CHAPTER 1: INTRODUCTION

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1. The Basket, Axiomatic and Stochastic Approaches to Index Number Theory

This is a book about index number theory in general and the construction of a Consumer Price Index (CPI) in particular. It turns out that there is no single approach to index number theory that experts agree is the "right" approach. Thus this volume will cover the four main approaches to index number theory that are used today by national and international statistical agencies. These four main approaches are as follows:

- The fixed basket approach (and averages of fixed baskets);
- The test or axiomatic approach;
- The stochastic approach and
- The economic approach.

In order to measure aggregate consumer price change between two periods, the *fixed basket approach* takes a "representative" basket of goods and services that households purchase in the two periods under consideration and prices out the cost of the basket using the prices of the current period for the numerator of the consumer price index and using the prices of the base period for the denominator of the index. This type of index dates back to the middle ages but it was studied in some detail by the English economist, Lowe, in the early 1800s and as a result is known as a Lowe Index. It is useful to introduce some notation at this point so that the basket approach can be explained more precisely. Suppose the base period is called period 0 and the current period is called period 1. Suppose that there are N goods and services that a specified group of households purchase in each period and the total quantity purchased by the households in period t of product n is q_{tn} for $t = 0,1$ and $n = 1,\ldots,N$. Suppose further that the average price for product n in period t is p_{tn} for $t = 0,1$ and $n = 1,\ldots,N$. Denote the set of period t prices as $p^t \equiv [p_{t1},\ldots,p_{tN}]$ and the corresponding set of period t quantities as $q^t \equiv [q_{t1},\ldots,q_{tN}]$ for $t = 0,1$. A possible choice of a "representative" basket of goods and services is the *base period quantity vector*, q^0. This leads to the *Laspeyres price index* $P_L(p^0,p^1,q^0) \equiv \Sigma_{n=1}^N p_{1n}q_{0n}/\Sigma_{n=1}^N p_{0n}q_{0n}$. Another possible choice of a "representative" basket of goods and services is the *current period quantity vector*, q^1. This leads to the *Paasche price index* $P_P(p^0,p^1,q^1) \equiv \Sigma_{n=1}^N p_{1n}q_{1n}/\Sigma_{n=1}^N p_{0n}q_{1n}$. Since each of these two quantity vectors is equally representative and they both measure overall inflation going from period 0 price to period 1 prices, it may make sense to take a symmetric average of these two estimates of overall inflation as our final point estimate for consumer price inflation over the two periods under consideration. This leads to the *Fisher price index*, $P_F(p^0,p^1,q^0,q^1) \equiv [P_L(p^0,p^1,q^0)P_P(p^0,p^1,q^1)]^{1/2}$, which is the geometric average of the Laspeyres and Paasche price indexes.[1] Another variant of the fixed basket approach to index number theory is to take the fixed basket as the product by product geometric average of the quantities consumed in periods 0 and 1. Thus the *Walsh price index* is defined as $P_W(p^0,p^1,q^0,q^1) \equiv \Sigma_{n=1}^N p_{1n}(q_{0n}q_{1n})^{1/2}/\Sigma_{n=1}^N p_{0n}(q_{0n}q_{1n})^{1/2}$. The fixed basket approach to index number theory is explained in some detail in chapter 2.

Note that these fixed basket indexes are all *functions* of the two price vectors, p^0 and p^1, and the two quantity vectors, q^0 and q^1, that pertain to the two periods under consideration; i.e., these price index functions are all of the form $P(p^0,p^1,q^0,q^1)$ where this *bilateral index number formula* is a function of the 4N variables contained in the vectors p^0,p^1,q^0,q^1.

The *axiomatic* or *test approach to index number theory* starts with a (unknown) bilateral index number formula, $P(p^0,p^1,q^0,q^1)$, and attempts to determine the functional form for the index number function by placing restrictions on the function or in other words, asking that the index function satisfy certain tests. An example of a test is the *weak identity test*; i.e., we ask that the index number function satisfy the following property: $P(p^0,p^1,q^0,q^1) = 1$ if $p^0 = p^1$ and $q^0 = q^1$. Thus if product prices are exactly the same in the two periods being compared and quantities consumed are exactly the same in the two periods being compared, then the price index should be equal to 1 (indicating that there is no inflation between the two periods). Another example of a test is the *strong identity test*; i.e., we ask that the index number function satisfy the following property: $P(p^0,p^1,q^0,q^1) = 1$ if $p^0 = p^1$. Thus if product prices are exactly the same in the two periods being compared, then the price index should be equal to 1 even if the quantity vectors are different over the two periods under consideration. Another test is *linearly homogeneity in the prices of period 1*; i.e., we ask that the index function $P(p^0,p^1,q^0,q^1)$ satisfy the following property: $P(p^0,\lambda p^1,q^0,q^1) = \lambda P(p^0,p^1,q^0,q^1)$ for all numbers $\lambda > 0$. If $P(p^0,p^1,q^0,q^1)$ satisfies this property, then if all prices in period 1 double, then the price level in period 1 also doubles. The test approach to index number theory is explained in some detail in chapter 3 of this volume. It turns out that the Fisher index which emerged as a "best" index

[1] Detailed references to the works of Laspeyres, Paasche and Fisher will be found in the subsequent chapters. A similar comment applies to other authors who will be mentioned in this introductory chapter.

from the viewpoint of the fixed basket approach to index number theory also emerges as a "best" index from the viewpoint of the test approach.[2]

The *stochastic approach to index number theory* dates back to the work of the English economists Jevons and Edgeworth in the 1800's. This approach to index number theory will be explained in detail in chapter 4. However, a brief outline of this approach follows below.

Using the notation for prices defined above, the simplest example of the stochastic approach works as follows. Treat each price ratio for product n, p_{1n}/p_{0n}, as an estimate of general inflation going from period 0 to period 1.[3] Thus a statistical model for this situation might be the following one:

(1) $p_{1n}/p_{0n} = \alpha + e_{tn}$; $\qquad n = 1,\ldots,N$

where α is the general measure of inflation going from period 0 to 1 and the e_{tn} are independently distributed error terms with 0 means and constant variances. Now choose α as the solution to the following least squares minimization problem:

(2) $\min_\alpha \{\Sigma_{n=1}^N e_{tn}^2\} = \min_\alpha \{\Sigma_{n=1}^N [(p_{1n}/p_{0n}) - \alpha]^2\}$.

