Can Sticky Quantities Explain Export Insensitivity to Exchange Rates?*

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Abstract

Why are trade flows insensitive to movements in real exchange rates? We use micro data on exports for Ireland to show that this insensitivity persists at the firm-level intensive margin, and that sticky prices and/or markup adjustment alone cannot explain it: some quantity stickiness is also necessary. We propose customer base as a potential source of quantity stickiness. We use a quantitative model of the firm problem calibrated to micro data to show that customer base can reduce the sensitivity of trade flows to real exchange rates. This is especially true if there are substantial costs of adjusting investment in customer base, and if these costs are incurred in the foreign market.

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1 Introduction

There is ample evidence from both macro and firm-level data that exports are insensitive to movements in real exchange rates, a fact which is key to explaining the disconnect between real exchange rates and other macroeconomic aggregates. A large literature explores the possibility that price stickiness and pricing-to-market are responsible for this insensitivity. A much smaller literature explores the possibility that this insensitivity is due to what we call sticky quantities, i.e. frictions in the adjustment of quantity demanded conditional on prices. The contribution of this paper is to provide both reduced form and quantitative evidence that sticky quantities can play a role in explaining why firm-level exports (and hence aggregate exports) are insensitive to movements in exchange rates.

We first use firm and customs micro data for Ireland to estimate the sensitivity of export prices, quantities, and revenue to real exchange rates and tariffs, after conditioning on destination market real demand. Our identification strategy relies on examining exports by the same firm of the same product to different destination markets, thereby controlling for marginal cost. Prices do respond to movements in real exchange rates, but the extent of pricing-to-market is relatively modest. Domestic currency prices respond to real exchange rate movements with an elasticity of 0.18, implying relatively large movements in destination currency prices in response to real exchange rates (passthrough is 0.82). However export quantities are very insensitive to movements in real exchange rates: the elasticity of export quantities with respect to real exchange rates is 0.32. The elasticity of export revenue with respect to real exchange rates is 0.5, the sum of price and quantity responses. Interestingly, export quantities are quite sensitive to tariffs, although in principle, real exchange rates and tariffs should shift the relative prices faced by buyers in a similar way. The elasticity of export quantities with respect to tariff changes is -3.10, while there is no pricing-to-market in response to tariff changes, so revenue inherits this elasticity.

Can pricing-to-market due to sticky prices and/or markup adjustment in responses to real exchange rate changes explain revenue responses, and in particular the insensitivity of exports to real exchange rates? Under standard assumptions about demand, and ignoring a possible role for customer base, the estimated elasticities of quantities and prices with respect to real exchange rates imply a price elasticity of demand of -0.39. This is well above reasonable values for the price elasticity of demand, which should be below -1 for a monopolist to be profit-maximizing. In contrast, since there is no pricing-to-market in response to tariff changes, the price elasticity of demand is equal to the quantity elasticity with respect to tariffs, i.e. -3.10. Clearly these two values of the price elasticity of demand are inconsistent, suggesting that sticky prices and markup adjustment alone cannot explain why exports are so insensitive to real exchange rates.

However the assumption that there is no role for customer base is at odds with recent evidence that current sales depend not just on current prices, but also on past investments of the firm, e.g. through marketing and advertising activities: see Foster, Haltiwanger, and Syverson (2016), Fitzgerald, Haller & Yedid-Levi (2023), and Argente, Fitzgerald, Moreira & Priolo (2023). We argue that the dependence of quantities on customer base can generate quantity stickiness in response to real exchange rates, but not tariffs, driving a wedge between quantity responses to real exchange rates and tariffs.

There are two channels through which this may operate. First, if customer base does not fully depreciate between one period and the next, the firm problem is forward-looking. Optimal investment in customer base in response to a certain shock then depends not just on how the shock affects flow profits today, but also how profits are affected in the future. Firms optimally invest more in response to shocks that are perceived to be persistent (e.g. tariff changes) than shocks with a similar impact on profits today that are mean-reverting (e.g. changes in real exchange rates).[1] This effect is magnified if there are adjustment costs of investment in customer base.

The second channel through which customer base may mute export responses to real exchange rates, but not tariffs, is through a potential for positive comovement between the benefits of investment in customer base, and the costs. Suppose that investments in customer base take place in the foreign market. In this case, the benefit of investing in customer base increases when the foreign market undergoes a real appreciation. But this appreciation also increases the cost of investing in customer base, thus dampening investment, and therefore quantity responses. In contrast, if the foreign market cuts tariffs on imports from the home country, this is unlikely to have a direct effect on the cost of investing in customer base, thus rationalizing differences in responses to real exchange rates and tariffs. This has a similar flavor to the distribution cost channel in Corsetti and Dedola (2005).

The rest of the paper is devoted to a quantitative assessment of these two possible implications of customer base. In order to focus on exporter responses, we take a partial equilibrium approach, which does not require us to take a stand on features of the international economy such as the nature of asset markets. We make use of the model of customer base accumulation by exporters estimated by Fitzgerald, Haller and Yedid-Levi (2023) to match facts about steady state exporter dynamics. We augment this model with the possi-

[1] This is the mechanism at the heart of Ruhl (2008), though in his case, fixed costs of export participation induce forward-looking responses of export participation, but not of the intensive margin of export sales.

bility of short term price stickiness, guided by the findings of Fitzgerald and Haller (2014) on the frequency of export price adjustment. We calculate numerically the responses of firms in this model when faced with a joint process for nominal exchange rates, foreign prices, and foreign demand (i.e. a VAR) estimated from the data. We also calculate numerically the responses when firms are faced with credibly announced deterministic tariff reduction paths that resemble the trade liberalization used to identify export responses to tariffs in our micro data. This allows us to compare model-implied revenue, quantity and price elasticities with those from the data.

As is already known, we find that export revenue and export quantities are less sensitive to real exchange rates if prices are sticky in foreign currency, and that these sensitivities are lower when prices adjust less frequently. What is new is that we find that export revenue and quantity elasticities to real exchange rates are decreasing in costs of adjusting investment in customer base, and are considerably lower if costs of accumulating customer base are incurred in the foreign market rather than the home market. In contrast, revenue and quantity elasticities with respect to tariffs are insensitive to costs of adjusting investment in customer base.

Our paper is most closely related to Drozd and Nosal (2012) who show that a quantitative two-country model with sticky quantities can account for several pricing puzzles in international macroeconomics, and to Corsetti and Dedola (2005) who argue that distribution costs may be important in explaining exchange rate disconnect. Our paper is related to a vast literature in macroeconomics and international macroeconomics on price stickiness surveyed by Burstein and Gopinath (2014). It is also related to papers in the macro and trade literatures which incorporate customer base, e.g. Arkolakis (2010), Eaton, Kortum and Kramarz (2011), Gourio and Rudanko (2014), Foster, Haltiwanger and Syverson (2016), Fitzgerald, Haller and Yedid-Levi (2023) and Argente, Fitzgerald, Moreira & Priolo (2023). Our empirical analysis is closely related to Berman, Martin and Mayer (2012). Relative to Ruhl (2008), we explore how the forward-looking nature of firm decisions can help explain the intensive margin, and not just the extensive margin of firm responses to real exchange rates and tariffs.

The next section describes the micro data. The third section presents evidence from these data on how export revenue, quantities and prices respond to real exchange rates and foreign demand, and also tariffs. The fourth section describes the model. The fifth section describes how we parameterize the model and the shock processes we feed in. The sixth section compares the elasticities of revenue, quantities and prices to exchange rates and tariffs obtained from the decision rules of the model with those in the data. The final section concludes.

2 Data

2.1 Micro data

We make use of two sources of confidential micro data made available to us by the Central Statistics Office (CSO) in Ireland: the Irish Census of Industrial Production (CIP), and Irish customs records. The data are described in detail in the appendix to Fitzgerald and Haller (2018).

2.1.1 Census of Industrial Production

The CIP, which covers manufacturing, mining, and utilities, takes place annually. Firms with three or more persons engaged are required to file returns.[2] We make use of data for the years 1996-2009 and for NACE Revision 1.1 sectors 10-40 (manufacturing, mining, and utilities). Of the variables collected in the CIP, those we make use of in this paper are the country of ownership, total revenue, employment, and an indicator for whether the firm has multiple plants in Ireland.

In constructing our sample for analysis, we drop firms with a zero value for total revenue or zero employees in more than half of their years in the sample. We perform some recoding of firm identifiers to maintain the panel dimension of the data, for example, in cases in which ownership changes.

2.1.2 Customs records

Our second source of data is customs records of Irish merchandise exports for the years 1996-2009. The value (euros) and quantity (tonnes)[3] of exports are available at the level of the VAT number, the Combined Nomenclature (CN) eight-digit product, and the destination market (country), aggregated to an annual frequency. These data are matched by the CSO to CIP firms using a correspondence between VAT numbers and CIP firm identifiers, along with other confidential information. The appendix to Fitzgerald and Haller (2018) provides summary statistics on this match.

[2] Multiplant firms also fill in returns at the level of individual plants, but we work with the firm-level data, since this is the level at which the match with customs records can be performed.

[3] The value is always available, but the quantity is missing for about 10% of export records.

In the European Union, data for intra-EU and extra-EU trade are collected separately, using two different systems called Intrastat and Extrastat. The reporting threshold for intra-EU exports (635,000 euro per year in total shipments within the EU) is different from that for extra-EU exports (254 euro per transaction).[4] The high threshold for intra-EU exports likely leads to censoring of exports by small exporters to the EU. However, since the threshold is not applied at the market level but to exports to the EU as a whole, we observe many firms exporting amounts below the 635,000 euro threshold to individual EU markets.

