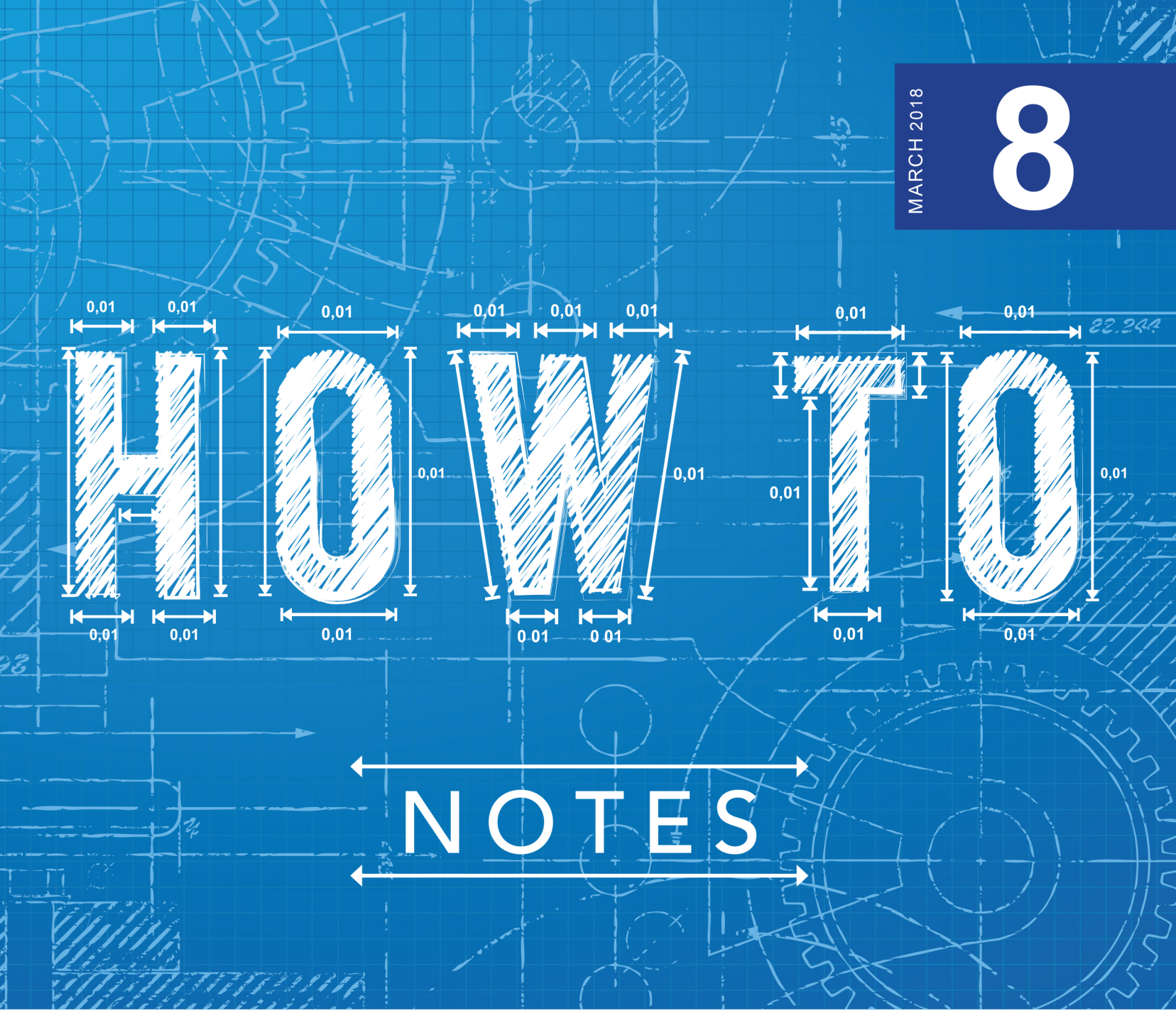

FISCAL POLICY

How to Calibrate Fiscal Rules

A Primer

FISCAL AFFAIRS DEPARTMENT | INTERNATIONAL MONETARY FUND

Cover Design: IMF Multimedia Services
Composition: The Grauel Group

Cataloging-in-Publication Data
Joint Bank-Fund Library

Names: Eyraud, Luc. | Baum, Anja, 1985- | Hodge, Andrew. | Jarmuzek, Mariusz. | Kim, Young. | Mbaye, Samba. | Türe, Elif. | International Monetary Fund. Fiscal Affairs Department.
Title: How to calibrate fiscal rules : a primer / this note was prepared by a team led by Luc Eyraud and including Anja Baum, Andrew Hodge, Mariusz Jarmuzek, Young Kim, Samba Mbaye, and Elif Ture.
Other titles: How to notes (International Monetary Fund)
Description: [Washington, DC] : Fiscal Affairs Department, International Monetary Fund, 2017. | How to notes / International Monetary Fund | December 2017. | Includes bibliographical references.

Identifiers: ISBN 978-1-48433-730-1
Subjects: LCSH: Fiscal policy—Rules and practice. | Calibration.

Classification: LCC HJ192.5.E972 2017

Publication orders may be placed online, by fax, or through the mail:
International Monetary Fund, Publication Services
PO Box 92780, Washington, DC 20090, U.S.A.
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HOW TO CALIBRATE FISCAL RULES: A PRIMER

This note provides guidance on how to *calibrate* fiscal rules; that is, how to determine the thresholds (ceiling, floor, or target) for specific fiscal aggregates constrained by rules. The note focuses, more specifically, on the calibration of the debt, balance, and expenditure rules.

It is one of two guidance notes on the design of fiscal rules; the other note focuses on rule *selection* (IMF, 2018). The two exercises are linked: if a fiscal framework had to be built from scratch, rules would need to be selected and calibrated at the same time. However, to simplify the analysis and because some countries already have fiscal rules embedded in their laws, this note examines the issue of calibration on its own.

The note is not exhaustive or definitive. There are many approaches to determining thresholds for rules. This note presents a selection of methods that are intuitive, are simple to implement, and leave room for policy judgment. The methods are based on past IMF work, including analytic and policy papers, technical assistance missions, and training. Future work might enhance or modify the framework presented here. Another important point is that the objective of this note is to provide practical guidance; it is not a substitute for the use of full-fledged macroeconomic models, which have the potential to capture all macroeconomic and fiscal implications of alternative calibrations.

The note is divided into four sections. The first section discusses general principles used to calibrate rules. The second section reports international evidence on the numerical ceilings used in existing rules. The third and fourth sections provide guidance on the calibration of the debt ceiling and the operational rules (fiscal balance and expenditure rules). EViews econometric and Excel files and manuals accompany this note to assist economists with country-specific calibration exercises. They are available upon request from the authors.

This note was prepared by a team led by Luc Eyraud and including Anja Baum, Andrew Hodge, Mariusz Jarmuzek, Young Kim, Samba Mbaye, and Elif Ture. The note received useful comments from IMF staff.

General Principles for Calibration of Rules

The calibration methodology presented in this note is based on four general principles:[1]

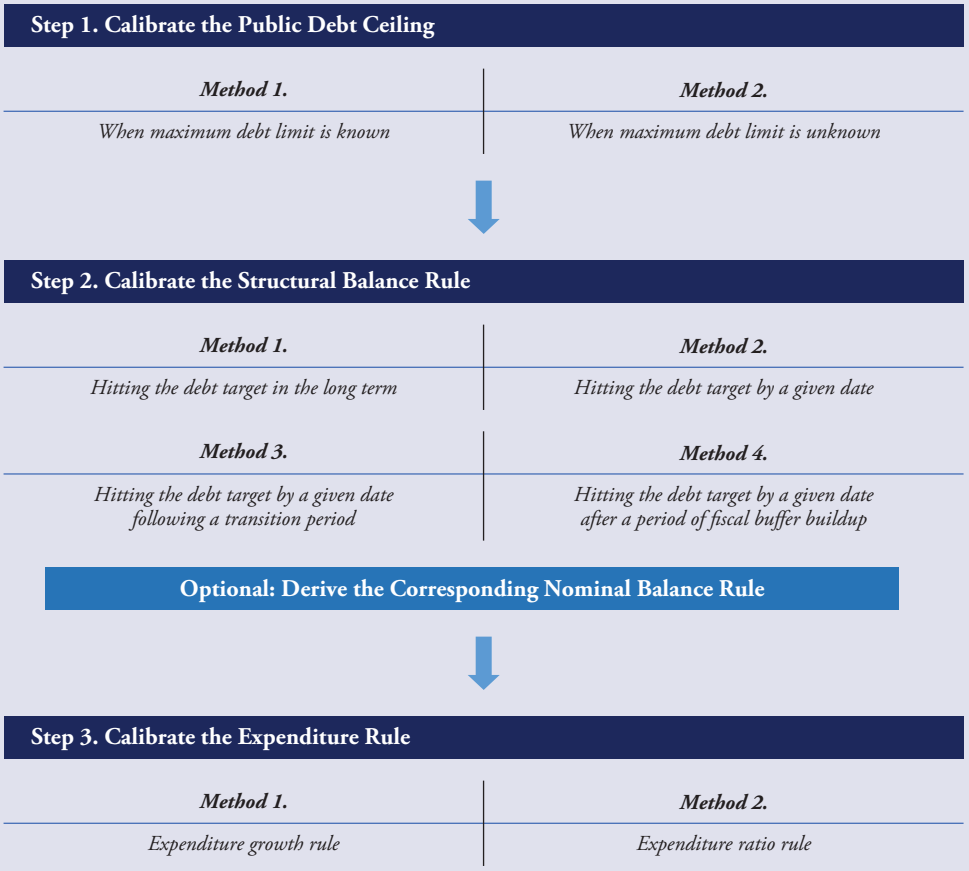
- *Calibration should be comprehensive and consistent*. Most countries have more than one fiscal rule (Schaechter and others 2012). To minimize the risks of inconsistency and conflict among rules, the fiscal framework should be assessed as a whole and the thresholds should be calibrated in a consistent manner. In particular, there should be a clear relationship between the debt and fiscal balance ceilings.
- *Calibration should be sequenced*. By analogy with monetary policy, a well-designed fiscal framework could set targets for both final and intermediate objectives of fiscal policy. Therefore, the framework could be structured around two types of rules (Eyraud and Wu 2015). The first type, called fiscal anchor, ensures that the framework achieves its final objective, which is to preserve fiscal sustainability; a natural candidate is a ceiling on the debt-to-GDP ratio.[2] The framework should also include operational rules that are under the control of governments while also having a close and predictable link to debt dynamics (for instance, a ceiling on the fiscal deficit or a cap on expenditure). One implication of this dual structure is that the calibration exercise should be sequenced (see Box 1 for a summary of the overall approach developed in this paper). The debt ceiling should preferably be set first, taking into account debt sustainability and the need to protect the country against adverse shocks. Then the operational rules can be calibrated from the debt ceiling.

[1]These principles apply to the exercise of rule calibration, not rule selection. The note "How to Select Fiscal Rules—A Primer" (IMF, 2018) presents criteria to assess the strengths and weaknesses of various fiscal rules, including their ability to strike a balance between sustainability and stabilization objectives, ease of monitoring, simplicity, resilience to shocks, and link to the budget process.

[2]While fiscal rules can serve different goals, the focus here is primarily on rules that promote fiscal sustainability.