The solution to the minimization problem defined by (2) is:

(3) $\alpha^* \equiv (1/N)\Sigma_{n=1}^N (p_{1n}/p_{0n}) \equiv P_C(p^0,p^1)$.

where the *Carli index*, $P_C(p^0,p^1)$, is defined as the *arithmetic average* of the N price ratios, p_{1n}/p_{0n}. Note that the Carli index depends only on prices for the two periods under consideration in contrast to the Fisher index, which depended on prices and quantities.

Now suppose we changed the stochastic specification from (1) to $\ln(p_{1n}/p_{0n}) = \ln\alpha + e_{tn}$ for $n = 1,\ldots,N$ where the e_{tn} are independently distributed error terms with 0 means and constant variances. Define $\beta \equiv \ln\alpha$ as the natural logarithm of α. The new least squares minimization problem is:

(4) $\min_\beta \{\Sigma_{n=1}^N [\ln(p_{1n}/p_{0n}) - \beta]^2\}$.

The solution to the minimization problem defined by (4) is:

(5) $\beta^* \equiv (1/N)\Sigma_{n=1}^N \ln(p_{1n}/p_{0n})$.

β^* is an estimator for the logarithm of the price index going from period 0 to period 1 prices. To obtain an estimator for the price index α^{**}, we need to exponentiate β^* to obtain α^{**}:

(6) $\alpha^{**} \equiv \exp[\beta^*] = \Pi_{n=1}^N (p_{1n}/p_{0n})^{1/N} \equiv P_J(p^0,p^1)$.

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[2] It should be noted that the "best" index cannot be unambiguously determined. When using the basket approach, there is a problem in choosing the "best" average of the Laspeyres and Paasche indexes. When using the test approach, there is a problem of deciding which tests are the most important ones for the index number formula to satisfy. Different sets of admissible tests will lead to different "best" index number formulae. When using the stochastic approach, different methods of averaging the prices or different stochastic specifications of the error terms will lead to different "best" indexes.

[3] It is assumed that all prices are positive in what follows.

Thus the new stochastic specification leads to a new estimator for the price index, the *geometric average* of the N individual price ratios, $P_J(p^0,p^1)$, which is the *Jevons index number formula*.

The Carli and Jevons index number formulae are examples of unweighted indexes.[4] Keynes criticized these indexes because they did not take into account the economic importance of each product in the budgets of the consumers of the N products. The economic importance of products can be taken into account by replacing the unweighted least squares minimization problems (2) or (4) by weighted least squares minimization problems. A measure of the economic importance of product n is its share of total consumer expenditures on the N goods and services in both periods 0 and 1. Define the inner product of the vectors p^t and q^t as $p^t \cdot q^t \equiv \Sigma_{n=1}^N p_{tn}q_{tn}$. The period t share of consumer expenditures on product n is defined as $s_{tn} \equiv p_{tn}q_{tn}/p^t \cdot q^t$ for n = 1,…,N and t = 0,1. Define the arithmetic average of the expenditure shares for product n over the two periods as:

$$(7)\ s(n) \equiv (½)s_{0n} + (½)s_{1n}\ ; \qquad n = 1,\ldots,N.$$

A useful weighted by economic importance version of the least squares minimization problem defined above by (4) is the following weighted least squares minimization problem:

$$(8)\ \min_\beta \{\Sigma_{n=1}^N s(n)[\ln(p_{1n}/p_{0n}) - \beta]^2\}.$$

The solution to the minimization problem defined by (8) is:

$$(9)\ \beta^{**} \equiv \Sigma_{n=1}^N s(n)\ln(p_{1n}/p_{0n}).$$

β^{**} is an estimator for the logarithm of the price index going from period 0 to period 1 prices. To obtain an estimator for the price index α^{***}, we need to exponentiate β^{**} to obtain α^{***}:

$$(10)\ \alpha^{***} \equiv \exp[\beta^{**}] = \Pi_{n=1}^N (p_{1n}/p_{0n})^{s(n)} \equiv P_T(p^0,p^1,q^0,q^1).$$

Thus the new stochastic specification leads to a new estimator for the price index, a *share weighted geometric average* of the N individual price ratios, $P_T(p^0,p^1,q^0,q^1)$, which is the *Törnqvist Theil index number formula*. This index emerges as a “best” index number formula from the viewpoint of the stochastic or descriptive statistics approach to index number theory. For a more detailed description of this third approach to index number theory, see chapter 4.

The economic approach to index number theory is the most complicated of the four main approaches to index number theory and it is explained in detail in chapter 5. An overview of the economic approach is presented in the following section.

2. The Economic Approach to Index Number Theory

Chapter 5 develops the economic approach to index number theory. The economic approach is based on the assumption that consumers choose their consumption bundles to maximize an index of well being or utility subject to a budget constraint. This approach to index number theory will appear to be rather unrealistic to many price statisticians. However, it is empirically observed that consumers will purchase more of a product when its price is relatively low and less of it when its price is relatively high. Thus as relative prices change, consumers substitute towards cheaper products for more expensive products in an attempt to maintain their standard of living. The

[4] Unweighted in this context means that the price ratios are equally weighted.

economic approach to index number theory takes these substitution effects into account and as a result provides a more realistic measure of consumer price inflation. Moreover, the economic approach to index number theory provides economists and policy makers with (approximate) measures of consumer welfare change; i.e., the economic approach to index number theory provides us not only with measures of household inflation but it also provides us with measures of real consumption. Finally, the economic approach is necessary in order to measure the effects of quality change and to take into account the welfare effects of new and disappearing products.

The consumer's period t budget constrained utility maximization problem is equivalent to the problem of minimizing the cost of achieving the period t level of utility when the economic approach is used. Suppose the consumer's utility function is f(q) where $q \equiv [q_1,\ldots,q_N]$ is a consumption vector. The consumer's *cost function*, C(u,p) that corresponds to the given utility function f(q) is defined as follows:

$$(11)\ C(u,p) \equiv \min_q \{p\cdot q : f(q) \geq u\}$$

where $p \equiv [p_1,\ldots,p_N]$ is a vector of positive prices that the consumer faces and $q \equiv [q_1,\ldots,q_N]$ is a nonnegative consumption vector. Thus the consumer chooses the consumption bundle which minimizes the cost of achieving the target utility level u and C(u,p) is the resulting minimum cost of achieving this target level of utility.

