A THEORY OF FEAR OF FLOATING*

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Abstract

Many central banks classified as flexible exchange rate regimes are reluctant to let the exchange rate fluctuate, a phenomenon known as "fear of floating". We present a simple theory where fear of floating emerges as an optimal policy outcome. The key feature of the model is an occasionally binding borrowing constraint linked to the exchange rate that introduces a feedback loop between aggregate demand and credit conditions. Contrary to the Mundellian paradigm, we show that a depreciation can be contractionary, and letting the exchange rate float can expose the economy to self-fulfilling crises.

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1 Introduction

According to the Mundell-Fleming paradigm, a floating exchange rate plays a pivotal role in stabilizing economic fluctuations in an open economy. By depreciating the exchange rate when a negative shock hits the economy, a central bank can shift demand towards domestic goods and help mitigate the recession. Yet, contrary to this policy prescription, many central banks in emerging markets are often reluctant to let the currency float, particularly when facing turbulence in financial markets (Calvo and Reinhart, 2002).[1] Why do central banks experience a "fear of floating"? Moreover, can a nominal exchange depreciation be contractionary?

This paper explores the idea that letting the exchange rate float may expose the economy to a self-fulfilling financial crisis, where a depreciation of the nominal exchange rate and a contraction in output and consumption mutually reinforce each other. In a simple model featuring household deleveraging and nominal rigidities, we establish that anchoring the nominal exchange rate can help prevent self-fulfilling financial crises. Fear of floating is thus an optimal policy outcome.

The model has two key elements. First, households face a borrowing limit linked to the value of their income. Second, nominal wages are rigid downward. The first feature implies that a reduction in household borrowing can lead to a deterioration of the market value of collateral and can induce self-fulfilling fluctuations (e.g., Schmitt-Grohé and Uribe, 2021). The second feature implies that monetary policy has implications for the real economy, which in turn interact with households' borrowing limits and aggregate demand.

In this environment, a nominal exchange rate depreciation tilts relative demand towards domestically produced goods through the standard expenditure switching channel. However, a depreciation also lowers the relative value of collateral, possibly leading to a contraction in the demand for domestic goods. We show that this second channel may dominate, and then a depreciation turns contractionary. Crucially, we show that anchoring the nominal exchange rate may help the economy avoid a self-fulfilling crisis. The logic is that by stabilizing the exchange rate, the central bank can prevent the feedback loop by which the reduction in the real value of income, aggregate demand and the depreciation mutually reinforce each other.

[1] Calvo and Reinhart (2002) state: "We find that countries that say they allow their exchange rate to float mostly do not—there seems to be an epidemic case of fear of floating."

Related literature. This paper contributes to a vast literature on optimal monetary policy in open economies. A fundamental theme in the literature going back to Mundell (1960) and Friedman (1953) is that a flexible exchange rate regime can insulate the economy from domestic and external shocks.[2] The overarching principle is that by varying the exchange rate—in particular, by depreciating during a recession—the government can adjust relative prices and stabilize output at the efficient level. Here, we present a model where letting the exchange rate float may exacerbate inefficient economic fluctuations and establish that depreciations can be contractionary.

There is also a large literature that has presented models where giving up monetary independence may be desirable. These benefits may emerge from a reduction in the inflationary bias generated by the time inconsistency problem of monetary policy (Alesina and Barro, 2002 and Chari, Dovis and Kehoe, 2020), reduction in transaction costs (Mundell, 1961), or larger risk sharing (Neumeyer, 1998; Arellano and Heathcote, 2010; Fornaro, 2022). Our contribution is to provide a distinct rationale for stabilizing the exchange rate, one that puts the lower vulnerability to self-fulfilling financial crises at the center stage.

We are also related to a literature on monetary policy in the presence of credit frictions and liability dollarization in open economies.[3] This literature has focused on comparing the performance of different monetary regimes in open economies where firms face financial market imperfections. Most of these studies have found that depreciations are expansionary (see, e.g., Gertler et al., 2007 and Céspedes et al., 2004): even though an increase in the exchange rate raises the value of the debt in local currency, a fixed exchange rate may actually exacerbate financial frictions by reducing firms' profitability and make financial constraints more binding. On the other hand, some important studies such as Aghion et al. (2004) and Cook (2004) , featuring sticky non-tradable prices and flexible wages, find that depreciations can be contractionary by tightening firms' balance sheet constraints. A key distinction in our mechanism behind contractionary depreciations is that it operates through households' deleveraging rather than firms' frictions on investment financing. In addition, we also show how a flexible exchange rate may exacerbate the vulnerability to a self-fulfilling financial crisis.

A few recent papers also study monetary policy in models with households' deleveraging (Ottonello, 2021, Farhi and Werning, 2016, Devereux, Young and Yu, 2019, Coulibaly,

[2]Modern treatments of this theme include Schmitt-Grohé and Uribe (2011) among others.

[3]Examples include Aghion, Bacchetta and Banerjee (2000), Aghion, Bacchetta and Banerjee (2004), Gertler, Gilchrist and Natalucci (2007), Caballero and Krishnamurthy (2001), Lahiri and Végh (2001), Cook (2004), Céspedes, Chang and Velasco (2004), Fornaro (2015),Cavallino and Sandri (2018), Gourinchas (2018), Du and Schreger (2016), Ottonello (2021), Coulibaly (2020).

2020, De Ferra, Mitman and Romei, 2020, Basu, Boz, Gopinath, Roch and Unsal, 2020). The key lesson from these studies is that implementing the full employment allocations, while feasible, is not necessarily optimal, either because of a deterioration of credit market access (e.g. Ottonello, 2021) or because of redistribution effects within households (De Ferra et al., 2020).[4] Our results uncover how a depreciation may turn contractionary and provide an argument for keeping the exchange rate fixed in the presence of self-fulfilling fluctuations.

This paper is related to the literature on aggregate demand externalities in the presence of constraints on monetary policy. In Schmitt-Grohé and Uribe (2016) and Farhi and Werning (2016), a fixed exchange rate plays a key role in preventing the government from stabilizing macroeconomic fluctuations and generate scope for macroprudential policy.[5] However, these papers abstract from the source of the rigidities in monetary policy. Our contribution is to provide a theory of why the government finds it optimal to keep the exchange rate fixed.

We are also related to a literature on financial fragility emerging from multiple equilibria. Our framework is most closely related to Schmitt-Grohé and Uribe (2021).[6] Different from their work, we consider a monetary model with nominal rigidities, which allows us to speak about how different exchange rate regimes affect the vulnerability to self-fulfilling financial crises.

Outline. Section 2 presents the model and Section 3 and 4.3 present the theoretical analysis on self-fulfilling crises and contractionary depreciations. Section 4.4 analyzes sophisticated monetary policy rules to rule out self-fulfilling crises. Section 5 concludes.

[4] In a different vein, Auclert, Rognlie, Souchier and Straub (2021) argue that a depreciation may be contractionary in the context of international terms of trade effects and heterogeneity.

[5] Relatedly, in Bianchi and Lorenzoni (2021), the government has a flexible exchange rate regime, but there is a utility cost from exchange rate fluctuations. Acharya and Bengui (2018), Fornaro and Romei (2019), and Bianchi and Coulibaly (2021) consider instead a zero lower bound constraint constraints (see also Korinek and Simsek (2016) for a closed economy analysis).

[6] Multiple equilibria also play a crucial role in early contributions by Chang and Velasco (2000, 2001), Aghion et al. (2000), Krugman (1999) as well as more recent work by Bocola and Lorenzoni (2020), among many others. Earlier work by Sachs (1984), Calvo (1988) and Obstfeld (1984) displayed a different form of multiple equilibria, emerging instead from a strategic game between the government and private agents. In this vein, the work by Corsetti, Pesenti and Roubini (1999) and Schneider and Tornell (2004) emphasize bailout expectations and Cole and Kehoe (1996) emphasize roll over problems (see also Bianchi and Mondragon, 2022).

2 Model

We consider a small open economy with two types of goods: tradables and non-tradables. Time is discrete and infinite. The economy features nominal rigidities and constraints on households' borrowing.

2.1 Households

There is a continuum of identical households of measure one. Households have preferences of the form

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \chi \log \left(\frac{M_{t+1}}{\mathcal{P}_t} \right) \right], \tag{1}$$

with $\chi \geq 0$ and where $\beta \in (0,1)$ is the discount factor. The consumption good c_t is a composite of tradable consumption c_t^T and non-tradable consumption c_t^N, according to a constant elasticity of substitution aggregator:

$$c_t = \left[\phi (c_t^T)^{\frac{\gamma-1}{\gamma}} + (1-\phi)(c_t^N)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \quad \text{where } \phi \in (0,1).$$

The elasticity of substitution between tradable and non-tradable consumption is γ. For convenience, we use $u(c^T, c^N)$ to denote the utility as a function of the two consumption goods. The real money holdings, $M_{t+1}/\mathcal{P}_t$, provide liquidity services to households that enter the utility function where M_{t+1} is the end-of-period money holdings and $\mathcal{P}_t$ is the ideal price index in period t. Denoting by P_t^N and P_t^T respectively the price of non-tradables and tradables (in terms of the domestic currency), the ideal price index satisfies,

$$\mathcal{P}_t = \left[\phi^\gamma \left(P_t^T \right)^{1-\gamma} + (1-\phi)^\gamma \left(P_t^N \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}.$$

We assume that the law of one price holds for the tradable good, that is, $P_t^T = e_t P_t^{T*}$, where e_t is the nominal exchange rate defined as the price of the foreign currency in terms of the domestic currency, and P_t^{T*} is the price of the tradable good denominated in foreign currency. We normalize the price of the tradable good in units of foreign currency to unity.

Households supply $\bar{h}$ units of labor inelastically. Because of the presence of downward wage rigidity and rationing (to be described below), each household's actual hours worked are given by $h_t \leq \bar{h}$, which is taken as given by the household. Each period households receive a wage rate, W_t, and government transfers, T_t, all expressed in terms of domestic currency, which serves as the numeraire, and receive y_t^T units of tradable goods. We assume that y_t^T is stochastic and follows a first-order Markov process. Households trade two types of one-period non-state-contingent bonds in credit markets: an foreign currency bond that pays a constant net return of R units of tradables, and a nominal domestic currency bond which pays $\tilde{R}_t$ in units of domestic currency. The budget constraint of the representative household is therefore given by

$$P_t^T c_t^T + P_t^N c_t^N + M_{t+1} + \tilde{b}_t + e_t b_t = P_t^T y_t^T + W_t h_t + M_t + \frac{\tilde{b}_{t+1}}{\tilde{R}_t} + \frac{e_t b_{t+1}}{R} + T_t. \tag{2}$$

where $\tilde{b}_t$ and b_t denote respectively the amount of domestic currency debt and foreign currency debt assumed in period $t-1$ and due in period t. The left-hand side represents total expenditures in tradable and non-tradable goods and purchases of bonds while the right-hand side represents total income, including the returns from bond issuance.

Households face a borrowing constraint that limits foreign currency debt to fraction κ of their individual current income:[7]

$$\frac{e_t b_{t+1}}{R} \leq \kappa \left[P_t^T y_t^T + W_t h_t \right]. \tag{3}$$

This borrowing constraint captures the idea that current earnings are a critical factor determining credit-market access (see e.g. Jappelli, 1990; Lian and Ma, 2020) and has been shown to be important for accounting for the dynamics of capital flows in emerging markets (e.g., Mendoza, 2002; Bianchi, 2011).[8] To ensure that the borrowing constraint is tighter than the natural debt limit, we assume $0 < \kappa < R/(R-1)$.

Optimality conditions. Optimality with respect to c_t^T and c_t^N imply that

$$\frac{c_t^N}{c_t^T} = \left(\frac{\phi}{1-\phi} \frac{P_t^N}{e_t} \right)^{-\gamma} \tag{4}$$

Denoting by $\mu \cdot u_T \geq 0$ the Lagrange multipliers on the collateral constraint where u_T is the marginal utility of tradable consumption, the Euler equation (5) equates the marginal

[7]The collateral constraint assumes that only foreign debt can be collateralized. We show in the appendix that all the results hold when a fraction $x \in [0, 1]$ of domestic can be collateralized as well.

[8]The credit constraint can be derived endogenously from a problem of limited enforcement under the assumption that households default occurs at the end of the current period and that upon default, households lose a fraction κ_t of the current income.

benefit of assuming more foreign currency debt with its marginal cost

$$(1-\mu_t)u_T(c_t^T, c_t^N) = \beta R u_T\left(c_{t+1}^T, c_{t+1}^N\right). \tag{5}$$

Similarly we have for nominal bonds

$$u_T(c_t^T, c_t^N) = \beta \tilde{R}_t \frac{e_t}{e_{t+1}} u_T\left(c_{t+1}^T, c_{t+1}^N\right). \tag{6}$$

Households' optimality condition for money balances yields the following money demand equation:

$$\frac{M_{t+1}}{\mathcal{P}_t} = \chi \frac{\tilde{R}_t}{U'(c_t)(\tilde{R}_t - 1)}, \tag{MD}$$

and relates the demand for cash holdings to the nominal interest rate and households' marginal utility of consumption. Taking households' wealth as given, condition (MD) states that an increase in the demand for money produces a decline in the nominal interest rate on domestic bonds $\tilde{R}_t$. Using the Euler equations for real bonds and nominal bonds, a non-arbitrage condition that equates the marginal benefits from buying the real and nominal bond can be derived. Together with the law of one price, this implies that the nominal exchange rate must satisfy the interest parity condition:

$$\frac{R}{1-\mu_t} = \tilde{R}_t \frac{e_t}{e_{t+1}}, \tag{IP}$$

that relates the return on foreign currency bonds, which includes the benefits of relaxing the credit constraint when it binds, to the return on domestic currency bonds and the expected depreciation of the domestic currency.

2.2 Firms and Nominal Rigidities

The non-tradable good is produced by a continuum of firms in a perfectly competitive market. Each firm produces a non-tradable good according to a linear production technology given by $y_t^N = n_t$ and obtains profits given by

$$\phi_t^N = P_t^N n_t - W_t n_t.$$

Given the linear production function, we obtain that in equilibrium $P_t^N = W_t$. An individual firm is therefore indifferent between any level of employment.

We assume there exists a minimum wage in nominal terms. Following Schmitt-Grohé and Uribe (2016), we assume that current nominal wage is bounded below by the previous period nominal wage, that is $W_t \geq W_{t-1}$. Similar to the notion of disequilibrium in models with rationing, the labor market is such that aggregate hours worked are the minimum between labor demand and labor supply: $h_t = \min\{n_t, \bar{h}\}$. If $W_t > W_{t-1}$, the aggregate number of hours worked equals the aggregate endowment of labor. If $h_t < \bar{h}$ it has to be that $W_t = W_{t-1}$. These conditions can be summarized as

$$(W_t - W_{t-1})(h_t - \bar{h}) = 0. \tag{7}$$

2.3 Monetary Policy

The central bank sets the money supply, M^s, and rebates all revenues from the net increase in money supply to the public in the form of lump-sum transfers (lump-sum tax if negative). The central bank's budget constraint at any point in time is given by

$$T_t = M^s_{t+1} - M^s_t.$$

We will consider different monetary policy regimes, and focus on the comparison between a fixed exchange rate regime and a flexible exchange rate regime. In the former, the government sets e and the money supply is endogenous, while in the latter, the government sets M and the exchange rate is endogenous.

2.4 Competitive Equilibrium

Market clearing for money requires that the supply of money by the central bank equals the demand for money by households: $M^s_{t+1} = M_{t+1}$. Market clearing for labor requires (7) and that the aggregate labor demand by firms equal the units of labor supplied by households:

$$h_t = n_t. \tag{8}$$

Market clearing for the non-tradable good requires that output be equal to the demand for non-tradables:

$$y^N_t = c^N_t. \tag{9}$$

We assume that the bond denominated in domestic currency is traded only domestically. Market clearing therefore implies

$$\tilde{b}_{t+1} = 0. \tag{10}$$

Combining the budget constraints of households, firms, and the central bank, as well as market clearing conditions, we arrive at the resource constraint for tradables, or the balance of payment condition:

$$c_t^T - y_t^T = \frac{b_{t+1}}{R} - b_t, \tag{11}$$

which says that the trade balance must be financed with net bond issuances. Given the central bank's monetary policy, an equilibrium is defined as follows.

