# The Fiscal Consequences of Missing an Inflation Target

Michele Andreolli and Hélène Rey*

November 2022

**Abstract**

The European Central Bank is unique in setting monetary policy for several sovereign states with heterogeneous debt levels and different maturity structures. The monetary-fiscal nexus is central to the functioning of the euro area. We focus on one particular aspect of that nexus, the effect of the reliability of the European Central Bank monetary policy on public finances. We show that when the ECB misses its inflation target this has large heterogeneous fiscal consequences for Euro Area countries.

*Michele Andreolli (London Business School) mandreolli@london.edu and Hélène Rey (London Business School, CEPR and NBER) hrey@london.edu.

# 1 Introduction

When the Euro was born, Mundell's optimal currency area theory gave a number of important insights (Mundell, 1961). A currency union would benefit from lower cross border transaction costs, in part due to lower exchange rate volatility. This would stimulate international trade (Rose, 2000) and international investment. A common currency would help complete the single market. But relinquishing monetary policy would also lead to important costs. When countries face asymmetric shocks, it would be harder for them to adjust, all the more so if labour mobility were low. The Euro Area labour mobility was not as high as in the United States due in particular to language barriers. In contrast, a non negligible amount of adjustment came from that channel in the US (Blanchard and Katz, 1992). Kenen (1969) added that the lack of fiscal transfers across countries would also be a problem. Against that view, Frankel and Rose (1998) argued in favour of the "endogeneity of optimum currency areas" making the point that a common monetary policy would lead to more symmetric demand shocks and therefore that concerns about abandoning an independent monetary policy may be overblown. In contrast Krugman (1993) pointed out that lower transaction costs in trade would lead to more sectoral specialisation across countries, mirroring what had happened in the United States. Portes and Rey (1998) emphasized the potential role of the Euro as an international currency if financial fragmentation in European bond and capital markets were to be reduced and the UK with its global financial centre were to become part of the Euro. The problem of running a single monetary policy with heterogeneous fiscal policies was thorny. The issue of moral hazard and the negative externality of the debt of one state on others was discussed and resulted in the adoption of the Maastricht criteria and later the Stability Pact, with the adoption of arbitrary numerical debt and deficit limits. Even though the importance of financial regulation was probably under-appreciated at the time, particularly the doom loop between sovereign and banking sector solvency, financial stability and the lender of last resort role of the ECB were examined and warnings issued in the first CEPR Monitoring the European Central Bank Report (Begg et al., 1998). But some topics, which turned out to be important *ex post* were never discussed at any length in the literature. Among those are the asymmetry of monetary policy transmission across Euro Area economies because of different financial and debt structures or some important aspects of the nexus between a common monetary policy and heterogeneous fiscal policies. For example, differences in the maturity of public debt have important effects on monetary policy transmission (see Andreolli, 2022). But debt of different durations also have important implications for the interactions between fiscal and monetary policies. We take up one of the dimensions of the monetary fiscal policy nexus in this paper. We start by documenting the important asymmetries in terms of debt levels and duration structures across Euro Area countries. We then show that in an environment where a credible Central Bank systematically undershoots its target for many years, heterogeneous duration structures lead to large differential fiscal effects across countries. In a context of declining real rates such as the one we had since the birth of the Euro, countries with long duration debt pay significantly higher service charges than countries with short dated debt, *ceteris paribus*. This is all the more true if the ECB, a credible central bank, persistently undershoots its target. Indeed, while short term bonds are priced using volatile expectations highly related to realized inflation, longer-dated bonds are priced using close-to-target inflation expectations. When realized inflation falls systematically short of the target for an extended period of time, countries with high debt levels and long debt duration pay a significantly higher fiscal costs than others. There are two effects which reinforce each other: a long maturity effect, which is a conscious choice in order to insure against a rise in rates - albeit a slow moving one as debt management offices, who pick debt maturity structures tend to exhibit a strong *status quo* bias in terms of debt structures; a mispricing effect due to the credibility of the Central Bank despite the persistent and one-sided deviations from target. Quantitatively they add up to several additional points of GDP in service charges and to large effects on debt accumulation -for some countries, in the order of 20 points of GDP. We highlight therefore another potential channel of divergence in the Euro Area, which is linked to heterogeneous debt durations and levels as well as persistent one-sided deviations from the inflation target. In particular, missing an inflation target persistently increases the gap between the fiscal positions of the long duration countries and those of the short duration countries. We also examine the recent period where the Central Bank overshoots its target significantly.

**Related Literature** We contribute mainly to two literatures: the optimal currency area literature, as described above, and the literature on inflation expectations and their implications for monetary-fiscal interactions.

A number of papers studied how inflation contributes to the dynamics of public debt, Hall and Sargent (2011) for the US, Ellison and Scott (2017) for the UK, Reinhart and Sbrancia (2015) for a panel of advanced and emerging economies, and Reichlin, Ricco and Tarbé (2021) for Euro Area countries. Hilscher, Raviv and Reis (2021) discussed under what conditions debt can be inflated away by combining inflation expectations embedded in option prices and the debt structure from an ex-ante perspective, we analyse the ex-post fiscal effects given inflation realisations. Jiang et al. (2020) show that across the Euro Area, convenience yields are strongly heterogeneous leading to large difference in borrowing costs, not explained by default risk and fundamentals. Wolf and Zessner-Spitzenberg (2021) show how hiking rates in a high debt world can increase default risks and therefore exacerbates the recessionary effects of the hike, which leads to heterogeneous responses of member countries to ECB monetary policy. Andreolli (2022) analyses empirically and theoretically how the maturity structure of public debt matters for the transmission of monetary policy. Furthermore, he proposes a new duration metric, Duration-to-GDP, that we use in this paper. That measure summarizes the insurance properties that long public debt gives to the fiscal authority against changes in interest rates. Hilscher, Raviv and Reis (2021) discuss how to extract risk-adjusted probabilities of large inflation surprises from option prices. Reis (2020) explores the discrepancy between household inflation expectations from surveys differ from market based inflation expectations. Coibion and Gorodnichenko (2015) and Candia, Coibion and Gorodnichenko (2022) shows how inflation expectations for households and firms are strongly related to salient prices such as the price of gasoline. Those expectations tend to be negatively correlated with GDP growth, contrarily to the survey of professional forecasters and the data unconditionally. The data seem to suggest there are therefore large deviations from the full information rational expectation (FIRE) framework. We show that long term inflation expectations of professional forecasters also exhibit large and persistent misses and that this can have large consequences for fiscal balances. Blanchard and Leigh (2013) show how persistent forecast errors can inform on the magnitude of the fiscal multiplier. Berger, Dell'Ariccia and Obstfeld (2018) analysed the quadrilemma of the Euro Area due to its particular fiscal and monetary framework. Auerbach and Obstfeld (2005) has shown in the context of liquidity traps the powerful effect of monetary policy on fiscal accounts. We are related to these themes but study the strong impact of a monetary policy anomaly (missing an inflation target) on public finances in the Euro Area.

The paper is organized as follows. Section 3 discusses how we measure inflation expectations. Section 2 introduces Duration-to-GDP to measure the insurance properties of long dated debt. Section 4 presents the main empirical results. Section 5 discusses the theoretical framework to analyse the empirical results and derives the theoretical results analytically. Section 6 shows the results of the counterfactual exercises by fitting the structural model to the data. Finally, Section 7 concludes[1].

# 2 Maturity of Debt and Public Finances: the Duration-to-GDP Measure

Debt management offices of sovereign states make the crucial decision of the maturity structure of the debt they issue. They tend to be conservative entities, who smooth the maturity structure of the debt and respond little to business cycle conditions. Issuers face the trade-off of insuring public finances against future adverse interest rates movements versus incurring higher interest costs when issuing at the longer end. Fixed-rate long-maturity debt allows issuers to lock-in an interest rate and removes uncertainty on the repayment schedule. Longer dated debt provides insurance for a longer time period. Holders of this debt however will see a change in its market value following fluctuations in interest rates. After an interest rate increase of one percentage point, the market value of one unit of debt will decline exactly by the Macaulay duration of that debt. If we take a simple case with a continuously compounded nominal interest rate $r_{t,j}$ at each maturity horizon $j$ and nominally-fixed public debt promises for each maturity $j$ being $d_{t,j}$, the market value of debt today is $D_t = \sum_{j=1}^{n} e^{-jr_{t,j}} d_{t,j}$. The Macaulay duration computes the percentage decline in the market value of debt following a one percentage point increase in interest rate equal across the yield curve $\partial r_t = \partial r_{t,j}\ \forall j$:

[1] Appendix A presents data sources and data processing steps. Appendix B presents a comparison of various long run inflation expectation measures. Appendix C presents the derivations for the analytical theoretical results. Appendix D shows the fit of the proposed geometric approximation for debt on bond-by-bond data. Appendix E presents the extension of the theoretical framework to a time varying maturity structure. Appendix F details the implementation of the counterfactual experiment and additional experiments.

$$MacDur_t = -\frac{\frac{\partial D_t}{\partial r_t}}{D_t} = \frac{\sum_{j=1}^{n} j e^{-j r_{t,j}} d_{t,j}}{\sum_{j=1}^{n} e^{-j r_{t,j}} d_{t,j}}$$

What matters for the issuer however is not the amount of insurance per unit of debt, but the amount of insurance in GDP units, which is more naturally linked to fiscal capacity. This is why we use the *Duration-to-GDP (DurGDP)* measure (proposed in Andreolli, 2022). As an example, if we take a government with public debt equal to 1 percent of GDP, it will be immaterial for debt servicing costs if debt is overnight or in consols (perpetual) bonds. In contrast, for a government with public debt of 100 % of GDP, the debt maturity will have first order effects on debt servicing costs. Following a one percent permanent increase in interest rates across the yield curve, with all debt overnight, debt servicing costs on existing debt would increase by one percent of GDP forever. On the other hand, debt service costs would not move at all on existing debt under a consol strategy. Equivalently, in the first case, the market value of public debt would remain constant at 100 percent of GDP. In the second case the market value of public debt would decline substantially, exactly by the Macaulay duration amount. After a one percent permanent increase in interest rates across the yield curve, debt duration measures *both the decline in market value of this debt and the net present value of debt servicing costs savings compared with overnight debt*[2]. *Duration-to-GDP* measures these savings not in terms of debt but in terms of debt servicing costs over GDP, which is what matters for the fiscal authority. Formally:

$$DurGDP_t = \frac{\sum_{j=1}^{n} j e^{-j r_{t,j}} d_{t,j}}{GDP_t}$$

$DurGDP_t$ is simply the product between Macaulay duration and debt to GDP.

We compiled a dataset for different European Union countries for Duration-to-GDP and for the United States and the United Kingdom for comparison purposes. Data sources and data processing steps are detailed in Appendix A. It is important to note that we are focusing

[2] Andreolli (2022) proves this equivalence formally in Proposition 1 of that paper. The key idea of the proof is a no-arbitrage argument in secondary markets and the fact that in the data public debt promises are approximately geometrically declining as maturity increases. This allows to prove the result in close form for any source of interest rate variation.

on public debt securities and not other types of public liabilities as we have maturity data for securities only. Table 1 presents average and latest values for Duration-to-GDP and its components for the countries in the dataset and Figure 1 presents the time series for a subset of countries[3].

Table 1: Duration-to-GDP and its components.

| Country | All Sample | | | Most Recent Values | | |
|---|---|---|---|---|---|---|
| | $DurGDP_t$ | $MacDur_t$ | $Debt_t$ | $DurGDP_t$ | $MacDur_t$ | $Debt_t$ |
| Austria | 4.7 | 7.3 | 63 | 7.5 | 10.3 | 73 |
| Belgium | 6.3 | 7.2 | 88 | 8.9 | 9.8 | 91 |
| Czech Republic | 2.1 | 5.9 | 27 | 2.5 | 6.5 | 39 |
| Denmark | 2.3 | 6.9 | 34 | 2.5 | 9.6 | 26 |
| Finland | 2.4 | 5.7 | 40 | 4.2 | 8.2 | 52 |
| France | 5.0 | 6.8 | 70 | 8.9 | 8.8 | 101 |
| Germany | 3.1 | 6.5 | 47 | 4.4 | 8.1 | 54 |
| Hungary | 2.8 | 4.4 | 56 | 3.6 | 5.3 | 67 |
| Ireland | 2.5 | 6.5 | 39 | 3.3 | 9.0 | 36 |
| Italy | 6.6 | 6.5 | 100 | 9.3 | 7.3 | 127 |
| Netherlands | 3.0 | 6.6 | 44 | 3.9 | 9.1 | 43 |
| Portugal | 3.6 | 5.7 | 61 | 5.5 | 7.1 | 78 |
| Spain | 4.1 | 6.4 | 61 | 8.0 | 7.9 | 102 |
| Sweden | 1.8 | 5.7 | 33 | 1.3 | 6.0 | 21 |
| United Kingdom | 6.0 | 10.0 | 55 | 12.3 | 13.3 | 93 |
| United States | 4.6 | 5.7 | 79 | 8.0 | 6.8 | 117 |

*Notes:* This table shows Duration-to-GDP in the second and fifth columns, Macaulay duration in the third and sixth columns, and public debt securities in the fourth and seventh columns. Macaulay duration source is the FTSE WGBI data and public debt securities is the OECD. Columns 2 to 4 show the average value on the 1999Q1 to 2022Q1 sample or longest available sample, whereas columns 5 to 7 show the most recent value: 2022Q1.

A few facts stand out. First, Duration-to-GDP (DurGDP) numbers are very heterogeneous across countries, both because of debt (securities) to GDP levels and because of different maturities of public debt. They range from a yearly average of 1.8% for Sweden to 6.6% for Italy for the period between 1999Q1 to 2021Q1. For Italy, this means that an unexpected decrease in interest rates by one percentage point increases the value of debt to GDP by 6.6 percent of GDP. If such a change were permanent, this would translate into higher debt servicing costs of a net present value of 6.6 percent of GDP. The effect of missing persistently on the downside the inflation target on average, with sticky long run expecta-

[3]Appendix Figure A.1 presents the time series for Duration-to-GDP and its constituents for all the countries in the dataset.

tions, which are anchored at the Central Bank target, is strongly related to Duration-to-GDP. Duration-to-GDP measures exactly the effect on the fiscal burden at the beginning of the deflationary period on legacy debt. Additionally, newly issued debt keeps being mispriced as long-run inflation expectations do not adjust; the fiscal burden of mispricing is also related to debt duration, as we prove formally in Section 5.

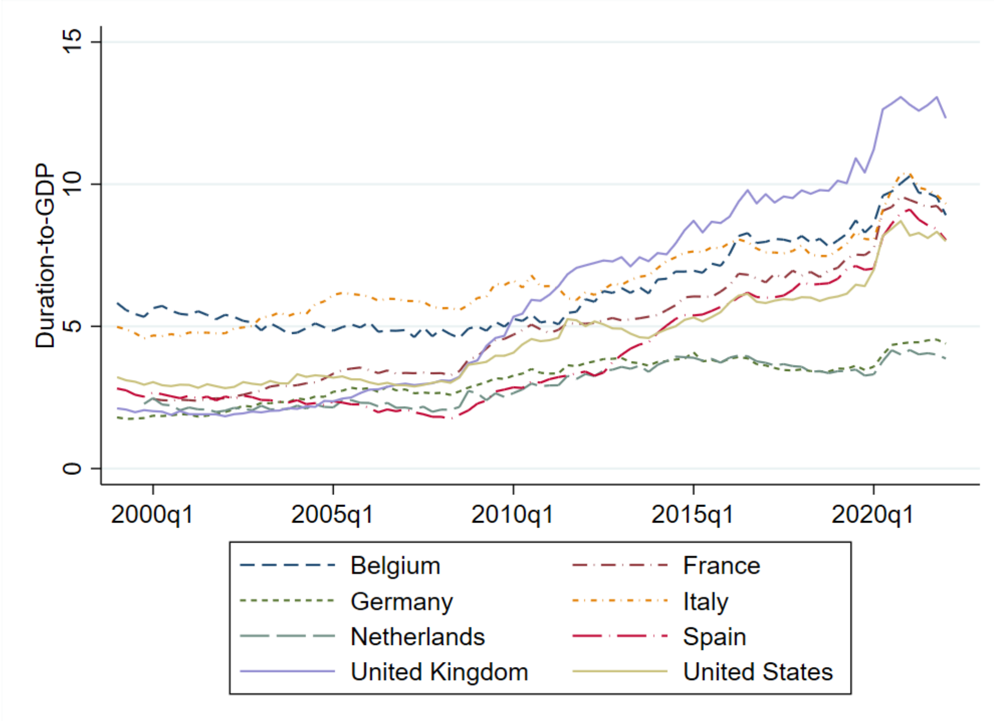
Second, recent values of DurGDP measured in 2022 Q1 are even more dispersed: Sweden's DurGDP has dropped to 1.3% while UK's is at 12.3%. For the UK Treasury, when the Bank of England undershoots the inflation target persistently it is very costly. The US has lower maturity than the big EU countries, but the relatively high debt level makes it similarly exposed to the average big European country. For Italy DurGDP has increased to 9.3%. DurGDP numbers are not that different for Austria (7.5 and Spain 8.0) despite the higher debt level of Spain (102%) compared to Austria (73%). This is because Austrian's debt duration is higher (10.3 versus 7.9 for Spain). Similarly, countries with comparable levels of debt to GDP such as France and Spain (both around 101/102 percent of GDP) can have different Duration-to-GDP: 8.8 for France and 8.0 for Spain because of different maturity structures[4].

Duration-to-GDP numbers have increased substantially in the past 20 years, especially in the recent period. This is due to both increases in debt levels and in Macaulay duration[5] as can be seen in Figure A.1. Starting from today, if interest increased permanently by one percentage point relative to current expectations, Italy will pay 9.3 percent of GDP less in net present value terms than if it had borrowed at shorter horizons, and France 8.9 percent of GDP less. There would be less variation for countries such as Germany (4.3 percent of GDP) or Finland (4.2 percent of GDP).

[4]This is an interesting comparison as if the ECB holds government debt by France and Spain in similar proportion to their GDP (or stock of debt), missing the inflation target would produce a bigger gain on debt holdings on French securities than Spanish ones. This could lead to substantial transfers across countries if all the profits were redistributed pro-rata by the capital key. In practice however most of the profits are captured by the National Treasuries via the National Central Banks and the redistribution is minimized

[5]Macaulay duration increases can also be decomposed in increases in maturity and decreases in interest rates, which have both happened on average in the last 20 years.

Figure 1: Duration-to-GDP




*Notes:* The time series presents the Duration-to-GDP metric for the largest Euro Area countries in addition to the US and the UK. Duration data is FTSE WGBI. Debt data are Government Debt Securities over GDP from the OECD. The sample goes from 1999Q1 to 2022Q1.

# 3 Measuring Long Run Inflation Expectations

We need to measure inflation expectations in the Euro Area. We can either use surveys or extract expectations from market data. The benefit of market based measures is that they embody the view of investors as reflected in actual market transactions. The downside is that one needs a model to extract inflation expectations from market data. Indeed measures of break-even inflation rates or inflation swap incorporate both inflation expectations and inflation risk premia, as well as liquidity premia. Those are extremely hard to disentangle. In contrast, survey data gives direct measures of inflation expectations. The potential drawback is that one can never be sure of the truthfulness of survey responses. We decide to use survey data in our baseline estimates as they have been shown to forecast inflation better than structural models (see Ang, Bekaert and Wei, 2007). Moreover, survey data can improve forecasting ability of structural yield curve models as argued by Kim and Orphanides (2012), Chernov and Mueller (2012). Many recent structural yield curve models directly employ survey data to improve their fit as Pennacchi (1991), Hördahl and Tristani (2018), Haubrich, Pennacchi and Ritchken (2012), Garcia and Werner (2010), Cohen, Hördahl and Xia (2018), Breach, D'Amico and Orphanides (2020). Stavrakeva and Tang (2021) show the usefulness of survey data to understand exchange rate dynamics. They also present evidence that positions taken by traders are consistent with their survey responses.

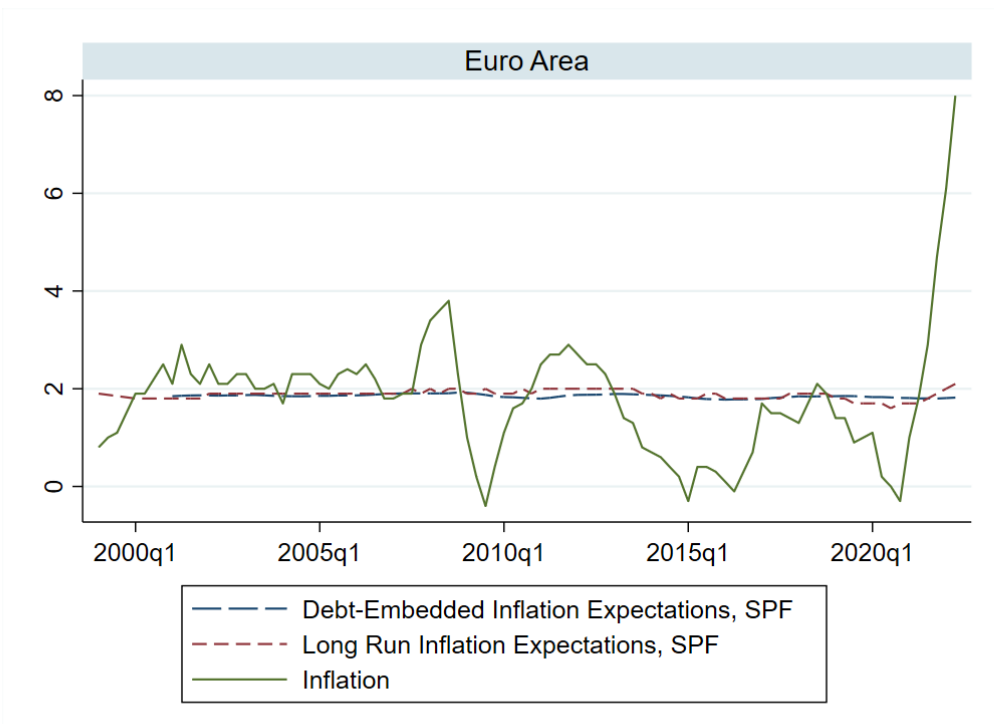
The next question is the appropriate horizon of the inflation expectations we need to use to understand the effects of missing inflation targets on government debt. What we need in our exercise are the inflation expectations embedded in current debt, that is for the debt outstanding today that was issued across all the past. Therefore, appropriate inflation expectations are the set of those prevailing $j$ periods ago for today: $\mathbb{E}_{t-j}(\pi_{t|t-j})$ for $j$ that goes from 1 to the maximum maturity of debt[6]. If all debt was one period (one quarter) debt, the relevant inflation expectations would simply be the short term ones, from the last quarter to the current one: $\mathbb{E}_{t-1}(\pi_{t|t-1})$. However, most public debt has a longer horizon than one quarter, as we showed in Section 2. Therefore, we need to do an issuance weighted average of past inflation expectations. We weight by how much debt outstanding today was issued $j$ periods ago: we call that measure "*debt-embedded inflation expectations*". Crucially, as maturity is relatively long, the most relevant inflation metric is the long run one, the one which is the most tightly linked to the inflation target.

