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Evaluating Changes in the Transmission Mechanism of
Government Spending Shocks

by Nooman Rebei

I N T E R N A T I O N A L M O N E T A R Y F U N D

IMF Working Paper

Middle East and Central Asia Department

Evaluating Changes in the Transmission Mechanism of Government Spending Shocks

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Authorized for distribution by Jean-François Dauphin

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Abstract

We empirically revisit the crowding-in effect of government spending on private consumption based on rolling windows of U.S. data. Results show that in earlier samples government spending is increasingly crowding in private consumption; however, this relation is reverted in the latest periods. We propose a model embedding non-separable public and private consumption in the utility function and rule-of-thumb consumers to assess the sources of non-monotonic changes in the transmission of the shock. The iterative full information estimation of the model reveals that changes in the co-movement between private and public spending is primarily driven by the fluctuations in the elasticity of substitution between private and public consumption, the share of financially constrained consumers, and the elasticity of intertemporal substitution.

JEL Classification Numbers: E2, E6, H3, H5

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1 Introduction

The topic of time-varying transmission of fiscal shocks has gained a lot of interest since many countries adopted active policies as a reaction to growth deceleration following the recent financial crisis. This issue engaged a debate among academics and policymakers about the effectiveness of fiscal policy at stimulating the economy. The literature has considered two main classes of explanations for the mitigated performance of such policies. The first class of explanations compare the impact of fiscal stimulus in different predefined economic episodes. In particular, several studies consider that fiscal multipliers are different during recession versus boom periods (see, for instance, [Fazzari et al., 2015](#); [Berger and Vavra, 2014](#); [Auerbach and Gorodnichenko, 2012](#); [Tagkalakis, 2008](#)), while others emphasize the changes in the performance of government spending shocks near the zero lower bound (see, for instance, [Ramey and Zubairy, 2014](#)). The second class of explanations do not take a stand on the origin of the change in the transmission of government spending shocks and rely solely on the observed data to distinguish episodes. For example, [Cimadomo and Bénassy-Quéré \(2012\)](#) estimate an empirical model based on rolling windows of data, then assess whether spending and tax multipliers have changed over time. Results suggest a decline in fiscal multipliers over time. Using vector autoregressions on U.S. time series for two subperiods, [Bilbiie et al. \(2008\)](#) find government spending shocks to have stronger effects on output, consumption, and wages in the earlier subperiod.

We contribute to the existing literature by empirically characterizing the evidence of time-varying propagation of government spending shocks with a particular interest in the responses of consumption. In this context, we propose a Bayesian vector autoregression model estimated on rolling windows of U.S. data spanning from 1950:I to 2015:IV. Initially, we estimate the model over the whole sample and results suggest that private consumption reacts positively and significantly to the shock, as extensively reported in the literature—see, for instance, [Blanchard and Perotti \(2002\)](#), [Fatas and Mihov \(2001\)](#), [Mountford and Uhlig \(2009\)](#), and [Fisher and Peters \(2010\)](#)—advocating for a Keynesian transmission of the shock. The rolling-window Bayesian estimation unveils non-monotonic behaviors of output and consumption following a public spending shock. In particular, for the periods centered around the 1960s and 1970s, the effect of the shock is mild but increasing over time. Subsequent rolling windows show a surge in U.S. fiscal multipliers and Keynesian behavior of private consumption. Then, the impact of government spending shocks drastically collapse during the periods centered around the 1990s. Interestingly, public spending starts crowding out private consumption during the latest periods including those following the financial crisis.

In the second part of the paper, we propose to identify the causes of the changes in the prop-

agation of government spending shocks to the economy in the context of an estimated dynamic and stochastic general equilibrium model. Different modifications to a standard RBC model have been proposed in the literature to account for the positive co-movement between public and private expenditures. [Linnemann and Schabert \(2006\)](#) model government spending as enhancing productivity of firms; [Galí et al. \(2007\)](#) combine rule-of thumb-consumers and nominal price rigidity; [Bouakez and Rebei \(2007\)](#) consider an environment where government spending directly influences utility of economic agents; and [Ravn et al. \(2012\)](#) propose a model with the deep habits mechanism. Based on Bayesian estimations of these models, [Kormilitsina and Zubairy \(2016\)](#) do not find support for the alternative assumptions in the data. Besides, under the estimated versions of the models private consumption tends to be crowded-out. This suggests an additional challenge to identifying the appropriate model potentially capable to generate the non-monotonic evolution of the impulse-response functions.

The strategy we pursue in this paper consists of combining two potential sources of generating positive responses of consumption while better fitting the data. Namely, we propose a model where government spending explicitly affects utility as in [Bouakez and Rebei \(2007\)](#) in addition to the presence of a share of financially constrained households as in [Galí et al. \(2007\)](#). Then, we proceed with iterative full information Bayesian estimations of the model based on the same rolling windows of data as in the BVAR; and we compute government spending shock's impulse-response functions for each window.

It is worth noting that from a theoretical standpoint, the model can imply both cases of crowding-in and crowding-out effects depending on the values of some key parameters. This turns out to be important to replicate the change in households behavior conditional on a government spending shock. Our findings suggest that the proposed model is able to account for the non-monotonic change in the co-movement between public and private consumption. Specifically, two key results seem to emerge. First, the size of the response consumption (and output) is (are) initially increasing owing to the change in the estimated value of elasticity of substitution between public and private spending. Second, the observed decline in the impact of government spending in the latest periods is mainly attributed to increased access to financial services and milder elasticity of intertemporal substitution.

The paper proceeds as follows. In Section 2, we introduce the econometric methodology in which impulse-response functions are allowed to vary across time and we characterize the stylized facts. In Section 3, we describe the theoretical model and discuss the endogenous propagation mechanisms of government spending shocks. Our estimation strategy is detailed in Section 4, in addition to a discussion of the results. Section 5 provides a conclusion.

2 Empirical Investigation

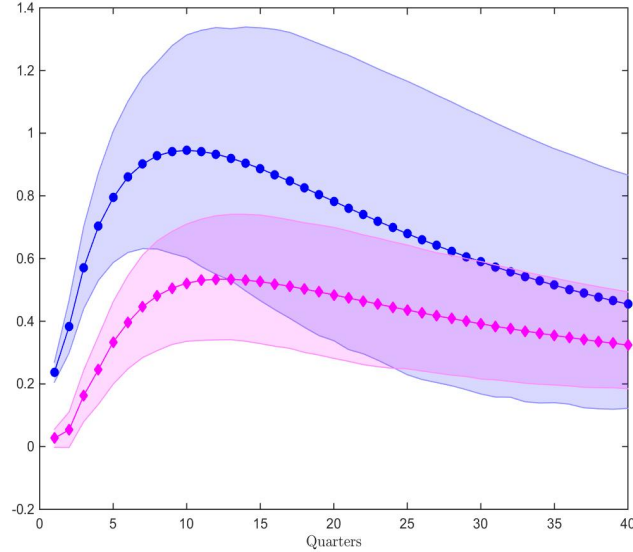
The structural form of the Vector Autoregression is:

$$AX_t = c + A_1X_{t-1} + \dots + A_pX_{t-p} + u_t \quad (1)$$

where X_t is a vector of endogenous variables, u_t is a vector of structural errors, and A is the matrix constituted by the contemporaneous effect among the endogenous variables.

The estimation of Equation (1) uses real GDP, Y_t , real government purchases, G_t , and real private consumption, C_t . All variables are in logs, expressed in seasonally adjusted per capita real form.¹ We consider quarterly data over the period 1950Q1–2015Q4 and we take a Bayesian approach to the estimation of the model specified above, implementing the Gibbs sampler for structural VARs. A Cholesky identification is imposed on the model with the following causality order $X_t = [G_t, Y_t, C_t]$ reflecting the principle that government spending is generally predetermined with respect to the contemporaneous realizations of other national account variables.²

Figure 1: Output and Consumption Responses (Sample: 1950:I–2015:IV)



Blue (Magenta) line with circles (diamonds) is the mean estimate of output (consumption) response. Shaded areas correspond to 90 percent confidence intervals.

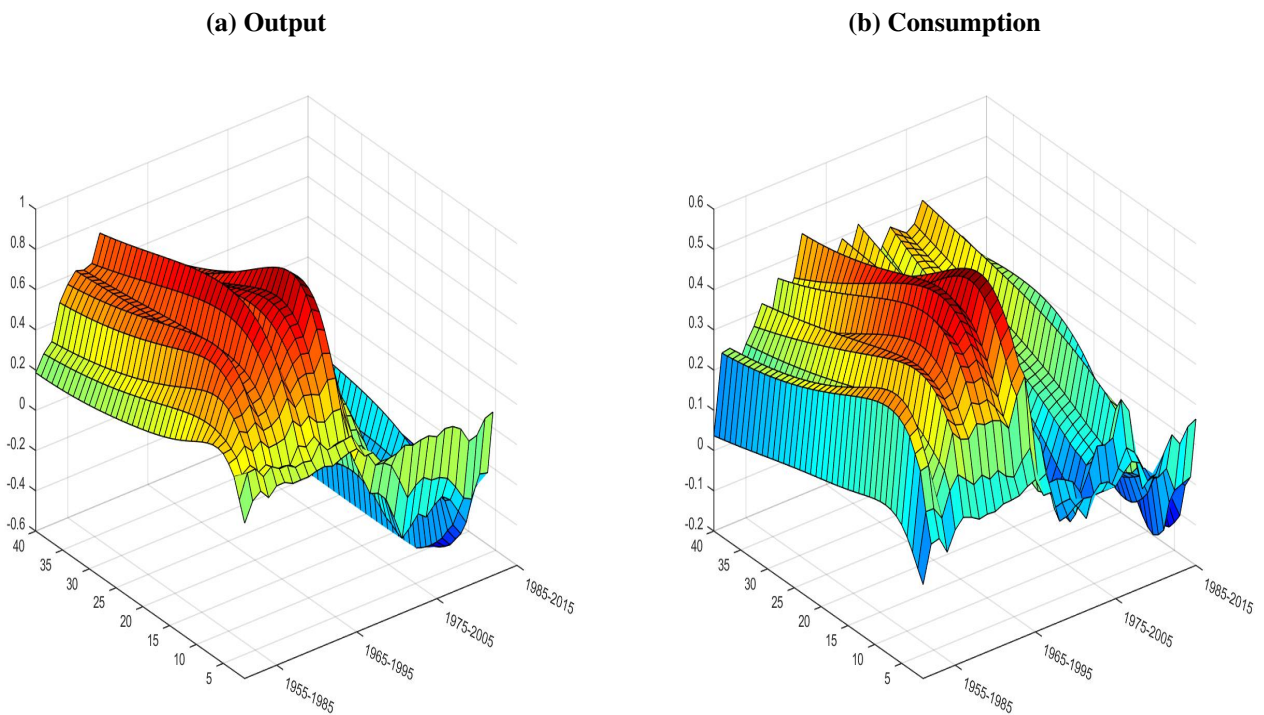
¹The time series come from FRED and correspond to: Real Gross Domestic Product (RGDPC1); Real Government Consumption (GDE1); and Real Personal Consumption Expenditures (PCECC96). All variables are expressed in billions of chained 2009 dollars and divided by the seasonally adjusted number of Civilian Labor Force (CLF16OV).