Combining firms' optimality condition, $W_t = P_t^N$, with households optimality condition (4), we arrive at an equation determining the aggregate demand for non-tradables as a function of the real wage, W_t/e_t, and the level of tradable consumption c^T,

$$c_t^N = \left(\frac{1-\phi}{\phi}\frac{e_t}{W_t}\right)^{\gamma} c_t^T. \tag{12}$$

Equations (11) and (12) will be playing a central role in the model dynamics. In the event of a deleveraging episode triggered by a binding credit constraint, the small open economy will have fewer tradable resources available. For a given relative price of non-tradables, this will lead to a reduction in the demand for non-tradable goods. With flexible wages, W_t would fall until $h_t = c^N = \bar{h}$. But if the wage is sticky and the central bank does not achieve a depreciation of the exchange rate, the economy will feature involuntary unemployment, which will in turn feed into a reduction in the borrowing capacity.

Definition 1 (Competitive Equilibrium)**.** Given an initial condition b_0 and W_{-1}, exogenous process $\{y_t^T\}_{t=0}^{\infty}$, and sequence of money supply $\{M_{t+1}^s\}_{t=0}^{\infty}$, an equilibrium is a stochastic sequence of prices $\{\tilde{R}_t, e_t, W_t, P_t^N\}$ and allocations $\{c_t^T, c_t^N, b_{t+1}, \tilde{b}_{t+1}, M_{t+1}, n_t, h_t\}_{t=0}^{\infty}$ such that

(i) Households optimize, and hence the following conditions hold: (4), (5), (12), (MD), (IP);

(ii) Firms' optimization

(iii) Market clearing

$$c_t^N = y_t^N, \;\; \tilde{b}_{t+1} = 0, h_t = n_t \;\; M_{t+1}^s = M_{t+1}$$

(iv) Labor market conditions (7), (8) and $W_t \geq W_{t-1}$ hold

2.5 Steady-State Equilibrium

We assume now that tradable output is constant, $y_t^T = y^T$ for all t and restrict our attention to the case in which $\beta R = 1$. We define a steady state equilibrium as a competitive equilibrium where all allocations are constant.

Definition 2 (Steady state equilibrium)**.** A steady state equilibrium is a competitive equilibrium where allocations are constant for all $t \geq 0$.

Notice that a constant consumption allocation under $\beta R = 1$ implies that the borrowing constraint is not binding. From the tradable resource constraint, using $b_{t+1} = b_0$, we obtain $c_t^T = y^T - (1-\beta)b_0$.

Absent a borrowing constraint, any initial values of debt lower than the natural debt limit would be consistent with a steady state equilibrium. Our goal next is to define the range of values of initial debt that are consistent with a steady state equilibrium in the presence of borrowing constraints. Toward this goal, we define the individual borrowing capacity in period t as

$$\bar{B}(b_{t+1}; b_t) = \kappa R \left[y^T + \frac{1-\phi}{\phi} \left(y^T - b_t + \frac{b_{t+1}}{R} \right)^{\frac{1}{\gamma}} (h_t)^{1-\frac{1}{\gamma}} \right] \tag{13}$$

We also let $\hat{b}$ denote the unique value of debt such that $\bar{B}(\hat{b}; \hat{b}) = \hat{b}$ when $h_t = \bar{h}$. The lemma below characterizes when a steady state equilibrium exists.

Lemma 1 (Existence of Steady-state Equilibrium)**.** *If $b_0 \leq \hat{b}$, we have that:*

i. the steady state equilibrium exists; and

ii. the optimal allocation satisfies $h_t = \bar{h}$. Moreover, a constant exchange rate such that

$$e_t \geq W_{t-1} \frac{\phi}{1-\phi} \left[\frac{y^T - (1-\beta)b_0}{\bar{h}} \right]^{-\frac{1}{\gamma}}.$$

implements this allocation.

Proof. See Appendix A.1 □

That is, when the initial and end-of-period debt level equals $\hat{b}$, we have that the borrowing constraint holds with equality. It follows then that for any level of debt $b_0 < \hat{b}$, the borrowing constraint is satisfied for $b_1 = b_0$ and a steady state equilibrium exists.

Moreover, at the steady state equilibrium, the optimal monetary policy implements full employment.

In a steady state equilibrium, we have that the borrowing constraint is slack. In this case, there is only one potential departure from the first-best allocation, the possibility of unemployment. It then follows that the optimal monetary policy achieves full employment, as the item (ii) of the lemma shows. Full employment is achieved by depreciating the currency enough so that the real wage falls and the nominal wage rigidity is not binding. Clearly, there is a wide range of monetary policies that deliver such an outcome. We focus on a policy that delivers zero inflation for $t = 0, 1, \ldots$ We do this partly for simplicity and partly to capture the traditional price stability objective of central banks. This implies that the central bank sets the exchange rate at a constant level given by

$$\bar{e} = W_{-1} \frac{\phi}{1-\phi} \left(\frac{c^T}{\bar{h}} \right)^{-\frac{1}{\gamma}}. \tag{14}$$

To be consistent with a constant path for the exchange rate, the central bank needs to set a constant money supply $\bar{M}$. Using (MD), we have that level of nominal money supply is given by

$$\frac{\bar{M}}{\chi} = \frac{W_{-1}}{u_N(c^T, \bar{h})} \frac{R}{R-1}. \tag{15}$$

Notice that the value of $\bar{e}$ and $\bar{M}$ depend on b_0. Namely, a higher b_0 implies a lower steady-level of consumption and therefore requires a higher $\bar{e}$ for given W_{-1}. Intuitively, when the level of consumption is lower, the real exchange rate is also lower, and achieving a reduction in the real wage requires a higher nominal exchange rate.

In the next section, we study how a steady state equilibrium may coexist with another equilibrium featuring deleveraging and a sudden stop in capital flows.

3 Self-Fulfilling Crises

In our economy, the amount that households can borrow is increasing in the price of non-tradable goods. Because the price of non-tradables is in turn increasing in the aggregate amount of borrowing, this implies that the borrowing capacity of an individual agent is increasing in the aggregate amount of borrowing. As shown formally Schmitt-Grohé and Uribe (2021), when this complementarity is strong enough, there is a possibility of multiple equilibria.[9] That is, for a range of initial debt values, a steady state equilibrium may coexist with another equilibrium in which households reduce their demand for borrowing, the real exchange rate depreciates, and tradable consumption falls.

In our model with a monetary non-neutrality because of nominal rigidities, monetary policy may affect the vulnerability to self-fulfilling crises. Our goal is to characterize precisely how the exchange rate regime determines this vulnerability and how this affect the choice of the optimal monetary policy.

We assume that the economy starts period 0 with an initial debt position $b_0 < \hat{b}$. As shown in Lemma 1, one possible competitive equilibrium in this case is the steady state equilibrium in which $b_{t+1} = b_0$ for all t, consumption is constant and the borrowing constraint does not bind. In addition, another equilibrium may also exist. We refer to a self-fulfilling crisis equilibrium as a competitive equilibrium featuring deleveraging and lower consumption in period 0. To facilitate the analysis, we focus on a situation where allocations are constant after period 1.[10]

Definition 3 (Self-fulfilling crisis equilibrium)**.** A self-fulfilling crisis equilibrium is a competitive equilibrium where $b_1 < b_0$.

The key to understand why this possibility may emerge is that the borrowing capacity of an individual household is increasing in the amount of aggregate borrowing. When aggregate borrowing increases, this raises the price of non-tradables, thereby relaxing the borrowing constraint. To the extent that a unit reduction in aggregate debt tightens by more than one unit the individual borrowing constraint, the economy may then feature multiple equilibria. That is, for given b_0, we may have a value of b^* such that $\bar{B}(b_0, b^*) = 0$ where $b^* < \hat{b}$.

The following assumption is therefore required for the existence of multiple equilibria, as we will see formally below.

Assumption 1. *The set of parameters satisfies*

$$\kappa \frac{1-\phi}{\phi} \left[\frac{y^T - (1-\beta)\hat{b}}{\bar{h}} \right]^{\frac{1}{\gamma}-1} > 1$$

Assumption 1 says that when evaluated at $b_0 = \hat{b}$, the slope of the individual borrowing limit with respect to aggregate borrowing is such that a decrease in aggregate borrowing

[9]See also Mendoza (1995) for an early discussion of this possibility. For related mechanisms in closed economy models leading to multiplicity see Stein (1995) and Brunnermeier and Pedersen (2009).

[10]This is without loss of generality absent uncertainty (see Schmitt-Grohé and Uribe, 2021).

reduces the individual borrowing capacity by more than one unit. *This assumption is assumed to be satisfied in the rest of the paper.*

3.1 The case of flexible wages

We start by characterizing the conditions for self-fulfilling crises under flexible wages. It is worth noting that given a competitive equilibrium with flexible wages, there exists a nominal exchange rate policy under sticky wages that implements the flexible wages allocation. Because the economy under flexible wages is always at full employment, we can refer interchangeably to this economy as one with flexible wages or one with a *full employment* policy. Echoing the results in Schmitt-Grohé and Uribe (2021), we have the following proposition.

Proposition 1 (Crises under Flexible Wages)**.** *Suppose Assumption 1 holds. Then, under flexible wages*

i. For any γ, if $b_0 \in ((1+\kappa)y^T, \hat{b})$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium. Moreover, we have that $\hat{b} > (1+\kappa)y^T$ and thus the interval is non-empty.

ii. Consider $\gamma \leq 1$. Then, if $b_0 \in [\underline{b}, (1+\kappa)y^T)$, there exists two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium , where $\underline{b}$ is given by

$$\underline{b} \equiv (1+\kappa)y^T - (1-\gamma)\left[\frac{\kappa(1-\phi)}{\gamma\phi}\right]^{\frac{\gamma}{\gamma-1}} \bar{h}$$

Moreover, if $b_0 < \underline{b}$, we have one and only one equilibrium (which corresponds to the steady state equilibrium).

iii. Consider $\gamma > 1$. Then, if $b_0 < (1+\kappa)y^T$ we have one and only one equilibrium (which corresponds to the steady state equilibrium).

Proof. See Appendix A.2 □

Proposition 1 provides necessary and sufficient conditions under which a self-fulfilling crisis can occur. When the initial debt belongs to the interval $[\underline{b}, (1+\kappa)y^T)$, we have two equilibria, the steady state equilibrium, and a crisis equilibrium. Moreover, for $\gamma < 1$, as we reduce further debt, we have a second self-fulfilling crisis equilibrium, as long as b_0 belongs to the interval $(\underline{b}, (1+\kappa)y^T)$.

The top panel of Figure 1 depicts the presence of multiple equilibria under flexible wages for $\gamma < 1$. (Figure 2 shows the case for $\gamma > 1$) The downward sloping line denotes the borrowing capacity for a range of values of debt under the steady state condition $b_1 = b_0$. The upward sloping dashed line denotes the individual borrowing capacity as a function of aggregate borrowing b_1, $\bar{B}(b_1, b_0)$, for a given initial borrowing level b_0, indicated in the plot. We consider a value of b_0 such that $\underline{a} < b_0 < \tilde{b}$.

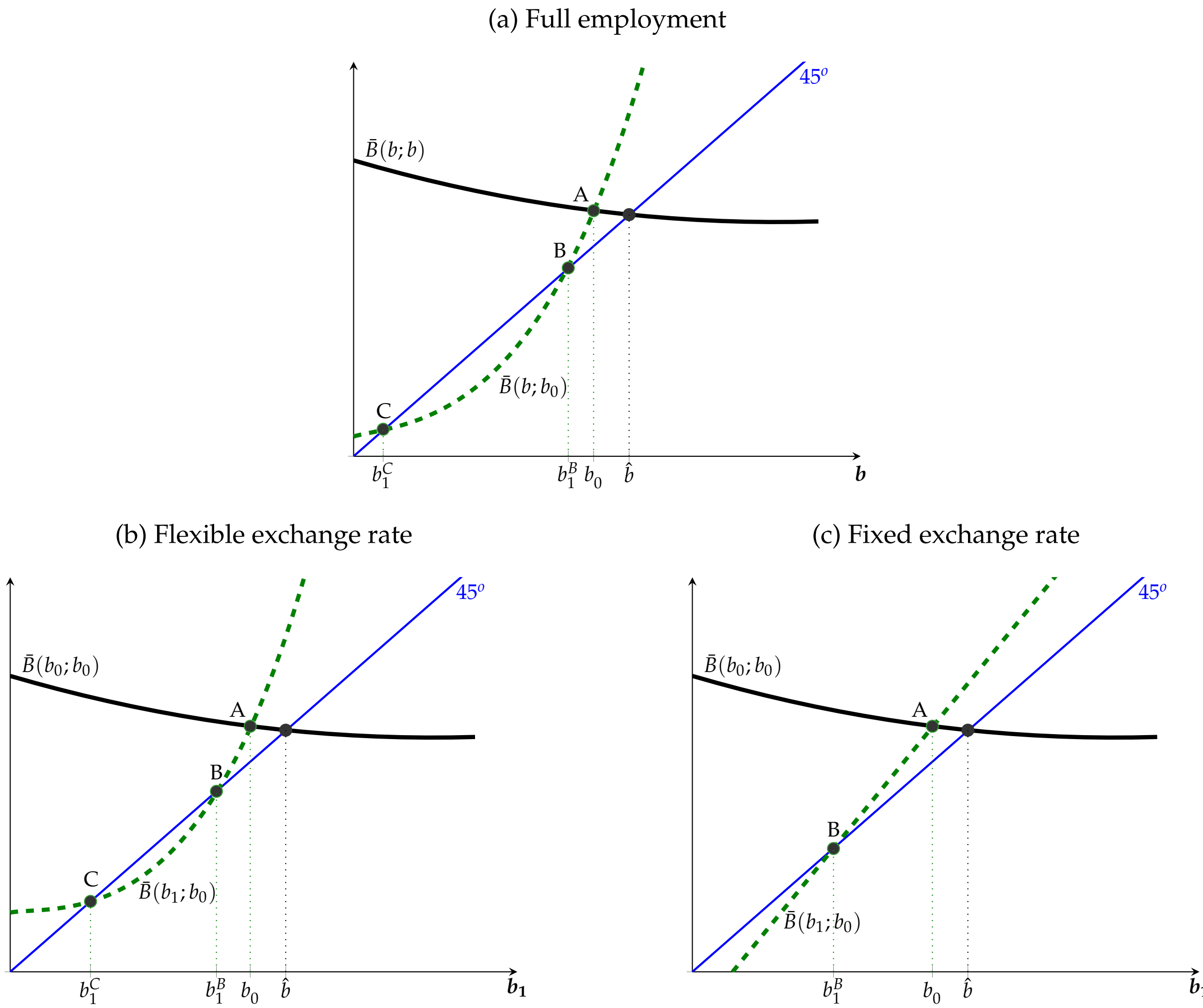


Figure 1: Multiple self-fulfilling crisis equilibria ($\gamma < 1$)

From Lemma 1 a steady-equilibrium exists. Point A on the downward sloping curve represents the steady state equilibrium. As one can see, the point is above the 45 degree line and therefore the borrowing constraint is not binding. Point B illustrates the When households feel pessimistic and decide to reduce their spending so as to reduce their debt, the fall in the real value of their labor income (expressed in units of tradables)

