0.0000
W34	-3036354.	300596.6	-10.10109	0.0000
W35	-3798200.	292802.0	-12.97191	0.0000
W36	-2846670.	298392.3	-9.540023	0.0000
W38	-2432497.	210901.2	-11.53382	0.0000
W39	-2863238.	198066.6	-14.45593	0.0000
W40	-3050904.	227287.5	-13.42311	0.0000
W41	649001.0	153681.1	4.223036	0.0000
W42	-524175.6	159347.9	-3.289504	0.0010
W43	-1975313.	156015.5	-12.66100	0.0000
W45	2692116.	229525.3	11.72906	0.0000
C_3	3259171.	457765.9	7.119734	0.0000
C_2	1371824.	454029.1	3.021445	0.0026
C1	-3530725.	464003.1	-7.609272	0.0000
C9	2365024.	498686.1	4.742511	0.0000
AR(1)	-0.126077	0.029875	-4.220136	0.0000
AR(2)	-0.241626	0.029239	-8.263846	0.0000
AR(3)	-0.104799	0.029107	-3.600460	0.0003
AR(9)	-0.084910	0.027983	-3.034307	0.0025
AR(15)	-0.096718	0.028680	-3.372337	0.0008
AR(22)	-0.079688	0.027953	-2.850776	0.0045
AR(25)	-0.066473	0.027755	-2.395008	0.0168
AR(26)	0.112937	0.028302	3.990483	0.0001
AR(109)	0.084671	0.027150	3.118656	0.0019
AR(110)	0.092451	0.027510	3.360672	0.0008
AR(121)	-0.067632	0.027149	-2.491096	0.0129
AR(141)	-0.108856	0.027081	-4.019667	0.0001
AR(145)	-0.117352	0.027841	-4.215112	0.0000
AR(160)	-0.093329	0.028011	-3.331830	0.0009

AR(169)	-0.067367	0.027706	-2.431470	0.0152
AR(173)	-0.085788	0.027194	-3.154652	0.0017
AR(180)	-0.102226	0.027804	-3.676653	0.0003
AR(202)	-0.064254	0.027695	-2.320067	0.0206
AR(212)	-0.092589	0.026813	-3.453129	0.0006
AR(221)	-0.088857	0.027772	-3.199497	0.0014
AR(243)	-0.069307	0.027257	-2.542685	0.0112
AR(260)	0.298500	0.029181	10.22913	0.0000
MA(260)	-0.892123	0.007123	-125.2501	0.0000

R-squared	0.927939	Mean dependent variable	60783.22
Adjusted R-squared	0.919730	S.D. dependent variable	2213940.
S.E. of regression	627252.9	Akaike info criterion	29.63351
Sum squared residuals	3.59E+14	Schwarz criterion	30.14156
Log likelihood	-14978.46	Hannan-Quinn criterion	29.82644
Durbin-Watson statistic	2.103759		

Sources: BCB and author’s estimates.

Appendix V. Kazakhstan ARIMA Model of Currency in Circulation—Regression Results

Variable	Coefficient	Std. Error	t-Statistic	Prob.
S	-2173.144	806.2254	-2.695454	0.0071
H	-10604.67	1084.354	-9.779712	0.0000
M2	4532.654	1264.057	3.585800	0.0004
M3	6925.541	1686.524	4.106400	0.0000
M4	3337.811	1388.684	2.403579	0.0164
T2	3872.691	1749.980	2.212991	0.0271
T3	12059.04	1996.092	6.041327	0.0000
T4	8161.730	1509.718	5.406130	0.0000
WED1	7075.015	1644.841	4.301338	0.0000
WED5	-4752.436	1558.074	-3.050199	0.0023
TH1	5799.708	1718.350	3.375162	0.0008
TH2	9552.187	1738.796	5.493565	0.0000
F2	6641.980	1371.445	4.843053	0.0000
W1	12307.90	2888.532	4.260953	0.0000
W46	5686.026	2686.372	2.116619	0.0345
W47	-13794.39	2922.722	-4.719707	0.0000
FRIDAY	11347.28	1181.856	9.601240	0.0000
AR(1)	0.428279	0.019568	21.88662	0.0000
AR(20)	0.103620	0.023788	4.356003	0.0000
AR(21)	-0.086910	0.023528	-3.693933	0.0002
AR(31)	-0.039572	0.016030	-2.468629	0.0137
AR(40)	0.086982	0.017850	4.872929	0.0000
AR(86)	-0.097740	0.022107	-4.421163	0.0000
AR(87)	0.072454	0.022592	3.207096	0.0014
AR(90)	0.051655	0.017933	2.880544	0.0041
AR(240)	0.142746	0.022973	6.213572	0.0000
AR(243)	-0.101612	0.021468	-4.733057	0.0000
AR(250)	0.135573	0.020331	6.668288	0.0000
AR(255)	-0.115754	0.022012	-5.258617	0.0000
AR(260)	0.617606	0.024059	25.67065	0.0000
MA(260)	-0.836121	0.008457	-98.86712	0.0000

