**Chapter 11: Lowe, Young and Superlative Indexes: Empirical Studies**

**CONSUMER PRICE INDEX THEORY**

**Chapter Titles**



# CHAPTER 11: LOWE, YOUNG AND SUPERLATIVE INDEXES: EMPIRICAL STUDIES[1]

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**Appendix to Chapter 11: Measuring Upper Level Substitution Bias**

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[1] The authors thank Ning Huang and Shaoxiong Wang for helpful comments.

## 1. Introduction

This chapter summarizes the results of calculations of Lowe, Young and superlative price indexes based on data from the Danish CPI. Section 2 lists the Lowe and Young indexes for 2014-2019. Section 3 presents estimates for annual superlative indexes for 2012-2018, and Section 4 compares annual superlative indexes with the corresponding Lowe and Young indexes for 2014-2018. Section 5 provides an overview of this and other empirical studies on Lowe, Young and superlative indexes. Lowe and Young indexes are the "practical" indexes that are used by most National Statistical Offices to produce their consumer price indexes. They utilize current monthly price indexes for the main categories of household consumption (called elementary indexes) and annual household expenditure weights for the same categories from a previous year. These data can be used retrospectively to construct annual superlative indexes. A superlative index is approximately free from substitution bias. Thus taking the difference between a superlative index and the "practical" index is a measure of upper level substitution bias for the practical index.

The data set consists of the weights and price indexes for 402 elementary aggregates used for calculating the Danish CPI for the period 2012-2019. The elementary price indexes cover the period January 2012 – December 2019. The annual expenditure weights are available for the years 2010-2018. The data set excludes elementary indexes that were not compiled throughout the period. The weight of the excluded elementary indexes amounts to approximately 5 % of the total weighting basis.

An Appendix uses the Danish data to compute some additional indexes including several multilateral indexes that use bilateral superlative indexes as building blocks.

## 2. Lowe and Young Price Indexes

Most countries calculate the CPI as an expenditure weighted arithmetic average of the elementary aggregate indexes that make up the CPI. Expenditure weights usually are only available with a time lag, so that the weight reference period precedes the price reference period when the weights are introduced into the CPI. If the weights are price-updated from the weight reference period to the price reference period, the resulting index will correspond to a Lowe price index. If the weights are used without price-updating, it will correspond to a Young price index.[2]

Figure 1 shows the monthly Lowe, Young and Geometric Young price indexes for 2014-2019. They are defined below by (1)-(3). The indexes are calculated as annually chained indexes with December as link month.[3] Expenditure weights are introduced with a lag of two years so that indexes for year t are based on expenditure weights for year t-2.[4] Hence, indexes for 2014 are based on weights for 2012; indexes for 2015 are based on weights for 2013 etc.

**Figure 1. Monthly Lowe and Young Indexes that are Chained Annually 2014-2019 (2014 = 100)**

[2] See definitions (1)-(3) below. The Lowe, Young and Geometric Young indexes will be defined in more detail in the Appendix. Diewert (2021) provides a detailed discussion of these indexes and their properties.
[3] In the Appendix, January is used as the link month.
[4] The Appendix also computes "true" Lowe and Young indexes as well as Lowe and Young indexes that use weights that are lagged one and two years.

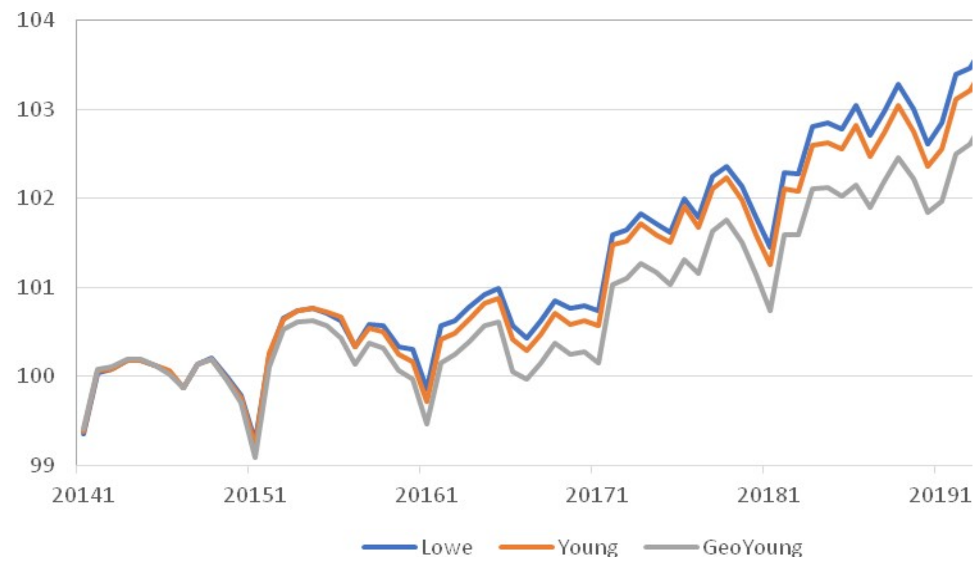


Table 1 shows the *annual* Lowe and Young price indexes for 2014-2019 and the annual rates of change. The annual price indexes are calculated as the arithmetic average of the monthly series. The annual rate of change is the rate of change between the annual indexes.

**Table 1. Lowe and Young Annual Price Indexes 2014-2019**

| | Annual Chained Indexes (2014 =100) | | | | | % change |
|---|---|---|---|---|---|---|
| | 2015 | 2016 | 2017 | 2018 | 2019 | 2014-2019 |
| **Lowe** | **100.43** | **100.65** | **101.79** | **102.67** | **103.48** | **3.48** |
| **Young** | **100.40** | **100.51** | **101.66** | **102.45** | **103.21** | **3.21** |
| **Young*** | **100.44** | **100.65** | **101.81** | **102.64** | **103.42** | **3.42** |
| **GeoYoung** | **100.24** | **100.21** | **101.19** | **101.91** | **102.59** | **2.59** |
| | **Annual rate of change (%)** | | | | | **Av. annual % change** |
| | **2015** | **2016** | **2017** | **2018** | **2019** | **2014-2019** |
| **Lowe** | **0.43** | **0.22** | **1.13** | **0.87** | **0.79** | **0.69** |
| **Young** | **0.40** | **0.11** | **1.15** | **0.78** | **0.74** | **0.63** |
| **Young*** | **0.44** | **0.21** | **1.15** | **0.82** | **0.76** | **0.67** |
| **GeoYoung** | **0.24** | **-0.03** | **0.98** | **0.71** | **0.66** | **0.51** |

From 2014 to 2019 the average annual rate of change of the Young index is 0.63 % against 0.69 % for Lowe index. Hence, the price-updating of weights from t-2 to December t-1 on average increases the annual rate of change of the index by 0.06 % point. The geometric Young index (GeoYoung) is below the Lowe and Young indexes, showing an annual rate of change of 0.51 %.

The Lowe price index is calculated by weighting together the elementary indices, $P^i$, with the price-updated weights:

(1) $P^{Lo}_{0,t} \equiv \sum_{i=1}^{402} w^i_{b(0)} P^i_{0,t}$, where $w^i_{b(0)} \equiv w^i_b P^i_{b,0} / \sum_{j=1}^{402} w^j_b P^j_{b,0}$ for i = 1,…,402.

The $w^i_{b(0)}$ are the weights from the weight reference period (b) price-updated to the price reference period (0) when the weights are introduced into the Lowe index. The weights are price-updated from average of year t-2 to December t-1 and applied for the index calculations for year t. For example, the Lowe index from January to December 2014 is calculated based on the weights from 2012 price-updated from average 2012 to December 2013.[5]

The Young index and the geometric Young index are calculated as the expenditure weighted arithmetic and geometric averages of the elementary price indexes:

(2) $P^{Yo}_{0,t} \equiv \Sigma_{i=1}^{402} w^i_b P^i_{0,t}$;

(3) $P^{GYo}_{0,i} \equiv \Pi_{i=1}^{402} {P_{0,t}}^{w^i_b}$.

Both the Young index and the geometric Young index are calculated based on the weights from year t–2 as they stand, without price-updating.

The index links from December to December are chained (multiplied) annually onto each other using the overlapping December as link month to obtain chained index series with a fixed index reference period. For example, the Young index for May 2017 with 2014 as index reference period is calculated as:

$$(4)\ P^{Yo}_{14:May17} \equiv P^{Yo}_{14:Dec14} \times P^{Yo}_{Dec14:Dec15} \times P^{Yo}_{Dec15:Dec16} \times P^{Yo}_{Dec16:May17}$$
$$= \Sigma_{i=1}^{402} w^i_{12} P^i_{14:Dec14} \times \Sigma_{i=1}^{402} w^i_{13} P^i_{Dec14:Dec15} \times \Sigma_{i=1}^{402} w^i_{14} P^i_{Dec15:Dec16} \times \Sigma_{i=1}^{402} w^i_{15} P^i_{Dec16:May17}.$$

In table 1, the index labelled Young* is based on the weights from year t–2 price-updated from average year t–1 to December t–1. For instance, weights from 2014 are price-updated from average of 2015 to December 2015 and used for the calculation of the index from January to December 2016. The Young* index lies between the Young and Lowe indexes, as could be expected. This approach is applied for calculating the Danish CPI. From 2014 – 2019, the Danish CPI increased by 3.47 % over the 6 years with an average annual increase of 0.68 %, compared to 3.42 % and 0.67 % for the Young* index calculated in this analysis.

The Young* index follows the requirement for the Harmonized Index of Consumer Prices (HICP) of the European Union.[6] The HICP is defined as an annually chain-linked Laspeyres-type index using December as link month. The weights should reflect the consumption pattern of year t – 1. However, in practice, year t – 1 expenditure data are not available for calculation of the index from January year t. To obtain the best possible estimate of the weights for year t – 1, these should be derived from consumption data for

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[5] Basically, the Lowe index is a fixed basket index that uses approximations to annual quantities as the “basket” in the numerator and denominator of the index. The basket is priced out at the prices of the current month in the numerator of the index and at the prices of the base period month in the denominator of the index. The price up-dating procedure deflates the annual weights by an annual price in order to obtain the annual “quantity” basket up to a factor of proportionality. The details of the up-dating procedure are explained more fully in the Appendix.

[6] See the *Harmonized Index of Consumer Prices (HICP) Methodological Manual,* Section 3.5.

year t – 2, the weight reference period. It is up to countries to decide whether to price-update the weights from t – 2 to t – 1, depending on which approach is considered to give the best estimate of the expenditure shares in year t – 1. In either case, the weights must be price-updated from year t – 1 to December t – 1.

## 3. Superlative Price Indexes

Following the 2004 *CPI Manual*, the Fisher, Walsh and Törnqvist price indexes are the preferred target indexes for the CPI and usually give very similar results:

> "Fisher, Walsh and Törnqvist price indices approximate each other very closely using "normal" time series data. This is a very convenient result since these three index number formulae repeatedly show up as being "best" in all the approaches to index number theory. Hence, this approximation result implies that it normally will not matter which of these indices is chosen as the preferred target index for a consumer price index." [7]

Fisher, Walsh and Törnqvist are superlative price indexes[8] that require weights from both the price reference period and the current period. When annual weights become available, it is possible to estimate a superlative CPI by aggregating the elementary indexes using weights from both periods.

Table 2 shows the annual Fisher, Walsh and Törnqvist price indexes for 2012-2018. The three indexes give almost identical results; all three show an average annual rate of change of 0.50 % over the period 2012-2018, which is approximately 0.1 to 0.2 percentage point per year below the "practical" indexes that were calculated in the previous section. Thus annual upper level substitution bias for the practical Danish indexes was fairly low over the sample period.

**Table 2. Annual Superlative Price Indexes 2012-2018**

| | **Annual chained indexes (2012=100)** | | | | | | **Direct index[1)]** |
|---|---|---|---|---|---|---|---|
| | **2013** | **2014** | **2015** | **2016** | **2017** | **2018** | **2012-2018** |
| **Fisher** | **100.67** | **101.14** | **101.33** | **101.43** | **102.41** | **103.06** | **103.31** |
| **Walsh** | **100.67** | **101.14** | **101.32** | **101.43** | **102.41** | **103.07** | **103.40** |
| **Törnqvist** | **100.66** | **101,14** | **101.32** | **101.42** | **102.41** | **103.06** | **103.38** |
| | **Annual rate of change (%)** | | | | | | **Av. annual % change** |
| | **2013** | **2014** | **2015** | **2016** | **2017** | **2018** | **2012-2018** |
| **Fisher** | **0.67** | **0.47** | **0.18** | **0.10** | **0.97** | **0.64** | **0.50** |
| **Walsh** | **0.67** | **0.47** | **0.18** | **0.10** | **0.97** | **0.64** | **0.50** |
| **Törnqvist** | **0.66** | **0.47** | **0.18** | **0.10** | **0.97** | **0.64** | **0.50** |

1) Direct Paasche and Laspeyres indexes for 2012-18 are 102.9 and 104.23, respectively.

The Fisher, Walsh and Törnqvist price indexes are estimated using the following formulas, where $w^i$ are the weights and $P^i$ the price indexes for the elementary aggregates:

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[7] ILO/IMF/OECD/UNECE/Eurostat/The World Bank (2004): *Consumer Price Index Manual. Theory and Practice*. International Labour Office, Geneva, p. 313.

[8] The theory and advantages of superlative indexes were developed by Diewert (1976).

(5) $P^F_{0,t} \equiv [(\Sigma_{i=1}^{402} \; w^i_0 \; P^t_{0,t})(\Sigma_{i=1}^{402} \; w^i_t \, (P^t_{0,t})^{-1})^{-1}]^{1/2}$ ;

(6) $P^W_{0,t} \equiv \Sigma_{i=1}^{402} \; w^i_W \; P^i_{0,t}$ where $w^i_W \equiv [\, w^i_0 \; w^i_t / P^i_{0,t}\,]^{1/2} / \Sigma_{j=1}^{402} [\, w^j_0 \; w^j_t / P^j_{0,t}\,]^{1/2}$ for i = 1,…,402;

(7) $P^T_{0,t} \equiv \Pi_{i=1}^{402} \, (P^i_{0,t})^{½(w^i_0 + w^i_t)}$.

When calculating the superlative indexes, the monthly elementary indexes are aggregated into annual averages (by taking the arithmetic mean)[9] to align with the annual weight reference periods. The chained superlative indexes are calculated by multiplying the annual links of the indexes.[10] For example, the chained Walsh index from 2012 to 2015 is calculated as:

(8) $P^W_{12:15} \equiv P^W_{12:13} \times P^W_{13:14} \times P^W_{14:15}$ .

The direct superlative indexes in Table 2 are calculated based on the expenditure weights for 2012 and 2018 and the chain-linked annual elementary indexes with 2012 = 100.

**4. Comparing Lowe, Young and Superlative Indexes**

Table 3 shows the superlative and Lowe and Young price indexes for the period 2014-2018. The Fisher, Walsh and Törnqvist indexes are almost identical, all with an average annual rate of change of 0.47 % over this period. The Lowe and Young indexes are calculated as explained above, i.e. as annually chained indexes with December as link month and with a two-year lag in the weight reference period, i.e., indexes for year t are based on consumption expenditure data for year t−2.

Over the period 2014 – 2018 the Lowe index exceeds the Young index, but the differences are small. The average annual rate of change of the Young index is 0.61 % against 0.66 % for the Lowe index.

**Table 3. Comparing Superlative, Lowe and Young Indexes 2014-2018**

| | Annual chained indexes (2014=100) | | | | % change | Av. annual % change |
|---|---|---|---|---|---|---|
| | 2015 | 2016 | 2017 | 2018 | 2014-18 | 2014-18 |
| **Fisher** | **100.18** | **100.28** | **101.26** | **101.90** | **1.90** | **0.47** |
| **Walsh** | **100.18** | **100.28** | **101.26** | **101.90** | **1.90** | **0.47** |
| **Törnqvist** | **100.18** | **100.28** | **101.25** | **101.90** | **1.90** | **0.47** |
| **Lowe** | **100.43** | **100.65** | **101.79** | **102.67** | **2.67** | **0.66** |
| **Young** | **100.40** | **100.51** | **101.66** | **102.45** | **2.45** | **0.61** |
| **GeoYoung** | **100.24** | **100.21** | **101.19** | **101.91** | **1.91** | **0.47** |

Compared to a superlative index, the Lowe shows an upward bias of 0.19 percentage points per year and Young shows an upward bias of 0.14 percentage points per year. The Geometric Young index gives similar results to the superlative indexes; i.e., for the particular data set used in this chapter, the Geometric Young index essentially eliminates upper level substitution bias. Since this index can be compiled using the same information that is used in compiling the Lowe and Young indexes, it would be of interest for

[9] In the Appendix, there is some discussion on the problems associated with aggregating monthly price indexes into annual indexes.
[10] The details associated with forming the annual Fisher indexes are explained in the Appendix.

other National Statistical Offices to carry out similar comparisons in order to determine whether upper level substitution bias was substantially reduced using the Geometric Young formula. The results to be presented in the following section indicate that there is a tendency for the Geometric Young formula to underestimate inflation as measured by a superlative index.

## 5. Overview of Empirical Studies on Substitution Bias

Table 4 summarises the results of this and six other studies of retrospective calculations comparing superlative price indexes to Lowe and Young indexes. More details about these studies are provided below.[11]

**Table 4. Comparing Empirical Studies of Superlative, Lowe and Young Indexes**

| | Average annual rate of change (%) | | | | Differences in annual rate of change (% point) | | | |
|---|---|---|---|---|---|---|---|---|
| | Lowe | Young | Geometric Young | Superlative index | Lowe – Young | Lowe – Superlative index | Young – Superlative index | Geometric Young – Superlative index |
| Denmark 2014-2018 | 0.66 | 0.61 | 0.47 | 0.47 | 0.05 | 0.19 | 0.14 | 0.00 |
| Denmark 1996-2003 (1) | 2.39 | 2.33 | 2.21 | 2.28 | 0.06 | 0.11 | 0.05 | -0.07 |
| Canada 1996-2005 (2) | 2.08 | 1.99 | 1.80 | 1.86 | 0.09 | 0.21 | 0.12 | -0.06 |
| Canada 2003-2011 (3) | 1.84 | 1.81 | 1.65 | 1.70 | 0.03 | 0.15 | 0.12 | -0.05 |
| USA 2001-2007 (4) | 2.50 | 2.42 | 2.12 | 2.24 | 0.08 | 0.26 | 0.18 | -0.12 |
| USA 2002-2010 (5) | 2.49 | 2.35 | 2.15 | 2.31 | 0.14 | 0.18 | 0.04 | -0.16 |
| New Zealand 2006-2008 (6) | 3.08 | 2.76 | 2.39 | 2.83 | 0.32 | 0.25 | -0.07 | -0.44 |

Based on the studies presented in Table 4 some general conclusions may be drawn:

- Lowe exceeds Young – price-updating expenditure shares increases the rate of change of the CPI.
- The Arithmetic Young exceeds the Geometric Young.
- The Fisher, Walsh and Törnqvist indexes give very similar results under normal conditions.
- The Lowe and Young indexes are biased upwards compared to a superlative price index, Lowe more than Young. There is one exception (New Zealand 2006-2008) where the Young index is below the superlative index.
- The Geometric Young is biased downwards compared to a superlative index, with one exception (Denmark 2014-2018) where it equals the superlative indexes.