Figure 2 shows, for the Euro Area and the period 1999Q1 to 2022Q2, the year-on-year (YoY) realized inflation of the Harmonised Consumer Price Index (HCPI), as well as long run inflation expectations (4/5 year-ahead inflation expectations on YoY HCPI from the Survey of Professional Forecasters) and our measure of debt-embedded inflation expectations. These inflation expectations are a testimony of the credibility of the ECB over that period as they hover tightly around 1.9% (the ECB target was close to but below 2% over most of that period). Moreover, they show that, quantitatively, the inflation expectation embedded in bond prices is chiefly the long run inflation expectation, or the inflation target. Realized inflation however is below target for a large part of the post 2008 period. The recent data show an uptick in expectations and a large increase in realised inflation. Figure 3 presents the same data for the US for comparison purposes. Inflation expectations there are also very well anchored during the period slightly above 2%, with, again, an uptick in the recent

[6] We define the cumulative inflation rate as $\pi_{t|t-j} \equiv \frac{P_t}{P_{t-j}}$, where $P_t$ is the appropriate price level.

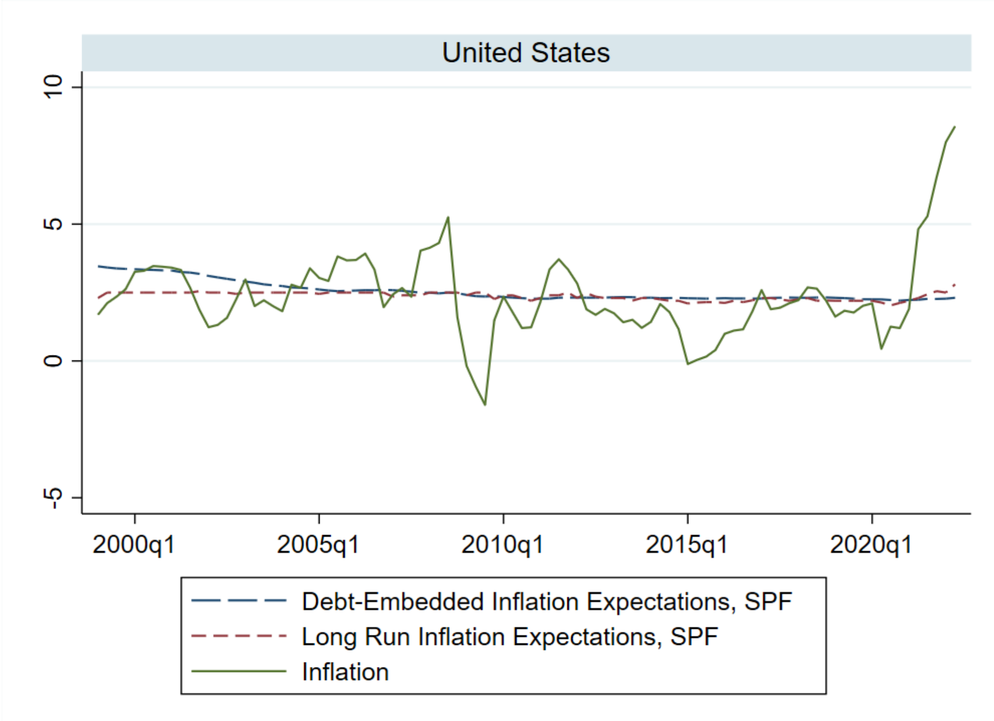
data. Appendix B presents the full description of the various inflation expectations and how we exactly construct inflation expectations embedded in debt. In Figures B.1 and B.2, we compare the long run inflation expectations from the SPF with alternative survey and market based metrics for long run inflation expectations. Overall, the market based ones are more volatile than the survey ones but still are centred around 2%. In Figure B.3, we add to the baseline figure, short run inflation expectations in SPF data. Short run expectations track quite closely realized inflation. We can see the whole set of inflation expectations at each horizon in the SPF in Figure B.4. The same pattern emerges, the shorter the horizon of the expectations, the closer they are to realized inflation, and as the horizon lengthens they approach the long run expectations. Figure B.5 compares inflation expectations embedded in debt from various sources. Overall they are all quite similar and this leads us to conclude that our results are not sensitive to using survey based expectations.

Figure 2: Inflation Expectations and Inflation in the Euro Area




*Notes:* This graph presents long run inflation expectations from the ECB Survey of Professional Forecasters (SPF) at 4/5 years ahead, the inflation expectations embedded on outstanding public debt at issuance, and realised inflation. Inflation and its expectations pertain to the Harmonised Index of Consumer Prices (HICP). The sample goes from 1999Q1 to 2022Q2.

Figure 3: Inflation Expectations and Inflation in the United States




*Notes:* This graph presents long run inflation expectations from the Philadelphia Fed Survey of Professional Forecasters (SPF), the inflation expectations embedded on outstanding public debt at issuance, and realised inflation. Inflation and its expectations pertain to the headline Consumer Price Index (Headline CPI). The sample goes from 1999Q1 to 2022Q2.

# 4 Empirical Results

## 4.1 Missing One's Inflation Targets

For a large part of the post 2008 period, the ECB and the Fed systematically undershot their inflation targets. Before the current post-Covid burst, inflation has been below long run inflation expectations for 75% of the time (from 2009q1 to 2020q1) in the Euro Area and 84% in the United States. This inflation anomaly was not only on the extensive margin, but also substantial as far as the intensive margin is concerned: inflation averaged 1.25% in the Euro Area and 1.59% in the United States compared with a long run inflation expectations of 1.88% (and debt-embedded expectations of 1.84% against a target of 1.9%) and 2.27% (and debt-embedded expectations of 2.31% against a target of 2%) respectively. If we include the recent period (up to 2022q2) average inflation is still below long run inflation expectations, at 1.50% vs 1.89% for the Euro Area and 2.03% vs 2.28% in the United States.

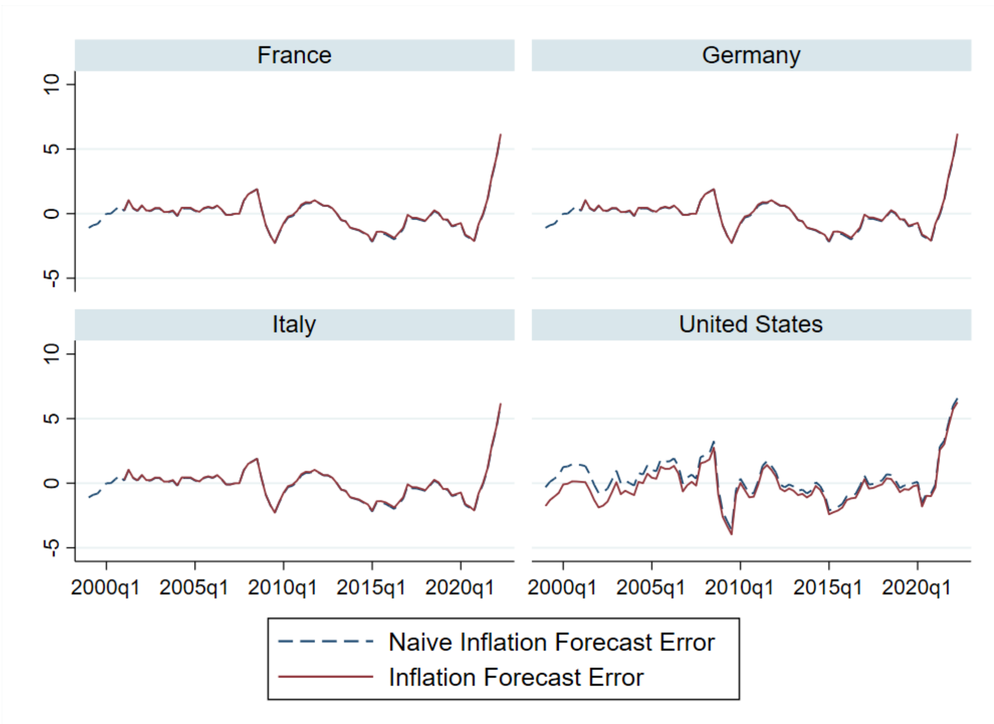
As what matters for our exercise are inflation expectations embedded in public debt, in Figure 4 we show the time series of the inflation forecast errors for France, Germany, Italy, and the United States. We use Euro Area wide HICP inflation for the European countries and assume it is a Euro Area-wide trader who is pricing the bonds. The debt embedded inflation expectation numbers could still differ across countries, as each country has a different maturity of its public debt, and therefore weights short and long run inflation expectations differently. However, given that debt is sufficiently long in all these countries, what matter, in each one of them, are the long run inflation expectations. Given the stability of long run expectations, we also show the forecast error on debt from a naive forecast, when all debt is priced with an inflation expectation of 1.9% in the Euro Area and 2% in the US. In the Euro Area countries we can see almost no difference vis-à-vis the forecast embedded in debt, again highlighting the strong credibility of the ECB target[7]. The US numbers are different from the European ones as the US has a different rate of inflation and of inflation expectations. However, the same overall pattern of persistent misses of the inflation target arises, albeit with a lower magnitude.

The main take-away is that inflation was below target for a long time and was below embedded long term inflation expectations for a long time. However, short term inflation expectations tracked inflation much better, consistent with the idea that the causes of inflation at a short frequency are relatively better understood than at a longer frequency. In Appendix Table B.1, we show that in the long dis-inflationary period from 2009q1 to 2020q1, inflation forecast errors embedded in debt were persistently negative at $-0.59$ and $-0.71$ percentage points for the Euro Area and for the United States respectively, and statistically different from zero. On the other hand, short run forecasts have been much closer to zero at $-0.04$ and $-0.13$ percentage points and not statistically different from zero.

In this section, we showed that the ECB and the Fed missed their inflation target for more than a decade and at the same time that expectations embedded in public debt have remained stable and close to their respective target. Next, we analyse what this implies for fiscal balances in Euro Area countries and in the US.

[7]In Appendix Figure B.6, we show that the same pattern of forecast errors holds for all the Euro Area countries in the dataset.

Figure 4: Inflation Forecasts Errors




*Notes:* This graph presents the forecast error on inflation expectations embedded on outstanding public debt at issuance. Inflation expectations come from the ECB (for the Euro Area on HICP inflation) and Philadelphia Fed (for the US on Headline CPI) Surveys of Professional Forecasters (SPF). The naive approach assumes an inflation expectation of 1.9% for the Euro Area and 2% for the US. The vertical axis units are percentage points. The sample goes from 1999Q1 to 2022Q2.

## 4.2 Fiscal Consequences of Missing Inflation Targets

In what follows, we take as given public debt amounts, maturity choices, and risk premia on public debt. We study the impact on debt burdens of a persistently too low inflation rate with strongly anchored inflation expectations, in other words we ask how much it costs to miss the inflation target persistently. The key difference across countries is the amount and the maturity of public debt outstanding.

To analyse the fiscal consequences, we conjure up the systematic forecasts errors in the debt-embedded inflation expectations, which we just calculated and our Duration-to-GDP measures, which give us the net present value of debt servicing cost increases compared to overnight debt, as a fraction of GDP, when interest rates go down by 1% permanently across the yield curve (or equivalently the increase in market value of the debt as a fraction of GDP). Nominal interest rates increase one for one with inflation expectations, keeping risk and liquidity premia constant. Hence a systematic positive mistake of size $\epsilon_t$ on embedded inflation expectations at date t (corresponding to a Central Bank undershooting its credible target) leads to an overestimate of long term nominal rates of the same amount. This has two effects on the fiscal burden. First, it increases the debt burden compared to correctly priced short debt on legacy debt, and this effect would happen even if long run expectations adjusted in the future. Moreover, it affects the pricing of newly issued debt, which do not incorporate the new lower inflation it its interest rates. This gives rise to a fiscal cost proportional to $\epsilon_t$ (measuring the extent to which interest rates are unexpectedly higher than warranted) multiplied by Duration-to-GDP. In Figure 5, we show the time series of these fiscal costs for France, Germany, Italy, and the United States[8]. We used forecast errors on inflation expectations embedded in debt as well as naive inflation expectations. At each point in time, this figure shows the net present value of debt servicing costs with the current debt profile compared with correctly priced short term debt in GDP units, if the decline in inflation compared to the expectation embedded in debt was permanent. Since short term debt is priced using approximately correct inflation expectations, we measure at each point in time the fiscal cost of the "mispricing" of longer dated debt, which is also proportional to the fiscal cost of missing the inflation target.

[8] In Appendix Figure B.7, we show the same figure for the other countries in the dataset and confirm the patterns we highlight for the biggest countries.

A few remarks are in order. First, if the misses had been temporary they would not have had large fiscal consequences, even with high inflation volatility. Long maturities help to smooth out variations in inflation from one period to the next. Moreover, if inflation expectations were right on average, misses would have mattered much less for fiscal balances, as the long low inflation regime would have had an effect only on the debt stock present at the beginning of the sample but not on all debt issued during in sample. However, the misses were not temporary, we had persistent, almost always one sided forecast errors for more than a decade, because of a persistent low inflation that was continuously expected to quickly revert to target. The second remark is that the numbers in Figure 5 are large in some periods, because of high debt, high maturity, and large misses; in 2020q4 in France the number reached more than 20% of GDP and was even higher in Italy and Belgium. This can be a first order phenomenon for public debt dynamics. Third, there is considerable heterogeneity across time and across countries. Across countries, numbers are lower for low debt and low maturity economies, such as Germany or the Netherlands compared to countries like Austria, Belgium, or Italy. Moreover, there is a large time variation, in the financial crisis period the numbers were lower for two reasons, first inflation misses were more modest (see Figures 4 and B.6), but also these misses mattered less as the level of public debt was generally lower and debt maturities were shorter(see Figures 1 and A.1).

In order to get a sense of the magnitude that this had on public debt since the creation of the Euro, we average out these numbers to see how persistent the misses were and the resulting fiscal costs with long term debt. For the 2000q1 to 2021q1 sample, Table 2 shows the average yearly cost with its heteroskedasticity robust standard error for debt embedded inflation expectations (columns 2 and 3) and for naive inflation expectations (columns 4 and 5). These numbers are large, positive, and almost all statistically different from zero[9]. On average, governments have been paying substantially more in ex-post real terms than they could have if they had borrowed with correctly priced short debt. The average value across all countries is 1.66% of GDP yearly, which is very large. This means that countries have been paying 1.66% of GDP more per year than they could have if they had borrowed with short debt with inflation expectations on average correct (the calculation assumes misses are permanent and fixes risk, term, and liquidy premia).

[9]The p-value is below 0.005 in all cases except in the naive forecast for the United States.

The large average number hides a considerable heterogeneity. Missing the inflation target with long debt has been costliest (a yearly 2.26% of GDP) for countries such as Italy or Belgium that have both high debt and high maturity and much lower for countries with low debt and low maturity as Germany of Finland (about 1 % of GDP). Note that the heterogeneity does not cut a simple North-South divide or high debt vs low debt countries. A country like Austria, with relatively low debt but high maturity has been hit more than Portugal, as Portugal had shorter maturity debt than Austria.

A key question is how the recent increase in inflation has changed the result. Inflation expectations tell us that the recent inflation surge was largely unexpected by forecasters and market participants. This means that bonds were priced with a much lower inflation expectation. Ex-post governments are insured against an interest rate increase and investors are facing large losses on their sovereign bond portfolios. If the inflation surge is high and persistent enough it could undo the higher fiscal costs of the previous decade. Has it? In Table, 3 covering the period 2000Q1 to 2022Q1, we can see that, so far, it has not. All the cost numbers are still positive albeit with a smaller magnitude and with lower statistical significance. The costs are still sizeable, with an average of 0.87% of GDP per year and in Italy or Belgium costs are still higher than 1.2% of GDP, which is far from negligible.

Additionally, in Appendix B and Appendix Table B.2 we show that the result on the fiscal cost of missing the inflation target is similar if we employ surveys, term structure models, or market based metrics directly to measure debt-embedded inflation expectations on US data[10]. This assuages the concern that the results are driven by the specific choice of inflation expectations.

We established that when credible Central Banks miss their inflation targets consistently for many periods there are heterogeneous and large fiscal costs. Why should we care? We provide a number of reasons which hold for any country and some which are specific to the Euro Area, with one monetary policy and several, fiscal and debt management authorities.

First, in a benchmark world where the Ricardian equivalence holds, these costs should not matter even in the presence of mispricing. Due to persistently low inflation and anchored inflation expectations, governments have sustained higher costs than expected in the past decade but at the same time bond holders have gained persistently on their bond portfolio.

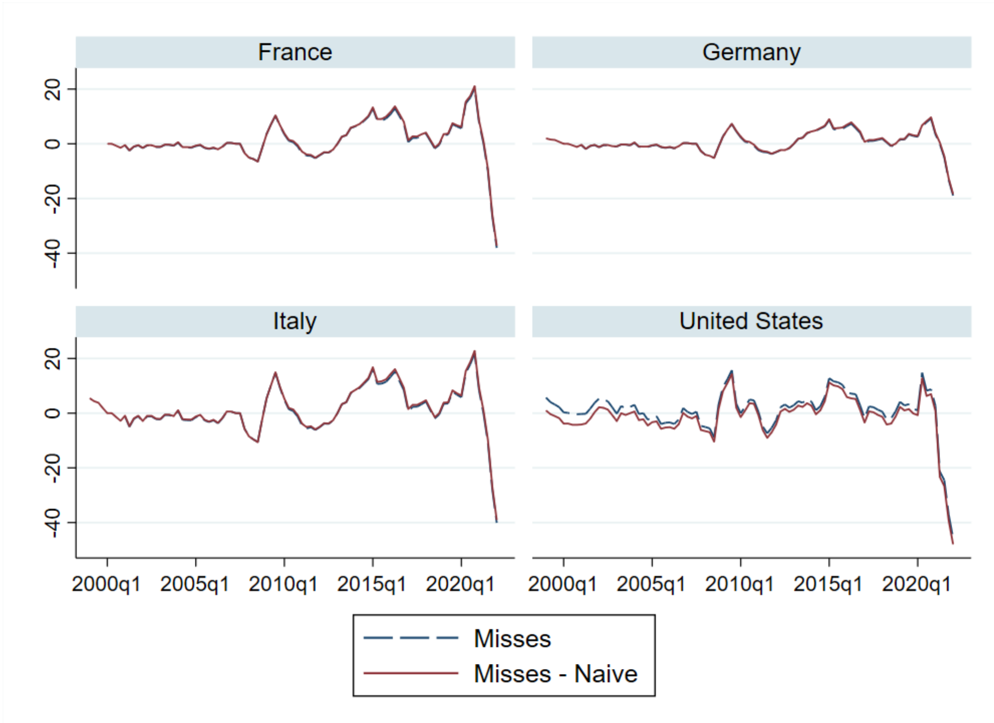
[10] For the Euro Area we do not have time series long enough for the alternative metrics.

The higher gains in the bond portfolio are exactly matched with higher taxation in net present value. With an infinitely lived representative agent, the bond gains and taxation losses accrue to the same agent, and with lump sum taxes there is no distortion from taxation. However, when Ricardian equivalence does not hold, gains and losses can accrue to different agents and have redistributive consequences. The losses due to lower inflation than expected accrue to the government, but only bondholders gain from these; in the United States, these are mainly rich older households (see Doepke and Schneider, 2006) and foreigners. In this paper, we focus on government debt, but if we look at the broader economy we can imagine even stronger redistributive impacts of missing persistently the inflation target. In the United States, the younger middle class who hold mortgages lose from lower inflation for example. At the same time, the government is a net loser from low unexpected inflation. Persistent inflation target misses in the past decade can be one of the contributors of the increase in inequality post-financial crisis. Furthermore, as the United States has relatively more fixed debt liabilities and more equity and FDI assets the foreigners tend to gain relatively from the misses in the inflation target (see Gourinchas and Rey, 2007), holding the exchange rate fixed.

In the Euro Area, all the distributional consequences of heterogeneity in the investor base holds as well. However, there are additional reasons why the misses have been consequential. First of all, the heterogeneous debt levels and maturities have generated heterogeneous fiscal losses across countries for the same monetary policy. If fiscal policy directly responds to debt levels due to the Stability and Growth Pact, the misses may constitute an amplification mechanism and, in some cases lead to austerity, for example. For a given level of debt, if the country had a higher maturity, the misses would have increased debt burdens due to ex-post higher real rates. Furthermore, public debt level and maturity can further affect the strength of the transmission of monetary policy due to different roll over needs, and this effect plays out in particular through real activity as shown by Andreolli (2022).

Finally, as explained before, Duration-to-GDP measures the net present value of interest rate servicing costs of long debt compared with correctly priced short debt in the case where inflation misses are permanent. Inflation misses were high and persistent, but not permanent. Moreover, the present exercise bundles together the effect of expectations with maturity. Therefore, we now fit a structural model to highlight and unbundle the key theo-

Figure 5: Fiscal Consequences of Missing Inflation Targets




*Notes:* This figure shows the fiscal costs of missing the inflation targets on public debt. The "Misses" lines show the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with SPF, OECD, and FTSE WGBI data. The "Misses - Naive" show the same multiplication, but with the naive inflation forecast. The vertical axis units are percent of GDP. A positive number implies that inflation is below its forecast and a negative number that inflation is above its forecast. The sample goes from 1999Q1 to 2022Q1.

retical mechanisms and to do a counterfactual analysis in the next sections.

# 5 Theoretical Framework

To better understand the monetary fiscal nexus we have been documenting, we build a partial equilibrium model enabling us to keep track of the costs of missing the inflation target and highlighting the role of maturity and inflation expectations. The law of motion of public debt that we set up has a good fit with actual public debt promises in the data. At the same time, we need to keep track only of a small number of state variables. This simplicity allows us to prove a number of results that guide the interpretation of our empirical exercises.

Table 2: Fiscal Consequences of Missing Inflation Targets - Up to the Inflation Increase

| Country | Cost | | Naive Cost | |
|---|---|---|---|---|
| | Mean | SE | Mean | SE |
| France | 2.16 | (0.61)*** | 2.43 | (0.63)*** |
| Germany | 1.06 | (0.36)*** | 1.23 | (0.37)** |
| Italy | 2.26 | (0.76)*** | 2.61 | (0.78)** |
| Belgium | 2.26 | (0.72)*** | 2.54 | (0.73)*** |
| Spain | 2.05 | (0.52)*** | 2.29 | (0.54)*** |
| Netherlands | 1.08 | (0.34)*** | 1.24 | (0.35)*** |
| Austria | 1.80 | (0.55)*** | 2.01 | (0.56)*** |
| Portugal | 1.31 | (0.42)*** | 1.54 | (0.43)*** |
| Ireland | 1.04 | (0.30)*** | 1.19 | (0.31)*** |
| Finland | 1.01 | (0.28)*** | 1.17 | (0.29)*** |
| United States | 2.23 | (0.54)*** | 0.35 | (0.55) |
| Average | 1.66 | (0.16)*** | 1.69 | (0.16)*** |

*Notes:* This table shows the fiscal costs missing the inflation targets on public debt. The second and third column show the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with SPF, OECD, and FTSE WGBI data. The fourth and fifth columns show the same multiplication, but with the naive inflation forecast. The second and fourth column show the average cost in GDP units. The third and fifth columns show the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. Each row shows a different country, except the last row which shows the simple unweighted average across all countries. The sample goes from 2001Q1 to 2021Q1. Legend: * p<.1; ** p<.05; *** p<.01.