Using the notation defined in the previous section, let q^t be the consumer's observed consumption vector in period t and suppose the consumer faces the price vector p^t in period t for t = 0,1. The consumer's period t level of utility is $u^t \equiv f(q^t)$ for t = 0,1. It is assumed that the consumer minimizes the cost of achieving his or her period t utility level u^t in periods 0 and 1. Thus we have the following equalities:

$$(12)\ p^t\cdot q^t = C(u^t,p^t)\ ; \qquad t = 0,1.$$

The *Konüs family of true cost of living indexes* that provides a measure of price inflation between periods 0 and 1, $P_K(u,p^0,p^1)$ is defined for each reference utility level u using the cost function as follows:

$$(13)\ P_K(u,p^0,p^1) \equiv C(u,p^1)/C(u,p^0).$$

Section 2 in chapter 5 develops the properties of this family of price indexes. Note that there is a possibly different true cost of living index for each choice of the reference level of utility, u. It is natural to choose u to be either $u^0 = f(q^0)$ or $u^1 = f(q^1)$ when making price comparisons between the two periods, 0 and 1. This leads to the *Laspeyres type true cost of living index*, $P_K(u^0,p^0,p^1) \equiv C(u^0,p^1)/C(u^0,p^0)$, and the *Paasche type true cost of living index*, $P_K(u^1,p^0,p^1) \equiv C(u^1,p^1)/C(u^1,p^0)$.

Section 3 makes an additional assumption; that f(q) is a *linearly homogeneous function* so that $f(\lambda q) = \lambda f(q)$ for all numbers $\lambda > 0$. This assumption is not supported by empirical evidence using aggregate price and quantity data. But it is a very useful assumption because it leads to true cost of living indexes $P_K(u,p^0,p^1)$ that are independent of the reference utility level u. It turns out that when the utility function is linearly homogeneous, the corresponding cost function C(u,p) factors into the product of the utility level u times the *unit cost function* c(p) which is equal to the minimum cost of achieving one unit of utility, C(1,p). Thus we have C(u,p) = uc(p) and hence:

$$(14)\ P_K(u,p^0,p^1) \equiv C(u,p^1)/C(u,p^0) = uc(p^1)/uc(p^0) = c(p^1)/c(p^0)$$

where $c(p^1)/c(p^0)$ is the ratio of unit cost in period 1 to unit cost in period 0.

Chapters 2, 3 and 4 defined various bilateral index number formula of the general form $P(p^0,p^1,q^0,q^1)$. Sections 5-9 in chapter 5 show that various true cost of living indexes of the form defined by (14) are equal to many of the bilateral index number formulae that were defined in previous chapters. These relationships are derived assuming that consumer preferences can be represented by certain specific functional forms for either the linearly homogeneous utility function $f(q)$ or the corresponding unit cost function $c(p)$. For example, if we assume that the consumer's unit cost function is a linear function of prices, so that $c(p) = \Sigma_{n=1}^N \alpha_n p_n$, then it can be shown that the Laspeyres price index, $P_L = p^1 \cdot q^0/p^0 \cdot q^0$, is *exactly* equal to the true cost of living index $c(p^1)/c(p^0)$. Thus the Laspeyres bilateral price index is an example of an *exact index number formula*. The theory of exact index number formulae was developed by Konüs and Byushgens and Pollak. If we can find a bilateral index number formula that is exact for a unit cost function $c(p)$ which can provide a second order Taylor series approximation to an arbitrary twice continuously differentiable unit cost function, then Diewert called the bilateral index a *superlative index*. Various examples of superlative index number formulae are given in chapter 5. It turns out that the Fisher, Törnqvist Theil and Walsh bilateral price indexes are all superlative indexes.

Section 8 in chapter 5 shows that these three indexes all approximate each other to the second order around a point where prices and quantities are equal (so that $p^0 = p^1$ and $q^0 = q^1$ at the point of approximation). This means that these three indexes will tend to approximate each other reasonably well, particularly if there is not too much variation in prices and quantities going from period 0 to period 1. From the viewpoint of the basket approaches to index number theory, the Fisher and Walsh indexes got good grades. The axiomatic approach to index number theory favoured the Fisher index while the stochastic approach to index number theory gave good grades to the Törnqvist Theil index. The Walsh index is a special case of a fixed basket index or Lowe index, which is an advantage since many price statisticians prefer fixed basket indexes because they are relatively easy to explain to the public. All three indexes are equally good from the viewpoint of the economic approach to index number theory. Thus it seems that it does not matter all that much on which approach to index number theory one takes: the four approaches lead to three specific index number formulae which will generate much the same answer in many situations.[5]

Section 11 of chapter 5 studies theoretical quantity indexes that are counterparts to the Konüs family of true cost of living indexes of the form $C(u,p^1)/C(u,p^0)$. The family of *Allen quantity indexes* is defined for each reference price vector, p, as follows:

$$(15)\quad Q_A(p,q^0,q^1) \equiv C(f(q^1),p)/C(f(q^0),p).$$

Thus the Allen quantity index is the ratio of the cost of achieving the period 1 level of utility, $f(q^1)$, to the cost of achieving the period 0 level of utility, $f(q^0)$, using the same reference price vector p in both the numerator and denominator of the ratio. The two most relevant choices for the reference price vector when comparing utility levels in periods 0 and 1 are the period 0 and 1 price vectors, p^0 and p^1. This leads to the *Laspeyres type Allen index*, $Q_A(p^0,q^0,q^1)$ and to the *Paasche type Allen index*, $Q_A(p^1,q^0,q^1)$. If we make the additional assumption that the utility function is linearly homogeneous, then we have $C(f(q^t),p) = f(q^t)c(p)$ for $t = 0,1$ and the Allen quantity index simplifies to the following utility ratio:

[5] This is true if there are no missing prices and fluctuations in prices and quantities are not too great. If there are missing prices or severe fluctuations in prices and quantities, then the three indexes can differ significantly.

(16) $Q_A(p,q^0,q^1) \equiv C(f(q^1),p)/C(f(q^0),p) = f(q^1)c(p)/f(q^0)c(p) = f(q^1)/f(q^0)$.

As was the case with the Konüs price index, it is possible to show that $Q_A(p,q^0,q^1) = f(q^1)/f(q^0)$ is exactly equal to various bilateral price indexes of the form $Q(p^0,p^1,q^0,q^1)$[6] for certain functional forms for f(q) and $Q(p^0,p^1,q^0,q^1)$.