### (1) ***Recalculations of the Danish CPI 1996-2006***. Carsten Boldsen Hansen. Paper presented at the 2007 Ottawa Group meeting

[11] Papers from the Ottawa Group are available from www.ottawagroup.org

This study uses the elementary indexes and weights for the Danish CPI to calculate Lowe, Young and superlative indexes. The Fisher, Walsh and Törnqvist indexes are almost identical. Over the period 1996-2003, the Walsh and Törnqvist indexes showed an average annual rate of change of 2.28 % while the Fisher annual rate was 2.27 %. For the Lowe and Young indexes, the weights were updated every third year; new weights were introduced with a varying lag of 2-3 years. Based on the series for 1996-2003, the annual Lowe index exceeded the corresponding Young index by 0.06 percentage points on average. The Lowe and Young indexes, on average, exceeded the annual rate of change of the Walsh by 0.11 percentage points and 0.05 percentage points respectively. The Geometric Young underestimated the annual rate of change of the Walsh index by 0.07 percentage points.

**(2)** ***Impact of the Price-Updating Weights Procedure on the Canadian Consumer Price Index*****. Ning Huang, Statistics Canada. Room document at the 2011 Ottawa Group meeting**

This study was based on data from the Canadian CPI for the period 1996-2005. In this period, the Canadian CPI was calculated as a chained index where weights were updated with intervals of 4 and 5 years with lags in the weight reference period of 2 years. For the period 1996-2005, the average annual rates of change for the Fisher, Walsh and Törnqvist indexes were 1.77 %, 1.86 % and 1.90 % respectively (Table 9). The significant different between Fisher and the two other indexes was explained to be caused by the sub-index for computers. When removing this sub-index from the calculations, the three superlative indexes gave similar results. For the same period the average annual rate of change of the Lowe index was 2.08 % against 1.99 % for the Young index and 1.80 % for the Geometric Young index (Table 9).[12]

**(3)** ***Choice of index number formula and the upper-level substitution bias in the Canadian CPI.*** **Ning Huang, Waruna Wimalaratne and Brent Pollard. Paper presented at the 2015 Ottawa Group meeting**

Based on Canadian data for 2003-2011, this paper examines superlative indexes and other symmetrically weighted indexes. Lowe and Young indexes are also compiled and the effect of different lags in the implementation of the expenditure weights in the calculation of the CPI are analysed. Table 5.4 of the paper compares a chained annual Fisher index to chained annual Lowe and Young indexes, compiled with a lag of one year in the introduction of the expenditure weights. The results of these calculations are reproduced in table 4 above.

**(4)** ***Reconsideration of Weighting and Updating Procedures in the US CPI.*** **John S. Greenlees and Elliot Williams. BLS working paper 431, 2009[13]**

The study was based on the data from the US Urban CPI for 2001-2007. In this period, the Urban CPI was calculated with biannual links and with a two year lag in the weight reference period. Based on data for the US Urban CPI for 2001-2007, the annual rate of change of the Törnqvist index was 2.24 %. For the same period the Young index showed an annual rate of change of 2.42 % and the Lowe rate was 2.50 % (Table 4). The geometric Young showed an annual rate of change of 2.12 % and hence it was well below the superlative index.

**(5)** ***Post-Laspeyres: The Case for a New Formula for Compiling Consumer Price Indices*****. Paul Armknecht and Mick Silver. Paper presented at the 2013 Ottawa Group meeting.**

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[12] For follow up studies on the Canadian CPI, see Huang, Wimalaratne and Pollard (2015) (2017).

[13] https://www.bls.gov/pir/journal/gj14.pdf

Based on data from the US Urban CPI this study calculated superlative indexes and alternative formulas for 2002 – 2010. In this period, the US CPI was calculated with biannual links. The weights covered two year periods, were updated every second year and were two years old when introduced into the CPI. The Fisher and the Törnqvist tracked each other very closely. Over the period 2002-2010, the Fisher price index increased by an annual average rate of change of 2.31 %, compared with 2.49 % for the Lowe index and 2.35 % for the Young index (page 13).

**(6) *New Zealand 2006 and 2008 Consumers Price Index Reviews: Price Updating. Chris Pike et al*. Room document at the 2009 Ottawa Group meeting**

Based on quarterly New Zealand CPI data for June 2006 to June 2008 with weights of 2003/04 and 2006/07, respectively, implemented in June 2006 and June 2008 quarters. For this period the average annual rate of change of the Lowe index was 3.08 % against 2.76 % for the Young index and 2.39 % for Geometric Young index (based on Table 10). A Fisher index for the same period showed an average annual rate of change of 2.83 % (page 24). This is the only study in which the Young index underestimated the superlative index.

The overall conclusion that can be drawn from this chapter is that it would be useful for National Statistical Offices to undertake similar retrospective studies in order to obtain approximate numerical estimates of the upper level substitution bias that might have been present in their CPIs.

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## Appendix: Supplementary Indexes for Denmark

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### A.1. Introduction

The main text of this chapter used monthly price indexes for 402 monthly elementary aggregates which are components of the Danish CPI for the 7 years 2012-2018. Annual expenditure weights for these 402 aggregates were also available for these years. Various monthly chained Young, and Lowe indexes were calculated using annual weights lagged one or two years since these types of indexes are used by national statistical offices to calculate their Consumer Price Indexes. The monthly price data were aggregated into yearly price data and then, along with the annual expenditure information, annual Lowe and Young indexes along with annual superlative Fisher, Törnqvist and Walsh indexes were calculated. It was found that the three superlative indexes were very close to each other, which is typically the case if the price and quantity data do not fluctuate too much.[14] The difference between these superlative indexes and the Lowe or Young indexes was used to form estimates of *upper level substitution bi*as for a national CPI that is based on the use of these monthly indexes that use lagged annual weights. The main text also reviewed recent studies on the magnitude of upper level substitution bias.[15]

The present Appendix uses the same data set to calculate various supplementary indexes. In section A.2, various monthly indexes are calculated that aggregate the 402 elementary indexes without using the annual weights. Thus these indexes use only monthly price information. It is of interest to calculate these unweighted indexes to see if weighting really matters. If unweighted indexes can adequately approximate an appropriate weighted index, then National Statistical Offices would not have to go to the expense of collecting household expenditure information. The three main unweighted indexes that are used at lower levels of aggregation by statistical offices in recent times are the Jevons, Dutot and Carli indexes.[16] These

[14] See Diewert (1978) who showed that these superlative indexes numerically approximate each other to the second order around an equal price and quantity point.

[15] A path breaking study on types of bias that might be associated with Lowe type Consumer Price Indexes and the possible magnitude of these types of bias was the Boskin Report; see Boskin, Dulberger, Gordon, Griliches and Jorgenson (1996). See Diewert (1998) for a follow up study on possible methods for measuring the various sources of bias.

[16] National Statistical Offices use unweighted indexes (which are called elementary indexes by NSOs) at the initial stages of aggregation. At the final stage of aggregation, NSOs always use price and expenditure weight information. In this Appendix, unweighted indexes are computed purely for illustrative purposes in order to see how close indexes that are computed using only price information can approximate various weighted indexes which are considered in the main text and in this Appendix.

indexes will be defined below along with other indexes that will be discussed below.[17] Comparing these indexes which do not use expenditure weights with indexes that do use weights will give readers some idea of the importance of weighting.

In section A.3, the monthly price indexes are aggregated into annual price indexes for the 402 classes of consumer goods and services. The annual expenditure shares for the 402 products are divided by the corresponding annual prices in order to generate 402 annual "quantities" or volumes for the 7 years of annual data. Using these 402 annual "prices" and "quantities", *annual standard fixed base and chained Laspeyres, Paasche and Fisher indexes* are calculated. Two multilateral indexes are also calculated: the *GEKS* and the *Relative Price Similarity Linked Predicted Share indexes*.[18]

Sections A.4 and A.5 calculate various *weighted month to month Consumer Price Indexes* using the same Danish data set. As was noted in the main text, national statistical offices cannot calculate month to month Consumer Price Indexes in real time using annual weights for the current year since these weights are only available with a lag of one or two years. However, annual weights for the current year can be used in retrospective index number studies so in section A.4, *Lowe, Young and Geometric Young indexes* are calculated using: (i) current year expenditure weights; (ii) weights lagged one year and (iii) weights lagged two years. These indexes which use lagged expenditure weights are "practical" Consumer Price Indexes.

Finally, in section A.5, the assumption is made that the annual expenditure shares can provide an approximation to monthly expenditure shares. Using this (problematic) assumption, monthly "quantities" or "volumes" can be computed and can be combined with the monthly price information to produce approximate *month to month fixed base and chained Laspeyres, Paasche and Fisher indexes*. These indexes can then be compared with the "practical" indexes calculated in section A.4. We also compute some multilateral indexes using the monthly price indexes and volume indexes.

Section A.6 draws some tentative conclusions from the above computations.

**A.2. Month to Month Aggregate Unweighted Indexes**

The monthly consumer price indexes for 402 aggregate product classes for Denmark for the years 2012-2018 were provided by Statistics Denmark. These indexes were normalized so that the price for each product class for January of 2012 was set equal to unity; i.e., each price index was divided by the corresponding index value for January of 2012. The resulting *normalized price for product class n in month t* is denoted by $p_{t,n}$ for t = 1,…,84. Thus t = 1 identifies the data for January of 2012, t = 2 corresponds to the data for February of 2012 and so on. Statistics Denmark also provided annual expenditure shares for each product class for the years 2012-2018 but this information will not be used in this section.

In the definitions below, N = 402 in our particular application. The *Jevons index* for month t, $P_J^t$, is defined as follows:

$$\text{(A1)}\quad P_J^t \equiv \Pi_{n=1}^N (p_{t,n}/p_{1,n})^{1/N} \qquad t = 1,\ldots,84$$
$$= \Pi_{n=1}^N (p_{t,n})^{1/N}$$

where the second equality follows from the fact that all prices have been normalized so that $p_{1,n} = 1$ for n = 1,…,N. Thus the Jevons fixed base index for month t is defined to be the geometric mean of the price

[17] The history and properties of these indexes are discussed in Diewert (2021a).
[18] These indexes are defined and discussed in Diewert (2021b).

ratios $p_{t,n}/p_{1,n}$. Since there are no missing products in this Danish data set, the fixed base and chained Jevons index are identical.

The fixed base *Dutot index* for month t, $P_D{}^t$, is defined as the arithmetic average of the prices in month t divided by the arithmetic average of the prices in month 1:

$$(A2)\ P_D{}^t \equiv \Sigma_{n=1}{}^N (1/N)p_{t,n}/\Sigma_{n=1}{}^N (1/N)p_{1,n} \qquad t = 1,\ldots,84$$
$$= \Sigma_{n=1}{}^N (1/N)p_{t,n}/\Sigma_{n=1}{}^N (1/N)1 \qquad \text{since } p_{1,n} = 1 \text{ for all n}$$
$$= \Sigma_{n=1}{}^N (1/N)p_{t,n}.$$

Again, since there are no missing products in the data set, the fixed base and chained Dutot index are identical.

The third commonly used elementary index is the *Carli in*dex. The *fixed base version* of this index for month t, $P_C{}^t$, is defined as the arithmetic mean of the long term relative prices, $p_{t,n}/p_{1,n}$:

$$(A3)\ P_C{}^t \equiv \Sigma_{n=1}{}^N (1/N)(p_{t,n}/p_{1,n}) \qquad t = 1,\ldots,84$$
$$= \Sigma_{n=1}{}^N (1/N)(p_{t,n}) \qquad \text{since } p_{1,n} = 1 \text{ for all n}$$
$$= P_D{}^t \qquad \text{using the third line in (A2).}$$

Thus if there are no missing prices for the window of data under consideration and all prices are normalized to equal 1 in the base month, then the fixed base Carli index for month t, $P_C{}^t$, is equal to the fixed base (and chained) Dutot index, $P_D{}^t$.

The definition of the chained Carli index for month t, $P_{CCh}{}^t$, is more complicated. First define the *Carli chain link index* between months t−1 and t, $P_{CLink}{}^t$, as follows:

$$(A4)\ P_{CLink}{}^t \equiv \Sigma_{n=1}{}^N (1/N)(p_{t,n}/p_{t-1,n}) \qquad t = 2,3,\ldots,84.$$

Using definition (A4), the *Carli chain linked indexes* for all months t in scope, $P_{CCh}{}^t$, are defined as follows:

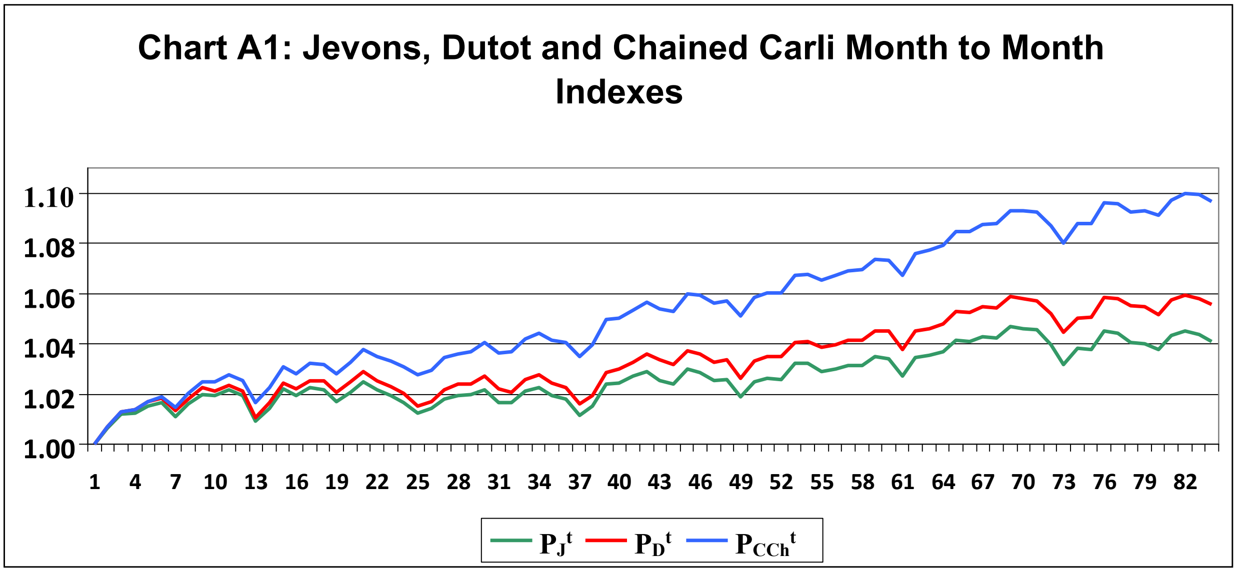
$$(A5)\ P_{CCh}{}^1 \equiv 1;\ P_{CCh}{}^t \equiv P_{CCh}{}^{t-1}\times P_{CLink}{}^t;\ t = 2,3,\ldots,84.$$

The indexes $P_J{}^t$, $P_D{}^t$ and $P_{CCh}{}^t$ for t = 84 are 1.04091, 1.05581 and 1.09678 respectively.[19] Chart A1 indicates that the chained Carli index finishes substantially above the Dutot index and the Dutot index finishes above the Jevons index. The choice of an elementary index number formula does matter.

It is not surprising that the chained Carli index finishes above the Jevons index (which is also a chained Jevons index) because the geometric mean of N price ratios will always be equal to or less than the arithmetic mean of the same N price ratios.[20] It is also the case that the geometric mean of N prices will always be equal to or less than the corresponding arithmetic mean of the same N prices and this explains why the Jevons index is less that the Dutot index when all prices are normalized to equal one in the base period.

The Jevons, Dutot and Chained Carli indexes for our Danish CPI data are plotted in Chart A1 below.

[19] The corresponding annualized average geometric growth rates for these indexes are as follows: $(P_J{}^{84})^{1/6} = 1.00671$, $(P_D{}^{84})^{1/6} = 1.00909$ and $(P_{CCh}{}^{84}) = 1.0155$.
[20] This follows from Schlömilch's inequality; see Hardy, Littlewood and Pólya (1934; 26) or Diewert (2021a).




When there are no missing prices, the Jevons and Dutot indexes both satisfy Walsh's (1901; 389), (1921; 540) Multiperiod Identity Test. This test is the following one: if the prices in period t are identical to the prices in period 1, then the index number formula should register a value of one to indicate that there is no change in the price level going from period 1 to t. The fixed base Carli index also satisfies this test but the chained Carli index does not. Thus the chained Carli index number formula is said to suffer from a *chain drift problem*.[21]

The economic approach to index number theory can be applied to bilateral indexes that utilize *both* price and quantity information; it cannot be applied if only price information is available. Thus the economic approach cannot determine which elementary index that utilizes only price information is "best". However, the test or axiomatic approach to index number theory can be applied to elementary indexes that utilize only price information. Since the chained Carli index does not satisfy the Multiperiod Identity Test but the Jevons and Dutot indexes do satisfy this important test, the Jevons and Dutot indexes are favoured over chained Carli indexes. However, since the Jevons index is invariant to changes in the units of measurement while the Dutot index does not satisfy this important test, the Jevons index probably emerges as a "best" index from the viewpoint of the test approach to index number theory when only price information is available.[22]

In addition to showing that the choice of an index number formula matters, Chart A1 shows that the Danish CPI data indicates the presence of a considerable amount of seasonality in the pattern of prices. Prices are generally very low in January and very high in October or November of each year.[23]

### A.3. Standard Annual Indexes

Statistics Denmark has provided estimated annual expenditure shares for the 402 elementary aggregates for the years 2012-2018. We will denote these years as years y = 1-7 in what follows. Denote the *annual*

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[21] For additional material and references to the literature on the chain drift problem, see Diewert (2021b).
[22] See Diewert (2021a) (2021b) for more complete discussions of the test approach to index number theory.
[23] However, other European countries (such as Belgium, Italy and the Netherlands) also have CPIs which exhibit similar amounts of seasonality.