## 5.1 Model Set-up

Public debt is issued via a bond with a geometrically decaying amortization. That bond pays a fixed net nominal interest rate $R_t^{new}$ on new issuances. The principal due decays at rate $\delta^d$. In each period the government issues $L_t$ nominal bonds, therefore the end of period stock of debt in each period can be written as the sum of remaining past issuances, which allows a recursive formulation. This specification also allows a recursive formulation for the average interest rate process on the debt stock[11]. If we define $F_t$ to be the debt payments on bonds

[11]Notice that all stock variables are defined as end-of-period variables, so that $D_t$ is the stock of bonds reflecting the choice $t$, whose new issuances will start at $t+1$. Interest rates are all defined in net terms rather than gross terms.

Table 3: Fiscal Consequences of Missing Inflation Targets - With the Recent Inflation Increase

| Country | Cost | | Naive Cost | |
|---|---|---|---|---|
| | Mean | SE | Mean | SE |
| France | 1.18 | (0.83) | 1.47 | (0.84)* |
| Germany | 0.57 | (0.46) | 0.76 | (0.46) |
| Italy | 1.23 | (0.95) | 1.61 | (0.96)* |
| Belgium | 1.27 | (0.91) | 1.56 | (0.92)* |
| Spain | 1.15 | (0.73) | 1.42 | (0.74)* |
| Netherlands | 0.64 | (0.42) | 0.82 | (0.42)* |
| Austria | 0.98 | (0.73) | 1.20 | (0.73) |
| Portugal | 0.70 | (0.55) | 0.94 | (0.55)* |
| Ireland | 0.66 | (0.36)* | 0.82 | (0.37)** |
| Finland | 0.54 | (0.39) | 0.71 | (0.40)* |
| United States | 0.61 | (0.98) | -1.28 | (0.98) |
| Average | 0.87 | (0.21)*** | 0.91 | (0.21)*** |

*Notes:* This table shows the fiscal costs missing the inflation targets on public debt. The second and third column show the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with SPF, OECD, and FTSE WGBI data. The fourth and fifth columns show the same multiplication, but with the naive inflation forecast. The second and fourth column show the average cost in GDP units. The third and fifth columns show the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. Each row shows a different country, except the last row which shows the simple unweighted average across all countries. The sample goes from 2001Q1 to 2022Q1. Legend: * p<.1; ** p<.05; *** p<.01.

in period $t$ we can characterize the debt dynamics system with three equations:

$$
\begin{aligned}
D_t &= L_t + (1-\delta^d)D_{t-1} \\
R_t^{ave} &= R_t^{new}\frac{L_t}{D_t} + R_{t-1}^{ave}\left(1-\frac{L_t}{D_t}\right) \\
F_t &= (R_{t-1}^{ave} + \delta^d)D_{t-1}
\end{aligned}
$$

This is a parsimonious system for the government debt dynamics. Crucially, we need to keep track only of two state variables: the debt stock and the average interest rate on debt. This is useful as it allows to prove results in close form and is not subject to the curse of dimensionality. Moreover, we do not lose much in terms of fitting actual debt promises: in Appendix D, we show that the approximation has a good fit on actual bond-by-bond data in the US.

We can rescale debt quantity variables in percent of GDP terms, with lower case letters being the value in GDP units $x_t \equiv \frac{X_t}{P_t Y_t}$ where $P_t$ is the aggregate price level for consumption goods, $Y_t$ is real GDP. Inflation is defined as $\pi_t \equiv \frac{P_t}{P_{t-1}}$ and real GDP growth as $g_t \equiv \frac{Y_t}{Y_{t-1}}$. The system becomes:

$$f_t = (R_{t-1}^{ave} + \delta^d)\frac{1}{g_t \pi_t} d_{t-1} \tag{1}$$

$$d_t = (1 - \delta^d)\frac{1}{g_t \pi_t} d_{t-1} + l_t \tag{2}$$

$$R_t^{ave} = \left(1 - \frac{l_t}{d_t}\right) R_{t-1}^{ave} + \frac{l_t}{d_t} R_t^{new} \tag{3}$$

This specification links the duration to $\delta^d$ and to $R_t^{new}$. To see this we can compute the Macaulay duration for any issuance:

$$Duration = \frac{\sum_{j=1}^{\infty} j \frac{(\delta^d + R_t^{new})(1-\delta^d)^{j-1}}{(1+R_t^{new})^j} L_t}{\sum_{j=1}^{\infty} \frac{(\delta^d + R_t^{new})(1-\delta^d)^{j-1}}{(1+R_t^{new})^j} L_t}$$

$$Duration = \frac{1 + R_t^{new}}{\delta^d + R_t^{new}}$$

We call $S_t$ the primary surplus, which we take as exogenous[12]. The funding needs of the government can be covered either by $S_t$ or by the issuance of new bonds :

$$F_t = S_t + L_t$$

In GDP units:

$$f_t = s_t + l_t \tag{4}$$

We make the simplifying assumption that risk neutral investors, with discount rate $\beta$,

[12]We can easily allow for non-bond funding by the government in this model, such as cash and non-marketable debt, and in that case we can redefine $S_t$ as the net resource needs the government uses to cover bond payments.

price long dated public debt. The Euler equation is then:

$$\frac{1}{(\delta^d + R_t^{new})} = \mathbb{E}_t\left(\frac{\beta}{\pi_{t+1}}\right) + \mathbb{E}_t\left[\beta\frac{1}{\pi_{t+1}}(1-\delta^d)\frac{1}{(\delta^d + R_{t+1}^{new})}\right]$$

If we iterate forward we can see that the price of long bonds is a weighted average of inflation expectations, with weights on long term expectations which increase with the duration of debt.

$$\frac{1}{(\delta^d + R_t^{new})} = \mathbb{E}_t\sum_{j=1}^{\infty}\left[\frac{1}{\pi_{t+j|t}}\beta^j(1-\delta^d)^{j-1}\right]$$

Where $\pi_{t+j|t}$ is $\frac{P_{t+j}}{P_t}$, the cumulative inflation up to period $j$[13].

One can show that the market value of public debt $q_t$ is simply a function of the interest rate on newly issued debt and average interest payments:

$$q_t = \frac{\delta^d + R_t^{ave}}{\delta^d + R_t^{new}}$$

In our partial equilibrium approach, the process of inflation is exogenous, as well as the process for inflation expectations: $\mathbb{E}_t[1/\pi_{t+j|t}]$. As stated above, we also assume for now that the net bond financing need/primary surplus is exogenous. Given initial conditions for the state variables $d_{-1}$ and $R_{-1}^{ave}$, the model has 6 endogenous variables $\{R_t^{new}, R_t^{ave}, l_t, f_t, d_t, q_t\}$

[13] We also decompose the cumulative inflation in its one period head components: $\pi_{t+j|t} = \prod_{k=1}^{j}\pi_{t+k}$.

and corresponding equations:

$$
\begin{aligned}
f_t &= (R^{ave}_{t-1} + \delta^d)\frac{1}{g_t\pi_t}d_{t-1} \\
d_t &= (1-\delta^d)\frac{1}{g_t\pi_t}d_{t-1} + l_t \\
R^{ave}_t &= \left(1 - \frac{l_t}{d_t}\right)R^{ave}_{t-1} + \frac{l_t}{d_t}R^{new}_t \\
f_t &= s_t + l_t \\
\frac{1}{(\delta^d + R^{new}_t)} &= \mathbb{E}_t \sum_{j=1}^{\infty}\left[\frac{1}{\pi_{t+j|t}}\beta^j(1-\delta^d)^{j-1}\right] \\
q_t &= \frac{\delta^d + R^{ave}_t}{\delta^d + R^{new}_t}
\end{aligned}
$$

We note that inflation expectations matter for bond pricing directly, while for the path of real debt only realized inflation matters.

## 5.2 Theoretical Results

Before fitting the model to actual data, we highlight how the debt structure and inflation expectations affect fiscal burdens in this simplified theoretical setting. We study an experiment which consists in a long lasting deflationary shock which gets incorporated in short run inflation expectations, but not in long run ones. Starting at period $t$ we have a change in inflation from period $t+1$ onward. One period-ahead inflation expectations adjust, starting from expectation in $t$ $\mathbb{E}_t(\pi_{t+1})$) and onwards ($\mathbb{E}_{t+j}(\pi_{t+j+1})$, $j \geq 1$). But they do not for horizons larger than 1, as they are pinned down by the inflation target. The debt structure matters because legacy debt has already been priced before any variation in inflation and inflation expectations and because long run expectations, which do not adjust, also affect the cost of newly issued debt. Let us assume that we issue new debt only to refinance the debt coming due: $L_{t+j} = \delta D_t$. [14] We assume that the transversality condition on debt holds, that is, we solve for an economy where $1 + R > \pi g$ as this simplifies the algebra and allows us to provide some economic intuition. However, the results do not hinge on this assumption. In

[14] In addition, we perform another exercise in Appendix C.1, where we assume the debt-to-GDP level to be constant at its steady state value, with the net resource needs/primary surpluses $s_t$ that adjust each period. That other experiment is akin to an extreme debt stabilization program.

fact, in the computational exercise in Section 6, we look at the impact on public debt at some point in the future under various scenarios and do not impose any transversality conditions. In particular, interest rates can be below the nominal growth rate of the economy as we iterate the budget constraint from a starting level of debt to a terminal one in finite time. Moreover, whereas here we make the stark assumption that one period ahead expectations adjust, but from two periods ahead onward they do not, in the computation section we allow for a more realistic term structure of inflation expectations.

#### 5.2.1 Fiscal burden, parallel derivatives and blips

We define the fiscal burden as the net present value of real primary surpluses/net resource needs $Y_t s_t$ evaluated with risk neutral utility, the same utility we use to price debt.

$$\sum_{j=1}^{\infty} \beta^j Y_{t+j} s_{t+j} \propto \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}$$

Under rational expectations pricing, the expectation of the fiscal burden is the market value of debt. We show our key results starting from the steady state where steady state variables do not have subscripts[15]. We define *parallel shifts or parallel derivatives* in future variables in the following way:

$$\frac{\partial y}{\{\partial x_{t+j}\}_{j=l}^{m}} \equiv \sum_{j=l}^{m} \frac{\partial y}{\partial x_{t+j}}$$

For any $y$ and $x$ variables[16].

**Result 1: Parallel shifts versus blips in expectations: effect on new issuances**

Our first result is that, if all inflation expectations long and short move (parallel shifts),

[15] In steady state interest rates on newly issued debt is equal to the rate on average debt so we simply write: $R = R^{new} = R^{ave}$.

[16] Formally, this "parallel" derivative is equivalent to a partial derivative to a latent variable $\chi$ which affects one to one all $x_{t+j}$ with $j = l, \ldots, m$, where then we use the chain rule on $y$: $\frac{\partial y}{\partial \chi} = \sum_{j=l}^{m} \frac{\partial y}{\partial x_{t+j}} \frac{\partial x_{t+j}}{\partial \chi} = \sum_{j=l}^{m} \frac{\partial y}{\partial x_{t+j}}$

then interest rates on newly issued debt will increase one to one[17]. But if only short run expectations move (blips), interest rates will increase by one over duration:

$$\frac{\partial R_{t+l}^{new}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}} = \frac{1+R}{\pi}$$
$$\frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} = \frac{1+R}{\pi}\frac{1}{\frac{1+R}{\delta^d+R}}$$

But maturity does not only matter for the pricing of newly issued bonds as we just showed. It also matters for the effective debt burden of legacy debt. If all debt is short term, it will be repriced in the next period, but if all legacy debt was priced with a 2% inflation expectation, its cost will linger on, even if newly issued debt is correctly priced. To see this, we proceed in steps. Let us show the impact of a parallel change in interest rates on newly issued debt on the average interest rates at various future horizons when the government only refinances the debt due, that is when $L_{t+j} = \delta D_t$.

$$\frac{\partial R_{t+k}^{ave}}{\{\partial R_{t+l}^{new}\}_{l=0}^{k}} = 1 - \left(1-\delta^d\right)^{k+1}$$

With long debt (low $\delta^d$), it takes a while for interest rates on newly issued debt to be incorporated on the average debt burden. With short debt ($\delta^d$ close to 1), newly issued debt is equivalent to average debt, so the parallel shift affects one to one right away the debt burden. We can use this result to study the effect of a change in interest rates on newly issued debt on the fiscal burden.

**Result 2: Effect of changes in interest rate on the fiscal burden**

$$\begin{aligned}\frac{\partial \sum_{j=1}^{\infty}\beta^j\prod_{k=1}^{j}g_{t+k}s_{t+j}}{\{\partial R_{t+l}^{new}\}_{l=0}^{\infty}} &= \sum_{j=1}^{\infty}\beta^j\prod_{k=1}^{j}g_{t+k}\frac{\partial s_{t+j}}{\partial R_{t+j-1}^{ave}}\frac{\partial R_{t+j-1}^{ave}}{\{\partial R_{t+l}^{new}\}_{l=0}^{k}}\\ &= D\left(\frac{1}{R}-\frac{1+R}{\delta^d+R}\frac{1-\delta^d}{1+R}\right)\end{aligned}$$

The first term of this expression $D$ simply scales the change by the steady state values of

[17]We present a detailed step-by-step derivation of each result we show in this section in Appendix C.

the fiscal burden. The first term in parentheses shows the permanent effect of a one percent increase in the debt burden, and the last term shows how much lower this will be if debt is longer maturity. Crucially, the first fraction of the second term is duration. The second term is simply a scaling factor that arises as we are looking at an announced shock that will happen in the following period.

We now combine the effect of a change in inflation expectations on interest rates, with the effect of the change in interest rates on the fiscal burden, to find the effect of inflation expectations on the fiscal burden. We consider two cases, in the first, only inflation expectations on next period adjust (blip) and in the second all future expectations adjust (parallel shift).

**Result 3: Effect of changes in inflation expectations (blips and parallel shifts) on the fiscal burden**

$$
\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} \\
&= \frac{D}{\pi} \frac{1+R}{R} \delta^d \\
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}} \\
&= \frac{D}{\pi} \frac{1+R}{R} \delta^d \frac{1+R}{\delta^d + R}
\end{aligned}
$$

If only short run expectations move, the effect on the debt burden of their increase is close to zero with long debt ($\delta^d$ small); not only legacy debt is insured, but also new debt does not change in cost. But if all expectations move, then only legacy debt is insured against the hike and new debt will slowly embed the new expectations.

In order to conclude our discussion, we now turn to the effect of inflation, holding inflation expectations constant, on the debt burden. In this case, nominal interest rates are fixed as what matters for newly issued rates are inflation expectations. As the government refinances the maturing debt, then also average interest rates do not depend directly on inflation. As interest rates are not affected by inflation, maturity also does not play a role and the overall effect on the fiscal burden is:

**Result 4: Effect of changes in inflation on the fiscal burden**

$$\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} = -\frac{D}{\pi}\frac{1+R}{R}$$

Hence the effect of inflation on the fiscal burden is larger the larger the debt stock, and as it is permanent, it will affect the whole future stream of surpluses.

We now turn to the main results of the theoretical exercise: how the debt burden varies under various maturity profiles when both inflation and expectations shocks hit the economy. We show below the effect when only short run inflation expectations move (blip) and then when all move (parallel shift):

**Result 5: Effect of changes in inflation and inflation expectations on the fiscal burden, blips and parallel shifts**

$$\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} &\equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} \\
&= -\frac{D}{\pi}\frac{1+R}{\delta^d + R}(1-\delta^d)\frac{R+\delta^d}{R} \\
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}} &\equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})} \\
&= -\frac{D}{\pi}\frac{1+R}{\delta^d + R}(1-\delta^d)
\end{aligned}$$

When debt is short ($\delta^d = 1$), the effect is always zero, there is no variation in debt burdens. After all, if there is an announced inflation tomorrow and short rates adjust, there will be zero effect of inflation in the case of short term debt. Moreover, as long as short run expectations are correct on average, as they generally are in the data, it will not matter that the central bank misses the inflation target. This underlies the reasoning in standard macroeconomic models with only short dated debt. Missing the inflation target can have repercussion on central bank credibility but is second order for fiscal issues. The picture completely changes when we look at long dated debt. There, even if all expectations adjust there will be an effect of a permanent shock to inflation, and this is *Duration-to-GDP*[18].

[18] As an example, if interest rates are 5% annualized, $R = 0.05/4$, with 6.75 years debt duration in a quarterly model $\delta^d$ equals 0.025, so that $1 - \delta^d$ is quite close to one.

When long run inflation expectations do not adjust, then the effect of the permanent shock is even higher, Duration-to-GDP gets multiplied by $\frac{R+\delta^d}{R}$[19]. These results are summarised in Table 4. Note that the results do not hinge on looking only at legacy debt, imposing $L_{t+j} = \delta D_t$. In Appendix C.1, we show that the same pattern arises under an alternative experiment, where the government keeps a constant debt-to-GDP ratio by adjusting primary surpluses. In Table C.1, we can see that the variation in the fiscal burden under short debt is zero and the same pattern arises with long debt: if all expectation adjust the variation is approximately Duration-to-GDP and if only short expectations adjust we also have a large (even larger!) multiplier. Finally, if public debt were all inflation-linked, then in that case there would not have been any effect on the fiscal burden of the exercises we study here.

Table 4: Inflation, Maturity, and the Fiscal Burden on Legacy Debt

| | Long run expectations do not adjust $\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}$ | Long run expectations adjust $\{\{\partial\pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}$ |
|---|---|---|
| Any Maturity $\delta^d$ | $-\frac{D}{\pi}\frac{1+R}{\delta^d+R}(1-\delta^d)\frac{R+\delta^d}{R}$ | $-\frac{D}{\pi}\frac{1+R}{\delta^d+R}(1-\delta^d)$ |
| Short Debt $\delta^d = 1$ | 0 | 0 |

*Notes:* This table shows the fiscal costs missing the inflation targets under different maturity and expectation regimes. We look at variation in the fiscal burden. We take a permanent increase in inflation from period $t+1$ onward, this increase is announced in period $t$. In the second column, the announcement is incorporated only on the short run expectations, that is in the following period expectations. In the third column, the announcement is incorporated in all future expectations. In the first row, we show the variation in the fiscal burden (the net present value of debt servicing costs) under any arbitrary maturity, with longer maturity being associated with a lower $\delta^d$. In the second row we show the effect under short debt, i.e. $\delta^d = 1$. In this experiment the government refinances only maturing debt.

The multipliers of Table C.1, describing the effect on the fiscal burden of changes in inflation accompanied by either parallel shifts or blips in inflation expectations, give us a clear picture of the economic effects at work. Quantitatively, they can be large. However, they have been derived using infinite horizon sums, with permanent changes in inflation, and with stark assumptions on the adjustment of inflation expectations. We refine our estimates in the next section.

# 6 Counterfactual Experiments

In this section, we fit the proposed model to actual data to perform a number of counterfactual experiments, in order to quantify the fiscal costs of missing the inflation target, in a context

[19]In our previous example, with $R = 0.05/4$ and $\delta^d = 0.025$, we would be multiplying Duration-to-GDP by 3. If interest rates are lower at $R = 0.025/4$, then the multiplying factor goes to 5.

where the Central Bank is credible. A key reason for doing the counterfactual exercises pertains to the role of Duration-to-GDP in our theoretical analysis. This metric measures the variation in the market value of public debt following a change in interest rate or a miss in inflation target, but it measures the net present value of debt servicing costs only if the change is permanent. Inflation has been away from its target persistently, but this has not been a permanent shift. Moreover, as we just showed in the previous section, if long run expectations do not adjust, then Duration-to-GDP is multiplied by an amplifying factor which depends on debt maturity. Therefore, it is essential to quantify how far we are from the *actual losses*, which occurred in a finite number of periods when we use Duration-to-GDP, a simple, transparent, but not all-encompassing metric.

We perform two experiments. The first one analyses how much debt would have been higher or lower at the end of the sample if maturity were short ($\delta = 1$); in that case the debt is better priced as short run expectations are close to FIRE in the data. The second one asks the same question but keeps the maturity structure of the debt as in the data and assumes perfect foresight for inflation expectations up to the last period of available data. In both these experiments, the Central Bank misses its inflation target persistently (as in the data). In order to perform those experiments, we extend the model to have a time varying maturity (new variable $\delta_t^{new}$ so that each bond vintage has a geometric structure - but the overall debt stock does not). In Appendix E, we prove that the same model with a time varying maturity retains a recursive structure when we take a first order Taylor approximation and in Appendix F we provide all the steps to we take to fit the model to the data. We assume an exogenous risk premium (or convenience yield or liquidity premium) in order to keep the model very simple and study the role of missing inflation targets in a transparent way. Our counterfactual experiment can therefore be thought as analysing the effect of persistent misses of the inflation target in presence of inflation expectations deviating from FIRE, keeping constant the risk premium (or convenience yield or liquidity premium). In practice, we may not be that far off in terms of inflation risk premium since the credibility of the Central Banks targets do not seem to have been eroded much according to expectations data. Maturity changes do have some effects on term premium, convenience yields and possibly liquidity premium. We abstract from those[20] .

[20]This means we are likely to overestimate premia in the case of short maturities and therefore to under-

We emphasize that we do not impose any transversality condition in this exercise. The reason is that the dynamic system has a finite number of periods, so that we do not need to take any sum to infinity. This means that we can be in an economy where interest rates are both above or below the growth rate of nominal GDP without any complication[21]. Results are presented in Table 5. The first two columns describe the excess debt accumulated and interest paid in the data compared to the short debt experiment. Compared to a situation in which all the debt issued would have been short term, given expectations observed in the data, the joint fiscal costs of missing the inflation target and the duration choice of countries are *very* large for the period 2001q1 to 2021q1[22]. They are as high as 31.2 percentage points of GDP for Italy, almost equally high for Belgium, 21.5 % for France. The lowest costs are for Germany and the Netherlands (13.9%) because of their low debt levels and low maturities. Hence during the declining real rate period of the post 2000s, the duration choice of countries and the inflation undershooting its credible target materially impacted the debt level of countries, especially if they started off with a high debt level and a relatively long maturity structure[23].

The last two columns describe the excess debt accumulated and interest paid compared to the perfect foresight experiment where maturity structures are the ones of the data but agents have perfect foresight when forming their inflation expectations. The fiscal costs of making systematic mistakes in forming expectations about inflation in an environment where the Central Bank misses its target persistently but retains credibility are non negligible, but nowhere near as high as in the first experiment. They amount to 5.1% for the US, 5.6% for Italy and are as low as 2.0 % for the Netherlands. For the United States this amounts to 1.1 trillion Dollars more in debt in 2021Q1 (using nominal GDP in 2021Q1) and 100 billion Euros for Italy (using 2021 nominal GDP). Thus, the role played by duration choices across countries in an environment of declining real rates interacts with the mistakes made when forecasting inflation to determine fiscal costs. Just like in our theoretical model, those mistakes, linked to the Central Bank missing its target, are more consequential in terms of

estimate the fiscal gap in our first experiment

[21]See Blanchard (2019) for a discussion of debt sustainability where nominal growth can be above interest rates.

[22]In Appendix Table F.1 we replicate this table with the whole sample. Results are very similar.

[23]Additionally, in Appendix Table F.2 we replicate the same exercise with alternative measures for inflation realisation (country specific HICP and GDP deflator) and show that the results are quite similar to the baseline.

debt accumulation the higher the Duration-to-GDP of the country.

