On the Optimal Speed of Sovereign Deleveraging with Precautionary Savings

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Abstract

We study the optimal path of reduction of government debt in an economy when the economy is depressed and there is significant sovereign default risk. We emphasize the endogenous response of savers to sovereign risk. We obtain two main results. First, this new macro-economic channel changes the tradeoff between the recessionary impact of fiscal consolidation and the risk of a future sovereign debt crisis. Second, we find that savers and borrowers almost always disagree about the optimal path of sovereign deleveraging.

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How fast should governments repay their debt? The question has been at the center of much policy debate in the aftermath of the Great Recession and of the Eurozone crisis.

One side of the tradeoff is rather straightforward. In any non-Ricardian model, fiscal consolidation can depress aggregate demand and decrease employment. Whether this is good or bad depends on the state of the economy. If the economy is already depressed, fiscal consolidation is pro-cyclical and it can have large negative welfare consequences.

The other side of the trade-off is more complex to analyze. Clearly, the point of fiscal consolidation is to lower future default risk. The key issue is then to understand how sovereign risk affects the economy. In benchmark models of external debt the cost of default is a temporary exclusion from international financial markets. But exclusion does not last long in practice, and official lenders typically step in when private lenders pull out. The literature then typically assumes that sovereign default creates a large exogenous loss in output (Eaton and Gersovitz, 1982; Arellano, 2008).

A more recent literature, motivated in parts by the Eurozone crisis, has emphasized the impact of sovereign risk on the funding costs of financial intermediaries. There are two main channels, a macroeconomic channel and a financial channel. Fiscal austerity lowers output and increases credit risk in the private sector (households and businesses), which hurts the banks and prevents them from making new loans. Gourinchas et al. (2016) quantify this macro channel in the case of Greece. Note that the price of government bonds only matters to the extent that high spreads induce the government to raise taxes and cut spending. Alternatively, sovereign credit risk can directly hurt levered financial intermediaries, either directly because they hold government bonds (Gennaioli et al., 2014; Bocola, 2016; Perez, 2016), or indirectly because the state insures some of their liabilities (e.g., deposit insurance).

The financial channel is plausible in the short run, but it only operates if one assumes that intermediaries cannot raise capital. If banks can raise capital then what matters is the pricing kernel of their shareholders. Another issue with this channel is that most of the public debt is not held by banks or levered institutions, but rather by pension funds and separate accounts of insurance companies. In that case, one should really model the savers as being exposed to sovereign risk. Finally, even if we consider banks, lending surveys suggest that the drop in credit supply was rather temporary, and that a significant part of the slowdown in credit comes from low credit demand.

It is therefore important to consider models where domestic sovereign exposure is not concentrated in levered financial institutions, but rather born by domestic savers. As it turns out, however, this idea already delivers rich dynamics and new insights. First, it clarifies the role of non-Ricardian features. We usually think of Ricardian equivalence as saying that the timing of taxes does not matter, but in fact it also says that sovereign risk does not matter. A direct corollary of Ricardian equivalence is that sovereign defaults are irrelevant, both ex-ante and ex-post. In a Ricardian model, if the government imposes a haircut of 30% on its debt, nothing happens, because this is exactly compensated by a 30% decrease in the net present value of taxes.

We consider a model where some agents are constrained in their ability to borrow, which breaks Ricardian equivalence. This activates both sides of the tradeoff. On the one hand, an increase in taxes used to repay the debt of the government has a negative impact on the disposable income of constrained agents, and thus on aggregate demand for goods and services.

On the other hand, the larger is the debt of the government, the higher is the probability of a default. Government default represents a net loss for holders of government debt. In our model the holders of (most of) government debt are domestic savers. The risk of sovereign default increases their precautionary savings, which hurts aggregate demand. An important point here is that this risk is linked to the size of the haircut even if default does not create exogenous deadweight losses.

We therefore obtain a new tradeoff between the contractionary effects of fiscal consolidation and the risk of a sovereign debt crisis. When the risk of default is very responsive to the level of debt, our model predicts that austerity can be expansionary. An increase in taxes can lead to a decrease in precautionary savings that is strong enough to offset the direct effect on the disposable income of constrained agents.

We study a sequence of shocks that captures the timing of events during the Great Recession and the Eurozone crisis. The economy starts in steady state. The first shocks is the start of private deleveraging which forces the constrained agents to pay back some of their debts. This leads to a decrease in aggregate demand, as in Eggertsson and Krugman (2012). We consider an economy where wages are sticky and the nominal interest rates does not adjust, either because it is set outside the country (eurozone) or because of the ZLB. Private deleveraging then creates a recession.

The second shock that hits the economy is a sovereign risk shock, modeled as an increase in the risk of government default. At this point we study several paths for sovereign deleveraging. The government can start immediately, or it can wait until private deleveraging is over. We estimate the output losses for each strategy and the welfare of savers and borrowers. Expansionary austerity does not arise in our calibrated model but a striking feature of the simulations is that borrowers and savers almost always disagree about the path of deleveraging. Borrowers prefer delayed deleveraging, while savers prefer early deleveraging, even though they understand that this will reduce their labor earnings. Our mode can therefore shed light on the political tensions that have appeared in almost all countries regarding fiscal policy.

Most of our analysis uses a closed economy limit where all the government debt is held by domestic savers. We also consider an extension where foreigners hold some of the debt.

Discussion of the Literature The literature on sovereign debt usually assumes that sovereign bonds are priced by deep-pocket investors, often risk-neutral and interpreted as international lenders. In the benchmark models of Aguiar and Gopinath (2006) and Arellano (2008), for instance, the government trades one period discount bonds with risk neutral competitive foreign creditors. As a result the price of the bond is $q_t = \frac{1-\pi_t \hbar}{1+r}$ where π is the probability of default and $\hbar$ is the haircut in case of default. The assumption that matters is not that investors are risk neutral, since we can always reinterpret the model as being written under the risk neutral measure of foreign lenders, or, equivalently, assume that their pricing kernel is correlated with the country's risk.[1]

By contrast, our key mechanism is that sovereign debt is held (in large parts) by domestic agents who are risk averse. In addition the precautionary savings interacts with the non-Ricardian features, so that agents are not only averse to deadweight losses, but also to haircuts, which is clearly an important empirical feature. In that sense our model resembles the models where sovereign risk hurts directly the balance sheet of levered financial intermediaries (Gennaioli et al., 2014; Bocola, 2016; Perez, 2016). Bocola (2016) models the direct exposure of banks. He decomposes the impact in two channels. First, asset losses can create a binding constraint on banks, leading to a decline in credit supply. But there is also a precautionary channel: even if the funding constraint of banks is not currently binding, it might bind in the future, and banks can decide to reduce their lending as a precautionary measure. The main result in Bocola (2016) is that the precautionary channel can be significant (up to 40% of the entire effect).

The predictions of models based on levered intermediaries' exposures, however, are very sensitive to the details of financial contracts available to intermediaries. Amplification only happens when intermediaries issue non-contingent debt and cannot be recapitalized. In the short run the assumption of constant bank capital is realistic, but less so as time passes. Our model can thus be thought of as

[1] Arellano (2008) extends her basic framework to risk averse lenders and chooses the parameters of their pricing kernel to match the average spread.

a medium run model of sovereign risk.

To keep the model tractable, we assume an exogenous mapping from debt levels to default risk, while much of the literature focuses on the incentives to repay, as summarized in Aguiar and Amador (2014).

The literature has also analyzed the feedback from private credit risk to sovereign risk. There are also two main channels: a macroeconomic channel, and a financial guarantee channel. The macro channel is straightforward: an increase in private funding costs decreases investment and consumption by borrowers, which can lead to a recession and lower tax revenues, more transfer payments on automatic programs (e.g. unemployment insurance) and perhaps discretionary fiscal stimulus, all of which can increase sovereign debt (Martin and Philippon, 2014; Gourinchas et al., 2016). The guarantee channel applies mostly to explicit and implicit guarantees on financial intermediaries, ranging from deposit insurance to outright bailouts (Acharya et al., 2015).

Our tradeoff between causing a recession and risking a debt crisis hinges crucially on the size of the fiscal multiplier, which is an endogenous object. Recent research by Huidrom et al. (2016) and Huidrom et al. (2016) points to the level of government debt and the 'fiscal space' as central determinants of the multiplier, aside from the cycle. They find low or even negative multipliers when government debt is high. Our results suggest that precautionary behavior could be behind such low multipliers when sovereign default becomes a clear possibility. Also closely related is the work of Romei (2015), who looks at a similar problem of a government deciding how fast to pay down a given stock of debt. However, she is mostly interested in the distributional aspects of this deleveraging and not in the decision of how long to remain in a crisis-prone region (Cole and Kehoe, 2000), which is the focus here. This is also related to the recent work of Escolano and Gaspar (2016)

The remainder of the paper is organized as follows. Section 1 describes the macroeconomic setup of the model. Section 1 develops a simple 2-period model to build intuition for our results. Section 3 presents the full model. Section 5 concludes.

1 General Setup

We consider a small open economy under a fixed nominal exchange rate. This section introduces the basic features of our model.