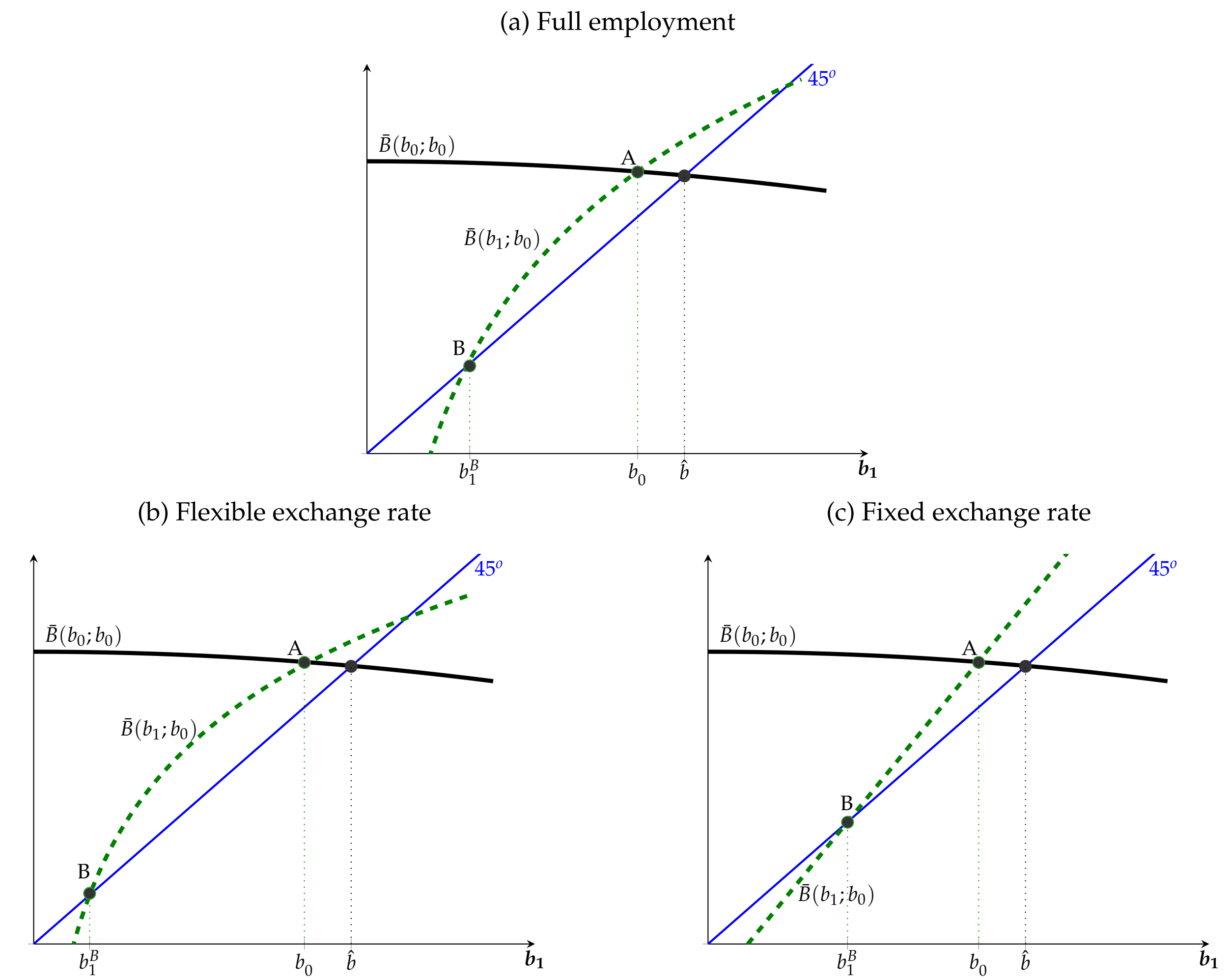


Figure 2: Multiple self-fulfilling crisis equilibria ($\gamma > 1$)

reduces the value of their collateral. Under Assumption 1, the decrease in value of the collateral is larger than the initial decrease in borrowing for a sufficient high level of initial debt leading to a binding borrowing constraint (Point B) which validates households' pessimistic outlook of the economy. Proposition 1 shows that minimum initial debt level for the existence of the self-fulfilling equilibrium (point B) depends on the elasticity of substitution across goods. In particular, this threshold is low when goods are weakly substitutable ($\gamma < 1$). Intuitively, the weaker is the degree of substitutability between tradable and non-tradable goods, the stronger is the fall in the real wage and thus the value of the collateral in response to a decrease in tradable consumption. For $\gamma < 1$, because the marginal response of the real wage is decreasing in tradable absorption and the slope of the borrowing capacity is larger than one when the economy is at point B, there exists existence of a second self-fulfilling equilibrium (point C) where the slope of the borrowing constraint is less than one.

We study next how monetary policy affect the vulnerability to self-fulfilling financial crisis equilibria in the presence of nominal rigidities.

3.2 The case with downward wage rigidity

As aforementioned, in the presence of nominal rigidities monetary policy can be designed to replicate the flexible wage allocation and achieve full employment. However such a policy leaves the economy vulnerable to self-fulfilling crises for large range of initial debt levels. While a monetary policy that sets the nominal exchange rate at its steady state level, given by (14), or fixes the money supply according to (15) delivers the same allocation in the steady state equilibrium, anchoring the nominal exchange rate or the supply of money has different implications for the dynamics of the key macroeconomic variables when the economy is subject to belief-driven fluctuations. We characterize in this section conditions under which self-fulfilling crisis equilibria can arise under a *fixed exchange rate* policy where $e_t = \bar{e}$ and under a *flexible exchange rate* policy where $M_t = \bar{M}$.

3.2.1 Flexible Exchange Rate

Consider a flexible exchange rate in which the central bank sets money supply $M_t = \bar{M}$ and lets the nominal exchange rate fluctuate freely. We focus on a flexible exchange rate regime where $\bar{M}$ corresponds to the efficient steady state level given by (15).

Lemma 2 (Unemployment under Flexible Exchange Rate)**.** *In a self-fulfilling crisis, the exchange rate depreciates at $t = 0$ and there is unemployment.*

Proof. See Appendix A.5 □

Given that households work less hours than their aggregate endowment of hours, the equilibrium in the labor market requires the nominal wage to be at minimum, i.e. $W_0 = W_{-1}$. The borrowing capacity function becomes

$$\bar{B}(b_{t+1}; b_t) = \kappa R \left[y^T + \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{W_{-1}}{e_0} \right)^{1-\gamma} \left(y^T - b_0 + \frac{b_1}{R} \right) \right] \tag{16}$$

where the equilibrium exchange rate e_0 is such that the demand for money (MD) equals the fixed supply of money $\bar{M}$. Under flexible exchange rates, the borrowing capacity is subject to two forces when households suddenly deleverage: the reduction in aggregate borrowing reduces output, causing a decline in the borrowing capacity, and the exchange rate depreciates in response to the decrease in tradable absorption, which in turn can either contribute to further tightening the borrowing constraint or help increase the borrowing capacity depending on the elasticity of substitution across goods γ. Based on these two forces, how does the flexible exchange rate regime affect vulnerability to crises?

Proposition 2 (Crises under Flexible Exchange Rate)**.** *Suppose Assumption 1 holds and wages must satisfy $W_t \geq W_{t-1}$. Under a flexible exchange rate (15).*

i. For any γ, if $b_0 \in ((1+\kappa)y^T, \hat{b})$, the steady-state equilibrium coexists with one and only one self-fulfilling crisis equilibrium. Moreover, we have that $\hat{b} > (1+\kappa)y^T$ and thus the interval is non-empty.

ii. Consider $\gamma \leq 1$. Then, if $b_0 \in [\underline{b}^m, (1+\kappa)y^T)$, there exists two self-fulfilling crisis equilibria that coexist with the steady-state equilibrium, where $\underline{b}^m > \underline{b}$ Moreover, if $b_0 < \underline{b}^m$, we have one and only one equilibrium (which corresponds to the steady state equilibrium).

iii. Consider $\gamma > 1$. Then, if $b_0 < (1+\kappa)y^T$ we have one and only one equilibrium (which corresponds to the steady state equilibrium).

Proof. See Appendix A.6 □

The characterization of the self-fulfilling crises equilibria under flexible exchange rate under full employment policy are qualitatively similar, as is also depicted in Figure 1 and 2. The crucial difference is that under $\gamma < 1$, the threshold for the existence of multiple equilibria now expands (item (ii)).

3.2.2 Fixed Exchange Rate

Consider now a fixed exchange rate regime in which the central bank sets the nominal exchange rate $e_t = \bar{e}$. We will focus on a fixed exchange rate regime where $\bar{e}$ corresponds to the efficient steady state level given by (14).[11]

Lemma 3 (Unemployment in Self-fulfilling crisis)**.** *In a self-fulfilling crisis, there is involuntary unemployment.*

[11]Notice here that the government has access to lump-sum taxes and so it is always able to implement the target level for the exchange rate (although it may not be able to uniquely implement the optimal allocations). One way to articulate this is by assuming the central bank promises to buy/sell in the market foreign currency at the given exchange rate.

Proof. See Appendix A.3 □

Under a fixed exchange rate, the real wage is downward rigid. When households become unexpectedly pessimistic and aggregate demand contracts, the contraction in demand for non-tradables translates one-to-one to a fall in production of non-tradable output causing involuntary unemployment. Given that households work less hours than their aggregate endowment of hours, equilibrium in the labor market requires the nominal wage to be at minimum, i.e. $W_0 = W_{-1}$. The relative price of non-tradables, W_0/e_0, is thus fixed and so a reduction in aggregate borrowing does not reduce W_0/e_0. As a result, the borrowing capacity function become,

$$\bar{B}(b_{t+1}; b_t) = \kappa R \left[y^T + \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma} \left(y^T - b_0 + \frac{b_1}{R} \right) \right]. \tag{17}$$

Even though the relative price of non-tradables does not respond to private agents' actions, a reduction in aggregate borrowing does affect output under both policies and hence this feeds back into the borrowing constraint. Based on this force, how does the exchange rate regime affect the vulnerability to crises? The next proposition sheds light on this question.

Proposition 3 (Crises under Fixed Exchange Rate)**.** *Under a fixed exchange rate policy, there is a nonempty region of debt levels $b_0 \in ((1+\kappa)y^T, \hat{b})$ for which a unique self-fulfilling crisis equilibrium coexists with the steady-state equilibrium. If $b_0 < (1+\kappa)y^T$, we have a unique equilibrium and this equilibrium is the steady state equilibrium.*

Proof. See Appendix A.4 □

Because the real wage does not respond to the fall in aggregate demand in period 0, a self-fulfilling crisis equilibrium can emerge as long as a unit cut in consumption in the steady state equilibrium brings down the value of the collateral by more than one unit (see the moddle panel of figure 1 and figure 2) and the outcome is feasible, i.e. is consumption at point B. Proposition 3 shows that the self-fulfilling crises equilibrium (point B) can only emerge when households do not have sufficient tradable resources $y^T + \kappa y^T$ to repay their due debt b_0.

4 To Fix or to Float?

Having characterized the outcomes under fixed and flexible exchange rates in the previous section, we now proceed to compare them in terms of output and welfare. We will argue that flexible exchange rates, rather than working as a shock absorber, may actually exacerbate economic fluctuations.

4.1 Comparison of Crises Region

We start by comparing the crisis regions under flexible and fixed exchange rates. Following Propositions 2 and 3, we have the following corollary.

Corollary 1 (Exchange rate regimes and vulnerability to crises)**.** *Let $\Omega_{fix}, \Omega_{flex}, \Omega_{full}$ be the regions of debt levels for which self-fulfilling crisis equilibria coexist with the steady-state equilibrium under fixed exchange rate, flexible exchange rate and flexible wage regimes. We have the following*

i. if $\gamma < 1$, the crisis region is smaller under fixed exchange rates: $\Omega_{fix} \subset \Omega_{flex} \subset \Omega_{full}$.

ii. if $\gamma \geq 1$, the crisis regions coincide:

The Corollary establishes that the crisis region under fixed exchange rate is always contained in the crisis region under flexible exchange rates. A paradox of exchange rate flexibility appears to emerge here: flexible exchange rates increase the vulnerability to belief-driven crises. Intuitively, when the elasticity of substitution across sectors is low ($\gamma < 1$) and households unexpectedly decide to cut consumption and increase savings, a depreciation of the nominal exchange rate puts downward pressure on the real wage and sharply reduces the value of the collateral. A more depreciated exchange rate amplifies the fall in the value of the collateral, leading to self-fulfilling crises even for relatively low initial debt levels. Corollary 1 thus sheds light on the role of exchange rate dynamics in coordinating households' expectations when faced with non-fundamental uncertainty.

To give more insights on how the exchange rate regime affects the behavior of the economy when subject to self-fulfilling crises, we numerically solve the model and present the policy functions in Figure 3. For low debt levels, the economy is at the steady-state equilibrium (dotted line) and this is the unique equilibrium. In this region, an increase in the initial debt position lowers consumption, the exchange rate depreciates and there is no unemployment. The dynamics of these macroeconomic variables are independent of the exchange rate regime. Under all policy regimes considered, for high debt levels a self-fulfilling crisis equilibrium emerges where consumption is increasing in the initial debt position. To see why, notice that for a given aggregate borrowing capacity a $1/R$ unit decrease in their initial debt level position implies a unit reduction in households'

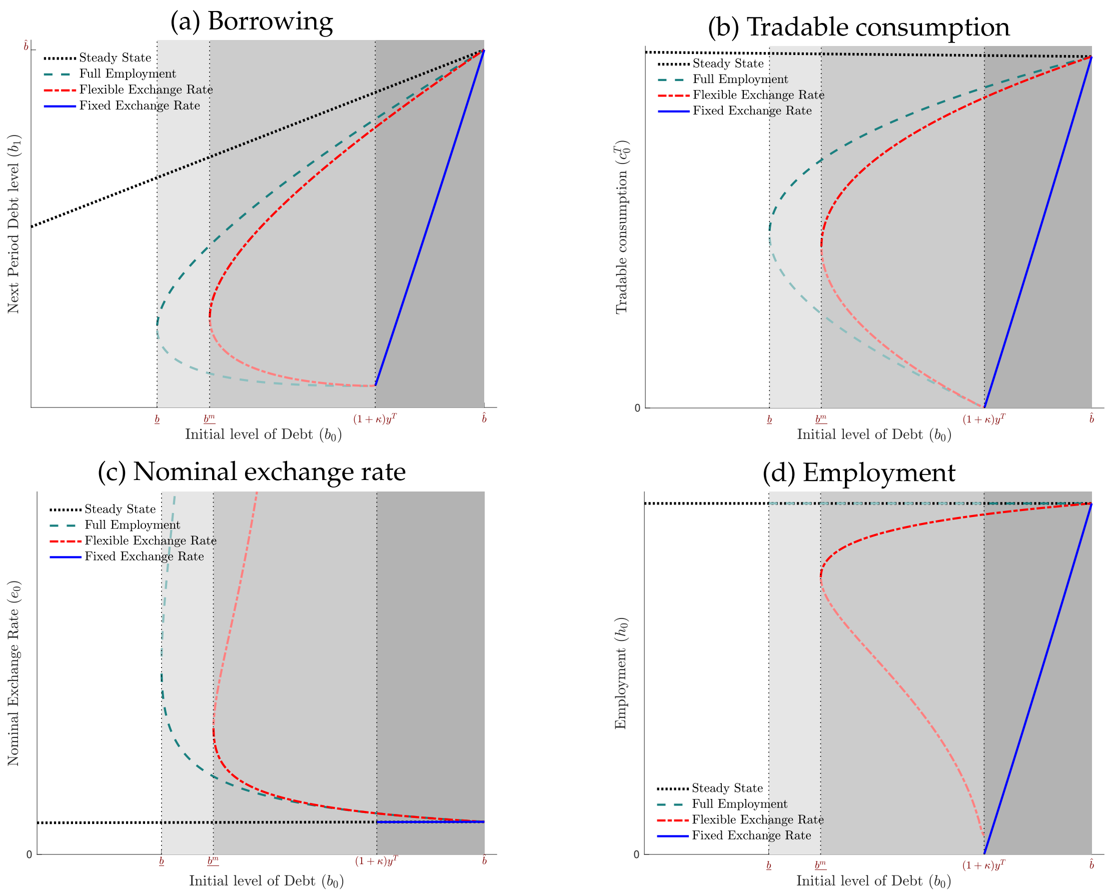


Figure 3: Policy functions for $\gamma < 1$

Note: Parameter values are $\phi = 0.25$, $\kappa = 0.5$, $W_{-1} = 1$, $R = 1.02$, $\beta = 1/R$, $\gamma = 0.4$.

borrowing. This in turn raises the aggregate borrowing capacity by more than one unit under assumption 1 which means that aggregate consumption must be reduced to satisfy the resource constraint (11). In response to the decrease in tradable consumption, the nominal exchange appreciates under flexible exchange rates. However, compared to its level under the full employment policy, the exchange rate is more depreciated under a flexible exchange rate where the economy experiences involuntary unemployment suggesting that depreciations may be contractionary.