Box 1. How to Calibrate Fiscal Rules in Three Steps

Box Figure 1. How to Calibrate Fiscal Rules in Three Steps

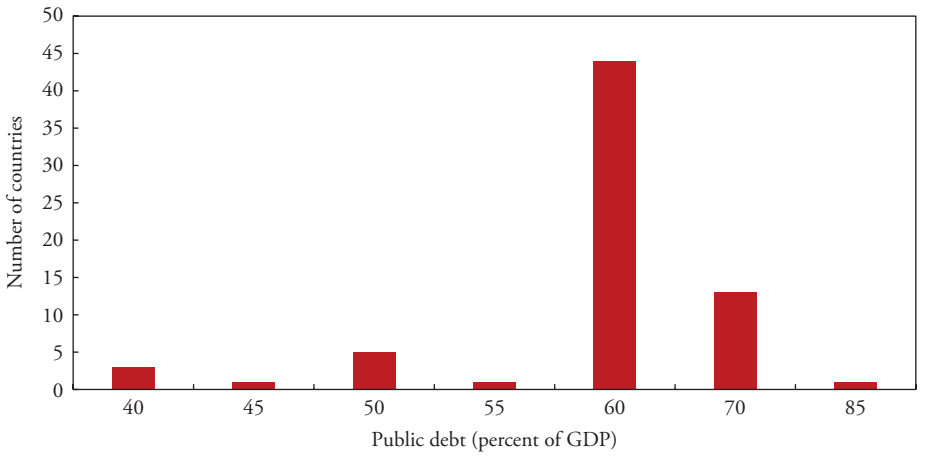


- *Calibration should be prudent*. Governments should take fiscal risks into account in setting fiscal targets and should preserve buffers to accommodate shocks. Not all risks can be mitigated, insured, or provisioned for through contingency funds in the budget; therefore, it is essential to create fiscal headroom by setting prudent debt and deficit ceilings (IMF 2016).
- *Calibration should be updated regularly but not too frequently*. Fiscal rules are designed to be robust to macroeconomic shocks, but conditions may evolve over time. For example, a government might change the way it responds to debt developments and the output gap. Countries that have had procyclical fiscal policies in the past may gradually move toward a countercyclical stance (Frankel, Vegh, and Vuletin 2013). The calibration should therefore be periodically updated. But the update process should not be too frequent; what separates rules from annual budget targets is that rules are long-lasting constraints on fiscal policy. According to the IMF definition, rules are numerical targets fixed in legislation that are binding for a minimum of three years. A review of fiscal rules could, for instance, be part of the electoral cycle (Schaechter and others 2012).

International Experience on Fiscal Rule Thresholds

Fiscal rules have become more common worldwide over the past two decades. In the early 1990s, only five countries had fiscal rules in place. By the end of 2015, the number of countries with at least one national or supranational fiscal rule surged to 92, of which more

Figure 1. Distribution of Public Debt Ceilings



Source: IMF Fiscal Rules Database.

than 60 percent were emerging market and developing economies.[3]

Ceilings on public debt are a common feature of rule-based fiscal frameworks. As of 2015, about 70 countries worldwide had a fiscal framework with an explicit cap on public debt. Debt rules are generally set in gross rather than net terms (gross debt minus financial assets) because it is hard to determine which government assets are truly liquid, particularly in times of financial stress. Also, the concept of net debt is less transparent than gross debt and more difficult to communicate to the public.

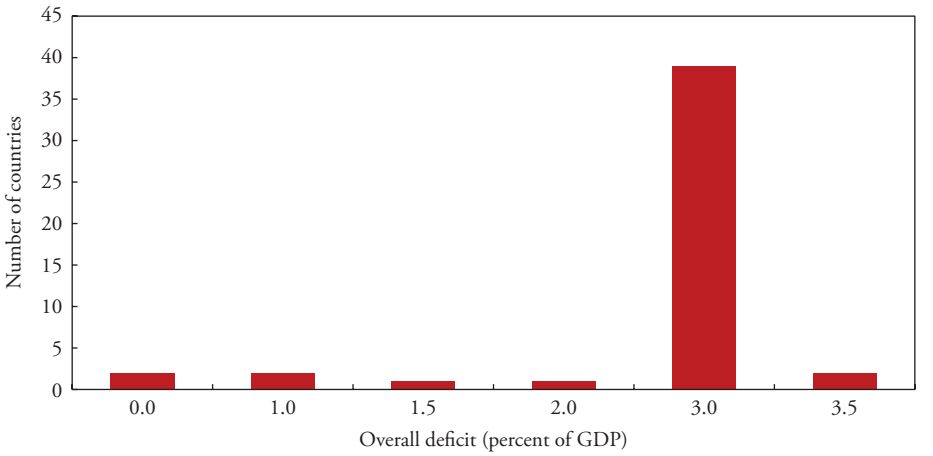
Gross debt ceilings can vary significantly across countries, but frequently range between 60 percent and 70 percent of GDP (Figure 1). The clustering of ceilings around these values reflects the strong representation of supranational rules. About three-quarters of countries with a debt ceiling are members of supranational unions. For instance, the European Union and the Eastern Caribbean Currency Union impose a debt ceiling of 60 percent of GDP, while the Central African Economic and Monetary Community and the West African Economic and Monetary Union both impose a cap of 70 percent of GDP on public debt. In the East African Community, member countries have adopted a debt ceiling of 50 percent of GDP in net present value terms during the convergence process toward the East African Monetary Union. Excluding supranational rules, the 60 percent threshold remains the most common among national debt rules.

Most fiscal frameworks rely on additional rules to operationalize their debt ceilings. This occurs because debt trajectories are not directly controlled by policymakers. More than 80 percent of countries with a debt ceiling also have rules imposing constraints on the (nominal or structural) budget balance; among those, almost a third also have expenditure ceilings in their fiscal frameworks. For ***nominal budget balance rules***, the 3 percent deficit ceiling is dominant throughout the world; it has been adopted by the European Union, the West African Economic and Monetary Union, and the East African Community (Figure 2). With regard to ***structural budget balance rules***,[4] ceilings have a wider distribution (Figure 3). The concentration of deficit ceilings between 0 and 1 reflects the adoption of medium-term budgetary objectives (MTOs) in European Union countries, as well as the structural balance ceiling (using average oil revenues instead of actual oil revenues) used in the Central African Economic and Monetary Community. A few countries impose two structural deficit ceilings—one at the supranational level and another at the national level—possibly with different thresholds. For instance, Germany has structural deficit ceilings of 0.5 percent of GDP (supranational medium-term objective) and 0.35 percent of GDP for the federal government (national rule).

[3]The stylized facts presented in this section are based on the 2017 version of the IMF Fiscal Rules Database, available at http://www.imf.org/external/datamapper/fiscalrules/map/map.htm. It covers 96 advanced, emerging, and developing economies.

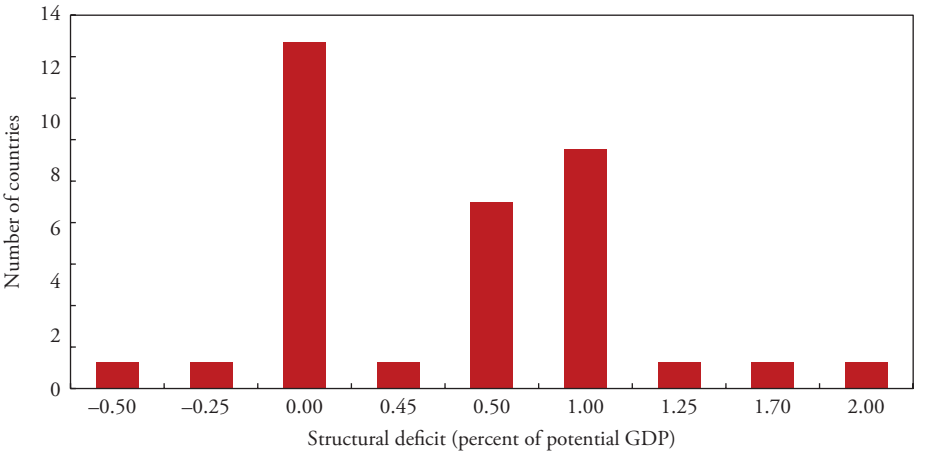
[4]In this note, *structural balance rules* are defined as rules that correct the nominal fiscal balance for cyclical and transitory factors (including, in some cases, the commodity price cycle).

Figure 2. Distribution of Nominal Deficit Ceilings



Source: IMF Fiscal Rules Database.

Figure 3. Distribution of Structural Deficit Ceilings



Source: IMF Fiscal Rules Database.

Expenditure rules are less common than debt or budget balance rules. These rules appear in several forms (Cordes and others 2015). In a majority of cases, expenditure rules consist of an explicit cap on nominal or real expenditure growth. Expenditure growth is capped either by a fixed numerical ceiling (in the range of 2 percent to 4 percent for real growth rules in our sample) or by a measure of medium- to long-term growth. In a few emerging market economies, expenditure rules apply to the expenditure-to-GDP ratio, with ceilings in the range of 30 percent to 40 percent of GDP. At the supranational level, only the European Union has imposed an expenditure rule on member states. This rule, called the *expenditure benchmark*, caps the annual growth of primary expenditure with long-term nominal GDP growth (European Commission, 2016). One of its characteristics is that it also corrects for revenue measures, meaning that spending growth cannot exceed medium-term growth unless the additional spending is matched by new discretionary revenue measures.

Calibrating the Public Debt Ceiling

This section outlines how to set the ceiling of the fiscal anchor (the debt ratio), with an approach based on precautionary considerations. Countries face sudden increases in debt because of negative macroeconomic shocks and the realization of contingent liabilities. One way to calibrate the debt ratio is to ensure with

high probability that debt is kept under control despite these shocks (for a general discussion, see IMF 2016).

Specifically, this section presents two alternative methods. The first method assumes that there is a known *maximum debt limit* beyond which debt dynamics spiral out of control. This method was developed by Debrun, Jarmuzek, and Shabunina (2017) for advanced and emerging market economies, while Baum and others (2017) adapted the method to low-income countries. In this approach, the debt rule ceiling should be set low enough to ensure with high probability that debt will remain below the debt limit even when negative shocks occur. The second method does not rely on an explicit debt limit; it selects the debt ceiling so that debt can be stabilized following negative shocks without breaching a *policy limit*—a maximum feasible level of the primary balance.

The debt ceiling set using either of these methods can be based on any institutional coverage (for example, central or general government).[5] However, it is important to ensure that the same institutional coverage is used throughout the calibration exercise. In other words, if the debt ceiling is computed for the central government, the related deficit and expenditure ceilings will also apply to the central government.

This section presents the two methods in an intuitive way. More details on the algorithms and formulas are provided in Appendix 1 and the manuals accompanying this note.

Method One: Calibrating the Debt Rule Ceiling When the Maximum Debt Limit Is Known

In this method, stochastic simulations are used to calibrate the debt rule ceiling by computing a safety margin below a known debt limit. The calibration is done in three steps. The first step is to identify the maximum debt limit. Second, the distribution of macroeconomic and fiscal shocks is estimated and used to simulate potential debt trajectories over a medium-term projection horizon. The results of these simulations are summarized in a fan chart. The third step identifies the debt rule ceiling, which is a sufficiently low starting level for debt (in the first year of the projection horizon) such that there is a safety margin and debt will remain below the maximum debt limit over the medium term with high probability, despite the potential for negative shocks. The debt rule ceiling is computed as the maximum debt limit minus the safety margin.

This method was developed by Debrun and others (2017) and Baum and others (2017). Earlier research on stochastic simulations of debt trajectories includes IMF (2003a), Ferrucci and Penalver (2003), Garcia and Rigobon (2004), and Celasun, Debrun, and Ostry (2007). The following paragraphs describe the three steps in greater detail.