An important feature of the customs data is that the eight-digit CN classification system changes every year. We concord the product-level data over time at the most disaggregated level possible following the approach of Pierce and Schott (2012) and Van Beveren, Bernard, and Vandenbussche (2012).

As a result, we have annual data on value and quantity of exports at the firm-product-market level, where the product is defined at the eight-digit (concorded) level, and the market refers to the destination country. We use this to construct a price (unit value) by dividing value by quantity, where available. In aggregate trade statistics, unit value data at the product level are notoriously noisy. However, conditioning on the exporting firm as well as the product considerably reduces this noise.

We restrict attention to 30 export markets which account for 94% of Irish merchandise exports over the sample period. The markets are: Australia, Austria, Belgium, Brazil, Canada, China, Denmark, Finland, France, Germany, Hong Kong, India, Italy, Japan, Malaysia, Mexico, Netherlands, New Zealand, Norway, Portugal, Saudi Arabia, South Africa, Spain, Sweden, Switzerland, Thailand, Turkey, U.A.E., U.K., and the U.S.

2.2 Macro data

We make use of data on two macro variables in our empirical analysis using the micro data. The first is the real consumption exchange rate between Ireland and the relevant destination market. The second is a measure of real local currency demand in the relevant destination market. Real exchange rates are constructed using data on annual average nominal exchange rates and CPIs from the IMF's *International Financial Statistics* (IFS). The bulk of the variation in real exchange rates is driven by variation in nominal exchange rates. Real demand in the target market is calculated as GDP less exports plus imports, all measured in current local currency, with this aggregate deflated by the relevant country's CPI. The

[4] Intra-EU exports below the threshold are recovered based on VAT returns. The destination market within the EU is not recorded for these returns.

National Accounts data are taken from the OECD's *National Accounts Statistics* where available, and otherwise from the World Bank's *World Development Indicators*. The CPIs are taken from IFS. We collect these variables for the markets described above. More details are provided in the appendix to Fitzgerald and Haller (2018).

In order to estimate the joint process for real exchange rates and foreign demand for our quantitative exercise, we require data at a higher frequency. We describe the quarterly data used for this exercise in Section 5.

2.3 Tariff data

We also make use of data on tariffs faced by Irish firms in the 30 markets of interest. These data are obtained from the WTO and other sources. Tariff data are reported by the WTO using the Harmonized System (HS) 6-digit classification. We restrict attention to HS6 product-market-years for which there are no non-ad-valorem tariffs,[5] and for which there is no sub-HS6 variation in ad valorem tariffs. The HS6 classification changes in 2002 and 2007. We concord the classification over the period 1996-2009 following the approach of Pierce and Schott (2012). To make use of the tariff data, we must also concord it with our export data. At a 6-digit level, the CN (export) classification corresponds to the HS classification. In some cases, our concordance of the CN classification over time results in "products" that cover multiple HS6 categories. We use export revenue at the firm-HS6-market-year level to construct a weighted average of tariffs across the relevant HS6 categories. Full details of the data sources and construction are provided in the appendix to Fitzgerald and Haller (2018).

3 Firm responses to macro variables and tariffs

We now use these data to estimate the elasticity of firm-product-market-level export revenue, quantity and price to real exchange rates, foreign demand and tariffs. We focus on the intensive margin of responses.[6] Our baseline estimating equation is:

$$w_t^{ijk} = c_t^{ij} + \gamma^{jk} + \boldsymbol{\lambda}'\mathbf{a}_t^{ijk} + \boldsymbol{\beta}'\left(\mathbf{z}_t^{jk} * long_t^{ijk}\right) + \boldsymbol{\phi}'\left(\mathbf{z}_t^{jk} * short_t^{ijk}\right) + \varepsilon_t^{ijk} \tag{1}$$

[5]Unlike ad-valorem tariffs, non-ad-valorem tariffs affect incentives to export differently depending on the firm's export price.

[6]Fitzgerald and Haller (2018) estimate participation responses, and find that both entry and exit are unresponsive to real exchange rates, while they are somewhat responsive to tariffs.

w_t^{ijk} is, in turn, log revenue, log quantity and log price for firm i selling product j in market k at time t. c_t^{ij} is a firm-product-year fixed effect, which controls for marginal cost. γ^{jk} is a product-market fixed effect. $\mathbf{a}_t^{ijk}$ is a vector of indicator variables for export history, i.e. indicator variables for the number of years since the most recent export entry (topcoded at 7 years), and an indicator for whether export entry is censored by the beginning of the sample. This vector captures not just the causal relationship between export histories and export revenue, quantity and price, but also the impact of persistent heterogeneity in idiosyncratic demand. $\mathbf{z}_t^{jk}$ is a vector, the elements of which are rer_t^k, which is the log of an index of the real exchange rate between the home market and market k, dem_t^k, which is the log of our index of real aggregate demand in market k, and $\tau_t^{jk} = \ln\left(1 + T_t^{jk}\right)$, which is the log gross ad valorem tariff faced by a firm exporting product j to market k. Finally, $long_t^{ijk}$ (for long export tenure) is an indicator variable, set equal to 1 for observations where market tenure is at least 6 years as well as observations in export spells whose entry is censored, and equal to zero otherwise, while $short_t^{ijk}$ is equal to $1 - long_t^{ijk}$.[7] We interact shocks with these variables to focus on elasticities for the observations least likely to be subject to selection bias by virtue of the fact that they have low exit probability.[8] The coefficients of interest are therefore $\boldsymbol{\beta}'$.

The results from estimating equation (1) are reported in Table 1. The first point to note about these results is that the elasticity of export revenue (in domestic currency) with respect to the real exchange rate is significantly different from zero, but less than one in absolute value. This is in line with estimates based on macro data. The second point to note is that there is pricing-to-market in response to real exchange rates. The elasticity of price in domestic currency with respect to the real exchange rate, which is equal to the markup elasticity because the firm-product-year fixed effect controls for marginal cost, is significantly greater than zero. The third point to note is that the elasticity of export revenue with respect to tariffs is negative, significantly different from zero, and significantly greater than one in absolute value. However there is no pricing-to-market in response to tariffs, as the coefficient on tariffs in the price equation is close to and not significantly different from zero.

[7]Since $long_t^{ijk}$ and $short_t^{ijk}$ are linear combinations of $\mathbf{a}_t^{ijk}$, their level effects are captured by the inclusion of $\mathbf{a}_t^{ijk}$ in the regression.

[8]See Fitzgerald and Haller (2018) for the associated exit equation.

Table 1: Revenue, price and quantity sensitivity to macro variables and tariffs

		(1)		(2)		(3)	
		Revenue		Quantity		Price	
		coeff	s.e.	coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	0.50	(0.08)**	0.32	(0.09)**	0.18	(0.04)**
	dem_t^k	0.43	(0.09)**	0.35	(0.09)**	0.08	(0.05)*
	τ_t^{jk}	-3.13	(0.65)**	-3.10	(0.67)**	-0.02	(0.35)
High exit prob	rer_t^k	0.47	(0.08)**	0.29	(0.09)**	0.17	(0.04)**
	dem_t^k	0.34	(0.09)**	0.26	(0.09)**	0.08	(0.05)*
	τ_t^{jk}	0.81	(0.56)	0.62	(0.54)	0.19	(0.31)
Export history controls		yes		yes		yes	
Firm-product-year f.e.		yes		yes		yes	
Product-market f.e.		yes		yes		yes	
N		184,890		184,890		184,890	
R^2		0.77		0.83		0.91	
R^2-adj		0.68		0.76		0.87	

Notes: Estimation method is OLS. Dependent variable is in turn log Euro revenue, log tonnes and log unit value at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Responses to exchange rates are broadly robust to estimation in differences, to dropping the interaction with the low exit probability indicator, and to dropping controls for export histories ($\mathbf{x}_t^{ik}$).[9] Responses to tariffs are sensitive to these changes: when estimating in differences and dropping the interaction with the low exit probability indicator, responses to tariffs are not significantly different from zero. This is not surprising given that selection bias is more likely for tariffs, and there is less underlying year-to-year variation in tariffs than in real exchange rates. There is some heterogeneity in behavior across firms of different sizes, domestic vs foreign-owned, and across different sectors, but in the main, these differences are not statistically significant. Moreover, our estimates of coefficients on real exchange rates are in line with those reported by Berman, Martin and Mayer (2012) based on French export data.

3.1 Interpretation

Can pricing-to-market account for the low elasticity of revenue with respect to real exchange rates, which contrasts with the high elasticity of revenue with respect to tariffs? Suppose that demand faced by firm i in market k at time t is given by:

$$Q_t^{ik} = Q_t^k d\left(\frac{\left(1 + T_t^{ik}\right) P_t^{ik} / E_t^k}{P_t^{k*}}\right) \Phi_t^{ik}$$

[9] All of the robustness checks are reported in the Appendix.

where Q_t^k is aggregate demand in market k, $d(\cdot)$ is a decreasing function, T_t^{ik} is the ad valorem tariff firm i faces in market k, P_t^{ik} is the price charged by firm i to buyers from market k expressed in home currency, E_t^k is the nominal exchange rate between the home market and market k, P_t^{k*} is the aggregate price level in market k expressed in the currency of market k, and Φ_t^{ik} is a demand shifter idiosyncratic to firm i and market k. Assuming that firm i faces the same marginal cost C_t^i for sales to all markets k, we can write P_t^{ik} as follows:

$$P_t^{ik} = \mu_t^{ik} C_t^i$$

where μ_t^{ik} is the gross markup over marginal cost. Normalizing the aggregate price level in the home market to one, we can define the real exchange rate as

$$RER_t^k = E_t^k P_t^{k*}$$

and write

$$Q_t^{ik} = Q_t^k d\left(\frac{\left(1+T_t^{ik}\right)\mu_t^{ik} C_t^i}{RER_t^k}\right)\Phi_t^{ik} \tag{2}$$

If we assume that the markup μ_t^{ik} may depend on the real exchange rate RER_t^k, but that idiosyncratic demand Φ_t^{ik} does not, we can take the partial derivative with respect to the real exchange rate to yield[10]:

$$\eta_{Q_t^{ik},RER_t^k} = \theta_t^{ik}\left(\eta_{\mu_t^{ik},RER_t^k} - 1\right) \tag{3}$$

where θ_t^{ik} is the price elasticity of demand, and $\eta_{Q_t^{ik},RER_t^k}$ and $\eta_{\mu_t^{ik},RER_t^k}$ denote the elasticities of quantity and the markup with respect to the real exchange rate, respectively.