4.2 Welfare

Now suppose that the economy starts with $b_0 < \hat{b}$ in period 0. Let π denote the probability that the economy ends in a self-fulfilling crisis equilibrium when the economy is in the vulnerable region. The dependence of the crisis region on the exchange rate regime allows us to determine the optimal exchange rate rate regime from a social welfare perspective as a function of debt levels.[12] Following Propositions 2 and 3, we present the results in the following corollary.

Proposition 4 (Fear of Floating)**.** *Let W_{fix} and W_{flex} be the welfare respectively under fixed and flexible exchange rate regimes. Then, for any $\pi > 0$ and any initial debt level $b_0 \leq (1+\kappa)y^T$, we have $W_{fix} \geq W_{flex}$, with strict inequality if and only if $\gamma < 1$.*

Proof. See Appendix A.7 □

Proposition 4 characterizes when it is optimal to fix the exchange rate. In line with the results presented above, when a fixed exchange rate ensures a unique equilibrium, it becomes optimal to fix the exchange rate. Letting the nominal exchange rate fluctuate leads to perverse movements in the relative price of non-tradables that make the economy vulnerable to a self-fulfilling financial crisis, and the desired outcome may not be attained.

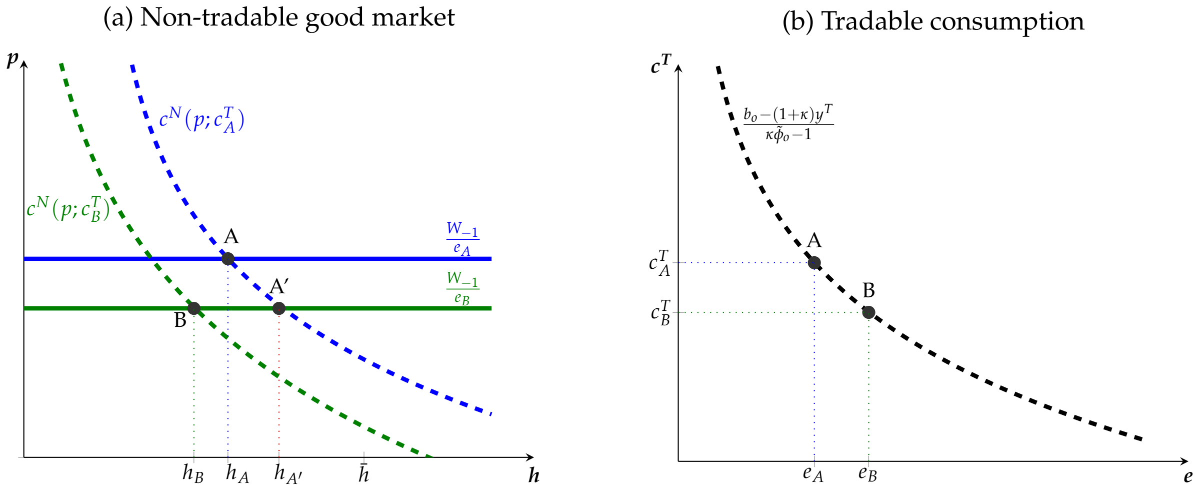
When both exchange rate regimes are subject to the possibility of self-fulfilling crisis, the welfare comparison is in general ambiguous. The welfare effects of a monetary policy conditional on a sunspot are fully determined by two macroeconomic variables: employment and debt. The key question is then what is the effect of an exchange rate depreciation on output and debt.

Consider first a scenario in which a depreciation expands output. If $\gamma > 1$, a flexible exchange rate would dominate a fixed one. This is because when tradable and non-tradable goods are substitutes, non-tradable output increases relatively more than the decline in the price and so the value of non-tradable output and therefore the value of the collateral goes up. On the other hand, as highlighted by Coulibaly (2020) and Ottonello (2021), if $\gamma < 1$, the government faces a trade-off, as the decline in the relative price of nontradables more than offsets the increase in output, and this contracts the borrowing capacity.

We argue, however, that an exchange rate depreciation may actually not expand output. Intuitively, notice that the overall effect of an exchange rate depreciation on output can be decomposed as

$$\frac{dc_0^N}{de_0} = \underbrace{\gamma \frac{c_0^N}{e_0}}_{\text{expend. switching}} + \underbrace{\frac{1}{R}\frac{c_0^N}{c_0^T} \cdot \frac{\partial c_0^N}{\partial b_1}}_{\text{expend. tilting}} \tag{18}$$

[12] For simplicity, we abstract from the utility of money balances to compute welfare, following the standard cashless limit.



Note: $c^N(p, c^T) \equiv \left(\frac{\phi}{1-\phi}p\right)^{-\gamma} c^T$ where p is the relative price of non-tradables.

Figure 4: A Case of Contractionary Depreciation under $\gamma > 1$

In the crisis equilibrium where the borrowing constraint binds, (18) becomes

$$\frac{d\log c_0^N}{d\log e_0} = \underbrace{\gamma}_{\text{expend. switching}} + \underbrace{(1-\gamma)\frac{\kappa\tilde{\phi}_0}{\kappa\tilde{\phi}_0 - 1}}_{\text{expend. tilting}} \tag{19}$$

where $\tilde{\phi}_0 \equiv \left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}$. Figure 4 illustrates these effects for $\gamma > 1$. The downward sloping broken line in the left panel represents the demand curve for non-tradable output and solid line represents the supply curve, both in terms of employment. The intersection, point A, indicates that the initial equilibrium demand for labor $h_A < \bar{h}$. Suppose that the exchange rate depreciates from e_A to $e_B > e_A$. If tradable absorption remains unchanged, the new intersection of the supply and demand curve would occur at point A' where output and employment are higher $h_{A'} > h_A$. However, the depreciation of the nominal exchange drives down the relative price of non-tradables and tightens the borrowing constraint. The resulting decline in tradable consumption from c_A^T to $c_B^T < c_A^T$ causes the demand curve to shift down and to the left. When the expenditure tilting effect is strong enough, the new intersection of the supply and demand curves occurs at point B, where output and employment fall below their initial level, $h_B < h_A$.

In the next section, we provide a formal characterization on when an exchange rate depreciation is contractionary and when it is expansionary.

4.3 Contractionary Depreciations

We have shown above that for a given money supply, a self-fulfilling crisis generates a depreciation of the nominal exchange rate. A distinct but related question is what are the effects of a policy-induced depreciation on the economy. For example, how does an increase in money supply affect output? We turn next to analyze this question.

Consider an initial $b_0 = \underline{b}$. Let $\underline{e}_0$ denote the exchange rate that implements full employment in a self-fulfilling crisis equilibrium. We have

$$\underline{e}_0 = W_{-1}\left[\frac{\kappa}{\gamma}\left(\frac{1-\phi}{\phi}\right)^{\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{20}$$

The next proposition highlights conditions under which an exchange rate depreciation is contractionary.

Proposition 5 (Contractionary Depreciations)**.** *Consider a self-fulfilling crisis equilibrium and two possible values for the exchange rate $e_0, \tilde{e}_0 \in (\underline{e}_0\gamma^{\frac{1}{1-\gamma}}, \underline{e}_0)$ such that $\tilde{e}_0 > e_0$ with $y^N(e_0) < \bar{h}$. Then, $y^N(\tilde{e}_0) < y^N(e_0)$ (i.e., depreciations are contractionary) if $\gamma > 1$ or if $\gamma < 1$ and $b_0 < (1+\kappa)y^T$. If $\gamma = 1$ depreciations are always expansionary.*

Proof. See Appendix A.8 □

To grasp the intuition of why a depreciation can be contractionary, let us assume that the wage rigidity is binding. Combining (12) with market clearing, we obtain

$$y_0^N = \left(\frac{1-\phi}{\phi}\frac{e_0}{W_{-1}}\right)^{\gamma} c_0^T. \tag{21}$$

Holding fixed c_0^T, a depreciation is always expansionary. This is because of the traditional expenditure switching effect: a higher exchange rate makes non-tradables relatively less expensive and increases demand. However, tradable consumption is not fixed. For a given nominal price of non-tradables, a depreciation reduces the real value of income and tightens the borrowing constraint. When the contraction in the borrowing constraint is sufficiently large, it can offset the expenditure switching effect and lead to a contraction in output. The proposition provides the conditions under which this happens.

Consider first the result in the proposition for the case with $\gamma \neq 1$. An exchange rate depreciation is associated with an increase in expenditures in non-tradables of magnitude $1-\gamma$. But a marginal increase in expenditures in non-tradables raises the value of collateral by a magnitude of κ times the relative expenditure share of non-tradables relative to tradables. This in turn leads to an increase in both tradable absorption and expenditures in non-tradables (holding constant e_0) in equal proportion, generating second-round effects on the value of the collateral and further spending. This indirect effect outweighs the expenditure switching effects.

In the case where the relative expenditure share in non-tradables relative to tradables is constant, that is $\gamma = 1$, changes in the nominal exchange rate do not affect the expenditures in non-tradables. As a result, in a self-fulfilling crisis a devaluation translates one-to-one into an increase in demand for non-tradables, leading to an expansion in output.

Figure 5 plots output and the next period debt level as a function of the nominal exchange rate for a given initial debt level b_0, calibrating the model using the parameter values described in section 4.1. The left panels shows that when the economy is in a crisis and the exchange rate lies within the lower and upper bounds defined in Proposition 5, a depreciation of the domestic currency lowers domestic output. Away from the crisis region, an exchange rate depreciation is expansionary. Figure 5 also confirms our findings that for $\gamma < 1$, fixing the nominal exchange rate at its steady-state level $\bar{e}$ helps keep the economy away from the crisis region and implements the first-best allocation. While for $\gamma > 1$ anchoring the nominal exchange rate does not eliminate the risk of self-fulfilling crisis equilibria, an important lesson from the bottom panels of Figure 5 is that a fixed exchange rate regime dominates a flexible exchange rate regime from a social welfare prospective. The depreciation of the nominal exchange rate under flexible exchange rates leads to more capital outflows and brings more unemployment.

We have focused on a contractionary depreciation under a self-fulfilling crisis equilibrium. In Appendix B, we extend this result to a configuration with a unique equilibrium in which the economy faces a binding borrowing constraint. The general result is that when an economy faces a binding borrowing constraint, it is possible that a depreciation leads to a contraction in aggregate demand and output.

4.4 Can Monetary Policy Avert Self-Fulfilling Crises?

The existence of a multiplicity of equilibria raises the question of whether sophisticated monetary policy rules can be designed to avoid self-fulfilling crisis equilibria.

We first consider an implementation via an employment policy rule. The idea here is that the central bank's monetary policy is set in targeting form and uses its policy instrument to reach its target. Since in the desired outcome (first-best allocation) the

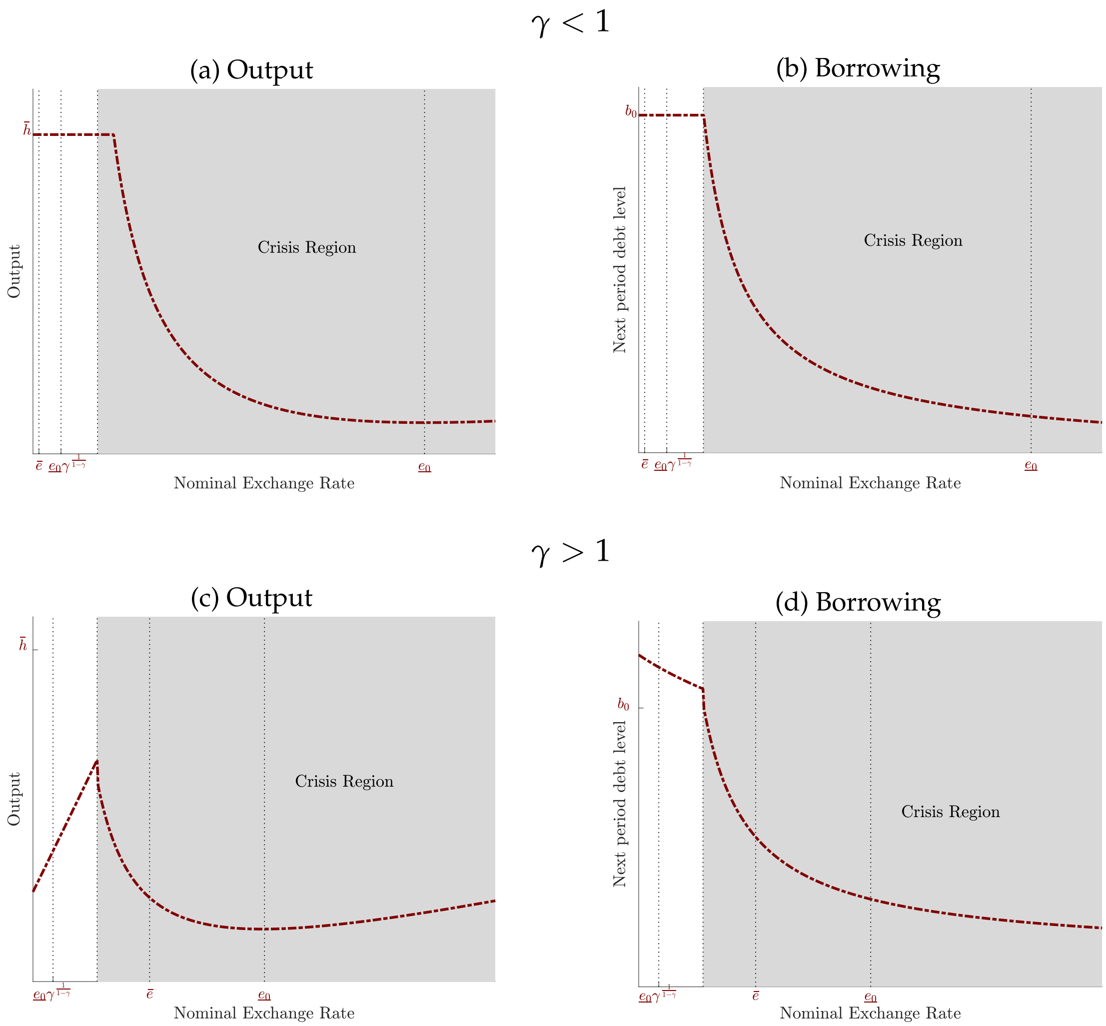


Figure 5: Contractionary Depreciations

Note: Parameter values are $\phi = 0.25$, $\kappa = 0.5$, $W_{-1} = 1$, $R = 1.02$, $\beta = 1/R$. For the left panels, we set $\gamma = 0.4$ and pick an initial debt level b_0 such that $b_0 < (1 + \kappa)y^T$. For the right panels, we set $\gamma = 1.4$ and an initial debt level such that a crisis equilibrium exists.

economy is at full employment, we look for a policy rule that implements full employment if households choose $b_1 = b_0$ and discourages deviations, that is $b_1 < b_0$. The next proposition describes this targeting rule.