1.1 Government

The government spends G_t on goods and services, levies lump-sum taxes T_t, and issues long term bonds. We model bonds with geometrically decaying face value as in Leland (1998). One unit of face value issued at time t pays a coupon $(1-\rho)^s\kappa$ in period $t+s+1$ as long as the government does not default. Let $B^g_{\$,t}$ be the face value in units of the common currency of debt outstanding at the end of time t. Because debt decays at rate δ, the amount of debt brought from the past is $(1-\rho)B^g_{\$,t-1}$. The net issuance is therefore $B^g_{\$,t}-(1-\rho)B^g_{\$,t-1}$. The appealing feature of Leland (1998) is that all debt trades at the same unit price, irrespective of when it was issued. Let q_t be the price of one unit of government debt. The nominal budget constraint of the government, conditional on not defaulting, is

$$q_t\left(B^g_{\$,t}-(1-\rho)B^g_{\$,t-1}\right)=\kappa B^g_{\$,t-1}+P_{H,t}\left(G_t-T_t\right),$$

where $P_{H,t}$ is the price index of home goods. It will be convenient to work with real variables, so we define real government debt $B^g_t\equiv\frac{B^g_{\$,t}}{P_{H,t}}$. We can then re-write the budget constraint (conditional on not defaulting) as

$$q_t\left(B^g_t-(1-\rho)\frac{B^g_{t-1}}{\Pi_{H,t}}\right)=\kappa\frac{B^g_{t-1}}{\Pi_{H,t}}+G_t-T_t, \tag{1}$$

where $\Pi_{H,t}\equiv\frac{P_{H,t}}{P_{H,t-1}}$ is the domestic (i.e. PPI) inflation rate from $t-1$ to t. This formula makes clear that unexpected inflation at time t lowers the real debt burden. We use this convention for all other nominal assets.

If the debt does not default, the return from holding the debt between t and $t+1$ is

$$\tilde{R}^{(g,0)}_{t+1}=\frac{\kappa+(1-\rho)\,q_{t+1}}{q_t}$$

Let r be the (constant) global risk free rate. The price $q^\star$ of risk-free debt must satisfies $\tilde{R}^{(g,0)}=1+r$ so

$$q^\star=\frac{\kappa}{r+\rho}$$

We normalize $\kappa=r+\rho$ so risk free debt trades at par, $q^\star=1$. We will discuss sovereign risk later.

1.2 Households

There is a continuum of households who differ in their discount rates: some are more patient that others. Household i seeks to maximize

$$\sum_{t=0}^{\infty} \beta_i^t \left(u\left(\mathbf{C}_t^i\right) - \kappa_n \frac{N_{i,t}^{1+\varphi}}{1+\varphi} \right), \tag{2}$$

where κ_n is a scaling factor and $\mathbf{C}_{i,t}$ is a bundle of home (H) and foreign goods (F), defined as in Gali and Monacelli (2008) by

$$\mathbf{C}_t^i \equiv \left[(1-\varpi)^{\frac{1}{\epsilon_h}} C_{H,t}^{i\,\frac{\epsilon_h - 1}{\epsilon_h}} + \varpi^{\frac{1}{\epsilon_h}} C_{F,t}^{i\,\frac{\epsilon_h - 1}{\epsilon_h}} \right]^{\frac{\epsilon_h}{\epsilon_h - 1}}, \tag{3}$$

where ϵ_h is the elasticity of substitution between home and foreign goods and ϖ is the degree of openness of the economy. As usual, the home consumer price index (CPI) is

$$\mathbf{P}_t \equiv \left[(1-\varpi) P_{H,t}^{1-\epsilon_h} + \varpi P_{F,t}^{1-\epsilon_h} \right]^{\frac{1}{1-\epsilon_h}}. \tag{4}$$

We index impatient households by b (as borrowers) and patient households by s (as savers). There is a mass χ of b-types and a mass $1-\chi$ of s-types with $\beta_s > \beta_b$. Let $B_{\$,t}^h$ be the nominal face value of the debt issued at t and due at $t+1$, and let $B_t^h \equiv \frac{B_{\$,t}^h}{P_{H,t}}$ be real debt in terms of home goods. The *borrower*s' budget constraint is

$$\mathbf{P}_t \mathbf{C}_t^b = W_t^b N_t^b + P_{H,t} \frac{B_t^h}{R_t^h} - P_{H,t-1} B_{t-1}^h - P_{H,t} T_t. \tag{5}$$

subject to a debt constraint

$$B_t^h \leq \bar{B}_t^h.$$

The *savers*' budget constraint is

$$\mathbf{P}_t \mathbf{C}_t^s = W_t^s N_t^s + \tilde{R}_t S_{t-1} - S_t - P_{H,t} T_t, \tag{6}$$

where $\tilde{R}_t$ is the nominal after-tax gross return on savings S_{t-1}. This return is a complex object since savers hold government bonds, private debt (of borrowers, directly in the benchmark model, or via intermediaries in an extension), foreign assets, and equity in corporate businesses. Several assets are traded in our economy. For any asset j that is traded, its return must satisfy

$$\mathbb{E}_t\left[\beta\frac{u'\left(\mathbf{C}_{t+1}^s\right)}{u'\left(\mathbf{C}_t^s\right)}\frac{\tilde{R}_{t+1}^{(j)}}{\mathbf{\Pi}_{t+1}}\right]=1, \tag{7}$$

where $\mathbf{\Pi}_{t+1}=\mathbf{P}_{t+1}/\mathbf{P}_t$ denotes the gross CPI inflation rate from t to $t+1$. Aggregating across types we get

$$\mathbf{C}_t \quad = \quad \chi\mathbf{C}_t^b+(1-\chi)\,\mathbf{C}_t^s.$$

From the CES bundle (3), we know that imports satisfy

$$C_{F,t} \quad = \quad \varpi\left(\frac{P_{F,t}}{\mathbf{P}_t}\right)^{-\epsilon_h}\mathbf{C}_t$$

and domestic consumption of home goods is $C_{H,t}=(1-\varpi)\left(\frac{P_{H,t}}{\mathbf{P}_t}\right)^{-\epsilon_h}\mathbf{C}_t$. Finally, under flexible wages, the labor supply condition is

$$\kappa_n N_{i,t}^{\varphi}=\frac{W_{i,t}}{\mathbf{P}_t}u'\left(\mathbf{C}_{i,t}\right).$$

We discuss wage and price rigidity later.

1.3 Production and Market Clearing

Production is linear in labor,

$$Y_{H,t}=\mathbf{N}_t-\delta_t\Delta,$$

where $\mathcal{H}_t$ is an indicator of sovereign default, Δ measure the deadweight loss from default, $\mathbf{N}_t$ is an index of labor supplied by borrowers and savers, as in Benigno et al. (2016)

$$\mathbf{N}_t\equiv N_{b,t}^{\chi}N_{s,t}^{1-\chi}. \tag{8}$$

This Cobb-Douglas specification, together with CARA preferences helps us obtain clear theoretical results. Firms minimize total labor costs $\chi W_{b,t}N_{b,t}+(1-\chi)\,W_{s,t}N_{s,t}$, which implies that per-capita labor incomes are the same for both types

$$W_{b,t}N_{b,t} = W_{s,t}N_{s,t} = \mathbf{W}_t\mathbf{N}_t,$$

and the wage index is defined as $\mathbf{W}_t \equiv W_{b,t}^{\chi}W_{s,t}^{1-\chi}$. Clearing the market for domestic goods requires

$$Y_{H,t} = C_{H,t} + G_t + \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\epsilon_f} C^*_{F,t}, \tag{9}$$

where $C^*_{F,t}$ is foreign demand, $P_{F,t}$ the foreign price index and ϵ_f is the demand elasticity. Finally, we make a technical assumption to ensure stationarity of net foreign assets (NFA).[2] We assume that there is a (small) impact of NFA on the country's borrowing (or saving) rate

$$\frac{\partial \log R_t}{\partial \log NFA_t} = -\epsilon_r,$$

where ϵ_r is a small but strictly positive number and the net foreign position evolves according to

$$\frac{NFA_t}{R_t} = NFA_{t-1} - \mathbf{P}_t\mathbf{C}_t + P_{H,t}\left(Y_t - G_t\right). \tag{10}$$

1.4 Steady State

We consider a steady state with stable prices at home and abroad: $\Pi = 1$. We normalize foreign prices to $P_F = 1$ and foreign demand to $C^*_F = \varpi$. We assume a debt pricing schedule where R is decreasing in NFA and $\beta R(0) = 1$. This ensures a unique steady state with $NFA = 0$, $\beta R = 1$, and

$$P_H Y \quad = \quad P_H G + \mathbf{PC}. \tag{11}$$

where $\mathbf{P} = \left[(1-\varpi) P_H^{1-\epsilon_h} + \varpi\right]^{\frac{1}{1-\epsilon_h}}$. Clearing the goods market requires

$$Y_H = C_H + G + P_H^{-\epsilon_f}\varpi, \tag{12}$$

and we know that

$$C_H = (1-\varpi)\left(\frac{P_H}{\mathbf{P}}\right)^{-\epsilon_h}\mathbf{C}. \tag{13}$$

[2] See Schmitt-Grohe and Uribe (2003) for a detailed discussion.

It is convenient to normalize the steady state so that all prices are equal to one. We thus choose the labor supply so that $P_H = 1$. Combining (11,12,13), this implies

$$\mathbf{C} = 1$$

and therefore

$$Y = \mathbf{N} = 1 + G.$$

We then choose the labor supply parameters (κ_n, φ) to support this production level.[3]

2 Simple Example with 2 Periods

We study here a simple model with two periods. The period $t = 1$ is the short run with fixed nominal prices and wages. The period $t = 2$ is the long run with flexible prices and wages. We start with the case without sovereign risk and we introduce government default later.