If we compare the numbers in the short-vs-long debt scenario (columns 2 and 3) with those in the perfect inflation foresight-vs-actual inflation expectations (columns 4 and 5), the second set of numbers are lower for three reasons. First, they capture "mistakes" in expectations, but do not adjust for the fact that realised lower inflation increases the real debt burden on legacy debt compared to a case where the inflation target would have been hit, - this is a key mechanism that we highlighted in Section 5. Second, they do not adjust for expectations pertaining to periods in the future[24], which is inconsequential in the scenario where the comparison is with short debt. Third, when we study the scenario under short debt, the debt burden falls faster because the secular decline in interest rates is immediately incorporated in average interest rates, whereas with long debt, even with perfect foresight, the decline is incorporated only on newly issued debt and not on legacy debt. Finally, there is a reason for which the effect in the short-vs-long debt comparison should be even higher: we used interest rates on 10 year bonds for newly issued debt, in order to make a *ceteris paribus* comparison; if we had used short (3 months) rates, the differential effect between the long and short borrowing profile would have been even higher as the yield curve was generally upward sloping. We emphasise that our calculations are all made in the context of the historical period under consideration, which features a secular decline in real rate, and most likely in a correlated way, realised inflation too low relative to target for an extended period of time, while medium and long run inflation expectations were well anchored. Our methodology however is general and can be used in any macroeconomic context.

Figure 6 shows the time series of debt, interest payments and inflation expectations in the data (baseline) and in the short debt experiment described above for the 3 biggest Euro Area economies and the US[25]. In all cases, the gap in terms of fiscal costs opens up after 2009 both because of the large increase in debt and because of the systematic undershooting of the inflation target and the related expectational mistakes.

Finally, we ask how the counterfactual exercise compares with the simple empirical results we presented in Section 4. The closest comparison between the computational exercise and the empirical result is comparing a short maturity regime with correctly priced inflation

[24] That is, we do not have realized inflation for 2030Q1!

[25] Appendix Figure F.1 shows the same figure for Belgium, Spain, the Netherlands, and Austria.

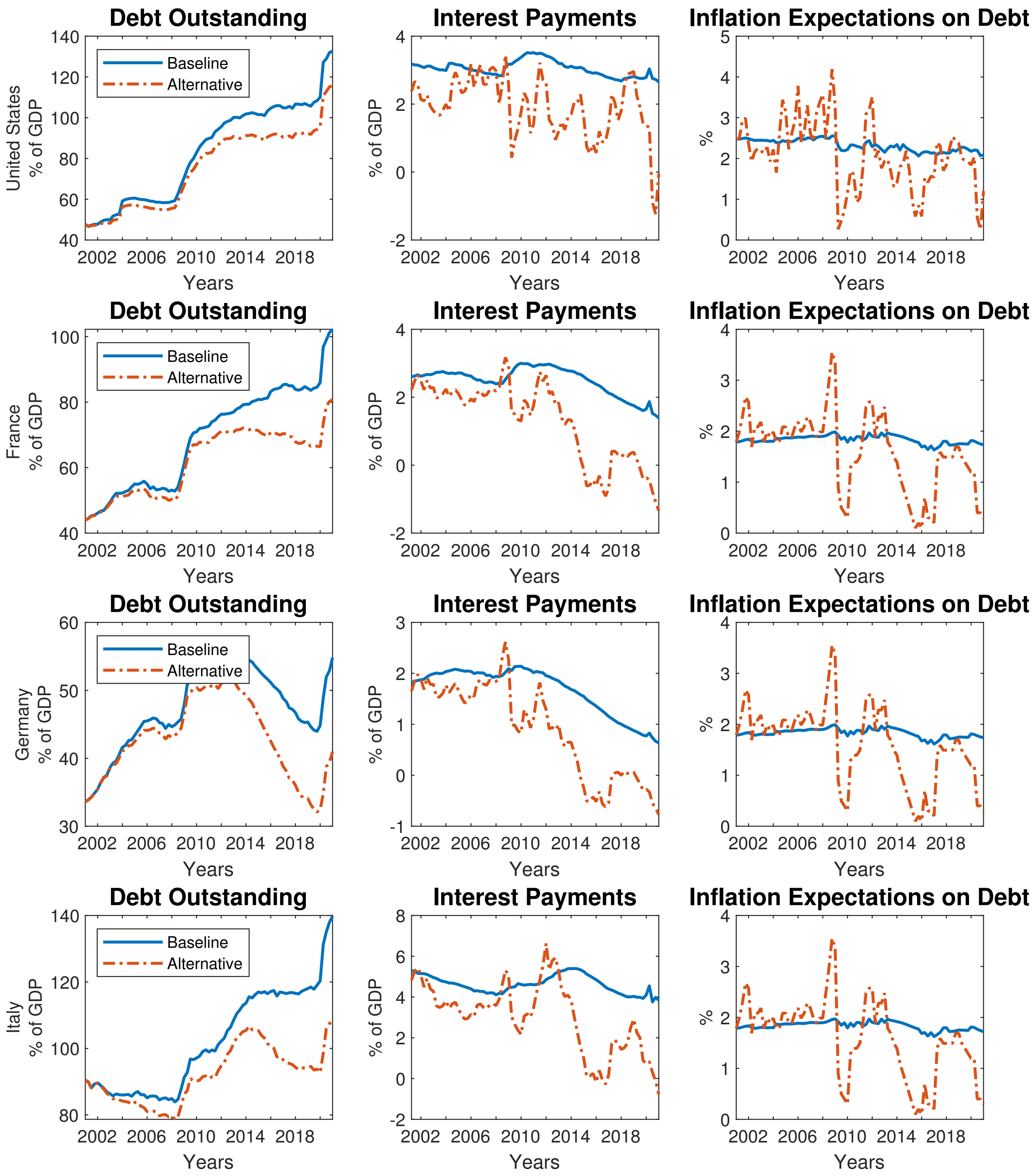
expectations and long maturity debt with actual debt embedded inflation expectations. In Table 6, we do exactly this. In column 2, we replicate the results presented in column 3 of Table 5, where we show the difference in average interest payments per year between the model fitted on actual data compared with a short maturity profile ($\delta^d = 1$). In columns 3 and 4, we replicate the results presented in columns 2 and 3 of Table 2 where we show the point estimate and the 95% confidence intervals of the multiplication of Duration-to-GDP with the forecast error of debt embedded inflation expectations; which shows the cost of missing the inflation target, with the existing debt structure, per year if the inflation mistake was permanent. We can see that the overall order of magnitude is quite similar in the two experiments. Moreover, as one should expect, the simple empirical exercise yields higher numbers compared to the counterfactual exercise; that is because inflation was below target persistently but the miss was not permanent. However, the 95% confidence intervals of the empirical exercise all contain the counterfactual exercise estimates, except for the United States, where the counterfactual estimate is just outside the interval. This means that the Duration-to-GDP metric, despite being very simple still tracks the order of magnitude of the cost of missing the inflation target quite well. An important conclusion of our paper is thus that the Duration-to-GDP metric is a useful benchmark measure for Central Banks and Debt Management Offices to gauge the effects of monetary policy on fiscal balances[26].

# 7 Conclusion

This paper shows that heterogeneity in the maturity structure and the level of debt, the Duration-to-GDP, is an important factor affecting the viability of an optimum currency area. When a Central Bank systematically misses its inflation target on one side for a long period, this can have large implications for the fiscal costs of the countries in the currency union. In case the central banks undershoots its target the larger the Duration-to-GDP, the larger the costs. And those costs are amplified when the Central Bank is credible as agents keep expecting a return to target, which is priced in long-dated bonds and persistently not happening. Fiscal costs of countries such as Italy or Belgium having chosen to issue relatively

[26] Andreolli (2022) shows that this metric is also a sufficient statistic to study the state dependent effects of regular monetary policy on fiscal burdens and to study the implication of this interrelationship for economic activity.

Figure 6: Fiscal Consequences of Missing Inflation Targets




*Notes:* This figure shows the results under the counterfactual exercise. It shows the path of debt, interest payments, and debt-embedded inflation expectations for the United States, France, Germany, and Italy. The blue solid line shows the path of these variables under the actual maturity structure $\delta_t^{ave}$, the red dot-dashed line shows the path under a counterfactual short debt ($\delta_t^{ave} = 1$). The sample goes from 2001Q1 to 2021Q1.

Table 5: Counterfactual Exercises

| Country | Short Debt | | Perfect Foresight | |
|---|---|---|---|---|
| | Debt | Interest Payments | Debt | Interest Payments |
| France | 21.4 | 1.20 | 3.7 | 0.21 |
| Germany | 13.9 | 0.78 | 2.6 | 0.15 |
| Italy | 31.2 | 1.63 | 5.6 | 0.29 |
| Belgium | 31.2 | 1.86 | 3.7 | 0.22 |
| Spain | 20.6 | 1.09 | 3.4 | 0.18 |
| Netherlands | 13.9 | 0.81 | 2.0 | 0.12 |
| Austria | 16.7 | 0.90 | 3.2 | 0.18 |
| United States | 17.0 | 1.14 | 5.1 | 0.34 |

*Notes:* This table shows the results under the counterfactual exercises. Columns 2 and 3 show the counterfactual fiscal burden under a short debt profile ($\delta^d = 1$). Columns 4 and 5 show the counterfactual fiscal burden under perfect foresight, that is in each period $t$ expectations are correct at all future horizon: $\mathbb{E}_t(\pi_{t+j}) = \pi_{t+j}$. For inflation expectations pertaining to periods that have not yet happened in the dataset (after 2021Q1) we use the appropriate inflation expectation. Columns 2 and 4 show the difference in debt-to-GDP level at the last period under the exercise compared to the case where we fit the model with actual data. Columns 3 and 5 show the difference in average interest payments per year under the exercise compared to the case where we fit the model with actual data. The sample goes from 2001Q1 to 2021Q1.

long maturity debt and having large debt levels are close to 30 points of GDP (accumulated debt) higher during the 2000-2021 period than if they had issued short term debt, whose price reflects realised inflation more closely than long dated instruments. importantly, we show that the Duration-to-GDP metric, despite its simplicity still tracks the order of magnitude of the cost of missing the inflation target quite well. An important conclusion of our paper is thus that the Duration-to-GDP metric is a useful benchmark measure for Central Banks and Debt Management Offices to gauge the effects of monetary policy on fiscal balances.

Table 6: Counterfactual Exercise Comparison with Empirical Result

| Country | Counterfactual Exercise | Empirical Result | |
|---|---|---|---|
| | Estimate | Estimate | Confidence Interval |
| France | 1.20 | 2.16 | [0.97 , 3.36] |
| Germany | 0.78 | 1.06 | [0.34 , 1.77] |
| Italy | 1.63 | 2.26 | [0.78 , 3.75] |
| Belgium | 1.86 | 2.26 | [0.86 , 3.67] |
| Spain | 1.09 | 2.05 | [1.03 , 3.07] |
| Netherlands | 0.81 | 1.08 | [0.41 , 1.75] |
| Austria | 0.90 | 1.80 | [0.72 , 2.88] |
| United States | 1.14 | 2.23 | [1.16 , 3.29] |

*Notes:* This table shows the comparison of a low maturity regime compared with a long maturity one in the counterfactual exercise and in the empirical results. Columns 2 shows the counterfactual fiscal burden under a short debt profile ($\delta^d = 1$) compared with the observed maturity. It shows the difference in average interest payments per year under the exercise compared to the case where we fit the model with actual data. Columns 3 and 4 show the simple empirical result, they show the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with SPF, OECD, and FTSE WGBI data. Column 3 shows the point estimate of the average yearly cost in GDP units. Column 4 shows the 95% confidence interval of the average and it is obtained by running a regression of the cost variable on a constant with White robust errors. The sample goes from 2001Q1 to 2021Q1.

# References


**Andreolli, Michele.** 2022. "The Maturity Structure of Public Debt and the Transmission of Monetary Policy."

**Ang, Andrew, Geert Bekaert, and Min Wei.** 2007. "Do macro variables, asset markets, or surveys forecast inflation better?" *Journal of monetary Economics*, 54(4): 1163–1212.

**Aruoba, Borağan.** 2020. "Term structures of inflation expectations and real interest rates." *Journal of Business & Economic Statistics*, 38(3): 542–553.

**Auerbach, Alan J, and Maurice Obstfeld.** 2005. "The case for open-market purchases in a liquidity trap." *American Economic Review*, 95(1): 110–137.

**Begg, David, Paul de Grauwe, Francesco Giavazzi, Charles Wylposz, and Harald Uhlig.** 1998. *MECB 1: The ECB: Safe at Any Speed?* Vol. Monitoring the European Central Bank, Centre for Economic Policy Research.

**Berger, Mr Helge, Mr Giovanni Dell'Ariccia, and Mr Maurice Obstfeld.** 2018. *Revisiting the economic case for fiscal union in the Euro area.* International Monetary Fund.

**Blanchard, Olivier.** 2019. "Public debt and low interest rates." *American Economic Review*, 109(4): 1197–1229.

**Blanchard, Olivier, and Lawrence Katz.** 1992. "Regional evolutions." *Brookings Papers on Economic Activity*, 23(1): 1–76.

**Blanchard, Olivier J, and Daniel Leigh.** 2013. "Growth forecast errors and fiscal multipliers." *American Economic Review*, 103(3): 117–20.

**Breach, Tomas, Stefania D'Amico, and Athanasios Orphanides.** 2020. "The term structure and inflation uncertainty." *Journal of Financial Economics*, 138(2): 388–414.

**Candia, Bernardo, Olivier Coibion, and Yuriy Gorodnichenko.** 2022. "The macroeconomic expectations of firms." National Bureau of Economic Research.

**Chernov, Mikhail, and Philippe Mueller.** 2012. "The term structure of inflation expectations." *Journal of financial economics*, 106(2): 367–394.

**Cohen, Benjamin H, Peter Hördahl, and Fan Dora Xia.** 2018. "Term premia: models and some stylised facts." *BIS Quarterly Review September*.

**Coibion, Olivier, and Yuriy Gorodnichenko.** 2015. "Is the Phillips curve alive and well after all? Inflation expectations and the missing disinflation." *American Economic Journal: Macroeconomics*, 7(1): 197–232.

**Doepke, Matthias, and Martin Schneider.** 2006. "Inflation and the redistribution of nominal wealth." *Journal of Political Economy*, 114(6): 1069–1097.

**Ellison, Martin, and Andrew Scott.** 2017. "Managing the UK National Debt 1694-2017."

**Frankel, Jeffrey A, and Andrew K Rose.** 1998. "The endogenity of the optimum currency area criteria." *The economic journal*, 108(449): 1009–1025.

**Garcia, Juan A, and Thomas Werner.** 2010. "Inflation risks and inflation risk premia."

**Gourinchas, Pierre-Olivier, and Helene Rey.** 2007. “International financial adjustment.” *Journal of political economy*, 115(4): 665–703.

**Grishchenko, Olesya, Sarah Mouabbi, and Jean-Paul Renne.** 2019. “Measuring inflation anchoring and uncertainty: A US and euro area comparison.” *Journal of Money, Credit and Banking*, 51(5): 1053–1096.

**Gürkaynak, Refet S, Brian Sack, and Jonathan H Wright.** 2010. “The TIPS yield curve and inflation compensation.” *American Economic Journal: Macroeconomics*, 2(1): 70–92.

**Hall, George J, and Thomas J Sargent.** 2011. “Interest rate risk and other determinants of post-WWII US government debt/GDP dynamics.” *American Economic Journal: Macroeconomics*, 3(3): 192–214.

**Haubrich, Joseph, George Pennacchi, and Peter Ritchken.** 2012. “Inflation expectations, real rates, and risk premia: Evidence from inflation swaps.” *The Review of Financial Studies*, 25(5): 1588–1629.

**Hilscher, Jens, Alon Raviv, and Ricardo Reis.** 2021. “Inflating Away the Public Debt? An Empirical Assessment.” *The Review of Financial Studies*.

**Hördahl, Peter, and Oreste Tristani.** 2018. “Inflation risk premia in the euro area and the United States.” *36th issue (September 2014) of the International Journal of Central Banking*.

**Jiang, Zhengyang, Hanno N Lustig, Stijn Van Nieuwerburgh, and Mindy Z Xiaolan.** 2020. “Bond convenience yields in the eurozone currency union.” *Columbia Business School Research Paper*.

**Kenen, Peter B.** 1969. “The theory of optimum currency areas: An eclectic view.” In *Monetary Problems of the International Economy.* , ed. R. Mundell and A. Swoboda, 41–60. University of Chicago Press.

**Kim, Don H, and Athanasios Orphanides.** 2012. “Term structure estimation with survey data on interest rate forecasts.” *Journal of Financial and Quantitative Analysis*, 241–272.

**Krugman, Paul.** 1993. "Lessons of Massachusetts for EMU." In *Adjustment and Growth in the European Monetary Union.* , ed. F. Torres and F. Giavazzi, 241–61. Cambridge University Press.

**Mundell, Robert A.** 1961. "A theory of optimum currency areas." *The American economic review*, 51(4): 657–665.

**Pennacchi, George G.** 1991. "Identifying the dynamics of real interest rates and inflation: Evidence using survey data." *The review of financial studies*, 4(1): 53–86.

**Portes, Richard, and Helene Rey.** 1998. "The emergence of the euro as an international currency."

**Reichlin, Lucrezia, Giovanni Ricco, and Matthieu Tarbé.** 2021. "Monetary-Fiscal Crosswinds in the European Monetary Union."

**Reinhart, Carmen M, and M Belen Sbrancia.** 2015. "The liquidation of government debt." *Economic Policy*, 30(82): 291–333.

**Reis, Ricardo.** 2020. *The people versus the markets: A parsimonious model of inflation expectations.* CFM, Centre for Macroeconomics.

**Rose, Andrew K.** 2000. "One money, one market: the effect of common currencies on trade." *Economic policy*, 15(30): 08–45.

**Stavrakeva, Vania, and Jenny Tang.** 2021. "A Fundamental Connection: Survey-based Exchange Rate Decomposition." *Unpublished working paper.*

**Wolf, Martin, and Leopold Zessner-Spitzenberg.** 2021. "Fear of Hiking? Monetary Policy and Sovereign Risk."

# A Data Construction

We compiled a dataset with debt to GDP from the OECD. We used general government debt securities to GDP, this excludes loans and other types of non-marketable debts[27]. We exclude these other categories as we do not have detailed duration information. By this token, our results are a lower bound on the effects of missing inflation targets as we focus only on a subset of the government liabilities. The duration of government debt data comes the FTSE WGBI Data. We used Macaulay duration.

This data is subject to a number of caveats. First of all, The OECD securities debt-to-GDP excludes holding by government entities, say social security and most importantly the central bank. A second caveat is that the FTSE WGBI data computes the duration measure on bonds (so excludes t-bills, and other debt). This implies that the measure is an upper bound as for public debt we use all bonds. To give a comparison for these two measures, overall General government securities debt-to-GDP (so not only Federal) in the US in 2019 was about 99% of GDP. All outstanding Federal bonds (so no t-bills or other debt) was around 70%, and the bond publicly held were 55%. Clearly the last one is too conservative, but the first one might be too high. As a comparison, in the current data for the US in 2019 we have Duration-to-GDP at 6.4% of GDP (the increase in the market value of public debt in GDP units in case there is a 1% decrease in interest rates across the yield curve), Andreolli (2022) computes the same measure from bond-by-bond data and finds 3.1% of debt to GDP in case of bonds only held by the general public and 4% of debt to GDP when including the FED holdings. This gives an idea of the degree of differences in magnitude for Duration-to-GDP depending on the source of the data. On the other hand, Macaulay duration for the US is quite similar when computed bond-by-bond and when computed with the FTSE WGBI index.

A second caveat pertains to countries under an IMF or Troika program. In these cases non-marketable loans become a significant part of public debt, and therefore the Macaulay duration of bonds becomes less informative of the duration risk of public liabilities. This is why we do not report Greek numbers and the numbers for other countries under these

[27] The OECD code for debt securities over GDP is SAF33; we exclude loans (SAF4), currency and deposits (AF2), Special Drawing Rights (AF12), other accounts payable (AF8), insurance, pensions and standardized guarantee schemes (AF6). The Total gross debt by general government has SAFGD code.

programs should be discounted for the program periods.

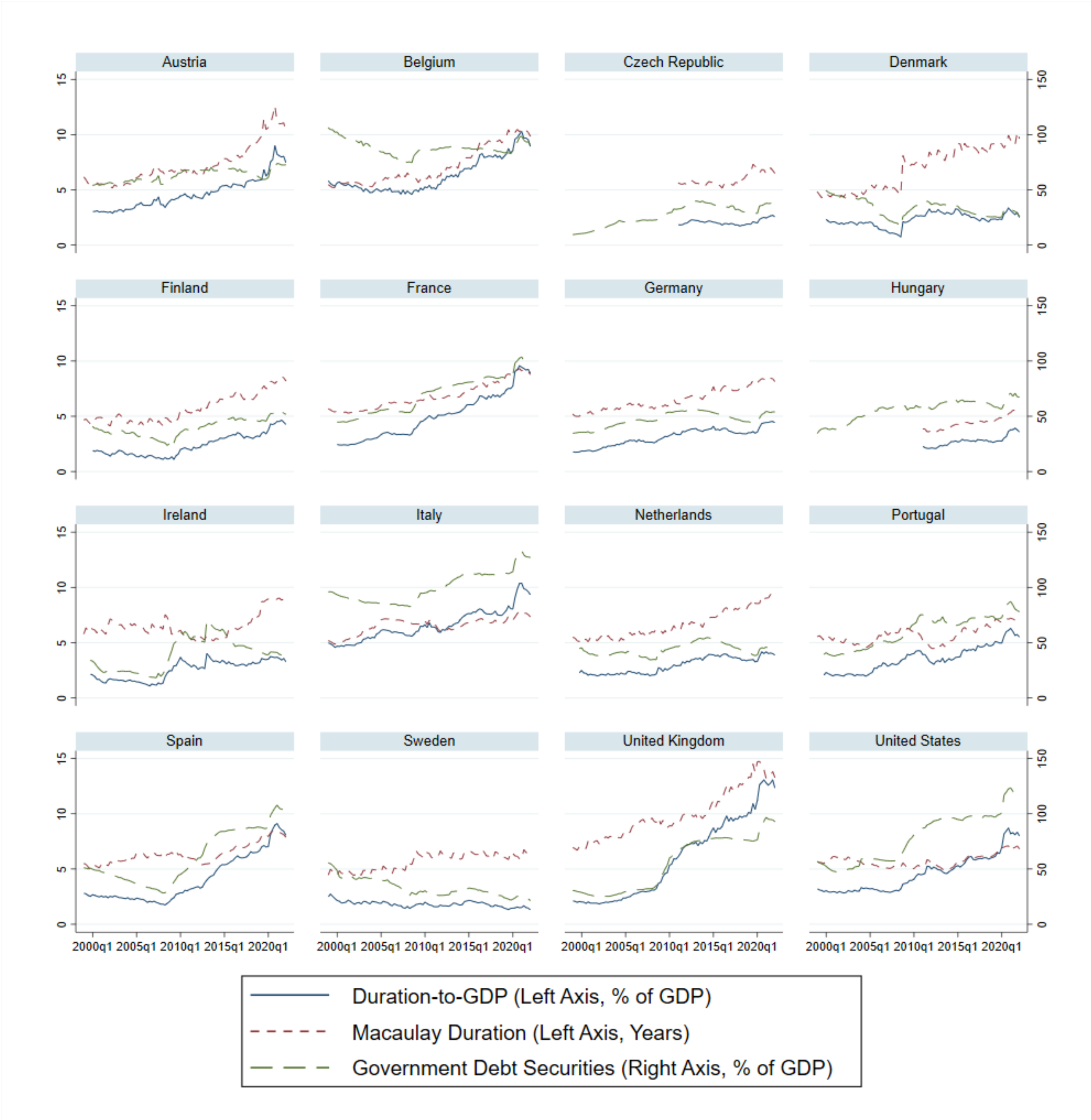
ECB reports the weighted average maturity (WAM) of its debt holdings in the context of the PSPP portfolio holdings (QE). We complement this with the average life for the overall stock of bonds from the FTSE WGBI data. Both these measures compute the maturity by taking the weighted years the principal of bond is due, weighted by amount outstanding (for the overall measure) or by ECB holdings of this bond (for the ECB holding measures).