Proposition 6 (Targeting Rule)**.** *A targeting rule that rules out the possibility of self-fulfilling crisis equilibria exists if and only if $\gamma > 1$. Moreover, for $\gamma > 1$ a simple targeting rule that rules out self-fulfilling crises equilibria is given by*

$$h(b_1, b_0) = \bar{h} + \phi_h(b_1 - b_0), \tag{22}$$

where $\phi_h > 0$ satisfies

$$u'\left(y^T - b_0 + \frac{b_1}{R}, \bar{h} + \phi_h(b_1 - b_0)\right) < u'\left(y^T - \frac{R-1}{R}b_1, \bar{h}\right).$$

Proof. See Appendix A.9 □

In contrast with the former policy $h_0 = \bar{h}$, it is the threat of a strong response to an eventual deviation from the efficient level of net foreign assets by the central bank that suffices to coordinate households' behavior and rule out any deviation from efficient net foreign assets in equilibrium. However, because of rationing in the labor market, a threat to raise employment above $\bar{h}$ is not feasible.

The next proposition describes a simple exchange rate rule that depends on the changes in the net foreign asset position

Proposition 7 (Exchange Rate Rule)**.** *There exists a simple exchange rate rule that rules out the possibility of sunspot equilibria. This policy rule is given by*

$$e(b_1, b_0) = \bar{e}\left[1 + \frac{\beta}{\bar{c}^T}(b_1 - b_0)\right]^{-\frac{1}{\gamma}}\left[\frac{b_1}{b_0} + \left(1 - \frac{b_1}{b_0}\right)\phi_e\frac{h_0}{\bar{h}}\right]^{\frac{1}{\gamma}}, \tag{23}$$

where $\phi_e = 0$ if $h_0 = \bar{h}$ and $\phi_e \geq 1$ otherwise.

Proof. See Appendix A.10 □

In the first-best allocation $b_1 = b_0$ which implies that $e(b_1, b_0) = \bar{e}$. The proposed exchange rate policy is thus consistent with the first-best allocation. If households feel pessimistic, the policy rule (23) prevents them from simultaneously deleveraging and working $\bar{h}$ units of labor. This is because if households deleverage while supplying $\bar{h}$ units of labor, by (23) the central bank will respond to the sudden bursts in capital outflows by setting the nominal exchange rate at a level that induces a fall in the nominal wage below W_{-1}, forcing firms to reduce their demand for labor to $h_0 < \bar{h}$. This policy rule also discourages households from deleveraging while supplying $h_0 < \bar{h}$ units of labor. When this happens, the central bank responds to the sudden bursts of capital outflows by setting the nominal exchange rate at a level that induces the nominal wage to rise above W_{-1}. The exchange rate policy rule thus implements the first-best allocation and discourages any deviations driven by non-fundamental uncertainty.

5 Conclusion

We provide a theory of fear of floating, the ubiquitous policy among central banks of preventing large fluctuations in exchange rates. In contrast to the Mundell-Fleming paradigm, we show that an exchange rate depreciation does not play the role of a shock absorber—rather it can be contactionary and make more the economy vulnerable to a self-fulfilling financial crisis.

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Appendix to "A Theory of Fear of Floating"

A Proofs

A.1 Proof of Lemma 1

The optimal policy problem consists in choosing e_t as well as $\{c_t^T, h_t, b_{t+1}, \mu_t\}$ to maximize households' lifetime utility subject to implemetability constraints describing the general equilibrium dynamics. Formally, the optimal policy problem is given by

$$\max_{e_t, c_t^T, h_t, b_{t+1}, \mu_t} \sum_{t=0}^{\infty} \beta^t u\left(c_t^T, h_t\right),$$

subject to

$$c_t^T = y^T - b_t + \frac{b_{t+1}}{R} \qquad (\times \lambda_t^*)$$

$$h_t \leq \bar{h} \qquad (\times \vartheta_t^*)$$

$$W_{t-1} \leq e_t \frac{1-\phi}{\phi}\left(\frac{c_t^T}{h_t}\right)^{\frac{1}{\gamma}} \qquad (\times \xi_t^*)$$

$$\frac{b_{t+1}}{R} = \kappa\left[y^T + \frac{1-\phi}{\phi}\left(\frac{c_t^T}{h_t}\right)^{\frac{1}{\gamma}} h_t\right] \qquad (\times \mu_t^*)$$

$$u_T(c_t^T, h_t) = \beta R u_T(c_{t+1}^T, h_{t+1}) + \mu_t \qquad (\times v^*)$$

$$0 = \mu_t \times \left\{\kappa\left[y^T + \frac{1-\phi}{\phi}\left(\frac{c_t^T}{h_t}\right)^{\frac{1}{\gamma}} h_t\right] - \frac{b_{t+1}}{R}\right\}. \qquad (\times \Lambda_t^*)$$

Defining $\bar{\mu}^* \equiv \mu^* + \mu\Lambda^*$, the first order conditions are given by:

$$\begin{aligned}
e_t &:: \quad \xi_t^* = 0 \\
h_t &:: \quad \vartheta_t^* = \left[1 + \frac{\gamma-1}{\gamma}\kappa\bar{\mu}_t^*\right] u_N(t) + \left(v_t^* - R v_{t-1}^*\right) u_{TN}(t) \\
c_t^T &:: \quad \lambda_t^* = \left[1 + \frac{1}{\gamma}\kappa\bar{\mu}_t^*\right] u_T(t) + \left(v_t^* - R v_{t-1}^*\right) u_{TT}(t) \\
b_{t+1} &:: \quad \lambda_t^* = \beta R \lambda_{t+1}^* + \bar{\mu}_t^* u_T(t) \\
\mu_t &:: \quad \Lambda_t^* \left\{\kappa\left[y^T + \frac{1-\phi}{\phi}\left(\frac{c_t^T}{h_t}\right)^{\frac{1}{\gamma}} h_t\right] - \frac{b_{t+1}}{R}\right\} - v_t^* = 0
\end{aligned} \qquad (A.1)$$

The optimality condition (A.1) says that the wage constraint is not binding under optimal monetary policy. Combining the first-order conditions with respect to c_t^T and b_{t+1}^* we arrive to

$$\frac{\kappa}{\gamma}\frac{p_t^N c_t^N}{c_t^T}\bar{\mu}_t^* + (v_t - Rv_{t-1})\frac{u_{TT}(t)}{u_T(t)} = \bar{\mu}_t^* + \frac{\kappa}{\gamma}\frac{p_{t+1}^N c_{t+1}^N}{c_{t+1}^T}\bar{\mu}_{t+1}^* + (v_{t+1} - Rv_t)\frac{u_{TT}(t+1)}{u_T(t+1)}$$

where p_t^N is the relative price of nontradables. In the stationary equilibrium, this equation yields $\mu^* = 0$. In the stationary equilibrium, the remaining optimality conditions become

$$\begin{aligned}
\vartheta^* &= u_N(c^T,h) - (R-1)u_{TN}(c^T,h)\ v^* \\
\lambda^* &= u_T(c^T,h) - (R-1)u_{TT}(c^T,h)\ v^* \\
v^* &= \Lambda^*\left\{\kappa\left[y^T + \frac{1-\phi}{\phi}\left(\frac{c^T}{h}\right)^{\frac{1}{\gamma}} h\right] - \frac{b}{R}\right\}
\end{aligned}$$

To show that there is no unemployment in the stationary equilibrium, first pick $\mu = 0$ (that is the slackness condition is satisfied). Then choose $\Lambda^* = 0$ which implies that $v^* = 0$. The optimality conditions become

$$\lambda = u_T(c^T,h) \tag{A.2}$$

$$\vartheta = u_N(c^T,h) > 0 \tag{A.3}$$

Condition (A.2) implies that the Euler equation is satisfied and condition (A.3) implies that the implementability constraint $h \leq \bar{h}$ is binding, that is $h = \bar{h}$. Thus, at the stationary equilibrium, optimal monetary policy targets full employment.

In a steady state equilibrium, $b_{t+1} = b_0$ for all t and by the resource constraint for tradable goods (11) $c^T = y^T - \frac{R-1}{R}b_0$. The equilibrium exists if the collateral constraint is satisfied,

$$\bar{B}(b_0;b_0) - b_0 = \kappa R\left[y^T + \frac{1-\phi}{\phi}\left(y^T - \frac{R-1}{R}b_0\right)^{\frac{1}{\gamma}}(\bar{h})^{\frac{\gamma-1}{\gamma}}\right] - b_0 \geq 0. \tag{A.4}$$

Because $\bar{B}'(b_0;b_0) - 1 < 0$ and $\bar{B}(\hat{b};\hat{b}) = \hat{b}$ by definition of $\hat{b}$, we have $\bar{B}(b_0;b_0) \geq b_0$ for any $b_0 \leq \hat{b}$ and $\kappa < \frac{R}{R-1}$ ensures that $c^T > 0$.

A.2 Proof of Proposition 1

Letting $\mathbf{F}(b_1; b_0)$ denote the excess borrowing capacity function defined as the difference between the borrowing capacity and level of borrowing, we have

$$\begin{aligned}\mathbf{F}(b_1; b_0) &= \bar{b}(b_1; b_0) - b_1 \\ &= \kappa R \left[y^T + \frac{1-\phi}{\phi} \left(y^T - b_0 + \frac{b_1}{R} \right)^{\frac{1}{\gamma}} (\bar{h})^{\frac{\gamma-1}{\gamma}} \right] - b_1\end{aligned}$$

Notice that b_1 is part of an equilibrium if these three conditions hold: $\mathbf{F}(b_1; b_0) \geq 0$, $b_1 \leq b_0$, and $\frac{b_1}{R} > b_0 - y^T$. The first condition implies that the borrowing constraint is satisfied. The second condition ensures that $\mu \geq 0$ and the last condition ensures that $c_0^T > 0$.

Case I: $\gamma \geq 1$. Because $\mathbf{F}$ is concave and $\mathbf{F}(b_0; b_0) > 0$, a self-fulfilling crises equilibrium exists (that is, there exists b_1 in $(R(b_0 - y^T), b_0)$ such that $\mathbf{F}(b_1; b_0) = 0$) if for the lowest value in the feasible domain of b_1, we have $\mathbf{F}(R(b_0 - y^T); b_0) = (1+\kappa)y^T - b_0 < 0$ which implies $b_0 > (1+\kappa)y^T$. Furthermore, because $\mathbf{F}$ changes sign over $(R(b_0 - y^T), b_0)$, the solution b_1 in $(R(b_0 - y^T), b_0)$ is unique by concavity of the function $\mathbf{F}$.[13] There is a unique self-fulfilling equilibrium for $b_0 > (1+\kappa)y^T$. Moreover, since $b_0 < \hat{b}$ from Lemma 1, it remains to show that $((1+\kappa)y^T, \hat{b})$ is non-empty.

Recall that $\mathbf{F}(\hat{b}, \hat{b}) = 0$ by definition of $\hat{b}$. Then, we have

$$\begin{aligned}\frac{\hat{b}}{R} &= \kappa y^T + \kappa \frac{1-\phi}{\phi} \left(y^T - \frac{R-1}{R} \hat{b} \right)^{\frac{1}{\gamma}} (\bar{h})^{\frac{\gamma-1}{\gamma}} \\ &= \kappa y^T + \hat{c}^T \kappa \frac{1-\phi}{\phi} \left[\frac{y^T - (1-\beta)\hat{b}}{\bar{h}} \right]^{\frac{1-\gamma}{\gamma}} \end{aligned} \tag{A.5}$$

Using the resource constraint $\hat{c}^T = y^T - \hat{b} + \frac{\hat{b}}{R}$ and substituting (A.5), we get

$$\left[1 - \kappa \frac{1-\phi}{\phi} \left(\frac{y^T - (1-\beta)\hat{b}}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} \right] \hat{c}^T = (1+\kappa)y^T - \hat{b} \tag{A.6}$$

From Assumption 1, it follows that the left-hand side of equation (A.6) is negative. Therefore $(1+\kappa)y^T < \hat{b}$. The interval $((1+\kappa)y^T, \hat{b})$ is thus non-empty.

[13] Note that because F is concave and changes sign over $(R(b_0 - y^T), b_0)$, the second solution when it exists is not feasible as it would satisfy $b_1 > b_0$.

Case II: $\gamma < 1$. Because $\mathbf{F}$ is convex and $\mathbf{F}(b_0; b_0) > 0$, when a self-fulfilling crises equilibrium exists (that is, there exists b_1 in $(R(b_0 - y^T), b_0)$ such that $\mathbf{F}(b_1; b_0) = 0$), we know that for b_1 such that $\mathbf{F}(b_1; b_0) = 0$ we have $\mathbf{F}'(b_1; b_0) \geq 0$.

$$\mathbf{F}'(b_1; b_0) \geq 0 \Leftrightarrow \frac{\kappa}{\gamma} \frac{1-\phi}{\phi} \left(\frac{c_0^T}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} \geq 1 \tag{A.7}$$

$$\Leftrightarrow c_0^T \geq \bar{h} \left[\frac{\kappa(1-\phi)}{\gamma\phi} \right]^{\frac{\gamma}{\gamma-1}}, \tag{A.8}$$

Using the resource constraint for tradable goods $b_0 = y^T + \frac{b_1}{R} - c_0^T$ then plugging in $\mathbf{F}(b_1; b_0) = 0$ to substitute for b_1 yields

$$b_0 = y^T + \kappa y^T + \kappa \frac{1-\phi}{\phi} \left(\frac{c_0^T}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} c_0^T - c_0^T \tag{A.9}$$

Notice that the right hand side of (A.9) is increasing in c_0^T as the derivative with respect to c_0^T corresponds to $\mathbf{F}'(b_1; b_0) \geq 0$. Hence, plugging (A.8) into (A.9) we arrive to

$$b_0 \geq (1+\kappa)y^T - (1-\gamma) \left[\frac{\kappa(1-\phi)}{\gamma\phi} \right]^{\frac{\gamma}{\gamma-1}} \bar{h} \equiv \underline{b}. \tag{A.10}$$

Moreover, because $\mathbf{F}$ is convex $\mathbf{F}(b_1; b_0) = 0$ has two solutions if and only if $\mathbf{F}(b_1; b_0)$ has at least one solution and for lowest value in the feasible domain of b_1 (i.e. $b_1 = R(b_0 - y^T)$), $\mathbf{F}(b_1; b_0) > 0$. We have

$$\mathbf{F}(R(b_0 - y^T); b_0) = (1+\kappa)\kappa y^T - b_0 > 0 \;\Leftrightarrow\; b_0 < (1+\kappa)\kappa y^T$$

Therefore, $\mathbf{F}(b_1; b_0) = 0$ has two solutions for $b_0 \in \left[\underline{b}, (1+\kappa)y^T\right)$ and a unique solution for $b_0 \in \left[(1+\kappa)y^T, \hat{b}\right)$ where we $b_0 < \hat{b}$ from Lemma 1. Furthermore, as shown above the interval $[(1+\kappa)y^T, \hat{b})$ is non-empty under Assumption 1.

A.3 Proof of Lemma 3

To see why under a commitment to the exchange rate policy (14), $e = \bar{e}$, there is involuntary unemployment $h_0 < \bar{h}$ in the self-fulfilling crisis equilibrium when it exists, assume by contradiction that $h_0 = \bar{h}$. Because $b_1 < b_0$ in the self-fulfilling crisis equilibrium, we have $c_0^T < y^T - \frac{R-1}{R} b_0$ and the equilibrium wage satisfies

$$W_0 = \bar{e}\, \frac{1-\phi}{\phi} \left(\frac{c_0^T}{\bar{h}} \right)^{\frac{1}{\gamma}} < \bar{e}\, \frac{1-\phi}{\phi} \left(\frac{y^T - \frac{R-1}{R} b_0}{\bar{h}} \right)^{\frac{1}{\gamma}} = W_{-1}.$$

We thus proved that $h_0 = \bar{h}$ in the crises leads to $W_0 < W_{-1}$ which violates the constraint on the nominal wage. Therefore, $h_0 < \bar{h}$ in the self-fulfilling crises equilibrium.