²Bouakez et al. (2014) identify fiscal policy shocks by exploiting the conditional heteroscedasticity of the structural disturbances and find similar impulse-response functions following public spending shocks compared to the Cholesky identification scheme.

Figure 1 replicates the stylized facts regarding the dynamic effects of government spending shocks as reported by Blanchard and Perotti (2002), Fatas and Mihov (2001), Mountford and Uhlig (2009), and Fisher and Peters (2010)—a rise in government spending has an expansionary effect on output and crowds in private consumption.

In order to explore the stability of the result over time, we estimate the empirical model on rolling-windows of data with fixed length—120 observations—starting from 1950:I and repeating the estimation moving the starting date by one year.³ As reported in Figure 2, results reveal significant change in the reaction of output and consumption over time. Adopting samples that cover periods starting date from 1950s until mid-1960s, the BVAR generates increasingly positive reactions of output and consumption to public expenditure shocks. However, the results vanish once more recent observations are included; and the impulse-response functions become significantly negative and remain so until the last sampling window (1985:I–2015:IV).

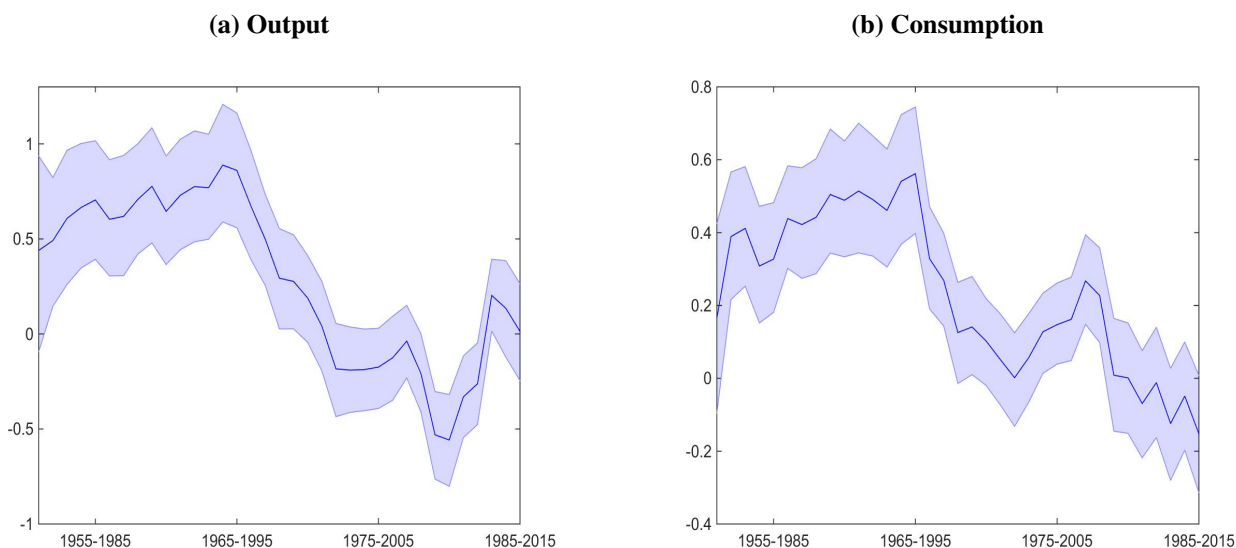
Figure 2: Impact of Government Spending Shock (3-variable BVAR)



Focusing on the second year impact of the shock, Figure 3 distinctly shows that both the responses of output and private consumption significantly change over time.

³Here, considering quarterly data for 30 years is assumed delivering reliable estimation results. Extending the number of observations per window to 160 delivered very similar results.

Figure 3: Second Year Impact of the Shock



Shaded areas correspond to 90 percent confidence intervals.

Using vector autoregressions for 1957—1979 and 1983—2004 [Bilbiie et al. \(2008\)](#) also find that government spending shocks have stronger effects on output and consumption in the earlier period. Only a decline in the responsiveness of private consumption is observed, although remaining positive. The advantage in our approach is twofold. First, the number of observations per window is larger allowing for more precise estimates.⁴ Second, the empirical procedure adopted in this paper does not require an ad hoc split of our dataset. Actually, a break date may be improperly imposed or not precisely estimated, which may contaminate the estimation results and the conclusions. Besides, if the change in the transmission of public spending shocks is non-monotonic, this would require specifying at least two break dates which is not necessarily supported by the data.

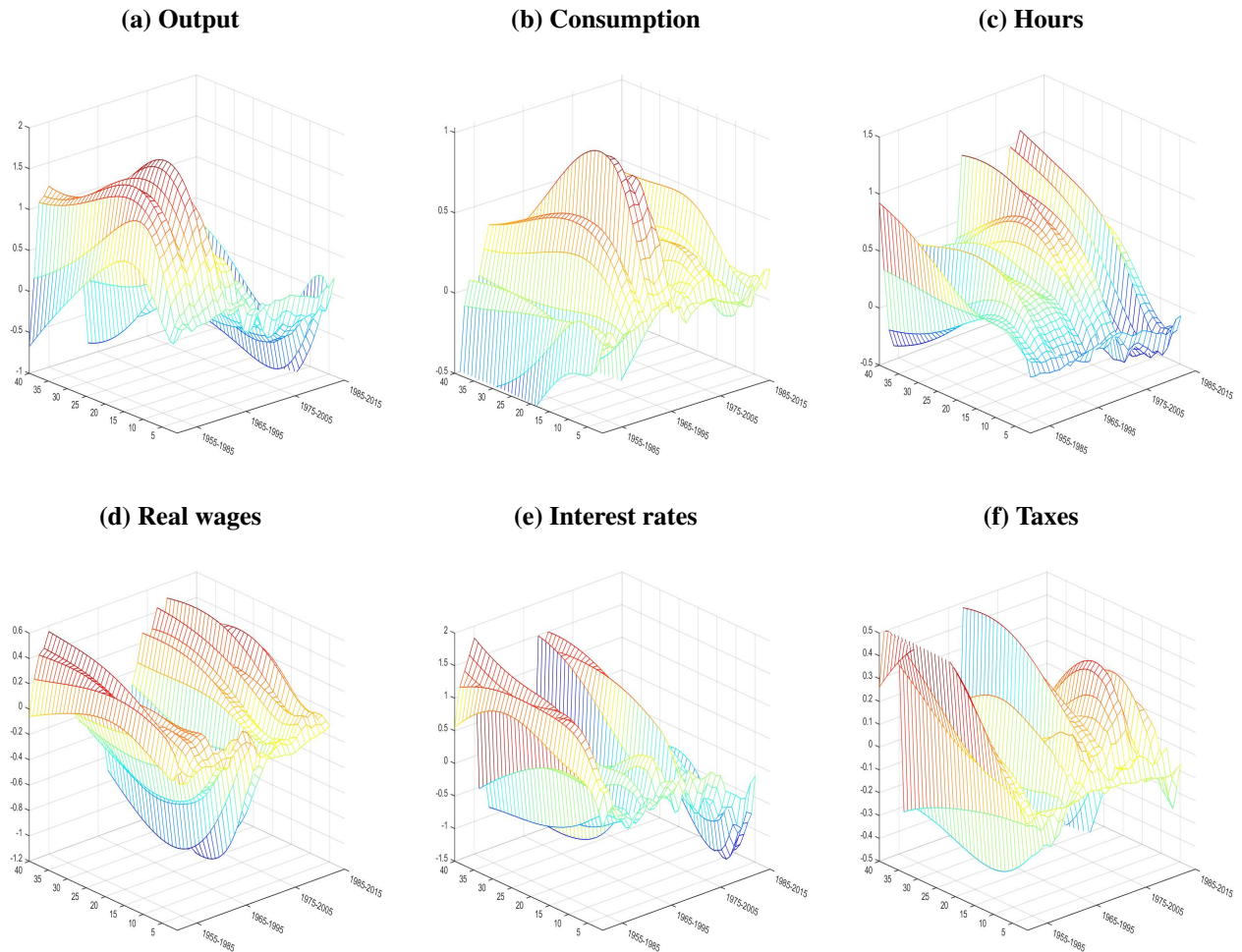
As a robustness check, we extend the model to include additional macroeconomic indicators to enrich the structure and dynamics of the empirical model. Namely, as in [Perotti \(2005\)](#) and [Ramey \(2011\)](#), we incorporate total hours worked, real wages, interest rates, and total taxes to the initial set of variables.⁵ Figure 4 shows that the result remains qualitatively the same. Further, real

⁴The change in the responses in output and consumption responses reported by [Bilbiie et al. \(2008\)](#) is not statistically significant, partly due to the relatively short observations per sample.