*expenditure share* for product class n in year y as $S_{y,n}$ for y = 1,…,7 and n = 1,…,N = 402. We need to define *annual prices* for the 402 products, $p_{y,n}{}^{*}$, that will match up with these annual expenditure shares. It turns out that it is not a trivial matter to construct annual prices from monthly prices.

If monthly price and quantity (or volume) information is available and there is seasonality in prices and quantities, then Mudgett (1955) and Stone (1956) recommended that an annual index should treat each product in each season as a separate product in the annual index number formula.[24] Diewert, Finkel, Sayag and White (2022) showed how this suggestion could be implemented for various index number formulae, provided that monthly price and quantity information is available.[25] Since monthly quantity or expenditure information on the 402 product classes is not available, this suggestion cannot be implemented using the Danish data.

Another approach to the problem of aggregating data over months to form annual indexes is to form annual unit value prices for each product. Purchases of a product over a time period may take place at different prices so the question arises: how should these possibly different prices be aggregated into a single price that is representative of all transaction prices made during the period? Walsh (1901; 96) (1921; 88) was the first to provide an answer to this question: he suggested that the appropriate price was the *unit value price* which is equal to the total value of transactions for the product under consideration divided by the total quantity transacted. The advantage of using a unit value price as the representative price is that the corresponding aggregate quantity is equal to the total quantity transacted during the period. This same aggregation strategy can be applied to the problem of aggregating over months. Thus let $p_{y,m,n}$ be the monthly unit value price for product n in month m of year y and let $q_{y,m,n}$ be the corresponding monthly total quantity transacted for product n in month m of year y. Then the corresponding *annual unit value price* for product n in year y, $p_{y,n}$, is defined as follows:

$$\text{(A5)}\quad P_{y,n} \equiv \Sigma_{m=1}{}^{12}\, p_{y,m,n}q_{y,m,n}/\Sigma_{m=1}{}^{M}\, q_{y,m,n} = \Sigma_{m=1}{}^{12}\, p_{y,m,n}q_{y,m,n}/Q_{y,n}\ ; \qquad y = 1,\ldots,7;\ n = 1,\ldots,402.$$

The *aggregate annual quantity* for product n in year y is defined as:

$$\text{(A6)}\quad Q_{y,n} \equiv \Sigma_{m=1}{}^{12}\, q_{y,m,n}\ ; \qquad y = 1,\ldots,7;\ n = 1,\ldots,402.$$

(A5) and (A6) define theoretical annual prices and quantities for each year y and each product n. Define the *annual price and quantity vectors* for year y as $P^y \equiv [P_{y,1},\ldots,P_{y,}]$ and $Q^y \equiv [Q_{y,1},\ldots,Q_{y,N}]$ for N = 402. Define total consumption for year y as $P^y\cdot Q^y \equiv \Sigma_{n=1}{}^{N}\, P_{y,n}Q_{y,n}$ and define the *annual sh*are for product n of total consumption in year y as:

$$\text{(A7)}\quad S_{y,n} \equiv P_{y,n}Q_{y,n}/P^y\cdot Q^y\ ; \qquad y = 1,\ldots,7;\ n = 1,\ldots,402.$$

Using definitions (A5)-(A7), it can be seen that if we divide each annual expenditure share $S_{y,n}$ by the corresponding annual unit value price $P_{y,n}$ defined by (A5), we obtain the annual quantity $Q_{y,n}$ defined by (A6) divided by total year y consumption, $P^y\cdot Q^y$; i.e., we have the following relationships:

$$\text{(A8)}\quad S_{y,n}/P_{y,n} = [P_{y,n}Q_{y,n}/P^y\cdot Q^y]/P_{y,n} = Q_{y,n}/P^y\cdot Q^y\ ; \qquad y = 1,\ldots,7;\ n = 1,\ldots,402.$$

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[24] Diewert (1983) showed how this approach to the construction of Mudgett Stone annual indexes could be extended to provide an annualized price comparison of the data for a current rolling year (12 consecutive months of data) to a base year.

[25] Using their Israeli data set, these authors showed that different methods of aggregation over months gave rise to substantially different annual indexes. The Mudgett-Stone approach to forming annual indexes is our preferred approach from a theoretical point of view. However, this approach needs some modification if there is substantial price change *within* the year as might be caused by a hyperinflation; see Hill (1996).

The above algebra shows that deflating an annual expenditure share by an appropriate annual price will lead to a "quantity" that is equal to the "true" annual quantity transacted divided by total annual consumption. The problem with the above algebra starts at definition (A5) which defined the annual unit value price for each product. In order to actually calculate these annual prices, $P_{y,n}$, it is necessary to have information on the corresponding annual quantities transacted, the $Q_{y,n}$. But this information is not available.

In order to form approximations to the "true" annual product prices and quantities, some additional assumptions must be made. Our *first additional assumption* is that for each product, purchases are distributed evenly over each month in each year. This assumption implies the following equations:

$$\text{(A9)}\quad q_{y,m,n}/Q_{y,n} = 1/12 \; ; \qquad y = 1,\ldots,7;\ m = 1,\ldots,12;\ n = 1,\ldots,402.$$

Upon substituting assumptions (A9) into definitions (A5), we obtain the following equations:

$$\text{(A10)}\quad P_{y,n} = \Sigma_{m=1}^{12}\, p_{y,m,n}q_{y,m,n}/Q_{y,n} \; ; \qquad y = 1,\ldots,7;\ n = 1,\ldots,402$$
$$= \Sigma_{m=1}^{12}\, (1/12)p_{y,m,n}.$$

Thus under assumption (A9), the annual unit value price for product n is simply the arithmetic average of the monthly unit value prices.

Our *second additional assumption* is that the monthly elementary price indexes that have been constructed by Statistics Denmark (the observable $p_{y,m,n}$) are adequate approximations to the monthly unit value prices (normalized to equal unity in month 1).[26] This assumption along with our previous assumption (A9) that implied equations (A10) means that taking the arithmetic average of the monthly Danish elementary indexes is an appropriate annual price index. In fact, many statistical agencies (including Statistics Denmark) use simple averages of their monthly elementary indexes as appropriate annual elementary indexes. Our discussion above simply indicates to readers that these annual indexes are not necessarily accurate approximations to "true" annual indexes that are based on alternative methodologies. In any case, in this section, we will construct *annual product prices* using the prices $p_{y,n}^*$ defined by the second line (A10). Thus define the year y annual price for product n, $p_{y,n}^*$, as follows:

$$\text{(A11)}\quad p_{y,n}^* \equiv \Sigma_{m=1}^{12}\, (1/12)p_{y,m,n} \; ; \qquad y = 1,\ldots,7;\ n = 1,\ldots,402.$$

The corresponding *annual product quantities* (or volumes) $q_{y,n}^*$ that will be used in this section are defined as follows:

$$\text{(A12)}\quad q_{y,n}^* \equiv S_{y,n}/p_{y,n}^* \qquad y = 1,\ldots,7;\ n = 1,\ldots,402.$$

Using equations (A8) and our assumptions, it can be seen that these annual "quantities" $q_{y,n}^*$ defined by (A12) are approximately equal to the true quantities transacted in year y divided by total consumption in year y, $P^y \cdot Q^y$.[27]

[26] These normalizations simply change the units of measurement for the product groups.
[27] Note that the annual share vectors that are generated by the price and quantity vectors $p^{y*}$ and $q^{y*}$ are equal to the Statistics Denmark share vectors $S^y \equiv [S_{y,1},\ldots,S_{y,402}]$ for y = 1,…,7.

In the indexes and tables which follow, the underlying annual price and quantity data used to generate the indexes will be the $p_{y,n}^*$ and $q_{y,n}^*$ defined by (A11) and (A12). The year y price and quantity vectors are defined as $p^{y*} \equiv [p_{y,1}^*,\ldots, p_{y,402}^*]$ and $q^{y*} \equiv [p_{y,1}^*,\ldots,q_{y,402}^*]$ for y = 1,…,7.

In making a price comparison between two periods, the Laspeyres and Paasche indexes are fundamental because they simply do a ratio comparison of the cost of a fixed reference quantity vector at the prices of the comparison period in the numerator and at the base period prices in the denominator. The Laspeyres index chooses the quantity vector that was consumed in the base period as the reference quantity vector and the Paasche index chooses the comparison period quantity vector. These indexes are both meaningful and easy to explain to the public. In general they will give different answers. If it is necessary to give a single estimate for inflation over the two periods being compared, then it is useful to take a symmetric average of the Laspeyres and Paasche indexes as the single estimate. It turns out that the geometric average of these two indexes has the "best" properties from the viewpoint of the test approach to index number theory which is the Fisher (1922) ideal index.[28] The Fisher index also has good properties from the viewpoint of the economic approach to index number theory. Thus in this section, we use the Danish CPI data to calculate annual Laspeyres, Paasche and Fisher indexes using the $p_y^*$ and $q_y^*$ as the underlying price and quantity data.[29]

The *fixed base Laspeyres, Paasche and Fisher indexes* for year y, $P_L^y$, $P_P^y$ and $P_F^y$, are defined as follows:

(A13) $P_L^y \equiv p^{y*}\cdot q^{1*}/p^{1*}\cdot q^{1*}$ ; y = 1,…,7;
(A14) $P_P^y \equiv p^{y*}\cdot q^{y*}/p^{1*}\cdot q^{y*}$ ; y = 1,…,7;
(A15) $P_F^y \equiv [P_L^y\, P_P^y]^{1/2}$ ; y = 1,…,7.

The above indexes are listed in Table A2 below.

In order to define chained indexes, it is useful to define the following *Laspeyres, Paasche and Fisher bilateral annual indexes* that compare the prices of year y relative to the base year z as follows:

(A16) $P_L(y/z) \equiv p^{y*}\cdot q^{z*}/p^{z*}\cdot q^{z*}$ ; y = 1,…,7; z = 1,…,7;
(A17) $P_P(y/z) \equiv p^{y*}\cdot q^{y*}/p^{z*}\cdot q^{y*}$ ; y = 1,…,7; z = 1,…,7;
(A18) $P_F(y/z) \equiv [P_L(y/z)P_P(y/z)]^{1/2}$ ; y = 1,…,7; z = 1,…,7.

The *annual chained Laspeyres, Paasche and Fisher indexes* are defined as follows for year 1:

(A19) $P_{LCH}^{1*} \equiv 1$ ; $P_{PCH}^{1*} \equiv 1$ ; $P_{FCH}^{1*} \equiv 1$ .

For years y following year 1, the above indexes are defined recursively using the bilateral maximum overlap annual indexes defined above by (A16)-(A19) as follows:

(A20) $P_{LCH}^y \equiv P_{LCH}^{y-1}\, P_L(y/(y-1))$ ; y = 2,...,7;
(A21) $P_{PCH}^y \equiv P_{PCH}^{y-1}\, P_P(y/(y-1))$ ; y = 2,...,7;
(A22) $P_{FCH}^y \equiv P_{FCH}^{y-1}\, P_F(y/(y-1))$ ; y = 2,...,7.

The chained Laspeyres, Paasche and Fisher indexes are also plotted in Chart A2 below.

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[28] See Diewert (1997; 138).
[29] Since the underlying price and quantity data are not actual annual unit value prices or actual total annual quantities, it is more correct to say that we are calculating various annual indexes using the $p^{y*}$ and $q^{y*}$ as the underlying price and quantity data and the Laspeyres, Paasche and Fisher formulae applied to these data.

A problem with the chained indexes is that in general, they will not satisfy Walsh's Multiperiod Identity Test and hence they may be subject to a certain amount of chain drift. On the other hand, fixed base indexes compare the prices of all periods with the prices of period 1 and hence the prices of period 1 play an asymmetric role. Gini (1924) (1931) showed how to solve the above problems with fixed base and chained indexes by introducing the GEKS index. This index is equal to normalization of all possible "star" indexes; i.e., each period is chosen as the base period and the final index is the geometric mean of the star indexes. Formally, the *annual GEKS price level*s, $p_{GEKS}{}^{y}$, are defined as follows:

(A23) $p_{GEKS}{}^{y} \equiv [\prod_{z=1}{}^{7} P_F(y/z)]^{1/7}$ ; $\qquad y = 1,...,7.$

The *annual GEKS price index* $P_{GEKS}{}^{y*}$ is defined as the following normalization of the above GEKS price levels:

(A24) $P_{GEKS}{}^{y} \equiv p_{GEKS}{}^{y}/p_{GEKS}{}^{1}$ ; $\qquad y = 1,...,7.$

The GEKS index is also shown in Chart A2 below.

The final annual "standard" index that will be calculated in this section is another multilateral index: the *Predicted Share Relative Price Similarity Linked Price Index*, $P_S{}^{y}$. The idea behind this index is to use the Fisher index to link any two periods in the available data sample. However, rather than picking the first year in the sample as the base year and computing fixed base Fisher indexes or using chained Fisher indexes, a set of bilateral links is chosen to link pairs of observations which have the most similar structure of relative prices. The most similar price pairs of observations are combined to construct an overall price index. If prices in any two years are equal or proportional to each other, then any "reasonable" bilateral index number will register the value one if prices are equal and will register the proportionality factor if prices are proportional to each other. But if prices are not proportional, then how exactly should the lack of price proportionality be measured?

Recall that $S_{y,n}$ is the Statistics Denmark annual share of household consumption for product class n in year y. The annual prices and quantities for year y, $p_{y,n}{}^{*}$ and $q_{y,n}{}^{*}$ defined above by (A11) and (A12), satisfy the following equations:

(A25) $S_{y,n} = p_{y,n}{}^{*}q_{y,n}{}^{*}/p^{y*}\cdot q^{y*}$ ; $\qquad y = 1,\ldots,7;\ n = 1,\ldots,402.$

Now think of using the *prices* of year z, $p^{z*}$, and the *quantities* of year y, $q^{y*}$, to *predict* the actual year y, product n expenditure share $S_{y,n}$ given by (A25) for n = 1,...,402. Denote this *predicted share* by $S_{z,y,n}$ which is defined as follows:

(A26) $S_{z,y,n} \equiv p_{z,n}{}^{*}q_{y,n}{}^{*}/p^{z*}\cdot q^{y*}$ ; $\qquad z = 1,\ldots,7;\ y = 1,\ldots,7;\ n = 1,\ldots,402.$

If the prices in year y are proportional to the prices of year z so that $p^{z*} = \lambda p^{y*}$ where $\lambda$ is a positive number, then it can verified that the predicted shares defined by (A26) will be equal to the actual expenditure shares defined by (A25) for year y; i.e., for the two years defined by y and z, we will have $S_{y,n} = S_{z,y,n}$ for n = 1,...,N. The following *Predicted Share measure of relative price dissimilarity* between the prices of year y and the prices of year z, $\Delta_{PS}(p^{z*},p^{y*},q^{z*},q^{y*})$, is well defined even if some product prices and shares in the two years being compared are equal to zero:[30]

[30] For information on the properties of this measure of relative price dissimilarity, see Diewert (2021b).

$$(A27)\ \Delta_{PS}(p^{z*},p^{y*},q^{z*},q^{y*}) \equiv \Sigma_{n=1}^{402} [S_{y,n} - S_{z,y,n}]^2 + \Sigma_{n=1}^{402} [S_{z,n} - S_{y,z,n}]^2$$
$$= \Sigma_{n=1}^{402} [(p_{y,n}{}^*q_{y,n}{}^*/p^{y*}\cdot q^y) - (p_{z,n}{}^*q_{y,n}{}^*/p^{z*}\cdot q^y)]^2$$
$$+ \Sigma_{n=1}^{402} [(p_{z,n}{}^*q_{z,n}{}^*/p^{z*}\cdot q^z) - (p_{y,n}{}^*q_{z,n}{}^*/p^{y*}\cdot q^z)]^2$$

In general, $\Delta_{PS}(p^{z*},p^{y*},q^{z*},q^{y*})$ takes on values between 0 and 2. If $\Delta_{PS}(p^{z*},p^{y*},q^{z*},q^{y*}) = 0$, then it must be the case that relative prices are the same years z and y; i.e., we have $p^{z*} = \lambda p^{y*}$ for some $\lambda > 0$. A bigger value of $\Delta_{PS}(p^{z*},p^{y*},q^{z*},q^{y*})$ generally indicates bigger deviations from price proportionality.

To see how this predicted share measure of annual relative price dissimilarity turned out for our Danish annual data, see Table A1 below.

**Table A1: Predicted Share Measures of Price Dissimilarity for Denmark for Years 1-7**

| Year | y = 1 | y = 2 | y = 3 | y = 4 | y = 5 | y = 6 | y = 7 |
|---|---|---|---|---|---|---|---|
| z = 1 | 0.000000 | 0.000033 | 0.000090 | 0.000209 | 0.000363 | 0.000482 | 0.000575 |
| z = 2 | 0.000033 | 0.000000 | 0.000023 | 0.000111 | 0.000234 | 0.000322 | 0.000403 |
| z = 3 | 0.000090 | 0.000023 | 0.000000 | 0.000049 | 0.000137 | 0.000202 | 0.000265 |
| z = 4 | 0.000209 | 0.000111 | 0.000049 | 0.000000 | 0.000033 | 0.000081 | 0.000134 |
| z = 5 | 0.000363 | 0.000234 | 0.000137 | 0.000033 | 0.000000 | 0.000021 | 0.000058 |
| z = 6 | 0.000482 | 0.000322 | 0.000202 | 0.000081 | 0.000021 | 0.000000 | 0.000016 |
| z = 7 | 0.000575 | 0.000403 | 0.000265 | 0.000134 | 0.000058 | 0.000016 | 0.000000 |

The above matrix is used to construct $P_S{}^y$, the real time *similarity linked price index for the Danish annual data.* This index is constructed as follows. Set $P_S{}^1 \equiv 1$. The bilateral Fisher index linking year 2 to year 1, $P_F(2/1)$[31] is set equal to $P_S{}^2$. Now look down the y = 3 column in Table A1. We need to link year 3 to either year 1 or year 2. The dissimilarity measures for these two years are 0.000090 and 0.000023 respectively. The degree of relative price dissimilarity is far smaller for the link to year 2 than it is to year 1 (year 3 prices are much closer to being proportional to year 2 prices than to year 1 prices) so we use the Fisher link from period 2 to period 3, $P_F{}^1(3/2)$, to link period 3 to period 2. Thus the final year 3 similarity linked index for y = 3 is $P_S{}^3 \equiv P_S{}^2 \times P_F(3/2)$. Now we need to link year 4 to either year 1, 2 or 3. Look down the y = 4 column in Table A1 to find the lowest dissimilarity measure above the main diagonal of the matrix. The smallest of the 3 numbers 0.000209, 0.000111 and 0.000049 is 0.00049. Thus we link the year 4 data to the year 3 data using the Fisher link from year 3 to year 4, $P_F{}^1(4/3)$, and the year 4 similarity linked final index value is $P_S{}^4 \equiv P_S{}^1 \times P_F{}^1(4/3)$. *Thus for each year, as the new data become available, we use the Fisher bilateral index that links the new period to the previous period that has the lowest measure of relative price dissimilarity*. The final two bilateral links are year 5 to year 4 and year 6 to year 5. The resulting year 5 and 6 similarity linked index values are $P_S{}^{5,} \equiv P_S{}^4 \times P_F(5/4)$ and $P_S{}^6 \equiv P_S{}^6 \times P_F(6/5)$. The optimal set of *bilateral links* for the real time similarity linked indexes can be summarized as follows:

1 – 2 – 3 – 4 – 5 – 6.