2.1 Long Run Equilibrium

Let us consider first an equilibrium where the government and the households repay their debts. The budget constraints, assuming no default, are then

$$T_2 = G_2 + \frac{B_1^g}{\Pi_2^H},$$
$$\frac{\mathbf{P}_2}{P_{H,2}} \mathbf{C}_2^b = \frac{\mathbf{W}_2 \mathbf{N}_2}{P_{H,2}} - \frac{B_1^h}{\Pi_{H,2}} - T_2$$

Savers earn a (possibly random) return from lending to other households and to the government, and they receive dividends from firms. Optimal labor supply implies $\kappa_n N_{i,2}^{\varphi} = \frac{W_{i,2}}{\mathbf{P}_2} u'(C_{i,2})$ for each agent and the labor index is defined as $\mathbf{N}_2 = N_{b,2}^{\chi} N_{s,2}^{1-\chi}$. Aggregate consumption is $\mathbf{C}_t = \chi \mathbf{C}_t^b + (1-\chi)\mathbf{C}_t^s$,

[3] Assuming for simplicity that production subsidies undo any monopoly distortions, so $P_H = W$, this requires $\kappa_n (1+G)^{\varphi} = u'(1)$, so we need to set $\kappa_n = \frac{u'(1)}{(1+G)^{\varphi}}$. Note that this is simply a way to scale the steady state to obtain convenient relative prices.

and the equilibrium conditions are

$$\begin{aligned}
\mathbf{N}_2 &= (1-\varpi)\left(\frac{P_{H,2}}{\mathbf{P}_2}\right)^{-\epsilon_h}\mathbf{C}_2 + G_2 + \left(\frac{P_{H,2}}{P_{F,2}}\right)^{-\epsilon_f} C^*_{F,2} \\
\frac{\mathbf{P}_2}{P_{H,2}}\mathbf{C}^s_2 &= \frac{\mathbf{W}_2\mathbf{N}_2}{P_{H,2}} + \frac{\chi}{1-\chi}\frac{B^h_1}{\Pi_{H,2}} + \frac{1}{1-\chi}\frac{B^g_1}{\Pi_{H,2}} + \frac{1}{1-\chi}\frac{NFA_1}{P_{H,2}} - T_2 + \frac{\left(1-\frac{W_2}{P_{H,2}}\right)Y_2}{1-\chi}, \\
\mathbf{P}_2\mathbf{C}_2 &= P_{H,2}\left(\mathbf{N}_2 - G_2\right) + NFA_1,
\end{aligned}$$

and the price index is $\mathbf{P}_2 = \left[\varpi + (1-\varpi)P_{H,2}^{1-\epsilon_h}\right]^{\frac{1}{1-\epsilon_h}}$. At time 2 we consider a model with flexible (and competitive) wages and prices, so $P_{H,2} = W_2$. And we use the fact that $W_{b,2}N_{b,2} = W_{s,2}N_{s,2} = \mathbf{W}_2\mathbf{N}_2$ to write the equilibrium conditions as

$$\begin{aligned}
\kappa_n \mathbf{N}_2^{\varphi} &= \left(u'\left(C_{b,2}\right)\right)^{\chi}\left(u'\left(C_{s,2}\right)\right)^{1-\chi} \\
\frac{\mathbf{P}_2}{P_{H,2}}\mathbf{C}^b_2 &= \mathbf{N}_2 - G_2 - \frac{B^h_1 + B^g_1}{\Pi_{H,2}} \\
\frac{\mathbf{P}_2}{P_{H,2}}\mathbf{C}^s_2 &= \mathbf{N}_2 - G_2 + \frac{\chi}{1-\chi}\frac{B^h_1 + B^g_1}{\Pi_{H,2}} + \frac{1}{1-\chi}\frac{NFA_1}{P_{H,2}}
\end{aligned}$$

2.2 Closed Economy and CARA Preferences

We use CARA preferences to obtain closed-form solutions.

$$u(C) = \frac{-1}{\gamma}\exp(-\gamma C)$$

Under CARA, we therefore get a simple aggregation result:

$$\begin{aligned}
\log\left(\kappa_n N_2^{\varphi}\right) &= -\gamma\frac{P_{H,2}}{\mathbf{P}_2}\left(\chi\left(N_2 - G_2 - \frac{B^h_1 + B^g_1}{\Pi_2}\right) + (1-\chi)\left(N_2 - G_2 + \frac{\chi}{1-\chi}\frac{B^h_1 + B^g_1}{\Pi_2} + \frac{1}{1-\chi}\frac{NFA_1}{P_{H,2}}\right)\right) \\
&= -\gamma\frac{P_{H,2}}{\mathbf{P}_2}\left(N_2 - G_2 + \frac{NFA_1}{P_{H,2}}\right)
\end{aligned}$$

In general the equilibrium at time 2 depends on the net foreign assets that the agents bring into the period. As they get richer, they consume more and work less. To keep the analysis simple, we focus here on the closed economy limit.

Closed economy limit Let us consider the closed economy limit where $\varpi \to 0$, and thus $\frac{P_{H,2}}{\mathbf{P}_2} = 1$ and $NFA_1 = 0$. Aggregate labor supply is independent of the distribution of debt balances among households and simply solves

$$\bar{N}(G) : \log \kappa_n + \varphi \log \bar{N} = -\gamma \left(\bar{N} - G\right)$$

In the steady state above, we choose κ_n so that $\bar{N} = 1 + G$ and therefore $\log \kappa_n = -\gamma - \varphi \log (1 + G)$. Once we have solved for the aggregate, we easily obtain the consumption of each group as

$$C_2^s = 1 + \frac{\chi}{1 - \chi} \frac{B_1^h + B_1^g}{\Pi_{H,2}} \tag{14}$$

and

$$C_2^b = 1 - \frac{B_1^h + B_1^g}{\Pi_{H,2}}$$

The nice feature of CARA/Cobb-Douglas is a clear dichotomy between the aggregate and the distributional consequences of debt balances. We use this CARA/ closed economy setup in the rest of this section.

2.3 Short Run: Fixed Price Equilibrium

Consider now the equilibrium at time 1 with exogenous prices and wages. The market clearing condition is

$$\mathbf{N}_1 = \mathbf{C}_1 + G_1$$

The government starts with B_0^g debt outstanding and the borrowers with B_0^h so the budget constraints are

$$\begin{aligned}
\frac{B_1^g}{R_1^g} &= G_1 - T_1 + \frac{B_0^g}{\Pi_1} \\
C_1^b &= \mathbf{w}_1 \mathbf{N}_1 + \frac{B_1^h}{R_1^h} - \frac{B_0^h}{\Pi_1} - T_1 \\
C_1^s &= \mathbf{w}_1 \mathbf{N}_1 + \frac{\tilde{R}_1}{\Pi_1} S_0 - S_1 - T_1 + \frac{(1 - \mathbf{w}_1) Y_1}{1 - \chi}
\end{aligned}$$

where w_1 is the real wage, $\tilde{R}_1$ is the nominal rate of return earned by savers, who also receive dividends from firms $(1-\mathbf{w}_1)\,Y_1$. Borrowers are subject to a borrowing limit

$$B_1^h \leq \bar{B}_1^h$$

Prices and wages are exogenous at time 1 and we ignore the labor supply curves. The savers' Euler equation is

$$\mathbb{E}_1\left[\beta \frac{u'(C_2^s)}{u'(C_1^s)} \frac{\tilde{R}_2}{\Pi_2}\right] = 1$$

where $\tilde{R}_2$ is the nominal return earned by savers at time 2. The return can be random if there is credit risk and/or aggregate uncertainty. In this section, however, we consider the case where all debts are risk free so R_1 is the same for all for households and for the government, and since there is no risk we have $R_1 = \tilde{R}_2$. We normalize $\Pi_1 = 1$. The equilibrium conditions become

$$\begin{aligned}
\frac{B_1^g}{R_1^g} &= G_1 - T_1 + B_0^g \\
(1-\chi)\,S_1 &= \frac{B_1^g}{R_1^g} + \chi \frac{B_1^h}{R_1^h} \\
(1-\chi)\,\tilde{R}_1 S_0 &= B_0^g + \chi B_0^h \\
C_1^s &= \mathbf{w}_1 \mathbf{N}_1 + \tilde{R}_1 S_0 - S_1 - T_1 + \frac{1-\mathbf{w}_1}{1-\chi} Y_1
\end{aligned}$$

The government chooses T_1 and the private debt limit is exogenous $\bar{B}_1^h$. Using market clearing at time 1, we can solve for the equilibrium as a function of R_1 and T_1. Equilibrium in financial market at time 1 requires

$$\tilde{R}_1 S_0 - S_1 = \frac{1}{1-\chi}\left(B_0^g - \frac{B_1^g}{R_1^g}\right) + \frac{\chi}{1-\chi}\left(B_0^h - \frac{B_1^h}{R_1^h}\right)$$

which then implies

$$C_1^s = \left(\mathbf{w}_1 + \frac{1-\mathbf{w}_1}{1-\chi}\right)\mathbf{N}_1 - G_1 - \frac{\chi}{1-\chi}\left(\frac{B_1^g}{R_1^g} - B_0^g + \frac{\bar{B}_1^h}{R_1^h} - B_0^h\right) \tag{15}$$

this gives us C_1^s as a function of N_1 and exogenous driving forces and pre-determined variables. The first two terms of the equations capture the classic Ricardian terms: Savers earn labor income and receive dividends, and they pay for government spending G_1. The last term is the non-Ricardian one. Savers must finance net lending to the government $\frac{B_1^g}{R_1^g} - B_0^g = G_1 - T_1$ and to the private sector

$\frac{\bar{B}_1^h}{R_1^h} - B_0^h$. Ricardian equivalence holds when $\chi = 0$, in which case T_1 does not matter for C_1. Otherwise, an increase in T_1 decreases the consumption of impatient agents, and given N_1, it must increase the consumption of savers.