⁵Time series come from FRED database and correspond to: Nonfarm Business Sector: Hours of All Persons (HOANBS); Nonfarm Business Sector: Real Compensation Per Hour (COMPRNFB) divided by the GDP Implicit Price Deflator (GDPDEF); 3-month Treasury Bill (TB3MS); and Government Current Tax Receipts (W054RC1Q027SBEA) divided by the GDP Implicit Price Deflator (GDPDEF) and the number of Civilian Labor

wage responses exhibit an inverted hump-shaped change over time. In the middle of the sample, real wages are negatively overshooting following a government spending shock. This result can be attributed to the crowding-in phenomena during the same periods, which generates a surge in aggregate demand and high inflation.

Figure 4: Impact of Government Spending Shock (7-variable BVAR)



Overall, the full sample estimation supports the Keynesian transmission of a government spending shock. However, the co-movement between private and public spending evolves non-monotonically over time. In particular, during the periods centered around the 1960s and 1970s, the effect of the shock is mild. Subsequent rolling windows show uprising U.S. fiscal multipliers owing to an exacerbated Keynesian behavior of private consumption. Then, the impact of government spending Force (CLF16OV).

shocks drastically collapses during the periods centered around the 1990s; although, it is less pronounced than in the case of the initial three-variable BVAR.

In the following section we discuss the rationale behind the changes in output and consumption impulse-response functions over time.

3 Theoretical Analysis

In this section we investigate the sources of private consumption reaction decline following a surge in government expenditures. As previously shown in the literature, the standard RBC model is not capable of generating positive reaction of consumption. To resolve this inconsistency, we consider a model embedding two sources of the crowding-in effect: the presence of financially constrained households combined with sticky prices (see Galí et al., 2007; Coenen and Straub, 2005); and government spending in the utility function (see Bouakez and Rebei, 2007). Six types of agents, who take decisions based on optimization or predefined rules, are introduced in the benchmark model: households with and without access to financial services; competitive firms; monopolistically competitive firms; a government; and a monetary authority.

3.1 Optimizing households

At each period t , the representative household sells labor services, measured in hours worked, H_t^o , and rents the capital stock inherited from the previous period, K_t^o , to monopolistically competitive firms producing intermediary goods. Labor services are sold at nominal wage, W_t , and R_t^k is the nominal rental rate of capital. As owners of those firms, optimizing households are also entitled to nominal dividend payments, D_t^o . In the course of each period a household receives lump-sum transfers, Tr_t , and pays income taxes at an average rate τ_t to the government. Then, the net of tax income is used to consume, C_t^o , invest in physical capital, I_t^o , and save in risk free bonds, B_t^o .

As proposed by Bouakez and Rebei (2007), the effective consumption, X_t^o , is assumed to be a constant-elasticity-of-substitution index of private consumption, C_t^o , and government spending, G_t , which takes the form

$$X_t^o = \left[\kappa (C_t^o)^{\frac{\nu-1}{\nu}} + (1 - \kappa) (G_t)^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \quad (2)$$

where κ is the weight of private consumption in the effective consumption index and $\nu > 0$ is the elasticity of substitution between private consumption and government spending.

Formally, the representative optimizing household's optimization problem is:

$$\max_{\{C_t^o, B_t^o, H_t^o, K_{t+1}^o, I_t^o\}} E_0 \sum_{t=0}^{\infty} \beta^t \iota_t \left[\frac{(X_t^o)^{1-\sigma}}{1-\sigma} - \mu \frac{(H_t^o)^{1+\varphi}}{(1+\varphi)} \right],$$

subject to Equation (2) and

$$P_t C_t^o + P_t I_t^o + \frac{B_t^o}{R_t} \leq (1 - \tau_t) W_t H_t^o + (1 - \tau_t) R_t^k K_t^o + B_{t-1}^o + (1 - \tau_t) D_t^o + T r_t, \quad (3)$$

$$K_{t+1}^o = (1 - \delta) K_t^o + \left[A_{I,t} - \Gamma \left(\frac{I_t^o}{I_{t-1}^o} \right) \right] I_t^o, \quad (4)$$

where P_t is the final good price index and R_t is the interest rate on government bonds. The function $\Gamma(\cdot)$ captures an incurred cost when investment is changed over time, which satisfies the following properties: $\Gamma(1) = 0$, $\Gamma'(1) = 0$, and $\chi = \Gamma''(1) > 0$. The parameters $\beta \in (0, 1)$, $\sigma > 0$, $\varphi > 0$, and $\delta \in (0, 1)$ correspond to the subjective discount factor, the inverse of the intertemporal elasticity of substitution, the inverse of the Frisch elasticity, and the depreciation rate of capital, respectively.

The two exogenous variables, ι_t and $A_{I,t}$, represent shocks to the discount rate that affects the intertemporal rate of substitution between consumption in different periods and shocks to investment productivity, respectively. Both shocks are assumed to follow a first-order autoregressive process with i.i.d. normal error terms such that $\iota_t = (\iota_{t-1})^{\rho_\iota} \exp(\varepsilon_{\iota,t})$, where $0 < \rho_\iota < 1$ and $\varepsilon_{\iota,t} \sim N(0, \sigma_\iota)$; and, similarly, $A_{I,t} = (A_{I,t-1})^{\rho_I} \exp(\varepsilon_{I,t})$, where $0 < \rho_I < 1$ and $\varepsilon_{I,t} \sim N(0, \sigma_I)$.

The first-order conditions associated with the optimal choices of C_t^o , B_t^o , H_t^o , K_{t+1}^o , and I_t^o are respectively given by:

$$\Lambda_t = \kappa \iota_t (X_t^o)^{-\sigma} \left(\frac{X_t^o}{C_t^o} \right)^{\frac{1}{\nu}}, \quad (5)$$

$$\Lambda_t = \beta E_t \Lambda_{t+1} \frac{R_t}{\pi_{t+1}}, \quad (6)$$

$$\Lambda_t (1 - \tau_t) w_t = \iota_t \mu (H_t^o)^\varphi, \quad (7)$$

$$\Lambda_t q_t = \beta E_t \Lambda_{t+1} \left[(1 - \tau_{t+1}) r_{t+1}^k + (1 - \delta) q_{t+1} \right], \quad (8)$$

$$\Lambda_t = q_t \Lambda_t \left[A_{I,t} - \Gamma \left(\frac{I_t^o}{I_{t-1}^o} \right) - \frac{I_t^o}{I_{t-1}^o} \Gamma' \left(\frac{I_t^o}{I_{t-1}^o} \right) \right] + \beta E_t q_{t+1} \Lambda_{t+1} \left[\left(\frac{I_{t+1}^o}{I_t^o} \right)^2 \Gamma' \left(\frac{I_{t+1}^o}{I_t^o} \right) \right], \quad (9)$$

where Λ_t is the nonnegative Lagrange multiplier associated with the budget constraint, $r_t^k = R_t^k / P_t$, $w_t = W_t / P_t$, and q_t is the Tobin's q .

3.2 Rule-of-thumb households

Rule-of-thumb households neither hold bonds nor invest in physical capital. Further, we assume that households do not optimise neither intertemporally nor intratemporally. Hence, they simply consume their labor revenue net of taxes augmented with the transfers received from the government in the course of each period.

$$P_t C_t^r = (1 - \tau_t) W_t H_t^r + T r_t, \quad (10)$$

We follow [Galí et al. \(2007\)](#) in assuming that the rule-of-thumb households work exactly the same hours as the optimizing consumers, implying

$$H_t^r = H_t^o \equiv H_t. \quad (11)$$

3.3 Firms

The final output, Y_t , is defined as a composite of the differentiated finished goods, $Y_t(j)$, $j \in (0, 1)$ denoting a type of finished good,

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\vartheta-1}{\vartheta}} dj \right)^{\frac{\vartheta}{\vartheta-1}},$$

where ϑ is the elasticity of substitution between differentiated finished goods.

Given prices P_t and $P_t(j)$, the finished-good-producing firm j maximizes its profits choosing the production of finished goods, $Y_t(j)$. It solves the following problem

$$\max_{Y_t(j)} P_t \left(\int_0^1 Y_t(j)^{\frac{\vartheta-1}{\vartheta}} dj \right)^{\frac{\vartheta}{\vartheta-1}} - \int_0^1 P_t(j) Y_t(j) dj.$$

Profit maximization leads to the following first-order condition for the demand of finished good j

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\vartheta} Y_t, \quad (12)$$

where the price index of finished goods is

$$P_t = \left(\int_0^1 P_t(j)^{1-\vartheta} dj \right)^{\frac{1}{1-\vartheta}}.$$

The functional form of the differentiated good production is as follows:

$$Y_t(j) = A_t K_t(j)^\alpha H_t(j)^\alpha, \quad (13)$$

We assume the technology to have a permanent impact on real variables. Namely, it follows an $I(1)$ stochastic process $A_t = A_{t-1} u_{A,t}$, with an autocorrelated shock such that: $u_{A,t} = (u_{A,t-1})^{\rho_A} \exp(\varepsilon_{A,t})$, where $0 < \rho_A < 1$ and $\varepsilon_{A,t} \sim N(0, \sigma_A)$.

Firms are price-takers in the markets for inputs and monopolistic competitors in the markets for products. At each processing stage, nominal prices are chosen optimally in a randomly staggered fashion. At the beginning of each period, a fraction $(1 - \theta)$ of final-stage producers can change their prices.

The first-order conditions for the finished-good producing firm j with respect to labor and capital are

$$w_t = (1 - \alpha) \zeta_t(j) \frac{Y_t(j)}{H_t(j)}, \quad (14)$$

$$r_t = \alpha \zeta_t(j) \frac{Y_t(j)}{K_t(j)}, \quad (15)$$

where $\zeta_t(j)$ is firm j 's real marginal cost.