Step 1: Setting the Debt Limit

In the first step, an assumption is made on the maximum debt limit, which is country-specific and can depend on many factors. There are many ways to set the maximum level of debt that a country does not wish to cross. Given the lack of consensus in the literature, a sensitivity analysis based on alternative debt limit estimates could be warranted. The following are two possible approaches.[6]

- *Risk of debt distress*. The debt limit could be thought of as the level beyond which it is believed that a debt distress episode will occur with heightened probability (for example, default, restructuring, or large increases in sovereign spreads). For instance, for emerging market economies and advanced economies, the IMF Debt Sustainability Analysis (DSA) framework for Market Access Countries uses benchmarks of 70 percent and 85 percent of GDP, respectively (IMF 2013a). For low-income countries, the IMF DSA framework has benchmarks for public debt in nominal terms in the range of 49 percent to 75 percent,[7] depending on institutional quality (IMF and World Bank 2012).
- *Risk of growth slowdown*. The debt limit can be chosen as the level beyond which it is believed growth will decline. For instance, Cecchetti, Mohanty, and Zampolli (2011) find that debt becomes a drag on growth when it exceeds around 85 percent of GDP in Organisation for Economic Co-operation and Development (OECD) countries. See Cottarelli

[5]IMF (2009) provides guidance about the appropriate institutional coverage of fiscal rules.

[6]For a conceptual framework on debt limits as well as empirical estimates, see Ghosh and others (2013).

[7]The corresponding benchmarks for the *present value* of public debt range from 38 percent to 74 percent of GDP: IMF(2013b). For low-income countries, the present value of debt can differ from its nominal value, particularly in countries that rely on concessional external debt, where the nominal value may be a poor indicator of the debt service burden in the near term.

Box 2. Simulating Macroeconomic Variables

The econometric files accompanying this note allow three possibilities for simulating macroeconomic variables over the medium-term forecast horizon.

VAR forecasts. An econometric Vector Autoregression (VAR) model is estimated for key macroeconomic variables at a quarterly frequency. Multiple potential trajectories for these variables are obtained by generating forecasts with the estimated VAR model, adding shocks each period. The shocks are drawn from a distribution calibrated using the estimated VAR residuals (and their estimated variance-covariance matrix). The VAR methodology works best when quarterly macroeconomic data are available, providing a sufficiently large sample size for econometric estimation.

Drawing directly from the joint distribution. If only annual data are available, a joint normal (or Student's *t*) distribution of the macroeconomic variables can be calibrated using the historical sample mean, variance, and covariance of these variables as parameters. Multiple potential trajectories of the macroeconomic variables are generated by drawing shocks directly from the joint distribution.

Forecasting with ad hoc path of macroeconomic variables. The user may specify a mean for each of the macroeconomic variables in each period of the forecast horizon. These mean values may correspond to the user's own forecast. Multiple potential trajectories of the macroeconomic variables can then be generated by adding shocks around the manually entered mean values. The shocks are produced by drawing from the joint distribution of the variables (described in the previous paragraph).

and Jaramillo (2012) for further discussion of the relationship between public debt and growth.

Step 2: Estimating the Effect of Shocks on Debt

The second step is to perform stochastic simulations to gauge the potential impact of macroeconomic and fiscal shocks on debt over the medium term. This requires estimating the joint distribution of macroeconomic variables (Box 2).[8] The set of variables to include in the joint distribution varies by country group. For advanced and emerging market economies, the econometric files accompanying this note use GDP growth, interest rates on government debt, and the exchange rate; for low-income developing countries, the terms of trade gap and disbursements of foreign loans are also included. Multiple simulations are carried out using the joint distribution.[9] Each simulation produces a path for macroeconomic variables over a medium-term projection horizon, where the variables have been subject to shocks in each period.

Medium-term debt trajectories consistent with each simulated path of macroeconomic variables are obtained from the system of simultaneous equations formed by the debt accumulation equation (that is, the government budget constraint) and a Fiscal Reaction Function (FRF) in which the level of the primary balance may respond to the level of debt and realizations of macroeconomic variables (see Box 3 for a discussion of the choice of the function). The fiscal reaction function includes a fiscal shock realized each period.[10] Debt trajectories produced using stochastic simulations can be summarized in a fan chart.

Step 3: Calibrating the Debt Ceiling

The final step is to calibrate the debt rule ceiling. This is done by choosing an initial level of debt (in the first projection year) so that debt remains below the maximum debt limit with a chosen probability over the medium term, despite the occurrence of negative

[8]The joint distribution can be specified using either a Vector Autoregression (VAR) or by directly calibrating a joint normal (or Student's *t*) distribution based on historical co-movement between variables. Alternatively, if these variables are expected to have different means to those observed in the past, the programs attached to this note allow the user to specify manually the mean of the forecast variables around which shocks are added. See Box 2 and Appendix 1 for more details.

[9]In both Methods One and Two, the macroeconomic shocks are drawn from symmetric (normal or Student's *t*) distributions. This does not reflect empirical evidence that shocks can be skewed to the downside in reality (Escolano and Gaspar 2016). Nonetheless, the fan charts may be asymmetric, because the impact of the shocks on debt paths (in the form of automatic debt dynamics) depends on the level of debt. For example, an adverse shock on the interest-growth differential will increase debt by more when the initial debt level is higher.

[10]The distribution of fiscal shocks is calibrated based on deviations between actual fiscal responses observed (that is, actual levels of the primary balance) and the fiscal response predicted by the FRF over the sample.

Box 3. Specifying the Fiscal Reaction Function

A fiscal reaction function (FRF) is a rule linking a particular level of the primary balance to prevailing macroeconomic and fiscal conditions. Different specifications of the FRF can be used depending on the scope of the calibration exercise:

- If the fiscal framework has only one rule—a debt rule—it makes sense to calibrate the debt ceiling using an FRF based on past behavior (Option 1). This is because the fiscal response is not constrained by any operational rule.
- If the fiscal framework includes at least one operational rule, the FRF should, in principle, behave according to this rule. In practice, this would create some circularity: the calibration of the debt rule would depend on the choice of the operational rule and, conversely, a given debt ceiling would translate into a specific target for the operational rule. To avoid this circularity, a simple and practical solution is to use a normative FRF that departs from the country's historical behavior and ensures that the policy response is "well behaved" and consistent with debt sustainability (Option 2). The fiscal reaction function is normative in the sense that it ensures that debt would converge to a long-term target level in absence of further shocks.
- A third option is to set an ad hoc path for the primary balance path over the projection period (Option 3).

Option 1. Estimated FRF Based on Past Behavior

The FRF applicable for either an advanced or emerging market economy is based on the specification of Bohn (1998). Other research on fiscal reaction functions in advanced or emerging market economies includes Abiad and Ostry (2005), Celasun and Kang (2006), and IMF (2003b).

The coefficients of the FRF are estimated econometrically to capture historical fiscal behavior. Econometric estimation is carried out using separate panels for advanced or emerging market economies, so that the estimated FRF coefficients differ among income groups. The specification to be estimated is

$$pb_{it} = \alpha_i + \beta_1 pb_{it-1} + \beta_2 ygap_{it} D_{it} + \beta_3 ygap_{it}(1 - D_{it}) + \rho d_{it-1} + \varepsilon_{it} \quad (3.1)$$

where pb_{it} is the primary balance (as a ratio of GDP) of country i in year t, d_{it} is debt (as a ratio of GDP), $ygap_{it}$ is the output gap, D_{it} is an indicator variable taking the value of 1 when the output gap is positive, α_i is the country-specific intercept term (fixed effect), and ε_{it} is a random error term, $\varepsilon_{it} \sim N(0, \sigma^2)$. The FRF allows for an asymmetric response to the output gap, so that the primary balance may deteriorate more when the output gap is negative than it improves when the gap is positive ($\beta_3 > \beta_2$). The output gap is projected over the forecast horizon using GDP growth forecasts obtained from simulations (based on the joint distribution of macroeconomic variables) combined with an Hodrick-Prescott filter to estimate potential output.

A less standard FRF can be used to capture fiscal behavior in low-income countries. Econometric evidence suggests that the primary balance may not react to public debt and the output gap in low-income countries (Baum and others 2017). Terms of trade movements are important for commodity-exporting countries that rely on commodity-based revenue. Disbursements of external financing can be treated as exogenous determinants of the primary balance, which tends to fluctuate with the availability of project-linked external financing and budget support. Thus,

$$pb_{it} = \alpha_i + \beta_1 pb_{it-1} + \beta_2 totgap_{it} D^c_{it} + \beta_3 totgap_{it}(1 - D^c_{it}) + \beta_4 extdis_{it} + \varepsilon_{it} \quad (3.2)$$

where $totgap_{it}$ is the deviation of terms of trade from trend, D^C_{it} is an indicator variable for commodity exporters, and $extdis_{it}$ are disbursements of external public debt (as a ratio of GDP). When using this FRF, terms of trade and disbursements of external financing are included in the joint distribution of macroeconomic variables. Simulations based on this joint distribution are used to project the terms of trade gap and external financing disbursements.

Each type of FRF includes a fiscal shock realized each period. The distribution of fiscal shocks is calibrated on the basis of the estimated residuals of the FRF; these residuals correspond to the deviations between actual fiscal responses observed (that is, actual levels of the primary balance) and the fiscal response predicted by the FRF in the sample.

Option 2. Normative FRF

The normative FRF captures the fiscal behavior necessary to stabilize debt at a long-term target level after shocks dissipate. The specification is the same as equation (3.1) above, and parameters are estimated econometrically using panel data, except for ρ. This parameter is calibrated to ensure that debt will converge to a long-term target level d^* in the absence

Box 3. Specifying the Fiscal Reaction Function *(continued)*

of further shocks. The long-term target should be a sustainable level of debt that is politically acceptable in the country-specific context. For advanced and emerging market economies, one option is to set the long-term target level at 60 percent of GDP. The appropriate setting for ρ is the value consistent with a steady state of the system of simultaneous equations formed by the debt accumulation equation and fiscal reaction function (3.1) when debt is set equal to its long-term target level d^*, with growth g and the real interest rate on government debt r set to their long-term steady state values. Algebraically, the appropriate value of ρ can be expressed as

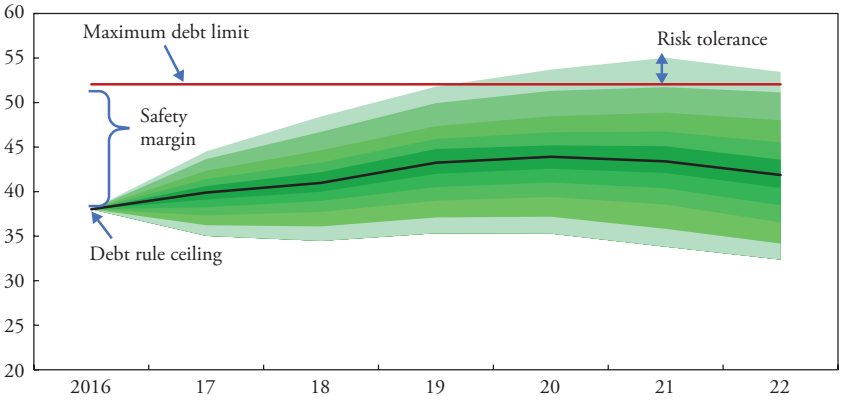
$$\rho = (1 - \beta_1)\left(\frac{r - g}{1 + g}\right) - \frac{\alpha_i}{d^*} \qquad (3.3)$$

The long-term steady state values for growth and the real interest rate can be proxied by imposing steady state on an estimated VAR model of the type described in Box 2 (that is, imposing that lagged values of variables must equal current values) and then solving for the vector of steady state values in terms of the estimated VAR coefficients. The econometric files accompanying this note compute these steady state values automatically. If the interest-growth differential is low and the long-term debt target d^* is close to the current level of debt, it is possible that the value of ρ computed using the formula takes on a smaller (positive) value than in the estimated FRF from option 1. In this case, debt will converge to the long-term target (in the absence of shocks) even if fiscal policy is less responsive to debt than it has been in the past.

Option 3. Ad Hoc Primary Balance Path

It is also possible for the user to impose a mean value for the primary balance in each period over the medium-term horizon. This may correspond to the user's baseline forecast (for instance, based on IMF *World Economic Outlook* projections). A fiscal shock can be added to this mean value each period to capture uncertainty about future fiscal behavior.