R-squared	0.748668	Mean dependent variable	72.84277
Adjusted R-squared	0.741218	S.D. dependent variable	9345.914
S.E. of regression	4754.329	Akaike info criterion	19.80077
Sum squared residuals	2.29E+10	Schwarz criterion	19.94789
Log likelihood	-10295.10	Hannan-Quinn criterion	19.85657
Durbin-Watson statistic	2.166946		

Sources: NBK and author's estimates.

Appendix VI. Morocco ARIMA Model of Currency in Circulation—Regression Results

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MONTH	22.90060	9.852181	2.324420	0.0203
H	-200.8700	29.11243	-6.899801	0.0000
S	213.3831	30.13089	7.081872	0.0000
S1	155.3291	30.82931	5.038357	0.0000
MONDAY	95.29994	19.95113	4.776668	0.0000
WEDNESDAY	-331.2694	17.29620	-19.15272	0.0000
FRIDAY	185.2504	20.83604	8.890866	0.0000
M2	244.0248	75.62887	3.226609	0.0013
T4	187.2851	57.23728	3.272083	0.0011
W27	243.8771	113.4847	2.148987	0.0318
W28	362.1811	116.1466	3.118311	0.0019
W29	235.7071	97.81401	2.409748	0.0161
AF1	722.7536	88.13524	8.200506	0.0000
AF2	1339.386	91.91315	14.57230	0.0000
AF3	951.3863	91.39197	10.40996	0.0000
AF4	280.7579	87.52496	3.207746	0.0014
AA_1	1080.499	92.60996	11.66720	0.0000
AA_2	1987.013	92.71267	21.43195	0.0000
AA_3	1603.770	91.50602	17.52639	0.0000
AA_4	1330.060	91.48371	14.53877	0.0000
AA_5	1136.380	91.72948	12.38838	0.0000
AA_6	690.7200	91.29693	7.565644	0.0000
AA_7	506.2355	90.85203	5.572088	0.0000
AA_8	377.0366	90.82744	4.151131	0.0000
AA_9	239.1390	91.86135	2.603261	0.0093
AA_10	200.7271	88.37275	2.271369	0.0233
AA1	-535.0637	89.43439	-5.982752	0.0000
AA2	-630.7206	92.33846	-6.830529	0.0000
AA3	-644.0249	91.54607	-7.034981	0.0000
AA4	-570.3301	91.74881	-6.216213	0.0000
AA5	-527.5888	92.04464	-5.731880	0.0000
AA6	-418.5062	91.42293	-4.577694	0.0000
AA7	-185.7298	90.87084	-2.043887	0.0412
AA8	-295.5614	91.82959	-3.218586	0.0013
AA9	-283.9825	88.28493	-3.216659	0.0013
AR(1)	0.338927	0.023967	14.14165	0.0000
AR(2)	-0.066124	0.019313	-3.423837	0.0006
AR(20)	0.032177	0.013666	2.354602	0.0187
AR(260)	0.676819	0.018990	35.64051	0.0000
MA(260)	-0.853959	0.007080	-120.6159	0.0000

R-squared	0.805276	Mean dependent variable	57.19665
Adjusted R-squared	0.799527	S.D. dependent variable	398.4656
S.E. of regression	178.4099	Akaike info criterion	13.23499
Sum squared residuals	42047565	Schwarz criterion	13.38829
Log likelihood	-8966.414	Hannan-Quinn criterion	13.29238
Durbin-Watson statistic	1.813386		

Sources: BAM and author’s estimates.