In the computational exercise we use 10 years benchmark rates on sovereign debt to measure interest rates. The data source is Refinitiv.

## A.1 Additional Figures

In this subsection, we add additional figures with a larger number of countries to provide a broader picture of the main results. In Figure A.1 we present Duration-to-GDP and its components (Macaulay duration and Public Securities Debt ) for all the countries in the dataset.

Figure A.1: Duration-to-GDP for all countries




*Notes:* The time series presents Duration-to-GDP (in the blue solid line), Macaulay duration (in the red short dashed line), and government debt securities (in the green long dashed line) for all the countries in the dataset. Macaulay duration data is FTSE WGBI. Debt data are government debt securities over GDP from the OECD. The sample goes from 1999Q1 to 2022Q1.

# B Comparison of Inflation Expectation Measures

In the baseline exercise we use the survey of professional forecasters (SPF) measures for long run inflation expectations due to their coverage and ability to forecast inflation (see Ang, Bekaert and Wei, 2007). Moreover, these metrics directly measure inflation expectations and do not need a structural model to disentangle expectations from risk premia, liquidity premia, or convenience yields as market based metrics. However, it is important to show how this measure compares other metrics for long run inflation expectations. Grishchenko, Mouabbi and Renne (2019) combines several inflation survey sources to fit a yield curve of inflation expectations for the US and the Euro Area. Similarly, Aruoba (2020) does a similar exercise for the US. For the US, we also present the Cleveland Fed Expected Inflation Term Structure by Haubrich, Pennacchi and Ritchken (2012) which fits a structural model with data from inflation swap rates, nominal Treasury rates, and survey forecasts of inflation. Finally, we also present row market based metrics: forwards on inflation liked swaps (for the Euro Area and the US) and on break even inflation rates from nominal and inflation linked treasuries (for the US). These metrics should not be used as inflation expectations directly as they incorporate inflation risk premia, liquidity premia, and convenience yields. However, they are still useful to look at, as they are based on market prices and we show robustness in our results to employing them directly.

Figures B.1 and B.2 present the results of this comparison for the Euro Area and for the US respectively. We can see that most survey based metrics track quite closely the SPF, and the market based metrics are a bit more volatile but still track the SPF and yield large mispricing as in the baseline.

Moreover, to understand the inflation expectations embedded in bond prices, long run inflation expectations are the key expectations given the relatively long maturity of public debt. However, it is important to see how they are related to shorter horizon expectations. For this reason, in Figure B.4, we plot the mean inflation expectations in the ECB and Fed SPF. From this picture we can see that the stability of long run inflation expectations does not carry through to the shorter horizon metrics, as they track variation in inflation more closely.

With respect to inflation embedded in bond prices we take a issuance weighted average of inflation rates and use the following formula:

$$\sum_{j=1}^{J-1} \mathbb{E}_{t-j}(\pi^a_{t|t-j}) \frac{1}{Mat_t} \left(1 - \frac{1}{Mat_t}\right)^{j-1} + \mathbb{E}_{t-J}(\pi^a_{t|t-J}) \left(1 - \frac{1}{Mat_t}\right)^{J}$$

Where $\pi^a_{t|t-j}$ is the annualized inflation rate from period $t-j$ to $t$, $Mat_t$ is the annualized maturity of public debt, measured with average life from the WGBI FTSE data. $J$ is the maximum inflation expectation horizon we have available (we set $J$ to 10 years). This formula comes directly from the theoretical framework, with a constant fraction of debt being repaid each period[28]. The formula implies that we assign the long term inflation expectation $\pi^a_{t|t-J}$ in period $t-J$ to all debt issued before period $t-J$[29]. We use $j$ to be an annual horizon, as we have annual inflation expectations in the ECB SPF. This means that $\mathbb{E}_{t-1}(\pi^a_{t|t-1})$ is the average, next year inflation expectation prevailing in the past 4 quarters, $\mathbb{E}_{t-2}(\pi^a_{t|t-2})$ is the two year head inflation expectations on average in the past 5 to 8 quarters, and so on.

In the baseline debt-embedded inflation expectations we study in the empirical exercise, we use the raw SPF data with the following assumptions: for $\mathbb{E}_{t-1}(\pi^a_{t|t-1})$ we use one year head inflation expectations, for $\mathbb{E}_{t-2}(\pi^a_{t|t-2})$ we use two years head inflation expectations, and for any $\mathbb{E}_{t-j}(\pi^a_{t|t-j})$ with $j > 2$ we use the long run inflation expectation. We are implicitly assuming that all inflation expectations above 2 years are the long run ones. In order to assuage concerns about this assumption, we also employ the inflation expectations of Grishchenko, Mouabbi and Renne (2019), Aruoba (2020), and Haubrich, Pennacchi and Ritchken (2012) who provide a full term structure of inflation expectations. Finally, for the United States, where we have a long time series of TIPS and nominal yield curves, we also computed the debt-embedded inflation expectations using break even inflation rates from the Fed (based on Gürkaynak, Sack and Wright 2010). We follow the Fed practice and start the sample in 2004q1 due to illiquidity in the TIPS market in the first years in which these securities were introduced. Moreover, we do not use the data for the last two quarters of

[28] Appendix D shows that this is indeed a good approximation of US public debt promises with bond-by-bond data.

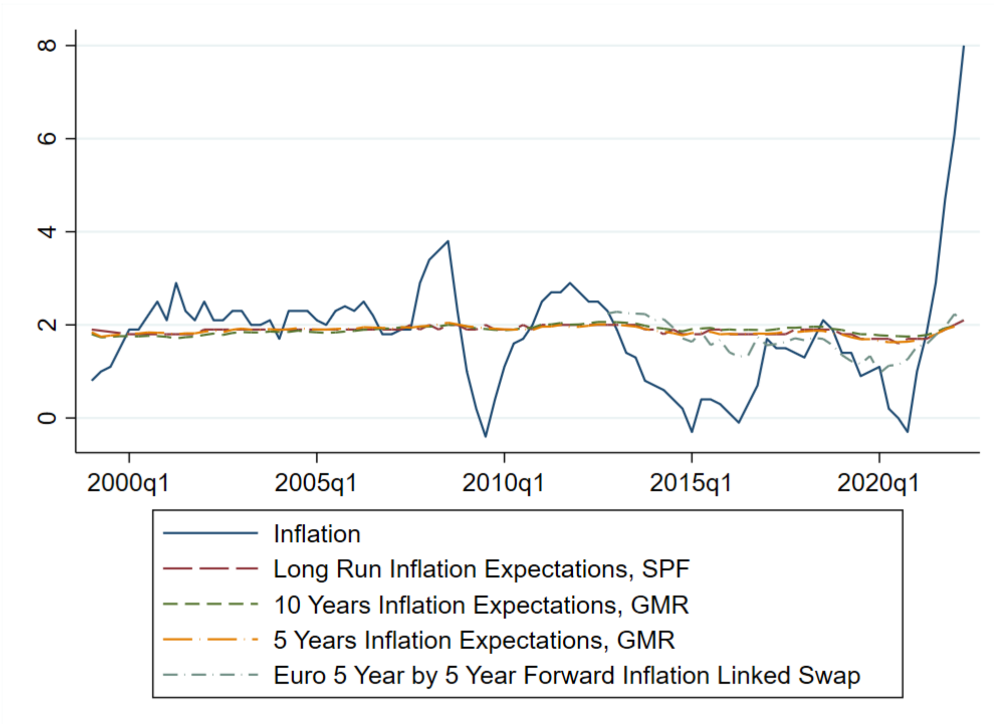
[29] Notice that this assumption attenuates our results. In the Euro Area, the first, in 1999q1, long term first inflation in the SPF was 1.9%, and we assign this to all the debt which was issued before that. However, it is likely that in a large number of Euro Area countries long run inflation expectations were higher than 1.9% in the decades before the creation of the Euro.

2008 (but use the value of 2008q2), when the financial crisis created affected the functioning of the TIPS market due to very high illiquidity (this phenomenon was also highlighted by Reis 2020) and therefore made the break even inflation even a worse predictor of inflation. Moreover, we exploit the all the break even rates, going from 2 years ahead to 20 years ahead. As we do not have one year ahead break even inflation we use the two years ahead metric. We plot the comparison in Figure B.5, together with actual inflation and the current long run inflation expectations from the SPF. All these metrics are very close to each other, implying we can use directly the SPF data. Moreover, the conclusion from Section 3 remains: all inflation expectations embedded in debt are very close to the current long-run inflation expectations, as proof of the credibility of the inflation target in the past two decades.

Figure B.6 and Table B.1 show the inflation forecast errors. Figure B.6 shows the time series of the inflation forecast errors for all Euro Area countries and for the United States measured with expectations embedded in current debt and with the naive inflation expectation: 1.9% in the Euro Area and 2.0% in the United States. Table B.1 shows the average inflation forecast errors in the Euro Area and in the United States for various samples and different expectations: in addition to the aforementioned expectations on debt and naive ones, it shows the forecast errors on short run inflation expectations. We can see that in the low inflation decade forecast errors on debt have been large at $-0.59$ and $-0.71$ percentage points in the Euro Area and in the United States respectively. Both these numbers are statistically different from zero. Whereas, when we measure short run inflation expectations, with lagged (one quarter) inflation expectations on current year inflation, the forecasts errors are $-0.04$ and $-0.13$ percentage points and not statistically different from zero. That is, short run inflation expectations tracked inflation much more than long run expectations and debt-embedded inflation expectations.

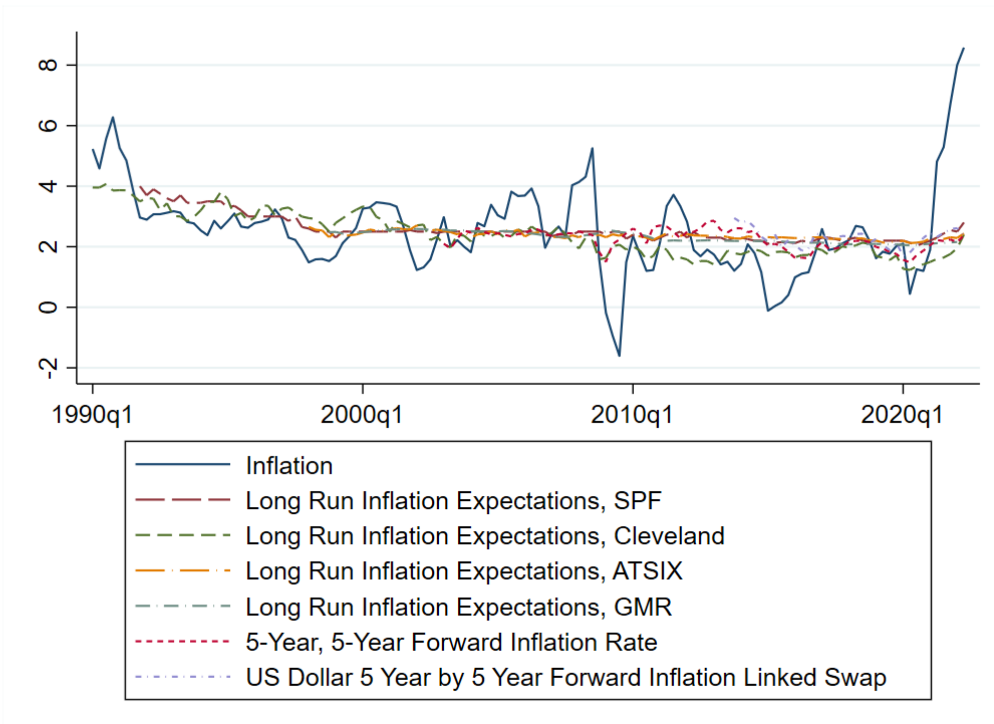
As a final step, we replicate the results on the fiscal costs of missing the inflation target for the US with different metrics for debt-embedded inflation expectations. The reason is that for the US we have a long time series for many inflation expectations metrics: pure survey ones, term structure model ones, and pure market based ones. In Table B.2 we replicate Table 2 with all the debt-embedded inflation expectations presented in Figure B.5. We can see that except with the naive forecast (2% always), all the metrics yield a statistical significant costs (the Cleveland Fed one only marginally). Crucially, the order of magnitude is similar in all the strongly significant results. Even in the case where we use the Break Even inflation rate directly to compute inflation expectation the cost is 1.7% of GDP, despite the presence of the risk premium. For the Euro Area the results with using the Grishchenko, Mouabbi and Renne (2019) are virtually identical for each country to the baseline ones with the SPF and are available upon request.

Figure B.1: Long Run Inflation Expectations Comparison and Inflation in the Euro Area



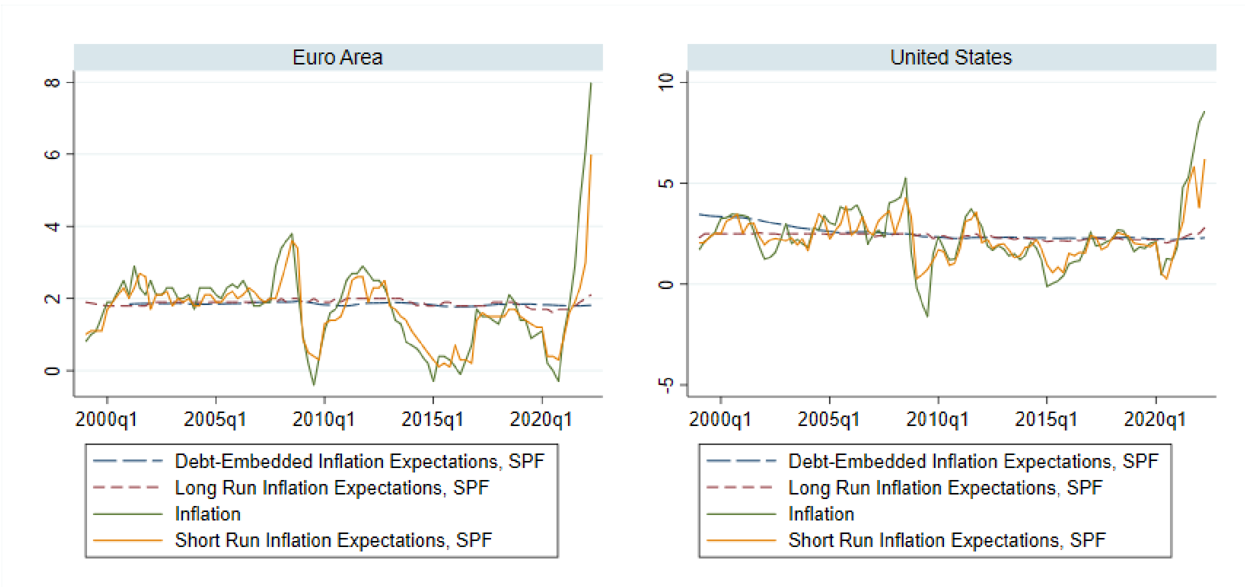

*Notes:* This graph presents long run inflation expectation measures and realised inflation for the Euro Area. The SPF series is the 4/5 years ahead YoY inflation from the ECB Survey of Professional Forecasters (SPF). 10 (5) Years GMR is the 10 (5) years ahead inflation expectation from the Grishchenko, Mouabbi and Renne (2019) measure. Inflation and its expectations pertain to the Harmonised Index of Consumer Prices (HICP). The sample goes from 1999Q1 to 2022Q2. The 5 year by 5 year forward inflation liked swap sample starts in 2012Q4.

Figure B.2: Long Run Inflation Expectations Comparison and Inflation in the United States




*Notes:* This graph presents long run inflation expectation measures and YoY CPI realised inflation for the United States. The SPF series is the 10 years average inflation from the Philadelphia Fed Survey of Professional Forecasters (SPF). Cleveland is the Cleveland FED 10 year ahead expected inflation, ATSIX is the mean inflation expectation 10 years ahead from Aruoba (2020). GMR is the 10 years ahead inflation expectation from the Grishchenko, Mouabbi and Renne (2019) measure. 5-Year, 5-Year Forward Inflation Rate is measured with nominal and inflation linked treasury rates. The sample goes from 1990q1 to 2022Q2. The 5 year by 5 year forward inflation liked swap sample starts in 2012Q4.

Figure B.3: Short vs Long Run Inflation Expectations and Inflation

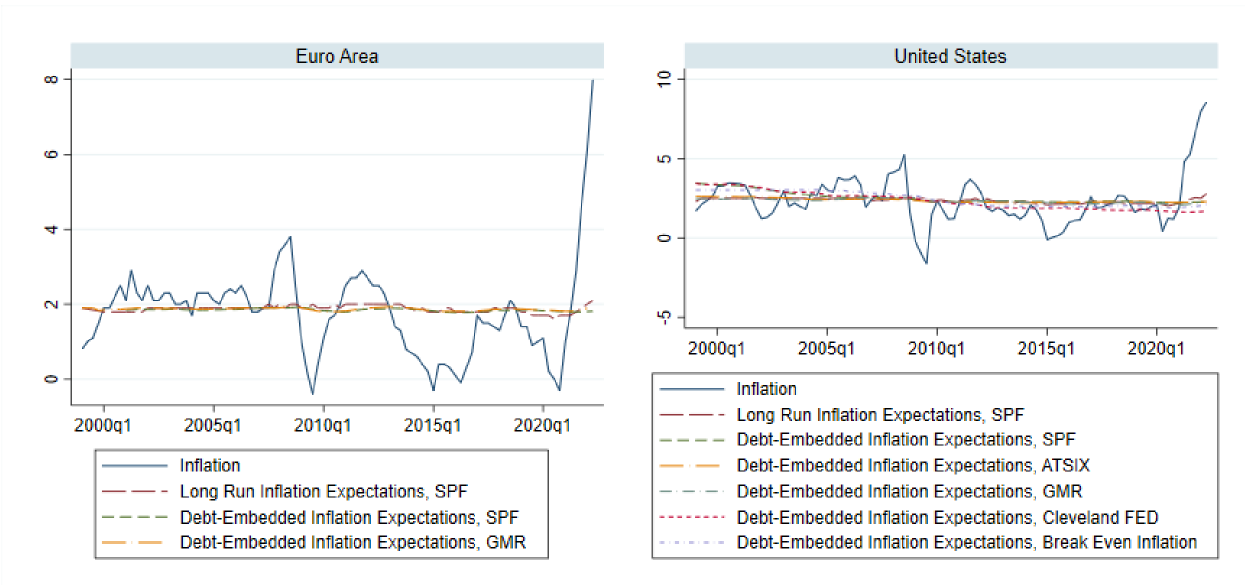



*Notes:* This graph presents long run inflation expectations, the inflation expectations embedded on outstanding public debt at issuance, short run inflation expectations for the current year, and realised inflation. The first panel shows the data for the Euro Area, there inflation expectation comes from the ECB Survey of Professional Forecasters (SPF), long run expectations are measured with 4/5 years ahead, and inflation and its expectations pertain to the Harmonised Index of Consumer Prices (HICP). The second panel shows the data for the United States, there inflation expectations come from the Philadelphia Fed Survey of Professional Forecasters (SPF), long run expectations are measured with 10 years average inflation, and inflation and its expectations pertain to the Consumer Price Index (CPI). The sample goes from 1999Q1 to 2022Q2.

Figure B.4: SPF Inflation Expectations Comparison and Inflation

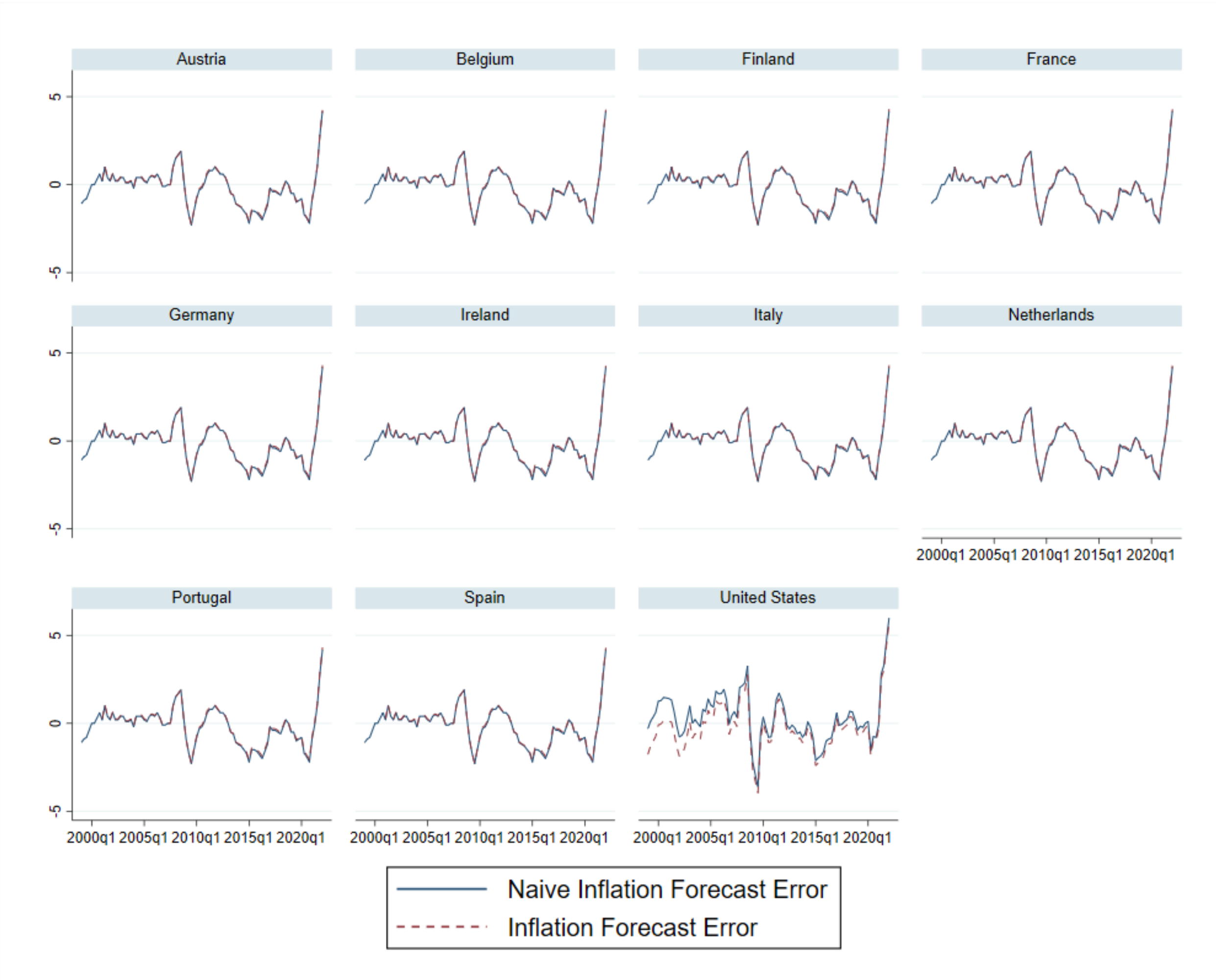

*Notes:* This graph presents all the Survey of Professional Forecasters (SPF) inflation expectations across horizons for the Euro Area (inflation is HCPI) in the first panel and for the United States (inflation is CPI) in the second panel. The sample goes from 1999q1 to 2022Q2.