Figure 4. Method 1: Public Debt Fan Chart
(Percent of GDP)



Source: IMF Staff Calculations.
Note: Debt is projected over a six-year horizon from 2017-2022. The last year before projections begin is 2016.

shocks.[11] Changing the starting debt level would shift and tilt the entire fan chart. The tolerated probability of breaching the debt limit can be chosen based on the level of risk a government is willing to accept. A probability of 10 percent is used as a baseline. Figure 4 shows an example of a debt ceiling calibration over a six-year horizon, with a safety margin below the maximum debt limit. If the current debt level is above the debt ceiling, then the country is not maintaining a sufficient safety margin given the degree of risk tolerance. The debt ceiling and the size of the safety margin will

[11]The stochastic simulations are based on symmetric laws and draw both positive and negative shocks, but only negative shocks matter for the calibration method. Indeed, the debt ceiling is calibrated so that debt remains below the maximum debt limit, except in a small percentage of cases when particularly bad combinations of shocks are realized.

be sensitive to a number of key parameters. For example, the safety margin will be larger if: (1) the level of risk aversion[12] increases (as debt must be lower to reduce the probability of breaching the limit); (2) the amount of macroeconomic volatility implied by the estimated joint distribution increases (because larger shocks can generate larger increases in debt, a larger safety margin is required); (3) the response of the primary balance to changes in debt becomes weaker, as reflected in the parameters of the FRF (a larger safety margin is required when the government is not acting strongly enough to offset the impact on debt of negative shocks); or (4) the length of the medium-term projection horizon increases (over a longer horizon, a larger margin is required to ensure with high probability that debt remains below the limit, as there is greater uncertainty about outcomes farther into the future).

An important question is how to deal with contingent liabilities, which can be significant and relatively frequently realized (Bova and others 2016). In the methodology presented above, debt projections will likely reflect "normal" contingent liability realizations through two channels:

- *Above-the-line contingent liabilities*. The FRF includes the impact of fiscal shocks. These shocks, which are drawn from their historical distribution, are affected by the materialization of past contingent liabilities, provided they were recorded above the line (generally, under transfers) and transmitted to the primary balance.
- *Below-the-line contingent liabilities*. The debt accumulation equation includes stock-flow adjustments, which are simulated over the forecast horizon using their historical distribution in the codes accompanying this note. Thus, debt simulations will reflect the historical pattern of the realization of contingent liabilities, provided they were recorded below the line.

If the researcher expects contingent liabilities—either above or below the line—to be larger than those experienced historically, this information could be captured by manually adjusting the stock-flow adjustments in the debt accumulation equation (see manuals attached to this note).

[12]In this simple framework, a higher degree of risk aversion is equivalent to a smaller required probability of debt breaching the maximum limit following negative shocks (for instance, a threshold of 5 percent or 1 percent rather than 10 percent).

Method Two: Calibrating the Debt Rule Ceiling When the Maximum Debt Limit Is Unknown

The second method calibrates the debt ceiling in cases in which the maximum debt limit is unknown. This method is most suitable for advanced economies with unconstrained market access, where considerable uncertainty might exist about how much debt can be sustained.[13]

At the core of this method is the assumption that the primary balance is bounded upward. A primary surplus above a certain bound may be unachievable for various reasons, including political and public resistance to spending cuts or the fact that additional revenue-raising measures eventually become ineffective (the peak of the Laffer curve has been passed). Given that countries cannot promise to do whatever it takes to ensure debt sustainability under all circumstances, there is necessarily a level of public debt, above which debt stabilization becomes "impossible."

The method is implemented in three steps. First, the maximum feasible primary balance is identified. Second, stochastic simulations are used to obtain multiple trajectories of macroeconomic variables, the primary balance, and debt, using an FRF that describes fiscal behavior in response to debt (see the options in Box 3). Finally, the debt rule ceiling is identified as the initial value of debt, to ensure with high probability that debt can be stabilized following negative shocks without breaching the maximum feasible primary balance.[14] The following paragraphs describe the three steps in greater detail.

Step 1: Setting the Maximum Primary Balance

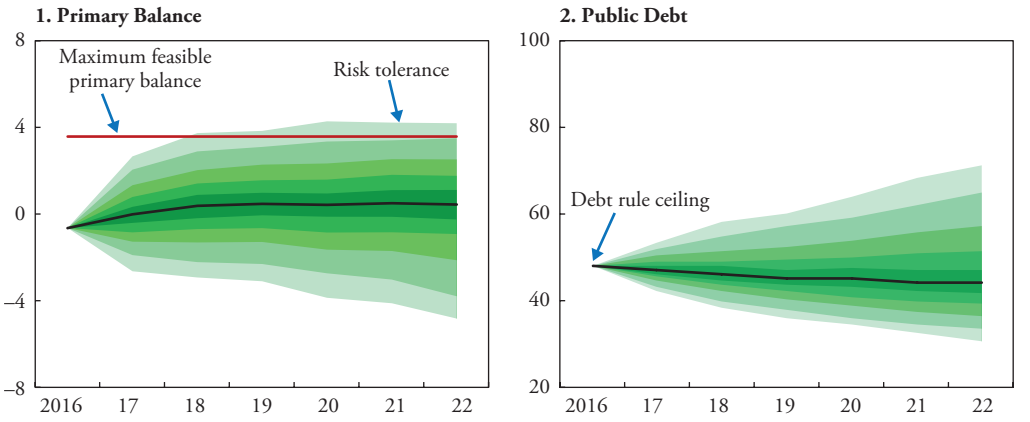
The first step is to identify the maximum feasible primary balance.[15] The calibration can be

[13]This method was developed by an IMF staff team that included X. Debrun, M. Jarmuzek, C. Lonkeng, S. Basu, N. End, W. Shi, J. Sin, and F. Toscani.

[14]Caution should be applied when using Method Two with an *estimated* (rather than *normative*) fiscal reaction function. If the estimated reaction function is explosive (reflecting undisciplined fiscal behavior in the past), the initial debt level, computed with Method Two, cannot be considered safe, given that it would place debt on an unsustainable path. In this case, the *normative or calibrated* reaction function should be used.

[15]In this method, it is possible to back out the debt level consistent with the maximum feasible primary balance, but this debt level is not an absolute upper limit for debt as in Method One. It is the highest debt level that can be stabilized without breaching the maximum feasible primary balance given current macroeconomic conditions. If the interest-growth differential increases, the maximum debt mechanically declines.

Figure 5. Method 2: Primary Balance and Debt Fan Charts
(Percent of GDP)



Source: IMF Staff Calculations
Note: The simulations are based on a normative fiscal reaction function. The primary balance and debt are projected over a six-year horizon from 2017-2022. The last year before projections begin is 2016.

country-specific or it can be based on cross-country historical experience of the largest primary balances countries have been able to achieve over certain periods. For advanced economies, it is possible to assume a maximum feasible primary surplus of about 4 percent of GDP; for emerging markets, a surplus of 2 percent of GDP could be appropriate (see Escolano and others 2014).

Tailoring the choice of maximum feasible primary balance to a particular country should be based on considerations of what primary balances can be sustained over a number of years in the prevailing macroeconomic circumstances. IMF (2013c) finds that large primary surpluses may be easier to achieve than to maintain. The median primary surplus achieved in a sample of 43 advanced and emerging market economies since 1950 was found to be lower when it was measured using five-year moving averages than using primary balances in individual years. There is also evidence that primary surpluses are harder to achieve when growth performance is below trend.

Step 2: Estimating the Effect of Shocks on the Primary Balance and Debt

The second step is to use stochastic simulations to determine trajectories of the primary balance and debt when shocks occur. In the files attached to this note, an estimated VAR (subject to shocks) is used to forecast multiple trajectories of macroeconomic variables over a medium-term projection horizon, similar to Method One. The corresponding trajectories of the primary balance and debt are computed using the system of simultaneous equations formed by the debt accumulation equation (that is, government budget constraint) and a fiscal reaction function (see Box 3 on the choice of an FRF). The potential trajectories for the primary balance and debt under shocks can be summarized using separate fan charts.

Step 3: Calibrating the Debt Ceiling

The final step calibrates the debt rule ceiling (initial debt level) that ensures with high probability that the maximum feasible primary balance is not breached over the medium term, even when negative shocks occur. Because the limit is on the primary balance and the fiscal rule is on debt, the two charts must be considered concurrently (Figure 5). The procedure is iterative: it starts with a certain level of debt in the first year and computes the corresponding debt trajectories under various shocks. If the primary balances required to stabilize debt breach the maximum feasible primary balance in a large number of these debt trajectories (say, more than 5 percent or 10 percent), the initial debt level is lowered and the exercise is repeated until most of the primary balance fan chart falls below the feasible maximum over the projection horizon.

The chosen debt rule ceiling will be sensitive to a number of key parameters. A lower debt ceiling will be required when there is higher risk aversion, because

lower debt reduces the probability of exceeding the maximum primary balance. A lower debt ceiling will also be implied by higher macroeconomic volatility, since the primary balance would need to be higher to stabilize debt when negative shocks are larger. Finally, a lower long-term target level of debt may require a lower debt ceiling, in cases where the normative fiscal reaction function is used (see Box 3). The normative FRF will embody a stronger response of the primary balance to the current level of debt when the long-term target is lower; so maintaining a lower debt ceiling will help keep the required primary balances below the maximum feasible.

Limitations of the Proposed Approach

The tools proposed in this note do not substitute for the use of full-fledged macroeconomic models that can better capture structural and nonlinear relationships among variables. This framework for calibrating the debt ceiling is made tractable by several simplifying assumptions:[16]

- Macroeconomic variables are projected using a simple VAR econometric model subject to shocks (drawn from a symmetric joint normal/Student's *t* distribution) or by drawing directly from the joint distribution if quarterly data are unavailable. Thus, the macroeconomic simulations are informed by historical data and cannot capture the impact of recent or expected structural changes in the economy.
- Structural/behavioral equations from economic theory (for example, aggregate demand curve or monetary policy rule) are not used to project macroeconomic variables, potentially missing valuable information on how the economy would behave in the future.
- A VAR econometric model has a simple linear structure. This structure may fail to capture nonlinearities among macroeconomic variables, such as changes in the relationship between interest rates and growth throughout the business cycle.
- The VAR and the joint normal/Student's *t* distribution are based on the assumption that macroeconomic shocks are symmetric. In reality, shocks may be skewed in the adverse direction. Also, the distribution of the shocks may not capture tail events well.
- Data constraints may prevent precise estimation of the VAR or calibration of the joint distribution, if only short time series for the relevant variables are available.
- The fiscal reaction function used to project fiscal variables contains only a small set of independent variables and ignores potential nonlinearities and breaks in the reaction of fiscal policy to debt.
- Fiscal variables (for example, the primary balance or debt) are not included in the VAR or the joint distribution, meaning that there is no feedback from fiscal policy changes to macroeconomic variables, in particular GDP.

Another important limitation is that the calibration methods presented above rely exclusively on the need to protect a country's fiscal position against negative shocks. Maintaining prudent debt levels is crucial to guarantee fiscal sustainability, but this is not the only criterion that can or should be taken into account when setting a debt target. In many countries, increasing public investment and funding education and health care are also priorities, and at least part of these expenditures must be financed through public debt.[17]

Therefore, there is an inherent trade-off when deciding on the level of the debt ceiling. Higher debt increases vulnerability to shocks and can undermine market confidence and lead to fiscal distress. But the debt ceiling should not be too low, to allow space for financing development needs. More complex models are needed to reflect the trade-off between risk management and development perspectives. By ignoring this trade-off, the methods proposed in this note may be biased toward austerity, at least for some emerging and developing economies.

[16]In the econometric files accompanying this note, users can make ad hoc adjustments to forecasts of macroeconomic variables and the primary balance to incorporate country-specific knowledge and judgment, which may help overcome some of the limitations. See Boxes 2 and 3 for more information.