Section 12 in chapter 5 provides a brief discussion on the problems associated with constructing true cost of living indexes when there are taste changes. The period t cost function for the consumer is the function $C^t(u,p)$ for t = 0,1; i.e., we allow the cost function to change going from period 0 to 1, reflecting a change in the consumer's preferences over different combinations of the N consumer goods and services. The true cost of living index using the preferences of period 0 and the reference utility level of period 0 is $C^0(u^0,p^1)/C^0(u^0,p^0)$ while the true cost of living index using the preferences of period 1 and the reference utility level of period 1 is $C^1(u^1,p^1)/C^1(u^1,p^0)$. Each of these measures is of interest and each measure is equally valid. If we require a single estimate for real price change between the two periods, then taking the geometric average of these two estimates seems to be a reasonable procedure. The analysis in section 12 shows how the Törnqvist Theil index could provide an observable approximation to this average measure of price change. However, it should be noted that the results in section 12 do not allow for completely general taste changes. The issue of taste change is of some importance since the Covid pandemic which started in 2020 surely changed consumer preferences. In order to model the transition from pre-Covid preferences to Covid preferences while allowing for completely different preferences, it seems that econometric methods would have to be used where separate preferences are estimated for both the pre-Covid period and the Covid period. National Statistical Offices are not well equipped to undertake econometric investigations. This is an area where further research is required.

Section 13 of chapter 5 explores the conditional cost of living concept. In this section, it is assumed that the consumer's preference function, f(q,z), is defined over a vector of market goods and services q and a vector of environmental or household demographic variables, z. The consumer's minimum (market) cost of achieving the utility level u given that he or she faces the vector of market prices p is the cost $C(u,p,z) \equiv \min_q \{p\cdot q : f(q,z) \geq u\}$. Pollak's *conditional cost of living index* is the cost ratio $C(u,p^1,z)/C(u,p^0,z)$. Thus this consumer price index is conditional not only on the chosen utility level, u, but it also depends on the environmental vector z. As usual, two special cases of this family of indexes is of interest when comparing the prices of periods 0 and 1: (i) the *Laspeyres type conditional index* $C(u^0,p^1,z^0)/C(u^0,p^0,z^0)$ and the *Paasche type conditional index* $C(u^1,p^1,z^1)/C(u^1,p^0,z^1)$. The main result in section 13 shows that for a certain fairly general functional form for the conditional cost function, one can show that the Törnqvist Theil price index $P_T(p^0,p^1,q^0,q^1)$ defined above is exactly equal to the geometric mean of the theoretical Laspeyres and Paasche type conditional cost of living indexes; i.e., we have $P_T(p^0,p^1,q^0,q^1) = \{[C(u^0,p^1,z^0)/C(u^0,p^0,z^0)][C(u^1,p^1,z^1)/C(u^1,p^0,z^1)]\}^{1/2}$.

Section 14 provides a framework for dealing with new and disappearing products in a consumer price index. When a new product appears for the first time, there is no price in a prior period for that product so typically, the new product is ignored in the consumer price index for the period of introduction. From the viewpoint of the economic approach to index number theory, an appropriate price for the new product in the prior base period of the index is the consumer's *reservation price* for the product. It is the price which is just high enough to deter the consumer from purchasing the

[6] Various bilateral quantity indexes can be obtained from counterpart bilateral price indexes by interchanging the role of prices and quantities. Thus the bilateral Laspeyres and Paasche quantity indexes are defined as $Q_L \equiv p^0\cdot q^1/p^0\cdot q^0$ and $Q_P \equiv p^1\cdot q^1/p^1\cdot q^0$. The Fisher quantity index is defined as $Q_F \equiv [Q_LQ_P]^{1/2}$.

product during the base period. This reservation price concept is due to Hicks and it is explained more fully in section 14 of chapter 5 and in even more detail later in chapter 8.

Section 15 of chapter 5 introduces *household production* and the consumer's *allocation of time* into the consumer price index framework. Up to this point, the economic approach to index number theory assumes that the consumer chooses its consumption vector to maximize a utility function subject to a budget constraint. But in reality, consumers get utility by spending time on enjoying their purchases subject to a budget constraint and a time constraint. The addition of the time constraint to the consumer's utility maximization problem greatly complicates the construction of a consumer price index. Section 15 follows the path breaking work of Becker in adding the time constraint to the consumer's utility maximization problem. This section is perhaps the most complicated section in the entire volume. National Statistical Offices have not really embraced the integration of the time constraint with the budget constraint due to the complexity of integration and perhaps due also to the difficulty in collecting data on the household's allocation of time across various activities. Section 15 does provide a framework for organizing the data and integrating the time constraint with the budget constraint.

Up to this point, the economic approach to index number theory has presented the theory as it applies to a single household. Section 16 looks at methods to aggregate over households to form what Pollak calls a *social cost of living index*. The material presented in this section assumes that price and quantity information is available for individual households. Section 16 also discusses *democratic* and *plutocratic price indexes*.

Section 17 looks at the problems associated with aggregating over households in order to form measures of economy wide real consumption. The analysis in this section supports the construction of aggregate Fisher quantity indexes.

Section 18 generalizes the discussion in section 17 in order to discuss alternative measures of social welfare and the relationship of measures of income inequality to social welfare. The work of Atkinson, Kolm, Sen, Jorgenson and Schreyer figures prominently in this section. A simple social welfare function is suggested that is equal to the product of per capita real consumption times one minus the Gini coefficient for the distribution of real income in the economy.

Section 19 addresses an important shortcoming of most of the analysis presented in chapters 2-5 up to this point: it is usually assumed that all prices are positive in the two periods being compared. However, the shorter is the length of the time period[7] and the longer is the time series of prices and quantities, the greater will be the likelihood of a *lack of matching of prices*. This problem is due to the following explanatory factors:

- The existence of seasonal products that are only available in certain seasons.
- When a durable good or storable product goes on sale, consumers can purchase multiple units of the product during the sale period and then purchase zero units of the product for subsequent periods until their inventory of the product is depleted or the durable good is worn out.
- Product churn; i.e., producers are constantly modifying their products and replacing "old" products with perhaps slightly different "new" products.

[7] If the time period is very short (for example, one day) hardly any purchases made by a single household will be matched over a sequence of days.

Section 19 discusses possible solutions to the lack of matching problem. In addition, chapters 7-10 all deal with missing prices in more detail.