The first-order condition with respect to $P_t(j)$ is

$$\tilde{P}_t(j) = \frac{\sigma}{\sigma - 1} \frac{E_t \sum_{q=0}^{\infty} (\beta \theta)^q \frac{\Lambda_{t+q}}{\Lambda_t} \zeta_t(j) Y_{t+q}(j)}{E_t \sum_{q=0}^{\infty} (\beta \theta)^q \frac{\Lambda_{t+q}}{\Lambda_t} Y_{t+q}(j) \frac{1}{P_{t+q}}}, \quad (16)$$

where $\tilde{P}_t(j)$ is the optimal price of firms allowed to change their prices at period t .

At the symmetric equilibrium, the aggregate price of finished goods is

$$P_t = \left[\theta P_{t-1}^{1-\vartheta} + (1 - \theta) \tilde{P}_t^{1-\vartheta} \right]^{\frac{1}{1-\vartheta}}. \quad (17)$$

3.4 Government and monetary policy

At each period, the fiscal authority purchases the final good, issues new bonds, raises taxes, and pays lump-sum transfers to households. The government's intertemporal budget constraint has the following form:

$$P_t G_t + B_{t-1} + T r_t = \tau_t P_t Y_t + \frac{B_t}{R_t}. \quad (18)$$

Galí et al. (2007) argues that when rule-of-thumb consumers are introduced in the model the time paths of government debt and taxes matter for the dynamics of the model. Hence, we introduce a simple tax rule that has following log-linear form:

$$\hat{tr}_t = \phi_g \hat{g}_t + \phi_b \hat{b}_t, \quad (19)$$

where the arguments of the fiscal rule are defined as follows: $\hat{tr}_t = \frac{tr_t - tr}{y}$, $\hat{g}_t = \frac{g_t - g}{y}$, and $\hat{b}_t = \frac{b_t - b}{y}$; while setting $y_t = \frac{Y_t}{A_t}$, $tr_t = \frac{Tr_t}{A_t}$, $g_t = \frac{G_t}{A_t}$, and $b_t = \frac{B_t}{A_t P_t}$. To capture the role of income taxes as automatic stabilizers, the income tax is allowed to respond to the state of the economy.⁶ This rule reflects the concern of the government about fiscal sustainability over time, which does not allow debt (as a share of output) to overshoot with respect to its steady-state level. Stationary government spending evolve following the standard rule: $g_t = g^{1-\rho_g} (g_{t-1})^{\rho_g} \exp(\varepsilon_{g,t})$, where $0 < \rho_g < 1$ and $\varepsilon_{g,t} \sim N(0, \sigma_g)$.

Finally, we assume that monetary policy is characterized by a Taylor rule with interest rate smoothing, which determines the nominal interest rate in the economy as a function of inflation and output deviations from their steady-state levels in addition to the previous period nominal interest rate. Formally, the monetary rule is defined as follows:

$$R_t = (R_{t-1})^{\rho_R} \left(\frac{\pi_t}{\pi} \right)^{\rho_\pi} \left(\frac{y_t}{y} \right)^{\rho_y} \exp(\varepsilon_{R,t}) \quad (20)$$

where $\varepsilon_{R,t}$ is a serially uncorrelated shock, which is assumed to follow a zero-mean normal distribution with a standard deviation σ_R .

3.5 Aggregation

As households in each of the two groups are identical, the aggregate level in per-capita consumption is specified as follows:

$$C_t = (1 - \gamma)C_t^o + \gamma C_t^r, \quad (21)$$

Since only optimizing households hold financial and physical assets, we obtain the following conditions for aggregate holdings of bonds, capital stock, investment, and dividends

$$B_t = (1 - \gamma)B_t^o, \quad (22)$$

⁶For simplicity, we assume that lump-sum taxes are constant ($\tau_t = \tau$); otherwise, an additional ad hoc fiscal rule should be introduced to the set of equilibrium conditions.

$$K_t = (1 - \gamma)K_t^o, \quad (23)$$

$$I_t = (1 - \gamma)I_t^o, \quad (24)$$

and

$$D_t = (1 - \gamma)D_t^o. \quad (25)$$

3.6 Theoretical results and interpretation

The simulations are conducted based on an initial calibration of the structural parameters. We calibrate the deep parameters to values similar to those encountered in the literature by considering that a period corresponds to one quarter. Namely, the subjective discount rate, β , is set to 0.99. The share of private consumption in the effective consumption basket is calibrated at 0.7. The inverse of Frisch elasticity of labor supply is set to 0.5. The preference parameter η is chosen so that the fraction of hours worked in the deterministic steady state is equal to 0.25. The depreciation rate, δ , is chosen to be 0.025, implying an average annual depreciation rate of capital equal to 10 percent. The share of physical capital in production, α , is assumed to be 0.3. The elasticity of substitution between intermediary goods, ϑ , is equal to 6. The steady-state shares of government spending and debt relative to total output are set to 0.2 and 4×0.6 , respectively; and the average income tax rate is assumed to be equal to 0.2. The autocorrelation of government spending is set to 0.9. We choose the standard values of 0.5, 1.5, and 0.2 for the Taylor rule parameters ρ_R , ρ_π , and ρ_y , respectively. With regard to the remaining parameters—the elasticity of substitution between private and public goods, ν , the inverse of the elasticity of intertemporal substitution, σ , the share of rule-of-thumb consumers, γ , and the Calvo pricing parameter, θ —we run sensitivity analysis based on their calibration to characterize the transmission of the shocks.

We identify two channels through which government spending can impact private consumption in this model. First, the impact of higher spending on wages and taxes directly impact private demand (i.e., the wealth effect). In a standard RBC model, the wealth effect is negative, which crowds out private consumption. In our model, the mechanism through which the wealth effect is activated is tributary to the degree of price rigidity and the share of rule-of-thumb consumers. Second, government spending affects the marginal utility of consumption since it is explicitly introduced in the utility function (i.e., the utility effect). The utility effect depends on the parameters governing the share of private spending in the effective consumption, the elasticity of substitution between private and public spending, and the elasticity of intertemporal substitution. To illustrate

the second propagation mechanism, we derive the effect of a change in government spending on the marginal utility for a given level of consumption from the log-linearized version of Equation (2) and Equation (5), which yields

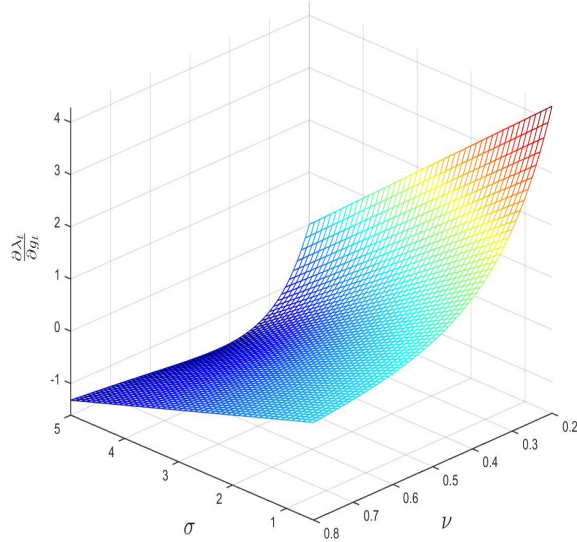
$$\frac{\partial \lambda_t}{\partial g_t} = \left(\frac{1}{\nu} - \sigma \right) (1 - \kappa) \left(\frac{g}{x} \right)^{\frac{\nu-1}{\nu}}$$

where the parameters g and x are the steady-state values of government spending and effective consumption, respectively.

Clearly, this equation shows that the necessary condition for government spending to increase the marginal utility of consumption is when the elasticity of substitution, ν , is lower than the elasticity of intertemporal substitution, $1/\sigma$; meaning that private and public spending are Edgeworth complements. Further, in order to offset the negative wealth effect, a combination of sufficiently low η and σ may lead to a crowding-in effect following a government pending shock.

Figure 5 depicts the impact of the two parameters ν and σ on the marginal utility of consumption. Note that since the elasticity of substitution between private and public consumption enters in the calculation of the steady state—namely, the value of x —, its impact is nonlinear (i.e., at low values of ν the crowding-in phenomena becomes predominant).⁷

Figure 5: Partial derivative of the marginal utility



⁷In particular, solving for the steady state of the model yields the following expression of the marginal utility partial derivative: $\frac{\partial \lambda_t}{\partial g_t} = \left(\frac{1}{\nu} - \sigma \right) (1 - \kappa) \left[\kappa \left(\frac{1 - \delta \alpha^{\frac{\nu-1}{\nu}} \frac{1-\tau}{g/y} - 1 \right)^{\frac{\nu-1}{\nu}} + (1 + \kappa) \right]^{-1}$.

Bouakez and Rebei (2007) find similar results with a log-utility function; but, one can notice the importance of the elasticity of intertemporal substitution in determining the sign and magnitude of the utility effect at different values of v . In particular, if the parameter σ increases (i.e., lower intertemporal elasticity), households become more risk-averse towards consumption fluctuations and, therefore, are more reluctant to consumption adjustments, which dampens the crowding-in effect in response to an increase of public spending.

In order to examine the properties of our model, it is useful to illustrate the sensitivity of the impulse-response functions under alternative calibration scenarios. Figure 6 and Figure 7 display the contour plot of the impact of a government spending shock as we alternatively change the calibration of the share of rule-of-thumb behavior and the degree of price stickiness, γ and θ ; then, the calibration of the elasticities affecting the utility function, v and σ . As the strongest impact of the shock generally occurs at the first period in the benchmark model, the contour plot would generally reflect the contemporaneous reaction to the shock, although not necessarily.