Figure B.5: Inflation Expectations on Debt Comparison and Inflation in the Euro Area and in the United States



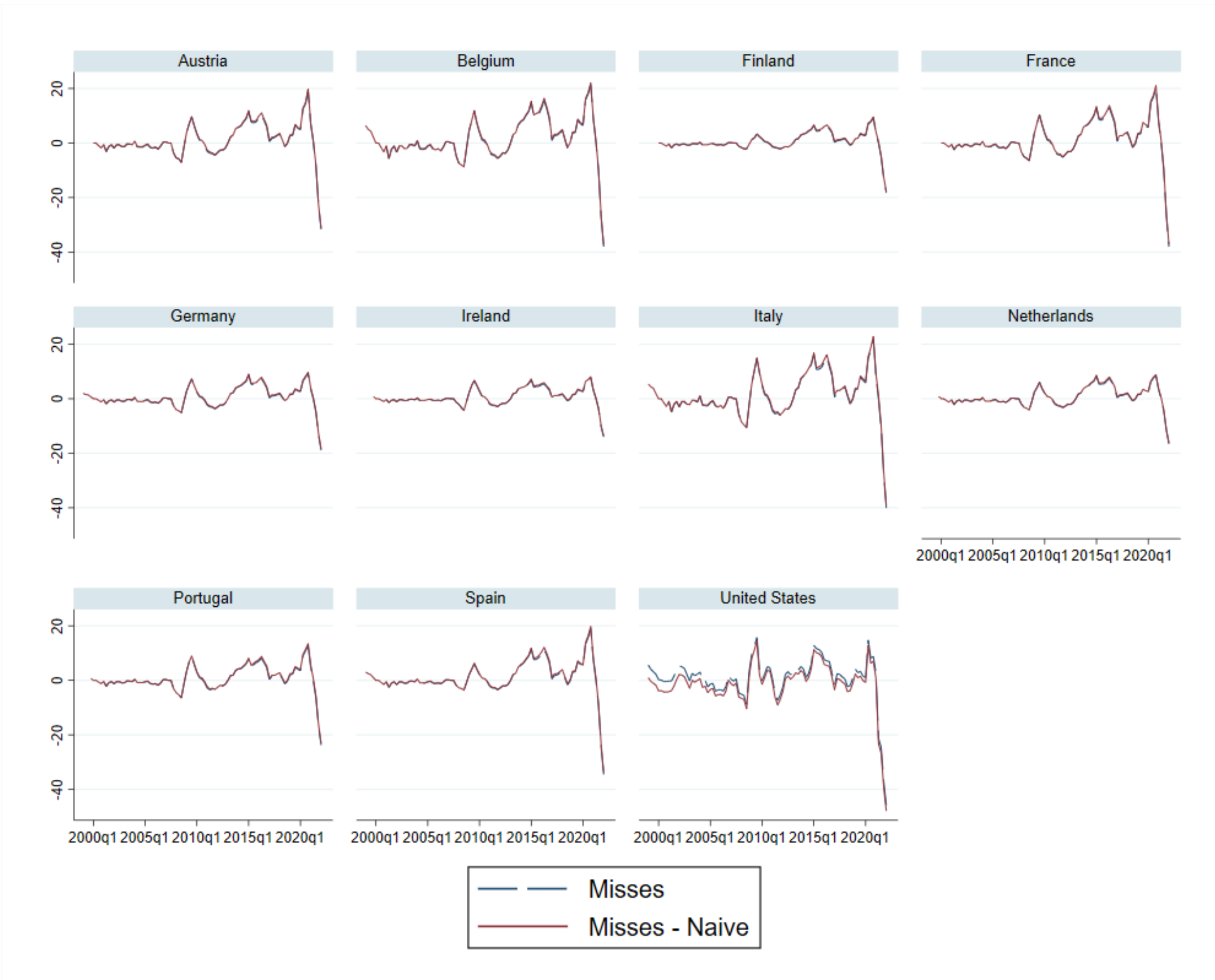

*Notes:* This graph presents debt-embedded inflation expectations and YoY inflation for the Euro Area (HCPI) and for the United States (CPI). The long run inflation expectations are the Survey of Professional Forecasters (SPF) series, the 4/5 years ahead YoY inflation from the ECB for the Euro Area and the 10 years average inflation from the Philadelphia Fed for the United States. Cleveland is the Cleveland FED model, ATSIX is the mean inflation expectation from Aruoba (2020). GMR is the inflation expectation from the Grishchenko, Mouabbi and Renne (2019) measure. Break Even Inflation uses the Break Even Inflation from the difference between Nominal and TIPS Treasuries. The sample goes from 1990q1 to 2022Q1.

Figure B.6: Inflation Forecasts Errors for All Euro Area Countries and for the United States




*Notes:* This graph presents the forecast error on debt-embedded inflation expectations. Inflation expectations come from the ECB (for the Euro Area on HICP inflation) and Philadelphia Fed (for the US on Headline CPI) Surveys of Professional Forecasters (SPF). The naive approach assumes a inflation expectation of 1.9% for the Euro Area and 2% for the US. The vertical axis units are percentage points. The sample goes from 1999Q1 to 2022Q2.

Figure B.7: Fiscal Consequences of Missing Inflation Targets for All Euro Area Countries and for the United States




*Notes:* This figure shows the fiscal costs of missing the inflation targets on public debt. The "Misses" lines show the multiplication between Duration-to-GDP and the forecast error on debt-embedded inflation expectations with SPF, OECD, and FTSE WGBI data. The "Misses - Naive" show the same multiplication, but with the naive inflation forecast. The vertical axis units are percent of GDP. A positive number implies that inflation is below its forecast and a negative number that inflation is above its forecast. The sample goes from 1999Q1 to 2022Q1.

Table B.1: Inflation Forecasts Errors

| Sample | Forecast | Euro Area | | United States | |
|---|---|---|---|---|---|
| | | Mean | SE | Mean | SE |
| From 2001q1 to 2020q1 | On Debt | -0.15 | (0.10) | -0.38 | (0.13) |
| | Naive | -0.20 | (0.10) | 0.11 | (0.14) |
| | Short Run | 0.03 | (0.07) | -0.07 | (0.10) |
| From 2009q1 to 2020q1 | On Debt | -0.59 | (0.13) | -0.71 | (0.16) |
| | Naive | -0.65 | (0.13) | -0.41 | (0.16) |
| | Short Run | -0.06 | (0.09) | -0.18 | (0.13) |
| From 2001q1 to 2022q2 | On Debt | -0.04 | (0.14) | -0.13 | (0.17) |
| | Naive | -0.09 | (0.14) | 0.33 | (0.17) |
| | Short Run | 0.17 | (0.10) | 0.10 | (0.12) |

*Notes:* This table shows the average inflation forecast errors in the Euro Area and in the United States. The first columns shows the sample under study. The second column shows the inflation forecast choice: "On Debt" is the debt-embedded inflation forecast, "Naive" is an inflation forecast of 1.9% in the Euro Area and 2.0% in the United States, and "Short Run" is the lagged (one quarter) forecast for current year inflation. All forecasts are based on the Survey of Professional Forecasters. The forecast error is the current inflation minus the inflation forecast. The third and fifth column show the average inflation forecast error in percentage points. The fourth and sixth columns show the standard error of the average and are obtained by running a regression of the inflation forecast error variable on a constant with White robust errors.

Table B.2: Fiscal Consequences of Missing Inflation Targets - Metrics Comparison for the United States

| Method | Cost | |
|---|---|---|
| | Mean | SE |
| Baseline | 2.23 | (0.54)*** |
| Naive | 0.35 | (0.55) |
| ATSIX | 1.82 | (0.55)*** |
| Cleveland Fed | 1.01 | (0.51)* |
| GMR | 1.70 | (0.54)*** |
| Break Even Inflation Rate | 1.70 | (0.51)*** |

*Notes:* This table shows the fiscal costs missing the inflation targets on public debt for the United States. It shows the multiplication between Duration-to-GDP and the forecast error on inflation expectations embedded on outstanding public debt at issuance with different methodologies to compute inflation expectations. Duration-to-GDP is computed with OECD, and FTSE WGBI data. The first row uses SPF data for inflation expectations, the second uses naive (2%) expectations, the third uses inflation expectations coming from Aruoba (2020), the fifth from the Cleveland Fed model, the sixth from Grishchenko, Mouabbi and Renne (2019), and finally the last row uses break even inflation rates as inflation expectations. The second and fourth column shows the average cost in GDP units. The third column shows the standard error of the average and are obtained by running a regression of the cost variable on a constant with White robust errors. The sample goes from 2001Q1 to 2021Q1. Legend: * p<.1; ** p<.05; *** p<.01.

# C Theoretical Results Derivation

In this appendix, we present the detailed derivations for the theoretical results in section 5. We start with the first experiment, where we highlight the costs on legacy debt, with the government refinancing only the maturing debt. This is equivalent in having $L_{t+j} = \delta D_{t+j} = \delta D_t$. The system we study becomes:

$$
\begin{aligned}
L_t &= \delta^d D \\
R_t^{ave} &= \left(1 - \frac{L_t}{D}\right) R_{t-1}^{ave} + \frac{L_t}{D} R_t^{new} \\
S_t &= R_{t-1}^{ave} D \\
\frac{1}{(\delta^d + R_t^{new})} &= \mathbb{E}_t \sum_{j=1}^{\infty} \left[ \frac{1}{\prod_{k=1}^{j} \pi_{t+k}} \beta^j (1 - \delta^d)^{j-1} \right]
\end{aligned}
$$

First, let's show the effect of changes inflation expectations in this framework. We establish the effect of a changing the expectations of inflation at some future horizon on newly issued bonds:

$$
\begin{aligned}
\frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})} &= (\delta^d + R)^2 \sum_{j=m}^{\infty} \pi^{-j-1} \beta^j (1 - \delta^d)^{j-1} \\
&= (\delta^d + R)^2 \pi^{-1} (1 - \delta)^{-1} \beta \sum_{j=m}^{\infty} \pi^{-j} \beta^j (1 - \delta^d)^j \\
&= (\delta^d + R)^2 \pi^{-1} (1 - \delta)^{-1} \frac{\left(\frac{\beta(1-\delta)}{\pi}\right)^m}{1 - \frac{\beta(1-\delta)}{\pi}} \\
&= (\delta^d + R)^2 \pi^{-1} (1 - \delta)^{-1} \frac{\left(\frac{1-\delta}{1+R}\right)^m}{1 - \frac{1-\delta}{1+R}} \\
&= \frac{\delta^d + R}{1 + R} \pi^{-1} (1 - \delta)^{-1} \left(\frac{1 - \delta}{1 + R}\right)^m
\end{aligned}
$$

With this result we can highlight the effect of next period expectations only:

$$
\begin{aligned}
\frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} &= \frac{\delta^d + R}{1 + R} \pi^{-1} (1 - \delta)^{-1} \left(\frac{1 - \delta}{1 + R}\right)^1 \\
&= \frac{1 + R}{\pi} \frac{\delta^d + R}{1 + R}
\end{aligned}
$$

That is, the interest rate changes by one over duration, when only short run expectation move. Similarly, we can compute the effect of shifting all future inflation expectation at the same time.

$$
\begin{aligned}
\frac{\partial R^{new}_{t+l}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}} &= (\delta^d + R)^2 \sum_{j=1}^{\infty} j\pi^{-j-1}\beta^j(1-\delta^d)^{j-1} \\
&= (\delta^d + R)^2 \pi^{-2}\beta \sum_{j=0}^{\infty} j \left(\frac{\beta(1-\delta^d)}{\pi}\right)^{j-1} \\
&= (\delta^d + R)^2 \pi^{-2}\beta \frac{1}{\left(1 - \frac{\beta(1-\delta^d)}{\pi}\right)^2} \\
&= (\delta^d + R)^2 \pi^{-2}\beta \frac{(1+R)^2}{(R+\delta^d)^2} \\
&= \pi^{-2}\beta(1+R)^2 \\
&= \frac{1+R}{\pi}
\end{aligned}
$$

If we have a parallel shift in inflation expectations, the change in interest rates is one to one irrespective of maturity. Next we show the effect of a change in newly issued debt from period $t+1$ on average interest rates in the future.

$$
\frac{\partial R^{ave}_{t+k}}{\{\partial R^{new}_{t+l}\}_{l=0}^{k}} = \sum_{l=0}^{k} (1-\delta^d)^{k-l} \frac{1}{\prod_{m=l}^{k} \pi_{t+m} g_{t+m}} \frac{l_{t+l}}{d_{t+k}}
$$

From steady state:

$$
\begin{aligned}
\frac{\partial R^{ave}_{t+k}}{\{\partial R^{new}_{t+l}\}_{l=0}^{k}} &= \sum_{l=0}^{k} \left(1-\delta^d\right)^{k-l} \delta^d \\
&= \frac{1 - \left(1-\delta^d\right)^{k+1}}{1-(1-\delta^d)} \delta^d \\
&= 1 - \left(1-\delta^d\right)^{k+1}
\end{aligned}
$$

Next, solve for the effect of a change in newly issued rates on the fiscal burden.

$$
\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial R_{t+l}^{new}\}_{l=0}^{\infty}} &= \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} \frac{\partial s_{t+j}}{\partial R_{t+j-1}^{ave}} \frac{\partial R_{t+j-1}^{ave}}{\{\partial R_{t+l}^{new}\}_{l=0}^{k}} \\
&= \sum_{j=1}^{\infty} \left(\beta \frac{1}{\pi}\right)^j D \left(1 - \left(1-\delta^d\right)^j\right) \\
&= D \left( \frac{\frac{1}{1+R}}{1 - \frac{1}{1+R}} - \frac{\frac{1-\delta^d}{1+R}}{1 - \frac{1-\delta^d}{1+R}} \right) \\
&= D \left( \frac{1}{R} - \frac{1-\delta^d}{\delta^d + R} \right) \\
&= D \frac{\delta}{R} \frac{1+R}{\delta^d + R}
\end{aligned}
$$

We find the effect of inflation expectations on the fiscal burden. Notice that the effect of a change in expectations in a given future horizon is the same irrespective of the period we are in, therefore the expression simplifies. We consider two cases, first if only inflation expectations on next period adjust:

$$
\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} \\
&= D \frac{\delta^d}{R} \frac{1+R}{\delta^d + R} \frac{1+R}{\pi} \frac{\delta^d + R}{1+R} \\
&= \frac{D}{\pi} \frac{1+R}{R} \delta^d
\end{aligned}
$$

And then if all future expectations adjust:

$$
\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}} \\
&= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{1+R}{\pi} \\
&= D \frac{\delta^d}{R} \frac{1+R}{\delta^d + R} \frac{1+R}{\pi} \\
&= \frac{D}{\pi} \frac{1+R}{R} \delta^d \frac{1+R}{\delta^d + R}
\end{aligned}
$$

We can see that even in the case in which all future expectations adjust the debt burden varies by less in the long debt case, as the legacy debt has its interest rate fixed. In the next step, We show the effect of inflation on the debt burden. First, we show that inflation does not have a direct effect on average interest rates. We can simply see that by substituting the issuance policy of refinancing only maturing debt in the law of motion of average interest rates:

$$\begin{aligned}
R_t^{ave} &= \left(1 - \frac{L_t}{D}\right) R_{t-1}^{ave} + \frac{L_t}{D} R_t^{new} \\
R_t^{ave} &= \left(1 - \delta^d\right) R_{t-1}^{ave} + \delta^d R_t^{new}
\end{aligned}$$

As interest rate on newly issued debt are not affected by realized inflation, holding expectation constant, then also average rates will not be as they are only a function of these new rates. With this result, we find the effects of future inflation at any horizon on primary surpluses:

$$\begin{aligned}
\frac{\partial s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} &= \sum_{l=1}^{\infty} \frac{\partial s_{t+j}}{\partial \pi_{t+l}} \\
&= -R_{t+j-1}^{ave} D \frac{1}{\prod_{k=1}^{j} g_{t+k}\pi_{t+k}} \sum_{l=1}^{j} \frac{1}{\pi_{t+l}} \\
&= -RD\frac{1}{\pi} j \left(\frac{1}{\pi g}\right)^j
\end{aligned}$$

Next, let's study what happens if inflation increases from tomorrow $t+1$ onward on the fiscal burden.

$$\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} &= \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} \frac{\partial s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} \\
&= -\sum_{j=1}^{\infty} (\beta g)^j \, RD \frac{1}{\pi} j \left(\frac{1}{\pi g}\right)^j \\
&= -\frac{D}{\pi} R \sum_{j=1}^{\infty} \left(\frac{1}{1+R}\right)^j j \\
&= -\frac{D}{\pi} \frac{R}{1+R} \sum_{j=0}^{\infty} \left(\frac{1}{1+R}\right)^{j-1} j \\
&= -\frac{D}{\pi} \frac{1+R}{R}
\end{aligned}$$

Finally, we combine the inflation and the expectations results to study the overall effect on the fiscal burden. We start with the scenario when all inflation expectations adjust.

$$\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}} &\equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})} \\
&= -\frac{D}{\pi} \frac{1+R}{R} + \frac{D}{\pi} \frac{1+R}{R} \delta^d \frac{1+R}{\delta^d + R} \\
&= -\frac{D}{\pi} \frac{1+R}{R} \frac{R + \delta^d - \delta^d - R\delta^d}{\delta^d + R} \\
&= -\frac{D}{\pi} \frac{1+R}{\delta^d + R} (1 - \delta^d)
\end{aligned}$$

Next when only short run expectations adjust:

$$\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} &\equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} \\
&= -\frac{D}{\pi} \frac{1+R}{R} + \frac{D}{\pi} \frac{1+R}{R} \delta^d \\
&= -\frac{D}{\pi} \frac{1+R}{R+\delta^d} \frac{R+\delta^d}{R} (1 - \delta^d)
\end{aligned}$$

## C.1 Solution under constant debt to GDP

In this subsection, we show the same derivations under the second exercise we perform, a constant debt to GDP scenario. The system we study becomes:

$$
\begin{aligned}
f_t &= (R_{t-1}^{ave} + \delta^d)\frac{1}{g_t \pi_t} d \\
d &= (1-\delta^d)\frac{1}{g_t \pi_t} d + l_t \\
R_t^{ave} &= \left(1 - \frac{l_t}{d}\right) R_{t-1}^{ave} + \frac{l_t}{d} R_t^{new} \\
f_t &= s_t + l_t \\
\frac{1}{(\delta^d + R_t^{new})} &= \mathbb{E}_t \sum_{j=1}^{\infty} \left[ \frac{1}{\prod_{k=1}^{j} \pi_{t+k}} \beta^j (1-\delta^d)^{j-1} \right]
\end{aligned}
$$

Some key steady state relationships:

$$
\begin{aligned}
R = R^{new} = R^{ave} &= \frac{\pi}{\beta} - 1 \\
R + \delta^d &= \frac{\pi}{\beta}\left(1 - (1-\delta^d)\frac{\beta}{\pi}\right) \\
d &= \frac{1}{\left(1 - \frac{1-\delta^d}{\pi g}\right)} l \\
d &= s \frac{\frac{\pi g}{1+R}}{1 - \frac{\pi g}{1+R}}
\end{aligned}
$$

First establish the effect of a change in interest rates on newly issued debt an average interest rates:

$$
\frac{\partial R_{t+k}^{ave}}{\{\partial R_{t+l}^{new}\}_{l=0}^{k}} = \sum_{l=0}^{k} (1-\delta^d)^{k-l} \frac{1}{\prod_{m=l}^{k} \pi_{t+m} g_{t+m}} \frac{l_{t+l}}{d_{t+k}}
$$

From steady state:

$$\frac{\partial R^{ave}_{t+k}}{\{\partial R^{new}_{t+l}\}^{k}_{l=0}} = \sum_{l=0}^{k}\left(\frac{1-\delta^d}{\pi g}\right)^{k-l}\left(1-\frac{1-\delta^d}{\pi g}\right)$$
$$= \frac{1-\left(\frac{1-\delta^d}{\pi g}\right)^{k+1}}{1-\frac{1-\delta^d}{\pi g}}\left(1-\frac{1-\delta^d}{\pi g}\right)$$
$$= \left(1-\left(\frac{1-\delta^d}{\pi g}\right)^{k+1}\right)$$

Next look at the effect of changing interest rates on newly issued rates on the fiscal burden.

$$\frac{\partial\sum_{j=1}^{\infty}\beta^j\prod_{k=1}^{j}g_{t+k}s_{t+j}}{\{\partial R^{new}_{t+l}\}^{\infty}_{l=0}} = \sum_{j=1}^{\infty}\beta^j\prod_{k=1}^{j}g_{t+k}\frac{\partial s_{t+j}}{\partial R^{ave}_{t+j-1}}\frac{\partial R^{ave}_{t+j-1}}{\{\partial R^{new}_{t+l}\}^{k}_{l=0}}$$
$$= \sum_{j=1}^{\infty}(\beta g)^j\frac{1}{g\pi}d\left(1-\left(\frac{1-\delta^d}{\pi g}\right)^{j}\right)$$
$$= d\frac{1}{g\pi}\left(\frac{\frac{\pi g}{1+R}}{1-\frac{\pi g}{1+R}}-\frac{\frac{1-\delta^d}{1+R}}{1-\frac{1-\delta^d}{1+R}}\right)$$
$$= d\frac{1}{1+R}\frac{1}{\left(1-\frac{\pi g}{1+R}\right)}\frac{1-\frac{1-\delta^d}{\pi g}}{\left(1-\frac{1-\delta^d}{1+R}\right)}$$
$$= \frac{d}{1+R}\frac{1}{\left(1-\frac{\pi g}{1+R}\right)}\frac{1+R}{\delta^d+R}\left(1-\frac{1-\delta^d}{\pi g}\right)$$

The first term of this expression simply scales the change by the steady state values of the fiscal burden and of the interest rate. Notice that, as we care about the effect as a percentage of GDP, the debt level $d$ shows how big it will be. The second term shows that the effect is permanent as it is one over one minus the discount factor. Maturity matters chiefly as the third element is duration. The last term also depends on maturity and scales duration by the fact that with positive nominal growth legacy nominal debt will not have an effect as large as in a no growth economy.