Appendix VII. New Zealand ARIMA Model of Currency in Circulation—Regression Results

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MONTH	1.493665	0.119722	12.47608	0.0000
MONDAY	-9.231777	1.193763	-7.733342	0.0000
TUESDAY	-10.43762	1.196131	-8.726153	0.0000
WEDNESDAY	-8.023829	1.123481	-7.141937	0.0000
FRIDAY	-19.76918	1.124588	-17.57904	0.0000
M1	19.28935	2.323473	8.301948	0.0000
T2	16.33036	5.502787	2.967653	0.0031
T4	23.04182	5.564054	4.141193	0.0000
TH1	20.67596	3.816438	5.417606	0.0000
TH4	13.25062	3.845394	3.445843	0.0006
F2	-14.17081	3.252311	-4.357151	0.0000
W1	59.43353	12.16764	4.884558	0.0000
W2	79.55021	17.62239	4.514155	0.0000
W46	13.23950	2.749884	4.814565	0.0000
C_3	41.80081	4.129616	10.12220	0.0000
C_6	24.46098	4.011631	6.097514	0.0000
C_8	14.13786	4.272577	3.308978	0.0010
C_12	8.534900	4.239624	2.013126	0.0443
C1	-22.99154	4.279038	-5.373063	0.0000
C4	-27.61013	4.482296	-6.159820	0.0000
C5	-35.64586	11.32861	-3.146535	0.0017
C6	-57.98100	12.69245	-4.568149	0.0000
C7	-73.43441	12.90440	-5.690649	0.0000
C8	-109.1587	12.86747	-8.483310	0.0000
C9	-105.8669	12.91558	-8.196834	0.0000
C10	-122.9535	17.14077	-7.173159	0.0000
C11	-115.5805	18.05175	-6.402731	0.0000
C12	-113.7686	18.17846	-6.258432	0.0000
C13	-102.8814	18.16861	-5.662589	0.0000
C14	-97.70983	18.20298	-5.367792	0.0000
C15	-92.39099	18.11805	-5.099390	0.0000
E_1	14.57095	4.175553	3.489585	0.0005
E_3	29.32017	4.712127	6.222280	0.0000
E_4	35.26128	4.740170	7.438821	0.0000
E_5	41.51883	4.762392	8.718062	0.0000
E_6	23.77389	4.822830	4.929447	0.0000
E_7	12.84009	4.609730	2.785433	0.0054
E2	-10.72453	4.185984	-2.562010	0.0105
E3	-24.19218	4.338271	-5.576455	0.0000
E4	-23.53013	4.240648	-5.548711	0.0000
E6	-13.43170	4.100269	-3.275810	0.0011
E7	-11.13869	4.341613	-2.565564	0.0104
E8	-20.99435	4.409155	-4.761535	0.0000
E9	-18.44959	4.198292	-4.394546	0.0000

AR(1)	0.315085	0.029357	10.73282	0.0000
AR(2)	0.067136	0.028736	2.336269	0.0196
AR(5)	0.111316	0.027942	3.983741	0.0001
AR(7)	-0.060875	0.028058	-2.169634	0.0302
AR(35)	0.066319	0.027009	2.455465	0.0142
AR(77)	0.147271	0.034943	4.214651	0.0000
MA(15)	-0.134845	0.023356	-5.773560	0.0000
MA(77)	-0.174602	0.028609	-6.103063	0.0000
MA(173)	0.080942	0.019135	4.229957	0.0000
MA(199)	-0.071679	0.018498	-3.875020	0.0001
MA(204)	0.111615	0.014764	7.559692	0.0000
MA(206)	0.077509	0.015321	5.058910	0.0000
MA(216)	0.096450	0.017048	5.657403	0.0000
MA(220)	0.229144	0.023439	9.776031	0.0000
MA(235)	0.300717	0.026004	11.56413	0.0000
MA(236)	-0.211290	0.017931	-11.78379	0.0000

R-squared	0.726685	Mean dependent var	1.012188
Adjusted R-squared	0.713336	S.D. dependent var	18.09531
S.E. of regression	9.688419	Akaike info criterion	7.425902
Sum squared resid	113389.5	Schwarz criterion	7.669366
Log likelihood	-4648.022	Hannan-Quinn criter.	7.517363
Durbin-Watson stat	1.976281		

Sources: RBNZ and author’s estimates.