2.4 Equilibrium without default

The link between the two periods comes from the Euler equation

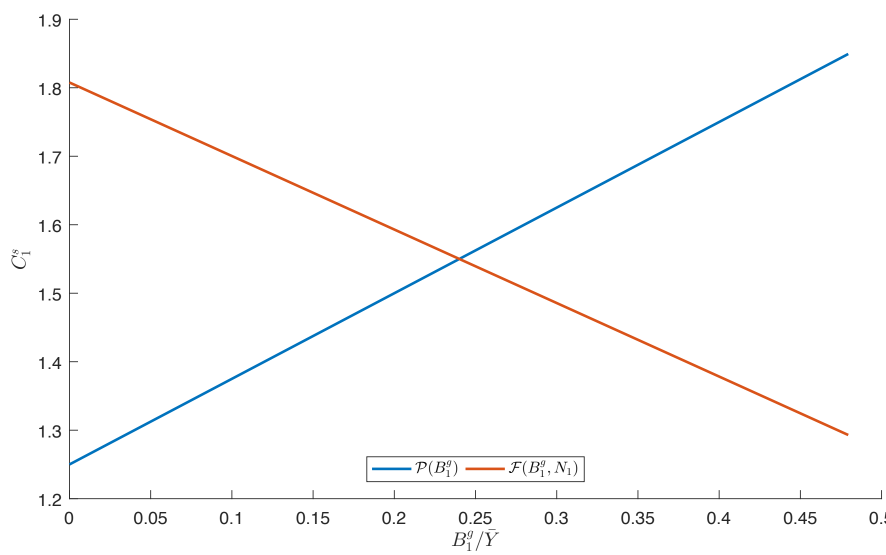
$$u'(C_1^s) = \beta \frac{R_1}{\Pi_2} u'(C_2^s) \tag{16}$$

Without default risk we have $R_1^g = R_1$ and we can write (14) as

$$C_2^s = 1 + \frac{\chi}{1-\chi} \frac{\bar{B}_1^h + B_1^g}{\Pi_{H,2}} \tag{17}$$

The equilibrium is characterized by equations (15,16,17) together with a specification of inflation and monetary policy. Consistent with our assumption of a small (closed) economy in a currency union, we consider the case $\Pi_{H,2} = \Pi_2 = 1$ and $\beta R_1 = 1$.[4] This equilibrium is depicted in Figure 1.

Figure 1: Equilibrium without default risk



[4]The same equations can also be used to think about a closed economy with independent monetary policy. For instance, we can look for the policies that implement $N_1 = \bar{N}(G_1)$: given T_1 and Π_2, the monetary policy rate R_1. Alternatively, we can consider an economy in a liquidity trap at time 1, $R_1 = 1$. We can think about forward guidance and commitment to a future Π_2. Or we can assume no commitment, normalize $\Pi_2 = 1$, and consider the equilibrium as a function of T_1.

We can describe the equilibrium with two curves. The financial wealth curve comes from the Euler equation (16) and the equilibrium budget constraint (17) of the savers. It describes a schedule $C_1^s = \mathcal{P}(B_1^g)$ which is increasing B_1^g:

$$\mathcal{P}(B_1^g) \equiv 1 + \frac{\chi}{1-\chi}\left(\bar{B}_1^h + B_1^g\right). \tag{18}$$

The funding curve $C_1^s = \mathcal{F}(B_1^g; \mathbf{N}_1)$ is simply equation (15) and it describes a schedule which is decreasing in B_1^g and increasing in $\mathbf{N}_1$

$$\mathcal{F}(B_1^g; \mathbf{N}_1) \equiv \left(\mathbf{w}_1 + \frac{1-\mathbf{w}_1}{1-\chi}\right)\mathbf{N}_1 - G_1 - \frac{\chi}{1-\chi}\left(\frac{B_1^g}{R_1^g} - B_0^g + \frac{\bar{B}_1^h}{R_1^h} - B_0^h\right) \tag{19}$$

The equilibrium $\mathbf{N}_1(B_1^g)$ is given by the solution

$$\mathcal{P}(B_1^g) = \mathcal{F}(B_1^g; \mathbf{N}_1).$$

Note that in the simple model considered here we can obtain a closed form solution for $\mathbf{N}_1$ as a function of B_1^g:

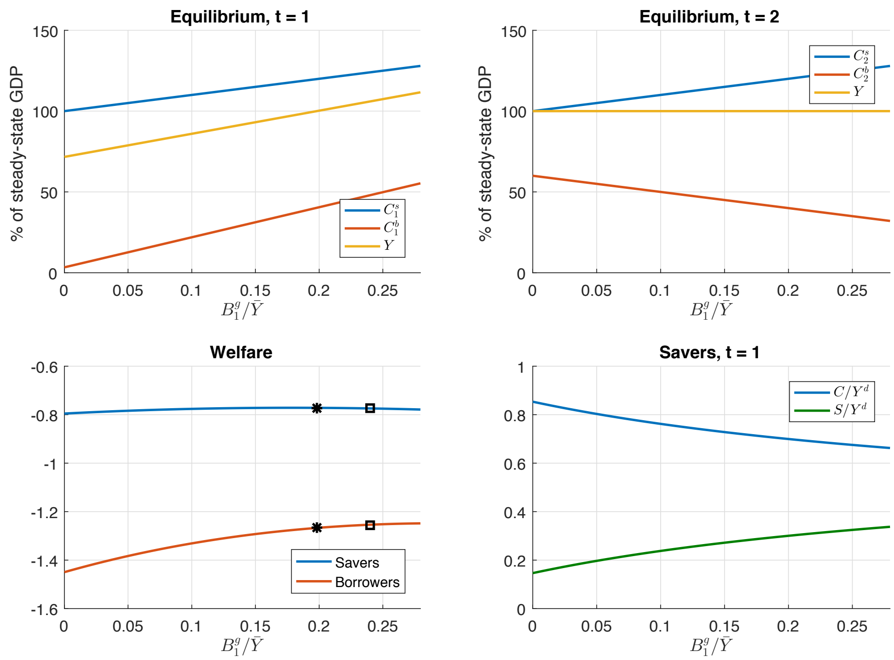
$$\left(1 + \frac{\chi}{1-\chi}(1-\mathbf{w}_1)\right)\mathbf{N}_1 = 1 + G_1 + \frac{\chi}{1-\chi}\left((1+\beta)\left(\bar{B}_1^h + B_1^g\right) - B_0^g - B_0^h\right) \tag{20}$$

We have the neoclassical terms first, then the non Ricardian terms that depend on $\chi > 0$. The multiplier on government debt is $\frac{\chi}{1-\chi}(1+\beta)$. The term $\frac{\chi}{1-\chi}$ is the fundamental non Ricardian factor. But because it appears both in the wealth equation (18), and in the funding equation (19) the total multiplier is $1+\beta$ times the non Ricardian factor.

Equation (20) also allows us to change the state of the economy. We can create a demand driven recession with private deleveraging or with low real wages. The economy can be depressed when $(1+\beta)\bar{B}_1^h - B_0^h < 0$ because this affects the consumption of constrained agents. Low real wages (or high profits $1-\mathbf{w}_1$) also depress the economy because the savers earn the profits but have a smaller propensity to consume than the borrowers.

Figure (2) summarizes simulation results as a function of debt left over at the end of the first period scaled by potential (flexible-price) GDP. The upper panels describe equilibrium output and the distribution of consumption in periods 1 and 2. The lower panels show the level of welfare of both types of agents, as well as the savings and consumption rates of savers as a function of their disposable

Figure 2: Deleveraging without Sovereign Risk (low wage recession)



Note: Black stars correspond to full employment in period 1. Black squares correspond to taxes that keep debt constant.

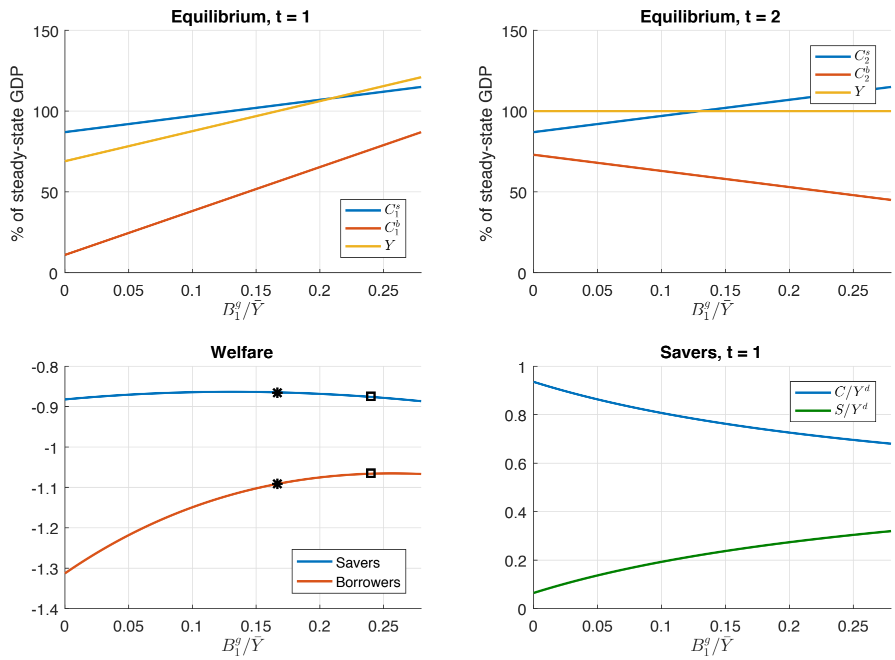
income in period 1.

Borrowers' consumption is increasing in debt in the short run (as taxes decrease) and decreasing in debt in the long run (as debt is repaid). Government borrowing is expansionary in the short run, and for the parameter values that match debt ratios in advanced economies,[5] the recession can be large. Borrowers strictly prefer low taxes and savers are less sensitive: They prefer to keep debt roughly constant. Both agents dislike high taxes in period 1 because it increases the recessionary pressures in an economy that is already depressed.

The recession in Figure (2) is induced by low real wages in the first period. Our main case is a recession induced by a private deleveraging shock by which $(1+\beta)\,\bar{B}^h_1 - B^h_0 < 0$. Figure (3) shows that the qualitative properties are similar. In this case the welfare gap savers and borrowers is smaller. Savers also seem to prefer positive taxes in the first period.

[5] see Table 1 in the Appendix.