Now, rearranging this expression, and substituting in our estimates of the quantity and markup elasticities from Table 1, we obtain

$$\theta_t^{ik} = \frac{\eta_{Q_t^{ik},RER_t^k}}{\eta_{\mu_t^{ik},RER_t^k} - 1} = \frac{0.32}{0.18-1} = -0.39$$

This value is clearly greater than -1.[11] The results reported in Appendix A indicate that

[10] Appendix B contains the detailed derivations

[11] Since the quantity and markup elasticities are estimated, the price elasticity of demand is a nonlinear function of random variables $\eta_{Q_t^{ik},RER_t^k}$ and $\eta_{\mu_t^{ik},RER_t^k}$. This implies that strictly speaking we should use the Delta method to recover the price elasticity of dmeand from equation (3). Doing so yields a range of $[-0.39, 0.40]$ for θ_t^{ik}, depending on the assumed correlation between $\eta_{Q_t^{ik},RER_t^k}$ and $\eta_{\mu_t^{ik},RER_t^k}$. The value is always significantly greater than zero and significantly less than one. Details are reported in the Appendix.

the implied price elasticity of demand is greater than -1 for all of the cuts of the data we have tried (firm size, ownership, sector). This is not driven by our use of Irish data. If we use instead the quantity and markup elasticities estimated by Berman, Martin and Mayer (2012), we obtain values for the price elasticity of demand in the range $[-0.55, -0.25]$. In contrast to research that makes use of macro data, these elasticities are based on firm-level data, so these values which are low in absolute value cannot be an artifact of aggregation. Note that a price elasticity of demand which is less than 1 in absolute value is inconsistent with optimizing behavior by monopolistically competitive firms.

Now a similar expression to (3) can be derived in the case of ad valorem tariffs:

$$\eta_{Q_t^{ik},1+T_t^{jk}} = \theta_t^{ik}\left(\eta_{\mu_t^{ik},1+T_t^{jk}} + 1\right)$$

where $\eta_{Q_t^{ik},1+T_t^{jk}}$ and $\eta_{\mu_t^{ik},1+T_t^{jk}}$ denote the elasticities of quantity and the markup with respect to the tariff, respectively. Again rearranging and substituting in our estimates of the relevant elasticities from Table 1, we obtain[12]

$$\theta_t^{ik} = \frac{\eta_{Q_t^{ik},1+T_t^{jk}}}{\eta_{\mu_t^{ik},1+T_t^{jk}} + 1} = \frac{-3.10}{-0.02+1} = -3.16$$

There is an order of magnitude in the difference between the elasticity of demand implied by tariff variation and that implied by real exchange rate variation. This is despite the fact that the underlying quantity and markup elasticities are obtained using responses of the same firms, and shocks to real exchange rates and tariffs at the same frequency (i.e. annual). The fact that there is pricing-to-market in response to real exchange rates, but not in response to tariffs, is not sufficient to account for the difference in revenue elasticities. This points to some explanation based on differential adjustment of quantities conditional on price (and therefore markup) adjustment.

We propose the following potential resolution of this puzzle. If Φ_t^{ik} in expression (2) is not exogenous idiosyncratic demand, but is the outcome of optimizing behavior by firms (as in the case of endogenously accumulated customer base), then we obtain:

$$\theta_t^{ik} = \frac{\eta_{Q_t^{ik},RER_t^k} - \eta_{\Phi_t^{ik},RER_t^k}}{\eta_{\mu_t^{ik},RER_t^k} - 1} = \frac{\eta_{Q_t^{ik},1+T_t^{jk}} - \eta_{\Phi_t^{ik},1+T_t^{jk}}}{\eta_{\mu_t^{ik},1+T_t^{jk}} + 1}$$

where $\eta_{\Phi_t^{ik},RER_t^k}$ is the elasticity of Φ_t^{ik} with respect to the real exchange rate, while $\eta_{\Phi_t^{ik},1+T_t^{jk}}$

[12] Values calculated using the delta method and associated standard errors are reported in the Appendix.

is the elasticity of Φ_t^{ik} with respect to tariffs. As long as $\eta_{\Phi_t^{ik},1+T_t^{jk}}$ is negative, and larger in absolute value than $\eta_{\Phi_t^{ik},RER_t^k}$, we can potentially reconcile the same price elasticity of demand with different (absolute value) estimates of quantity elasticities with respect to real exchange rates and quantity elasticities with respect to tariffs.

Reconciling our estimates of quantity and markup elasticities with respect to real exchange rates with a value for the price elasticity of demand that is greater than 1 in absolute value may be more challenging. It requires in addition that $\eta_{\Phi_t^{ik},RER_t^k} < 0$. Is it plausible that customer base would be *decreasing* in the real exchange rate? If a real exchange rate depreciation makes it more expensive to acquire customers, or if a real depreciation forecasts a negative evolution in future aggregate demand in market k, this could potentially be the case. Our quantitative exercise allows us to examine this possibility.

4 Model

The basic exporter problem is as in Fitzgerald, Haller and Yedid-Levi (2023), where it is structurally estimated to match moments of exporter dynamics. Here we simplify by conditioning on export participation. This is consistent with our focus in the empirical analysis on export relationships with a long history of participation at the firm-product-market level. We augment the model by allowing for sticky prices à la Rotemberg (1982).

The key elements of the model are as follows. Firms face the same cost of production irrespective of the market they are serving, though this cost can move around over time with the domestic price of labor. Firms can increase sales in a market in two ways: by charging low prices, and by investing in accumulable market-specific customer base through expenditures on marketing and advertising. These investments are subject to adjustment costs. Costs associated with investing in customer base may be incurred in the home market or the foreign market. As long as customer base does not fully depreciate, this adds a forward-looking dimension to the firm's problem. Quadratic price adjustment costs also make firm decisions forward-looking.

4.1 Demand

As in the empirical analysis, i indexes firms, and k indexes markets. Firm i's demand in market k has four components. It depends on aggregate demand in market k, and on the consumer price of its good relative to the aggregate price level in market k. In addition, it depends on the fraction of customers it reaches, which is a function of its "customer base."

Finally, there is an idiosyncratic component to demand. If prices are set in domestic currency, we have:

$$Q_t^{ik} = Q_t^k \left(\frac{\left(1 + T_t^{ik}\right) P_t^{ik} / E_t^k}{P_t^{k*}} \right)^{-\theta} \left(D_t^{ik}\right)^{\alpha} \exp\left(\varepsilon_t^{ik}\right). \tag{4}$$

Here, Q_t^k is aggregate real demand in market k, T_t^{ik} is the ad valorem tariff faced by firm i in market k, P_t^{k*} is the aggregate price level in market k, expressed in country k's currency, P_t^{ik} is the price the firm charges to customers from market k, expressed in home currency, and E_t^k is the nominal exchange rate between the home market and market k. If prices are set in foreign currency, P_t^{ik}/E_t^k is replaced by P_t^{ik*}, the price charged to customers from market k, expressed in foreign currency. D_t^{ik} is customer base. If $0 < \alpha < 1$, there are diminishing returns to customer base, and the optimal customer base conditional on export participation is finite and positive. ε_t^{ik} is idiosyncratic demand.

Note that this specification for demand is consistent with the assumptions in the previous section, with an isoelastic form for $d(\cdot)$, and $\Phi_t^{ik} = \left(D_t^{ik}\right)^{\alpha} \exp\left(\varepsilon_t^{ik}\right)$.

4.2 Costs

Firm i faces marginal cost of production $C_t^i = W_t/\omega^i$ in units of home currency, where W_t is nominal labor cost in the home country, expressed in domestic currency, and ω^i is labor productivity for firm i. This is the same for all markets the firm serves.

4.3 Accumulation of customer base

Customer base accumulates according to:

$$D_t^{ik} = (1 - \delta) D_{t-1}^{ik} + A_t^{ik}, \tag{5}$$

where δ is the depreciation rate, and A_t^{ik} is the increment to customer base due to marketing and advertising activities undertaken by the firm. Expenditure on investment in customer base is given by $W_t^{\gamma_D} \left(E_t W_t^{k*}\right)^{1-\gamma_D} c\left(D_t^{ik}, A_t^{ik}\right)$, where:

$$c\left(D_t^{ik}, A_t^{ik}\right) = A_t^{ik} + \phi \frac{\left(A_t^{ik}\right)^2}{D_t^{ik}}$$

and $\gamma_D \in [0,1]$. This formulation includes a convex adjustment cost.[13] W_t^{k*} is nominal labor cost in country k, expressed in country k's currency. If $\gamma_D = 1$, investment costs are incurred in the home market, while if $\gamma_D = 0$, costs are incurred in the foreign market. If implemented as part of a broad-based trade liberalization, lower tariffs might affect foreign labor cost. However we believe this effect is likely to be of second order importance.