Appendix VIII. Sudan ARIMA Model of Currency in Circulation—Regression Results

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DAY	-1.863491	0.289089	-6.446082	0.0000
S(-1)	71.43419	20.26931	3.524254	0.0005
SUNDAY	108.7029	12.18282	8.922635	0.0000
WEDNESDAY	90.82763	12.56259	7.230006	0.0000
MON5	-243.7973	54.76926	-4.451353	0.0000
W5	83.97357	21.36826	3.929826	0.0001
W9	45.15275	20.92127	2.158222	0.0314
W13	45.86034	20.92032	2.192143	0.0289
W29	-235.3133	22.44648	-10.48330	0.0000
W30	-80.34377	20.75716	-3.870653	0.0001
W31	-62.99848	21.65877	-2.908682	0.0038
W48	64.51586	21.38705	3.016585	0.0027
FITR_1	643.2825	54.34184	11.83770	0.0000
FITR_2	411.1399	55.86069	7.360093	0.0000
FITR_3	380.2881	53.81831	7.066148	0.0000
FITR_4	277.1591	55.99680	4.949551	0.0000
ADHA_1	408.7681	52.59646	7.771779	0.0000
ADHA_2	238.1701	54.55351	4.365807	0.0000
ADHA_3	308.4898	53.20117	5.798552	0.0000
ADHA_4	332.0770	53.67513	6.186795	0.0000
ADHA_6	391.9273	53.91422	7.269461	0.0000
ADHA_7	-156.7157	53.93275	-2.905762	0.0038
ADHA3	-152.9064	53.83508	-2.840274	0.0047
ADHA4	-353.5587	52.97900	-6.673563	0.0000
ADHA5	-94.06754	52.68151	-1.785589	0.0748
ADHA7	-268.5866	51.97756	-5.167357	0.0000
AR(1)	-0.195444	0.038022	-5.140317	0.0000
AR(5)	0.173715	0.037461	4.637185	0.0000
AR(7)	-0.118985	0.037943	-3.135920	0.0018
AR(8)	-0.158408	0.037434	-4.231691	0.0000
AR(10)	-0.106730	0.036614	-2.915016	0.0037
AR(34)	-0.137654	0.037149	-3.705474	0.0002
AR(58)	-0.126114	0.036249	-3.479075	0.0005
AR(61)	0.132937	0.037112	3.582064	0.0004
AR(63)	0.154949	0.037107	4.175767	0.0000
AR(87)	-0.107046	0.037174	-2.879633	0.0042
AR(105)	-0.113127	0.037722	-2.999000	0.0029
AR(133)	0.101055	0.036084	2.800562	0.0053
AR(168)	-0.120273	0.036351	-3.308696	0.0010
AR(190)	-0.141373	0.036415	-3.882275	0.0001
AR(193)	0.095827	0.036402	2.632434	0.0088
AR(195)	0.117499	0.036303	3.236651	0.0013
AR(196)	0.134100	0.037628	3.563849	0.0004
AR(209)	0.079870	0.035942	2.222200	0.0267
AR(215)	0.137768	0.036866	3.736976	0.0002

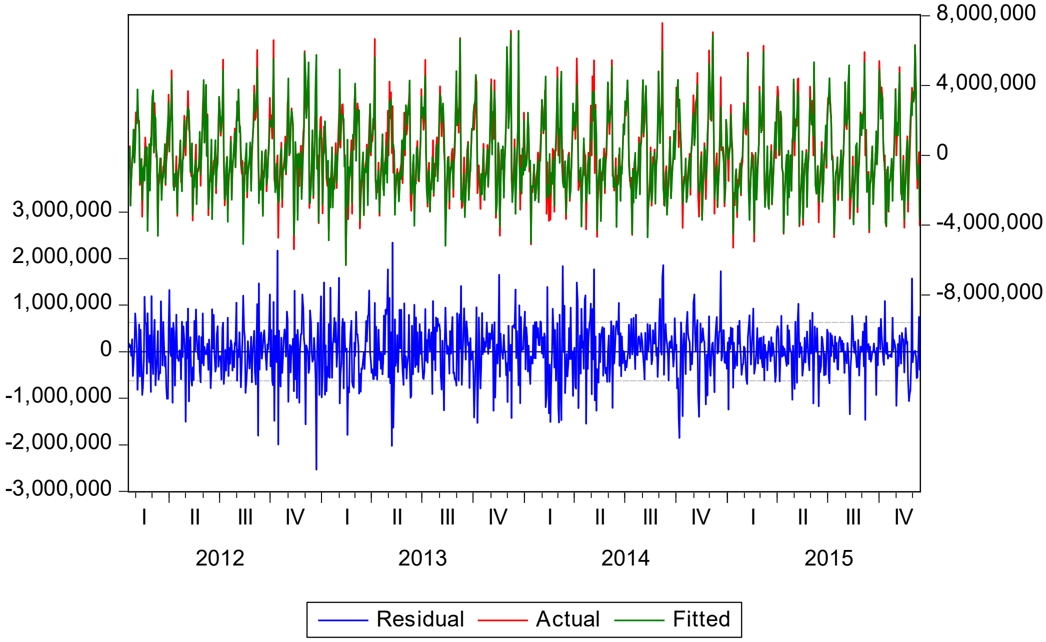
AR(233)	0.128890	0.034643	3.720533	0.0002
AR(248)	-0.104968	0.036174	-2.901776	0.0039
AR(260)	0.187514	0.036719	5.106762	0.0000
MA(260)	0.921162	0.006725	136.9807	0.0000