3. Elementary Indexes

Chapter 6 discusses the problems associated with choosing a bilateral index number formula at the first stage of aggregation in constructing a consumer price index. In particular, the problem of formula choice is discussed when only price information is available. This type of index is frequently called an *elementary index*. In this case, the economic approach to index number theory cannot be applied and so only the test and stochastic approaches to index number theory can be used in this "prices only" context. Examples of elementary indexes that use only price information for the two periods under consideration are the Carli, Dutot and Jevons indexes. The Carli and Jevons indexes were defined earlier by (5) and (6) and were equal to the arithmetic and geometric averages of the price ratios, p_{1n}/p_{0n}. The *Dutot index* is defined as follows using our usual notation:

$$(17)\ P_D(p^0,p^1) \equiv [\Sigma_{n=1}^{N} (1/N)p_{1n}]/[\Sigma_{n=1}^{N} (1/N)p_{0n}]\ .$$

Thus the Dutot index is a ratio of the average price in period 1 to the average price in period 0. Unfortunately, this index is not invariant to changes in the units of measurement whereas the Carli and Jevons indexes are invariant. Applying the test approach to bilateral index number theory in the prices only case leads to the Jevons index as being a "best" index.

In this chapter, it is assumed that all prices are positive and this is a limitation of the analysis. The problems associated with missing prices will be discussed in chapters 7 and 9.

Chapter 6 also discusses in some detail some of the problems associated with determining the *scope of an index*. For example, how exactly should a product be defined? Should the elementary aggregate have a regional or type of household dimension in addition to a product dimension? Should prices be collected from households directly or from outlets servicing households?

An appendix to chapter 6 discusses another interesting measurement problem that arises at the first stage of aggregation. Some retail outlets charge a *fixed monthly or annual fee* in order to give customers access to their products (or to give members a lower price on products). For example, telecommunications firms often charge a fixed monthly access fee that is independent of the usage of their services. The appendix discusses alternative methods for treating these fixed access fees in a consumer price index.

4. The Chain Drift Problem and Multilateral Indexes

Chapter 7 deals with possible solutions to the *chain drift problem* which will be defined below. This chapter also addresses the problems associated with missing prices.

Up to this point, we have focussed on the problem of measuring consumer price inflation over two periods, a base period 0 and a current period 1; i.e., we have been discussing bilateral index number theory. In chapter 7, the focus is on constructing an index over many periods. A simple method for adapting bilateral index number theory to the case of many periods is to choose the bilateral index number formula, fix the base period and as new data become available, we simply compute the bilateral index linking the current period to the base period. This method generates a sequence of *fixed base index numbers*. The problem with this strategy is that the structure of the economy changes over time with new products appearing and old products disappearing that one is eventually forced to give up on the use of fixed base index numbers.

An alternative to fixed base index numbers is to use *chained indexes*. Chained indexes work as follows. Suppose we have chosen a suitable bilateral index number formula, say $P(p^0,p^1,q^0,q^1)$ and we want to use this formula to compute chained indexes over time periods 1,2,…,T. Suppose the price and quantity vectors for period t are p^t and q^t for t = 1,…,T. For period 1, we set the price level P^1 equal to 1. For period 2, we set the price level P^2 equal to the index $P(p^1,p^2,q^1,q^2)$. For period 3, set the price level P^3 equal to $P(p^2,p^3,q^2,q^3)$ times the price level in period 2, P^2. Thus we have $P^3 = P^2 \times P(p^2,p^3,q^2,q^3) = 1 \times P(p^1,p^2,q^1,q^2) \times P(p^2,p^3,q^2,q^3)$. Similarly, the price level in period 4 is $P^4 = P^3 \times P(p^3,p^4,q^3,q^4) = 1 \times P(p^1,p^2,q^1,q^2) \times P(p^2,p^3,q^2,q^3) \times P(p^3,p^4,q^3,q^4)$. And so on. Thus we build up the overall price change going from period 1 to period T by multiplying together the period to period chain links $P(p^{t-1},p^t,q^{t-1},q^t)$ that link the prices of period t to the prices of the previous period t−1. Thus it appears that chained indexes will be more reliable than fixed base indexes when there is a great deal of product churn because the chained indexes will have more matched prices on average than fixed base indexes. The use of chained indexes was endorsed by Marshall many years ago.

The 2004 *Consumer Price Index Manual* endorsed the use of a superlative index number formula (like the Fisher bilateral price index) at the first stages of aggregation if price and quantity data were available and the *Manual* also endorsed the use of chained indexes. However, when this chaining strategy was implemented at the first stage of aggregation when price and quantity data were available (say at the level of a retail outlet for some class of products), it was found that the resulting indexes often led to unusually low price levels as time marched on. A way of formalizing this *chain drift problem* is to look at a series of chained indexes over T periods and add the data of the base period as an artificial final period T+1. Ideally, we would like the resulting period T+1 price level to equal the period 1 price level; i.e., we would like the bilateral index number formula to satisfy Walsh's *Multiperiod Identity Test*. In recent years, with the increasing availability of retail scanner data, statistical agencies have found that this test often fails quite spectacularly.

The chain drift problem is mainly caused by consumer stockpiling behaviour. Thus when the price of a product is particularly low, consumers tend to purchase a large amount of it (thus driving the overall price index down) but when prices return to normal in a subsequent time period, consumers purchase less than the normal amount of the product and this means that the index does not recover to its pre-sale level. An example in chapter 7 illustrates this phenomenon. Thus the normal case of chain drift is downward chain drift. But upward chain drift can also occur (due to incomplete adjustment on the part of households). The case of upward chain drift is also explained in chapter7.

As statistical agencies and academics worked on constructing subindexes of the consumer price index using scanner data over the past 20 years, they discovered that the use of multilateral indexes could reduce chain drift. A *multilateral index* simultaneously determines price levels for a window of say T periods so the attention shifts to the determination of price levels rather than rates of inflation over two periods (the bilateral approach to index number theory). Multilateral price indexes were introduced by Gini in order to measure price levels across the different regions of Italy. Balk was an early pioneer in applying multilateral methods in the time series context. Ivancic, Diewert and Fox stimulated general interest in the use of *rolling window multilateral index number methods* in the time series context.

Chapter 7 has treatments of several related topics:

- A comparison of likely differences in many bilateral and multilateral index number formulae is made under the assumption that there are long run trends in prices.