4.4 Price setting

We assume there is a quadratic cost of adjusting prices in market k. If prices are sticky in domestic currency, this cost is given by

$$W_t^{\gamma_P}\left(E_t W_t^{k*}\right)^{1-\gamma_P} g\left(P_t^{ik}, P_{t-1}^{ik}\right) = W_t^{\gamma_P}\left(E_t W_t^{k*}\right)^{1-\gamma_P} \chi\left(\frac{P_t^{ik} - P_{t-1}^{ik}}{P_{t-1}^{ik}}\right)^2.$$

If prices are sticky in foreign currency, this cost is given by

$$W_t^{\gamma_P}\left(E_t W_t^{k*}\right)^{1-\gamma_P} g\left(P_t^{ik*}, P_{t-1}^{ik*}\right) = W_t^{\gamma_P}\left(E_t W_t^{k*}\right)^{1-\gamma_P} \chi\left(\frac{P_t^{ik*} - P_{t-1}^{ik*}}{P_{t-1}^{ik*}}\right)^2.$$

As in the case of costs of investing, $\gamma_P \in [0,1]$. If $\gamma_P = 1$, costs of adjusting prices are incurred in the home market. If $\gamma_P = 0$, costs of adjusting prices are incurred in the foreign market.

If $\chi = 0$, prices are flexible, it does not matter what is the currency of invoicing, and the optimal price is:

$$P_t^{ik} = \frac{\theta}{\theta - 1} C_t^i.$$

4.5 Information

When making choices about participation, investment, and prices, firms observe the current value of the individual idiosyncratic state variable, ε_t^{ik}, as well as knowing the process from which it is drawn. Firms also observe current realizations of macro variables $Z_t^k = \left\{W_t, E_t^k, P_t^{k*}, W_t^{k*}, Q_t^k\right\}$ before making choices, and they use the empirical joint process for these shocks to form expectations about future values of these variables. With respect to tariffs, we describe our experiment in detail below, including what firms know, and when they know it.

[13] Fitzgerald et al. (2023) assume irreversibility, but we simplify for this analysis by allowing investment in customer base to be reversible.

5 Model parameters and shock processes

5.1 Model parameters

Fitzgerald, Haller and Yedid-Levi (2023) fix β and estimate $\{\alpha, \delta, \phi\}$ to match moments of the post-entry dynamics of export participation, export quantities and export prices using a bi-annual (6-month) model. We use their estimates of these parameters, modified for a quarterly frequency, to line up with the frequency of time series macro data. We assume a value for θ consistent with a long-run trade elasticity of 3 (i.e. $\theta/(1-\alpha)=3$).[14] Fitzgerald and Haller (2014) report monthly frequencies of price adjustment for Irish exporters to the UK by currency of invoicing. Let ρ_d be the implied fraction of domestic-invoiced prices that are adjusted at a quarterly rate, and let ρ_f be the implied fraction of foreign-invoiced prices that are adjusted at a quarterly rate. Using the approach of Keen and Wang (2007), we can combine the frequency of price adjustment with a value for the price elasticity of demand θ to obtain a value for χ, the parameter governing the cost of adjusting prices using the relationship:

$$\chi = \frac{2(\theta-1)(1-\rho)}{\rho(1-\beta(1-\rho))}.$$

Baseline parameter values are reported in Table 2.

Table 2: Parameter values for the quantitative exercise

β	α	δ	ϕ	θ	ρ_d	ρ_f
$1.05^{-0.25}$	0.41	0.38	0.73	$3(1-\alpha)$	0.407	0.295

Notes: β is as assumed by Fitzgerald, Haller and Yedid-Levi (2023), adjusted to a quarterly frequency. α is as estimated by Fitzgerald, Haller and Yedid-Levi (2023). δ and ϕ are as estimated by Fitzgerald, Haller and Yedid-Levi (2023), but adjusted for a quarterly frequency. θ is consistent with a long-run trade elasticity of 3. ρ_d is the quarterly frequency of price adjustment for exports invoiced in domestic currency implied by the monthly frequency reported in Fitzgerald and Haller (2014), while ρ_f is the corresponding frequency for exports invoiced in Sterling.

We perform our quantitative exercises for the polar cases where $\gamma_D = 0$ (costs of accumulating customer base in the foreign country only) and $\gamma_D = 1$ (costs of accumulating customer base in the home country only). We assume throughout that all costs of adjusting prices are incurred in the same currency as costs of accumulating customer base, i.e. $\gamma_P = \gamma_D$.

Fitzgerald, Haller and Yedid-Levi (2023) also estimate a process for firm-market level idiosyncratic demand ε_t^{ik}. However we simplify our quantitative exercise by ignoring this idiosyncratic heterogeneity.

[14] This is based on the coefficient on tariffs in Table 1.

5.2 Macro shock processes

The macro variables that enter the firm's dynamic problem in Section 4 are the domestic nominal wage, nominal exchange rate with market k, consumer prices in market k, nominal wage in market k, and real demand in market k, i.e. $Z_t^k = \{W_t, E_t^k, P_t^{k*}, W_t^{k*}, Q_t^k\}$. Country-level data on average wages at the quarterly frequency is very sparse. So we use quarterly data on the nominal exchange rate, consumer prices, and real output to estimate a VAR in z_t^k, where:

$$z_t^k = \begin{bmatrix} \ln E_t^k \\ \ln P_t^{k*} - \ln P_t \\ \ln Q_t^k - \ln Q_t \end{bmatrix}.$$

E_t^k is the quarterly average nominal exchange rate between country k and Ireland, P_t is the Irish CPI, and Q_t is real demand in Ireland. P_t^{k*} and Q_t^k are the corresponding variables for country k. We then set $W_t^{k*} = P_t^{k*}$ in our quantitative exercises.

We obtain quarterly data on nominal exchange rates, the CPI and real GDP from International Financial Statistics, 1990 Q1-2022 Q4 for the following countries (the exact time-period varies by country): Ireland, United States, United Kingdom, Japan, Canada, Germany, Netherlands, Belgium, France, Italy, Sweden, Norway, and Switzerland.[15] These are all important export destinations for Ireland over the sample period.

We use these data to estimate VARs country-by-country:

$$z_t^k = \alpha_k + B_k z_{t-1}^k + \gamma_k t + \nu_t^k.$$

For each country k, we use the estimated matrix of coefficients to recover the fitted values of ν_t^k, and calculate the associated variance-covariance matrix, Σ_k. The estimates of B_k and Σ_k are reported in Tables 3 and 4. We confirm that for each matrix B_k, all eigenvalues are less than 1 in absolute value.

[15]For Switzerland, we use real GDP data from the OECD, as the IFS data ends in 1999.

Table 3: VAR estimates I

dep	indep	US	UK	CHE	JPN	CAN	DEU
$\ln E_t^k$	$\ln E_{t-1}^k$	0.928**	0.971**	0.871**	0.907**	0.876**	0.860**
	$\ln \frac{P_{t-1}^{k*}}{P_{t-1}}$	0.298**	0.119**	0.137**	0.040	0.041	-0.007
	$\ln \frac{Q_{t-1}^{k*}}{Q_{t-1}}$	0.024	0.016	0.044**	0.024	0.021	-0.003
$\ln \frac{P_t^{k*}}{P_t}$	$\ln E_{t-1}^k$	-0.014**	-0.047**	-0.021*	-0.019**	-0.048**	0.005
	$\ln \frac{P_{t-1}^{k*}}{P_{t-1}}$	1.005**	0.974**	0.997**	1.005**	0.977**	0.988**
	$\ln \frac{Q_{t-1}^{k*}}{Q_{t-1}}$	0.008	0.003	0.026**	0.042**	0.025**	0.020**
$\ln \frac{Q_t^{k*}}{Q_t}$	$\ln E_{t-1}^k$	-0.039	-0.054	-0.008	0.027	0.012	0.057
	$\ln \frac{P_{t-1}^{k*}}{P_{t-1}}$	-0.106	-0.134**	-0.157**	-0.121**	-0.150**	-0.124**
	$\ln \frac{Q_{t-1}^{k*}}{Q_{t-1}}$	0.932**	0.931**	0.980**	0.953**	0.950**	0.970**
		Variance-covariance matrix of error terms					
	σ_{EE}	0.00130	0.00069	0.00058	0.00243	0.00110	0.00008
	σ_{EP}	-0.00007	-0.00004	-0.00003	0.00001	-0.00003	-0.00001
	σ_{EQ}	-0.00018	-0.00012	-0.00011	0.00002	-0.00002	0.00001
	σ_{PP}	0.00004	0.00002	0.00004	0.00004	0.00003	0.00004
	σ_{PQ}	-0.00002	-0.00001	-0.00001	-0.00001	-0.00001	-0.00002
	σ_{QQ}	0.00083	0.00112	0.00079	0.00092	0.00092	0.00093
	N	111	111	111	111	111	111

Notes: Estimation method is OLS. VARs estimated country-by-country. Estimating equation includes a time trend in each case. ** statistically different from zero at 5% level. * statistically different from zero at 10% level.