R-squared	0.874755	Mean dependent variable	13.62905
Adjusted R-squared	0.862072	S.D. dependent variable	133.4980
S.E. of regression	49.57935	Akaike info criterion	10.73403
Sum squared residuals	1165145.	Schwarz criterion	11.13311
Log likelihood	-2757.949	Hannan-Quinn criterion	10.89033
Durbin-Watson statistic	2.041079		

Sources: Bank of Sudan and author’s estimates.

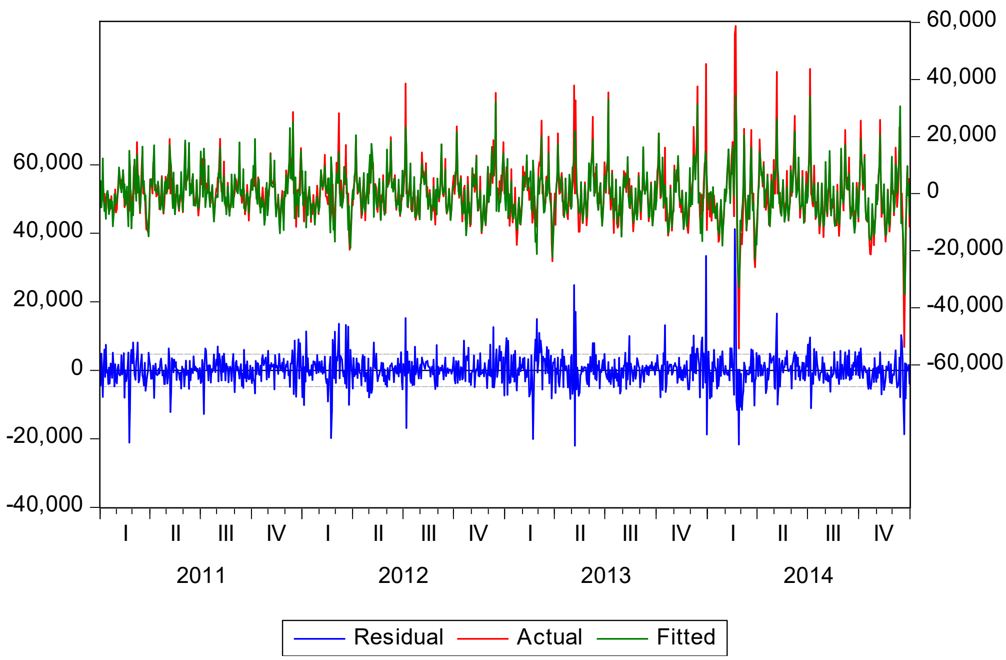
Appendix IX. ARIMA Model Residuals

Brazil



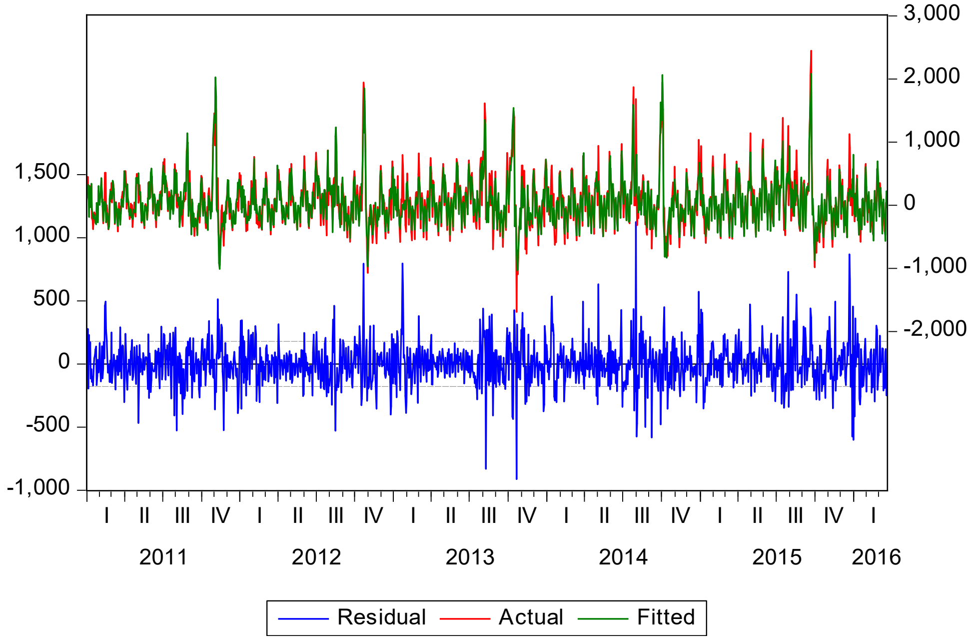
Sources: BCB and author's estimates.