[17]Economic theory suggests that public investment should be primarily financed by debt issuance rather than taxes (Ostry, Ghosh, and Espinoza 2015). One reason is that the distortions brought by taxation are smaller when tax increases are smoothed over time through debt finance. In addition, public investment projects are expected to generate gains over several years and to benefit future generations; therefore, their full cost should not be borne only by current taxpayers.

Framework for Commodity Exporters

In the methods presented above, the fiscal rule associated with the final objective of fiscal policy (that is, debt sustainability) is the *gross* debt rule. But *net* debt (debt minus financial assets) may be a more relevant anchor in countries with large financial assets. For instance, in resource-rich countries it is common to focus on net wealth, measured as net financial wealth (financial assets minus debt) plus resource wealth—the present value of future resource revenues (Baunsgaard and others, 2012). The calibration of the net wealth (or net financial wealth) target can be based on one of the two following approaches:

Long-Term Sustainability Approach

An important challenge is to decide how to allocate net wealth across generations, given that natural resources are exhaustible. Various models have been developed to calibrate the appropriate level of net wealth in resource-rich countries, assuming the need to achieve fiscal sustainability and intergenerational equity (see a review in IMF 2012).

- *Fixed net wealth benchmark*. In the standard model of the Permanent Income Hypothesis (PIH), intergenerational equity is achieved by preserving government net wealth at its initial level, so that future generations will enjoy a similar amount of wealth as the current generation. Several variations of this model exist, depending on whether net wealth is preserved in real terms, real terms per capita, or as a share of GDP. The general idea is that if only a fraction of resource revenues is spent every period, financial savings will increase sufficiently to make up for the depletion of resource wealth. Total net wealth is therefore kept constant, although its composition changes over time: the share of resource wealth (present value of future resource revenues) will decline over time, but this decline will be perfectly offset by an increase in net financial wealth.
- *Variable net wealth path to accommodate higher investment*. Alternative models, also based on the intertemporal budget constraint, relax the assumption of constant net wealth. Extensions of the PIH model allow for a temporary scaling-up of public investment. Net wealth initially declines during the scaling-up period, because saving is lower than under the PIH and the stock of financial assets does not increase quickly enough to offset the decline in resource wealth. These models formalize the trade-off faced by commodity producers between investing resource revenues in real or financial assets. A higher targeted stock of real assets is likely to come at the expense of a lower stock of net financial assets. Models with fiscal multipliers somehow mitigate this trade-off, because higher public investment generates higher financial savings through its positive impact on GDP growth and nonresource revenues. In these models, a rule on net wealth can be introduced only after the scaling-up period of investment; that is, once net wealth stabilizes again.

Risk-Based Approach

Uncertainty is another important consideration when calibrating net wealth targets in resource-rich countries. These countries need larger and more durable buffers than other countries because economic shocks can be large and highly persistent. The amount of the required savings depends on the degree of resource dependence, the level of risk the country is facing, and its risk tolerance.

Several methods exist to compute the level of net financial wealth that countries should maintain as a precautionary buffer—a buffer that can be tapped in bad times to support spending when resource revenues fall short. For instance, IMF (2012) proposes to use a value-at-risk (VaR) approach and a model-based approach to estimate the adequacy of buffers. Both methods estimate the minimum buffer that can absorb tail risks in resource revenue volatility. Specifically, the buffer should be set high enough to ensure with high probability that it is not fully depleted over the forecast horizon and, therefore, expenditure cuts will not be needed. Another method was developed in "The Commodities Roller Coaster" (IMF 2015): the amount of financial savings is calibrated to ensure that investment returns on financial assets are sufficient to avoid large fiscal adjustment in the event that commodity prices fall. To illustrate, IMF (2015) computes, for three major oil exporters, the level of financial assets that would be sufficient to generate investment returns to cover half the lost revenue over five years with 75 percent to 90 percent probability.

From the Debt Rule to the Operational Rules

Debt and deficits are tied through an accounting identity. A country's debt is the cumulative stock of past deficit flows, while the (overall) deficit captures

the annual change in the country's debt. In practice, currency fluctuations, nondebt financing of deficits, and accumulation of financial assets can all temporarily alter the one-to-one link between debt and deficits, but in general the debt path closely follows that of the deficit. In the same way, deficits are inherently tied to government spending, with cuts to the deficit often requiring cuts to spending. As a result, any target set on the debt path implicitly puts constraints on the deficit and, ultimately, on spending. Therefore, one needs to ensure consistency between the debt rule and the operational rules placed on the deficit and spending.

In the methods presented below, the time horizon is generally longer than the time horizon used to calibrate the debt ceiling in the previous section (which was six years by default). Using identical time horizons in both exercises would be unwarranted and potentially misleading. The calibration of the debt ceiling is based on the prudence principle that debt dynamics should remain under control even if negative shocks occur repeatedly over the medium term. It would not be reasonable to extend this time horizon beyond five or six years. A scenario of repeated negative shocks over the long term (say, 20 years) would not only be unrealistic but would also certainly result in a safe debt level equal to zero. On the contrary, the time horizon used to calibrate the operational rules is a policy decision reflecting national preferences. The question is whether a government wishes to attain the safe debt target asymptotically in the long term or over a shorter horizon.

From the Public Debt Ceiling to the Deficit Ceiling[18]

The derivation of the deficit ceiling from the debt ceiling can be done flexibly, depending on the timing and sequencing of the desired adjustment path. In what follows we will discuss four main approaches, each adding a layer of flexibility: (1) a constant balance target that guides debt to its ceiling in the long term, (2) a constant balance target that guides debt to its ceiling by a given date, (3) a constant balance target that guides debt to its ceiling by a given date following a transition period, and (4) a constant balance target that guides debt to its ceiling by a given date, while creating space for long-term increases in age-related government spending.

Approach 1: Convergence in the Long Term

This approach derives the constant fiscal balance[19] that, if maintained, would lead to a gradual convergence toward the debt target in the long term. Equation (1) lays out the basic formula (derived in Appendix 2) that could be used for the calibration of both the overall balance and the primary balance targets as a share of GDP (b^*), for a given debt-to-GDP target (d^*) and parameter λ:

$$b^* = \lambda d^* \tag{1}$$

Note that λ is alternatively equal to $\frac{-\gamma}{1+\gamma}$ in the case of an overall balance target, and $\frac{i-\gamma}{1+\gamma}$ in the case of a primary balance target, where γ stands for the nominal GDP growth over the long term and i is the nominal interest rate paid on public debt.[20]

Equation (1) suggests that, for a hypothetical country that grows by 5 percent in nominal terms and pays 3 percent of nominal interest rate on its debt over the long term, a 60 percent of GDP debt target would imply an overall deficit target of about 2.9 percent of GDP (as shown in Figure 6) or a primary deficit target of about 1.1 percent of GDP.

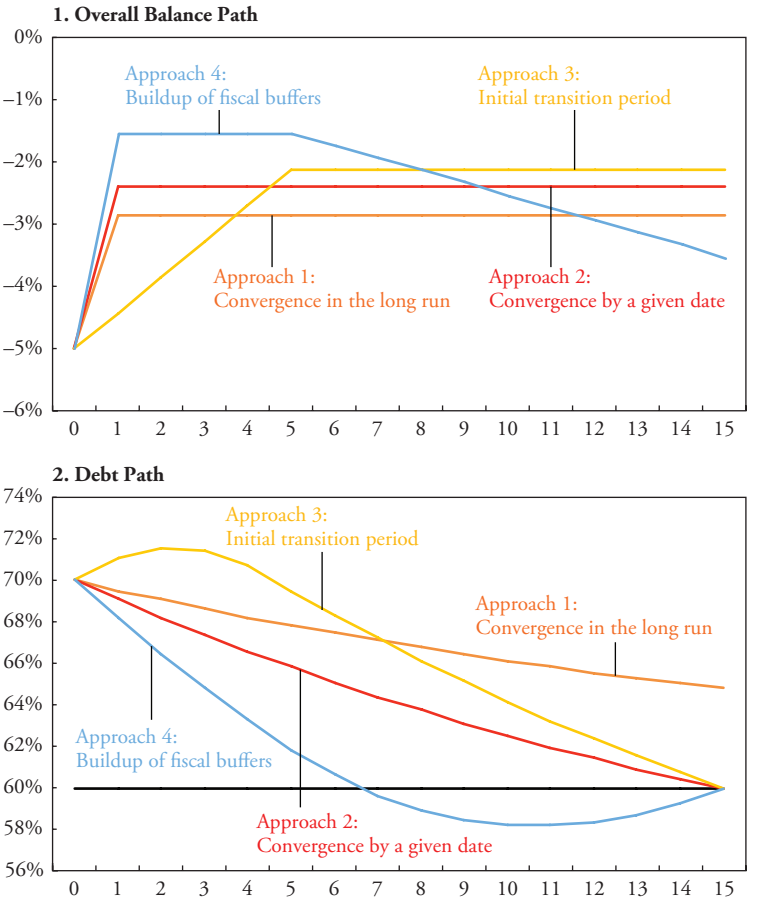
This is the standard approach to calibrating the budget balance rule from a given debt target. It has, for example, largely inspired the calibration of the European Union's framework of fiscal rules. Depending on the debt target and the initial fiscal balance, this formula can entail either an instantaneous consolidation or expansion of the fiscal position. A positive feature of this approach is that it is simple and compels an immediate adjustment of fiscal policy toward the debt target. On the downside, convergence to the debt target can be very slow. For example, it would take about 15 years for our hypothetical country's debt to complete half the distance from an initial debt ratio of 70 percent to a target of 60 percent. In addition, this approach relies on the simplifying assumption that λ is

[18]In countries with exhaustible natural resources, the framework would need to be modified to take into account future resource revenues (Baunsgaard and others 2012; IMF 2012). In this context, the PIH, Modified PIH, and Fiscal Sustainability Frameworks provide benchmarks to calibrate the threshold of fiscal balance rule and, indirectly, estimates of the sustainable level of expenditure.

[19]In the rest of this section, the formulas are based on fiscal *balances*. The results should be interpreted as a *deficit* ceiling or a *deficit* target when the balance is negative.

[20]Growth and interest rates could be replaced by their real-term values when deriving λ in equation (1), under the simplifying assumptions that nominal interest rates and nominal GDP are deflated with a similar deflator and that there is no difference between actual and expected inflation. Note that the derivations implicitly assume that either the share of foreign currency debt is low, or the effective exchange rate is largely stable over the long term.

Figure 6. Illustrative Case for Fiscal Balance Calibration



Source: IMF Staff.
Assumptions: nominal growth, 5%; interest rate, 3%; initial debt, 70%; debt target, 60%; initial balance, –5%.

constant over the long term; in reality, growth and the interest–growth rate differential vary with the level of public debt (IMF 2010, 2013c).

The balance target derived in this approach is also known as the "debt stabilizing fiscal balance," which, in the case of the *primary balance*, is somewhat of a misnomer. Indeed, constant primary balance rules imply explosive debt paths (that is, diverging to +/-∞), except when (1) GDP grows faster than the interest rate paid on debt or (2) the starting debt level is already at the target $d_0 = d^*$ (Escolano 2010). Consider the case in which growth is below the interest rate and the initial debt ratio is above the target. In this case, the increase in nominal debt (owing to the financing of interest payments) would be larger than the increase in nominal GDP. This would raise the debt-to-GDP ratio indefinitely, as the primary balance is kept constant at the target ratio and does not adjust to offset the rising debt service. In contrast, a constant *overall balance* rule would place the debt on a convergent path: if the overall balance is set at the constant level b^*, the debt ratio will asymptotically converge to d^* from any initial debt level (under the assumption that nominal growth γ is positive).