Figure 6 illustrates the magnitude of the wealth effect as a function of the parameters θ and γ .⁸ In the presence of sticky prices an increase in government spending increases aggregate demand, which in turn raises the real wage. Higher current labor income stimulates the consumption of non-optimizing households, and if the weight of those consumers is sufficiently large, aggregate consumption also increases. Hence, the model would produce a crowding in of consumption when both θ and γ are sufficiently high. One can notice that government spending multiplier becomes higher than one around the same area.

Figure 7 helps to visualize the utility effect.⁹ As pointed out earlier, complementarity between private and public spending associated with a high elasticity of intertemporal substitution contributes to dampening the negative wealth effect of higher public spending. Besides, when the parameter v is sufficiently low (i.e., below 0.5), consumption is crowded in regardless of the values of the rest of parameters, including σ . Similarly, the dynamic reaction of output is inversely proportional to the degree of public-private substitutability. Further, under extreme values of the key parameters—very high v and σ —the model can generate negative response of aggregate output (not illustrated).

⁸For a better understanding of the wealth effect, we calibrate the share private consumption if the effective consumption bundle to 1 (i.e., $X_t \equiv C_t$). Therefore, any crowding in effect should be attributed to the presence of rule-of-thumb behavior.

⁹As in the previous scenario, we set the share of rule-of thumb consumers to 0 in order to identify the crowding-in effect that is tributary solely to the utility channel.

Figure 6: Impact of a government spending shock as a function of θ and γ

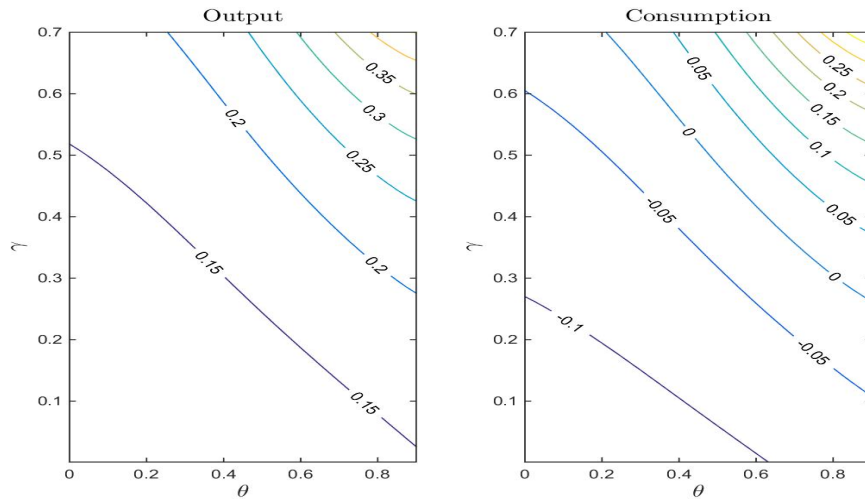
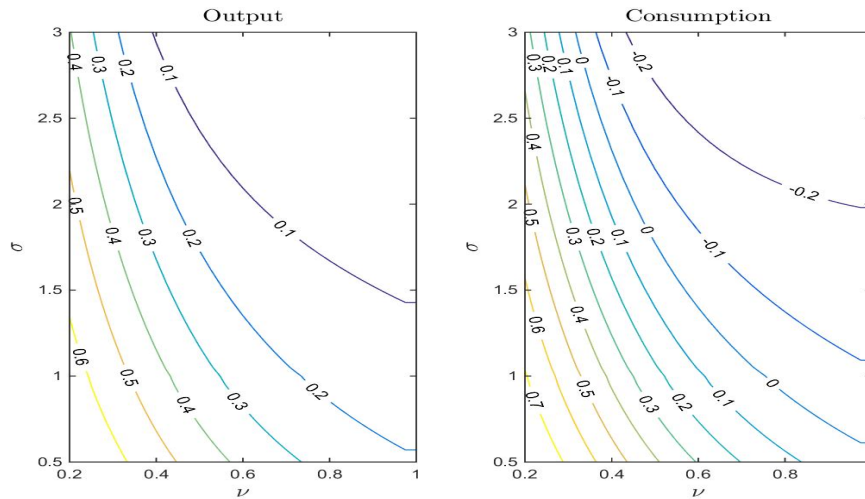


Figure 7: Impact of a government spending shock as a function of ν and σ



4 Estimated Model

4.1 Estimation strategy

In order to account for the sources of change in the transmission of public spending shock, we implement a Bayesian estimation applied to the log-linearized model on rolling-window data as proposed by [Canova \(2006\)](#), [Canova and Sala \(2009\)](#), and [Castelnuovo \(2012\)](#). In particular, we estimate the model using the 1950:I-1980:IV window of data, then we move the first and last ob-

servations of the window by one year exactly as in the BVAR estimation. As in Bayesian practice, the likelihood function and the prior distribution of the parameters are combined to calculate the posterior distribution. The posterior Kernel is then simulated numerically using the Metropolis-Hasting algorithm with 150,000 replications. We use a similar set of variables as in the empirical section: output, consumption, and government spending; but, variables are transformed to become stationary as in the theoretical framework—we use the share government spending to output and the growth rate of consumption. In addition, we incorporate the 3-month treasury bill interest rates and consumer price inflation rates in the set of observed variables in order to have additional information allowing for a better identification of several structural parameters (e.g., Taylor rule parameter and price stickiness). As a reminder, the model comprises five structural sources of fluctuations—shocks to: (i) discount rate; (ii) investment productivity; (iii) trending technology; (iv) government spending; and (v) monetary policy. We also include measurement errors corresponding to each of the observables used in the estimated.

4.2 Estimation results

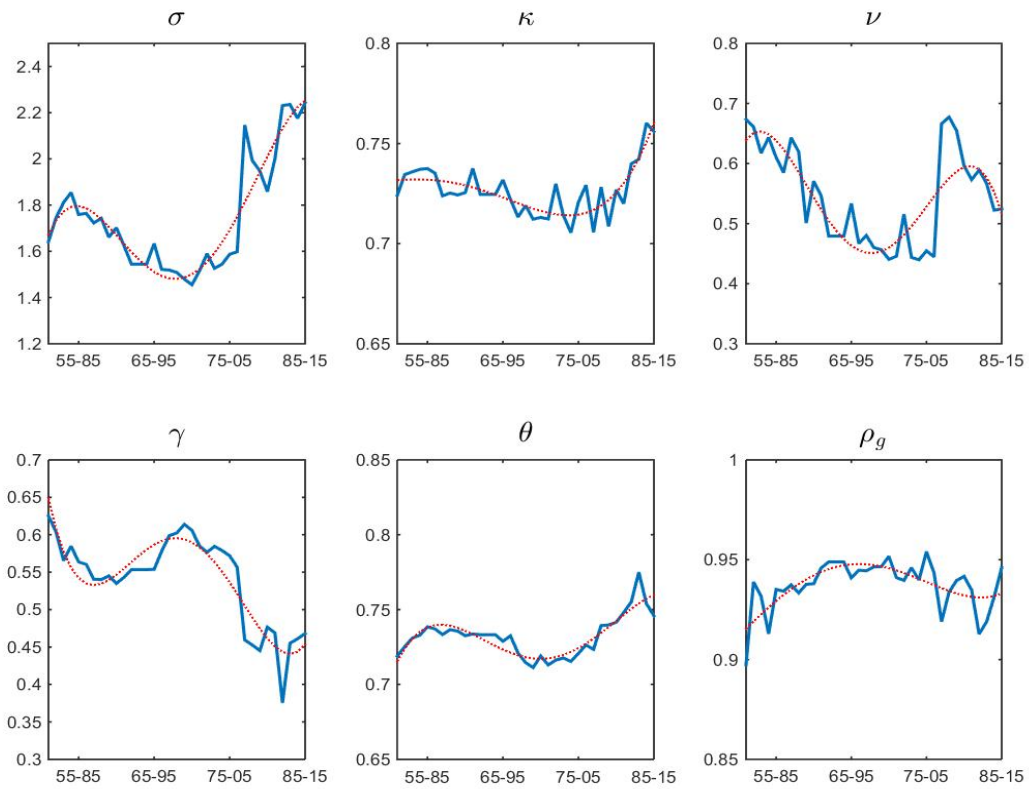
Detailed results of the recursive estimation of the parameters of the model over 36-years rolling samples of data are presented in Section A (to save space Table 1 shows results every three years). Focusing on the parameters that are important for the transmission of government spending shocks (see Figure 8), a number of findings are worth emphasizing. Figure 8 shows that the degree of price stickiness is stable over time—the posterior average stays in a narrow band ranging between 0.7 and 0.8. Interestingly, the persistence of government spending, ρ_g , is stable over time—between 0.9 and 0.95—, which excludes the prospect of attributing changes in the impulse-response functions to an exogenous source.¹⁰

It is also notable that we estimate pronounced changes in the parameters defining the structure of the utility function; namely, the intertemporal elasticity of substitution and the elasticity of substitution between private and public consumption. The inverse elasticity of intertemporal substitution, σ , fluctuates quite a bit, showing a particularly large increase from a trough during 1970:I–2000:IV of about 1.45 to a peak of about 2.24 during 1985:V–2015:IV. The parameter gov-

¹⁰Additional parameters can affect the transmission of the shock, but at a lesser extent. Galí et al. (2007) argue that lowering the share of government spending that is instantaneously financed by lump-sum taxes, as captured by parameter ϕ_g , tends to bolster disposable income and thus consumption especially in the case of rule-of-thumb consumers. Further, the Taylor rule parameters influence the dynamic reactions of interest rates and inflation; and consequently define the wealth effect stance. Estimation results show that these parameters are relatively stable and counterfactual exercises reveal that they do not considerably alter the impulse-response functions.

erning the share of private spending in the consumption bundle, κ , is pretty flat for most of the sample, but then shows a little blip from an early-sample average of 0.72 to about 0.76. The elasticity of private and public consumption, ν , shows a marked fall until late 1970s when it troughs around 0.44; then, it peaks at around 0.68 towards the end of the sample, before falling back to about 0.52 at the very last rolling window of data. Finally, the share of rule-of-thumb consumers, γ , falls steadily through the sample period, from about 0.62 in 1950:I–1980:IV to about 0.36 in early 2000s.^{11, 12}

Figure 8: Iterative estimation



Blue solid lines are the posterior medians of the structural parameter. Red dotted lines correspond to the polynomial trend.