A.4 Proof of Proposition 3

Letting $\mathbf{F}(b_1; b_0)$ denote the excess borrowing capacity function defined as the difference between the borrowing capacity and level of borrowing, we have

$$\begin{aligned}\mathbf{F}(b_1; b_0) &= \bar{b}(b_1; b_0) - b_1 \\ &= \kappa R \left[y^T + \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma} \left(y^T - b_0 + \frac{b_1}{R} \right) \right] - b_1 \end{aligned} \tag{A.11}$$

We have

$$\mathbf{F}'(b_1; b_0) \equiv \frac{\partial \mathbf{F}(b_1; b_0)}{\partial b_1} = \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma} - 1$$

Notice that b_1 is part of an equilibrium if $\mathbf{F}(b_1; b_0) = 0$, $b_0 > b_1$, and $\frac{b_1}{R} > b_0 - y^T$. The first condition is such that the constraint holds with equality. The second condition ensures that $\mu > 0$ and the last condition ensures that $c_0^T > 0$. Because $\mathbf{F}(b_0; b_0) > 0$, that is the borrowing constraint does not bind in the stationary equilibrium, a sufficient condition for non-existence b_1 that satisfies the first two conditions is $\mathbf{F}'(b_1; b_0) < 0$.

Case I: $\gamma \leq 1$. For $\gamma \leq 1$, after substituting for $W_{-1}/\bar{e}$ using (14) we get

$$\begin{aligned}\mathbf{F}'(b_1; b_0) &= \kappa \frac{1-\phi}{\phi} \left(\frac{y^T - \frac{R-1}{R} b_0}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} - 1 \\ &> \kappa \left(\frac{1-\phi}{\phi} \right)^{\frac{1}{\gamma}} \left(\frac{y^T - \frac{R-1}{R} \hat{b}}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} - 1 > 0 \end{aligned} \tag{A.12}$$

where the last inequality uses assumption 1. (A.12). Combined with $\mathbf{F}(b_0; b_0) > 0$, this implies that there exists $b_1 < b_0$ such that $\mathbf{F}(b_1; b_0) = 0$. It remains to check condition under which $c_0^T > 0$ in a self-fulfilling crises equilibrium. Solving for $\frac{b_1}{R}$ using $F(b_1; b_0) = 0$ and substituting into the resource constraint $c_0^T = y^T - b_0 + \frac{b_1}{R}$ we get

$$c_0^T = \frac{b_0 - (1+\kappa) y^T}{\kappa \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{W_{-1}}{\bar{e}} \right)^{1-\gamma} - 1} \tag{A.13}$$

$c_0^T > 0$ if and only if $b_0 > (1+\kappa)y^T$. Since $b_0 < \hat{b}$, it follows that a self-fulfilling crisis equilibrium coexists with the stationary equilibrium for any $b_0 \in ((1+\kappa)y^T, \hat{b})$. Furthermore, as shown in Appendix A.2, the interval $((1+\kappa)y^T, \hat{b})$ is non-empty under Assumption 1.

Case II: $\gamma > 1$. Consider now the case where $\gamma > 1$. Recall that since $F(b_0; b_0) > 0$ the stationary equilibrium is the unique equilibrium iff

$$\mathbf{F}'(b_1; b_0) \leq 0 \;\Leftrightarrow\; \kappa \frac{1-\phi}{\phi}\left(\frac{y^T - \frac{R-1}{R}b_0}{\bar{h}}\right)^{\frac{1}{\gamma}-1} - 1 < 0$$

$$\Leftrightarrow\; b_0 \leq \frac{R}{R-1}\left[y^T - \bar{h}\left(\kappa\frac{1-\phi}{\phi}\right)^{\frac{\gamma}{\gamma-1}}\right] \equiv \mathbb{B}_0 \tag{A.14}$$

For b_0 such that $\mathbf{F}'(b_1; b_0) > 0$ (i.e., for b_0 not satisfying (A.14)), because $\mathbf{F}(b_0; b_0) > 0$ we know that there exists $b_1 < b_0$ such that $\mathbf{F}(b_1; b_0) = 0$. From (A.13), we have that $c_0^T > 0$ if and only if $b_0 > (1+\kappa)y^T$. Moreover, since $b_0 < \hat{b}$ from Lemma 1, it follows that a self-fulfilling crisis equilibrium exists if and only $b_0 \in (\max\{(1+\kappa)y^T, \mathbb{B}_0\}, \hat{b})$. In addition, it is straightforward to see that $\max\{(1+\kappa)y^T, \mathbb{B}_0\} = \mathbb{B}_0$ would imply that

$$\frac{R-1}{R}(1+\kappa) < 1 - \frac{\bar{h}}{y^T}\left(\kappa\frac{1-\phi}{\phi}\right)^{\frac{\gamma}{\gamma-1}}.$$

This inequality never holds since $\frac{R-1}{R}(1+\kappa) > 1$. Therefore, a self-fulfilling financial crisis equilibrium coexists with the stationary equilibrium if and only $b_0 \in ((1+\kappa)y^T, \hat{b})$. Furthermore, as shown in Appendix A.2, the interval $((1+\kappa)y^T, \hat{b})$ is non-empty under Assumption 1.

A.5 Proof of Lemma 2

Unemployment under flexible exchange rate. To see why under a commitment to the money supply policy (15), $M = \bar{M}$, there is involuntary unemployment $h_0 < \bar{h}$ in the self-fulfilling crisis equilibrium when it exists, assume by contradiction that $h_0 = \bar{h}$ in this equilibrium. The demand for money in period 0 in the stationary equilibrium and in the self-fulfilling crises are

$$\frac{\chi W_{-1}}{\bar{M}} = \left[1 - \frac{1}{R}\right] u_N\left(y^T - \frac{R-1}{R}b_0, \bar{h}\right) \tag{A.15a}$$

$$\frac{\chi W_0}{\bar{M}} = \left[1 - \frac{1}{\tilde{R}_0}\right] u_N\left(y^T - b_0 + \frac{b_1}{R}, \bar{h}\right) \tag{A.15b}$$

We now discuss the two cases depending on the sign of u_{TN}.

Consider first that $u_{TN} \geq 0$, that is $\gamma \leq 1$. Then, it must be that $\tilde{R}_0 \geq R$. This is because for $\tilde{R}_0 < R$ we have $W_0 < W_{-1}$ by (A.15a) and (A.15b) which violates the constraint on the nominal wage. Given that $\tilde{R}_0 \geq R$, from the (IP) condition $e_0 \leq \frac{e_1}{1-\mu_0}$. Substituting that into the demand for nontradables (12) we get

$$h_0 = \left[\frac{1-\phi}{\phi}\frac{e_0}{W_0}\right]^{\gamma} c_0^T \leq \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\frac{1}{1-\mu_0}\right]^{\gamma} c_0^T$$

Using the Euler equation for foreign bonds (5) to substitute for μ_0 we arrive to

$$h_0 \leq \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\right]^{\gamma} c_1^T \left(\frac{c_0}{c_1}\right)^{1-\gamma} < \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\right]^{\gamma} c_1^T = \bar{h} \tag{A.16}$$

which contradicts $h = \bar{h}$. Therefore, $h_0 < \bar{h}$ in the self-fulfilling crises equilibrium.

Next, consider the case where $u_{TN} < 0$, that is $\gamma > 1$. From (A.15b) and the demand for money in period 1,

$$\chi W_0 = M_1 \left[1 - \frac{1}{\tilde{R}_0}\right] u_N \left(y^T - b_1 + \frac{b_1}{R}, \bar{h}\right), \tag{A.17}$$

because $b_1 < b_0$ we have $M_1 \frac{R-1}{R} > \bar{M}\frac{\tilde{R}_0-1}{\tilde{R}_0}$. Plugging $\frac{W_t}{e_t} = \frac{u_N(c_t^T, c_t^N)}{u_T(c^T, c_t^N)}$ into (A.17) and using the Euler equation for foreign bonds (5) we get

$$\begin{aligned} \frac{e_1}{1-\mu_0} &= M_1 \left[1 - \frac{1}{R}\right](1-\mu_0) u_T\left(c_1^T, \bar{h}\right) \\ &= M_1 \frac{R-1}{R} u_T\left(c_0^T, \bar{h}\right) > \bar{M}\frac{\tilde{R}_0 - 1}{\tilde{R}_0} u_T\left(c_0^T, \bar{h}\right) = e_0 \end{aligned}$$

Substituting into the demand for non-tradables in period 0 we arrive to

$$\begin{aligned} h_0 &= \left[\frac{1-\phi}{\phi}\frac{e_0}{W_0}\right]^{\gamma} c_0^T < \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\frac{1}{1-\mu_0}\right]^{\gamma} c_0^T = \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\right]^{\gamma} c_1^T \left(\frac{c_0}{c_1}\right) \\ &< \left[\frac{1-\phi}{\phi}\frac{e_1}{W_0}\right]^{\gamma} c_1^T = \bar{h} \end{aligned}$$

which contradicts $h = \bar{h}$. Therefore, $h_0 < \bar{h}$ in the self-fulfilling crises equilibrium.

Exchange depreciation. From the money demand equation (MD) we have

$$\frac{\chi e_0}{\bar{M}} = \frac{\tilde{R}_0 - 1}{\tilde{R}_0} u_T(c_0^T, c_0^N)$$

Substituting for the (IP), we get

$$\frac{\chi e_0}{\bar{M}} = \left[1 - \frac{(1-\mu_0)e_0}{Re_1}\right] u_T(c_0^T, c_0^N)$$
$$= \left[1 - \frac{e_0}{Re_1}\frac{u_T(c_1^T,\bar{h})}{u_T\left(c_0^T, c_0^N\right)}\right] u_T(c_0^T, c_0^N) \tag{A.18}$$

where the second equality uses the Euler equation for foreign bonds (5). Note that equilibrium in period 1 requires the relative price to be equal to the marginal rate of substitution, that is $\frac{e_1}{W_0} = \frac{u_T(c_1^T,\bar{h})}{u_N(c_1^T,\bar{h})}$. Plugging this equation into (A.18) we get

$$\frac{\chi e_0}{\bar{M}} = \left[1 - \frac{e_0}{RW_0}\frac{u_N(c_1^T,\bar{h})}{u_T(c_0^T, c_0^N)}\right] u_T(c_0^T, c_0^N)$$
$$\frac{\chi}{\bar{M}} = \left[\frac{1}{e_0}u_T(c_0^T, c_0^N) - \frac{1}{RW_0}u_N(c_1^T,\bar{h})\right] \tag{A.19}$$

Finally, using the present value resource constraints between period 0 and period 1 on the one hand and the demand for non-tradable goods on the other hand, that is

$$c_1^T = \frac{1}{\beta}y^T - \frac{1-\beta}{\beta}(c_0^T + b_0) \quad \text{and} \quad c_0^N = \left(\frac{1-\phi}{\phi}\frac{e_0}{W_0}\right)^{\gamma} c_0^T$$

equation (A.19) can be rewritten as an implicit function relating the nominal exchange e_0 to the level of tradable consumption c_0^T,

$$\frac{\chi}{\bar{M}} = \frac{1}{e_0}u_T\left(c_0^T, \left(\frac{1-\phi}{\phi}\frac{e_0}{W_0}\right)^{\gamma} c_0^T\right) - \frac{1}{RW_0}u_N\left(Ry^T - (R-1)(c_0^T + b_0), \bar{h}\right) \tag{A.20}$$

Recall that since there is unemployment in the self-fulfilling crises equilibrium under flexible exchange rate, we have $W_0 = W_{-1}$. Hence, totally differentiating (A.20) yields

$$[1-(1-\gamma)(1-\tilde{\phi}_0)]\frac{de_0}{dc_0^T} = -\left[1 - \left(1-\frac{1}{\gamma}\right)\tilde{\phi}_1\frac{1-\beta}{\beta\tilde{R}_0}\frac{c_0^T}{c_1^T}\right]\frac{e_0}{c_0^T} < 0 \tag{A.21}$$

where $\tilde{\phi}_t \equiv e_t c_t^T/(e_t c_t^T + W_t c_t^N) < 1$ is the share of expenditures in tradable goods. It is straightforward to see that for $\gamma \leq 1$ then $de_0/dc_0^T < 0$. To show see why $de_0/dc_0^T < 0$ for $\gamma > 1$, note that $\left(1-\gamma^{-1}\right)\tilde{\phi}_1 c_0^T/c_1^T < 1$, it remains to show that $(1-\beta)/(\beta\tilde{R}_0) \leq 1$. First, note that if $\tilde{R}_0 \geq R$ then $\beta\tilde{R}_0 = \beta R \geq 1$. Next, if $\tilde{R}_0 < R$, note that the Euler equation for nominal bond requires $\mathcal{P}_1 c_1 = \beta\tilde{R}_0\mathcal{P}_0 c_0$. Using the demand for money, we have $\frac{\bar{M}}{\mathcal{P}_0 c_0} = \chi\frac{\tilde{R}_0}{\tilde{R}_0 - 1}$ and $\frac{\bar{M}_1}{\mathcal{P}_1 c_1} = \chi\frac{R}{R-1}$. Because the left-hand side is decreasing in $\tilde{R}$ and by (15) this implies that $\mathcal{P}_1 c_1 > \mathcal{P}_0 c_0$. Thus, from $\mathcal{P}_1 c_1 = \beta\tilde{R}_0\mathcal{P}_0 c_0$, we obtain $\beta\tilde{R}_0 > 1$.

A.6 Proof of Proposition 2

Letting $\mathbf{F}(b_1; b_0)$ denote the excess borrowing capacity function defined as the difference between the borrowing capacity and level of borrowing, we have

$$\begin{aligned}\mathbf{F}(b_1; b_0) &= \bar{b}(b_1; b_0) - b_1 \\ &= \kappa R \left[y^T + \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{W_{-1}}{e_0}\right)^{1-\gamma} \left(y^T - b_0 + \frac{b_1}{R}\right) \right] - b_1\end{aligned}$$

where the nominal exchange rate e_0 is determined by the following implicit equation

$$e_0 = \frac{\bar{M}}{\chi}\left(1 - \frac{1}{\tilde{R}_0}\right) u_T \left(y^T - b_0 + \frac{b_1}{R}, \left(\frac{1-\phi}{\phi}\frac{e_0}{W_{-1}}\right)^{\gamma}\left(y^T - b_0 + \frac{b_1}{R}\right)\right) \tag{A.22}$$

The first and second derivative of $\mathbf{F}$ with respect to b_1, $\mathbf{F}' \equiv \frac{d\mathbf{F}(b_1;b_0)}{db_1}$ and $\mathbf{F}'' \equiv \frac{d^2\mathbf{F}(b_1;b_0)}{db_1^2}$, are given by

$$\begin{aligned}\mathbf{F}' &= \kappa \frac{1-\phi}{\phi}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\left(1 - (1-\gamma)\frac{c_0^T}{e_0}\frac{de_0}{dc_0^T}\right) - 1 \\ \mathbf{F}'' &= (1-\gamma)\,\kappa\frac{1-\phi}{\phi}\left(\frac{W_{-1}}{e_0}\right)^{1-\gamma}\left[-\left(1-(1-\gamma)\frac{c_0^T}{e_0}\frac{de_0}{dc_0^T}\right)\frac{1}{e_0}\frac{de_0}{dc_0^T} - \frac{d}{dc_0^T}\left(\frac{c_0^T}{e_0}\frac{de_0}{dc_0^T}\right)\right]\end{aligned}$$

From (A.21) we have that

$$\begin{aligned}1-(1-\gamma)\frac{c_0^T}{e_0}\frac{de_0}{dc_0^T} &= \frac{1+(1-\gamma)\tilde{\phi}_0 + \frac{1}{\gamma}(1-\gamma)^2\tilde{\phi}_1\frac{R-1}{\tilde{R}_0}\frac{c_0^T}{c_1^T}}{1-(1-\gamma)(1-\tilde{\phi}_0)} > 0 \\ &= \frac{1}{\gamma} + \frac{(1-\gamma)^2\tilde{\phi}_0}{1-(1-\gamma)(1-\tilde{\phi}_0)}\left[-1 + \frac{c_0^T}{c_1^T}\frac{\tilde{\phi}_1}{\tilde{\phi}_0}\frac{R-1}{\tilde{R}_0}\right]\end{aligned} \tag{A.23}$$

Noting that $\frac{\tilde{\phi}_1 c_0^T}{\tilde{\phi}_0 c_1^T} = \frac{(\mathcal{P}_0/e_0)c_0}{(\mathcal{P}_1/e_1)c_1} = \beta R(1-\mu_0)$ where the last equality stems from the Euler equation for real bond, we arrive to

$$1-(1-\gamma)\frac{c_0^T}{e_0}\frac{de_0}{dc_0^T} = \frac{1}{\gamma} + \frac{(1-\gamma)^2\tilde{\phi}_0}{1-(1-\gamma)(1-\tilde{\phi}_0)}\left[-1+(1-\mu_0)\frac{R-1}{\tilde{R}_0}\right] < \frac{1}{\gamma} \tag{A.24}$$

Differentiating (A.21), we obtain after some algebraic manipulation,

$$\frac{d}{dc_0^T}\left(\frac{c_0^T}{e_0}\frac{de_0}{dc_0^T}\right) = -\left(1-\frac{1}{\gamma}\right)^2 c_0^T \left\{ (1-\tilde{\phi}_0)\,\tilde{\phi}_0 \left(\frac{\gamma}{e_0}\frac{de_0}{dc_0^T}\right)^2 \right.$$
$$\left. + (R-1)^2 \left[\frac{\tilde{\phi}_1\left(\tilde{R}_0 - \tilde{\phi}_1\right)}{\left(\tilde{R}_0 c_1^T\right)^2} - (1-\tilde{\phi}_0)\,\frac{\tilde{R}_0 - 1}{R-1}\frac{\gamma}{e_0}\frac{de_0}{dc_0^T} \right] \right\} < 0 \tag{A.25}$$

Using (A.23) and (A.25), it is straightforward to see that sign of $\frac{d^2\mathbf{F}(b_1;b_0)}{db_1^2} \propto$ sign of $1-\gamma$. Notice again that b_1 is part of an equilibrium if $\mathbf{F}(b_1;b_0)=0$, $b_1 < b_0$, and $\frac{b_1}{R} > b_0 - y^T$.