Thus for the Danish annual data*, the real time similarity linked indexes coincide with the Fisher chained indexes*; i.e., we have $P_S{}^y = P_{FCh}{}^y$ for y = 1,…,7.

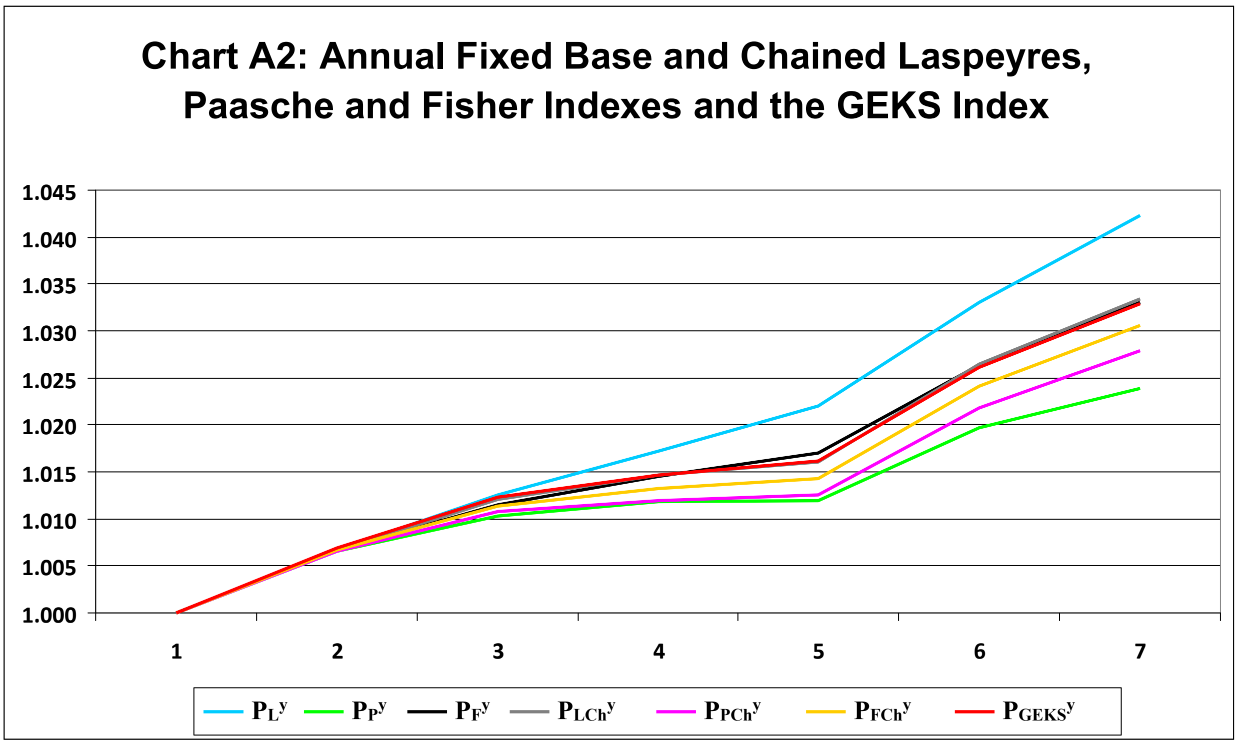
The annual fixed base Laspeyres, Paasche and Fisher indexes, $P_L{}^y$, $P_P{}^y$, $P_F{}^y$, the chained Laspeyres, Paasche and Fisher indexes, $P_{LCh}{}^y$, $P_{PCh}{}^y$, $P_{FCh}{}^y$, the GEKS index $P_{GEKS}{}^y$ and the Predicted Share Similarity Linked index $P_S{}^y$ are listed below in Table A1 and plotted on Chart A2.

[31] $P_F(2/1)$ is defined by (A18), $P_F(y/z) \equiv [P_L(y/z)P_P(y/z)]^{1/2}$, with y =2 and z = 1.

**Table A2: Annual Fixed Base and Chained Laspeyres, Paasche and Fisher Indexes, GEKS Indexes And Real Time Similarity Linked Indexes**

| Year y | $P_L^y$ | $P_P^y$ | $P_F^y$ | $P_{LCh}^y$ | $P_{PCh}^y$ | $P_{FCh}^y$ | $P_{GEKS}^y$ | $P_S^y$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 1.00672 | 1.00659 | 1.00666 | 1.00672 | 1.00659 | 1.00666 | 1.00688 | 1.00666 |
| 3 | 1.01253 | 1.01033 | 1.01143 | 1.01209 | 1.01075 | 1.01142 | 1.01226 | 1.01142 |
| 4 | 1.01727 | 1.01186 | 1.01456 | 1.01466 | 1.01191 | 1.01329 | 1.01459 | 1.01329 |
| 5 | 1.02198 | 1.01195 | 1.01695 | 1.01606 | 1.01255 | 1.01430 | 1.01621 | 1.01430 |
| 6 | 1.03302 | 1.01974 | 1.02636 | 1.02645 | 1.02181 | 1.02413 | 1.02615 | 1.02413 |
| 7 | 1.04230 | 1.02390 | 1.03306 | 1.03338 | 1.02791 | 1.03064 | 1.03292 | 1.03064 |
| G. Rate | 1.00693 | 1.00394 | 1.00544 | 1.00549 | 1.00460 | 1.00504 | 1.00541 | 1.00504 |

The last row in Table A2 lists the geometric average rate of growth of the relevant index over the seven year period; i.e., the average geometric growth rate for the fixed base Laspeyres index, $P_L^y$, was 1.00693 = $1.04230^{1/6}$ which translates into an average inflation rate of 0.693 percent per year.




The similarity linked index $P_S^y$ turned out to equal the chained Fisher index $P_{FCh}^t$. $P_S^y$ is a preferred index since it satisfies the Multiperiod Identity Test and it (theoretically) can be implemented in real time provided that household expenditure information is available in real time. The GEKS index $P_{GEKS}^y$ also satisfies the Multiperiod Identity Test and it also does not depend on the choice of a base period. It cannot be implemented in real time but Rolling Window versions of this index can be implemented in real time.[32] Note that $P_S^y$ lies in the middle of the various indexes that are plotted on Chart A2 and $P_{GEKS}^y$ lies a bit

[32] See Ivancic, Diewert and Fox (2011) on Rolling Window GEKS. The pros and cons of various multilateral index number formulae are discussed in Diewert (2021b).

above $P_S^y$. The Fisher fixed base index $P_F^y$ can hardly be distinguished from $P_{GEKS}^y$. The outlier indexes are the fixed base Laspeyres and Paasche indexes; $P_L^y$ is on average 0.693 − 0.504 = 0.189 percentage points above our preferred chained Fisher and Similarity Linked indexes while $P_P^y$ is on average 0.110 percentage points below $P_{FCh}^y$ and $P_S^y$.

The average difference between the growth rates for the fixed base Laspeyres and the chained Fisher indexes is 0.189 percentage points while the difference between the chained Laspeyres and the chained Fisher indexes is only 0.045 percentage points. Thus substitution bias using the fixed base Laspeyres formula is much larger than the substitution bias using the chained Laspeyres index.

Finally, note that the average difference between the fixed base Laspeyres and Paasche annual growth rates is 0.299 percentage points while the average difference between the chained Laspeyres and Paasche growth rates is only 0.089 percentage points. Thus for the Danish data, chaining reduces the spread between the Laspeyres and Paasche formulae. This is an indication that it is probably preferable to use chained Fisher indexes rather than fixed base Fisher indexes.[33]

To conclude this section, we use the annual price data $p_{y,n}^*$ to calculate annual fixed base Jevons, Dutot and Carli indexes, $P_J^y$, $P_D^y$ and $P_C^y$. Since there are no missing observations, the fixed base Jevons and Dutot indexes coincide with their chained counterparts. However, since the annual average product prices no longer equal 1 for year 1, it is no longer the case that $P_D^y = P_C^y$. Thus the fixed base annual Carli index, $P_C^y$, must be calculated separately. The chained annual Carli index for year y is denoted by $P_{CCh}^y$. These indexes are listed in Table A3 below and are plotted (along with $P_S^y$ for comparison purposes) in Chart A3 below.

**Table A3: Annual Jevons, Dutot, Fixed Base and Chained Carli Indexes and the Annual Predicted Share Index**

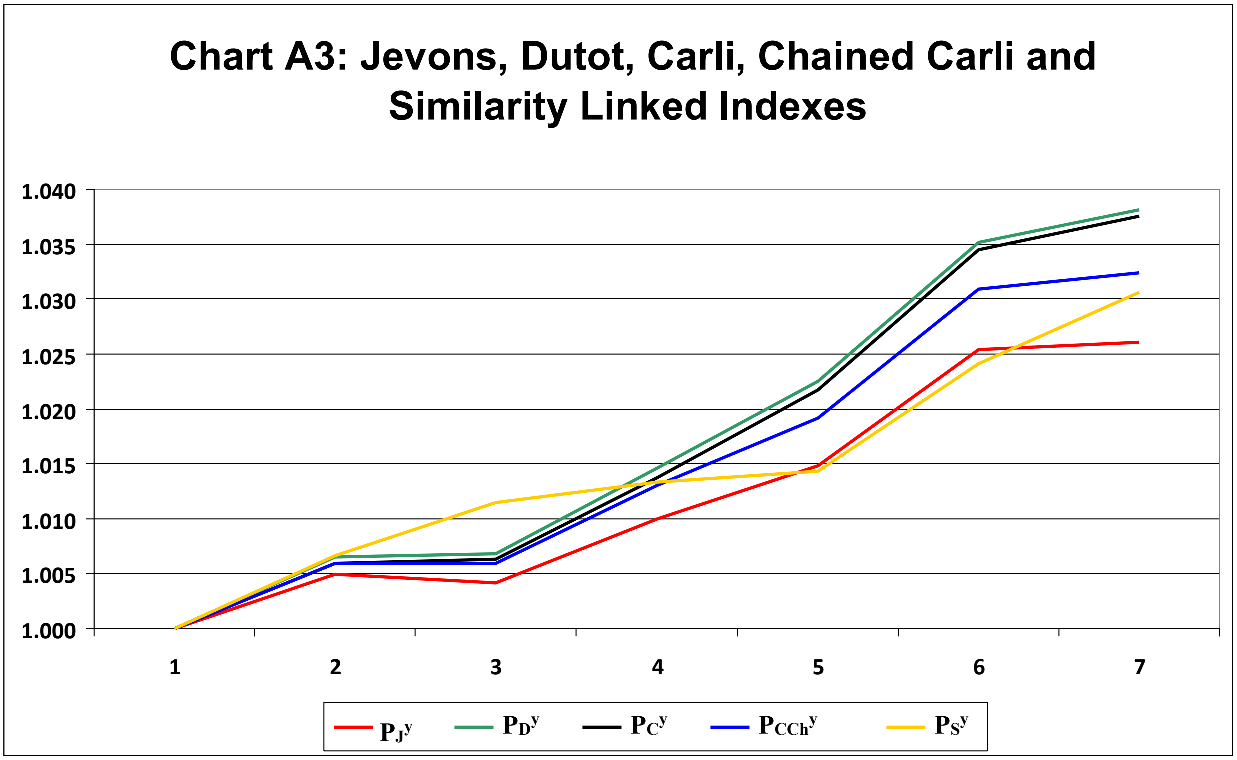
| Year y | $P_J^y$ | $P_D^y$ | $P_C^y$ | $P_{CCh}^y$ | $P_S^y$ |
|---:|---:|---:|---:|---:|---:|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 1.00495 | 1.00650 | 1.00595 | 1.00595 | 1.00666 |
| 3 | 1.00418 | 1.00682 | 1.00631 | 1.00590 | 1.01142 |
| 4 | 1.00999 | 1.01462 | 1.01376 | 1.01302 | 1.01329 |
| 5 | 1.01478 | 1.02252 | 1.02171 | 1.01917 | 1.01430 |
| 6 | 1.02543 | 1.03520 | 1.03446 | 1.03094 | 1.02413 |
| 7 | 1.02608 | 1.03815 | 1.03758 | 1.03240 | 1.03064 |
| G. Rate | 1.00430 | 1.00626 | 1.00617 | 1.00533 | 1.00504 |

It can be seen that the growth rate for the annual Jevons index $P_J^y$ is on average 0.074 percentage points below the growth rate of our preferred similarity linked index $P_S^y$ while the growth rates for the Dutot index $P_D^y$ and the fixed base Carli index $P_C^y$ are about 0.12 percentage points above the growth rate for $P_S^y$ on average.[34] The growth rate for the chained Carli index is only about 0.03 percentage points above the $P_S^y$ growth rate on average. However, for several years, the chained Carli differed substantially from the similarity linked index. Thus it can be seen that weighting does matter: the unweighted indexes are not completely reliable but they can approximate trend inflation.

[33] Chaining tends to be the preferred option if the underlying data have smooth trends; see Diewert (1978) (2021b) and Hill (1988).

[34] Since the annual prices were not normalized to equal one in the first year, the fixed base Carli index is no longer exactly equal to the Dutot index. However, since the annual product prices for the first year are approximately equal to one, the fixed base Carli is approximately equal to the fixed base (and chained) Dutot index.

As was noted in section A.2 above, the month to month Jevons index ended up at 1.04091 whereas in Table A3 above, it can be seen that the annual Jevons index ended up much lower at 1.02608, a gap of 1.5 percentage points. This large difference is due to the substantial seasonality in the monthly prices: the January prices were always unusually low relative to average prices for the year and this seasonality in prices is what explains the large difference.




It can be seen that all four elementary indexes capture the trend in the similarity linked indexes $P_S^y$ fairly well. It also can be seen that the Dutot indexes are quite close to the fixed base Carli indexes; this is to be expected since the year one annual prices for the 402 products are fairly close to unity. While none of the annual elementary indexes were very close to our best weighted index $P_S^y$ (which was also equal to the chained Fisher index) for all years, it can be seen that the Jevons index is reasonably close at the end of the sample period and probably provides the best approximation to $P_S^y$.

In the following section, we return to the construction of month to month indexes that use the annual expenditure shares to weight the 402 elementary prices.

### A.4. Month to Month Indexes Using Annual Weights

National statistical offices in general do not calculate their consumer price indexes using the standard index number formulae that are listed in the previous sections. They use annual expenditure shares $S_{y,n}$ or annual "quantities" $q_{y,n}^*$ like those defined by (A11) in the previous section along with monthly prices. They use these prices and quantities in modifications of what are called Lowe (1823) or Young (1812) indexes in the index number literature. The modifications involve a mixture of the use of fixed base and chained indexes as was explained in the main text. In this Appendix, we will explain in more detail how exactly these "practical" indexes are constructed.

The basic *Lowe index* is similar to the Laspeyres index in that it prices out a basket of goods and services at the prices of month t in the numerator of the index and divides by the value of the same basket valued at the prices of month 1. It is different from the Laspeyres index because the quantity basket is not necessarily equal to the basket that was consumed in month 1.

Recall that the price of product n in month t for the Danish data was denoted by $p_{t,n}$ for t = 1,…,84 and n = 1,…,402 and the vector of month t prices was defined as $p^t \equiv [p_{t,1},\ldots,p_{t,402}]$ for t = 1,…,84. The expenditure share for product n in year y was defined as $S_{y,n}$ for y = 1,..,7 and n = 1,…,402. In the previous section, $S_{y,n}$ was deflated by the corresponding annual price $p_{y,n}{}^*$ to form the annual "quantity" $q_{y,n}{}^*$. Define the annual quantity vector for year y as $q^{y*} \equiv [q_{y,1}{}^*,\ldots, q_{y,402}]$ for years y = 1,…,7.

The monthly *Lowe price index*, $P_{Lo}{}^t$, for the first 13 months in the data set is defined as follows:[35]

$$\text{(A27)}\quad P_{Lo}{}^t = p^t\cdot q^{1*}/p^1\cdot q^{1*}\ ; \qquad t = 1,\ldots,13.$$

Thus the cost of the year 1 annual basket of commodities $q^{1*}$ valued at the prices of month t, $p^t\cdot q^{1*}$, is divided by the cost of the year 1 annual basket valued at the prices of January in year 1, $p^1\cdot q^{1*}$, to give us the Lowe index for month t, $P_{Lo}{}^t$, for the first 13 months in the data window.[36]

In earlier years, many National Statistical Offices did not change the annual basket for their Lowe indexes for many years. However, in recent times, most countries using the Lowe index methodology for their CPIs update their annual baskets every year. Thus their Lowe indexes are a mixture of fixed base and chained Lowe indexes. For the version of the Lowe index used in this Appendix, the annual basket will be changed in January of each year. Thus (A27) defines our Lowe index for Denmark for the first 13 months in our data window. For the remaining months, $P_{Lo}{}^t$, is defined as follows:

$$\text{(A28)}\quad P_{Lo}{}^t = P_{Lo}{}^{13}\, p^t\cdot q^{2*}/p^{13}\cdot q^{2*} \qquad t = 13,\ldots,25;$$
$$P_{Lo}{}^t = P_{Lo}{}^{25}\, p^t\cdot q^{3*}/p^{25}\cdot q^{3*}\ ; \qquad t = 25,\ldots,37;$$
$$P_{Lo}{}^t = P_{Lo}{}^{37}\, p^t\cdot q^{4*}/p^{37}\cdot q^{4*}\ ; \qquad t = 37,\ldots,49;$$
$$P_{Lo}{}^t = P_{Lo}{}^{49}\, p^t\cdot q^{5*}/p^{49}\cdot q^{5*}\ ; \qquad t = 49,\ldots,61;$$
$$P_{Lo}{}^t = P_{Lo}{}^{61}\, p^t\cdot q^{6*}/p^{61}\cdot q^{6*}\ ; \qquad t = 61,\ldots,73;$$
$$P_{Lo}{}^t = P_{Lo}{}^{73}\, p^t\cdot q^{7*}/p^{73}\cdot q^{7*}\ ; \qquad t = 73,\ldots,84.$$

These Lowe indexes could be constructed by national offices retrospectively but they cannot be calculated in real time. Thus, in practice, the annual baskets used in the Lowe formula are lagged one or two years. For illustrative purposes, we will use the year 1 basket as in definitions (A27) for the first year of our data set and then lag the annual basket by one year in subsequent years. Thus our Lowe indexes using one year lagged annual weights, $P_{Lo1}{}^t$, are defined as follows:

$$\text{(A29)}\quad P_{Lo1}{}^t = p^t\cdot q^{1*}/p^1\cdot q^{1*}\ ; \qquad t = 1,\ldots,13;$$
$$P_{Lo1}{}^t = P_{Lo1}{}^{13}\, p^t\cdot q^{1*}/p^{13}\cdot q^{1*}\ ; \qquad t = 13,\ldots,25;$$

[35] Note that these Lowe indexes can be interpreted as weighted Dutot indexes.