Figure 3: Deleveraging without Sovereign Risk (private deleveraging recession)



2.5 Sovereign Risk

Let us now introduce sovereign risk. We assume that the government can default between time 1 and time 2 and that the risk of default increases with the debt burden B_1^g.[6] Let δ be a indicator of default. The probability of default is given by the increasing function $\pi(\cdot)$:

$$\Pr(\delta = 1) = \pi\left(B_1^g; \epsilon\right),$$

where we think of ϵ as an exogenous shifter of credit risk which is useful for comparative statics. In case of default, the government imposes a haircut $\hbar$ and repays only $(1 - \hbar) B_1^g$. In addition, we introduce a deadweight loss to output of Δ (which may be zero), which changes the market clearing condition as

$$\mathbf{N}_2 - \delta\Delta = C_2 + G_2.$$

[6] Equivalently, we could normalize by GDP or we could a limit on how much the government can tax $T_2 = G_2 + \frac{B_1^g}{\Pi_2}$ at time 2.

We can solve for the equilibrium labor supply at time 2 as a function of the occurrence of default

$$\mathbf{N}_2 = \bar{\mathbf{N}}(\delta) : \log \kappa_n + \varphi \log \mathbf{N} = -\gamma (\mathbf{N} - G_2 - \delta \Delta)$$

and G_2 is fixed so we drop it from the list of arguments, and as before we normalize the preferences so that $\bar{\mathbf{N}}(0) = 1 + G$. The important point is that redistributive shocks do not affect the aggregate labor index, so $\mathbf{N}$ does not depend on $\hbar$. The consumption of savers is random for two reasons, the deadweight loss$\Delta$and the haircut $\hbar$:

$$C_2^s(\delta) = \bar{\mathbf{N}}(\delta) - G_2 - \delta\Delta + \frac{\chi}{1-\chi} \frac{B_1^h + (1-\delta\hbar) B_1^g}{\Pi_2}$$

At time 1 the savers understand that sovereign debt is risky, which induces *precautionary savings*. Savers have a portfolio. They can save risk free at rate R_1, either abroad or by lending to borrowers as here, or by depositing money to intermediaries who then lend to borrowers, as in the dynamic extension below. Their Euler equation implies

$$u'(C_1^s) = \mathbb{E}_1 \left[\frac{\beta R_1}{\Pi_2} u'(C_2^s) \right]$$

We consider a small economy with exogenous monetary policy, and we set $\beta R_1 = 1$ and $\Pi_2 = 1$. The Euler equation becomes

$$\begin{aligned} -\gamma C_1^s &= \log \left((1-\pi) e^{-\gamma C_2^s(0)} + \pi e^{-\gamma C_2^s(1)} \right) \\ &= \log \left((1-\pi) e^{-\gamma \left(\bar{N}(0) - G_2 + \frac{\chi}{1-\chi} \left(B_1^h + B_1^g \right) \right)} + \pi e^{-\gamma \left(\bar{N}(1) - G_2 - \Delta + \frac{\chi}{1-\chi} \left(B_1^h + (1-\hbar) B_1^g \right) \right)} \right) \end{aligned}$$

This defines a new wealth function $C_1^s = \mathcal{P}(B_1^g; \pi)$ which is increasing in B_1^g and decreasing in π

$$\mathcal{P}(B_1^g; \pi) \equiv 1 + \frac{\chi}{1-\chi} \left(B_1^h + B_1^g \right) - \frac{1}{\gamma} \log \left(1 - \pi + \pi e^{\gamma \left(\bar{N}(0) + \Delta - \bar{N}(1) + \frac{\chi}{1-\chi} \hbar B_1^g \right)} \right) \tag{21}$$

Note that $\bar{N}(0) + \Delta - \bar{N}(1) + \frac{\chi}{1-\chi} \hbar B_1^g > 0$, so $\log \left(1 - \pi + \pi e^{\gamma \left(\bar{N}(0) + \Delta - \bar{N}(1) + \hbar \frac{\chi}{1-\chi} B_1^g \right)} \right)$ is increasing in π. If we specify the schedule $\pi(B_1^g; \epsilon)$ we can then solve for

$$C_1^s = \mathcal{P}(B_1^g; \pi(B_1^g; \epsilon))$$

which is decreasing in ϵ. The schedule as a function of B_1^g is both lower and flatter than before because of the default risk. The direct multiplier is still $\frac{\chi}{1-\chi}$ but an increase in B_1^g has two other effects via credit risk. For a given π it increases the losses in the bad state $\frac{\chi}{1-\chi}\hbar B_1^g$. It also increases π. Both effects lower the value of debt and therefore consumption. If these effects are very strong (π a step function for instance), then it is possible for the schedule $\mathcal{P}\left(B_1^g;\pi\left(B_1^g;\epsilon\right)\right)$ to be *decreasing* in B_1^g, at least locally.

The funding constraint $C_1^s = \mathcal{F}\left(B_1^g, \epsilon; \mathbf{N}_1\right)$ is then

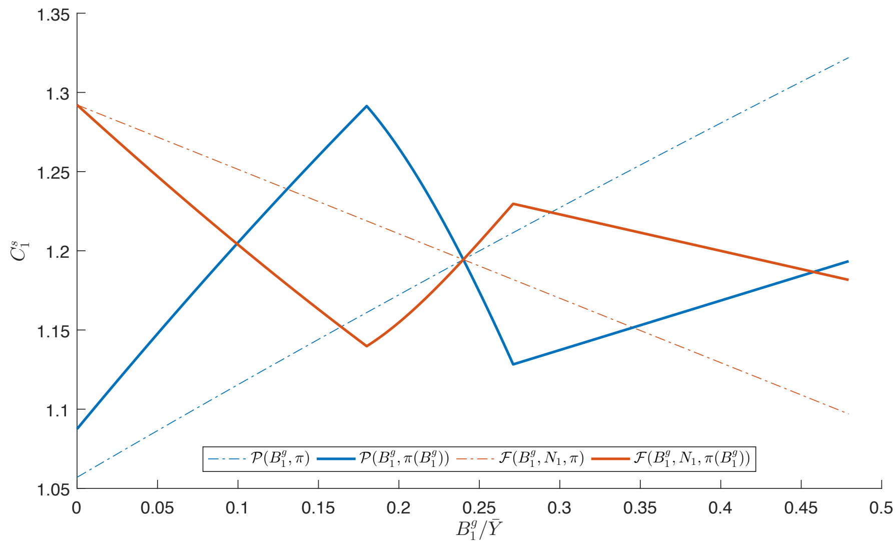
$$\mathcal{F}\left(B_1^g, \epsilon; \mathbf{N}_1\right) \equiv \left(\mathbf{w}_1 + \frac{1-\mathbf{w}_1}{1-\chi}\right)\mathbf{N}_1 - G_1 - \frac{\chi}{1-\chi}\left(q\left(B_1^g;\epsilon\right)\frac{B_1^g}{R_1} - B_0^g + \frac{\bar{B}_1^h}{R_1} - B_0^h\right) \tag{22}$$

where the price of government bonds $q\left(B_1^g;\epsilon\right)$ is priced by savers as

$$\begin{aligned} q_1 &= \mathbb{E}_1\left[\beta\frac{u'\left(C_{s,2}\right)}{u'\left(C_{s,1}\right)}\left(1-\delta\hbar\right)\right] \\ &= \frac{1}{R_1} - \hbar\mathbb{E}_1\left[\beta\frac{u'\left(C_{s,2}\right)}{u'\left(C_{s,1}\right)}\delta\right] \\ &= \frac{1}{R_1} - \beta\hbar\pi\left(B_1^g;\epsilon\right)e^{\gamma\left(C_1^s - C_2^s(1)\right)} \end{aligned}$$

We can see that $q\left(B_1^g;\epsilon\right)$ is decreasing in both arguments. As a result, the funding schedule (22) is increasing in ϵ and less steep as a function of B_1^g than before. Again, if the price effect is strong, we can get the funding curve to be locally *increasing* in B_1^g.

Figure 4: Equilibrium with Sovereign Default Risk



Note: Equilibrium is unique conditional on a choice of B^g. But three choices of B^g are consistent with the same level of employment $\mathbf{N}_1$.

An equilibrium must satisfy

$$\mathcal{P}\left(B_1^g; \pi\left(B_1^g; \epsilon\right)\right) = \mathcal{F}\left(B_1^g, \epsilon; \mathbf{N}_1\right)$$

Figure 4 shows the equilibrium. The straight dashed lines are drawn for fixed π, equal to the equilibrium value. This is the case where credit risk is not responsive to leverage. The solid lines correspond to the equilibrium pricing function calibrated below. Note that our timing convention implies a unique equilibrium. The equilibrium is unique because we assume that the government chooses B_1^g, and then that the markets price the bonds knowing B_1^g and therefore, implicitly, that the government stands ready to adjust taxes to obtain B_1^g for any price. The alternative timing/commitment assumption of Calvo can yield multiple equilibria (see Lorenzoni and Werning (2013) for a discussion).

But Figure 4 makes clear that there are strong complementarities in the model. More precisely, the figure shows that there are three levels of B_1^g that are consistent with the same output in the first period. Aggregate efficiency is of course higher when debt is lower, because, for given N_1, lower debt reduces default risk and expected deadweight losses. This is not a Pareto-improvement per-se because the borrowers might prefer default and lower taxes. To make it Pareto superior we would need to let the government adjust relative transfers at time 2 based upon whether default occurs or not.