Kazakhstan



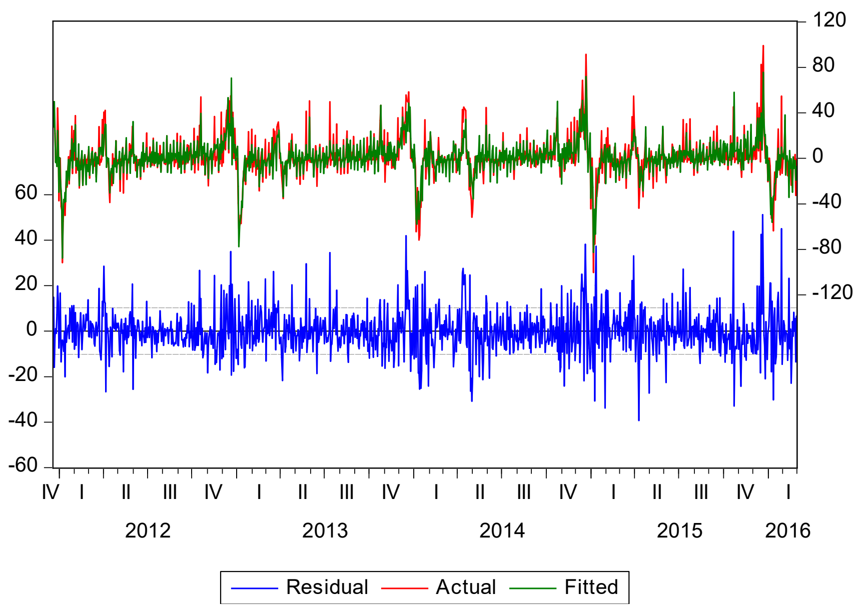
Sources: NBK and author's estimates.

Morocco



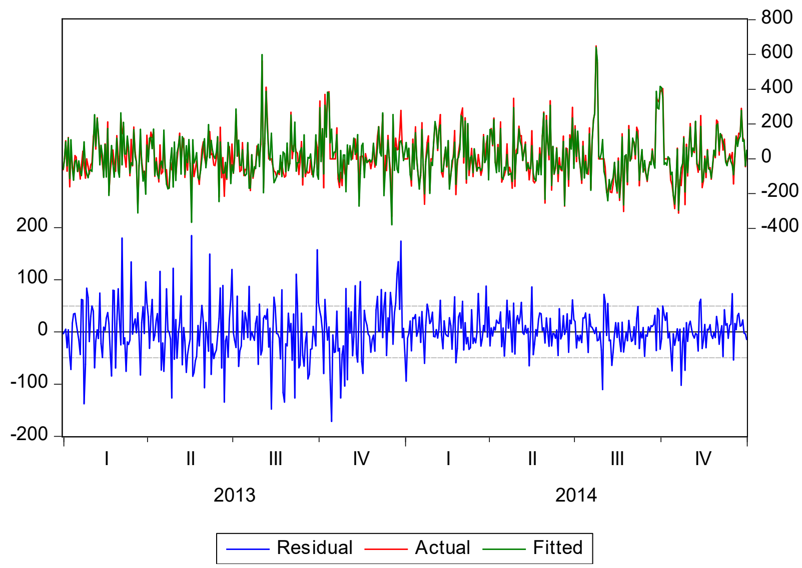
Sources: BAM and author's estimates.

New Zealand



Sources: RBNZ and author's estimates.

Sudan



Sources: Bank of Sudan and author's estimates.

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