JAPAN-IMF SCHOLARSHIP PROGRAM FOR ASIA 2024

Basic Mathematics Aptitude Test

(Full score: 40)

Please Note:

■ You have 60 minutes to complete.

■ No calculators are allowed.

■ Please show all your work and write your answers in the designated space.

Thank you.

Country: _____

Reference Number: _____

Name: _____

In each question below, choose the correct answer from A-E (2 points for each question):

1. Calculate $5\times3 + 15\div 3 \div (-5) - (-6)$.

A. 4
B. 6
C. 8
D. 11
E. 20

2. Calculate $\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$.

A. $\frac{2}{\sqrt{5}}$
B. 2
C. 5
D. $\frac{10+\sqrt{5}}{2}$
E. $\frac{10+2\sqrt{5}}{2}$

3. Suppose that

$$f(x) = \frac{e^{\ln x^2}}{e^{\ln x}}$$

Which of the following is true?

A. $f(x) = x^2$
B. $f(x) = 2x$
C. $f(x) = x$
D. $f(x) = -x$
E. $f(x) = \ln x$

4. Suppose that

$$\begin{cases} f(Q) = A - BQ \\ g(Q) = C + DQ \end{cases},$$

where A, B, C and D are constants and $B \neq -D$. Find the value of Q at which the two functions intersect.

A. $Q = \frac{C-A}{D+B}$
B. $Q = \frac{A-C}{D+B}$
C. $Q = \frac{D+B}{A+C}$
D. $Q = \frac{A+C}{D+B}$
E. $Q = \frac{A+D}{C-B}$

5. Solve for x in the following equation: $2x^2 + 7x - 4 = 0$.

A. $x = -\frac{7}{4}$

B. $x = \frac{-7 \pm \sqrt{17}}{4}$

C. $x = \frac{7 \pm \sqrt{17}}{4}$

D. x = -0.5 and x = 4.

E. x = 0.5 and x = -4.

6. Solve for x in the following equation. $2^{3x} - 7(2^{2x}) - 2^{x+3} = 0$

A. x = 0

B. x = 1

C. x = 2

D. x = 3

E. x = 4

7. Find the set of natural number(s) of x that satisfy the following inequality:

$$7x^2 - 91x + 280 < 0$$

A. {5}

B. {5,6}

C. {6,7}

D. {6,7,8}

E. {7,8,9}

8. Given the sets $S_1 = \{1,4,8\}, S_2 = \{1,7,8\}$, and $S_3 = \{3,8\}$, find: $(S_1 \cap S_2) \cup S_3$, where $\cup$ and $\cap$ denote union and intersection of sets, respectively.

A. {1,3}
B. {1,4,7}
C. {1,3,8}
D. {3,4,8}
E. {4,7,8}

9. In a class of 50 students, 30 take Microeconomics, 23 take Macroeconomics, and 12 take both. What is the probability that a randomly selected student takes neither Microeconomics nor Macroeconomics?

A. 18 percent.
B. 22 percent.
C. 24 percent.
D. 36 percent.
E. 41 percent.

10. If an individual starts a business, there is a 30% chance of earning $100,000, a 50% chance of earning $200,000, and a 20% chance of incurring a loss of $100,000 in the first year. What would be the expected income for the first year?

A. $100,000
B. $110,000
C. $130,000
D. $150,000
E. $170,000

11. Find the first-order derivative of the function $f(x) = x \log_e x$.

A. $f'(x) = 1$
B. $f'(x) = x$
C. $f'(x) = \log_e x$
D. $f'(x) = \log_e x + 1$
E. $f'(x) = \log_e x + x$

12. Find the cross partial derivative $\frac{\partial^2 f(x,y)}{\partial x \partial y}$ of $f(x, y) = e^{2y} \log_e(3x)$.

A. $\frac{e^{2y}}{x}$

B. $\frac{2e^{2y}}{x}$

C. $-\frac{e^{2y}}{x^2}$

D. $2\, e^{2y} \log_e(3x)$
E. $4\, e^{2y} \log_e(3x)$

13. Evaluate the integral $\int_{-2}^{2} (4x^2 + 3x) dx$.

A. 0
B. 12
C. $\frac{50}{3}$
D. $\frac{64}{3}$
E. $\frac{100}{3}$

14. Coordinates of points A and B in the three-dimensional space are given by A = (1, 4, -2) and B = (3, 4, 5). Find the distance between A and B.

A. 7
B. $\sqrt{53}$
C. $\sqrt{56}$
D. 8
E. $\sqrt{59}$

15. Evaluate the following sum $S_n = a + \frac{a}{1+r} + \frac{a}{(1+r)^2} + \frac{a}{(1+r)^3} + \cdots + \frac{a}{(1+r)^{n-1}}$, when a=6, r=0.05, and $n \to \infty$.

A. 120
B. 123
C. 126
D. 129
E. 132

16. Find the quadratic approximation of the following function at the point $x \approx 0$.

$$f(x) = 4\log_e(1 + x) + 6$$

A. $6 + 4x - 4x^2$
B. $6 + 4x - 2x^2$
C. $6 + 4x$
D. $6 + 4x + 2x^2$
E. $6 + 4x + 8x^2$

17. Find the absolute minima (as an ordered pair of *x* and *y*) for the function $y = 3x^4 + 8x^3 - 18x^2$ over (-∞, ∞).

A. (-5, -425)
B. (-3,-135)
C. (-1,-23)
D. (0, 0)
E. (3,-297)

18. Find the values of *x* and *y* that solve the following problem:
maximize $\log_e(2x) + \log_e(3y + 6)$ subject to $x + 2y = 8$.

A. $(x, y) = (1,3.5)$
B. $(x, y) = (2,3)$
C. $(x, y) = (3,2.5)$
D. $(x, y) = (4,2)$
E. None of the above.

19. Find the determinant of matrix $\begin{pmatrix} -1 & 1 & 6 & 8 \\ 9 & 1 & 0 & 9 \\ 2 & 0 & 0 & 4 \\ 0 & 0 & 0 & 3 \end{pmatrix}$.

A. -36
B. -9
C. 0
D. 18
E. 72

20. Let X' denote the transpose matrix of matrix X, and X^{-1} denote the inverse matrix. Calculate $(Z'Z)^{-1}$ for the following non-square matrix $Z = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$.

A. $\begin{bmatrix} \frac{3}{7} & -\frac{1}{7} \\ -\frac{1}{7} & \frac{3}{14} \end{bmatrix}$ B. $\begin{bmatrix} \frac{1}{2} & -\frac{1}{6} \\ -\frac{1}{6} & \frac{1}{4} \end{bmatrix}$ C. $\begin{bmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{8} \end{bmatrix}$ D. $\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$ E. $\begin{bmatrix} \frac{3}{2} & -\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{4} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$