Calibration Figure (5) summarizes our results when we introduce sovereign risk. The critical feature added here is the π function which describes the probability of default.[7] To calibrate it, we estimate an equation for average sovereign spreads in the eurozone on data from Martin and Philippon (2014), to get

$$Spread_t = 0.01 \cdot \mathbf{1}\left(B^g_{t-2} < 0.9\right) B^g_{t-2} + 0.2 \cdot \mathbf{1}\left(B^g_{t-2} > 0.9\right)\left(B^g_{t-2} - 0.9\right)$$

where $Spread_t$ is the annual spread over the german interest rate, and B^g_t is government debt rebased by potential GDP. These numbers imply an essentially flat default probability until debt riches 90% of GDP. On the other hand, when debt/GDP is around 1, an increase of debt of 10% of GDP would move spreads by around 2.5%. Let us assume that the duration of debt is 5 years and that the loss rate in case of default is $\hbar = 0.5$. This gives us a π function of the form:

$$\pi = \frac{5}{0.5}\left(\frac{1}{100} \cdot \frac{B^g_1}{\bar{Y}/5} + (0.2 - .01)\left(\frac{B^g_1}{\bar{Y}/5} - 0.9\right)\mathbf{1}\left(\frac{B^g_1}{\bar{Y}/5} > 0.9\right)\right).$$

Figure 5: Deleveraging with Sovereign Risk

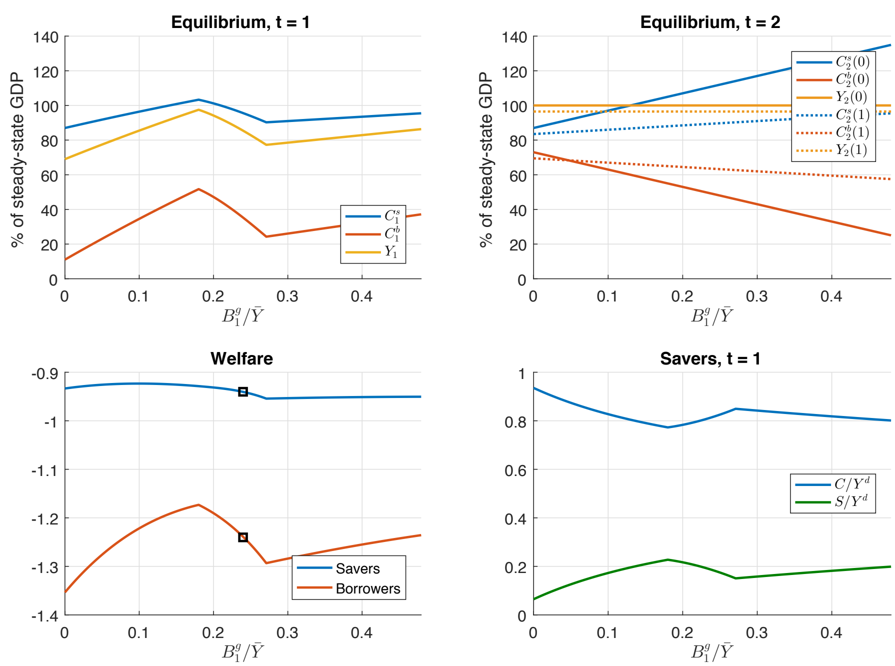


Figure (5) reveals some new dynamics: savers support tax increases in the region where such taxes prove effective in reducing the probability of default. In this region, moreover, tax increases are

[7] Also, the panel marked $T = 2$ now includes output and consumption when default happens, in dashed lines.

expansionary. The reason why this happens can be linked to the precautionary behavior of savers. Indeed, in the critical intermediate region, the savers exhibit a much higher marginal propensity to consume, as they expect the high consumption and no default state to happen with ever higher probability. Therefore, when the government raises taxes in this region, the savers' consumption response more than compensates for the borrowers' spending cuts.

3 Dynamic Closed Economy

We consider a model with an infinite horizon but truncated in the sense that after some (large) T we assume that the economy is in its flexible price steady state without default risk. To compute the solution we start from period T. In all periods $t < T$, we assume that prices, wages, and the nominal (risk free) interest rate are fixed. The model is calibrated at quarterly frequency (one period is one quarter)

3.1 Government Default

The government can default once (and only once) at any time $t < T$ and that the risk of default increases with the debt burden B_t^g. Let $\mathcal{H}_t$ be the history of default up to time t: $\mathcal{H}_t = 0$ if and only if there has been no default up to and including time t. Note that our earlier assumption that risk disappears after T is simply that $\Pr\left(\mathcal{H}_s = 0 \mid \mathcal{H}_T = 0\right) = 1$ for all s. In case of default, the government imposes a haircut $\hbar$ and its budget constraint becomes

$$T_t^d + q_t^d \left(B_t^{g,d} - (1-\hbar)(1-\rho) B_{t-1}^g \right) = G_t + (1-\hbar)\kappa B_{t-1}^g$$

where q_t^d and $B_t^{g,d}$ are the price and the amount of new debt after default, and T_t^d is the level of taxes after default. In addition, default creates a permanent deadweight loss of output Δ so the resource constraint is

$$\mathbf{N}_t = C_t + G_t + \mathcal{H}_t \Delta.$$

3.2 Long Run Equilibrium

For $t \geq T$, there is not default risk so the price of government debt is $q^{\star} = 1$. The government keeps the level of debt constant so taxes are $T = G + rB^g$. The borrower's budget constraint is

$$\mathbf{C}_t^b = \frac{W_t^b}{P_t} N_t^b + \frac{B_t^h}{R_t^h} - \frac{B_{t-1}^h}{\Pi_{H,t}} - T_t.$$

In steady state we have

$$\mathbf{C}^b = \frac{W^b N^b}{P} - \frac{r}{1+r} B^h - T.$$

The net payment of each borrower is $\frac{r}{1+r} B^h$. From our earlier analysis we know that savers will consume

$$C_T^s\left(\mathcal{H}_T\right) = \mathbf{N}\left(\mathcal{H}_T\right) - G + r\frac{\chi}{1-\chi}\left(\frac{B_{T-1}^h}{1+r} + B_{T-1}^g\right)$$

We choose the parameter κ_n to normalize aggregate consumption without default: $\mathbf{C}(0) = 1$ and $\mathbf{N}(0) = 1 + G$ in the flexible price equilibrium. In case of default we have $\mathbf{N}_T^1 = \bar{\mathbf{N}}(\Delta)$ that solves

$$\frac{\log(\kappa_n) + \log(\bar{\mathbf{N}}(\Delta))}{\gamma} = -\bar{\mathbf{N}}(\Delta) + G + \Delta$$

3.3 Dynamics

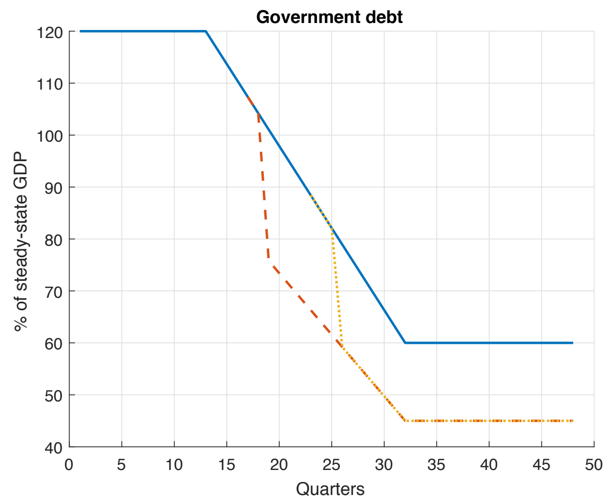
We take the path of private debt $\bar{B}_t^h$ as exogenous and we use it to create a demand-driven recession in the economy, similar to the recessions that have been observed in many countries in 2008-2009. In the benchmark model, the path of private deleveraging is independent of fiscal policy and of government default. Wages are fixed until period T when they become flexible again.

We then need to specify fiscal policy. The simplest way to do so is to set a path for B_t^g which, given constant government spending $G_t = G$, implies a path for taxes T_t. We denote the path of sovereign debt without default as $B_t^{g,0}$. In case of default, debt is reduced by the haircut $\hbar$ but the path of debt dynamics does not change. In other words, $B_t^{g,1} = (1-\hbar)\, B_t^{g,0}$. The actual path is

$$B_t^g\left(\mathcal{H}_t\right) = \left(1 - \mathcal{H}_t\right) B_t^{g,0} + \mathcal{H}_t B_t^{g,1}.$$

Figure 6 shows the path of public deleveraging with and without default.

Figure 6: Sample Paths of Public Debt with and without Default



Note: Solid line is path without default. Dashed and dotted lines are two examples of default paths.

3.3.1 After default ($\mathcal{H}_t = 1$)

The budget constraint of the government after default is

$$T_t^1 + q_t^1 (1 - \hbar) \left(B_t^{g,0} - (1 - \rho) B_{t-1}^{g,0} \right) = G + (1 - \hbar) \kappa B_{t-1}^{g,0}$$

Since there is no further risk of default we have $q_t^1 = q^\star = 1$ The resource constraint after default is

$$\mathbf{N}_t^1 - \Delta = \mathbf{C}_t^1 + G_t$$

because of the deadweight loss Δ. Savers earn labor income and receive dividends from the firms so their per-capita income is $W_t^s N_t^s + \left(\frac{N_t - \Delta - W_t N_t}{1-\chi} \right) = N_t \left(W_t + \frac{1-W_t}{1-\chi} \right) - \frac{\Delta}{1-\chi}$. Because there is no further risk and since $\beta R = 1$, their Euler equation $u' \left(C_t^{s,d} \right) = \beta R \mathbb{E}_t \left[u' \left(C_{t+1}^{s,d} \right) \right]$ implies

$$C_t^{s,1} = C_{t+1}^{s,1}$$

while borrowers's consumption is given by

$$C_t^{b,1} + \bar{B}_{t-1}^h + T_t^1 = \mathbf{N}_t^1 + \frac{\bar{B}_t^h}{R}$$

Our assumptions that the path of sovereign debt, the haircut, and the deadweight loss Δ are unaffected by default implies that the equilibrium path after default does not depend on when default took place.