Approach 2: Convergence by a Given Date

This approach is similar to the first one, except that the balance rule (b^*) is calibrated so that the debt ratio hits its target (d^*) after N years. Equation (2) below lays out the basic formula in this approach given the initial level of debt d_0, λ, N, and d^* (see Appendix 2 for details).

$$b^* = \frac{\lambda}{(1+\lambda)^N - 1}\left[d_0 (1+\lambda)^N - d^*\right] \qquad (2)$$

Note that λ is equal to $\frac{-\gamma}{1+\gamma}$ in the case of an overall balance target and $\frac{i-\gamma}{1+\gamma}$ in the case of a primary balance target, as before.

This approach would be recommended, for example, for countries looking to operationalize multiyear government plans that set specific fiscal targets to be achieved by a given date. Its main appeal resides in the fact that it forces convergence to be as quick as desired. As a result, it usually requires a larger fiscal effort relative to the previous approach. For example, to ensure that our hypothetical country in Figure 6 hits the 60 percent target within a 15-year span starting from an initial debt of 70 percent of GDP, one would need to maintain an overall deficit of 2.4 percent or a primary deficit of 0.6 percent of GDP (versus 2.9 percent and 1.1 percent, respectively, in the previous approach).[21]

Approach 3: Convergence by a Given Date Following a Transition Period

The two previous approaches implicitly assume an instantaneous adjustment of the fiscal balance to its target; but in some cases this would imply an overly strong contraction or expansion of fiscal policy, making the move economically and politically difficult. This third approach adds another layer of flexibility by allowing for an initial transition period in which the balance gradually converges to its target (through a gradual consolidation or expansion, depending on its starting level). In this approach, we assume that the number of years of the transition period is exogenously given (equal to T).

Equation (3) illustrates a special case in which the balance is adjusted annually by a constant amount α until it reaches the target b_T^* after T years. If b_T^* is maintained afterward, this will ensure convergence to the debt target by the end of year N ($N \geq T$).[22] In this case, the path for the balance ratio would be

$$b_t = \begin{cases} \alpha t + b_0, \text{ when } 0 < t < T \\ \alpha T + b_0 = b_T^*, \text{ when } T \leq t \leq N \end{cases} \quad (3)$$

[21]Note however that, in this approach, the derived balance target will need to be adjusted to the debt-stabilizing balance after the debt ratio reaches its target. Otherwise, the debt ratio would keep diverging.

[22]A linear adjustment schedule is not crucial to the outcome; what matters is the total adjustment αT at the end of year T that brings the balance to the target.

Equation (4) depicts the main calibration formula, which includes an additional term A compared with the "instantaneous adjustment" in the second approach (see Appendix 2 for details). This additional term captures the effect on the balance of delaying the adjustment: to hit the debt target by a given year, a higher balance is needed after the transition period when there is gradual rather than instantaneous fiscal tightening.

$$b_T^* = \frac{\lambda}{(1+\lambda)^N - 1}\left[d_0\,(1+\lambda)^N - d_N^* + A(T, b_0, N, \lambda, d_0, d_N^*)\right] \quad (4)$$

Note that λ is alternatively equal to $\frac{-\gamma}{1+\gamma}$ in the case of an overall balance target, and $\frac{i-\gamma}{1+\gamma}$ in the case of a primary balance target, as before.

Figure 6 illustrates the overall balance and debt paths in this approach based on the hypothetical country case discussed above. It is assumed that the transition period for the balance ratio is $T = 5$ years and the convergence horizon for the debt ratio is $N = 15$ years, as in the second approach. In contrast to the "instantaneous adjustment" scenario in the second approach, the adjustment is backloaded, with initially higher deficits and debt followed by much tighter policies to hit the debt target within the same horizon. This method could, for instance, be used to calibrate convergence criteria for countries that are willing to join a monetary union by a certain date, as was the case at the inception of the euro area. It can also be used to calibrate country-specific fiscal adjustment programs. It allows more flexibility in the transition to the balance target, while still forcing the convergence to the debt target to be as quick as desired. But that flexibility comes at the cost of sustaining a higher fiscal position after the transition period (captured by the term A).

Approach 4: Convergence by a Given Date Following the Buildup of Fiscal Buffers

This approach offers another layer of flexibility in the calibration of the balance rule by allowing for a balance path that accommodates expected increases in future spending. It is recommended for countries facing the prospect of aging costs but could also be suitable with regard to other long-term costs, such as those related to the environment. In an economy with an aging population, targeting the constant fiscal balance derived in the second approach (equation 2)

would become more and more difficult over time, as age-related budgetary spending rises, gradually crowding out non-age-related spending. Therefore, fiscal effort should be frontloaded to build up buffers that will be used to absorb future spending pressures and help smooth the burden of adjustment over time. In a sense, this is the opposite problem of the third approach with back-loaded adjustment, in that compliance with the balance rule is relatively easier in the short-to-medium term but would become markedly harder in the long term.

For simplicity, we separate two time horizons in our simulations. In the short-to-medium term, age-related costs are assumed to be stable, so the government can target a fixed balance without difficulty. In the long term, the ratio of age-related spending to GDP increases, which translates into a steady deterioration of the fiscal balance. Importantly, the simulation assumes unchanged policies, meaning that, in the long term, the fiscal balance *excluding additional age-related spending* (relative to the initial level) is constant. If this balance b^* is held constant, the debt ratio would converge toward its target by the end of year N, even after accounting for higher age-related costs.[23]

Equation (5) illustrates the balance path of an economy in which age-related costs are projected to increase after P years ($P < N$).

$$b_t = \begin{cases} b^*, \text{ when } 0 < t < P \\ b^* - \Delta A_t, \text{ when } P \leq t \leq N \end{cases} \quad (5)$$

where $\Delta A_t = A_t - A_0$ denotes the *incremental* aging costs in a given year, meaning the difference between total age-related costs A_t relative to their size in the initial period $A_0 = A_{P-1}$.[24]

Equation (6) shows the main formula deriving the constant fiscal balance in this approach using similar notations as above, given d_0, λ, N, d_N^*, and ΔA_t (see Appendix 2 for details). Note that equation (6) is an adaptation of the long-term adjustment need formula of the European Commission, also known as the "S2 indicator" (European Commission 2015).

$$b^* = \frac{\lambda}{(1+\lambda)^N - 1}\left[d_0\,(1+\lambda)^N - d_N^* + S\right] \quad (6)$$

where $S = \sum_{t=1}^{N}(1+\lambda)^{N-t}\Delta A_t$ denotes the value in year N of cumulative future increases in long-term age-related spending through N.[25] The constant balance, excluding incremental aging costs, b^*, includes an extra term $S(\Delta A_t)$ compared with the "instantaneous adjustment" formula (equation 2), which captures the additional and upfront adjustment to prepare for future increases in age-related spending. As before, λ is equal to $\frac{-\gamma}{1+\gamma}$ in the case of an overall balance target and $\frac{i-\gamma}{1+\gamma}$ in the case of a primary balance target.

Figure 6 illustrates the overall deficit and debt paths in this approach, under the simplifying assumption that long-term age-related costs are defined as $A_t = \delta(t+1-P) + A_0$ for $t \geq P$, implying that these costs increase linearly by δ over time starting from year $P = 6$.[26] All other parameters are kept the same as before. In the early years, the country runs lower deficits than would be needed absent long-term aging costs (that is, relative to the "convergence by a given date" scenario), effectively building up fiscal buffers. But as age-related costs increase beginning in the sixth year, the fiscal balance path progressively deteriorates to accommodate these costs. Still, the debt ratio hits its target by the end of the period N. In the scenario presented in Figure 6, the debt ratio declines below target in the later years to accommodate future increases in deficits that will bring debt back to target by the end of the forecast horizon. Note that this approach is usually suited to a much longer term perspective (decades) than the ones discussed above. We have used the same time horizon in Figure 6 ($N = 15$ years) for ease of comparison across approaches.

A broadly similar approach is also used to calibrate the Medium-Term Objectives (MTO) of the European fiscal framework (European Commission 2016). In particular, the MTO formula includes a term that covers a fraction of the present value of the projected increase in age-related expenditure.

[23]Age-related spending (especially pension expenditures) is largely predefined on the basis of pension parameters and demographics, and is not tied closely to economic activity. Thus, it can be projected more reliably.

[24]Note that $b_t = b^* - \Delta A_t$, throughout the whole period N; however, since $\Delta A_t = 0$ until year $P - 1 (A_0 = A_{P-1})$, we can set $b_t = b^*$ before year P.

[25]Accordingly, the present discounted value of S would be $S/(1+\lambda)^N = \sum_{t=1}^{N}(1+\lambda)^{-t}\Delta A_t$.

[26]In the simulation, δ is set at about 0.2 percent of GDP per year starting in year $P = 6$, so the total increase in age-related spending would be 2 percent of GDP until year $N = 15$ (within a 10-year horizon), which corresponds to the average increase in pension and health care spending in advanced and emerging market economies. Thus, in present value terms, the cumulative increase in age-related spending through year N would be $S/(1+\lambda)^N = 22$ percent of GDP.

From the Structural Deficit Ceiling to the Nominal Deficit Ceiling

The previous formulas are used to ensure that the debt and balance rules are consistent in the long term; that is, when nominal aggregates are equal to their steady-state structural values. It is thus reasonable to assume that these formulas calibrate the structural balance rule.[27]

It can be useful to derive from the structural balance rule a corresponding threshold for the nominal balance rule. This is particularly important in countries that have both nominal and structural balance rules, to minimize the risk of conflict between them. For a given structural balance target, the nominal deficit ceiling should be sufficiently high to allow automatic stabilizers to fully operate during a typical economic downturn (this means that the nominal deficit should be allowed to increase in response to the cyclical decline in revenues). But the nominal deficit ceiling should not be so high that it permits discretionary fiscal expansions that are inconsistent with the structural balance ceiling.

Equation (7) details how to compute a constant nominal balance target, nb^*, for a given structural balance target, sb^* (See Appendix 2 for its derivation).

$$nb^* = sb^* - OG_{max}[r(1 - \eta) - e(1 - \kappa)] \qquad (7)$$

where OG_{max} stands for the maximum output gap during a typical downturn (it is a negative number); r and η are, respectively, the revenue ratio and its elasticity relative to output; and e and κ are, respectively, the spending ratio and its elasticity relative to output.

Estimates of revenue and spending elasticities generally yield values close to 1 and 0, respectively; hence the following is a common proxy for the link between structural and nominal balances:

$$nb^* = sb^* + OG_{max}.e \qquad (8)$$

The structural balance target sb^* is expressed as a ratio to potential output while the nominal balance target nb^* is expressed as a ratio to current output (see Fedelino and others 2009 and Escolano 2010 for a detailed discussion). For a government targeting a structural deficit of 1 percent of potential GDP, spending 40 percent of GDP on average, and facing an output gap of –2 percent, equation (8) would imply a constant nominal deficit ceiling of 2 percent of GDP.

This formula is, for instance, used to ensure consistency between the nominal deficit ceiling and the Medium-Term Objective (in structural terms) in the European fiscal framework (European Commission 2016).

From the Structural Balance Ceiling to the Expenditure Ceiling

The final step in deriving operational rules is the calibration of expenditure rules, which can be defined in terms of either expenditure growth or an expenditure ratio. For simplicity, we assume that there is no cyclical component to expenditure, meaning that expenditure does not respond automatically to economic conditions (as would be the case if there were an unemployment benefit scheme, for instance) and that automatic stabilizers operate on the revenue side only. This is consistent with empirical evidence showing that revenues are far more sensitive than expenditure to the business cycle (Price, Dang, and Guillemette 2014). In this case, there is no difference between observed/nominal expenditure and structural expenditure. We also assume that the country has a given structural tax ratio r^s (computed as the ratio of structural revenues to potential GDP), which remains constant unless there is a change in tax policy.