¹¹At the the very end of the sample the posterior average of the parameter γ increases by about 10 percent, which coincides with the inclusion of the financial crisis period data in the estimation sample, reflecting a deterioration of access to financial services.

¹²The posterior estimates of the share of rule-of-thumb-consumers are consistent with estimates obtained by [Campbell and Mankiw \(1989\)](#) for the fraction of liquidity-constrained households in the United States for the pre-1990 period.

Surprisingly, changes in the posterior averages of the parameters σ and ν occur starting around the estimation window 1975:I–2005:IV, approximately 5 years after the break identified in the BVAR estimation. Further, the decline in the share of financially constrained households seems synchronized with the break in the empirical impulse-response functions.

Finally, to assess the performance of the benchmark model relative to standard one, we estimated a version of the model without rule-of-thumb consumers. Results show that the model embedding limited access to financial services is preferable under the assumed priors. The Bayes factor is largely in favor of our benchmark model, where the marginal data density is 14.57 larger on a log-scale which translates into a posterior odds ratio largely in favor of the rule-of-thumb assumption.

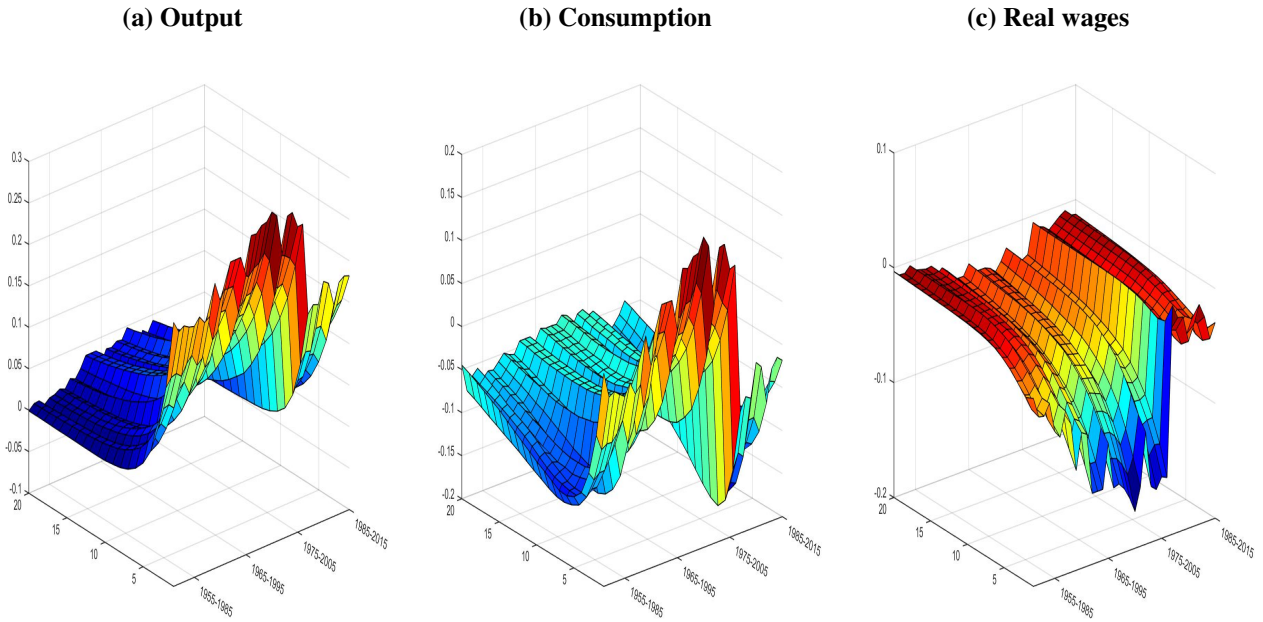
4.3 Time varying impulse-response functions

To assess the impact of the change in the deep parameters on the endogenous propagation mechanisms of government spending shock, we simulate the model based on the posterior distributions of the estimated parameters in each rolling window. Figure 9 shows the 50th percentile of the impulse-response functions per window.

To replicate the stylised facts discussed in Section 2, the parameters that govern the transmission of the shock are expected to raise the crowding-in effect at the beginning of the sample then the crowding-out effect should materialise at the end of the sample. As described in the theoretical section, the crowding-in effect is yielded by (i) a high share of rule-of-thumb consumers, γ ; (ii) a low elasticity of substitution between private and public consumption, ν ; and (iii) a high elasticity of intertemporal substitution, $1/\sigma$. Note that in the early samples posterior averages of these parameters clearly generate an increasing reaction of private consumption owing to high share of rule-of-thumb consumers, a mild substitutability between private and public consumption. From the middle panel of Figure 9, we learn that consumption responds more sharply as the posterior average of the parameter ν declines over time; and the response function becomes clearly positive up to late 1970s. Another distinct feature of Figure 9 is that the non-monotonic change in the impulse-response functions is qualitatively well captured in the estimated theoretical model, where the initial increase of the crowding-in effect seems tributary to the decline of the estimated elasticity of substitution and the higher share of financially constrained households. Then, the crowding-out effect observed in the late samples seems to be yielded by the drop in the elasticity of intertemporal substitution. A more careful analysis of the sources of fluctuations in the co-movements between private and public spending will be discussed in the counterfactual exercises.

Defining the government spending multiplier as the ratio of the impact response of output to the impact response of government spending, one can notice that it is positively correlated with the magnitude of the crowing-in effect on private consumption. The left panel of Figure 9 reveals that the multiplier was higher than 1 in the early samples- with a peak of 1.5 during 1969:I–1999:IV; then, dropped below 1 at the very end of the sample with a trough of 0.9 during 1985:I–2015:IV.

Figure 9: Theoretical model: Impulse-response functions

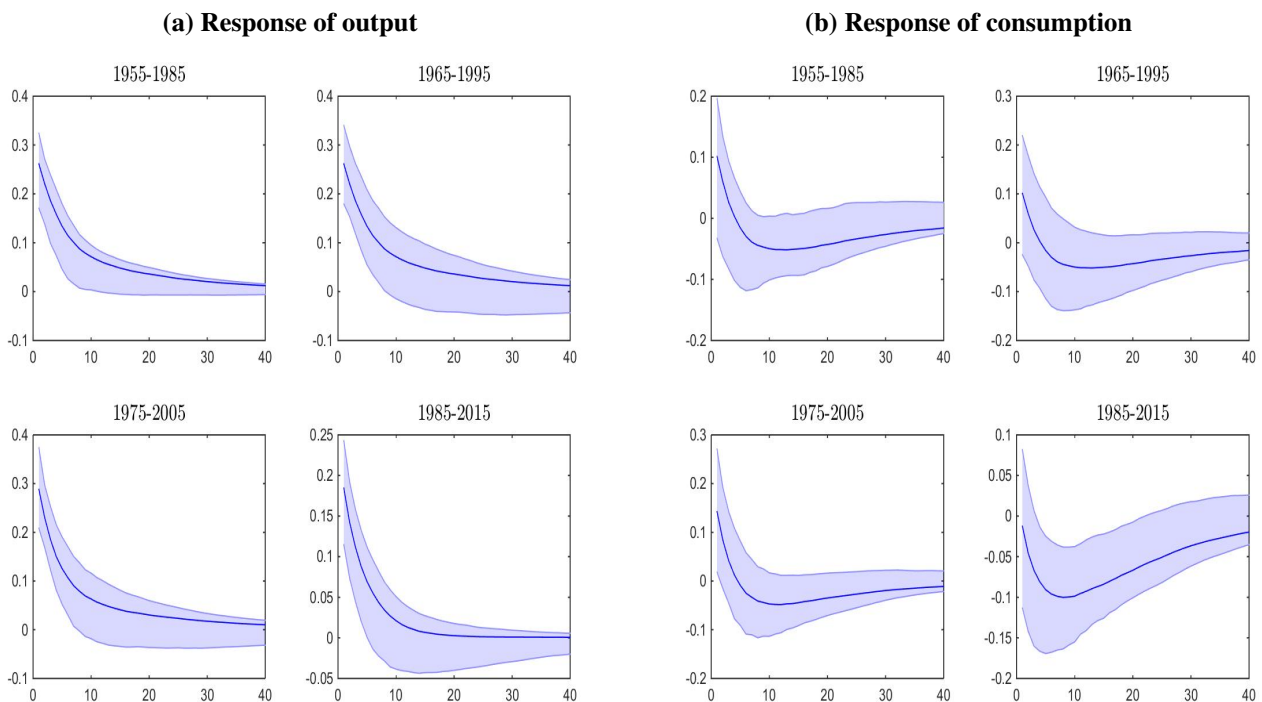


The response of real wages is primarily sensitive to the degree of price rigidity, θ , and the inverse of Frisch elasticity, φ . The higher the degree of price stickiness and the Frisch elasticity the more positive real wages would respond to a government spending shock. The estimation of the model generates a low posterior average labor supply elasticity ranging between 0.24 and 0.48.¹³ Following an increase in aggregate demand, labor supply increases as well as the marginal utility of consumption. It is clear from Equation (7) that if the labor supply elasticity is sufficiently low, the real wage should decline to compensate for higher marginal utility of consumption. Note that in a plain RBC model, real wages react positively to a public spending shock as labor demand increases and the marginal utility of consumption declines. Therefore, the decline in real wages, as reported in the right panel of Figure 9, is exacerbated during the periods where the crowing-in effect is peaking.