Lemma 1. *Let $N_t^{1,t'}$ denote employment at time t in an economy where default occurred at time $t' < t$. Then for all (t', t'') we have*

$$N_t^{1,t'} = N_t^{1,t''}$$

As a result we only need to solve for one default path. At period T, we can solve for the equilibrium with flexible wages. Aggregate employment is $\mathbf{N}_T(1)$ and consumption of the savers is

$$C_T^{s,1} = \mathbf{N}_T(1) - G + r\frac{\chi}{1-\chi}\left(\frac{B_{T-1}^h}{1+r} + (1-\hbar)B_{T-1}^{g,0}\right)$$

This consumption is constant after default so we must have

$$C_t^{s,1} = C_T^{s,1}$$

3.3.2 Before default

Since we specify a path for B_t^g, this implies a path for the default probability $\pi(B_t^g)$. We want to compute consumption along the no-default path, $C_t^{s,0}$. We know it at time $t = T-1$ since we know C_T^s. Now suppose we know $C_{t+1}^{s,0}$. Since we know consumption in case of default $C_{t+1}^{s,1}$ from the previous section, we can solve for $C_t^{s,0}$ using the Euler equation on risk-free debt, which, given $\beta R = 1$, is simply

$$u'\left(C_t^{s,0}\right) = (1-\pi_t)\,u'\left(C_{t+1}^{s,0}\right) + \pi_t u'\left(C_{t+1}^{s,1}\right)$$

We thus obtain the pricing kernel that allows us to price undefaulted government debt as

$$q_t^0 = (1-\pi_t)\frac{\beta u'\left(C_{t+1}^{s,0}\right)}{u'\left(C_t^{s,0}\right)}\left(\kappa + (1-\rho)\,q_{t+1}^0\right) + \pi_t\frac{\beta u'\left(C_{t+1}^{s,1}\right)}{u'\left(C_t^{s,0}\right)}(1-\hbar)\left(\kappa + (1-\rho)\,q^\star\right)$$

Plugging this price into the government budget constraint yields taxes

$$T_t^0 + q_t^0\left(B_t^{g,0} - (1-\rho)\,B_{t-1}^{g,0}\right) = G + \kappa B_{t-1}^{g,0}$$

Taxes give us output via the budget constraint of the savers

$$C_t^{s,0} + T_t^0 + \frac{\chi}{1-\chi}\left(\bar{B}_t^h - R\bar{B}_{t-1}^h\right) + \frac{1}{1-\chi} q_t^0 \left(B_t^g - (1-\rho)\, B_{t-1}^g\right) = \mathbf{N}_t^0 + \frac{\kappa}{1-\chi} B_{t-1}^g$$

Finally, we obtain the consumption of borrowers as

$$C_t^{b,0} + T_t^0 = \mathbf{N}_t^0 + \bar{B}_t^h - R\bar{B}_{t-1}^h.$$

3.4 Simulations

Let us now consider the dynamics of the model. The risk of default is stochastic and given by

$$\Pr\left(\delta_t = 1\right) = \epsilon_t \pi \left(B_t^g\right)$$

where ϵ_t is an exogenous process. We are interested in public deleveraging in an economy that is already depressed. We do so by considering the following sequence of shocks (recall that one period is one quarter):

- At $t = 0$ the economy is in steady state with $\epsilon_0 = 0$.
- At $t = 1$ private deleveraging starts and last for 5 years, until $t = 20$.
- At $t = 5$, there is a shock to sovereign risk: $\epsilon_5 = 1$.
  - The government commits to a 10-year deleveraging path, and we consider 4 starting dates: either at $t = 5$ (immediate), or $t = 13$ (halfway through private deleveraging, or $t = 21$ (after private deleveraging is over), or never $t = \infty$.

Figure shows the path of private $\bar{B}_t^h$ and public debt conditional on no default $B_t^{g,0}$. We consider a model with standard parameters and a risk aversion of 2.

Figure 7: Deleveraging Paths for Private and Public Debts

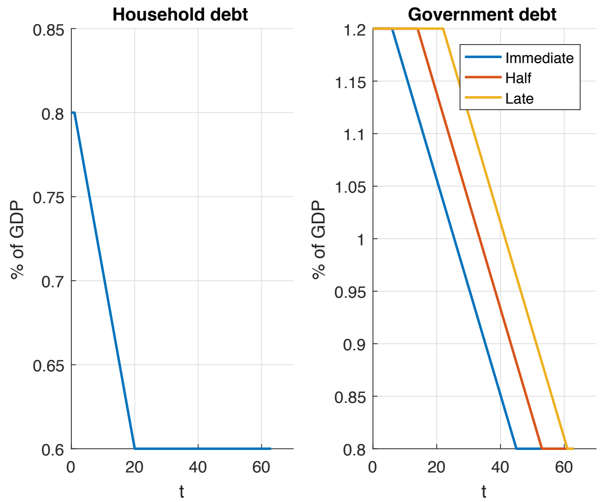
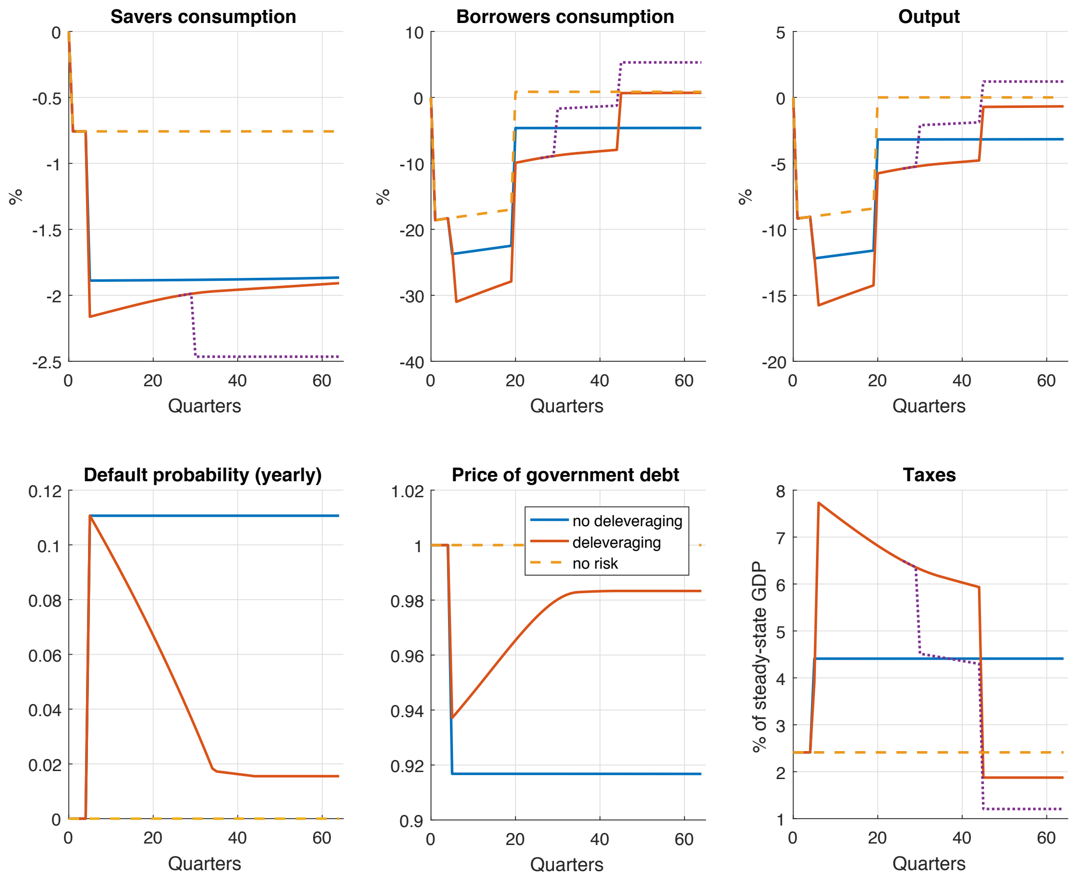


Figure 8 plots the paths of real outcomes, starting with savers's consumption. Private deleveraging implies smaller interest payments from borrowers to savers in the long run. Since our savers are permanent income agents they lower their consumption. With r=2% and a decrease of 0.2 GDP this predicts roughly a 40 basis points drop in consumption, just from the long run effect. In addition, there is the capitalized value of lost output, which is of the same order of magnitude. So absent all other shocks, savers consumption drops by a bit less than 1%.

Figure 8: Dynamics with Early Sovereign Deleveraging

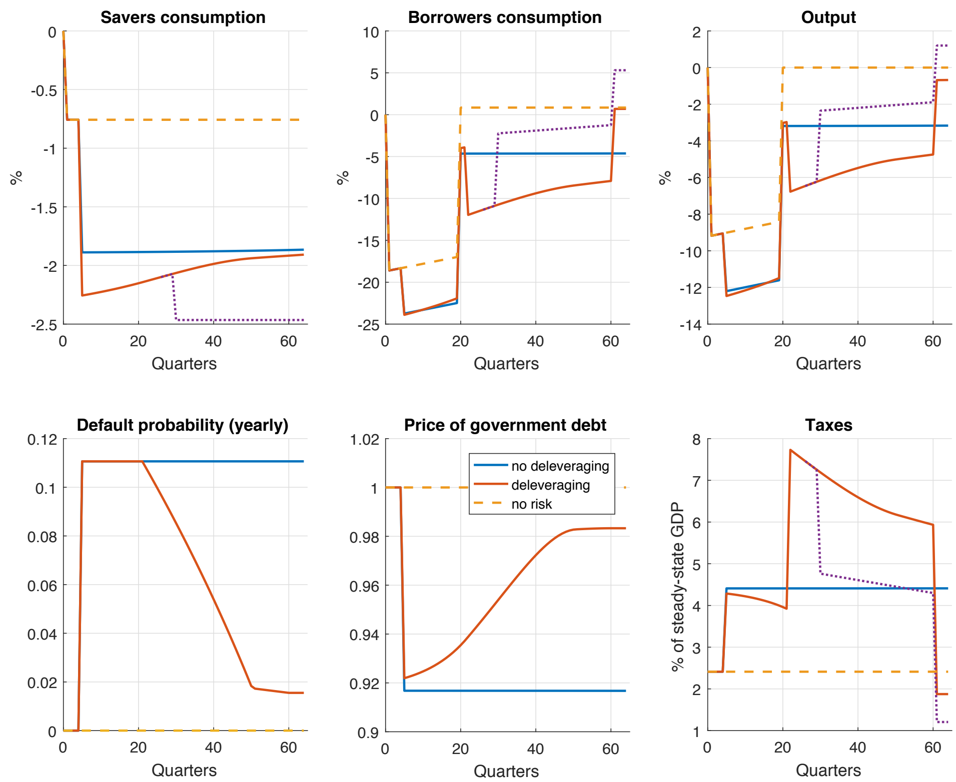


Notes: Each period is one quarter. Private deleveraging occurs in all cases, from period 1 to period 20, and reduces household debt from 0.8 GDP to 0.6 GDP. Yellow dashed line has no sovereign risk or deleveraging. Solid blue line has sovereign risk, but no deleveraging, and plots the paths conditional on no default. Solid red line has sovereign risk and deleveraging, and plots the paths conditional on no default. The dashed purple line is one path where sovereign default occurs in period 30.