Under these assumptions, for a given structural balance ratio sb, the implied expenditure ratio e^s (in percentage of potential GDP) is equal to:

$$e^s = r^s - sb$$

which implies that:

$$\Delta e^s = \Delta r^s - \Delta sb \qquad (9)$$

If there is no tax policy change ($\Delta r^s = 0$) and if the country already complies with the structural balance rule ($sb = sb^*$ and $\Delta sb = 0$), the two equations show that the structural balance rule can be interpreted as (1) a constant ratio of spending-to-potential GDP $\Delta e^s = 0$ or (2) a rule in which spending growth is equal to potential GDP growth. In other words, if the country is already at a structural position consistent with the structural balance rule, nominal spending

[27]In principle, the deficit ceilings derived from the various approaches discussed here could be applied to either the nominal balance or the structural balance. When applied to the *structural* balance, the budget balance ceiling is binding on average over the cycle (although the nominal balance can deviate and fluctuate with the cycle). When applied to the *nomina*l balance, however, the ceiling is constantly binding and must be enforced every year. This means that the fiscal deficit consistent with the debt target is more demanding when the formulas are applied to the nominal balance than to the structural balance.

should grow at the same pace as nominal potential GDP.

For instance, for a country that is collecting 40 percent of potential GDP in revenues and whose potential GDP is growing in nominal terms by 4 percent a year on average, maintaining a structural deficit of 1 percent of potential GDP would be consistent with either (1) an expenditure-ratio ceiling of 41 percent of potential GDP or (2) a ceiling of 4 percent applied to nominal expenditure growth.

This basic framework can be expanded in two ways:

- The framework can account for structural changes in revenue mobilization ($\Delta r^s \neq 0$ in equation 9). If a country takes new discretionary revenue measures (either revenue enhancing or revenue diminishing), the expenditure ceiling should be adjusted upward or downward to ensure that the structural balance remains at its targeted value (sb^*) and that debt remains on its targeted path.[28] Allowing for this flexibility in the expenditure rule is key to preserving incentives for taking new revenue-enhancing measures as well as to ensuring that the adoption of new tax expenditures (or other revenue-decreasing measures) does not derail the debt path.
- The framework can also be adapted to fit transition periods toward the structural balance target if the country does not comply with the rule from the start ($sb \neq sb^*$). In this case, the growth of government spending should be temporarily maintained below or above trend GDP to guide the structural balance toward its target. The size of the wedge between spending growth and trend growth would have to be calibrated to engineer the required change in the structural stance.

Both of these extensions of the framework are featured in the "expenditure benchmark" implemented in the European Union. European Commission (2016) provides more details on the calculations.

[28]Another option is to combine the expenditure rule with a debt-break mechanism that forces a correction of the rule for past deviations from the debt path (see Debrun, Epstein, and Symansky 2008 for a detailed discussion of the case of Israel).

Appendix 1. Deriving the Debt Rule Threshold

Method One

This method requires the characterization of the joint distribution of the macroeconomic variables needed to project the public debt ratio. For advanced and emerging market economies, these variables are growth, the average interest rate on debt, and the exchange rate. For low-income countries, the terms of trade and external financing disbursements are also required. Characterizing the joint distribution is done either by (1) estimating a vector autoregression (VAR) or (2) directly calibrating a joint multivariate distribution based on historical co-movement between variables. A VAR can be used when quarterly data are available, so that there are sufficient observations for econometric estimation. The direct calibration of a joint multivariate distribution may be most appropriate where only annual data are available.

Indirect Estimation with a VAR

If quarterly data are available, an unrestricted VAR can be estimated for each country, describing the joint dynamics of the macroeconomic variables needed to project public debt. The econometric model is

$$X_t = A_0 + \sum_{j=1}^{p} A_j X_{t-j} + \varepsilon_t$$

where X_t is the k-dimensional vector of macroeconomic variables and ε_t is a k-dimensional vector of normally distributed shocks: $\varepsilon_t \sim N(0, \Omega)$.

The estimated variance-covariance matrix of the VAR, $\hat{\Omega}$ is then used to generate N sequences of macroeconomic shocks ε_t over the six-year projection horizon (N is a large number of simulations; for example, more than 1,000). For each of the N simulations of shocks, the estimated VAR model is used to forecast macroeconomic variables X_t over the six-year projection horizon, adding the generated shocks to the VAR model each year as the error term. Using a VAR to make projections is ideal, as the lagged effect of macroeconomic shocks can be taken into account through the autoregressive structure of the model.

Direct Calibration of a Multivariate Distribution

As an alternative to VAR estimation, a simpler approach can be used if quarterly macroeconomic data are not available. A multivariate normal (or Student's t) distribution of key macroeconomic variables can be calibrated based on historical comovements of macroeconomic variables. N sequences of six-year projections can be obtained by drawing repeatedly from this distribution.

A multivariate normal distribution of a k-dimensional vector of macroeconomic variables can be written as

$$x \sim N_k(\mu, \Sigma)$$

with the k-dimensional mean vector

$$\mu = (E[X_1], E[X_2], \ldots, E[X_k])$$

and the $k \times k$ covariance matrix

$$\Sigma = (cov(X_i, X_j)), \text{ for all } i = 1,2, \ldots, k; \quad j = 1,2, \ldots, .k$$

The parameters μ, Σ can be calibrated based on the historical mean, variance, and covariance of macroeconomic variables. Alternatively, the econometric programs accompanying this note allow for the use of a multivariate Student's t distribution, which has wider tails. (See manuals describing the files for more information.)

Calibrating the Debt Ceiling

The debt ceiling is calibrated as follows:

1. A set of macroeconomic variables is forecast over a six-year projection horizon N times by either (a) using the estimated VAR, including shocks each period; or alternatively by (b) drawing directly from the calibrated multivariate distribution of macroeconomic variables each year.
2. The N sets of macroeconomic variable forecasts are used to generate N trajectories of the primary balance, using a fiscal reaction function (FRF) and the previous year level of debt (see Box 3 in the text for FRF options). In the econometric programs accompanying this note, annual changes in the primary balance implied by the FRF are constrained (that is, they cannot exceed certain limits) on the basis of historical experience to ensure that projected primary balances are realistic. Fiscal shocks can also be added directly in the FRF. The distribution of fiscal shocks is calibrated based on estimated deviations between actual fiscal responses observed (that is, actual levels of the primary balance) and the fiscal response predicted by the FRF within the sample.
3. The N corresponding trajectories of debt (starting at the current debt level) are obtained by the system of simultaneous equations formed by the debt accu-

mulation equation (government budget constraint) and the FRF. The debt accumulation equation is

$$d_t = \left(1 + \left(\frac{r_t - g_t}{1 + g_t}\right)\right) d_{t-1} - pb_t + SFA_t$$

where d_t is debt (as a ratio of GDP), r_t is the average effective real interest rate on debt, g_t is the real GDP growth rate, pb_t is the primary balance (as a ratio of GDP) and SFA_t is the stock-flow adjustment (as a ratio of GDP). The debt accumulation equation includes a constant stock flow adjustment each period that could potentially account for realization of contingent liabilities.

4. If the 95th debt percentile (or other chosen percentile, given risk tolerance) of the debt ratio distribution is significantly below the maximum debt limit (MDL) in all years of the projection horizon [or is significantly above the MDL in at least one year], the starting level of debt is increased [decreased] by a small amount (0.3 percent), and steps 1–3 are repeated based on the new starting level.

Steps 1–4 are repeated until the 95th percentile of the debt level falls into a small interval around the MDL in at least one year of the medium-term projection horizon: $Debt^{95} \in [MDL - 0.4; MDL + 0.4]$, without significantly breaching the MDL in any year. The starting level of debt satisfying this criterion is called the debt ceiling; that is, the level of debt from which its projection does not exceed the MDL with 95 percent likelihood over the medium-term projection horizon. The safety margin is computed as the MDL minus the debt ceiling.

The fan charts can also be used to determine the probability of breaching the maximum debt limit, conditional on any starting level. For example, using the current debt level as the starting level, the fan chart can be used to determine the probability that debt will exceed the debt limit in all years over the projection horizon. This can be useful to gauge the extent of risk associated with a country's current debt position.

Method Two

The debt ceiling is calibrated as follows:

1. A set of macroeconomic variables is forecast over a six-year projection horizon N times using an estimated VAR, including shocks each period (the VAR estimation is similar to that in Method One).
2. The N sets of forecasts are used to generate N trajectories of the primary balance, using either an estimated or normative fiscal reaction function (see Box 3 in the text).
3. The N corresponding trajectories of debt (starting at the current debt level) are obtained by the system of simultaneous equations formed by the debt accumulation equation (government budget constraint) and the fiscal reaction function (which depends on the lagged value of debt).
4. If the 95th percentile of primary balances (or other chosen percentile, given risk tolerance) is significantly below the maximum feasible primary surplus (MFPS) (that, is, falls below the interval $pb^{95} \in [MFPS - 0.4; MFPS + 0.4]$) in all years of the projection horizon [or is significantly above the MFPS in any one year], the starting level of debt is increased [decreased] by 0.3 percent, and steps 1–3 are repeated based on the new starting level. For advanced economies a typical MFPS would be about 4 percent of GDP, while for emerging market economies it would be 2 percent of GDP (see Escolano and others 2014).

Steps 1–4 are repeated until the 95th percentile of the primary balance falls into the small interval around the MFPS in at least one year of the medium-term projection horizon: $pb^{95} \in [MFPS - 0.4; MFPS + 0.4]$, without significantly breaching the MFPS in any year. The starting level of debt satisfying this criterion is called the debt ceiling.

Appendix 2. Deriving the Operational Rule Thresholds

From the Public Debt Ceiling to the Deficit Ceiling

We start with the accounting identity relating the gross debt to overall balance (OB) and primary balance (PB), also known as the debt dynamics equation:[1]

$$D_t = D_{t-1} - OB_t \Rightarrow d_t = \frac{1}{(1+\gamma_t)} d_{t-1} - ob_t$$

$$D_t = (1+i_t) D_{t-1} - PB_t \Rightarrow d_t = \frac{(1+i_t)}{(1+\gamma_t)} d_{t-1} - pb_t$$

where D_t is the nominal level of debt at the end of period t, i_t is the effective nominal interest rate paid on the inherited debt D_{t-1}, and γ_t is the nominal growth rate of GDP, with small letters denoting a variable as a share of GDP. We can rewrite this debt dynamics equation as

$$d_t = (1+\lambda_t^j) d_{t-1} - b_t^j \text{ for } j \in \{o,p\}$$

$$1+\lambda_t^j = \begin{cases} \frac{1}{(1+\gamma_t)}, & \text{when } b_t^j = ob_t \text{ for } j = o \\ \frac{(1+i_t)}{(1+\gamma_t)}, & \text{when } b_t^j = pb_t \text{ for } j = p \end{cases}$$

with b_t^j denoting the balance ratio in period t, which becomes the overall balance if $j = o$ and primary balance if $j = p$ together with the corresponding growth-adjusted interest rate $\lambda_t^o = \frac{-\gamma_t}{(1+\gamma_t)}$ if $j = o$ and $\lambda_t^p = \frac{i_t - \gamma_t}{(1+\gamma_t)}$ if $j = p$. In the rest of the appendix, we will drop the index j and derive the relevant equations for the balance ratio; one could retrieve the relevant equations for the overall and primary balances using corresponding λ_t^j for $j = \{o, b\}$:

$$d_t = (1+\lambda_t) d_{t-1} - b_t \qquad \text{(A.2.1)}$$

Approach 1: Convergence in the Long Term

Using equation (A.2.1) and assuming $\lambda_t = \lambda$ is constant in the long term, the constant balance ratio b^* compatible with achieving a constant debt ratio d^* in the long term would be

$$b^* = \lambda d^* \qquad \text{(A.2.2)}$$

Specifically, the constant overall and primary balance ratios are

$$ob^* = \frac{-\gamma}{1+\gamma} d^*$$

and

$$pb^* = \frac{i-\gamma}{1+\gamma} d^*$$

Using equation (A.2.2), the *instantaneous* adjustment to the balance ratio to reach a constant debt ratio d^* over the long term can be decomposed as

$$b^* - b_0 = [\lambda d_0 - b_0] - \lambda(d_0 - d^*)$$

where the first term on the right-hand side corresponds to the gap to the debt-stabilizing balance ratio (at the initial level of debt), and the second term corresponds to the additional adjustment due to the debt target d^* being (potentially) different from the initial debt ratio d_0.[2] While the pace of adjustment is instantaneous in this particular example, the convergence to the desired debt ratio would be asymptotic.