- The problems raised by missing prices are discussed briefly but they are discussed in much more detail in chapters 8 and 9.
- The main multilateral indexes are defined and compared which includes the GEKS, Geary Khamis, Time Product Dummy and Relative Price Similarity Linked indexes.
- A new test approach to multilateral indexes is developed where the object is to construct price levels rather than a bilateral ratio of prices.
- The various indexes defined in the chapter are computed for a small scanner data set which is listed in an appendix to the chapter.

From the viewpoint of the axiomatic approach to multilateral index number theory, the "best" index is the SPQ similarity linked indexes. The basic idea behind similarity linked price indexes is as follows. If prices between two periods are proportional, then any "reasonable" price index will be equal to the factor of proportionality. Hence, define a suitable measure of relative price dissimilarity between the prices of any two periods. The nonnegative dissimilarity measure must have the property that if prices in the two periods are proportional, then the dissimilarity measure is equal to 0. Larger measures of dissimilarity indicate larger deviations from price proportionality. When the prices of the current period become available, calculate the chosen measure of relative price dissimilarity with the prices of each prior period in a window of observations. The prior period with the lowest measure of dissimilarity is chosen as the link observation and the bilateral Fisher index for the current period relative to the chosen link period is calculated and it is used to update the price level of the link period. This method of linking leads to price levels which will always satisfy Walsh's Multiperiod Identity Test for prices. Using the predicted share measure of relative price dissimilarity is recommended because it can deal with missing prices (and zero quantities) and it penalizes a lack of matching of prices between the two periods. The SPQ similarity linking method is a bit more complicated but it leads to quantity indexes (as well as price indexes) which satisfy a Multiperiod Identity Test for quantities.

5. Quality Adjustment Methods

Chapter 8 presents a general framework for measuring the effects of quality change in a consumer price index context. Most of the existing methods for adjusting for quality change can be regarded as special cases of this framework.

Here is the basic problem: a new product suddenly appears. It could be a genuinely new product or a possibly improved version of an existing product. How can we capture the possible benefits of this new product in a consumer price index or in the companion index of real consumption?

It is not possible to measure the effects of quality change without using the economic approach to index number theory since we are trying to measure the benefit or utility of the new product relative to existing products. We use the utility function $f(q)$ that was defined in the beginning of section 2 above. Again, we assume that the utility function is linearly homogeneous so that it satisfies the property $f(\lambda q) = f((\lambda q_1,\lambda q_2,\ldots,\lambda q_N) = \lambda f(q)$ for all numbers $\lambda > 0$. The chapter develops alternative quality adjustment methods that depend on alternative assumptions about the functional form of the utility function.

Chapter 8 studies four types of model depending on the assumptions made about $f(q)$:

- $f(q)$ is a linear function of the form $f(q) = \alpha \cdot q \equiv \Sigma_{n=1}^{N} \alpha_n q_n$. This class of models includes methods used by statistical agencies as well as the *time dummy hedonic regression model* studied in chapters 6 and 7.
- $f(q)$ is again a linear function of q but the coefficients α_n are now functions of various amounts of K price determining characteristics, $z_1,\ldots,z_K$. Thus we have $f(q,z) = \Sigma_{n=1}^{N} \alpha_n(z_{n1},z_{n2},\ldots,z_{nK})q_n$ where the $z_{n1},z_{n2},\ldots,z_{nK}$ are the amounts of characteristic 1, 2,…,K that one unit of product n contains for n = 1,…,N. This class of models leads to *general hedonic regression models with characteristics*.
- $f(q)$ is a *Constant Elasticity of Substitution* (CES) *utility function*. This class of utility functions includes the linear utility function defined above as a special case but it is more flexible; i.e., it is consistent with a wider range of consumer substitution responses to changes in prices. Feenstra worked out a very elegant method for dealing with this case that does not require extensive econometric estimation; only an estimate for the elasticity of substitution is required in order to implement Feenstra's method.
- $f(q)$ is a more general functional form that allows for a wider range of consumer responses to changes in prices. This framework has been used by Hausman and Diewert and Feenstra.

The problem with the CES approach is that the reservation prices that this approach generates for missing products are infinite. Typically, it does not require an infinite price to discourage a consumer from purchasing a product. The fourth class of methods that uses more flexible functional forms generates finite reservation prices but has the disadvantage that the associated econometric estimation is quite complex and difficult to implement at scale.

Section 11 of chapter 8 discusses some additional approaches to the treatment of quality change such as clustering.

6. Seasonal Products

Chapter 9 deals with the problem of seasonal products. A seasonal good or service has regular fluctuations in prices and quantities that are synchronized with the seasons of the year. A *strongly seasonal product* is a seasonal product that is not available in all months of the year. Thus strongly seasonal products create a missing price problem for the seasons where the product is simply not available.

The problems associated with missing prices are addressed in chapters 7 and 8 and so the methods for dealing with missing prices suggested in these chapters can be used to deal with missing prices in the strongly seasonal context. However, the fact that there is a degree of regularity in the appearance and disappearance of strongly seasonal products means that alternative methods for dealing with missing seasonal prices can be devised. In particular, indexes which match the prices and quantities of December for the current year to the prices and quantities of December for the base year are likely to have fewer missing prices and quantities that an index the compares the prices of the current month with the prices of the previous month. In other words, year over year December indexes and year over year January indexes are likely to be more reliable than an index that compares January prices to February prices. Thus sections 2 and 3 of chapter 9 present the algebra for constructing year over year indexes for each month of the year. Section 2 uses carry forward prices for any prices that are missing when a product is not present. A carry forward price is simply the last observed price for the product which is used as an imputed price for the product when it is missing.[8] Since seasons are not perfectly synchronized with months of the year, carry forward prices can occur in the year over year context.[9] In general, it is not a good idea to use carry forward prices, particularly in conditions of general inflation (or deflation), since the carry forward price in the inflation context will tend to give the index a downward bias. Hence in section 3, the various year over year monthly indexes are recalculated using only matched prices in the two periods being compared.