Table 4: VAR estimates II

dep	indep	NLD	BEL	FRA	ITA	SWE	NOR
$\ln E_t^k$	$\ln E_{t-1}^k$	0.658**	0.864**	0.840**	0.717**	0.845**	0.892**
	$\ln \frac{P_{t-1}^{k*}}{P_{t-1}}$	-0.034**	-0.009	-0.010	-0.020	0.036	-0.053
	$\ln \frac{Q_{t-1}^{k*}}{Q_{t-1}}$	-0.009	-0.002	-0.002	-0.002	0.064**	0.023
$\ln \frac{P_t^{k*}}{P_t}$	$\ln E_{t-1}^k$	-0.014	0.031	0.017	-0.110	-0.069**	-0.091**
	$\ln \frac{P_{t-1}^{k*}}{P_{t-1}}$	1.008**	0.991**	0.991**	0.974**	0.997**	0.937**
	$\ln \frac{Q_{t-1}^{k*}}{Q_{t-1}}$	0.020**	0.014**	0.022**	0.015**	0.027**	0.052**
$\ln \frac{Q_t^{k*}}{Q_t}$	$\ln E_{t-1}^k$	0.072	0.055	0.041	0.045	-0.002	0.102
	$\ln \frac{P_{t-1}^{k*}}{P_{t-1}}$	-0.169**	-0.127**	-0.142**	-0.155*	-0.118**	-0.059
	$\ln \frac{Q_{t-1}^{k*}}{Q_{t-1}}$	0.935**	0.967**	0.963**	0.961**	0.963**	0.913**
		Variance-covariance matrix of error terms					
	σ_{EE}	0.00005	0.00008	0.00006	0.00004	0.00049	0.00060
	σ_{EP}	-0.00001	-0.00001	0.00000	0.00000	-0.00004	-0.00004
	σ_{EQ}	0.00001	0.00000	0.00001	0.00001	0.00006	-0.00009
	σ_{PP}	0.00005	0.00005	0.00003	0.00004	0.00003	0.00006
	σ_{PQ}	-0.00002	-0.00001	-0.00001	-0.00002	-0.00001	0.00002
	σ_{QQ}	0.00087	0.00082	0.00093	0.00091	0.00081	0.00097
	N	107	111	111	107	111	111

Notes: Estimation method is OLS. VARs estimated country-by-country. Estimating equation includes a time trend in each case. ** statistically different from zero at 5% level. * statistically different from zero at 10% level.

5.3 Tariff paths

For tariffs, we implement an exercise designed to mimic the Uruguay Round trade liberalization which is used to identify firm-level responses to tariffs in the micro data. We assume that for some long period of time, ad valorem tariffs are fixed, and expected to remain constant forever. Unexpectedly, there is a credible announcement that tariffs will follow a particular declining path over a period of years, after which they will remain constant forever. We take 182 different tariff paths of length 14 years from the data described in Section 2. These are drawn from MFN tariffs charged by the US, Canada, Australia, Japan and New Zealand, for (concorded) products which Ireland exports to these destinations, and for which the maximum tariff over the period 1996-2009 is positive.

6 Quantitative exercise and results

With parameters and shock processes in hand, we can investigate the ability of the model to explain the responses of exports to real exchange rates and tariffs.

For macro shocks, we use our VAR estimates to simulate time series of the vector z_t^k of length 60 quarters, using in turn the parameters for the United States, the United Kingdom, Japan, Canada, Germany, Netherlands, Belgium, France, Italy, Sweden, Norway, and Switzerland. We use Dynare to solve for firm responses to each of the simulated series, starting from the deterministic steady state. As already noted, since we are unable to include wages in the quarterly VAR, we assume that $W_t^{k*} = P_t^{k*}$. In addition, we assume that $P_t = W_t = 1$, $\forall t$, since domestic factors are differenced out in the empirical analysis by comparing across markets within a firm. We also assume that tariffs are constant in this exercise. This yields a time series for the vector $\begin{bmatrix} \ln R_t^k & \ln Q_t^k & \ln P_t^k \end{bmatrix}$.

We then aggregate both macro variables and firm responses from a quarterly to an annual frequency. For the macro variables, we construct the quarterly real exchange rate, and average it across quarters, and sum aggregate demand across quarters. For the firm responses, we sum across quarterly revenue and quantity, and construct the price as the ratio of annual revenue to annual quantity. We then use these annualized variables to run a regression similar to the regression we run in the actual data, pooling across all the "countries" and including "country" fixed effects:

$$w_t^k = c_t + \gamma^k + \boldsymbol{\beta}' \boldsymbol{x}_t^k + \varepsilon_t^k, \tag{6}$$

where w_t^k is, in turn, log revenue, log quantity, and log price, and $\boldsymbol{x}_t^k$ contains the log real exchange rate and log foreign demand. We repeat this exercise for 50 different draws of the 60-quarter 12-country time series, and collect the median of the parameter estimates.

For tariffs, we randomly draw 12 paths from the 182 tariff paths of length 14 years described above. We convert these annual tariff paths to a quarterly frequency, assuming that tariffs are constant within a calendar year. We assume that tariffs will be fixed at the terminal value forever beyond the end of the 14-year horizon. We then use Dynare to solve for firm responses to each of the tariff paths, starting from the deterministic steady state, and assuming that firms are certain about the future path of tariffs. We assume that all macro variables are constant in this exercise. This again yields a time series for the vector $\begin{bmatrix} \ln R_t^k & \ln Q_t^k & \ln P_t^k \end{bmatrix}$.

We then aggregate firm responses from a quarterly to an annual frequency as in the case of the macro shock exercise. We use these annualized firm responses to run a regression similar to the regression we run in the actual data, pooling across all the "countries" or "products" and including the appropriate fixed effects:

$$w_t^k = \gamma^k + \beta \ln\left(1 + T_t^k\right) + \varepsilon_t^k, \tag{7}$$

where w_t^k is, in turn, log revenue, log quantity, and log price. We repeat this exercise for 50 different random draws of 12 tariff paths, and collect the median of the parameter estimates.

6.1 Baseline results: macro shocks

Table 5 reports the results from the quantitative exercise involving macro shocks, i.e. the median parameter estimates based on 50 simulations of the panel of 12 countries of the relevant model and parameterization. We vary the invoice currency, the currency in which investment in customer base takes place, ϕ, which governs the adjustment cost of investing in customer base, and ρ, which governs the frequency of price adjustment. The row with our benchmark values of $\{\phi, \rho\}$ is highlighted. Note that when prices are sticky in home currency, parameter estimates are invariant to the degree of price stickiness, and are identical to the parameter estimates when prices are fully flexible ($\rho = 1$) so we do not report a separate table for home currency price stickiness. Estimates of the relevant coefficients from the data (i.e. Table 1) are reproduced in the last row of the table for comparison.

The results can be summarized as follows. Revenue and quantity elasticities with respect to exchange rates are lower when prices are sticky in foreign currency than in home currency. Conditional on being sticky in foreign currency, revenue and quantity elasticities are lower the stickier are prices. Revenue and quantity elasticities with respect to real exchange rates are lower when investment in customer base is in foreign currency rather than in home currency, and when costs of adjustment for this investment are high. Exchange rate passthrough (equal to 1 minus the price elasticity with respect to the real exchange rate) is, as is already well known, lower when prices are stickier (lower ρ).

When prices are flexible ($\rho = 1$), the revenue elasticity with respect to the real exchange rate is always above the price elasticity of demand, illustrating the limits of the ability of sticky quantities alone to account for export insensitivity to exchange rates. For combinations of high levels of price stickiness and high levels of investment adjustment costs, the revenue elasticity with respect to real exchange rate is below the price elasticity of demand (1.77). But the revenue elasticity with respect to the real exchange rate does not fall below 1 for any of the values of ϕ and ρ considered.

Table 5: Elasticities from quantitative exercise: foreign currency price stickiness

		Invest at home						Invest abroad					
		Rev		Qty		Price		Rev		Qty		Price	
ϕ	ρ	*rer*	*q*	*rer*	*q*	*rer*	*q*	*rer*	*q*	*rer*	*q*	*rer*	*q*
0	0.100	2.48	1.68	1.80	1.67	0.68	0.02	1.71	1.72	1.02	1.70	0.68	0.02
0	0.295	2.59	1.69	2.06	1.69	0.53	0.00	1.82	1.73	1.29	1.73	0.53	0.00
0	0.407	2.66	1.70	2.21	1.70	0.45	0.00	1.89	1.74	1.45	1.74	0.44	0.00
0	0.500	2.71	1.70	2.33	1.71	0.38	-0.01	1.95	1.74	1.57	1.75	0.37	-0.01
0	1[§]	3.00	1.70	3.00	1.70	0.00	0.00	2.24	1.73	2.24	1.73	0.00	0.00
0.73	0.100	2.37	1.68	1.68	1.62	0.69	0.02	1.67	1.64	0.98	1.66	0.69	0.02
0.73	**0.295**	2.47	1.69	1.92	1.65	0.55	0.00	1.77	1.65	1.22	1.68	0.55	0.00
0.73	0.407	2.54	1.70	2.07	1.66	0.47	0.00	1.84	1.66	1.37	1.70	0.46	0.00
0.73	0.500	2.59	1.70	2.20	1.66	0.40	-0.01	1.89	1.65	1.50	1.71	0.39	-0.01
0.73	1[§]	2.90	1.69	2.90	1.65	0.00	0.00	2.20	1.65	2.20	1.69	0.00	0.00
1.46	0.100	2.33	1.63	1.64	1.61	0.69	0.02	1.65	1.66	0.96	1.64	0.69	0.02
1.46	0.295	2.42	1.63	1.86	1.63	0.56	0.00	1.75	1.67	1.18	1.67	0.56	0.01
1.46	0.407	2.48	1.63	2.00	1.64	0.48	0.00	1.80	1.68	1.33	1.68	0.48	0.00
1.46	0.500	2.53	1.64	2.12	1.65	0.42	0.00	1.86	1.68	1.45	1.69	0.41	-0.01
1.46	1[§]	2.86	1.63	2.86	1.63	0.00	0.00	2.18	1.67	2.18	1.67	0.00	0.00
3	0.100	2.28	1.61	1.59	1.58	0.69	0.02	1.63	1.65	0.93	1.62	0.69	0.02
3	0.295	2.36	1.61	1.78	1.60	0.58	0.01	1.72	1.66	1.14	1.65	0.58	0.01
3	0.407	2.42	1.61	1.92	1.61	0.51	0.00	1.77	1.66	1.26	1.66	0.50	0.00
3	0.500	2.47	1.61	2.03	1.62	0.44	0.00	1.82	1.66	1.38	1.67	0.44	0.00
3	1[§]	2.82	1.61	2.82	1.61	0.00	0.00	2.16	1.66	2.16	1.66	0.00	0.00
15	0.100	2.21	1.58	1.51	1.55	0.70	0.02	1.59	1.62	0.89	1.60	0.70	0.02
15	0.295	2.26	1.58	1.63	1.57	0.64	0.01	1.65	1.63	1.01	1.61	0.64	0.01
15	0.407	2.31	1.58	1.72	1.57	0.58	0.01	1.69	1.63	1.11	1.63	0.58	0.01
15	0.500	2.35	1.58	1.83	1.58	0.53	0.00	1.72	1.64	1.20	1.64	0.53	0.00
15	1[§]	2.76	1.58	2.76	1.58	0.00	0.00	2.14	1.63	2.14	1.63	0.00	0.00
Data		0.50	0.43	0.32	0.35	0.18	0.08	0.50	0.43	0.32	0.35	0.18	0.08