We now combine the effect of a change in the inflation expectation on interest rates with the effect of the change in interest rates on the fiscal burden to find the effect of inflation expectations on the fiscal burden. Notice that the effect of a change in expectations in a given future horizon is the same irrespective of the period we are in, therefore the expression simplifies. We consider two cases, first if only inflation expectations on next period adjust:

$$\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\partial d_t}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})} \\
&= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{1+R}{\pi} \frac{\delta^d + R}{1+R} \\
&= \frac{d}{1+R} \frac{1}{\left(1 - \frac{\pi g}{1+R}\right)} \frac{1+R}{\delta^d + R} \left(1 - \frac{1-\delta^d}{\pi g}\right) \frac{1+R}{\pi} \frac{\delta^d + R}{1+R} \\
&= \frac{d}{\pi} \frac{1}{\left(1 - \frac{\pi g}{1+R}\right)} \left(1 - \frac{1-\delta^d}{\pi g}\right)
\end{aligned}$$

And then if all future expectations adjust:

$$\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}\}_{l=0}^{\infty}} &= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{\partial R_{t+l}^{new}}{\{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{m=1}^{\infty}} \\
&= \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial R_{t+l}^{new}} \frac{1+R}{\pi} \\
&= \frac{d}{1+R} \frac{1}{\left(1 - \frac{\pi g}{1+R}\right)} \frac{1+R}{\delta^d + R} \left(1 - \frac{1-\delta^d}{\pi g}\right) \frac{1+R}{\pi} \\
&= \frac{d}{\pi} \frac{1}{\left(1 - \frac{\pi g}{1+R}\right)} \frac{1+R}{\delta^d + R} \left(1 - \frac{1-\delta^d}{\pi g}\right)
\end{aligned}$$

We can see that even in the case in which all future expectations adjust the debt burden varies by less in the long debt case, as the legacy debt has its interest rate fixed. We now move to the effect of inflation on the debt burden. As a first step, find the effects of inflation on primary surpluses:

$$\begin{aligned}
\frac{\partial s_{t+j}}{\partial \pi_{t+j}} &= \frac{\partial f_{t+j}}{\partial \pi_{t+j}} - \frac{\partial l_{t+j}}{\partial \pi_{t+j}} \\
&= -(R_{t+j-1}^{ave} + \delta^d) \frac{1}{g_{t+j} \pi_{t+j}^2} d + \frac{1}{g_{t+j} \pi_{t+j}} d \frac{\partial R_{t+j-1}^{ave}}{\partial \pi_{t+j}} - (1-\delta^d) \frac{1}{g_{t+j} \pi_{t+j}^2} d \\
&= -(R_{t+j-1}^{ave} + 1) \frac{1}{g_{t+j} \pi_{t+j}^2} d + \frac{1}{g_{t+j} \pi_{t+j}} d \frac{\partial R_{t+j-1}^{ave}}{\partial \pi_{t+j}} \\
&= -(R_{t+j-1}^{ave} + 1) \frac{1}{g_{t+j} \pi_{t+j}^2} d
\end{aligned}$$

Where the last step uses the fact that average interest rates prevailing in the last period are not affected by inflation in the current period, again holding expectation constant. Next, we show that changes in inflation do not have an effect also on the contemporaneous average interest rate if we are evaluating the change from steady state.

$$\begin{aligned}
\frac{\partial R^{ave}_{t+j}}{\partial \pi_{t+j}} &= \left(1 - \frac{l_t}{d}\right) \frac{\partial R^{ave}_{t+j-1}}{\partial \pi_{t+j}} + \frac{l_t}{d} \frac{\partial R^{new}_{t+j}}{\partial \pi_{t+j}} + \frac{1}{d} \left(R^{new}_{t+j} - R^{ave}_{t+j-1}\right) \frac{\partial l_{t+j}}{\partial \pi_{t+j}} \\
&= \frac{1}{d} \left(R - R\right) \frac{\partial l_{t+j}}{\partial \pi_{t+j}} \\
&= 0
\end{aligned}$$

Inflation in the past will not affect average interest rates as they are have an effect on average interest rates today given their effect on contemporaneous average interest rates, which we just showed to be zero. Armed with this results, we can show the effect of inflation on the surplus at any horizon following the shock:

$$\begin{aligned}
\frac{\partial s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} &= \sum_{l=1}^{\infty} \frac{\partial s_{t+j}}{\partial \pi_{t+l}} \\
&= \frac{\partial s_{t+j}}{\partial \pi_{t+j}} \\
&= -(R^{ave}_{t+j-1} + 1) \frac{1}{g_{t+j} \pi^2_{t+j}} d + \frac{1}{g_{t+j} \pi_{t+j}} d \frac{\partial R^{ave}_{t+j-1}}{\partial \pi_{t+j}} \\
&= \frac{1 + R}{\pi} \frac{1}{g\pi} d
\end{aligned}$$

Next, let's study what happens if inflation increases from tomorrow $t + 1$ onward on the fiscal burden.

$$
\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} &= \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} \frac{\partial s_{t+j}}{\{\partial \pi_{t+l}\}_{l=1}^{\infty}} \\
&= -\sum_{j=1}^{\infty} (\beta g)^j \frac{1+R}{\pi} \frac{1}{g\pi} d \\
&= -\frac{\beta g}{1-\beta g} \frac{1+R}{\pi} \frac{1}{g\pi} d \\
&= -\frac{\frac{\pi g}{1+R}}{1-\frac{\pi g}{1+R}} \frac{1+R}{\pi} \frac{1}{g\pi} d \\
&= -d \frac{1}{\pi} \frac{1}{1-\frac{\pi g}{1+R}}
\end{aligned}
$$

Finally, we combine the effects to study the joint effect of inflation and expectations under the two scenarios. Start with the case when all inflation expectation adjust to the new level of inflation.

$$
\begin{aligned}
\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}} &\equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \sum_{m=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+m})} \\
&= -d \frac{1}{\pi} \frac{1}{1-\frac{g\pi}{1+R}} \left[ 1 - \frac{1+R}{\delta^d + R} \left( 1 - \frac{1-\delta^d}{\pi g} \right) \right] \\
&= -d \frac{1}{\pi} \frac{1}{1-\frac{g\pi}{1+R}} \left[ \frac{(\delta^d + R)\pi g - (1+R)(\pi g - 1 + \delta^d)}{(\delta^d + R)\pi g} \right] \\
&= -d \frac{1}{\pi} \frac{1}{1-\frac{g\pi}{1+R}} \left[ \frac{\delta^d \pi g - R(-1+\delta^d) - (\pi g - 1 + \delta^d)}{(\delta^d + R)\pi g} \right] \\
&= -d \frac{1}{\pi} \frac{1}{1-\frac{g\pi}{1+R}} \left[ \frac{(1-\delta^d)\left(\frac{1+R}{\pi g} - 1\right)}{\delta^d + R} \right] \\
&= -d \frac{1}{\pi} \frac{1}{1-\frac{g\pi}{1+R}} \frac{1+R}{\delta^d + R} \frac{1-\delta^d}{\pi g} \left( 1 - \frac{\pi g}{1+R} \right) \\
&= -d \frac{1}{\pi} \frac{1+R}{\delta^d + R} \frac{1-\delta^d}{\pi g}
\end{aligned}
$$

Next when only short run expectations adjust:

$$\frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}} \equiv \sum_{l=1}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \pi_{t+l}} + \sum_{l=0}^{\infty} \frac{\partial \sum_{j=1}^{\infty} \beta^j \prod_{k=1}^{j} g_{t+k} s_{t+j}}{\partial \mathbb{E}_{t+l}(\pi_{t+l+1})}$$
$$= -d\frac{1}{\pi}\frac{1}{1-\frac{g\pi}{1+R}}\left[1-\left(1-\frac{1-\delta^d}{\pi g}\right)\right]$$
$$= -d\frac{1}{\pi}\frac{1}{1-\frac{g\pi}{1+R}}\frac{1-\delta^d}{\pi g}$$
$$= -d\frac{1}{\pi}\frac{1}{R+1-g\pi}\frac{1+R}{\delta^d+R}(\delta^d+R)\frac{1-\delta^d}{\pi g}$$
$$= -d\frac{1}{\pi}\frac{R+\delta^d}{R-(g\pi-1)}\frac{1+R}{\delta^d+R}\frac{1-\delta^d}{\pi g}$$

These results are summarized in Table C.1. In the first row we show the effect under any arbitrary maturity and in the second row we show the same effect under short debt. Overall the same pattern emerges. When debt is short, it does not matter if the inflation target is missed or not, the fiscal burden does not vary. When debt is long there are potentially large variations in the fiscal burden. When all expectations adjust the variation is approximately Duration-to-GDP. With long debt $\frac{1-\delta^d}{\pi g}$ is close to one. When only short run inflation expectation move, the fiscal burden varies a lot more. The multiplicative factor $\frac{R+\delta^d}{R-(g\pi-1)}$ is larger than in the legacy debt only case as we are subtracting the net nominal growth rate of the economy $g\pi - 1$ from net nominal interest rates $R$.

Table C.1: Inflation, Maturity, and Fiscal Burdens with Debt Stabilization

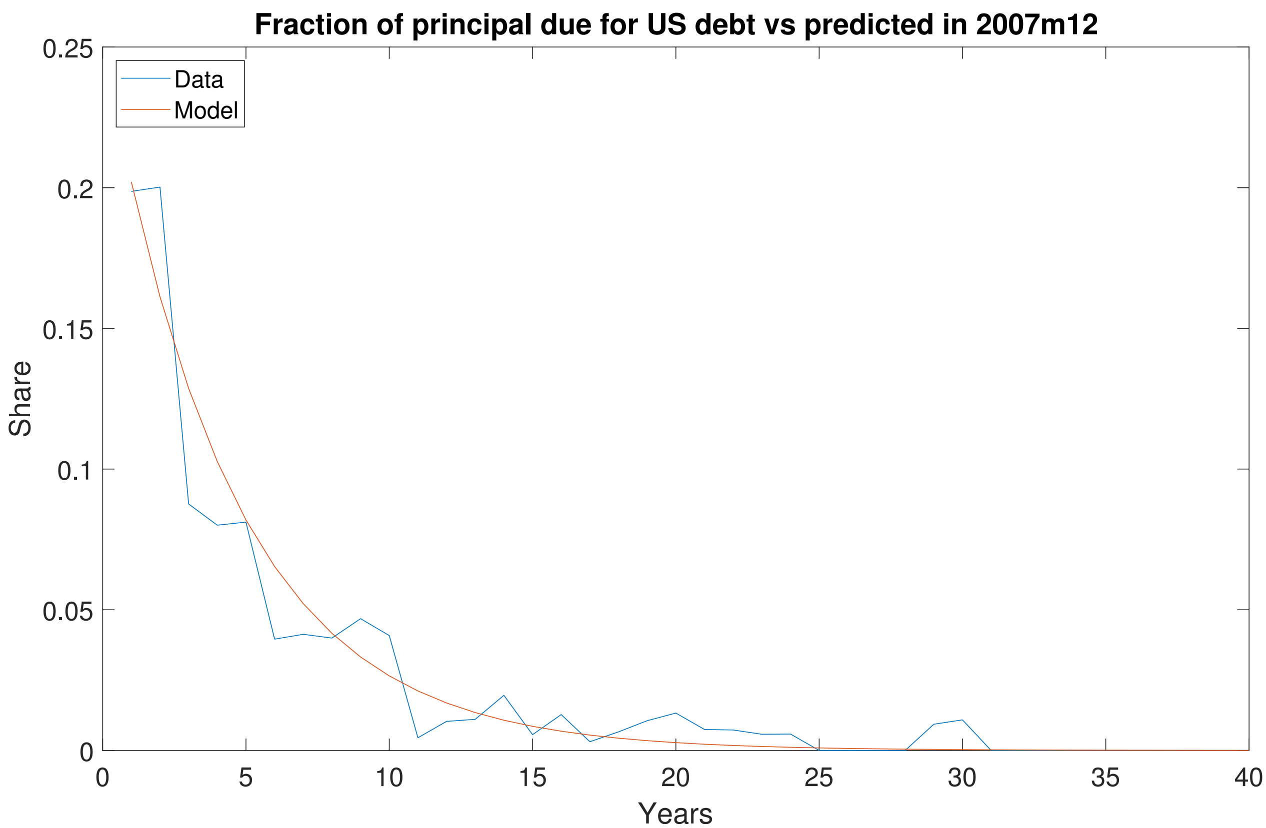
| | Long run expectations do not adjust $\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+1})\}_{l=0}^{\infty}$ | Long run expectations adjust $\{\{\partial \pi_{t+1+l}, \mathbb{E}_{t+l}(\pi_{t+l+m})\}_{l=0}^{\infty}\}_{m=1}^{\infty}$ |
|---|---|---|
| Any Maturity $\delta^d$ | $-\frac{d}{\pi}\frac{1+R}{\delta^d+R}\frac{1-\delta^d}{\pi g}\frac{R+\delta^d}{R-(g\pi-1)}$ | $-\frac{d}{\pi}\frac{1+R}{\delta^d+R}\frac{1-\delta^d}{\pi g}$ |
| Short Debt $\delta^d = 1$ | 0 | 0 |

*Notes:* This table shows the fiscal costs missing the inflation targets under different maturity and expectation regimes. We take a permanent increase in inflation from period $t+1$ onward, this increase is announced in period $t$. In the second column, the announcement is incorporated only on the short run expectations, that is in the following period expectations. In the third column, the announcement is incorporated in all future expectations. In the first row, we show the variation in the fiscal burden (the net present value of debt servicing costs) under any arbitrary maturity, with longer maturity being associated with a lower $\delta^d$. In the second row we show the effect under short debt, i.e. $\delta^d = 1$. In this experiment the government keeps a constant debt to GDP ratio $d$.

# D Data Fit of Geometric Approximation Model

The proposed geometric approximation model has the benefit of fully characterizing public debt and its law of motion with a small number of state variables. This allows to prove a number of results, as the market value to interest rate saving mapping that Duration-to-GDP allows and to avoid the curse of dimensionality that models with unrestricted maturity choices face. However, an important question that arises from this model is how good it is as an approximation for actual public debt promises. We cannot answer this question with the dataset used in this paper, as we have only information on averages: Macaulay duration and average life. However, we can use the results from Andreolli (2022) on US data with bond-by-bond data. He shows that the model has a high fit on actual public debt promises: he computes the $R^2$ in each period of his US estimation sample on annual model predicted principal debt promises on actual promises. The average R2 is high at 0.9001 and its standard deviation is low at 0.0306. He uses bond-by-bond data, which are nominally fixed rate, marketable bonds held by the public. The full description of the data construction can be found in Appendix A of Andreolli (2022). Figure D.1 shows on a given period (December 2007) the model predictions vis-à-vis the data. The key take away is that this model does a good job in approximating the actual maturity choices of the US treasury, so that we do not lose much in precision but we gain in tractability and in being able to use average maturity data.

Figure D.1: Geometric Approximation Fit




*Notes:* The figure presents the fraction of principal debt promises in the data and in the geometric approximation model for the United States in December 2007. The data comes from bond-by-bond data which are nominally fixed rate, marketable bonds held by the public. The model fits this data with the maturity parameter.

# E Time Varying Maturity and Recursive Formulation

In order to fit the quantitative model to the data we need to accommodate for the existence of a time varying maturity structure. In this appendix, we show how to extend the framework from first principles while retaining a recursive structure under a first order approximation. Let's assume that in each period the government issues new debt $L_t$ with a geometrically decaying principal with an interest rate $R_t^{new}$ and a maturity parameter $\delta_t^{new}$. With this formulation, each bond vintage has a geometric structure, but the overall debt stock does not as we do not have a unique discounting parameter. However, we show that for a small deviation from steady state the overall debt stock retains a geometric structure with the average principal repayment. Let's start by defining the non-approximated stock of debt, average principal due, average interest rate on debt:

$$\tilde{D}_t \equiv \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j L_{t-j}$$
$$\tilde{\delta}_t^{ave}\tilde{D}_t \equiv \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j L_{t-j}\delta_{t-j}^{new}$$
$$\tilde{R}_t^{ave}\tilde{D}_t \equiv \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j L_{t-j}R_{t-j}^{new}$$

Divide by real GDP ($Y_t$) and the price level ($P_t$) at various horizons and define cumulative growth: $g_{t|t-j} \equiv \frac{Y_t}{Y_{t-j}}$ and cumulative inflation: $\pi_{t|t-j} \equiv \frac{P_t}{P_{t-j}}$. Lower case letters define issuance and debt over nominal GDP ($l_t \equiv \frac{L_t}{Y_tP_t}$ and $\tilde{d}_t \equiv \frac{\tilde{D}_t}{Y_tP_t}$).

$$\tilde{d}_t \equiv \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j \frac{1}{g_{t|t-j}\pi_{t|t-j}} l_{t-j}$$
$$\tilde{\delta}_t^{ave}\tilde{d}_t \equiv \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j \frac{1}{g_{t|t-j}\pi_{t|t-j}} l_{t-j}\delta_{t-j}^{new}$$
$$\tilde{R}_t^{ave}\tilde{d}_t \equiv \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j \frac{1}{g_{t|t-j}\pi_{t|t-j}} l_{t-j}R_{t-j}^{new}$$

We are going to guess a recursive formulation for the three variables:

$$d_t = l_t + \frac{1-\delta^{ave}_{t-1}}{\pi_{t|t-1} g_{t|t-1}} d_{t-1} \tag{5}$$

$$\delta^{ave}_t d_t = l_t \delta^{new}_t + \frac{1-\delta^{ave}_{t-1}}{\pi_{t|t-1} g_{t|t-1}} \delta^{ave}_{t-1} d_{t-1} \tag{6}$$

$$R^{ave}_t d_t = l_t R^{new}_t + \frac{1-\delta^{ave}_{t-1}}{\pi_{t|t-1} g_{t|t-1}} R^{ave}_{t-1} d_{t-1} \tag{7}$$

Notice that in steady state the two formulations are equivalent with $\tilde{\delta}^{ave} = \delta^{ave} = \delta^{new} = \delta$, $\tilde{d} = d = l \frac{1}{1-\frac{1-\delta}{\pi g}}$ (with no nominal growth $\pi g = 1$, this simplifies to $d = l/\delta$), and $\tilde{R}^{ave} = R^{ave} = R^{new} = R$, where a variable without subscript indicates the steady state value. Start by taking a first order Taylor approximation of the recursive formulation, where we linearize debt, issuance, interest rates and the decaying parameter and log-linearize growth, and inflation. $\hat{x}_t \equiv x_t - x$ for $l$, $d$, $R$, and $\delta$ and $\hat{x}_t \equiv \frac{x_t - x}{x}$ for $\pi$ and $g$. The law of motion of the debt stock:

$$\hat{d}_t = \hat{l}_t + \frac{1-\delta}{\pi g} \hat{d}_{t-1} - \frac{1-\delta}{\pi g} d(\hat{\pi}_{t|t-1} + \hat{g}_{t|t-1}) - \frac{1}{\pi g} d\hat{\delta}^{ave}_{t-1}$$

The law of motion of the average fraction of debt to be repaid:

$$\begin{aligned} d\hat{\delta}^{ave}_t + \delta \hat{d}_t &= l\hat{\delta}^{new}_t + \delta \left[ \hat{l}_t + \frac{1-\delta}{\pi g} \hat{d}_{t-1} - \frac{1-\delta}{\pi g} d(\hat{\pi}_{t|t-1} + \hat{g}_{t|t-1}) - \frac{1}{\pi g} d\hat{\delta}^{ave}_{t-1} \right] + \frac{1-\delta}{\pi g} d\hat{\delta}^{ave}_{t-1} \\ d\hat{\delta}^{ave}_t &= l\hat{\delta}^{new}_t + \frac{1-\delta}{\pi g} d\hat{\delta}^{ave}_{t-1} \\ \hat{\delta}^{ave}_t &= \left(1 - \frac{1-\delta}{\pi g}\right) \hat{\delta}^{new}_t + \frac{1-\delta}{\pi g} \hat{\delta}^{ave}_{t-1} \end{aligned}$$

The law of motion of the average interest rate:

$$\begin{aligned} d\hat{R}^{ave}_t + R\hat{d}_t &= l\hat{R}^{new}_t + R \left[ \hat{l}_t + \frac{1-\delta}{\pi g} \hat{d}_{t-1} - \frac{1-\delta}{\pi g} d(\hat{\pi}_{t|t-1} + \hat{g}_{t|t-1}) - \frac{1}{\pi g} d\hat{\delta}^{ave}_{t-1} \right] + \frac{1-\delta}{\pi g} d\hat{R}^{ave}_{t-1} \\ \hat{R}^{ave}_t &= \left(1 - \frac{1-\delta}{\pi g}\right) \hat{R}^{new}_t + \frac{1-\delta}{\pi g} \hat{R}^{ave}_{t-1} \end{aligned}$$

We now move to the variables without the recursiveness guess. Start again with the law of motion of the stock of debt.

$$
\begin{aligned}
\tilde{d}_t &= \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j \frac{1}{g_{t|t-j}\pi_{t|t-j}} l_{t-j} \\
\tilde{d}_t &= \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j \frac{1}{\prod_{k=0}^{j-1} g_{t-k|t-k-1}\pi_{t-k|t-k-1}} l_{t-j} \\
\hat{\tilde{d}}_t &= \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j \hat{l}_{t-j} - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \sum_{k=0}^{j-1}\left(\hat{g}_{t-k|t-k-1}+\hat{\pi}_{t-k|t-k-1}\right) - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \frac{1}{1-\delta} j \hat{\delta}_{t-j}^{new}
\end{aligned}
$$

Use this expression in the law of motion of the average principal repayment.

$$
\begin{aligned}
\tilde{\delta}_t^{ave}\tilde{d}_t &= \sum_{j=0}^{\infty}(1-\delta_{t-j}^{new})^j \frac{1}{\prod_{k=0}^{j-1} g_{t-k|t-k-1}\pi_{t-k|t-k-1}} l_{t-j}\delta_{t-j}^{new} \\
\tilde{d}\hat{\tilde{\delta}}_t^{ave} + \delta\hat{\tilde{d}}_t &= \delta\left[\sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j \hat{l}_{t-j} - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \sum_{k=0}^{j-1}\left(\hat{g}_{t-k|t-k-1}+\hat{\pi}_{t-k|t-k-1}\right)\right. \\
&\quad \left. - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \frac{1}{1-\delta} j \hat{\delta}_{t-j}^{new}\right] + \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \hat{\delta}_{t-j}^{new} \\
\tilde{d}\hat{\tilde{\delta}}_t^{ave} &= \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \hat{\delta}_{t-j}^{new} \\
\tilde{d}\hat{\tilde{\delta}}_t^{ave} &= l\hat{\delta}_t^{new} + \left(\frac{1-\delta}{\pi g}\right)\tilde{d}\hat{\tilde{\delta}}_{t-1}^{ave} \\
\hat{\tilde{\delta}}_t^{ave} &= \left(1-\frac{1-\delta}{\pi g}\right)\hat{\delta}_t^{new} + \left(\frac{1-\delta}{\pi g}\right)\hat{\tilde{\delta}}_{t-1}^{ave}
\end{aligned}
$$

This last expression shows that $\hat{\tilde{\delta}}_t^{ave}$ has a recursive structure and it is equal to the expression for $\hat{\delta}_t^{ave}$. We are going to use this result back in the last term of the debt stock formula:

$$
\begin{aligned}
\sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{j} l\frac{1}{1-\delta}j\hat{\delta}_{t-j}^{new} &= \sum_{k=1}^{\infty}\sum_{j=k}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{j} l\frac{1}{1-\delta}\hat{\delta}_{t-j}^{new} \\
&= \frac{1}{1-\delta}\sum_{k=1}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{k}\sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{j} l\hat{\delta}_{t-k-j}^{new} \\
&= \frac{1}{1-\delta}\sum_{k=1}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{k}\tilde{d}\hat{\tilde{\delta}}_{t-k}^{ave} \\
&= \frac{1}{1-\delta}\frac{1}{1-\frac{1-\delta}{\pi g}}\sum_{k=1}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{k} l\hat{\tilde{\delta}}_{t-k}^{ave} \\
&= \frac{1}{1-\delta}\frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}}\sum_{k=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{k} l\hat{\tilde{\delta}}_{t-1-k}^{ave}
\end{aligned}
$$