Approach 2: Convergence by a Given Date

Under the assumption that λ_t is time-varying, equation (A.2.1) has the following solution

$$d_N = d_0 \prod_{t=1}^{N} (1+\lambda_t) - \sum_{t=1}^{N} \left[\prod_{i=t+1}^{N} (1+\lambda_i) \right] b_t$$

Assuming that λ_t is time-invariant, the solution to equation (A.2.1) becomes

$$d_N = d_0 (1+\lambda)^N - \Sigma_{t=1}^{N} (1+\lambda)^{N-t} b_t \qquad \text{(A.2.3)}$$

Given the initial debt level d_0, the constant balance ratio b^* that achieves a target debt ratio d_N^* in N periods $N < \infty$ can be derived from (A.2.3) as

$$b^* = \frac{1}{\Sigma_{t=1}^{N} (1+\lambda)^{N-t}} (d_0 (1+\lambda)^N - d_N^*)$$

which leads to

$$b^* = \frac{\lambda}{(1+\lambda)^N - 1} (d_0 (1+\lambda)^N - d_N^*) \qquad \text{(A.2.4)}$$

Rearranging equation (A.2.4) to decompose the components of adjustment as before, we get

$$b^* - b_0 = [\lambda d_0 - b_0] + \frac{d_0 - d_N^*}{((1+\lambda)^N - 1)/\lambda} \qquad \text{(A.2.5)}$$

Setting $N = 1$ in equation (A.2.5), for example, would give $b^* - b_0 = [\lambda d_0 - b_0] + [d_0 - d_N^*]$, where the *instantaneous* adjustment to the balance ratio required to bring the debt ratio to d^* in 1 period would be the sum of (1) the gap to the debt-stabilizing balance ratio (at the initial debt level) and (2) an additional term

[1] If gross debt becomes negative, it will be interpreted as assets.

[2] Note that when $\lambda < 0$ and $d^* < d_0$, the second term is positive, implying additional adjustment relative to the adjustment needed to stabilize the debt at its initial level. However, if $\lambda > 0$, the debt path becomes unstable, diverging to $+\infty$ if $d^* < d_0$ and $-\infty$ if $d^* > d_0$.

reflecting the distance between the initial and desired debt ratios.

Approach 3: Convergence by a Given Date Following a Transition Period

Let the initial balance ratio b_0 in period $t = 0$ be gradually adjusted until period $T < N$, and achieve b_T^* in period T, which is the constant balance ratio that brings the debt ratio to d_N^* by period N. Assuming a linear adjustment schedule for the balance ratio would give

$$b_t = \begin{cases} \alpha t + b_0, & \text{when } 0 < t < T \\ \alpha T + b_0 = b_T^*, & \text{when } T \le t \le N \end{cases} \quad \text{(A.2.6)}$$

with α denoting the annual adjustment of the fiscal balance from period 0 to T. In this case, equation (A.2.3) becomes

$$d_N = d_0 (1+\lambda)^N - \sum_{t=1}^{T} (1+\lambda)^{N-t}(\alpha t + b_0) - \sum_{t=T+1}^{N} (1+\lambda)^{N-t}(\alpha T + b_0)$$

Rearranging[3] to decompose the components of the total adjustment $b_T^* - b_0 = \alpha T$ leads to

$$\alpha T = \frac{d_0((1+\lambda)^N - 1)}{\Sigma_{t=1}^{N}(1+\lambda)^{N-t}} - b_0 + \frac{\alpha \Sigma_{t=1}^{T}(1+\lambda)^{N-t}(T-t)}{\Sigma_{t=1}^{N}(1+\lambda)^{N-t}} + \frac{d_0 - d_N^*}{\Sigma_{t=1}^{N}(1+\lambda)^{N-t}}$$

which simplifies as

$$\alpha T = \underbrace{[\lambda d_0 - b_0]}_{\text{I}} + \underbrace{\frac{\alpha \Sigma_{t=1}^{T}(1+\lambda)^{N-t}(T-t)}{((1+\lambda)^N - 1)/\lambda}}_{\text{II}} + \underbrace{\frac{d_0 - d_N^*}{((1+\lambda)^N - 1)/\lambda}}_{\text{III}} \quad \text{(A.2.7)}$$

The total adjustment $b_T^* - b_0$ is therefore given by the sum of (I) the gap to the debt-stabilizing balance ratio (at the initial debt level), (II) an additional term reflecting the need to adjust more when the adjustment is gradual rather than instantaneous, and (III) the required extra adjustment due to the distance from the debt target. Note that if $T = 1$, equation (A.2.7) collapses to equation (A.2.5) with an instantaneous adjustment to the constant balance ratio required to achieve d_N^*, which could be very costly. As T increases, the yearly adjustment α falls:[4]

$$\alpha = \frac{\left(\lambda d_0 - b_0 + \frac{d_0 - d_N^*}{((1+\lambda)^N - 1)/\lambda}\right)}{\left(T - \frac{T(1+\lambda)^{N-T}((1+\lambda)^T - 1)/\lambda - \Sigma_{t=1}^{T}(1+\lambda)^{N-t}t}{((1+\lambda)^N - 1)/\lambda}\right)}$$

This $\alpha(T, b_0, N, \lambda, d_0, d_N^*)$ pins down the path for the balance ratio as depicted in (A.2.6), and using (A.2.7) we would have

$$b_T^* = \alpha T + b_0 = \frac{\lambda}{(1+\lambda)^N - 1}[d_0(1+\lambda)^N - d_N^* + A(T, b_0, N, \lambda, d_0, d_N^*)]$$

where $A(T, b_0, N, \lambda, d_0, d_N^*) = \alpha \sum_{t=1}^{T}(1+\lambda)^{N-t}(T-t)$.

Approach 4: Convergence by a Given Date Following a Buildup of Fiscal Buffers

Let $\Delta A_t = A_t - A_0$ be the change in age-related spending relative to period t_0, and b^* be the constant balance ratio excluding the incremental change in age-related spending ΔA_t that, if maintained, would ensure convergence to the debt target d_N^* by the end of period N.[5] In other words, the balance ratio follows this path:

$$b_t = b^* - \Delta A_t \quad \text{(A.2.8)}$$

In this case, equation (A.2.3) becomes

$$d_N^* = d_0(1+\lambda)^N - \sum_{t=1}^{N}(1+\lambda)^{N-t}b^* + S$$

with the analytical solution

$$b^* = \frac{\lambda}{(1+\lambda)^N - 1}(d_0(1+\lambda)^N - d_N^* + S) \quad \text{(A.2.9)}$$

where $S = \sum_{t=1}^{N}(1+\lambda)^{N-t}\Delta A_t$ is the value in year N of the cumulative increase in long-term age-related costs through N. Similar to (A.2.7) we could decompose the components of the total adjustment for a given sequence of ΔA_t as

$$b^* - b_0 = [\lambda d_0 - b_0] + \frac{d_0 - d_N^*}{((1+\lambda)^N - 1/\lambda}$$

[3]On the right-hand side, add and subtract d_0 and rewrite $\alpha t = \alpha T - \alpha(T-t)$. Move d_N to the right-hand side and the terms with αT to the left-hand side. Divide both sides by $\sum_{t=1}^{N}(1+\lambda)^{N-t} = ((1+\lambda)^N - 1)/\lambda$, simplify, and group as shown.

[4]In equation (A.2.7), $\sum_{t=1}^{T}(1+\lambda)^{N-t}t = (1+\lambda)^{N+1}/\lambda^2 [1 - \frac{1+\lambda(T+1)}{(1+\lambda)^{T+1}}]$.

[5]Note that, here we present a general framework in which A_t can differ from A_0 in any given year $t > 0$. In the note, however, we presented simulations based on a special case in which $A_t = A_0$ until year $P-1$ and starts increasing from year P.

$$+ \underbrace{\frac{S}{((1+\lambda)^N - 1)/\lambda}}_{\text{IV}}$$

where (IV) is the additional required upfront adjustment due to costs of an aging population.

Note that if we impose the simplifying assumption that age-related costs increase linearly by an annual amount δ starting from year $P \le N$, i.e., $A_t = \delta(t + 1 - P) + A_0$ for $t \ge P$, the analytical solution for S can be derived as

$$S = (1+\lambda)^{(N+1-(P-1))}/\lambda^2 \left[1 - \frac{1 + \lambda(N + 1 - (P-1))}{(1+\lambda)^{N+1-(P-1)}}\right]\delta$$

Under the special case in our simulations, in which $A_t = A_0$ until year $P - 1$ and starts increasing from year P, the balance ratio would remain constant at b^* as given by (A.2.9) until year $P - 1$ then would gradually decline by ΔA_t starting from year P as given by (A.2.8).

From the Structural Deficit Ceiling to the Nominal Deficit Ceiling

Let Y and Y^s denote the actual and potential nominal GDP, R and R^s the levels of actual and structural revenues, E and E^s the levels of actual and structural expenditures, and $NB = R - E$ and $SB = R^s - E^s$ the nominal and structural balance. Let $OG = \frac{Y - Y^s}{Y^s}$ be the output gap, which implies $\frac{Y^s}{Y} = \frac{1}{1+OG}$. Note that structural revenues are defined as $\frac{R^s}{R} = \left(\frac{Y^s}{Y}\right)^{\eta}$ and structural expenditures as $\frac{E^s}{E} = \left(\frac{Y^s}{Y}\right)^{\kappa}$, where η and κ are the elasticities of revenue and expenditures with respect to the output, which measure the relationship between the cyclical components of revenues and expenditure relative to the cyclical component of output. In this case, the structural balance as a ratio of potential GDP (sb) is defined as

$$\begin{aligned} sb &= \frac{SB}{Y^s} = \frac{R^s}{Y^s} - \frac{E^s}{Y^s} = \frac{R}{Y^s}\frac{R^s}{R} - \frac{E}{Y^s}\frac{E^s}{E} = \frac{R}{Y^s}\left(\frac{Y^s}{Y}\right)^{\eta} \\ &\quad - \frac{E}{Y^s}\left(\frac{Y^s}{Y}\right)^{\kappa} \\ &= \frac{R}{Y/(1+OG)}\left(\frac{1}{1+OG}\right)^{\eta} - \frac{E}{Y/(1+OG)}\left(\frac{1}{1+OG}\right)^{\kappa} \\ &= \frac{R}{Y}(1+OG)^{1-\eta} - \frac{E}{Y}(1+OG)^{1-\kappa} \qquad \text{(A.2.10)} \end{aligned}$$

For a small enough OG, one can approximate $(1+OG)^{1-x} \approx 1 + (1-x)OG$ for $x = \{\eta, \kappa\}$.

In this case, equation (A.2.10) becomes

$$\begin{aligned} sb &= r(1 + (1-\eta)OG) - e(1 + (1-\kappa)OG) \\ &= (r - e) + OG[r(1-\eta) - e(1-\kappa)] \\ &= nb + OG[r(1-\eta) - e(1-\kappa)] \qquad \text{(A.2.11)} \end{aligned}$$

where r and e respectively denote revenue and expenditures as a share of GDP, with the nominal fiscal balance (nb) as a share of GDP given by $nb = r - e$.

Using (A.2.11) and taking a medium-term view (averages of revenue and expenditure ratios), one could derive from a given structural balance target sb^* a corresponding nominal balance target nb^* (and reciprocally) as

$$nb^* = sb^* - OG[r(1-\eta) - e(1-\kappa)]$$

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