Notes: Table reports median estimates of coefficients on log real exchange rate and log real demand based on estimating equation (6) in the 50 sets of simulated data for each set of parameter values and assumptions about currency of invoicing and investment expenditure. [§]When $\rho = 1$, prices are fully flexible. Results when prices are sticky in domestic currency are identical to this case, and are invariant to the degree of domestic currency price stickiness. Row with benchmark values of $\{\phi, \rho\}$ is highlighted. Bottom row of the table reports corresponding data estimates taken from Table 1.

6.2 Baseline results: tariffs

Table 6 reports the results from the quantitative exercise involving tariffs. Results are invariant to price stickiness, since the pre-tariff price is the price that is assumed to be sticky. Given our perfect foresight experiment, elasticities of export revenue with respect to tariffs are insensitive to the degree of quantity stickiness, and are always close to the (assumed) trade elasticity of -3.

Table 6: Elasticities from quantitative exercise: tariffs

ϕ	Rev	Qty	Price
0	-3.00	-3.00	0.00
0.73	-3.00	-3.00	0.00
1.46	-3.00	-3.00	0.00
3	-2.99	-2.99	0.00
15	-2.99	-2.99	0.00
Data	-3.13	-3.10	-0.02

Notes: Table reports median estimates of coefficients on log real exchange rate and log real demand based on estimating equation (7) in the 50 sets of simulated data for each set of parameter values. Bottom row of the table reports corresponding data estimates taken from Table 1.

6.3 How important is shock persistence?

The three variables in our estimated VARs have high persistence. The average across the 12 countries for which we estimate the VARs of the autoregressive coefficients on the log nominal exchange rate is 0.85. The average of the autoregressive coefficients on the log difference between foreign and Irish CPI is 0.99. The average of the autoregressive coefficients on the log difference between foreign and Irish real GDP is 0.95. We investigate the role of shock persistence in firm responses to macro shocks by repeating our baseline quantitative exercise, but replacing these three autoregressive coefficients in the country-level VAR coefficient matrices B_k with 0.65. Note that we do this both for firm beliefs, and for the process from which simulated shocks are drawn. We perform this exercise only for the case of foreign currency price stickiness and investment in customer base in the foreign country, since we know that this gives a lower bound on revenue and quantity elasticities. Table 7 reports the results.

Table 7: Elasticities from quantitative exercise with low persistence shocks

		Invest abroad					
		Rev		Qty		Price	
ϕ	η	rer	q	rer	q	rer	q
0	0.100	1.12	1.75	0.11	1.77	1.00	-0.02
0	0.295	1.15	1.75	0.18	1.77	0.97	-0.02
0	0.407	1.18	1.75	0.26	1.77	0.92	-0.02
0	0.500	1.22	1.75	0.35	1.77	0.87	-0.02
0	1	1.89	1.74	1.89	1.74	0.00	0.00
0.73	0.100	1.03	1.49	0.03	1.51	1.00	-0.02
0.73	0.295	1.06	1.50	0.08	1.52	0.97	-0.02
0.73	0.407	1.09	1.50	0.15	1.52	0.93	-0.02
0.73	0.500	1.12	1.50	0.24	1.52	0.89	-0.02
0.73	1	1.81	1.48	1.81	1.48	0.00	0.00
1.46	0.100	1.00	1.43	0.00	1.44	1.00	-0.02
1.46	0.295	1.02	1.43	0.05	1.45	0.98	-0.02
1.46	0.407	1.05	1.43	0.11	1.45	0.94	-0.02
1.46	0.500	1.08	1.43	0.18	1.45	0.90	-0.02
1.46	1	1.78	1.41	1.78	1.41	0.00	0.00
3	0.100	0.97	1.37	-0.03	1.39	1.00	-0.02
3	0.295	0.99	1.37	0.01	1.39	0.98	-0.02
3	0.407	1.01	1.37	0.06	1.39	0.95	-0.02
3	0.500	1.04	1.37	0.12	1.39	0.92	-0.02
3	1	1.75	1.36	1.75	1.36	0.00	0.00
15	0.100	0.93	1.30	-0.07	1.32	1.00	-0.02
15	0.295	0.94	1.30	-0.05	1.32	1.00	-0.02
15	0.407	0.95	1.30	-0.03	1.32	0.98	-0.02
15	0.500	0.96	1.30	0.01	1.33	0.96	-0.02
15	1	1.71	1.29	1.71	1.29	0.00	0.00
Data		0.50	0.43	0.32	0.35	0.18	0.08

Notes: Table reports median estimates of coefficients on log real exchange rate and log real demand based on estimating equation (6) in the 50 sets of simulated data for each set of parameter values, where the autoregressive coefficients in the driving shock processes are replaced by 0.65. Prices are assumed sticky in foreign currency. Bottom row of the table reports corresponding data estimates. Bottom row of the table reports corresponding data estimates taken from Table 1.

In this exercise, we do see elasticities of revenue with respect to "real exchange rates" that are close to 1, and even falling below 1 for combinations of very high investment adjustment costs and very low frequencies of price adjustment. This makes it clear that part of the challenge in explaining export insensitivity to real exchange rates is the fact that real exchange rates are so persistent. This also suggests that future research may want to explore the role of firms' beliefs about the persistence of exchange rates in explaining their behavior. Note that for the stochastic process followed by shocks to matter, firm behavior must be forward-looking. In this sense, frictions in adjusting quantities are still key to firm responses.

6.4 Summary

Our results show that sticky quantities, in the form of costly accumulation of customer base, can drive a wedge between the elasticity of export revenue with respect to real exchange rates and the elasticity with respect to tariffs. This wedge is greater if investments in customer base are incurred in the currency of the destination market, and the higher are the costs of adjusting investment in customer base. In robustness analysis, we also verify that this wedge is increasing in the persistence of customer base (i.e. the lower is δ) and in the marginal product of customer base (α). We also confirm, as is already known, that destination currency price stickiness and lower frequency of price adjustment reduce the elasticity of export revenue with respect to real exchange rates.

Taking the joint process for real exchange rates and foreign demand from the data, our model of customer base accumulation is not able to fully rationalize the low elasticity of export revenue with respect to real exchange rates that we observe in the data. This is because (as is well known) exchange rates are very persistent. We show that within the context of our model, lower persistence of real exchange rates implies lower sensitivity of exports.

7 Conclusion

In this paper, we argue that sticky prices and markup adjustment are not sufficient to account for the insensitivity of exports to real exchange rates. We show using micro data for Ireland that even conditional on the behavior of markups, export quantities are insensitive to real exchange rates. These elasticities are estimated at the firm-product level, but they imply a value for the price elasticity of demand inconsistent with profit maximization. We argue that this suggests market-specific quantity stickiness may play a role in the insensitivity of exports to real exchange rates.

We then perform a quantitative exploration of a particular model of quantity stickiness. In our model, firms invest in customer base which shifts their demand conditional on price. In addition, prices are sticky. We parameterize the model based on evidence from micro data on exporter dynamics and export price stickiness. We use the parameterized model to simulate firm responses to macro shocks (nominal exchange rates, the aggregate price level, and foreign demand) based on VAR estimates, and perfectly anticipated tariff paths that resemble trade liberalizations from the data. Our model can deliver a lower sensitivity of export revenue to real exchange rates than to tariffs. This elasticity gap is bigger in the case of foreign currency price stickiness, a low frequency of price adjustment, costs of accumulating customer base that are incurred in the destination market, and high costs of adjusting investment in customer base. However, although our model can deliver an elasticity of export revenue with respect to tariffs that is close to what we estimate in the data, it cannot match the low estimated elasticity of export revenue to real exchange rates. More research is indicated to understand the role that e.g. firm beliefs about the stochastic process of macro variables can play in explaining the sensitivity of exports to these variables in the presence of frictions in adjusting quantities as well as prices.