Use a similar method to express the cumulative inflation and growth:

$$
\begin{aligned}
&\sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{j} l\sum_{k=0}^{j-1}\left(\hat{g}_{t-k|t-k-1}+\hat{\pi}_{t-k|t-k-1}\right) = \\
&= 0 + l\left(\frac{1-\delta}{\pi g}\right)\left(\hat{g}_{t|t-1}+\hat{\pi}_{t|t-1}\right) + l\left(\frac{1-\delta}{\pi g}\right)^{2}\left(\hat{g}_{t|t-1}+\hat{\pi}_{t|t-1}+\hat{g}_{t-1|t-2}+\hat{\pi}_{t-1|t-2}\right) \\
&+ l\left(\frac{1-\delta}{\pi g}\right)^{3}\left(\hat{g}_{t|t-1}+\hat{\pi}_{t|t-1}+\hat{g}_{t-1|t-2}+\hat{\pi}_{t-1|t-2}+\hat{g}_{t-2|t-3}+\hat{\pi}_{t-2|t-3}\right) + \dots \\
&= l\frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}}\left(\frac{1-\delta}{\pi g}\right)^{0}\left(\hat{g}_{t|t-1}+\hat{\pi}_{t|t-1}\right) + l\frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}}\left(\frac{1-\delta}{\pi g}\right)^{1}\left(\hat{g}_{t-1|t-2}+\hat{\pi}_{t-1|t-2}\right) \\
&+ l\frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}}\left(\frac{1-\delta}{\pi g}\right)^{2}\left(\hat{g}_{t-2|t-3}+\hat{\pi}_{t-2|t-3}\right) + \dots \\
&= l\frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}}\sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^{j}\left(\hat{g}_{t-j|t-j-1}+\hat{\pi}_{t-j|t-j-1}\right)
\end{aligned}
$$

Plug these results in the debt expression and obtain in a recursive formulation:

$$\hat{\tilde{d}}_t = \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j \hat{l}_{t-j} - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \sum_{k=0}^{j-1}\left(\hat{g}_{t-k|t-k-1} + \hat{\pi}_{t-k|t-k-1}\right) - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \frac{1}{1-\delta} j \hat{\delta}^{new}_{t-j}$$

$$\hat{\tilde{d}}_t = \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j \hat{l}_{t-j} - l \frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}} \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j \left(\hat{g}_{t-j|t-j-1} + \hat{\pi}_{t-j|t-j-1}\right)$$

$$- \frac{1}{1-\delta}\frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}} \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \hat{\tilde{\delta}}^{ave}_{t-1-j}$$

$$\hat{\tilde{d}}_t = \hat{l}_t - l \frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}} (\hat{g}_{t|t-1} + \hat{\pi}_{t|t-1}) - \frac{1}{1-\delta}\frac{\frac{1-\delta}{\pi g}}{1-\frac{1-\delta}{\pi g}} l \hat{\tilde{\delta}}^{ave}_{t-1} + \left(\frac{1-\delta}{\pi g}\right)\hat{\tilde{d}}_{t-1}$$

$$\hat{\tilde{d}}_t = \hat{l}_t - \frac{1-\delta}{\pi g}\tilde{d}(\hat{g}_{t|t-1} + \hat{\pi}_{t|t-1}) - \frac{1}{\pi g}\tilde{d}\hat{\tilde{\delta}}^{ave}_{t-1} + \left(\frac{1-\delta}{\pi g}\right)\hat{\tilde{d}}_{t-1}$$

Which has the same recursive structure and it is equal to $\hat{d}_t$. We can show with similar steps the same result for the average interest rate:

$$\tilde{R}^{ave}_t \tilde{d}_t = \sum_{j=0}^{\infty}(1-\delta^{new}_{t-j})^j \frac{1}{g_{t|t-j}\pi_{t|t-j}} l_{t-j} R^{new}_{t-j}$$

$$\tilde{R}^{ave}_t \tilde{d}_t = \sum_{j=0}^{\infty}(1-\delta^{new}_{t-j})^j \frac{1}{\prod_{k=0}^{j-1} g_{t-k|t-k-1}\pi_{t-k|t-k-1}} l_{t-j} R^{new}_{t-j}$$

$$\tilde{d}\hat{\tilde{R}}^{ave}_t + R\hat{\tilde{d}}_t = R\left[\sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j \hat{l}_{t-j} - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \sum_{k=0}^{j-1}\left(\hat{g}_{t-k|t-k-1} + \hat{\pi}_{t-k|t-k-1}\right)\right.$$

$$\left. - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \frac{1}{1-\delta} j \hat{\delta}^{new}_{t-j}\right] + \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \hat{R}^{new}_{t-j}$$

$$\tilde{d}\hat{\tilde{R}}^{ave}_t = \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \hat{R}^{new}_{t-j}$$

$$\hat{\tilde{R}}^{ave}_t = \left(1-\frac{1-\delta}{\pi g}\right)\hat{R}^{new}_t + \left(\frac{1-\delta}{\pi g}\right)\hat{\tilde{R}}^{ave}_{t-1}$$

This means that the recursive formulation works up to a first order approximation even if the time varying maturity does not yield an exact recursive structure.

## E.1 Note on Secondary Market Price

We can show that the secondary market price of the overall debt stock behaves as in the constant maturity case in the linearized equilibrium. First of all define the secondary market price of the overall stock of public debt the price that multiplies the book value of debt such that the product equal to the issuance weighted sum of secondary market prices of various debt vintages ($q_t^{t-j}$ is the price at time $t$ of debt issued in period $t-j$).

$$\tilde{q}_t^D \tilde{D}_t \equiv \sum_{j=0}^{\infty} (1-\delta_{t-j}^{new})^j L_{t-j} q_t^{t-j}$$

In GDP units:

$$\tilde{q}_t^D \tilde{d}_t \equiv \sum_{j=0}^{\infty} (1-\delta_{t-j}^{new})^j \frac{1}{g_{t|t-j}\pi_{t|t-j}} l_{t-j} q_t^{t-j}$$

As long as no-arbitrage across vintages holds, then Euler equation, for an arbitrary unique stochastic discount factor is:

$$q_t^{t-j} = \mathbb{E}_t \left[ \sum_{k=1}^{\infty} \frac{M_{t+k|t}}{\pi_{t+k|t}} \left( (1-\delta_{t-j}^{new}) q_{t+1}^{t-j} + R_{t-j}^{new} + \delta_{t-j}^{new} \right) \right]$$
$$q_t^{t-j} = \mathbb{E}_t \left[ \sum_{k=1}^{\infty} \frac{M_{t+k|t}}{\pi_{t+k|t}} (1-\delta_{t-j}^{new})^{k-1} \left( R_{t-j}^{new} + \delta_{t-j}^{new} \right) \right]$$

In state state:

$$\frac{1}{R+\delta} = \left[ \sum_{k=1}^{\infty} \frac{M(k)}{\pi^k} (1-\delta)^{k-1} \right]$$

Where $M(k)$ is the steady state stochastic discount factor $k$ periods ahead. Notice that we do not assume an exponential form for the stochastic discount factor, so that we allow for any arbitrary yield curve shape in steady state.

Take a first order Taylor approximation around the steady state of the above expression:

$$\begin{aligned}
\frac{q_t^{t-j}}{R_{t-j}^{new}+\delta_{t-j}^{new}} &= \mathbb{E}_t\left[\sum_{k=1}^{\infty}\frac{M_{t+k|t}}{\pi_{t+k|t}}(1-\delta_{t-j}^{new})^{k-1}\right] \\
\hat{q}_t^{t-j}\frac{1}{R+\delta}-\frac{1}{(R+\delta)^2}(\hat{R}_{t-j}^{new}+\hat{\delta}_{t-j}^{new}) &= \mathbb{E}_t\left[\sum_{k=1}^{\infty}\frac{M(k)}{\pi^k}(1-\delta)^{k-1}(\hat{M}_{t+k|t}-\hat{\pi}_{t+k|t})\right] \\
&\quad -\sum_{k=1}^{\infty}\frac{M(k)}{\pi^k}(1-\delta)^{k-2}(k-1)\hat{\delta}_{t-j}^{new} \\
\hat{q}_t^{t-j}\frac{1}{R+\delta}-\frac{1}{(R+\delta)^2}(\hat{R}_{t-j}^{new}+\hat{\delta}_{t-j}^{new}) &= \mathbb{E}_t\left[\sum_{k=1}^{\infty}\frac{M(k)}{\pi^k}(1-\delta)^{k-1}(\hat{M}_{t+k|t}-\hat{\pi}_{t+k|t})\right] \\
&\quad -\hat{\delta}_{t-j}^{new}\frac{d}{d(1-\delta)}\sum_{k=1}^{\infty}\frac{M(k)}{\pi^k}(1-\delta)^{k-1} \\
\hat{q}_t^{t-j}\frac{1}{R+\delta}-\frac{1}{(R+\delta)^2}(\hat{R}_{t-j}^{new}+\hat{\delta}_{t-j}^{new}) &= \mathbb{E}_t\left[\sum_{k=1}^{\infty}\frac{M(k)}{\pi^k}(1-\delta)^{k-1}(\hat{M}_{t+k|t}-\hat{\pi}_{t+k|t})\right] \\
&\quad -\hat{\delta}_{t-j}^{new}\frac{d}{d(1-\delta)}\frac{1}{R+\delta} \\
\hat{q}_t^{t-j}\frac{1}{R+\delta}-\frac{1}{(R+\delta)^2}(\hat{R}_{t-j}^{new}+\hat{\delta}_{t-j}^{new}) &= \mathbb{E}_t\left[\sum_{k=1}^{\infty}\frac{M(k)}{\pi^k}(1-\delta)^{k-1}(\hat{M}_{t+k|t}-\hat{\pi}_{t+k|t})\right] \\
&\quad -\hat{\delta}_{t-j}^{new}\frac{1}{(R+\delta)^2} \\
\hat{q}_t^{t-j} &= \frac{1}{(R+\delta)}\hat{R}_{t-j}^{new}+(R+\delta)\mathbb{E}_t\left[\sum_{k=1}^{\infty}\frac{M(k)}{\pi^k}(1-\delta)^{k-1}(\hat{M}_{t+k|t}-\hat{\pi}_{t+k|t})\right]
\end{aligned}$$

Where $\frac{d}{d(1-\delta)}$ is the derivative with respect to $1-\delta$. Notice that the last term of the final expression is equal across vintages $j$. We can use this result together with the fact that the price of the current vintage ($j=0$) is always zero, as the interest rate on newly issued debt $R_t^{new}$ adjusts to express the price only in terms of vintage interest rates compared to the current one:

$$\hat{q}_t^{t-j} = \frac{1}{(R+\delta)}\hat{R}_{t-j}^{new}-\frac{1}{(R+\delta)}\hat{R}_t^{new}$$

Armed with this last result we can linearize the expression for secondary market price of overall debt:

$$
\begin{aligned}
\tilde{q}_t^D \tilde{d}_t &\equiv \sum_{j=0}^{\infty} (1-\delta_{t-j}^{new})^j \frac{1}{g_{t|t-j}\pi_{t|t-j}} l_{t-j} q_t^{t-j} \\
\tilde{d}\hat{\tilde{q}}_t^D + q\hat{\tilde{d}}_t &= q\left[\sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j \hat{l}_{t-j} - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \sum_{k=0}^{j-1}\left(\hat{g}_{t-k|t-k-1} + \hat{\pi}_{t-k|t-k-1}\right)\right. \\
&\quad \left. - \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \frac{1}{1-\delta} j \hat{\delta}_{t-j}^{new}\right] + \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \hat{q}_t^{t-j} \\
\tilde{d}\hat{\tilde{q}}_t^D &= \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \hat{q}_t^{t-j} \\
\tilde{d}\hat{\tilde{q}}_t^D &= \sum_{j=0}^{\infty}\left(\frac{1-\delta}{\pi g}\right)^j l \left(\frac{1}{(R+\delta)}\hat{R}_{t-j}^{new} - \frac{1}{(R+\delta)}\hat{R}_t^{new}\right) \\
\tilde{d}\hat{\tilde{q}}_t^D &= \tilde{d}\left(\frac{1}{(R+\delta)}\hat{R}_t^{ave} - \frac{1}{(R+\delta)}\hat{R}_t^{new}\right) \\
\hat{\tilde{q}}_t^D &= \frac{1}{(R+\delta)}\hat{R}_t^{ave} - \frac{1}{(R+\delta)}\hat{R}_t^{new}
\end{aligned}
$$

Which says that the secondary market price will be higher if the average interest rate is with respect to the interest rate on newly issued debt. Notice that this is the same expression as under fixed maturity, and variation in maturity do no affect the secondary market price up to a first order approximation.

# F Counterfactual Experiments Details

In this appendix, we describe how we fit the data in the theoretical model and perform additional experiments.

We use the same dataset as in the empirical exercises. for the main three state variables, bond debt-to-GDP $d_t$, average interest rates at book value $R_t^{ave}$, and average fraction of bond debt due in each period $\delta_t^{ave}$, we use seasonally adjusted government securities debt over GDP from OECD data, government bonds average coupon from WGBI data, and (one over) government bonds average life from WGBI data. We adjust all data to quarterly frequency. Real GDP growth $g_t$ (OECD data for European Countries and Fred for the US) and inflation $\pi_t$ (HICP for the Euro Area and CPI for the US) are QoQ rates. With respect to inflation expectations in the baseline, we use raw SPF data and we assign forecasts in the following way. For the Euro Area expectations, we use current year expectations for the following two quarters, one year ahead expectation for three and four quarters ahead, two years ahead expectations for five to eight quarters ahead, and long run expectations from nine quarters ahead onwards. For the US, we use the same data, but as two years ahead expectations are not recorded in the SPF we use inflation in two calendar years from now for 5 to 8 quarters ahead when available and QoQ inflation 4 quarters ahead when not available.

We measure the interest rate on newly issued debt $R_t^{new}$ with the benchmark interest rate on a 10 years government bond, we use this rate as it is the most liquid and it is available for all countries in the sample for the all sample. We compute issuance over GDP $l_t$ and (one over) maturity on newly issued debt $\delta_t^{new}$ from equations (5) and (6). As this is "flow" derived data from "stock" original data, it is imprecise. Therefore, we filter $\delta_t^{new}$ with a HP filter with a low smoothing parameter (10) and keep the trend component[30]. We ensure that the maximum average maturity for newly issued debt is 20 years (that is, we allow for longer debt, up to consols, but the average maturity of debt issued in a quarter does not exceed

[30] Additionally, we also applied a moving average filter to the data and the results are quite similar to the ones presented in the paper. They are available upon request. Moreover, we also tried to apply the same procedure for interest rates on newly issued debt $R_t^{new}$, by computing it from equation (7), applying the same HP filter with a low smoothing parameter (10) and keeping the trend component consistent with the idea that Debt Management Offices do not usually change abruptly the maturity of newly issued debt. The benefit of this procedure is that we do not need to use the 10 year rate but the cost is that the newly issued rate can be a bit jumpy given that it is a flow measure derived from a stock one. Overall the results are very similar to the ones presented and are available upon request.

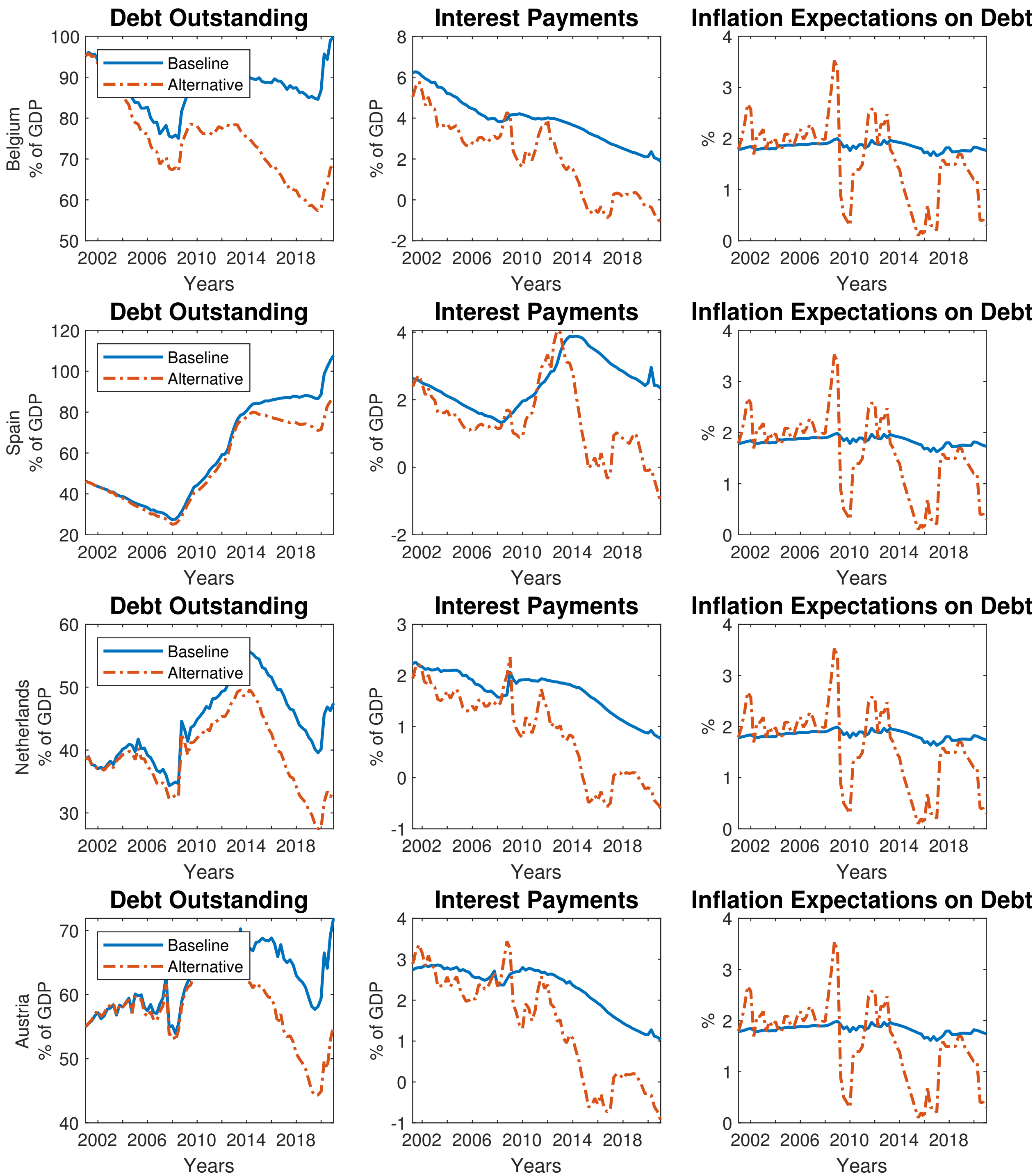
that threshold, as in the data).

The only parameter we calibrate is $\beta$ as 0.995. With this parameter, inflation expectation data, maturity on newly issued debt, and interest rates on newly issued debt we compute the risk premium (or convenience yield or liquidity premium) as the difference between interest rates on newly issued debt and the interest rate that would prevail under risk neutral pricing from the Euler equation. Finally, we also find the net resource needs $s_t$ from equations (5), (6), (7), and the budget constraint.

Table 5 shows the results of this exercise in the baseline sample. Additionally, Table F.1 shows the same results in the whole sample, from 2000Q1 to 2022Q1. The results are very similar.

In our baseline exercise we use Euro Area wide HICP inflation for each Euro Area country for inflation realisations. We do this as this maps directly to the Survey of Professional Forecasters inflation expectations. However, an important question is how using country specific inflation can affect affect results, as country specific inflation is finally what affects debt dynamics. In Table F.2 we rerun the baseline exercise (short vs long debt) under country specific HICP in columns 4 and 5 and under the country specific GDP deflator in columns 6 and 7. We can see how the numbers are very similar across the various scenarios and therefore conclude that the results are not strongly affected by the inflation measure we employ.

Figure F.1: Fiscal Consequences of Missing Inflation Targets - Extended Country Sample




*Notes:* This figure shows the results under the counterfactual exercise. It shows the path of debt, interest payments, and debt-embedded inflation expectations for the Belgium, Spain, the Netherlands, and Austria. The blue solid line shows the path of these variables under the actual maturity structure $\delta_t^{ave}$, the red dot-dashed line shows the path under a counterfactual short debt ($\delta_t^{ave} = 1$). The sample goes from 2001Q1 to 2021Q1.

Table F.1: Counterfactual Exercises - with Recent Period

| Country | Short Debt | | Perfect Foresight | |
|---|---|---|---|---|
| | Debt | Interest Payments | Debt | Interest Payments |
| France | 20.6 | 1.21 | 2.2 | 0.14 |
| Germany | 13.2 | 0.78 | 1.7 | 0.11 |
| Italy | 30.2 | 1.69 | 3.3 | 0.19 |
| Belgium | 29.7 | 1.86 | 2.5 | 0.16 |
| Spain | 20.0 | 1.13 | 1.9 | 0.11 |
| Netherlands | 13.0 | 0.80 | 1.3 | 0.08 |
| Austria | 15.4 | 0.90 | 2.0 | 0.13 |
| United States | 14.4 | 1.05 | 2.6 | 0.20 |

*Notes:* This table shows the results under the counterfactual exercises. Columns 2 and 3 show the counterfactual fiscal burden under a short debt profile ($\delta^d = 1$). Columns 4 and 5 show the counterfactual fiscal burden under perfect foresight, that is in each period $t$ expectations are correct at all future horizon: $\mathbb{E}_t(\pi_{t+j}) = \pi_{t+j}$. For inflation expectations pertaining to periods that have not yet happened in the dataset (after 2022Q1) we use the appropriate inflation expectation. Columns 2 and 4 show the difference in debt-to-GDP level at the last period under the exercise compared to the case where we fit the model with actual data. Columns 3 and 5 show the difference in average interest payments per year under the exercise compared to the case where we fit the model with actual data. The sample goes from 2001Q1 to 2022Q1.

Table F.2: Counterfactual Exercises - Alternative Inflation Measures

| Country | Baseline | | CS HICP | | GDP Deflator | |
|---|---|---|---|---|---|---|
| | Debt | Interest Payments | Debt | Interest Payments | Debt | Interest Payments |
| France | 21.4 | 1.20 | 21.6 | 1.21 | 21.5 | 1.21 |
| Germany | 13.9 | 0.78 | 13.7 | 0.78 | 13.3 | 0.78 |
| Italy | 31.2 | 1.63 | 31.9 | 1.64 | 30.5 | 1.60 |
| Belgium | 31.2 | 1.86 | 30.3 | 1.85 | 29.1 | 1.80 |
| Spain | 20.6 | 1.09 | 20.8 | 1.09 | 19.5 | 1.03 |
| Netherlands | 13.9 | 0.81 | 13.6 | 0.80 | 13.6 | 0.81 |
| Austria | 16.7 | 0.90 | 15.0 | 0.84 | 16.1 | 0.88 |
| United States | 17.0 | 1.14 | | | 17.2 | 1.14 |

*Notes:* This table shows the results under the counterfactual exercises under different measures for realised inflation. All show the counterfactual fiscal burden under a short debt profile ($\delta^d = 1$) compared to the actual maturity. Columns 2 and 3 show the baseline metric for realized inflation Euro Area wide HICP for the Euro Area countries and CPI for the US. Columns 4 and 5 show use Country Specific (CS) HICP for Euro Area countries realised inflation. Columns 6 and 7 show use country specific GDP deflator for Euro Area countries and for the United Stated for realised inflation. Columns 2, 4, and 6 show the difference in debt-to-GDP level at the last period. Columns 3, 5, and 7 show the difference in average interest payments per year. The sample goes from 2001Q1 to 2022Q1.