Sections 4 and 5 of chapter 9 construct annual indexes in the seasonal products context. The most accurate method for constructing an annual index in this context is to treat each product in each period (month or quarter) as a separate annual product. This type of annual index was first suggested by Mudgett and Stone in separate publications. Section 4 uses the carry forward prices constructed in section 2 in order to calculate various Mudgett Stone annual indexes while section 5 uses only matched prices in constructing the various annual indexes. National Statistical Offices do not use the Mudgett Stone methodology when they construct annual consumer price indexes; instead they usually just take the arithmetic average of their monthly year over year indexes to construct an annual index (or they take the arithmetic average of their month to month consumer price indexes for the calendar year). Using our Israeli data on fresh fruits, we found that there was a substantial amount of bias using the usual method for forming annual indexes compared to the Mudgett Stone method. The bias is due to the fact that expenditures on strongly seasonal products are not spread evenly over all months.

Sections 6 and 7 construct month to month price indexes using various index number formulae. The computations in section 6 used carry forward prices for missing prices[10] while the computations in section 7 used only matched product prices. Our “best” month to month indexes used the Predicted Share methodology (explained in chapter 7) for linking the current month to the previous month that had the most similar structure of relative prices. The downward bias in using carry forward prices (instead of matched prices) in the context of month to month indexes was much more pronounced than it was in the context of constructing year over year monthly indexes.

Up to this point, the various indexes used monthly price and quantity data. It is of interest to use only the price data to construct various month to month indexes. Thus sections 8 and 9 construct various indexes such as the Carli, Dutot and Jevons indexes (and the Time Product Dummy indexes that use only price data). These indexes that use only price information can then be compared with our “best” similarity linked indexes which used monthly price and quantity information. Section 8 used month to month carry forward prices while section 9 used only prices that were actually observed. Section 9 also modified the Predicted Share relative price similarity linking methodology to the *prices only situation*. In place of the maximum overlap bilateral Fisher index (which was used to link the current period prices to the prices of a prior period with the most similar structure of relative prices), the maximum overlap Jevons index was used to link the current period prices to the prices of a prior period. This *new prices only multilateral method* seemed to work well using the Israeli data set in the sense that seasonal fluctuations were muted using the prices only Predicted

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[8] A carry forward price for the year over year monthly indexes will in general be different from a carry forward price for a month to month index.

[9] For example, weather can delay or bring forward harvests of fresh fruits and vegetables. Similarly snowfall conditions can delay the opening of ski lifts and so on.

[10] Section 7 uses month to month carry forward prices for missing products which are different from the carry forward prices used in section 2. For the Israeli data set, the probability that a month to month price for the same product was missing turned out to be 0.447. There were very few missing prices in the year over year context that was used in section 2.

Share indexes and these indexes were reasonably close to our "best" Predicted Share similarity linked indexes that used both price and quantity information.[11]

Section 10 looked at various annual basket indexes and compared these indexes to our "best" similarity linked indexes. Several of these annual basket indexes captured trend inflation rather well but the seasonal fluctuations were often very large (and in opposite directions) compared to our "best" index. It seems to be worthwhile for National Statistical Offices to invest in obtaining monthly expenditure weights in order to produce more accurate price indexes for seasonal commodity groups.

Finally, in section 11, some of the problems associated with measuring trend inflation and seasonal adjustment are discussed. The basic problem is that it is difficult to distinguish trend inflation from changes in seasonal price patterns. Thus our focus in chapter 9 is on obtaining measures of price change before seasonal adjustment. There are many methods suggested in the literature on how to seasonally adjust an economic time series. But all of these methods require as an input an unadjusted series. Thus producing the best possible unadjusted series should be the main task of a National Statistical Office.

7. The Treatment of Durable Goods and Owner Occupied Housing

Chapter 10 looks at the treatment of durable goods in a Consumer Price Index. The basic problem is the following one. When a household purchases a durable good,[12] a certain price is paid for the ownership of it in the period of purchase. However, the benefits from the use of the durable good persist into the future for many periods and thus, it does not seem to be fair to charge the entire purchase price of the durable to its period of purchase. But how exactly are we to decompose the purchase price of the durable into period by period charges for the use of the durable over its useful lifetime? This is the *fundamental problem of accounting*.

There is no universally agreed answer to this problem. However, there are *three main approaches* to addressing this problem:

- The *acquisitions approach*. This approach simply allocates the entire purchase price to the period of purchase.
- The *rental equivalence approach*. If rental markets for the durable good exist, then use the current period rental price as an imputed price for the use of the durable in the current period. By using the services of the durable during a period, the owner forgoes the opportunity cost of renting the services of the durable to another user.
- The *user cost approach*. This is the financial opportunity cost of using the services of the durable which will be explained in more detail below.

It is obvious that the acquisitions approach will not measure the service flow yielded by ownership of a durable good beyond the first period and so for durables which have a long useful life (such as housing), the acquisitions approach will not be suitable for measuring the flow of utility to the consumer that using the services of the durable good generates.

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[11] This new similarity linked multilateral method that used only price information also penalized a lack of price matching.

[12] A durable good provides a flow of services to the household that persists for more than one accounting period.

The rental equivalence approach does measure the utility flow generated by using the services of the durable good but it may fail in situations where the rental market is thin or nonexistent or heavily regulated by the government.

The user cost approach is more complicated than the first two approaches. Here is an explanation of how the approach works. Suppose that a household purchases a new unit of the durable good at the beginning of period t at the price P_0^t. The 0 subscript indicates that the age of the purchased good is 0, indicating that it is a new unit of the durable good. The consumer uses the services of the durable during period t. At the end of period t (which is the beginning of period t+1), the consumer observes that the used durable good could be sold at the price P_1^{t+1}, where the subscript 1 indicates that the durable good has been used for 1 period so it is now a second hand good. Thus it appears that the consumer's total cost of using the services of the durable good during period t is simply the purchase price less the selling price or $P_0^t - P_1^{t+1}$. But in purchasing the durable good at the beginning of period t, the household ties up its financial capital for the period and thus there is a *financial opportunity cost of holding the durable good* over the period. This financial opportunity cost is $r^tP_0^t$ where r^t is the relevant interest rate that the household faces.[13] Thus the full *user cost* is $U^t \equiv P_0^t(1+r^t) - P_1^{t+1}$. This user cost formula is not the usual one used by economists. Suppose that the price of a new unit of the durable good at the beginning of period t+1 is P_0^{t+1}. We can use the end of period t (or beginning of period t+1) prices P_0^{t+1} and P_1^{t+1} for a new and used unit of the durable good in order to define a *depreciation rate* for the durable good, δ^t, defined as follows: $(1-\delta^t) \equiv P_1^{t+1}/P_0^{t+1}$. We can also use the new good prices P_0^t and P_0^{t+1} in order to define the period t *asset appreciation rate*, i^t, as follows: $(1+i^t) \equiv P_1^{t+1}/P_0^{t+1}$. Using these definitions, we can express the end of period t price for a unit of the used durable, P_1^{t+1}, in terms of the price of a new unit of the durable at the beginning of period t, P_0^t, as follows:

(18) $P_1^{t+1} = (1-\delta^t)(1+i^t)P_0^t$.