Case I: $\gamma \geq 1$. Notice that for $\gamma \geq 1$ we have $\mathbf{F}''(b_1;b_0) \leq 0$ (with equality iff $\frac{1}{\gamma}=1$). Because $\mathbf{F}$ is concave and $\mathbf{F}(b_0;b_0) > 0$, a self-fulfilling crises equilibrium exists (that is, there exists b_1 in $(R(b_0-y^T), b_0)$ such that $\mathbf{F}(b_1;b_0)=0$) if for the lowest value in the feasible domain of b_1, we have $\mathbf{F}(R(b_0-y^T);b_0) = (1+\kappa)y^T - b_0 < 0$ which implies $b_0 > (1+\kappa)y^T$. Furthermore, because F changes sign over $(R(b_0-y^T), b_0)$, the solution b_1 in $(R(b_0-y^T), b_0)$ is unique by concavity of the function $\mathbf{F}$. There is a unique self-fulfilling equilibrium for $b_0 > (1+\kappa)y^T$. Moreover, by Lemma 1 $b_0 < \hat{b}$ and as shown in Appendix A.2, the interval $((1+\kappa)y^T, \hat{b})$ is non-empty under Assumption 1.

Case II: $\gamma < 1$. In this case $\mathbf{F}''(b_1;b_0) < 0$ and because F is convex and $\mathbf{F}(b_0;b_0) > 0$, when a self-fulfilling crises equilibrium exists (that is, there exists b_1 in $(R(b_0-y^T), b_0)$ such that $\mathbf{F}(b_1;b_0)=0$), we know that for b_1 such that $\mathbf{F}(b_1;b_0)=0$ we have $\mathbf{F}'(b_1;b_0) \geq 0$. Letting $\psi_0 \equiv 1-(1-\gamma)\frac{c_0^T}{e_0}\frac{de_0}{dc_0^T}$. From (A.23) and (A.24), $\psi_0 \in (0, \frac{1}{\gamma})$ and we have

$$\mathbf{F}'(b_1;b_0) \geq 0 \Leftrightarrow \psi_0 \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{W_{-1}}{e_0}\right)^{1-\gamma} \geq 1$$
$$\Leftrightarrow \psi_0 \kappa \frac{1-\phi}{\phi} \left(\frac{c_0^T}{h_0}\right)^{\frac{\gamma-1}{\gamma}} \geq 1. \tag{A.26}$$

Using $b_0 = y^T + \frac{b_1}{R} - c_0^T$ and plugging in $\mathbf{F}(b_1;b_0)=0$ to substitute for $\frac{b_1}{R}$ we get

$$b_0 = y^T + \kappa y^T + \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{W_{-1}}{e_0}\right)^{1-\gamma} c_0^T - c_0^T \tag{A.27}$$

Notice that the right hand side of (A.27) is increasing in c_0^T as the derivative with respect to c_0^T corresponds to $\mathbf{F}'(b_1; b_0) \geq 0$. Hence, plugging (A.26) into (A.27) we arrive to

$$b_0 \geq (1+\kappa)y^T - (1-\psi_0)\left[\kappa\psi_0 \frac{1-\phi}{\phi}\right]^{\frac{\gamma}{\gamma-1}} h_0 \equiv \underline{b}^m. \tag{A.28}$$

Since $\psi_0 < 1/\gamma$, from (A.10) and (A.28) we get that $\underline{b}^m < \underline{b}$. Moreover, because F is convex $\mathbf{F}(b_1; b_0) = 0$ has two solutions if and only if $\mathbf{F}(b_1; b_0)$ has a solution and for lowest value in the feasible domain of b_1, i.e. $b_1 = R(b_0 - y^T)$, we have

$$\mathbf{F}(R(b_0 - y^T); b_0) = (1+\kappa)\kappa y^T - b_0 > 0 \ \Leftrightarrow \ b_0 < (1+\kappa)\kappa y^T$$

Therefore, $\mathbf{F}(b_1; b_0) = 0$ has two solutions for $b_0 \in \left[\underline{b}^m, (1+\kappa)y^T\right)$ and a unique solution for $b_0 \in \left[(1+\kappa)y^T, \hat{b}\right)$ where we $b_0 < \hat{b}$ from Lemma 1. Furthermore, as shown in Appendix A.2, the interval $((1+\kappa)y^T, \hat{b})$ is non-empty under Assumption 1.

A.7 Proof of Proposition 4

We assume that there is non-zero probability $\pi > 0$ that the economy ends in a self-fulfilling crisis equilibrium when the economy is in the vulnerable region. Recall that the allocation in the steady-state equilibrium is efficient. We denote by

$$\bar{W} = u\left(y^T - \frac{R-1}{R} b_0, \bar{h}\right) \tag{A.29}$$

$$W_C^p = u\left(y^T - b_0 + \frac{1}{R} b_1^p, h_0^p\right) + \frac{\beta}{1-\beta} u\left(y^T - \frac{R-1}{R} b_1^p, \bar{h}\right) \tag{A.30}$$

the social welfare respectively in the steady-state equilibrium and in a self-fulfilling crises equilibrium under the exchange rate regime $p \in \{\text{fix}, \text{flex}\}$. By definition $W_C^p < \bar{W}$ for any $p \in \{\text{fix}, \text{flex}\}$. Consider that $b_0 < (1+\kappa)y^T$. From Proposition 3, there is a unique equilibrium under fixed exchange rate and this equilibrium is the steady equilibrium. Thus, $W^{\text{fix}} = \bar{W}$. Under flexible exchange rate, however, two self-fulfilling crises equilibria exist for $\gamma < 1$. Thus, $W^{\text{flex}} = (1-\pi)\bar{W} + \pi W_C^{\text{flex}}$ if $\gamma < 1$ and $W^{\text{flex}} = \bar{W}$ otherwise. Therefore, we have $W^{\text{fix}} = W^{\text{flex}}$ for $\gamma \geq 1$ and

$$W^{\text{fix}} > W^{\text{flex}} \text{ for } \gamma < 1.$$

A.8 Proof of Proposition 5

We start by showing the full employment nominal exchange rate in a self-fulfilling crises when the economy starts at $b_0 = \underline{b}$ is given

$$\underline{e}_0 = W_{-1} \frac{\phi}{1-\phi} \left(\frac{y^T - b_0 + \underline{b}/R}{\bar{h}} \right)^{-\frac{1}{\gamma}}$$
$$= W_{-1} \left[\frac{\kappa}{\gamma} \left(\frac{1-\phi}{\phi} \right)^{\gamma} \right]^{\frac{1}{1-\gamma}} \tag{A.31}$$

To see this notice that the level of tradable consumption in a self-fulfilling crises for $b_0 = \underline{b}$ under a full employment policy is

$$\underline{c}_0^T = y^T - \underline{b} + \kappa y^T + \kappa \frac{1-\phi}{\phi} \left(\frac{\underline{c}_0^T}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} \underline{c}_0^T \tag{A.32}$$

From (A.7) we have that $\frac{\kappa}{\gamma} \frac{1-\phi}{\phi} \left(\frac{\underline{c}_0^T}{\bar{h}} \right)^{\frac{1-\gamma}{\gamma}} = 1$. Substituting it into (A.32) yields

$$(1-\gamma)\underline{c}_0^T = (1+\kappa)y^T - \underline{b} = (1-\gamma) \left[\frac{\kappa(1-\phi)}{\gamma\phi} \right]^{\frac{\gamma}{\gamma-1}} \bar{h}$$
$$\underline{c}_0^T = \left[\frac{\kappa(1-\phi)}{\gamma\phi} \right]^{\frac{\gamma}{\gamma-1}} \bar{h} \tag{A.33}$$

where the first equality uses (A.10) to substitute for $\underline{b}$.

We now turn to proving the results from Proposition 5. Recall that in a self-fulfilling crises equilibrium the collateral constraint binds. Using the aggregate demand for non-tradables (12) plugging in the next period debt level

$$y_0^N = \left(\frac{\phi}{1-\phi} \frac{W_{-1}}{e_0} \right)^{-\gamma} \left[y_0^T - b_0 + \frac{b_1}{R} \right] \quad \text{and} \quad \frac{b_1}{R} = \kappa \left[y_0^T + \left(\frac{W_{-1}}{e_0} \right) y_0^N \right],$$

yields (after using the equilibrium condition $c_0^N = y_0^N$)

$$y_0^N = \left[1 - \kappa \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} \right]^{-1} \left[\frac{1-\phi}{\phi} \frac{e_0}{W_{-1}} \right]^{\gamma} \left[(1+\kappa)\, y^T - b_0 \right], \tag{A.34}$$

Differentiating (A.34) with respect to e_0 we get

$$\frac{dy_0^N}{de_0} = \left[1 - \kappa \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} \right]^{-1} \left[\gamma - \kappa \left(\frac{1-\phi}{\phi} \right)^{\gamma} \left(\frac{e_0}{W_{-1}} \right)^{\gamma-1} \right] \frac{y_0^N}{e_0}. \tag{A.35}$$

To show that a depreciation is contractionary for any $e_0 \in (\underline{e}_0 \gamma^{\frac{1}{1-\gamma}}, \underline{e}_0)$ one need to show that (i) $0 < y_0^N \leq \bar{h}$ and (ii) $dy_0^N/de_0 < 0$. The first condition ensures that y_0^N is well defined and the second condition ensures y_0^N is monotonic and decreasing with e_0 that is for any $e_0, e_0^2 \in (\underline{e}_0\gamma^{\frac{1}{1-\gamma}}, \underline{e}_0)$ such that $e_0 > e_0^2$ we have $y^N(e_0) < y^N(e_0^2)$. We consider two cases.

Case I: $\gamma > 1$. We first consider the case where $\gamma > 1$. As shown in Propositions 1-2, the region of debt levels for which a self-fulfilling crises exists (regardless of the monetary policy regime considered) is $b_0 \in ((1+\kappa)y^T, \hat{b}]$. Because for any $e_0 \in (\underline{e}_0\gamma^{\frac{1}{1-\gamma}}, \underline{e}_0)$ we have $e_0 > \underline{e}_0\gamma^{\frac{1}{1-\gamma}}$, it follows that

$$\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} > \frac{\kappa}{\gamma}\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{\underline{e}_0}{W_{-1}}\right)^{\gamma-1} = 1 \tag{A.36}$$

where the equality uses (A.32) to substitute for $\underline{e}_0$. Using (A.36) and $b_0 > (1+\kappa)y^T$, we obtain from (A.34) that

$$y_0^N = \left[\frac{1-\phi}{\phi}\frac{e_0}{W_{-1}}\right]^{\gamma}\left[1-\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}\right]^{-1}\left[(1+\kappa)y^T - b_0\right] > 0$$

It remains to show that $\frac{dy_0^N}{de_0} < 0$. From $e_0 < \underline{e}_0$, we have

$$\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} < \kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{\underline{e}_0}{W_{-1}}\right)^{\gamma-1} = \gamma \tag{A.37}$$

where the equality uses again (A.32) to substitute for $\underline{e}_0$. Using now the inequalities (A.36) and (A.37) we arrive to

$$\frac{dy_0^N}{de_0} = \left[1-\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}\right]^{-1}\left[\gamma-\kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}\right]\frac{y_0^N}{e_0} < 0$$

Therefore, for any $e_0, e_0^2 \in (\underline{e}_0\gamma^{\frac{1}{1-\gamma}}, \underline{e}_0)$ such that $e_0 > e_0^2$ we have $y^N(e_0) < y^N(e_0^2)$.

Case II: $\gamma < 1$. We now consider the case where $\gamma < 1$. Notice that in this case, as shown in Propositions 1-2, self-fulfilling crises equilibrium can emerge for both $b_0 > (1+\kappa)y^T$ and $b_0 < (1+\kappa)y^T$. If $b_0 > (1+\kappa)y^T$

$$\begin{aligned} y_0^N > 0 \Rightarrow \kappa\left(\frac{1-\phi}{\phi}\right)^{\gamma}\left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} > 1 > \gamma \\ \Rightarrow e_0 < \underline{e}_0\gamma^{\frac{1}{1-\gamma}} \end{aligned} \tag{A.38}$$

and (A.38) implies that

$$\frac{dy_0^N}{de_0} = \left[1 - \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}\right]^{-1} \left[\gamma - \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}\right] \frac{y_0^N}{e_0} > 0.$$

For $b_0 > (1+\kappa)y^T$, consider $e_0 \in (\underline{e}_0 \gamma^{\frac{1}{1-\gamma}}, \underline{e}_0)$. We have $e_0 > \underline{e}_0 \gamma^{\frac{1}{1-\gamma}}$ which implies

$$\kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} > \frac{\kappa}{\gamma} \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{\underline{e}_0}{W_{-1}}\right)^{\gamma-1} = 1 \tag{A.39}$$

and using (A.39) along with $b_0 > (1+\kappa)y^T$ we get from (A.34) that $y_0^N > 0$. It remains to show that $\frac{dy_0^N}{de_0} < 0$. From $e_0 < \underline{e}_0$ we have

$$\kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1} > \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{\underline{e}_0}{W_{-1}}\right)^{\gamma-1} = \gamma \tag{A.40}$$

Using now both inequalities (A.36) and (A.40) we arrive to

$$\frac{dy_0^N}{de_0} = \left[1 - \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}\right]^{-1} \left[\gamma - \kappa \left(\frac{1-\phi}{\phi}\right)^{\gamma} \left(\frac{e_0}{W_{-1}}\right)^{\gamma-1}\right] \frac{y_0^N}{e_0} < 0$$

Therefore, for any $e_0, e_0^2 \in (\underline{e}_0 \gamma^{\frac{1}{1-\gamma}}, \underline{e}_0)$ such that $e_0 > e_0^2$ we have $y^N(e_0) < y^N(e_0^2)$.

A.9 Proof of Proposition 6

We start by showing that $\gamma < 1$ is a necessary and sufficient condition for the existence of a targeting rule that rules of self-fulfilling crisis equilibria and then showing that the proposed rule (22) rules out self-fulfilling crisis equilibra.