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A Robustness

Table 8: Revenue, price and quantity sensitivity to macro shocks and tariffs: estimation in differences

		(1)		(2)		(3)	
		Δ ln Revenue		Δ ln Quantity		Δ ln Price	
		coeff	s.e.	coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	0.48	(0.17)**	0.07	(0.17)	0.38	(0.09)**
	dem_t^k	0.65	(0.25)**	0.64	(0.26)**	-0.02	(0.14)
	τ_t^{jk}	-0.54	(1.60)	-0.71	(1.64)	-0.21	(0.86)
High exit prob	rer_t^k	0.19	(0.20)	0.14	(0.20)	0.04	(0.11)
	dem_t^k	1.07	(0.28)**	1.10	(0.27)**	-0.04	(0.16)
	τ_t^{jk}	0.80	(1.55)	0.74	(1.71)	-0.16	(1.09)
Export history controls		yes		yes		yes	
Firm-product-year f.e.		yes		yes		yes	
Product-market f.e.		no		no		no	
N		113,843		113,556		113,556	
R^2		0.38		0.39		0.35	
R^2-adj		0.22		0.24		0.19	

Notes: Estimation method is OLS. Dependent variable is in turn change in log Euro revenue, log tonnes and log unit value at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 9: Revenue, price and quantity sensitivity to macro shocks: no interaction with spell length

	(1)		(2)		(3)	
	Revenue		Quantity		Price	
	coeff	s.e.	coeff	s.e.	coeff	s.e.
rer_t^k	0.47	(0.08)**	0.30	(0.09)**	0.17	(0.04)**
dem_t^k	0.40	(0.09)**	0.32	(0.09)**	0.08	(0.05)*
τ_t^{jk}	-0.37	(0.53)	-0.50	(0.53)	0.12	(0.29)
Export history controls	yes		yes		yes	
Firm-product-year f.e.	yes		yes		yes	
Product-market f.e.	yes		yes		yes	
N	184,890		184,890		184,890	
R^2	0.77		0.83		0.91	
R^2-adj	0.68		0.76		0.87	

Notes: Estimation method is OLS. Dependent variable is in turn log Euro revenue, log tonnes and log unit value at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 10: Revenue, price and quantity sensitivity to macro shocks: no interaction with spell length, no trajectories

	(1)		(2)		(3)	
	Revenue		Quantity		Price	
	coeff	s.e.	coeff	s.e.	coeff	s.e.
rer_t^k	0.53	(0.09)**	0.35	(0.09)**	0.18	(0.04)**
dem_t^k	0.56	(0.09)**	0.48	(0.10)**	0.08	(0.05)*
τ_t^{jk}	-0.37	(0.55)	-0.49	(0.54)	0.12	(0.29)
Export history controls	no		no		no	
Firm-product-year f.e.	yes		yes		yes	
Product-market f.e.	yes		yes		yes	
N	184,890		184,890		184,890	
R^2	0.75		0.82		0.91	
R^2-adj	0.65		0.74		0.87	

Notes: Estimation method is OLS. Dependent variable is in turn log Euro revenue, log tonnes and log unit value at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 11: Quantity sensitivity to macro shocks: Firm size

		(1)		(2)		(3)	
		Small		Medium		Large	
		coeff	s.e.	coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	0.17	(0.14)	0.44	(0.20)**	0.32	(0.15)**
	dem_t^k	0.08	(0.15)	0.37	(0.21)*	0.61	(0.17)**
	τ_t^{jk}	-0.18	(1.39)	-4.97	(1.50)**	-4.20	(1.10)**
High exit prob	rer_t^k	0.13	(0.14)	0.43	(0.20)**	0.29	(0.15)*
	dem_t^k	0.03	(0.15)	0.33	(0.16)**	0.54	(0.17)**
	τ_t^{jk}	1.08	(1.11)	-0.60	(1.22)	0.47	(0.91)
Export history controls		yes		yes		yes	
Firm-product-year f.e.		yes		yes		yes	
Product-market f.e.		yes		yes		yes	
N		70,357		43,428		61,752	
R^2		0.86		0.85		0.82	
R^2-adj		0.79		0.78		0.75	

Notes: Small: <100 employees. Medium: 100-249 employees. Large: 250+ employees. Estimation method is OLS. Dependent variable is log tonnes at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 12: Price sensitivity to macro shocks: Firm size

		(1)		(2)		(3)	
		Small		Medium		Large	
		coeff	s.e.	coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	0.14	(0.07)**	0.20	(0.09)**	0.19	(0.08)**
	dem_t^k	0.17	(0.07)**	0.06	(0.10)	0.05	(0.09)
	τ_t^{jk}	0.40	(0.74)	-0.24	(0.61)	0.16	(0.60)
High exit prob	rer_t^k	0.14	(0.07)**	0.21	(0.09)**	0.18	(0.08)**
	dem_t^k	0.17	(0.07)**	0.07	(0.10)	0.05	(0.09)
	τ_t^{jk}	0.13	(0.55)	0.75	(0.65)	-0.46	(0.54)
Export history controls		yes		yes		yes	
Firm-product-year f.e.		yes		yes		yes	
Product-market f.e.		yes		yes		yes	
N		70,357		43,428		61,752	
R^2		0.92		0.93		0.89	
R^2-adj		0.89		0.89		0.84	

Notes: Small: <100 employees. Medium: 100-249 employees. Large: 250+ employees. Estimation method is OLS. Dependent variable is log unit value at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 13: Implied price elasticity of demand: Firm size

	Small	Medium	Large
rer_t^k	0.20	0.55	0.40
τ_t^{jk}	0.13	6.54	3.62

Table 14: Quantity sensitivity to macro shocks: Ownership

		(1)		(2)	
		Domestic		Foreign	
		coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	0.05	(0.14)	0.42	(0.11)**
	dem_t^k	0.14	(0.16)	0.42	(0.12)**
	τ_t^{jk}	-0.72	(1.30)	-3.60	(0.78)**
High exit prob	rer_t^k	0.04	(0.14)	0.39	(0.11)**
	dem_t^k	0.07	(0.16)	0.34	(0.12)**
	τ_t^{jk}	1.64	(1.22)	0.52	(0.65)
Export history controls		yes		yes	
Firm-product-year f.e.		yes		yes	
Product-market f.e.		yes		yes	
N		68,595		110,572	
R^2		0.88		0.80	
R^2-adj		0.82		0.73	

Notes: Estimation method is OLS. Dependent variable is log tonnes at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 15: Price sensitivity to macro shocks: Ownership

		(1)		(2)	
		Domestic		Foreign	
		coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	0.27	(0.06)**	0.16	(0.05)**
	dem_t^k	0.12	(0.07)**	0.08	(0.06)
	τ_t^{jk}	-0.17	(0.74)	-0.04	(0.40)
High exit prob	rer_t^k	0.28	(0.06)**	0.15	(0.05)**
	dem_t^k	0.13	(0.07)*	0.07	(0.06)
	τ_t^{jk}	-0.16	(0.65)	0.05	(0.38)
Export history controls		yes		yes	
Firm-product-year f.e.		yes		yes	
Product-market f.e.		yes		yes	
N		68,595		110,572	
R^2		0.94		0.87	
R^2-adj		0.91		0.82	

Notes: Estimation method is OLS. Dependent variable is log unit value at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 16: Implied price elasticity of demand: Ownership

θ_t^{ik}	Domestic	Foreign
rer_t^k	0.07	0.50
τ_t^{jk}	0.87	3.75

Table 17: Quantity sensitivity to macro shocks: Industry

		(1)		(2)		(3)		(4)		(5)	
		Cons food		Cons nonf nondur		Cons durables		Intermediates		Capital goods	
		coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	-0.17	(0.19)	0.58	(0.23)**	-0.36	(0.78)	0.28	(0.17)*	0.48	(0.14)**
	dem_t^k	0.27	(0.23)	0.18	(0.25)	1.76	(0.80)**	0.32	(0.19)	0.36	(0.15)**
	τ_t^{jk}	-1.24	(1.54)	-3.11	(1.60)*	-1.89	(4.60)	-3.21	(1.27)**	-3.21	(1.46)**
High exit prob	rer_t^k	-0.15	(0.19)	0.55	(0.23)**	-0.51	(0.78)	0.25	(0.17)	0.44	(0.14)**
	dem_t^k	0.26	(0.23)	0.07	(0.25)	1.69	(0.80)**	0.08	(0.19)	0.30	(0.15)**
	τ_t^{jk}	0.27	(1.30)	-1.09	(1.32)	4.79	(3.68)	1.42	(1.14)	0.95	(0.92)
Export history controls		yes		yes		yes		yes		yes	
Firm-product-year f.e.		yes		yes		yes		yes		yes	
Product-market f.e.		yes		yes		yes		yes		yes	
N		36,036		24,649		5,094		46,249		66,091	
R^2		0.78		0.80		0.83		0.85		0.77	
R^2-adj		0.67		0.73		0.72		0.78		0.69	

Notes: Estimation method is OLS. Dependent variable is log tonnes at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 18: Price sensitivity to macro shocks: Industry

		(1)		(2)		(3)		(4)		(5)	
		Cons food		Cons nonf nondur		Cons durables		Intermediates		Capital goods	
		coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.	coeff	s.e.
Low exit prob	rer_t^k	0.18	(0.06)**	0.01	(0.13)	0.14	(0.29)	0.30	(0.08)**	0.16	(0.07)**
	dem_t^k	0.03	(0.07)	0.37	(0.15)**	-0.19	(0.33)	0.24	(0.10)**	-0.04	(0.07)
	τ_t^{jk}	-0.05	(0.71)	1.98	(1.18)*	2.07	(2.14)	-0.61	(0.64)	-0.62	(0.53)
High exit prob	rer_t^k	0.19	(0.06)**	0.01	(0.13)	0.16	(0.29)	0.29	(0.08)**	0.15	(0.07)**
	dem_t^k	0.05	(0.07)	0.37	(0.15)**	-0.18	(0.33)	0.22	(0.10)**	-0.04	(0.07)
	τ_t^{jk}	0.31	(0.95)	1.87	(0.93)**	-0.83	(1.91)	-0.19	(0.62)	-0.26	(0.46)
Export history controls		yes		yes		yes		yes		yes	
Firm-product-year f.e.		yes		yes		yes		yes		yes	
Product-market f.e.		yes		yes		yes		yes		yes	
N		36,036		24,649		5,094		46,249		66,091	
R^2		0.89		0.81		0.86		0.92		0.85	
R^2-adj		0.84		0.74		0.77		0.88		0.79	