[36] The Lowe index is not as fundamental as the Laspeyres or Paasche indexes: households in month t do not (in general) consume the annual basket; they consume an appropriate monthly basket. If seasonality in prices and quantities is moderate and if consumption growth over the year is relatively even, then the Lowe index can provide an adequate approximation to the Laspeyres and Paasche indexes between months 1 and t. However, as was seen in the main text and in section A2 above, there is a great deal of seasonality in the Danish price data and so it is likely that there is a considerable amount of seasonality in consumption as well and hence the Lowe index may not provide a very good approximation to the underlying monthly Laspeyres and Paasche indexes.

$P_{Lo1}{}^{t} = P_{Lo1}{}^{25}\, p^{t}\cdot q^{2*}/p^{25}\cdot q^{2*}$ ; $t = 25,\ldots,37;$
$P_{Lo1}{}^{t} = P_{Lo1}{}^{37}\, p^{t}\cdot q^{3*}/p^{37}\cdot q^{3*}$ ; $t = 37,\ldots,49;$
$P_{Lo1}{}^{t} = P_{Lo1}{}^{49}\, p^{t}\cdot q^{4*}/p^{49}\cdot q^{4*}$ ; $t = 49,\ldots,61;$
$P_{Lo1}{}^{t} = P_{Lo1}{}^{61}\, p^{t}\cdot q^{5*}/p^{61}\cdot q^{5*}$ ; $t = 61,\ldots,73;$
$P_{Lo1}{}^{t} = P_{Lo1}{}^{73}\, p^{t}\cdot q^{6*}/p^{73}\cdot q^{6*}$ ; $t = 73,\ldots,84.$

Our illustrative Lowe indexes using two year lagged annual weights, $P_{Lo2}{}^{t}$, are defined as follows:

(A30) $P_{Lo2}{}^{t} = p^{t}\cdot q^{1*}/p^{1}\cdot q^{1*}$ ; $t = 1,\ldots,13;$
$P_{Lo2}{}^{t} = P_{Lo2}{}^{13}\, p^{t}\cdot q^{1*}/p^{13}\cdot q^{1*}$ ; $t = 13,\ldots,25;$
$P_{Lo2}{}^{t} = P_{Lo2}{}^{25}\, p^{t}\cdot q^{1*}/p^{25}\cdot q^{1*}$ ; $t = 25,\ldots,37;$
$P_{Lo2}{}^{t} = P_{Lo2}{}^{37}\, p^{t}\cdot q^{2*}/p^{37}\cdot q^{2*}$ ; $t = 37,\ldots,49;$
$P_{Lo2}{}^{t} = P_{Lo2}{}^{49}\, p^{t}\cdot q^{3*}/p^{49}\cdot q^{3*}$ ; $t = 49,\ldots,61;$
$P_{Lo2}{}^{t} = P_{Lo2}{}^{61}\, p^{t}\cdot q^{4*}/p^{61}\cdot q^{4*}$ ; $t = 61,\ldots,73;$
$P_{Lo2}{}^{t} = P_{Lo2}{}^{73}\, p^{t}\cdot q^{5*}/p^{73}\cdot q^{5*}$ ; $t = 73,\ldots,84.$

For years 1 and 2, the annual weights of year 1 are used in the above definitions. Starting at year 3, the annual weights are lagged by two years. The Lowe indexes $P_{Lo}{}^{t}$, $P_{Lo1}{}^{t}$ and $P_{Lo2}{}^{t}$ defined above by (A27) – (A30) are plotted on Chart A4 below.[37]

The *Young index* $P_{Y}{}^{t}$ for the first 13 months uses the annual expenditure shares of year 1, the $S_{1,n}$ for n = 1,…,402 as weights for the monthly prices of month t divided by the price of month t for each product n, the $p_{t,n}/p_{1,n}$, as follows:[38]

(A31) $P_{Y}{}^{t} \equiv \Sigma_{n=1}{}^{402}\, S_{1,n}(p_{t,n}/p_{1,n})$ ; $t = 1,\ldots,13.$

Thus for the first 13 month in our window of observations, the Young price index is equivalent to a weighted fixed base Carli index. For the version of the Young index used in this Appendix, the annual share weights will be changed in January of each year. Thus (A31) defines our Young index for Denmark for the first 13 months in our data window. For the remaining months, $P_{Y}{}^{t}$, is defined as follows:

(A32) $P_{Y}{}^{t} = P_{Y}{}^{13}\, \Sigma_{n=1}{}^{402}\, S_{2,n}(p_{t,n}/p_{13,n})$ ; $t = 13,\ldots,25;$
$P_{Y}{}^{t} = P_{Y}{}^{25}\, \Sigma_{n=1}{}^{402}\, S_{3,n}(p_{t,n}/p_{25,n})$ ; $t = 25,\ldots,37;$
$P_{Y}{}^{t} = P_{Y}{}^{37}\, \Sigma_{n=1}{}^{402}\, S_{4,n}(p_{t,n}/p_{37,n})$ ; $t = 37,\ldots,49;$
$P_{Y}{}^{t} = P_{Y}{}^{49}\, \Sigma_{n=1}{}^{402}\, S_{5,n}(p_{t,n}/p_{49,n})$ ; $t = 49,\ldots,61;$
$P_{Y}{}^{t} = P_{Y}{}^{61}\, \Sigma_{n=1}{}^{402}\, S_{6,n}(p_{t,n}/p_{61,n})$ ; $t = 61,\ldots,73;$
$P_{Y}{}^{t} = P_{Y}{}^{73}\, \Sigma_{n=1}{}^{402}\, S_{7,n}(p_{t,n}/p_{73,n})$ ; $t = 73,\ldots,84.$

As was the case with the Lowe index, the Young index cannot be calculated in real time. Thus real time Young indexes cannot use current year expenditure weights but must use weights that are lagged one or two years. In order to calculate Young indexes using one year lagged weights, we will use the year 1 basket as in definitions (A30) for the first year of our data set and then lag the annual basket by one year in subsequent years. Thus our Young indexes using one year lagged annual weights, $P_{Y1}{}^{t}$, are defined as follows:

[37] These partially chained Lowe indexes are chained every January. As was explained in the main text, Statistics Denmark does the annual chaining every December. Thus the Lowe indexes in this Appendix will not be equal to the Lowe indexes computed in the main text.
[38] Note that these Young indexes can be interpreted as weighted Carli indexes.

(A33) $P_{Y1}{}^{t} = \Sigma_{n=1}{}^{402} S_{1,n}(p_{t,n}/p_{1,n})$ ; $t = 1,\ldots,13$;
$P_{Y1}{}^{t} = P_{Y1}{}^{13} \Sigma_{n=1}{}^{402} S_{1,n}(p_{t,n}/p_{13,n})$ ; $t = 13,\ldots,25$;
$P_{Y1}{}^{t} = P_{Y1}{}^{25} \Sigma_{n=1}{}^{402} S_{2,n}(p_{t,n}/p_{25,n})$ ; $t = 25,\ldots,37$;
$P_{Y1}{}^{t} = P_{Y1}{}^{37} \Sigma_{n=1}{}^{402} S_{3,n}(p_{t,n}/p_{37,n})$ ; $t = 37,\ldots,49$;
$P_{Y1}{}^{t} = P_{Y1}{}^{49} \Sigma_{n=1}{}^{402} S_{4,n}(p_{t,n}/p_{49,n})$ ; $t = 49,\ldots,61$;
$P_{Y1}{}^{t} = P_{Y1}{}^{61} \Sigma_{n=1}{}^{402} S_{5,n}(p_{t,n}/p_{61,n})$ ; $t = 61,\ldots,73$;
$P_{Y1}{}^{t} = P_{Y1}{}^{73} \Sigma_{n=1}{}^{402} S_{6,n}(p_{t,n}/p_{73,n})$ ; $t = 73,\ldots,84$.

Our illustrative Young indexes using two year lagged annual weights, $P_{Y2}{}^{t}$, are defined as follows:

(A34) $P_{Y2}{}^{t} = \Sigma_{n=1}{}^{402} S_{1,n}(p_{t,n}/p_{1,n})$ ; $t = 1,\ldots,13$;
$P_{Y2}{}^{t} = P_{Y2}{}^{13} \Sigma_{n=1}{}^{402} S_{1,n}(p_{t,n}/p_{13,n})$ ; $t = 13,\ldots,25$;
$P_{Y2}{}^{t} = P_{Y2}{}^{25} \Sigma_{n=1}{}^{402} S_{1,n}(p_{t,n}/p_{25,n})$ ; $t = 25,\ldots,37$;
$P_{Y2}{}^{t} = P_{Y2}{}^{37} \Sigma_{n=1}{}^{402} S_{2,n}(p_{t,n}/p_{37,n})$ ; $t = 37,\ldots,49$;
$P_{Y2}{}^{t} = P_{Y2}{}^{49} \Sigma_{n=1}{}^{402} S_{3,n}(p_{t,n}/p_{49,n})$ ; $t = 49,\ldots,61$;
$P_{Y2}{}^{t} = P_{Y2}{}^{61} \Sigma_{n=1}{}^{402} S_{4,n}(p_{t,n}/p_{61,n})$ ; $t = 61,\ldots,73$;
$P_{Y2}{}^{t} = P_{Y2}{}^{73} \Sigma_{n=1}{}^{402} S_{5,n}(p_{t,n}/p_{73,n})$ ; $t = 73,\ldots,84$.

For years 1, 2 and 3, the annual weights of year 1 are used in the above definitions. Starting at year 3, the annual weights are lagged by two years. The Young indexes $P_{Y}{}^{t}$, $P_{Y1}{}^{t}$ and $P_{Y2}{}^{t}$ defined above by (A31) – (A33) are plotted on Chart A4 below.

In the main text, Lowe and Young indexes using expenditure weights lagged two years were calculated since these indexes are frequently used by national statistical agencies. The Geometric Young index has also been used by some Caribbean countries using lagged expenditure weights so this index was also considered in the main text. The logarithm of the *Geometric Young index*, $\ln P_{GY}{}^{t}$, using current annual expenditure weights for year 1, is defined as follows:[39]

(A35) $\ln P_{GY}{}^{t} \equiv \Sigma_{n=1}{}^{402} S_{1,n} \ln(p_{t,n}/p_{1,n})$ ; $t = 1,\ldots,13$.

For the version of the Geometric Young index used in this Appendix, the annual share weights will be changed in January of each year. Thus (A35) defines the logarithm of our *Geometric Young index* for Denmark for the first 13 months in our data window. For the remaining months, $\ln P_{Y}{}^{t}$, is defined as follows:

(A36) $\ln P_{GY}{}^{t} = \ln P_{GY}{}^{13} + \Sigma_{n=1}{}^{402} S_{2,n}\ln(p_{t,n}/p_{13,n})$ ; $t = 13,\ldots,25$;
$\ln P_{GY}{}^{t} = \ln P_{GY}{}^{25} + \Sigma_{n=1}{}^{402} S_{3,n}\ln(p_{t,n}/p_{25,n})$ ; $t = 25,\ldots,37$;
$\ln P_{GY}{}^{t} = \ln P_{GY}{}^{37} + \Sigma_{n=1}{}^{402} S_{4,n}\ln(p_{t,n}/p_{37,n})$ ; $t = 37,\ldots,49$;
$\ln P_{GY}{}^{t} = \ln P_{GY}{}^{49} + \Sigma_{n=1}{}^{402} S_{5,n}\ln(p_{t,n}/p_{49,n})$ ; $t = 49,\ldots,61$;
$\ln P_{GY}{}^{t} = \ln P_{GY}{}^{61} + \Sigma_{n=1}{}^{402} S_{6,n}\ln(p_{t,n}/p_{61,n})$ ; $t = 61,\ldots,73$;
$\ln P_{GY}{}^{t} = \ln P_{GY}{}^{73} + \Sigma_{n=1}{}^{402} S_{7,n}\ln(p_{t,n}/p_{73,n})$ ; $t = 73,\ldots,84$.

As was the case with the Lowe and Young indexes, the Geometric Young index cannot be implemented in real time. Our illustrative version of the *Geometric Young index that uses expenditure weights lagged one year*, $P_{GY1}$, has logarithms that are defined by (A35) and (A36) except the expenditure share weights in lines 1-6 of equations (A36) are replaced by the following annual weights: $S_{1,n}$, $S_{2,n}$, $S_{3,n}$, $S_{4,n}$, $S_{5,n}$, $S_{6,n}$. Our version of the *Geometric Young index that uses expenditure weights lagged two years*, $P_{GY2}$, has

[39] Note that these Geometric Young indexes can be interpreted as weighted Jevons indexes.

logarithms that are defined by (A35) and (A36) except the expenditure share weights in lines 1-6 of equations (A36) are replaced by the following annual weights: $S_{1,n}$, $S_{1,n}$, $S_{2,n}$, $S_{3,n}$, $S_{4,n}$, $S_{5,n}$. The Geometric Young indexes $P_{GY}^t$, $P_{GY1}^t$ and $P_{GY2}^t$ are listed in Table A4 and plotted on Chart A4 below. In addition to these nine indexes, real time Similarity Linked monthly price indexes, $P_S^t$, are also listed in Table A4 and plotted on Chart A4. These indexes are an approximation to "true" month to month similarity linked indexes, which have good axiomatic and economic properties. The $P_S^t$ will be defined formally in the following section.