Let $\hat{h}_0$ denote the employment level induced by the targeting rule off the equilibrium path. Because raising employment above $\bar{h}$ is not feasible, we have $\hat{h}_0 \leq \bar{h}$. Given this policy, a self-fulfilling crises equilibrium exists if and only if $b_1 < b_0$ satisfies

$$u_T \left(y^T - b_0 + \frac{b_1}{R}, \hat{h}_0\right) \geq u_T \left(y^T - \frac{R-1}{R} b_1, \bar{h}\right) \tag{A.41}$$

To see why any allocation $\{b_1\}$ that satisfies (A.41) also satisfy all the remaining equilibrium conditions (7), (11), (12), (MD), (IP). Given b_1 that satisfies (A.41), pick $c_0^T = y^T - b_0 + \frac{b_1}{R}$ to satisfy (11) and $W_0 = W_{-1}$ to satisfy (7). Pick $e_0 = W_{-1} \frac{\phi}{1-\phi} (c^T / \hat{h}_0)^{-1/\gamma}$ to satisfy (12). Then, choose $\tilde{R}_0 = R \frac{e_1}{e_0}$ with $e_1 \equiv W_{-1} \frac{\phi}{1-\phi} ((y^T - \frac{R-1}{R} b_1) / \bar{h})^{-1/\gamma}$ to satisfy (IP) and pick $M_0 = \chi e_0 (\tilde{R}_0 - 1) / (\tilde{R}_0 u_T(c_0^T, \hat{h}_0))$ to satisfy (MD).

It follows that a targeting rule that rules out self-fulfilling crises equilibria exist if and only if $\nexists\, b_1 < b_0$ that satisfies (A.41). In order words, for any $b_1 < b_0$ we need

$$u_T\left(y^T - b_0 + \frac{b_1}{R}, \hat{h}_0\right) < u_T\left(y^T - \frac{R-1}{R}b_1, \bar{h}\right). \tag{A.42}$$

Because $b_1 < b_0$, we have

$$u_T\left(y^T - b_0 + \frac{b_1}{R}, \bar{h}\right) > u_T\left(y^T - \frac{R-1}{R}b_1, \bar{h}\right),$$

and since $\hat{h}_0 \le \bar{h}$ it follows that a targeting rule that rules out self-fulfilling crises equilibria exists if and only if $u_{TN} > 0 \Leftrightarrow \gamma > 1$.

Consider now the following employment targeting rule (for $\gamma < 1$)

$$h(b_1, b_0) = \bar{h} + \phi_h(b_1 - b_0), \tag{A.43}$$

where $\phi_h > 0$ satisfies

$$u'\left(y^T - b_0 + \frac{b_1}{R}, \bar{h} + \phi_h(b_1 - b_0)\right) < u'\left(y^T - \frac{R-1}{R}b_1, \bar{h}\right).$$

Since $b_1 < b_0$ and $\phi_h > 0$, we have that $h_0 = h(b_1, b_0) < \bar{h}$. In addition, the policy rule $h(b_0, b_0) = 0$. Moreover, off the equilibrium path we have

$$u'\left(y^T - b_0 + \frac{b_1}{R}, h_0\right) < u'\left(y^T - \frac{R-1}{R}b_1, \bar{h}\right).$$

Thus the proposed targeting rule (A.43) implements the first-best allocation and rules out the possibility of self-fulfilling crisis equilibria.

A.10 Proof of Proposition 7

We start by rewriting the exchange rate rule here for convenience

$$\begin{aligned} e(b_1, b_0) &= \bar{e}\left[1 + \frac{\beta}{c^T}(b_1 - b_0)\right]^{-\frac{1}{\gamma}}\left[\frac{b_1}{b_0} + \left(1 - \frac{b_1}{b_0}\right)\phi_e\frac{h_0}{\bar{h}}\right]^{\frac{1}{\gamma}} \\ &= W_{-1}\left[\frac{1-\phi}{\phi}\left(y^T - b_0 + \frac{b_1}{R}\right)\right]^{-\frac{1}{\gamma}}\left[\frac{b_1}{b_0} + \left(1 - \frac{b_1}{b_0}\right)\phi_e\frac{h_0}{\bar{h}}\right]^{\frac{1}{\gamma}} \end{aligned} \tag{A.44}$$

To show that the policy (A.44) rules out self-fulfilling crises equilibria, one needs to show that this policy off the equilibrium path, if an household believes that all other households will choose (b_1, h_0) with $b_1 < b_0$ (i.e., they deleverage) that household finds it optimal to choose a different action, that is $(b_1^i, h_0^i) \neq (b_1, h_0)$.

First notice that given our central bank exchange rate rule, off the equilibrium path there is involuntary unemployment $h_t < \bar{h}$. To clearly see this, consider by contradiction that there is full employment off the equilibrium path. Then we have

$$e_0 = W_{-1} \left[\frac{1-\phi}{\phi} \left(y^T - b_0 + \frac{b_1}{R} \right) \right]^{-\frac{1}{\gamma}} \left[\frac{b_1}{b_0} \bar{h} + 0 \right]^{\frac{1}{\gamma}} \tag{A.45}$$

and the equilibrium wage would violate the wage rigidity constraints

$$W_0 = e_0 \left(\frac{1-\phi}{\phi} \frac{c_0^T}{\bar{h}} \right)^{\frac{1}{\gamma}} = W_{-1} \frac{b_1}{b_0} < W_{-1}.$$

So, assume that an household i believes that all other households will choose (b_1, h_0) with $b_1 < b_0$ and $h_0 < \bar{h}$. Given the central bank exchange rate rule (A.44), the equilibrium wage satisfies

$$W_0 = e(b_1, b_0) \left(\frac{1-\phi}{\phi} \frac{c_0^T}{h_0} \right)^{\frac{1}{\gamma}} = W_{-1} \left(\frac{\bar{h}}{h_0} \right)^{\frac{1}{\gamma}} \left[\frac{b_1}{b_0} + \left(1 - \frac{b_1}{b_0} \right) \phi_e \frac{h_0}{\bar{h}} \right]^{\frac{1}{\gamma}} \tag{A.46}$$

$$> W_{-1} \left[\frac{b_1}{b_0} \frac{\bar{h}}{h_0} + \left(1 - \frac{b_1}{b_0} \right) \right]^{\frac{1}{\gamma}}$$

$$= W_{-1} \left[1 + \frac{b_1}{b_0} \left(\frac{\bar{h}}{h_0} - 1 \right) \right]^{\frac{1}{\gamma}} > W_{-1} \tag{A.47}$$

Thus, the labor market slackness condition is not binding $\left(h_0 - \bar{h}\right)\left(W_0 - W_{-1}\right) > 0$. This implies that firms are willing to meet any increase in labor supply. Because the individual household i's welfare is strictly increasing in hours worked, the household will make himself strictly better off by increasing h_0 by $\varepsilon > 0$ arbitraly small.

Therefore, if an household believes that all other households will choose (b_1, h_0) with $b_1 < b_0$ that household finds it optimal to choose a different action. The exchange rate rule (A.44) thus rules out the possibility of self-fulfilling crises equilibria (i.e., equilibria with $b_1 < b_0$).

B Contractionary Depreciations with Fundamental Shocks

We discuss here conditions under which depreciations are contractionary when fundamental shocks lead to a binding borrowing constraint. That is, we dispense Assumption 1 and consider a situation in which there is a unique equilibrium, which features potentially a binding constraint.

Consider that the economy starts period 0 with an initial debt position b_0, there is no uncertainty and $\beta R = 1$. Assume that in period 0, there is a one-shot unexpected financial shock represented by a temporary decline in the collateral coefficient κ_0 such that

$$\kappa_0 < b_0 \left[y^T + \frac{1-\phi}{\phi} \left(y^T - \frac{R-1}{R} b_0 \right)^{\frac{1}{\gamma}} (\bar{h})^{1-\frac{1}{\gamma}} \right]^{-1} \tag{B.1}$$

i.e., the borrowing constraint binds. Proposition B.1 describes conditions under which depreciations are contractionary.

Proposition B.1 (Contractionary Depreciations)**.** *Suppose that κ_0 is such that the borrowing constraint binds and consider $e_0^1, e_0^2 \in (\underline{e}\gamma^{\frac{1}{1-\gamma}}, \underline{e})$ where $\bar{e}$ is given by* (14) *and*

$$\underline{e} \equiv W_{-1} \left[\frac{\kappa}{\gamma} \left(\frac{1-\phi}{\phi} \right)^{\gamma} \right]^{\frac{1}{1-\gamma}}. \tag{B.2}$$

such that $e_0^1 > e_0^2$. Then, $y^N(e_0^1) < y^N(e_0^2)$, that depreciations are contractionary, if one of the following conditions is satisfied

i. if $b_0 < (1+\kappa)y^T$ and $\gamma < 1$

ii. if $b_0 > (1+\kappa)y^T$ and $\gamma > 1$.

Proof. Assume that $\{\kappa_0\}$ is such that the inequality (B.1) is satisfied which implies that $b_1 = b_0$ violates the borrowing constraint. The borrowing constraints therefore binds given the fundamentals in period 0. Using the demand for nontradable goods and imposing that the borrowing constraint binds in period 0,

$$y_0^N = \left(\frac{\phi}{1-\phi} \frac{W_{-1}}{e_0} \right)^{-\gamma} \left[y_0^T - b_0 + \frac{b_1}{R} \right] \quad \text{and} \quad \frac{b_1}{R} = \kappa \left[y_0^T + \left(\frac{W_{-1}}{e_0} \right) y_0^N \right],$$

we obtain

$$y_0^N = \left[\left(\frac{\phi}{1-\phi} \frac{W_{-1}}{e_0} \right)^{\gamma} - \kappa \left(\frac{W_{-1}}{e_0} \right) \right]^{-1} \left[(1+\kappa) y^T - b_0 \right]. \tag{B.3}$$

Differentiating non-tradable output with respect to the nominal exchange rate yields

$$\frac{\partial y_0^N}{\partial e_0} = \frac{y_0^N}{e_0}\left[\left(\frac{\phi}{1-\phi}\frac{W_{-1}}{e_0}\right)^{\gamma} - \kappa\left(\frac{W_{-1}}{e_0}\right)\right]^{-1}\left[\gamma\left(\frac{\phi}{1-\phi}\frac{W_{-1}}{e_0}\right)^{\gamma} - \kappa\left(\frac{W_{-1}}{e_0}\right)\right] \tag{B.4}$$

Two cases can be distinguished depending on b_0. Consider first the case where $(1+\kappa)y^T - b_0 > 0$. Then from (B.3), $y_0^N > 0$ requires that

$$\left(\frac{\phi}{1-\phi}\frac{W_{-1}}{e_0}\right)^{\gamma} - \kappa\left(\frac{W_{-1}}{e_0}\right) > 0 \Rightarrow e_0 > W_{-1}\left(\kappa\right)^{\frac{1}{1-\gamma}}\left(\frac{1-\phi}{\phi}\right)^{\frac{\gamma}{1-\gamma}}.$$

Substituting it into (B.4), we have on the one hand that if $\gamma \geq 1$ then an exchange depreciation is always expansionary. On the other hand, from (B.4) we have that $\frac{\partial y_0^N}{\partial e_0} < 0$, i.e. an exchange rate depreciation is contractionary, if $\gamma < 1$ and

$$\gamma\left(\frac{\phi}{1-\phi}\frac{W_{-1}}{e_0}\right)^{\gamma} - \kappa\left(\frac{W_{-1}}{e_0}\right) < 0 \Rightarrow e_0 < W_{-1}\left(\frac{\kappa}{\gamma}\right)^{\frac{1}{1-\gamma}}\left(\frac{1-\phi}{\phi}\right)^{\frac{\gamma}{1-\gamma}}. \tag{B.5}$$

Defining $\underline{e}$ as in (B.2) and noting the value of the collateral $\kappa(y^T + \frac{W_{-1}}{e_0}y_0^N)$ falls when depreciations are contractionary, it follows that for $e_0^1, e_0^2 \in (\underline{e}\gamma^{\frac{1}{1-\gamma}}, \underline{e}]$ such that $e_0^1 > e_0^2$ we have $y^N(e_0^1) < y^N(e_0^2)$.

Consider now that case where $(1+\kappa)y^T - b_0 < 0$. Proceeding similar, we have (B.3) that $y_0^N > 0$ requires

$$\left(\frac{\phi}{1-\phi}\frac{W_{-1}}{e_0}\right)^{\gamma} - \kappa\left(\frac{W_{-1}}{e_0}\right) < 0 \Rightarrow e_0 > W_{-1}\left(\kappa\right)^{\frac{1}{1-\gamma}}\left(\frac{1-\phi}{\phi}\right)^{\frac{\gamma}{1-\gamma}}.$$

Substituting it into (B.4), we have on the one hand that if $\gamma \leq 1$ then an exchange depreciation is always expansionary. On the other hand, from (B.4) we have that $\frac{\partial y_0^N}{\partial e_0} < 0$, i.e. an exchange rate depreciation is contractionary, if $\gamma > 1$ and

$$\gamma\left(\frac{\phi}{1-\phi}\frac{W_{-1}}{e_0}\right)^{\gamma} - \kappa\left(\frac{W_{-1}}{e_0}\right) > 0 \Rightarrow e_0 < W_{-1}\left(\frac{\kappa}{\gamma}\right)^{\frac{1}{1-\gamma}}\left(\frac{1-\phi}{\phi}\right)^{\frac{\gamma}{1-\gamma}}. \tag{B.6}$$

□

Illustration. The parameter values are $\phi = 0.25$, $W_{-1} = 1$, $R = 1.02$, $\beta = 1/R$, $\gamma = 0.4$. The next figure plots policy functions for output and borrowing as a function of the nominal exchange rate for two credit regime: the normal credit regime where the value of collateral coefficient is set to $\kappa_h = 0.5$ and the tight credit regime where the value of

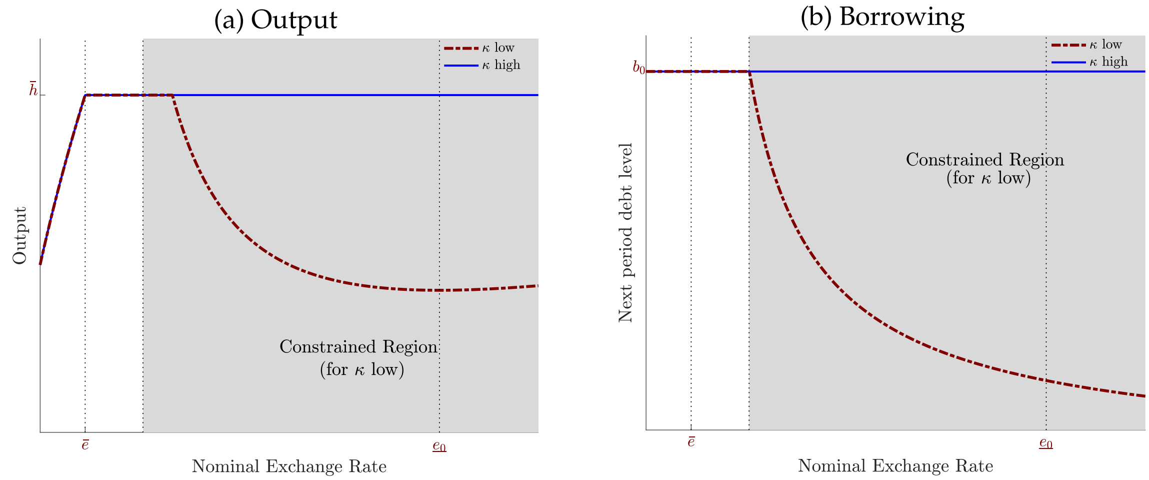

Figure B.1: Contractionary Depreciations with Fundamental Shocks

the collateral coefficient drops to $\kappa_l = 0.25$. In the normal credit regime, the borrowing constraint never binds.