¹³This estimate is consistent with the Real Business cycle literature which often adopts a relatively high Frisch elasticity (e.g., Prescott, 1986; King et al., 1988); and with the estimation results reported by Rebei (2014).

Figure 10 shows the degree of significance of the impulse responses where median, 5th percentile, and 95th percentile of the reaction functions are reported. As is clear from left panel of Figure 10, output reactions to a government spending shock do not seem to be significantly different over the different sampling windows. Although the model is unable to generate negative public spending multiplier in the late sampling windows, it still can replicate the decline in the response of output to a government spending shock. The right panel of Figure 10 shows that consumption reaction slowly increases at the early samples, then switches sign in a statistically significant fashion at the latest periods. Consequently, one can argue that the structural change in the degree of financial inclusion, the elasticity of infratemporal substitution, and the elasticity between private and public consumption qualitatively account for the change in the response of consumption.

Figure 10: Impact of government spending shock



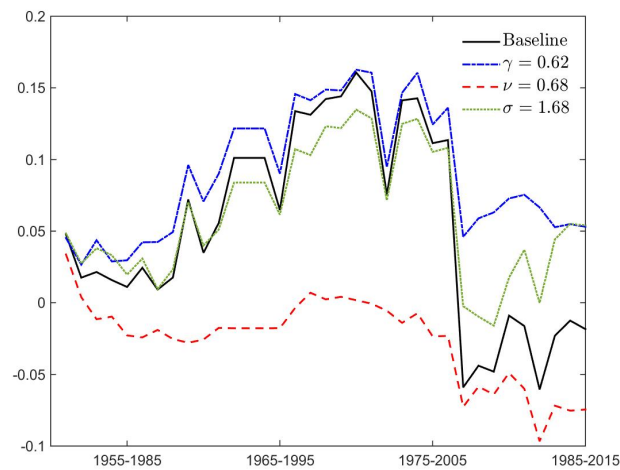
4.4 Some counterfactual exercises

Now, for the sake of understanding the main drivers of the change in the response of consumption to government spending shocks, we proceed with computing the impulse-response functions

assuming that the key parameters γ , ν , and σ are alternatively stable over the sampling windows.¹⁴

Figure 11 shows many interesting features from the counter-factual exercises.¹⁵ We notice that, until periods centered around the 1990s, the impact multipliers are not very sensitive to small changes in the share of rule-of-thumb consumers and the elasticity of intertemporal substitution consistent with the pattern of their posterior averages (see Figure 8). However, these parameters seem to exacerbate the revenue effect in the late sample leading to an abrupt decline of the response of consumption; thereby, further curbing the impact of a government spending shock. As discussed earlier, a positive co-movement of consumption and government spending requires a low elasticity of substitution between private and public consumption. Hence, once ν is set at its initial posterior average, the reaction of consumption is virtually negative during the whole sample as opposed to the benchmark case characterized with declining values of the elasticity.

Figure 11: Change in the contemporaneous reaction of consumption



To summarize, two key results seem to emerge. First, the size of the responses of consumption (and those of output) are initially increasing owing to the rising complementarity between public and private spending. Second, the decline in the impact of government spending in the recent years can be primarily attributed to the increasing access to financial services and the declining elasticity of intertemporal substitution.

¹⁴The parameters γ , ν , and σ are alternatively set to their initial posterior average corresponding to 0.62, 0.68, and 1.65, respectively.

¹⁵Notice that the greater the gap between the counter-factual and the baseline scenarios the most likely the change is attributable to the corresponding parameter.

4.5 Back to the BVAR estimation

Some care needs to be taken in comparing the impulse-response functions conditional to a particular shock generated by a BVAR versus those computed based on a theoretical model. In particular, several sources of discrepancies may lead to a misinterpretation of the results (e.g., the number of lags, the set of endogenous and state variables, and the nature of structural shocks and their identification). As an additional robustness exercise, we propose to use the same BVAR specification—same endogenous variables and number of lags—fed with simulated data from the model, then used to compute impulse-response functions under the same identification strategy. To do so, the model is simulated based on the posterior averages of the estimated parameters and series of 1,000 observations for the endogenous variables are generated. Following the estimation of the three-variable BVAR, impulse-response functions of output and consumption are computed; then, to conclude that the structural model is correct, its predictions about the time series properties of the data—here, in terms of impulse-response functions—need to match those based on actual data.

Figure 12: BVAR responses of consumption

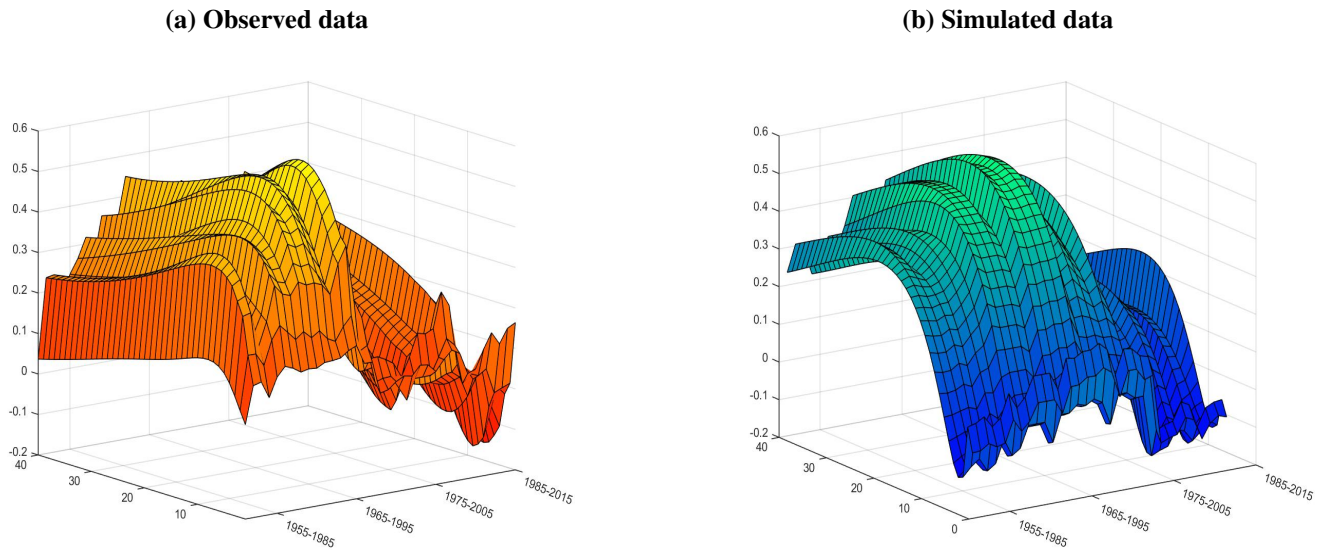


Figure 12 shows the change in the BVAR impulse-response functions of consumption over time using the observed data and the model-based simulated data when putting the parameters at their posterior averages for each rolling-window of data. Interestingly, the model now is able to mimic the hump shape and magnitude of the dynamic reactions of consumption to a government spending shock. In particular, the crowding-in effect gradually increases to reach its maximum around the sampling window 1970:I–2010:IV; then, vanishes afterwards leading to negative co-movement

during the recent periods with a trough of about -0.1 percent on impact during 1985:I–2015:IV.

5 Conclusion

We examine the stability of the positive correlation between private and public consumption conditional on a spending shock, a finding often documented in empirical research. We empirically show that the responses of output and consumption are unstable over time and exhibit two episodes. In the early sampling windows, the shock yields an increasing crowding-in effect with a surge in U.S. fiscal multipliers and Keynesian behavior of private consumption during the periods centered around the 1970s. Then, the correlation starts declining afterwards until generating negative reaction of consumption and a decline in public spending multiplier in the recent periods.

To account for the structural changes leading to the result, we propose a model featuring households limited access to financial services and non-separable private public spending. From a theoretical standpoint the model can imply both cases of crowding-in and crowding-out effects depending on the values of a set of deep parameters. The estimated model accounts for the observed increase followed by the decline of the reaction of output and consumption to public spending shocks. On the one hand, the decline of the elasticity of substitution between private and public spending accounts for bulk of change in the dynamics of responses in the early sampling windows. On the other hand, the increase in household access to financial services and the decline the elasticity of intertemporal substitution significantly reduce the expansionary effects of increased government purchases in the recent periods.