**Table A4: Lowe, Young, Geometric Young and Similarity Linked Monthly Indexes**

| Month | $P_{Lo}^t$ | $P_{Lo1}^t$ | $P_{Lo2}^t$ | $P_Y^t$ | $P_{Y1}^t$ | $P_{Y2}^t$ | $P_{GY}^t$ | $P_{GY1}^t$ | $P_{GY2}^t$ | $P_S^t$ |
|---:|---:|---:|---:|---:|---:|---:|---:|---:|---:|---:|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 1.01157 | 1.01157 | 1.01157 | 1.01208 | 1.01208 | 1.01208 | 1.01171 | 1.01171 | 1.01171 | 1.01171 |
| 3 | 1.01484 | 1.01484 | 1.01484 | 1.01577 | 1.01577 | 1.01577 | 1.01513 | 1.01513 | 1.01513 | 1.01514 |
| 4 | 1.01422 | 1.01422 | 1.01422 | 1.01540 | 1.01540 | 1.01540 | 1.01457 | 1.01457 | 1.01457 | 1.01458 |
| 5 | 1.01468 | 1.01468 | 1.01468 | 1.01595 | 1.01595 | 1.01595 | 1.01506 | 1.01506 | 1.01506 | 1.01507 |
| 6 | 1.01295 | 1.01295 | 1.01295 | 1.01427 | 1.01427 | 1.01427 | 1.01336 | 1.01336 | 1.01336 | 1.01337 |
| 7 | 1.01209 | 1.01209 | 1.01209 | 1.01310 | 1.01310 | 1.01310 | 1.01213 | 1.01213 | 1.01213 | 1.01211 |
| 8 | 1.01547 | 1.01547 | 1.01547 | 1.01666 | 1.01666 | 1.01666 | 1.01574 | 1.01574 | 1.01574 | 1.01573 |
| 9 | 1.01791 | 1.01791 | 1.01791 | 1.01953 | 1.01953 | 1.01953 | 1.01823 | 1.01823 | 1.01823 | 1.01823 |
| 10 | 1.01671 | 1.01671 | 1.01671 | 1.01830 | 1.01830 | 1.01830 | 1.01707 | 1.01707 | 1.01707 | 1.01707 |
| 11 | 1.01578 | 1.01578 | 1.01578 | 1.01728 | 1.01728 | 1.01728 | 1.01615 | 1.01615 | 1.01615 | 1.01616 |
| 12 | 1.01289 | 1.01289 | 1.01289 | 1.01432 | 1.01432 | 1.01432 | 1.01311 | 1.01311 | 1.01311 | 1.01312 |
| 13 | 1.01030 | 1.01030 | 1.01030 | 1.01117 | 1.01117 | 1.01117 | 1.00986 | 1.00986 | 1.00986 | 1.01059 |
| 14 | 1.02195 | 1.02152 | 1.02152 | 1.02369 | 1.02303 | 1.02303 | 1.02176 | 1.02115 | 1.02115 | 1.02252 |
| 15 | 1.02224 | 1.02268 | 1.02268 | 1.02453 | 1.02473 | 1.02473 | 1.02233 | 1.02251 | 1.02251 | 1.02309 |
| 16 | 1.02008 | 1.02022 | 1.02022 | 1.02255 | 1.02231 | 1.02231 | 1.02017 | 1.01991 | 1.01991 | 1.02092 |
| 17 | 1.02167 | 1.02190 | 1.02190 | 1.02404 | 1.02394 | 1.02394 | 1.02177 | 1.02165 | 1.02165 | 1.02253 |
| 18 | 1.02114 | 1.02132 | 1.02132 | 1.02336 | 1.02305 | 1.02305 | 1.02124 | 1.02091 | 1.02091 | 1.02200 |
| 19 | 1.01826 | 1.01809 | 1.01809 | 1.02044 | 1.01954 | 1.01954 | 1.01806 | 1.01714 | 1.01714 | 1.01879 |
| 20 | 1.01796 | 1.01854 | 1.01854 | 1.02054 | 1.02036 | 1.02036 | 1.01790 | 1.01768 | 1.01768 | 1.01864 |
| 21 | 1.02156 | 1.02218 | 1.02218 | 1.02460 | 1.02441 | 1.02441 | 1.02148 | 1.02123 | 1.02123 | 1.02222 |
| 22 | 1.02238 | 1.02269 | 1.02269 | 1.02535 | 1.02488 | 1.02488 | 1.02240 | 1.02188 | 1.02188 | 1.02314 |
| 23 | 1.02025 | 1.02104 | 1.02104 | 1.02311 | 1.02301 | 1.02301 | 1.02023 | 1.02010 | 1.02010 | 1.02097 |
| 24 | 1.01949 | 1.02031 | 1.02031 | 1.02221 | 1.02215 | 1.02215 | 1.01945 | 1.01936 | 1.01936 | 1.02019 |
| 25 | 1.01863 | 1.01926 | 1.01926 | 1.02080 | 1.02076 | 1.02076 | 1.01835 | 1.01828 | 1.01828 | 1.01830 |
| 26 | 1.02623 | 1.02669 | 1.02641 | 1.02875 | 1.02842 | 1.02798 | 1.02598 | 1.02563 | 1.02523 | 1.02593 |
| 27 | 1.02745 | 1.02650 | 1.02678 | 1.03037 | 1.02835 | 1.02846 | 1.02734 | 1.02541 | 1.02553 | 1.02730 |
| 28 | 1.02928 | 1.02804 | 1.02782 | 1.03237 | 1.02994 | 1.02948 | 1.02916 | 1.02682 | 1.02635 | 1.02912 |
| 29 | 1.02833 | 1.02787 | 1.02778 | 1.03135 | 1.02973 | 1.02945 | 1.02822 | 1.02665 | 1.02638 | 1.02818 |
| 30 | 1.02814 | 1.02773 | 1.02719 | 1.03115 | 1.02964 | 1.02880 | 1.02795 | 1.02639 | 1.02556 | 1.02791 |
| 31 | 1.02534 | 1.02678 | 1.02646 | 1.02817 | 1.02860 | 1.02795 | 1.02474 | 1.02522 | 1.02457 | 1.02470 |
| 32 | 1.02378 | 1.02450 | 1.02463 | 1.02668 | 1.02631 | 1.02617 | 1.02353 | 1.02312 | 1.02297 | 1.02349 |
| 33 | 1.02850 | 1.02723 | 1.02740 | 1.03179 | 1.02926 | 1.02918 | 1.02839 | 1.02590 | 1.02584 | 1.02835 |
| 34 | 1.02911 | 1.02779 | 1.02811 | 1.03237 | 1.02972 | 1.02973 | 1.02898 | 1.02639 | 1.02640 | 1.02895 |
| 35 | 1.02590 | 1.02513 | 1.02586 | 1.02909 | 1.02685 | 1.02727 | 1.02572 | 1.02352 | 1.02394 | 1.02568 |
| 36 | 1.02321 | 1.02273 | 1.02381 | 1.02635 | 1.02434 | 1.02507 | 1.02251 | 1.02055 | 1.02127 | 1.02251 |
| 37 | 1.01632 | 1.01743 | 1.01871 | 1.01909 | 1.01863 | 1.01955 | 1.01474 | 1.01432 | 1.01521 | 1.01494 |
| 38 | 1.02765 | 1.02772 | 1.02860 | 1.03123 | 1.03024 | 1.03071 | 1.02614 | 1.02516 | 1.02567 | 1.02633 |
| 39 | 1.03179 | 1.03253 | 1.03281 | 1.03597 | 1.03564 | 1.03527 | 1.03056 | 1.03022 | 1.02997 | 1.03077 |
| 40 | 1.03287 | 1.03427 | 1.03368 | 1.03721 | 1.03737 | 1.03620 | 1.03164 | 1.03176 | 1.03077 | 1.03184 |
| 41 | 1.03313 | 1.03448 | 1.03400 | 1.03749 | 1.03752 | 1.03645 | 1.03194 | 1.03194 | 1.03101 | 1.03214 |
| 42 | 1.03252 | 1.03375 | 1.03348 | 1.03698 | 1.03672 | 1.03584 | 1.03135 | 1.03106 | 1.03031 | 1.03156 |
| 43 | 1.03021 | 1.03074 | 1.03264 | 1.03440 | 1.03321 | 1.03474 | 1.02868 | 1.02753 | 1.02897 | 1.02887 |
| 44 | 1.02672 | 1.02724 | 1.02958 | 1.03087 | 1.02952 | 1.03137 | 1.02542 | 1.02410 | 1.02591 | 1.02561 |
| 45 | 1.03037 | 1.03172 | 1.03222 | 1.03486 | 1.03411 | 1.03396 | 1.02918 | 1.02840 | 1.02832 | 1.02939 |
| 46 | 1.02990 | 1.03150 | 1.03199 | 1.03432 | 1.03358 | 1.03336 | 1.02866 | 1.02789 | 1.02774 | 1.02887 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **47** | **1.02751** | **1.02902** | **1.02964** | **1.03186** | **1.03109** | **1.03076** | **1.02629** | **1.02549** | **1.02524** | **1.02649** |
| **48** | **1.02642** | **1.02835** | **1.02924** | **1.03073** | **1.02996** | **1.02987** | **1.02502** | **1.02424** | **1.02420** | **1.02522** |
| **49** | **1.02196** | **1.02387** | **1.02561** | **1.02549** | **1.02441** | **1.02514** | **1.02009** | **1.01903** | **1.01971** | **1.02010** |
| **50** | **1.03064** | **1.03161** | **1.03280** | **1.03504** | **1.03258** | **1.03283** | **1.02896** | **1.02657** | **1.02677** | **1.02897** |
| **51** | **1.03045** | **1.03203** | **1.03341** | **1.03524** | **1.03342** | **1.03401** | **1.02902** | **1.02727** | **1.02777** | **1.02903** |
| **52** | **1.03163** | **1.03349** | **1.03509** | **1.03650** | **1.03490** | **1.03561** | **1.03020** | **1.02867** | **1.02929** | **1.03021** |
| **53** | **1.03337** | **1.03510** | **1.03653** | **1.03836** | **1.03667** | **1.03754** | **1.03192** | **1.03030** | **1.03107** | **1.03193** |
| **54** | **1.03360** | **1.03579** | **1.03719** | **1.03853** | **1.03728** | **1.03793** | **1.03223** | **1.03102** | **1.03161** | **1.03224** |
| **55** | **1.02907** | **1.03126** | **1.03285** | **1.03407** | **1.03218** | **1.03249** | **1.02737** | **1.02553** | **1.02580** | **1.02733** |
| **56** | **1.02793** | **1.03001** | **1.03139** | **1.03328** | **1.03133** | **1.03181** | **1.02647** | **1.02455** | **1.02495** | **1.02644** |
| **57** | **1.02942** | **1.03174** | **1.03343** | **1.03496** | **1.03316** | **1.03381** | **1.02801** | **1.02625** | **1.02681** | **1.02798** |
| **58** | **1.03154** | **1.03411** | **1.03577** | **1.03712** | **1.03560** | **1.03626** | **1.03006** | **1.02855** | **1.02912** | **1.03003** |
| **59** | **1.02964** | **1.03301** | **1.03488** | **1.03510** | **1.03426** | **1.03497** | **1.02806** | **1.02723** | **1.02783** | **1.02803** |
| **60** | **1.02962** | **1.03345** | **1.03522** | **1.03509** | **1.03485** | **1.03543** | **1.02785** | **1.02758** | **1.02805** | **1.02782** |
| **61** | **1.02877** | **1.03279** | **1.03446** | **1.03375** | **1.03357** | **1.03402** | **1.02657** | **1.02630** | **1.02668** | **1.02666** |
| **62** | **1.03813** | **1.04260** | **1.04329** | **1.04406** | **1.04403** | **1.04365** | **1.03630** | **1.03617** | **1.03570** | **1.03636** |
| **63** | **1.03854** | **1.04293** | **1.04382** | **1.04471** | **1.04476** | **1.04468** | **1.03666** | **1.03661** | **1.03640** | **1.03672** |
| **64** | **1.04039** | **1.04482** | **1.04571** | **1.04685** | **1.04676** | **1.04663** | **1.03862** | **1.03843** | **1.03822** | **1.03869** |
| **65** | **1.03956** | **1.04376** | **1.04463** | **1.04604** | **1.04576** | **1.04550** | **1.03786** | **1.03749** | **1.03717** | **1.03793** |
| **66** | **1.03871** | **1.04298** | **1.04360** | **1.04540** | **1.04494** | **1.04432** | **1.03698** | **1.03649** | **1.03578** | **1.03705** |
| **67** | **1.04230** | **1.04733** | **1.04744** | **1.04910** | **1.04900** | **1.04774** | **1.04006** | **1.03985** | **1.03859** | **1.04007** |
| **68** | **1.04045** | **1.04510** | **1.04532** | **1.04716** | **1.04684** | **1.04564** | **1.03868** | **1.03830** | **1.03708** | **1.03875** |
| **69** | **1.04487** | **1.04951** | **1.04997** | **1.05180** | **1.05152** | **1.05061** | **1.04314** | **1.04283** | **1.04193** | **1.04322** |
| **70** | **1.04603** | **1.05087** | **1.05123** | **1.05306** | **1.05293** | **1.05198** | **1.04423** | **1.04404** | **1.04313** | **1.04432** |
| **71** | **1.04344** | **1.04805** | **1.04889** | **1.05026** | **1.04987** | **1.04941** | **1.04166** | **1.04120** | **1.04065** | **1.04172** |
| **72** | **1.03924** | **1.04339** | **1.04535** | **1.04570** | **1.04494** | **1.04545** | **1.03733** | **1.03650** | **1.03694** | **1.03742** |
| **73** | **1.03497** | **1.03987** | **1.04216** | **1.04085** | **1.04055** | **1.04134** | **1.03268** | **1.03231** | **1.03299** | **1.03286** |
| **74** | **1.04292** | **1.04782** | **1.05074** | **1.04965** | **1.04939** | **1.05076** | **1.04085** | **1.04050** | **1.04174** | **1.04095** |
| **75** | **1.04324** | **1.04830** | **1.05064** | **1.05027** | **1.05012** | **1.05101** | **1.04128** | **1.04098** | **1.04172** | **1.04140** |
| **76** | **1.04847** | **1.05368** | **1.05608** | **1.05575** | **1.05557** | **1.05646** | **1.04659** | **1.04629** | **1.04705** | **1.04671** |
| **77** | **1.04946** | **1.05485** | **1.05660** | **1.05688** | **1.05662** | **1.05673** | **1.04744** | **1.04709** | **1.04717** | **1.04756** |
| **78** | **1.04813** | **1.05382** | **1.05588** | **1.05559** | **1.05537** | **1.05562** | **1.04613** | **1.04589** | **1.04621** | **1.04625** |
| **79** | **1.04940** | **1.05477** | **1.05850** | **1.05697** | **1.05614** | **1.05763** | **1.04696** | **1.04611** | **1.04743** | **1.04667** |
| **80** | **1.04699** | **1.05228** | **1.05505** | **1.05457** | **1.05371** | **1.05445** | **1.04506** | **1.04416** | **1.04484** | **1.04477** |
| **81** | **1.04969** | **1.05519** | **1.05784** | **1.05750** | **1.05690** | **1.05765** | **1.04782** | **1.04713** | **1.04784** | **1.04753** |
| **82** | **1.05209** | **1.05757** | **1.06093** | **1.06008** | **1.05941** | **1.06078** | **1.05012** | **1.04938** | **1.05066** | **1.04983** |
| **83** | **1.04951** | **1.05497** | **1.05809** | **1.05718** | **1.05660** | **1.05784** | **1.04765** | **1.04697** | **1.04814** | **1.04735** |
| **84** | **1.04572** | **1.05119** | **1.05407** | **1.05312** | **1.05265** | **1.05384** | **1.04377** | **1.04322** | **1.04433** | **1.04347** |
| **G. Rate** | **1.00748** | **1.00836** | **1.00882** | **1.00866** | **1.00859** | **1.00878** | **1.00716** | **1.00708** | **1.00726** | **1.00712** |

All of the above indexes capture the trend in Danish CPI inflation reasonably well. However, the three "true" indexes that use current year annual weights do differ considerably at times. If we take the geometric average annual growth rates for "true" Lowe, Young and Geometric Young indexes, $P_{Lo}{}^{t}$, $P_{Y}{}^{t}$, $P_{GY}{}^{t}$, 1.00748,[40] 1.00866, 1.00716 , and subtract the average annual growth rate for the similarity linked indexes $P_{S}{}^{t}$, 1.00712, we find that the approximate annual substitution bias in the three "true" indexes over the entire sample period is 0.038, 0.154 and 0.004 percentage points per year respectively.[41]

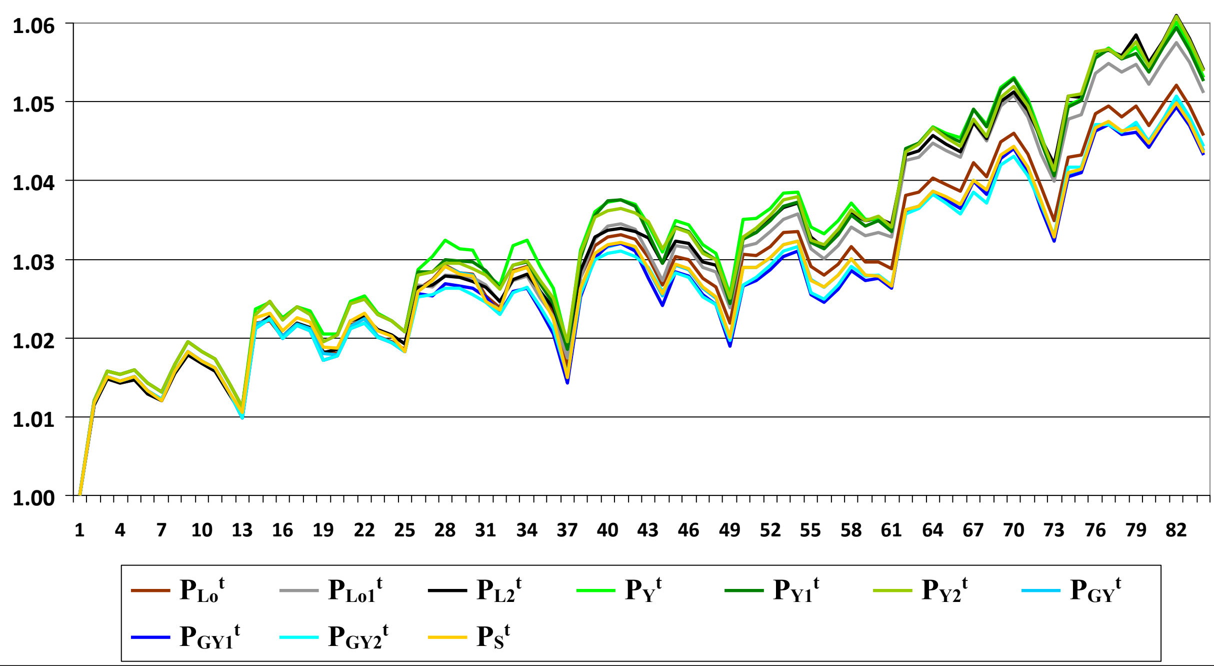
Looking at Table A4, it can be seen that the average annual inflation rate of the three Lowe indexes increase as the lag in the annual weights increases. The average growth rates of the $P_{Lo}{}^{t}$, $P_{Lo1}{}^{t}$ and $P_{Lo2}{}^{t}$ are 1.00748, 1.00836 and 1.0866. Thus the average annual substitution bias for the Lowe indexes increases from 0.038 percentage points per year for the current weight Lowe to 0.124 and 0.172 percentage points per year for the practical Lowe indexes that use weights that are one and two years old. The geometric

[40] To be precise, the geometric annual average growth rate for the "true" Lowe index $P_{Lo}{}^{t}$ is defined to be $(P_{Lo}{}^{84})^{1/6}$ = 1.00748 so $P_{Lo}{}^{84}$ = 1.04572 = $(1.00748)^{6}$.

[41] The lagged indexes are only approximations to the "true" lagged indexes since we use the year 1 expenditure weights in place of the lagged expenditure weights for years 1 and 2.

average annual growth rates of the $P_Y^t$, $P_{Y1}^t$ and $P_{Y2}^t$ less the corresponding average of the real time similarity linked indexes $P_S^t$ are 0.154, 0.147 and 0.164 percentage points respectively. Finally, the average annual geometric growth rates of the Geometric Young indexes, $P_{GY}^t$, $P_{GY1}^t$ and $P_{GY2}^t$, less the corresponding annual average of the real time similarity linked indexes are 0.004, –0.004 and 0.014 percentage points respectively. It can be seen that the three Geometric Young indexes are close to each other and have the smallest approximate substitution bias.

**Chart A4: Monthly Lowe, Young, Geometric Young and Similarity Linked Indexes**




At the end of the sample period, the highest line corresponds to $P_{Lo2}^t$ followed by the three Young indexes, $P_{Y2}^t$, $P_Y^t$ and $P_{Y1}^t$. These four high inflation indexes are tightly clustered and difficult to distinguish. These indexes are followed by the Lowe index that uses weights lagged one year, $P_{Lo1}^t$. There is a gap between these five indexes and the next index, which is the "true" Lowe index $P_{Lo}^t$. The final four indexes, $P_{GY2}^t$, $P_{GY}^t$, $P_S^t$ and $P_{Y1}^t$, are tightly clustered and are difficult to distinguish in Chart A4. The seasonality in the monthly data is again apparent.

For the Danish data under consideration, there appears to be upward substitution biases in the lagged Lowe and Young indexes while the lagged Geometric Young indexes appear to be largely free from substitution bias. These results are in agreement with the results in the main text.

In the following section, the construction of the similarity linked indexes $P_S^t$ will be explained.

### A.5. Month to Month Approximate Fisher and Similarity Linked Indexes

In this section, standard weighted month to month price indexes for Denmark will be constructed such as the Laspeyres, Paasche and Fisher indexes.[42] However, as was noted in earlier sections of this chapter, monthly information on quantities or expenditures on consumer goods and services is not available. Thus we will use the available annual expenditure information as *approximations* to actual monthly expenditures. Recall that the annual expenditure share on product n in year y was defined as $S_{y,n}$ for y = 1,…,7. The approximate *monthly expenditure share for product n in month t*, $s_{t,n}$, is defined as follows:

$$\text{(A37)}\quad \begin{aligned} s_{t,n} &\equiv S_{1,n}\,; && t = 1,\ldots,12;\ n = 1,\ldots,402;\\ s_{t,n} &\equiv S_{2,n}\,; && t = 13,\ldots,24;\ n = 1,\ldots,402;\\ s_{t,n} &\equiv S_{3,n}\,; && t = 25,\ldots,36;\ n = 1,\ldots,402;\\ s_{t,n} &\equiv S_{4,n}\,; && t = 37,\ldots,48;\ n = 1,\ldots,402;\\ s_{t,n} &\equiv S_{5,n}\,; && t = 49,\ldots,60;\ n = 1,\ldots,402;\\ s_{t,n} &\equiv S_{6,n}\,; && t = 61,\ldots,72;\ n = 1,\ldots,402;\\ s_{t,n} &\equiv S_{7,n}\,; && t = 73,\ldots,84;\ n = 1,\ldots,402. \end{aligned}$$

Recall that the official month t price index for product n (normalized to equal 1 in month 1) was defined as $p_{t,n}$ in section 2. This monthly price index is used to deflate the corresponding monthly expenditure to form an approximate month t, product n "quantity" (or volume), $q_{t,n}$; i.e., we have the following definitions:

$$\text{(A38)}\quad q_{t,n} \equiv s_{t,n}/p_{t,n}\,; \qquad t = 1,\ldots,84;\ n = 1,\ldots,402.$$

Define the month t price and quantity vectors as $p^t \equiv [p_{t,1},\ldots, p_{t,402}]$ and $q^t \equiv [q_{t,1},\ldots,q_{t,402}]$ for t = 1,…,84. Now repeat definitions (A13)-(A24) in section 3 to define the fixed base monthly Laspeyres, Paasche and Fisher indexes $P_L{}^t$, $P_P{}^t$ and $P_F{}^t$, the chained monthly Laspeyres, Paasche and Fisher indexes $P_{LCh}{}^t$, $P_{PCh}{}^t$ and $P_{FCh}{}^t$, and the monthly GEKS index $P_{GEKS}{}^t$. In forming these indexes using definitions (A13)-(A24), the monthly price vector $p^t$ replaces the annual price vector $p^{y*}$, the monthly quantity vector $q^t$ replaces the annual quantity vector $q^{y*}$ and t = 1,…,84 replaces y = 1,…,7. These monthly indexes are listed in Table A6 below and plotted on Chart A5.