Notes: Estimation method is OLS. Dependent variable is log tonnes at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

Table 19: Implied price elasticity of demand: Sector

θ_t^{ik}	(1)	(2)	(3)	(4)	(5)
rer_t^k	-0.21	0.59	-0.42	0.40	0.57
τ_t^{jk}	1.30	1.04	0.62	8.23	8.44

Table 20: Sensitivity to macro shocks: Split exchange rate in nominal and price parts

		(1)		(2)	
		Quantity		Price	
		coeff	s.e.	coeff	s.e.
Low exit prob	x_t^k	0.30	(0.09)**	0.24	(0.04)**
	p_t^k	0.62	(0.10)**	0.11	(0.05)**
	dem_t^k	0.40	(0.09)**	0.08	(0.05)*
	τ_t^{jk}	-2.98	(0.67)**	-0.08	(0.35)
High exit prob	x_t^k	0.27	(0.09)**	0.24	(0.04)**
	p_t^k	0.04	(0.11)	0.14	(0.05)**
	dem_t^k	0.32	(0.09)**	0.08	(0.05)*
	τ_t^{jk}	0.59	(0.54)	0.15	(0.31)
Export history controls		yes		yes	
Firm-product-year f.e.		yes		yes	
Product-market f.e.		yes		yes	
N		184,890		184,890	
R^2		0.83		0.91	
R^2-adj		0.76		0.87	

Notes: Estimation method is OLS. Dependent variable is log unit value at the level of the firm-product-market. Robust standard errors are calculated. ** indicates significance at the 5% level. * indicates significance at the 10% level.

B Derivation of elasticities

B.1 Implications of the estimates in a model without customer base

Start with a normalization of $P_t = 1$ so that the Real Exchange Rate is $RER_t^k = \frac{E_t^k P_t^{k*}}{P_t} = E_t^k P_t^{k*}$.

Ignoring aggregate shocks, the optimal pricing decition of a firm is a constant markup over cost:

$$Q_t^{ik} = Q_t^k d\left(\frac{P_t^{ik*}}{P_t^{k*}}\right)\Phi_t^{ik} = Q_t^k d\left(\frac{P_t^{ik}/E_t^k}{P_t^{k*}}\right)\Phi_t^{ik} = Q_t^k d\left(\frac{P_t^{ik}}{RER_t^k}\right)\Phi_t^{ik} = Q_t^k d\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)\Phi_t^{ik}$$

where μ_t^{ik} is (potentially) a function of RER_t^k.

We want to derive the elasticity of Q_t^{ik} with respect to RER_t^k.

$$\frac{\partial Q_t^{ik}}{\partial RER_t^k} = Q_t^k \Phi_t^{ik} d'\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)\left[\frac{\frac{\partial \mu_t^{ik}}{\partial RER_t^k} C_t^i RER_t^k - \mu_t^{ik}C_t^i}{\left(RER_t^k\right)^2}\right] =$$

$$Q_t^k \Phi_t^{ik} d'\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)\mu_t^{ik}C_t^i\left[\frac{\frac{\partial \mu_t^{ik}}{\partial RER_t^k}\frac{RER_t^k}{\mu_t^{ik}} - 1}{\left(RER_t^k\right)^2}\right] =$$

$$Q_t^k \Phi_t^{ik} d'\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)\frac{\mu_t^{ik}C_t^i}{RER_t^k}\left[\epsilon_{\mu,RER} - 1\right]\frac{1}{\left(RER_t^k\right)}$$

where $\epsilon_{\mu,RER}$ is the elasticity of the markup with respect to the real exchange rate.

Now note that the price elasticity of demand is defined as:

$$\theta_t^{ik} = \frac{\partial Q_t^{ik}}{\partial\left(\frac{P_t^{ik*}}{P_t^{k*}}\right)}\frac{\left(\frac{P_t^{ik*}}{P_t^{k*}}\right)}{Q_t^{ik}} = \frac{\partial Q_t^{ik}}{\partial\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)}\frac{\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)}{Q_t^{ik}}$$

implying that

$$Q_t^k \Phi_t^{ik} d'\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)\frac{\mu_t^{ik}C_t^i}{RER_t^k} = \theta_t^{ik}Q_t^{ik}$$

and we can substitute:

$$\frac{\partial Q_t^{ik}}{\partial RER_t^k} = \left[\eta_{\mu,RER} - 1\right]\theta_t^{ik}\frac{Q_t^{ik}}{\left(RER_t^k\right)}$$

$$\Leftrightarrow$$

$$\eta_{Q,RER} = \frac{\partial Q_t^{ik}}{\partial RER_t^k}\frac{RER_t^k}{Q_t^{ik}} = \left[\eta_{\mu,RER} - 1\right]\theta_t^{ik}$$

where $\eta_{Q,RER}$ is the elasticity of quantitiy with respect to RER. As our point estimates are the qunatity and markup elasticities, we can infer the implied price elasticity of demand:

$$\theta_t^{ik} = \frac{\eta_{Q,RER}}{\eta_{\mu,RER} - 1} = \frac{0.26}{0.17 - 1} = -0.313$$

B.2 Adding customer base

As before assume that demand is

$$Q_t^{ik} = Q_t^k d\left(\frac{\mu_t^{ik}C_t^i}{RER_t^k}\right)\Phi_t^{ik}$$

But now assume that Φ_t^{ik} is a function of RER_t^k.

In this case

$$\frac{\partial Q_t^{ik}}{\partial RER_t^k} = Q_t^k \left[\Phi_t^{ik} d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \left[\frac{\frac{\partial \mu_t^{ik}}{\partial RER_t^k} C_t^i RER_t^k - \mu_t^{ik} C_t^i}{\left(RER_t^k\right)^2} \right] + d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\partial \Phi_t^{ik}}{\partial RER_t^k} \right]$$

$$= Q_t^k \left[\Phi_t^{ik} d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \mu_t^{ik} C_t^i \left[\frac{\frac{\partial \mu_t^{ik}}{\partial RER_t^k} \frac{RER_t^k}{\mu_t^{ik}} - 1}{\left(RER_t^k\right)^2} \right] + d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\partial \Phi_t^{ik}}{\partial RER_t^k} \right]$$

$$= Q_t^k \left[\Phi_t^{ik} d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\mu_t^{ik} C_t^i}{RER_t^k} \left[\eta_{\mu,RER} - 1 \right] \frac{1}{\left(RER_t^k\right)} + d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\partial \Phi_t^{ik}}{\partial RER_t^k} \right]$$

$$= Q_t^k \frac{\Phi_t^{ik}}{\left(RER_t^k\right)} \left[d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\mu_t^{ik} C_t^i}{RER_t^k} \left[\eta_{\mu,RER} - 1 \right] + d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\partial \Phi_t^{ik}}{\partial RER_t^k} \frac{RER_t^k}{\Phi_t^{ik}} \right]$$

$$= Q_t^k \frac{\Phi_t^{ik}}{\left(RER_t^k\right)} \left[d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\mu_t^{ik} C_t^i}{RER_t^k} \left[\eta_{\mu,RER} - 1 \right] + d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \eta_{\Phi,RER} \right]$$

$$= Q_t^k \frac{\Phi_t^{ik}}{\left(RER_t^k\right)} d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \left[\frac{d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\mu_t^{ik} C_t^i}{RER_t^k} \left[\epsilon_{\mu,RER} - 1 \right]}{d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right)} + \eta_{\Phi,RER} \right]$$

$$\Leftrightarrow$$

$$\frac{\partial Q_t^{ik}}{\partial RER_t^k} \frac{RER_t^k}{Q_t^{ik}} = \left[\frac{d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\mu_t^{ik} C_t^i}{RER_t^k} \left[\eta_{\mu,RER} - 1 \right]}{d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right)} + \eta_{\Phi,RER} \right]$$

and note that the price elasticity of demand is

$$\theta_t^{ik} = \frac{d' \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right) \frac{\mu_t^{ik} C_t^i}{RER_t^k}}{d \left(\frac{\mu_t^{ik} C_t^i}{RER_t^k} \right)}$$

therefore we have:

$$\eta_{Q,RER} = \theta_t^{ik} \left[\eta_{\mu,RER} - 1 \right] + \eta_{\Phi,RER}$$

$$\Leftrightarrow$$

$$\theta_t^{ik} = \frac{\eta_{Q,RER} - \eta_{\Phi,RER}}{\eta_{\mu,RER} - 1}$$

C Price elasticity of demand estimates based on delta method

The price elasticity of demand is a nonlinear function of random variables:

$$\theta_t^{ik} = \frac{\eta_{Q_t^{ik},RER_t^k}}{\eta_{\mu_t^{ik},RER_t^k} - 1}$$

and similarly for tariffs. To use the delta method to calculate the mean and standard deviation of θ_t^{ik}, we need to take a stand on the covariance of the two elasticities. We do not estimate this covariance, but based on the fact that the correlation must lie in the range $[-1, 1]$, we can calculate a range of different means and standard deviations. The table below reports the means and standard deviations for correlations equal to $\{-1, 0, 1\}$:

Table 21: Price elasticity of demand using Delta method

$\rho(\eta_Q, \eta_P)$	-1	0	1
Variation		rer_t^k	
$\mu(\theta_t^{ik})$	-0.39	-0.39	-0.40
$\sigma(\theta_t^{ik})$	0.09	0.11	0.13
Variation		τ_t^{jk}	
$\mu(\theta_t^{ik})$	-3.33	-3.57	-3.81
$\sigma(\theta_t^{ik})$	1.81	1.32	0.44