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Appendices

A Estimation Results

Table 1: Recursive Bayesian Estimation

Prior distribution	Posterior averages and 90 percent confidence intervals															
	shape	mean	s.d.	1950 – 1980	1953 – 1983	1956 – 1986	1959 – 1989	1962 – 1992	1965 – 1995	1968 – 1998	1971 – 2001	1974 – 2004	1977 – 2007	1980 – 2010	1983 – 2013	1985 – 2015
ϕ	Gamma	0.5	0.2	0.248 [0.157,0.360]	0.275 [0.154,0.401]	0.310 [0.170,0.42]	0.323 [0.175,0.472]	0.301 [0.171,0.426]	0.296 [0.165,0.420]	0.282 [0.163,0.397]	0.319 [0.173,0.452]	0.316 [0.183,0.445]	0.482 [0.278,0.666]	0.482 [0.284,0.678]	0.420 [0.261,0.579]	0.525 [0.350,0.694]
σ	Normal	1.0	0.5	1.622 [1.035,2.183]	1.861 [1.208,2.429]	1.703 [1.073,2.352]	1.711 [1.041,2.386]	1.579 [0.941,2.190]	1.548 [0.926,2.167]	1.477 [0.824,2.101]	1.618 [0.973,2.136]	1.574 [0.973,2.136]	1.989 [1.345,2.587]	2.028 [1.394,2.657]	2.190 [1.607,2.777]	2.244 [1.637,2.838]
χ	Gamma	5.0	3.0	1.342 [0.196,2.628]	5.395 [0.354,9.409]	9.208 [4.657,13.512]	9.861 [4.293,15.091]	9.279 [4.150,14.433]	9.253 [4.186,14.606]	8.365 [3.108,12.892]	8.645 [3.673,14.128]	7.602 [2.707,12.279]	5.656 [2.497,8.881]	6.333 [2.609,9.977]	5.869 [1.589,9.640]	5.965 [1.389,10.770]
κ	Beta	0.7	0.1	0.720 [0.564,0.885]	0.725 [0.574,0.883]	0.714 [0.551,0.875]	0.720 [0.572,0.880]	0.718 [0.558,0.879]	0.715 [0.565,0.884]	0.706 [0.547,0.873]	0.721 [0.564,0.884]	0.711 [0.546,0.878]	0.713 [0.565,0.876]	0.716 [0.557,0.866]	0.746 [0.593,0.901]	0.747 [0.584,0.910]
ν	Gamma	0.7	0.2	0.707 [0.450,0.948]	0.689 [0.419,0.982]	0.663 [0.426,0.865]	0.640 [0.374,0.927]	0.497 [0.342,0.634]	0.498 [0.329,0.681]	0.476 [0.335,0.623]	0.572 [0.348,0.800]	0.475 [0.348,0.601]	0.708 [0.437,0.984]	0.619 [0.391,0.874]	0.591 [0.344,0.861]	0.575 [0.306,1.044]
γ	Beta	0.5	0.2	0.622 [0.489,0.749]	0.583 [0.462,0.705]	0.537 [0.414,0.657]	0.527 [0.405,0.674]	0.551 [0.422,0.677]	0.582 [0.472,0.702]	0.615 [0.505,0.731]	0.571 [0.454,0.677]	0.568 [0.440,0.696]	0.445 [0.333,0.554]	0.466 [0.347,0.575]	0.464 [0.338,0.591]	0.458 [0.331,0.579]
ϕ_λ	Gamma	0.1	0.1	0.118 [0.029,0.209]	0.118 [0.031,0.203]	0.123 [0.027,0.211]	0.119 [0.029,0.208]	0.112 [0.028,0.195]	0.115 [0.029,0.198]	0.115 [0.029,0.203]	0.125 [0.029,0.220]	0.107 [0.026,0.186]	0.109 [0.027,0.190]	0.112 [0.025,0.192]	0.111 [0.023,0.191]	0.111 [0.027,0.193]
ϕ_θ	Inv-gamma	0.5	4.0	0.228 [0.125,0.319]	0.210 [0.123,0.294]	0.272 [0.144,0.364]	0.260 [0.127,0.376]	0.191 [0.112,0.268]	0.203 [0.120,0.287]	0.192 [0.126,0.258]	0.249 [0.140,0.355]	0.180 [0.116,0.241]	0.397 [0.221,0.566]	0.330 [0.171,0.466]	0.152 [0.097,0.198]	0.143 [0.095,0.191]
θ	Beta	0.8	0.1	0.721 [0.648,0.795]	0.733 [0.663,0.803]	0.729 [0.662,0.792]	0.729 [0.661,0.802]	0.732 [0.667,0.802]	0.730 [0.666,0.797]	0.712 [0.644,0.785]	0.708 [0.641,0.778]	0.716 [0.647,0.788]	0.737 [0.670,0.804]	0.743 [0.674,0.816]	0.749 [0.666,0.834]	0.730 [0.654,0.812]
α	Beta	0.3	0.1	0.194 [0.150,0.236]	0.229 [0.183,0.274]	0.279 [0.235,0.328]	0.272 [0.220,0.317]	0.259 [0.213,0.305]	0.262 [0.216,0.306]	0.259 [0.216,0.302]	0.269 [0.218,0.319]	0.254 [0.212,0.298]	0.303 [0.246,0.358]	0.297 [0.238,0.354]	0.265 [0.200,0.319]	0.220 [0.172,0.267]
ρ_R	Beta	0.5	0.2	0.527 [0.421,0.646]	0.551 [0.457,0.678]	0.456 [0.334,0.573]	0.493 [0.371,0.609]	0.530 [0.413,0.647]	0.558 [0.450,0.679]	0.543 [0.416,0.655]	0.507 [0.384,0.631]	0.481 [0.364,0.595]	0.464 [0.362,0.568]	0.478 [0.376,0.584]	0.563 [0.464,0.661]	0.639 [0.555,0.706]
ρ_π	Normal	1.5	0.2	1.270 [1.035,1.496]	1.383 [1.128,1.612]	1.433 [1.228,1.649]	1.451 [1.228,1.669]	1.438 [1.234,1.670]	1.456 [1.235,1.685]	1.433 [1.203,1.672]	1.441 [1.190,1.660]	1.439 [1.172,1.713]	1.522 [1.325,1.716]	1.510 [1.293,1.709]	1.629 [1.401,1.855]	1.574 [1.298,1.825]
ρ_ν	Normal	0.1	0.1	0.196 [0.144,0.248]	0.205 [0.153,0.258]	0.200 [0.153,0.242]	0.208 [0.163,0.259]	0.218 [0.169,0.267]	0.223 [0.172,0.271]	0.239 [0.190,0.287]	0.235 [0.184,0.287]	0.245 [0.191,0.295]	0.225 [0.186,0.270]	0.214 [0.170,0.257]	0.165 [0.124,0.207]	0.153 [0.114,0.191]
ρ_λ	Beta	0.6	0.2	0.896 [0.844,0.946]	0.908 [0.852,0.966]	0.934 [0.892,0.977]	0.933 [0.888,0.977]	0.946 [0.912,0.979]	0.942 [0.908,0.979]	0.945 [0.911,0.981]	0.938 [0.899,0.979]	0.952 [0.919,0.985]	0.930 [0.896,0.974]	0.931 [0.881,0.978]	0.929 [0.888,0.975]	0.947 [0.916,0.979]
ρ_A	Beta	0.2	0.1	0.347 [0.156,0.531]	0.378 [0.194,0.561]	0.209 [0.052,0.365]	0.216 [0.043,0.375]	0.291 [0.087,0.483]	0.283 [0.081,0.483]	0.321 [0.122,0.514]	0.245 [0.034,0.451]	0.453 [0.242,0.682]	0.173 [0.037,0.301]	0.211 [0.042,0.373]	0.215 [0.053,0.361]	0.207 [0.041,0.354]
ρ_I	Beta	0.6	0.2	0.483 [0.311,0.655]	0.439 [0.299,0.568]	0.443 [0.322,0.558]	0.470 [0.323,0.610]	0.438 [0.308,0.557]	0.442 [0.329,0.562]	0.427 [0.317,0.538]	0.448 [0.331,0.566]	0.409 [0.303,0.521]	0.462 [0.315,0.602]	0.458 [0.346,0.570]	0.631 [0.321,0.742]	0.675 [0.570,0.776]
ρ_λ	Beta	0.6	0.2	0.961 [0.937,0.987]	0.945 [0.917,0.973]	0.918 [0.869,0.965]	0.918 [0.870,0.967]	0.921 [0.877,0.971]	0.922 [0.874,0.967]	0.927 [0.886,0.971]	0.914 [0.865,0.967]	0.935 [0.901,0.971]	0.922 [0.880,0.967]	0.946 [0.915,0.983]	0.952 [0.924,0.982]	0.933 [0.895,0.974]
σ_λ	Inv-gamma	0.01	4.0	0.0157 [0.018,0.018]	0.013 [0.011,0.015]	0.012 [0.010,0.014]	0.011 [0.009,0.013]	0.011 [0.009,0.013]	0.010 [0.008,0.012]	0.009 [0.007,0.011]	0.009 [0.008,0.011]	0.009 [0.008,0.011]	0.008 [0.007,0.010]	0.008 [0.007,0.010]	0.009 [0.007,0.010]	0.008 [0.007,0.009]
σ_A	Inv-gamma	0.01	4.0	0.0052 [0.007,0.007]	0.005 [0.004,0.006]	0.002 [0.001,0.003]	0.003 [0.001,0.005]	0.004 [0.002,0.006]	0.004 [0.002,0.006]	0.004 [0.003,0.005]	0.002 [0.001,0.004]	0.004 [0.003,0.005]	0.002 [0.001,0.002]	0.002 [0.001,0.003]	0.004 [0.003,0.005]	0.004 [0.003,0.005]
σ_I	Inv-gamma	0.01	4.0	0.0505 [0.088,0.088]	0.201 [0.029,0.335]	0.308 [0.176,0.441]	0.282 [0.148,0.412]	0.297 [0.163,0.425]	0.282 [0.158,0.406]	0.259 [0.135,0.375]	0.252 [0.119,0.357]	0.161 [0.069,0.246]	0.188 [0.095,0.278]	0.152 [0.045,0.198]	0.120 [0.032,0.172]	0.101 [0.032,0.172]
σ_R	Inv-gamma	0.005	4.0	0.0013 [0.002,0.002]	0.002 [0.001,0.002]	0.002 [0.001,0.002]	0.002 [0.001,0.002]	0.002 [0.001,0.002]	0.002 [0.001,0.002]	0.002 [0.001,0.002]	0.001 [0.001,0.002]	0.001 [0.001,0.002]	0.001 [0.001,0.002]	0.001 [0.001,0.002]	0.001 [0.001,0.001]	0.001 [0.001,0.001]
σ_I	Inv-gamma	0.005	4.0	0.0062 [0.010,0.010]	0.008 [0.005,0.012]	0.008 [0.005,0.010]	0.008 [0.005,0.011]	0.008 [0.005,0.011]	0.008 [0.005,0.011]	0.007 [0.005,0.011]	0.007 [0.004,0.009]	0.007 [0.004,0.010]	0.006 [0.004,0.008]	0.008 [0.004,0.011]	0.007 [0.004,0.011]	0.005 [0.003,0.008]