The task of defining monthly relative price similarity linked indexes remains. The definitions for the real time Predicted Share Similarity linked monthly price index $P_S{}^t$ is similar to the earlier definition of these indexes for the annual indexes. We use the *prices* of month r, $p^r$, and the *quantities* of month t, $q^t$, to *predict* the actual month t, product n expenditure shares $s_{t,n}$ defined by (A37) for n = 1,...,402. Denote this *predicted share* by $s_{r,t,n}$ which is defined as follows:

$$\text{(A39)}\quad s_{r,t,n} \equiv p_{r,n}q_{t,n}/p^r\cdot q^t\,; \qquad r = 1,\ldots,84;\ t = 1,\ldots,84;\ n = 1,\ldots,402.$$

If the prices in month r are proportional to the prices in month t so that $p^r = \lambda p^t$ where $\lambda$ is a positive number, then it can verified that the predicted shares defined by (A39) will be equal to the actual expenditure shares defined by (A37) for month t; i.e., for the two months defined by r and t, we will have $s_{t,n} = s_{r,t,n}$ for n = 1,...,402. The following *Predicted Share measure of relative price dissimilarity* between the prices of month r and the prices of month t, $\Delta_{PS}(p^r,p^t,q^r,q^t)$, is defined as follows:

$$\begin{aligned} \text{(A40)}\quad \Delta_{PS}(p^r,p^t\, q^r,q^t) &\equiv \Sigma_{n=1}{}^{402}\,[s_{t,n} - s_{r,t,n}]^2 + \Sigma_{n=1}{}^{402}\,[s_{r,n} - s_{t,r,n}]^2\\ &= \Sigma_{n=1}{}^{402}\,[(p_{t,n}q_{t,n}/p^t\cdot q^t) - (p_{r,n}q_{t,n}/p^r\cdot q^t)]^2\\ &\quad + \Sigma_{n=1}{}^{402}\,[(p_{r,n}q_{r,n}/p^r\cdot q^r) - (p_{t,n}q_{r,n}/p^t\cdot q^r)]^2. \end{aligned}$$

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42 It would be more accurate to call these indexes approximations to standard monthly indexes since accurate monthly quantity or expenditure information is not available.

To see how this predicted share measure of monthly relative price dissimilarity for months 1 to 12 turned out for our Danish data, see Table A5 below.[43]

**Table A5: Predicted Share Measures of Price Dissimilarity for Denmark for Months 1-12**

| r,t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00000 | 0.00017 | 0.00016 | 0.00021 | 0.00020 | 0.00024 | 0.00027 | 0.00024 | 0.00027 | 0.00028 | 0.00028 | 0.00036 |
| 2 | 0.00017 | 0.00000 | 0.00006 | 0.00012 | 0.00014 | 0.00016 | 0.00012 | 0.00017 | 0.00021 | 0.00020 | 0.00022 | 0.00028 |
| 3 | 0.00016 | 0.00006 | 0.00000 | 0.00005 | 0.00009 | 0.00014 | 0.00015 | 0.00012 | 0.00012 | 0.00013 | 0.00018 | 0.00026 |
| 4 | 0.00021 | 0.00012 | 0.00005 | 0.00000 | 0.00003 | 0.00008 | 0.00015 | 0.00008 | 0.00007 | 0.00009 | 0.00013 | 0.00019 |
| 5 | 0.00020 | 0.00014 | 0.00009 | 0.00003 | 0.00000 | 0.00004 | 0.00014 | 0.00009 | 0.00008 | 0.00008 | 0.00008 | 0.00012 |
| 6 | 0.00024 | 0.00016 | 0.00014 | 0.00008 | 0.00004 | 0.00000 | 0.00010 | 0.00009 | 0.00009 | 0.00007 | 0.00005 | 0.00006 |
| 7 | 0.00027 | 0.00012 | 0.00015 | 0.00015 | 0.00014 | 0.00010 | 0.00000 | 0.00006 | 0.00012 | 0.00012 | 0.00015 | 0.00018 |
| 8 | 0.00024 | 0.00017 | 0.00012 | 0.00008 | 0.00009 | 0.00009 | 0.00006 | 0.00000 | 0.00004 | 0.00006 | 0.00010 | 0.00015 |
| 9 | 0.00027 | 0.00021 | 0.00012 | 0.00007 | 0.00008 | 0.00009 | 0.00012 | 0.00004 | 0.00000 | 0.00002 | 0.00006 | 0.00011 |
| 10 | 0.00028 | 0.00020 | 0.00013 | 0.00009 | 0.00008 | 0.00007 | 0.00012 | 0.00006 | 0.00002 | 0.00000 | 0.00003 | 0.00007 |
| 11 | 0.00028 | 0.00022 | 0.00018 | 0.00013 | 0.00008 | 0.00005 | 0.00015 | 0.00010 | 0.00006 | 0.00003 | 0.00000 | 0.00002 |
| 12 | 0.00036 | 0.00028 | 0.00026 | 0.00019 | 0.00012 | 0.00006 | 0.00018 | 0.00015 | 0.00011 | 0.00007 | 0.00002 | 0.00000 |

The above matrix can be used to construct the real time *similarity linked price index for the Danish monthly data* $P_S^t$ for the first 12 months. This index is constructed in the same way that the annual indexes were constructed. Thus set $P_S^1 \equiv 1$. The bilateral Fisher index linking month 2 to month 1, $P_F(2/1)$ is set equal to $P_S^2$). Now look down the t = 3 column in Table A5. We need to link month 3 to either month 1 or month 2. The dissimilarity measures for these two months are 0.00016 and 0.00006 respectively. The degree of relative price dissimilarity is far smaller for the link to month 2 than it is to month 1 so we use the Fisher link from month 2 to month 3, $P_F^1(3/2)$, to link month 3 to month 2. The final month 3 similarity linked index for t = 4 is $P_S^3 \equiv P_S^2 \times P_F(3/2)$. The first three measures of dissimilarity in column 4 of Table A5 are 0.00021, 0.00012 and 0.00005. Thus it is optimal to link month 4 to month 3 and so on. It turns out that the optimal set of *bilateral links* for the real time similarity linked indexes for months 1 to 12 can be summarized as follows:

1 – 2 – 3 – 4 – 5 – 6 – 7 – 8 – 9 – 10 – 11 – 12.

Thus for the Danish monthly data*, the real time similarity linked indexes coincide with the Fisher chained indexes* for months 1-12 i.e., we have $P_S^t = P_{FCh}^t$ for t = 1,…,12.

It turns out that using the monthly Danish data, we found that most bilateral links were chain links. There were only 10 links that were not chained: 31 linked to 26, 33 linked to 28, 55 linked to 50, 59 linked to 57, 64 linked to 62, 67 linked to 55, 68 linked to 66, 74 linked to 68, 79 linked to 67 and 83 linked to 81. Two of these links (67-55 and 79-67) were year over year links. The *Real Time Similarity linked price index* $P_S^t$ is listed in Table A6 below and plotted on Chart A5.

There is one more month to month similarity linked index that is listed in Table A6: the *Modified Predicted Share similarity linked index*, $P_{SM}^t$. This index is an index that can be constructed in real time after one year of price and quantity data have been collected. Instead of using real time linking in the first year, Hill's (2001) Spanning Tree method of linking the first 12 months is used. Basically this method looks at the first 12 months of data as a whole and finds the path linking all 12 months that generates the lowest sum of bilateral measures of price dissimilarity. Thus this method of linking requires that the first 12 months of data be used as a "training" set of data where an initial set of bilateral links is determined

[43] In order to fit all 12 columns of dissimilarity measures for months 1-12 on a single page, the actual dissimilarity measures have been multiplied by 10 in Table A5.

simultaneously using already available historical data.[44] Using the information in Table A5 above, we find that the optimal path that makes simultaneous use of the data is the following set of bilateral links:

```
    1
    |
2 – 3 – 4 – 5 –  6
            |
7 – 8 – 9 – 10 – 11 – 12.
```

Thus month 1 is linked to month 3, month 3 is linked to months 2 and 4, month 5 is linked to months 6 and 10, month 10 is linked to months 9 and 11, month 12 is linked to month 11, month 9 is linked to month 10, month 8 is linked to month 9 and finally month 7 is linked to month 8. Once the first 12 observations have been linked, we use real time linking to calculate the remainder of the bilateral links for the Modified Similarity linked index, $P_{SM}^t$. The bilateral links for months 13 to 84 are exactly the same as the corresponding links for $P_S^t$ for t = 13,…,84. The Modified Similarity linked index $P_{SM}^t$ is listed in Table A6 below. It does not appear on Chart A5 because $P_{SM}^t$ cannot be distinguished from the real time Similarity linked index $P_S^t$ defined earlier.

**Table A6: Laspeyres, Paasche, Fisher Fixed Base and Chained Indexes, GEKS Index and Similarity Linked Indexes**

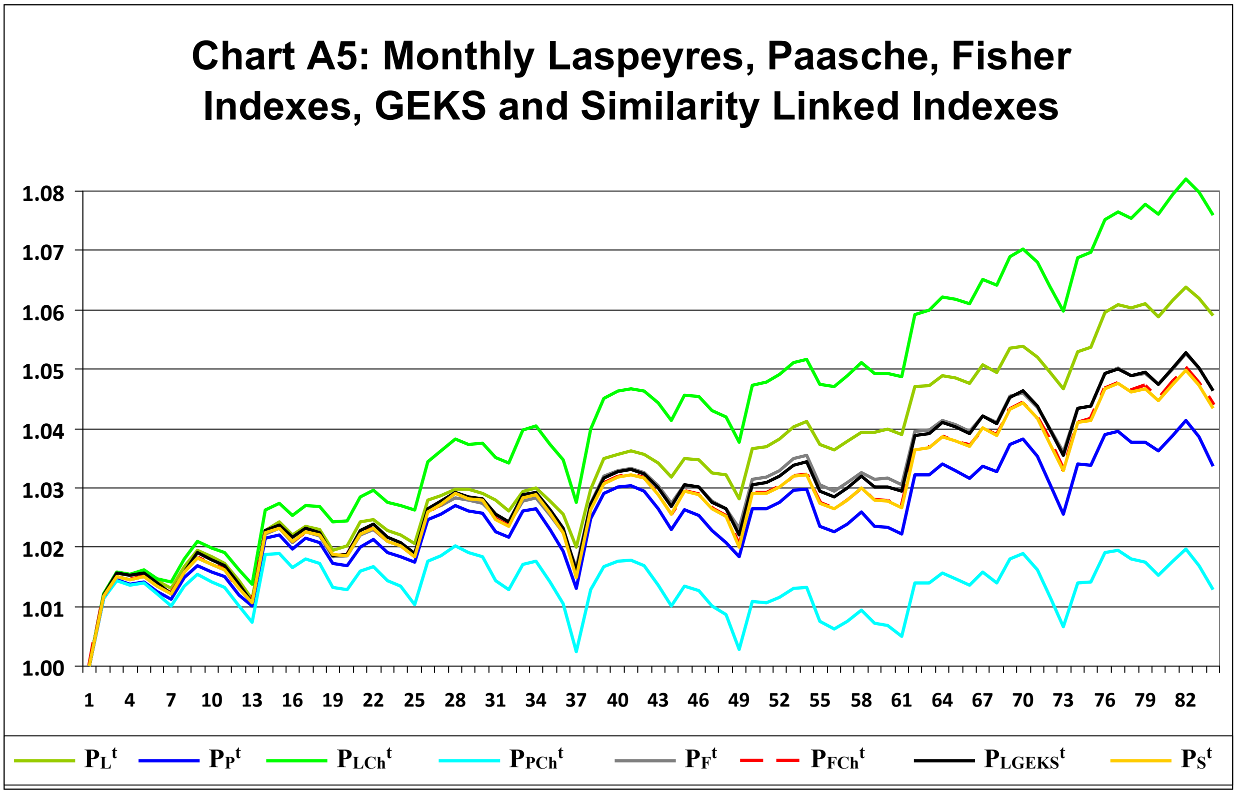
| Month t | $P_L^t$ | $P_P^t$ | $P_{LCh}^t$ | $P_{PCh}^t$ | $P_F^t$ | $P_{FCh}^t$ | $P_{GEKS}^t$ | $P_S^t$ | $P_{SM}^t$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 2 | 1.01208 | 1.01134 | 1.01208 | 1.01134 | 1.01171 | 1.01171 | 1.01193 | 1.01171 | 1.01173 |
| 3 | 1.01577 | 1.01456 | 1.01586 | 1.01443 | 1.01516 | 1.01514 | 1.01570 | 1.01514 | 1.01516 |
| 4 | 1.01540 | 1.01381 | 1.01550 | 1.01367 | 1.01460 | 1.01458 | 1.01518 | 1.01458 | 1.01460 |
| 5 | 1.01595 | 1.01422 | 1.01610 | 1.01403 | 1.01509 | 1.01507 | 1.01569 | 1.01507 | 1.01509 |
| 6 | 1.01427 | 1.01248 | 1.01463 | 1.01211 | 1.01337 | 1.01337 | 1.01390 | 1.01337 | 1.01339 |
| 7 | 1.01310 | 1.01116 | 1.01411 | 1.01012 | 1.01213 | 1.01211 | 1.01209 | 1.01211 | 1.01214 |
| 8 | 1.01666 | 1.01481 | 1.01809 | 1.01337 | 1.01573 | 1.01573 | 1.01580 | 1.01573 | 1.01575 |
| 9 | 1.01953 | 1.01694 | 1.02095 | 1.01552 | 1.01824 | 1.01823 | 1.01906 | 1.01823 | 1.01826 |
| 10 | 1.01830 | 1.01587 | 1.01992 | 1.01423 | 1.01708 | 1.01707 | 1.01788 | 1.01707 | 1.01710 |
| 11 | 1.01728 | 1.01507 | 1.01914 | 1.01319 | 1.01617 | 1.01616 | 1.01696 | 1.01616 | 1.01618 |
| 12 | 1.01432 | 1.01189 | 1.01622 | 1.01003 | 1.01311 | 1.01312 | 1.01381 | 1.01312 | 1.01314 |
| 13 | 1.01117 | 1.01012 | 1.01382 | 1.00738 | 1.01064 | 1.01059 | 1.01067 | 1.01059 | 1.01062 |
| 14 | 1.02286 | 1.02159 | 1.02637 | 1.01868 | 1.02223 | 1.02252 | 1.02282 | 1.02252 | 1.02254 |
| 15 | 1.02436 | 1.02213 | 1.02734 | 1.01886 | 1.02325 | 1.02309 | 1.02371 | 1.02309 | 1.02312 |
| 16 | 1.02198 | 1.01975 | 1.02534 | 1.01653 | 1.02087 | 1.02092 | 1.02162 | 1.02092 | 1.02095 |
| 17 | 1.02360 | 1.02145 | 1.02712 | 1.01796 | 1.02253 | 1.02253 | 1.02310 | 1.02253 | 1.02256 |
| 18 | 1.02300 | 1.02080 | 1.02678 | 1.01723 | 1.02190 | 1.02200 | 1.02240 | 1.02200 | 1.02202 |
| 19 | 1.01949 | 1.01727 | 1.02429 | 1.01332 | 1.01838 | 1.01879 | 1.01864 | 1.01879 | 1.01882 |
| 20 | 1.02019 | 1.01698 | 1.02449 | 1.01282 | 1.01858 | 1.01864 | 1.01882 | 1.01864 | 1.01866 |
| 21 | 1.02419 | 1.02009 | 1.02842 | 1.01605 | 1.02214 | 1.02222 | 1.02274 | 1.02222 | 1.02224 |
| 22 | 1.02462 | 1.02126 | 1.02962 | 1.01670 | 1.02294 | 1.02314 | 1.02397 | 1.02314 | 1.02317 |
| 23 | 1.02277 | 1.01911 | 1.02765 | 1.01433 | 1.02094 | 1.02097 | 1.02172 | 1.02097 | 1.02099 |
| 24 | 1.02202 | 1.01834 | 1.02701 | 1.01341 | 1.02018 | 1.02019 | 1.02078 | 1.02019 | 1.02022 |
| 25 | 1.02057 | 1.01739 | 1.02635 | 1.01031 | 1.01898 | 1.01830 | 1.01886 | 1.01830 | 1.01832 |
| 26 | 1.02791 | 1.02461 | 1.03434 | 1.01759 | 1.02626 | 1.02593 | 1.02653 | 1.02593 | 1.02596 |
| 27 | 1.02865 | 1.02548 | 1.03619 | 1.01849 | 1.02706 | 1.02730 | 1.02771 | 1.02730 | 1.02733 |
| 28 | 1.02982 | 1.02700 | 1.03818 | 1.02015 | 1.02841 | 1.02912 | 1.02926 | 1.02912 | 1.02915 |
| 29 | 1.02972 | 1.02616 | 1.03738 | 1.01906 | 1.02794 | 1.02818 | 1.02850 | 1.02818 | 1.02821 |
| 30 | 1.02908 | 1.02580 | 1.03750 | 1.01840 | 1.02744 | 1.02791 | 1.02809 | 1.02791 | 1.02793 |
| 31 | 1.02794 | 1.02257 | 1.03507 | 1.01441 | 1.02525 | 1.02469 | 1.02556 | 1.02470 | 1.02473 |
| 32 | 1.02616 | 1.02164 | 1.03422 | 1.01285 | 1.02390 | 1.02348 | 1.02433 | 1.02349 | 1.02352 |
| 33 | 1.02938 | 1.02612 | 1.03966 | 1.01716 | 1.02775 | 1.02835 | 1.02879 | 1.02835 | 1.02838 |
| 34 | 1.03005 | 1.02655 | 1.04041 | 1.01760 | 1.02830 | 1.02894 | 1.02931 | 1.02895 | 1.02897 |
| 35 | 1.02775 | 1.02305 | 1.03733 | 1.01414 | 1.02540 | 1.02567 | 1.02621 | 1.02568 | 1.02570 |