Using (18), the user cost U^t becomes the following expression:

(19) $$\begin{aligned} U^t &= P_0^t(1+r^t) - P_1^{t+1} \\ &= P_0^t(1+r^t) - (1-\delta^t)(1+i^t)P_0^t \\ &= [r^t - i^t + \delta^t(1+i^t)]P_0^t. \end{aligned}$$

Although the user cost concept is used by many National Statistical Offices in their productivity accounts in order to measure capital services if they provide measures of the Multifactor Productivity or Total Factor Productivity for their economy, countries have been reluctant to use the user cost methodology to measure the services of consumer durables in their Consumer Price Indexes. The problem is that it is not straightforward to determine what exactly is the appropriate interest rate r^t, depreciation rate δ^t and asset appreciation rate i^t to use in the user cost formula defined by (19) above. In particular, the use of ex post asset appreciation rates i^t in formula (19) is not recommended due to the volatility in asset prices; instead predicted or smoothed asset appreciation rates should probably be used. But this raises the question: which of many possible methods should be used in order to smooth asset appreciation rates? It is also difficult to determine the "right" opportunity cost of financial capital r^t and it is not easy to determine depreciation rates δ^t either. Nevertheless, sometimes countries are forced to use the user cost approach to value the

[13] If the household borrows money to finance the purchase of the consumer durable, then r^t is the rate of interest that the household pays in order to secure the loan. If the household does not have to borrow funds to finance the purchase and has investments, then the relevant rate of interest r^t is the rate of return on a marginal investment made at the beginning of period t.

services of Owner Occupied Housing due to the lack of comparable rental markets. Making somewhat arbitrary decisions about r^t , i^t and δ^t is acceptable if these decisions are explained to users. After all, user costs are routinely used by academic economists and by national statistical agencies that produce estimates of Multifactor Productivity.

National Statistical Offices for the most part just use the acquisitions approach to value the services of consumer durables in their Consumer Price Indexes. The exception to this rule is Owner Occupied Housing (OOH). For the most countries, the rental equivalence approach is used but for a few countries where rental markets are thin, a simplified user cost approach is used. Eurostat's Harmonized Index of Consumer Prices has simply omitted the services of OOH but this may change in the future.

Chapter 10 discusses the three main approaches to the treatment of durables in a CPI in more detail in sections 2-5. Sections 6-8 look at different models of depreciation which is of interest if the user cost approach to the treatment of durables is implemented. Section 9 shows how the acquisitions approach will in general understate the flow of services from the use of durable goods in the national accounts.

Section 10 of chapter 10 looks at the accounting problems that are posed by stockpiling behavior on the part of households.

Sections 11-18 deal with the complications associated with including housing services in a CPI. A main problem is that housing consists of two main assets: (i) the structure (which depreciates) and (ii) the land plot that supports the structure (which does not depreciate). Thus it is not possible to *exactly* match the prices of the same property over time due to depreciation of the structure (and possible renovations and additions to an existing structure). Thus we have a difficult quality adjustment problem. Hedonic regression techniques offer the best solution to these measurement problems.

Section 18 looks at a fourth approach to the treatment of OOH; namely the *payments approach*. Some of the problems associated with the use of this approach are discussed in this section.

8. Lowe, Young and Superlative Indexes: An Empirical Study for Denmark

Chapter 11 concludes this Consumer Price Index Theory volume by looking at the components of the Danish CPI and experimenting with alternative methods of aggregating the data. Thus Statistics Denmark has provided its data for 402 monthly elementary price indexes for the seven years 2012-2018 along with the annual basket weights used to aggregate these elementary indexes into an overall index.

This chapter computes various "practical" monthly indexes such as the annual basket Lowe and Young indexes along with various superlative annual indexes. Thus estimates of the amount of substitution bias in the annual indexes can be computed.

An Appendix to the Chapter computes various elementary indexes as well as some approximate Fisher and relative price similarity linked indexes using the same data set.

9. Conclusion

This volume started at the Ottawa Group meeting in Tokyo Japan in 2015. Thus it has been in progress for some seven years. The authors of the various chapters are as follows: W. Erwin

Diewert wrote chapters 1-8; the authors of chapter 9 are Diewert, Yoel Finkel, Doron Sayag and Graham White; the authors of chapter 10 are Diewert and Chihiro Shimizu; the authors of the main text of chapter 11 are Martin Nielsen, Martin Larsen and Carsten Boldsen; the author of the appendix to chapter 11 is Diewert. The overall editor of the volume is Diewert. The authors would like to thank all of the persons who provided comments on the various chapters; these commentators are listed in a footnote at the beginning of each chapter. Needless to say, the commentators are not responsible for any remaining errors. The authors want to thank Chihiro Shimizu in particular for carefully reading the manuscript of each chapter and checking each equation for errors.

The main purpose of the volume is to provide an overview of the main index number theories that have been suggested over the past 150 years that could be useful to statisticians that construct Consumer Price Indexes. The volume contains hundreds of references to papers and books on index number theory that could be useful to readers who want to explore the subject more deeply. This book could also be useful to academics who wish to learn more about the problems facing price statisticians in their attempts to produce accurate Consumer Price Indexes. There are many unsolved problems that could be usefully studied by academics. Parts of the various chapters could be used as supplementary reading material for courses in advanced macroeconomics.[14] This book could also be useful to private sector economists who are processing micro data into aggregates to be used by management.

This volume does require some mathematical background in order to follow completely all of the various arguments made in the text. Basically, matrix algebra and advanced calculus are used in many of the chapters. Some knowledge of microeconomics is also helpful.

An important topic which is not covered in the present volume is sampling theory. The application of multilateral indexes to cross country and cross region comparisons is also not covered but of course, multilateral indexes are studied in some detail in this volume.

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[14] Another fairly recent book that could be used to teach economists and economic statisticians the fundamentals of index number theory is Bert Balk's 2008 book, *Price and Quantity Index Numbers*. His book is particularly good on the early history of index number theory and on the test and stochastic approaches to index number theory. He does not cover the economic approach.