[44] Hill's method can be particularly useful if the monthly data exhibit substantial seasonal fluctuations.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **36** | **1.02554** | **1.01928** | **1.03473** | **1.01043** | **1.02241** | **1.02251** | **1.02323** | **1.02251** | **1.02254** |
| **37** | **1.01998** | **1.01314** | **1.02765** | **1.00238** | **1.01655** | **1.01493** | **1.01592** | **1.01494** | **1.01496** |
| **38** | **1.02982** | **1.02483** | **1.03990** | **1.01293** | **1.02733** | **1.02632** | **1.02714** | **1.02633** | **1.02636** |
| **39** | **1.03496** | **1.02911** | **1.04502** | **1.01670** | **1.03203** | **1.03076** | **1.03171** | **1.03077** | **1.03079** |
| **40** | **1.03570** | **1.03018** | **1.04629** | **1.01758** | **1.03293** | **1.03183** | **1.03272** | **1.03184** | **1.03187** |
| **41** | **1.03621** | **1.03041** | **1.04673** | **1.01775** | **1.03330** | **1.03213** | **1.03307** | **1.03214** | **1.03217** |
| **42** | **1.03567** | **1.02941** | **1.04637** | **1.01695** | **1.03253** | **1.03156** | **1.03239** | **1.03156** | **1.03159** |
| **43** | **1.03425** | **1.02645** | **1.04430** | **1.01366** | **1.03034** | **1.02886** | **1.03000** | **1.02887** | **1.02890** |
| **44** | **1.03176** | **1.02306** | **1.04132** | **1.01013** | **1.02740** | **1.02561** | **1.02687** | **1.02561** | **1.02564** |
| **45** | **1.03487** | **1.02626** | **1.04566** | **1.01336** | **1.03056** | **1.02938** | **1.03059** | **1.02939** | **1.02941** |
| **46** | **1.03482** | **1.02545** | **1.04535** | **1.01265** | **1.03012** | **1.02887** | **1.03013** | **1.02887** | **1.02890** |
| **47** | **1.03261** | **1.02277** | **1.04311** | **1.01013** | **1.02768** | **1.02649** | **1.02763** | **1.02649** | **1.02652** |
| **48** | **1.03220** | **1.02081** | **1.04199** | **1.00872** | **1.02649** | **1.02522** | **1.02646** | **1.02522** | **1.02525** |
| **49** | **1.02809** | **1.01841** | **1.03768** | **1.00281** | **1.02324** | **1.02009** | **1.02211** | **1.02010** | **1.02012** |
| **50** | **1.03660** | **1.02648** | **1.04734** | **1.01092** | **1.03153** | **1.02897** | **1.03060** | **1.02897** | **1.02900** |
| **51** | **1.03701** | **1.02657** | **1.04774** | **1.01066** | **1.03178** | **1.02903** | **1.03084** | **1.02903** | **1.02906** |
| **52** | **1.03830** | **1.02765** | **1.04911** | **1.01165** | **1.03296** | **1.03021** | **1.03204** | **1.03021** | **1.03024** |
| **53** | **1.04025** | **1.02957** | **1.05107** | **1.01314** | **1.03489** | **1.03193** | **1.03385** | **1.03193** | **1.03196** |
| **54** | **1.04119** | **1.02983** | **1.05160** | **1.01323** | **1.03549** | **1.03224** | **1.03430** | **1.03224** | **1.03227** |
| **55** | **1.03739** | **1.02355** | **1.04752** | **1.00754** | **1.03045** | **1.02734** | **1.02936** | **1.02733** | **1.02735** |
| **56** | **1.03645** | **1.02261** | **1.04709** | **1.00622** | **1.02950** | **1.02645** | **1.02843** | **1.02644** | **1.02647** |
| **57** | **1.03787** | **1.02394** | **1.04886** | **1.00753** | **1.03088** | **1.02799** | **1.02996** | **1.02798** | **1.02800** |
| **58** | **1.03934** | **1.02599** | **1.05115** | **1.00936** | **1.03264** | **1.03004** | **1.03199** | **1.03003** | **1.03006** |
| **59** | **1.03935** | **1.02346** | **1.04936** | **1.00715** | **1.03137** | **1.02804** | **1.03025** | **1.02803** | **1.02805** |
| **60** | **1.04000** | **1.02333** | **1.04936** | **1.00674** | **1.03163** | **1.02783** | **1.03023** | **1.02782** | **1.02785** |
| **61** | **1.03901** | **1.02218** | **1.04880** | **1.00500** | **1.03056** | **1.02667** | **1.02935** | **1.02666** | **1.02668** |
| **62** | **1.04704** | **1.03213** | **1.05927** | **1.01397** | **1.03956** | **1.03637** | **1.03875** | **1.03636** | **1.03639** |
| **63** | **1.04728** | **1.03218** | **1.06001** | **1.01397** | **1.03970** | **1.03674** | **1.03913** | **1.03672** | **1.03675** |
| **64** | **1.04883** | **1.03405** | **1.06224** | **1.01568** | **1.04142** | **1.03870** | **1.04100** | **1.03869** | **1.03872** |
| **65** | **1.04851** | **1.03296** | **1.06173** | **1.01468** | **1.04071** | **1.03794** | **1.04021** | **1.03793** | **1.03796** |
| **66** | **1.04755** | **1.03162** | **1.06109** | **1.01357** | **1.03955** | **1.03706** | **1.03915** | **1.03705** | **1.03707** |
| **67** | **1.05076** | **1.03365** | **1.06514** | **1.01573** | **1.04217** | **1.04014** | **1.04210** | **1.04007** | **1.04010** |
| **68** | **1.04944** | **1.03268** | **1.06415** | **1.01398** | **1.04102** | **1.03877** | **1.04074** | **1.03875** | **1.03878** |
| **69** | **1.05360** | **1.03726** | **1.06904** | **1.01804** | **1.04540** | **1.04323** | **1.04528** | **1.04322** | **1.04324** |
| **70** | **1.05397** | **1.03821** | **1.07033** | **1.01896** | **1.04606** | **1.04433** | **1.04634** | **1.04432** | **1.04434** |
| **71** | **1.05210** | **1.03527** | **1.06798** | **1.01614** | **1.04365** | **1.04174** | **1.04372** | **1.04172** | **1.04175** |
| **72** | **1.04938** | **1.03060** | **1.06388** | **1.01164** | **1.03995** | **1.03743** | **1.03964** | **1.03742** | **1.03744** |
| **73** | **1.04663** | **1.02558** | **1.05980** | **1.00663** | **1.03605** | **1.03287** | **1.03553** | **1.03286** | **1.03289** |
| **74** | **1.05304** | **1.03401** | **1.06876** | **1.01403** | **1.04348** | **1.04103** | **1.04346** | **1.04095** | **1.04098** |
| **75** | **1.05372** | **1.03375** | **1.06962** | **1.01407** | **1.04369** | **1.04148** | **1.04381** | **1.04140** | **1.04142** |
| **76** | **1.05965** | **1.03894** | **1.07525** | **1.01908** | **1.04925** | **1.04679** | **1.04927** | **1.04671** | **1.04673** |
| **77** | **1.06088** | **1.03960** | **1.07650** | **1.01955** | **1.05018** | **1.04764** | **1.05004** | **1.04756** | **1.04758** |
| **78** | **1.06027** | **1.03769** | **1.07542** | **1.01802** | **1.04892** | **1.04633** | **1.04883** | **1.04625** | **1.04628** |
| **79** | **1.06109** | **1.03763** | **1.07773** | **1.01752** | **1.04930** | **1.04719** | **1.04947** | **1.04667** | **1.04669** |
| **80** | **1.05888** | **1.03626** | **1.07620** | **1.01527** | **1.04751** | **1.04529** | **1.04741** | **1.04477** | **1.04479** |
| **81** | **1.06152** | **1.03874** | **1.07939** | **1.01763** | **1.05007** | **1.04806** | **1.05026** | **1.04753** | **1.04755** |
| **82** | **1.06384** | **1.04138** | **1.08203** | **1.01962** | **1.05255** | **1.05036** | **1.05272** | **1.04983** | **1.04986** |
| **83** | **1.06199** | **1.03855** | **1.07976** | **1.01694** | **1.05021** | **1.04788** | **1.05024** | **1.04735** | **1.04738** |
| **84** | **1.05906** | **1.03372** | **1.07603** | **1.01292** | **1.04632** | **1.04400** | **1.04637** | **1.04347** | **1.04350** |
| **G. Rate** | **1.00961** | **1.00554** | **1.01229** | **1.00214** | **1.00757** | **1.00720** | **1.00758** | **1.00712** | **1.00712** |

It can be seen that the two similarity linked indexes, $P_S^t$ and $P_{SM}^t$, approximate each other to the fourth decimal place. These indexes end up at 1.0435 (to four decimal places) and should have the least amount of upper level substitution bias for the Danish monthly data set. The average annual geometric growth of the real time monthly similarity linked indexes $P_S^t$ is 1.00712 or 0.712 percentage points per year. The fixed base and chained monthly Laspeyres indexes, $P_L^t$ and $P_{LCh}^t$, have average annual geometric growth rates equal to 1.00961 and 1.01229 respectively which indicates an average upward bias of 0.249 and 0.517 percentage points per year relative to the preferred similarity linked index $P_S^t$. Thus the behavior of the monthly chained Laspeyres index is very different from the behavior of the annual chained Laspeyres index: the annual fixed base and chained Laspeyres indexes, $P_L^y$ and $P_{LCh}^y$, had geometric average annual growth rates equal to 1.00693 and 1.00549 compared to 1.00504, the average annual growth rate for the annual similarity linked indexes, which indicate much smaller average annual upward biases of 0.189 and

0.045 percentage points respectively in the annual Laspeyres indexes. The monthly chained Laspeyres has a very large upward chain drift whereas the annual chained Laspeyres index has a very modest upward chain drift. The monthly fixed base and chained Paasche indexes, $P_P^t$ and $P_{PCh}^t$, have annual average growth rates equal to 1.00554 and 1.00214 which indicates an average downward bias of 0.158 and 0.498 percentage points respectively relative to the growth rate for $P_S^t$. The monthly chained Fisher index, $P_{FCh}^t$, is very close to the two monthly similarity linked indexes with an annual average growth rate of 1.00720. The annual average growth rates for the monthly fixed base Fisher index and the monthly GEKS index, 1.00757 and 1.00758 respectively, are a bit above the chained monthly fixed base Fisher growth rate.




On Chart A5, the top line is the monthly Chained Laspeyres index $P_{LCh}^t$, followed by the Fixed Base Laspeyres index $P_L^t$. The black line is the Fixed Base Fisher index which lies a bit above the real time Similarity Linked index $P_S^t$, which can barely be distinguished from the Chained Fisher index. The lowest line corresponds to the monthly Chained Paasche index which lies below the Fixed Base Paasche index. The seasonal fluctuations in the Danish data are substantial.

**A.6. Conclusion**

The main points that emerged in section A.2 are as follows:

- In situations where there are no missing prices and no expenditure or quantity information is available, the monthly Jevons index $P_J^t$ is a preferred index.
- The upper level monthly price data from Denmark for the years 2012 to 2018 exhibit substantial seasonal fluctuations.

The main findings in section A.3 were:

- It is not a trivial matter to aggregate monthly consumer prices into annual prices. The usual National Statistical Office practice of forming annual prices as the arithmetic average of monthly prices is not consistent with theoretical approaches to index number theory and is likely to be particularly inaccurate if there are strong seasonal fluctuations in monthly prices and quantities. Since the Danish monthly price data does exhibit strong seasonal fluctuations, it is likely that the corresponding monthly expenditure data also exhibits strong seasonal fluctuations.
- In this section, (approximate) Laspeyres, Paasche, Fisher fixed base and chained annual indexes, $P_L^y$, $P_P^y$, $P_F^y$ and $P_{LCh}^y$, $P_{PCh}^y$, $P_{FCh}^y$ were computed for the 7 years in the sample. The multilateral GEKS and Predicted Share Similarity Linked annual indexes, $P_{GEKS}^y$ and $P_S^y$, were also computed. The similarity linked annual indexes $P_S^y$ have good properties from the viewpoint of both the economic and test approaches to index number theory and so the bias in the remaining indexes was measured relative to this index. The annual chained Fisher indexes $P_F^y$ were found to be identical to the annual similarity linked indexes $P_F^y$.
- The fixed base annual Laspeyres index $P_L^y$ was on average 0.19 percentage points above our preferred chained Fisher and Similarity Linked indexes while the fixed base Paasche annual index $P_P^y$ was on average 0.045 percentage points below $P_{FCh}^y$ and $P_S^y$.
- The average difference between the fixed base Laspeyres and the chained Fisher indexes was 0.19 percentage points while the difference between the chained Laspeyres and the chained Fisher indexes was only 0.045 percentage points. Thus annual substitution bias using the fixed base Laspeyres formula is much larger than the substitution bias using the chained Laspeyres index.

The real time "practical" month to month consumer price indexes that National Statistical Offices are able to calculate at higher levels of aggregation use annual expenditure shares (or annual quantities) from a previous year and their monthly price indexes. The three main monthly indexes of this type that are used are the Lowe, Young and Geometric Young indexes. If current annual expenditure or quantity weights are used, these indexes are denoted by $P_{Lo}^t$, $P_Y^t$ and $P_{GY}^t$ respectively.[45] If the annual weights are lagged one year, these indexes are denoted by $P_{Lo1}^t$, $P_{Y1}^t$ and $P_{GY1}^t$. If the annual weights are lagged two years, these indexes are denoted by $P_{Lo2}^t$, $P_{Y2}^t$ and $P_{GY2}^t$. The upper level substitution bias in these indexes is measured relative to the monthly similarity linked indexes $P_S^t$, which were defined in section A5. The main findings in section A.4 were as follows:

- The monthly average upward substitution bias for the "true" Lowe indexes was close to 0.04 percentage points per year; for the "true" Young index, it was 0.15 percentage points per year while the "true" geometric Young indexes had a tiny upward substitution bias equal to 0.004 percentage points per year on average.
- The means of the three monthly Lowe indexes increased as the lag in the annual weights increased. The average substitution bias for the Lowe indexes increased from 0.04 percentage points per year for the current weight Lowe index to 0.17 percentage points per year for the practical Lowe index that uses weights that are two years old.
- The average substitution bias for the monthly Young indexes increased from 0.15 percentage points per year for the Young index that uses current expenditure weights to 0.17 percentage points per year for the practical Young index that uses weights that are two years old.
- The average substitution bias for the monthly Geometric Young indexes increased from 0.004 percentage points per year for the Geometric Young index that uses current expenditure weights to 0.014 percentage points per year for the practical Geometric Young index that uses weights that are two years old.

[45] Of course, these indexes cannot be calculated in real time so they are not really "practical".

- The three monthly Geometric Young indexes were close to each other and had the smallest approximate substitution bias.

In section A.5, (approximate) Laspeyres, Paasche, Fisher fixed base and chained monthly indexes, $P_L^t$, $P_P^t$, $P_F^t$ and $P_{LCh}^t$, $P_{PCh}^t$, $P_{FCh}^t$ were computed for the 84 months in the sample. The multilateral GEKS and Predicted Share Similarity Linked monthly indexes, $P_{GEKS}^t$ and $P_S^t$, were also computed. The main findings in section A.5 were as follows:

- The monthly chained Fisher indexes $P_F^t$ were not identical to the monthly similarity linked indexes $P_F^t$ but they are so close to each other that they cannot be distinguished from each other on a chart.
- The monthly fixed base and chained Laspeyres indexes, $P_L^t$ and $P_{LCh}^t$, had an average upward bias (relative to our preferred similarity linked indexes) of 0.25 and 0.52 percentage points per year over the sample period respectively. Thus the behavior of the monthly chained Laspeyres index is very different from the behavior of the annual chained Laspeyres index: the monthly chained Laspeyres had a very large upward chain drift whereas the annual chained Laspeyres index had a much smaller upward chain drift.
- The monthly fixed base and chained Paasche indexes, $P_P^t$ and $P_{PCh}^t$, had an average downward bias of 0.16 and 0.50 percentage points respectively.
- The monthly chained Fisher index, $P_{FCh}^t$, was very close to the monthly similarity linked indexes. Thus chained Fisher indexes performed well for this particular data set, both in the annual context as well as in the monthly context.
- The monthly fixed base Fisher index, $P_F^t$, was very close to the monthly GEKS index, $P_{GEKS}^t$, and these indexes are slightly above our preferred similarity linked indexes.

Some overall conclusions are as follows:

- National Statistical Offices could consider computing Geometric Young indexes for their official CPIs in place of the Lowe and Young indexes that are presently widely used. From the main text and this Appendix, it appears that the lagged Lowe and Young indexes have some measureable upward substitution bias while the Lagged Geometric Young index has perhaps a smaller amount of downward substitution bias.
- For countries that have substantial seasonal fluctuations in prices and quantities, the use of annual expenditure weights will lead to inaccurate monthly consumer price indexes. Moreover, taking an arithmetic average of monthly prices will lead to inaccurate annual prices, which in turn will lead to inaccurate estimates of household consumption.[46] Thus it would be very useful if countries would attempt to estimate monthly expenditure weights.
- It will not be possible to obtain current expenditure information by month for all categories of consumption. But typically, some expenditure information can be obtained on a delayed basis. Thus it would be useful for Statistical Offices to produce an *analytical CPI* that could be revised as more information becomes available. The U.S. Bureau of Labor Statistics produces alternative CPIs on a regular basis which indicates that it is possible to produce multiple consumer price indexes without confusing the public.

The last point is an important one, particularly in recent times when all economies have been affected by the Covid pandemic and developments in the Ukraine. A Lowe or Young consumer price index is very useful measure of consumer inflation provided that relative quantities grow in a proportional manner or provided that consumer expenditure shares are approximately constant across recent months and years.

[46] See Diewert, Finkel, Sayag and White (2022) on this point.

However, substantial changes in consumer expenditure shares can occur rather suddenly which greatly strengthens the case for having alternative, revisable CPIs that make use of weight information which is available on a delayed basis.[47]

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[47] See Diewert and Fox (2022a) (2022b) on this point.

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