Then, at t=5, we switch on the credit risk. The price of government debt drops by 8%. Savers make a capital loss and their consumption drops further. Then much depends on what the government does. If the government does nothing then saver's consumption remains constant on the no-default path, but with a high likelihood of jumping down in case of default. If the government reduces its leverage, the savers consumptions drops a bit more on impact because of recessionary effects, but then appreciates as time passes without default.

Borrowers' consumption follows mechanically the path of deleveraging, and output net of taxes. Note that they are hurt by the drop in savers' consumption at time 5. Their consumption jumps back up at time 20 when private deleveraging is over. Their consumption also jumps up after default because taxes go down. Note that so far we have assumed that borrowers are very impatient and never on their Euler equations. We need to change and solve for occasionally binding constraints, to get rid of the unrealistic upward jumps in consumption.

Figure 9: Dynamics with Late Sovereign Deleveraging

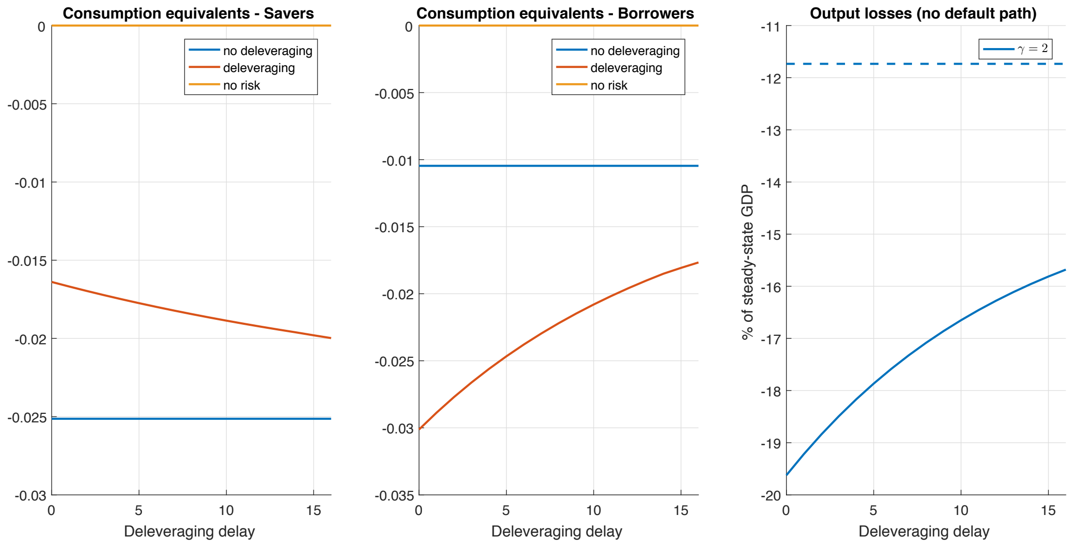


Notes: Each period is one quarter. Private deleveraging occurs in all cases, from period 1 to period 20, and reduces household debt from 0.8 GDP to 0.6 GDP. Yellow dashed line has no sovereign risk or deleveraging. Solid blue line has sovereign risk, but no deleveraging, and plots the paths conditional on no default. Solid red line has sovereign risk and deleveraging, and plots the paths conditional on no default. The dashed purple line is one path where sovereign default occurs in period 30.

Figure 9 shows the case where the government waits for private deleveraging to end before starting its own deleveraging. There is less austerity in the short run, but savers have to live with high credit risk for 5 years, which depresses their consumption.

Figure 10 compares the welfare losses as a function of the delay before deleveraging, compared to the no risk case (with only private deleveraging). The left panel shows that savers prefer an early deleveraging process, while borrowers prefer a late deleveraging process. The output loss is worse when deleveraging starts early.

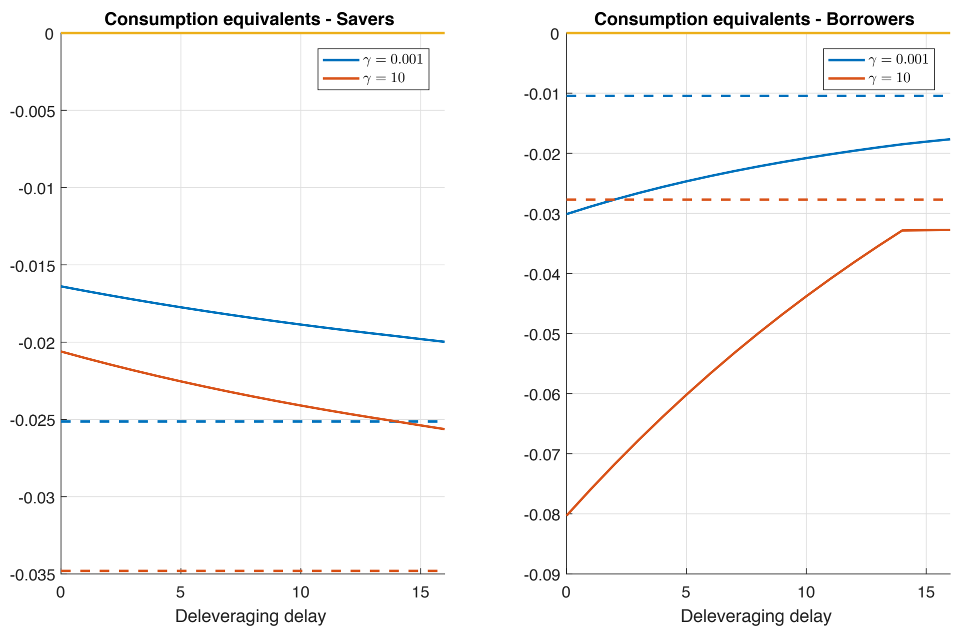
Figure 10: Welfare and Deleveraging Delay



Notes: Horizontal axis is the delay in quarter between the risk shock and the start of sovereign deleveraging. Vertical axis measures welfare in consumption equivalent units. Output losses are capitalized over 60 quarters with the borrower's discount factor.

Figure 11 compares the welfare losses in consumption equivalent for different delays and different values of the risk aversion parameter. When borrowers are risk averse they would choose deleveraging to start right after private deleveraging is over. This is the only point at which they would agree with the savers.

Figure 11: Welfare and Deleveraging Delay for Different Risk Aversions



4 Extensions

4.1 Epstein-Zin Preferences

We want to explore the role of risk aversion. As in models of news driven fluctuations, however, we need to separate risk aversion from inter temporal substitution.

[To be completed]

4.2 Dynamic Open Economy

We now consider the case where $\varpi > 0$ and foreigners also hold the bonds.

[To be completed]

5 Conclusion

We analyze the tradeoff between sovereign risk and fiscal austerity in an economy with heterogenous agents where domestic savers hold (most of) the government debt. The negative impact of fiscal austerity on growth is muted by the endogenous response of savers.

We find that borrowers and savers almost always disagree regarding the optimal path of sovereign deleveraging, even though they are equally exposed to the recessionary impact of fiscal austerity. This might explain why it is so difficult to find political consensus regarding fiscal policy in the aftermath of a financial crisis.

Appendix

A Calibration

Parameter	Description	Source/Target	Value
β	Five-year discount factor	Gourinchas et al. (2016)	0.97^5
β_b	Borrower's discount factor		0.5^5
χ	Proportion of borrowers	Gourinchas et al. (2016)	0.5
φ	Inverse labor supply elasticity	Gourinchas et al. (2016)	1
$\bar{N}$	Steady-state labor supply	Normalized	1
w_1	Wage rate	Induce recession	$0.7w_2$, w_2
w_2	Steady-state wage rate	Linear production	1
B_0^g	Initial Government debt	Gourinchas et al. (2016)	$1.2 \times \frac{\bar{N}}{5}$
B_0^h	Initial Household debt		$1 \times \frac{\bar{N}}{5}$
$\bar{B}$	Borrowing limit	Induce recession	B_0^h, $35\% B_0^h$
G_1	Government consumption		G_2
G_2	Government consumption in SS	Gourinchas et al. (2016)	$0.2 \times \frac{\bar{N}}{5}$
δ	Haircut		75%
Δ	Output cost of default		$0.1 \times \bar{N}$

Table 1: Parameter values

References

Acharya, V., I. Drechsler, and P. Schnabl (2015). A pyrrhic victory? - bank bailouts and sovereign credit risk. *Journal of Financial Economics 69*(6), 2689–2739.

Aguiar, M. and M. Amador (2014). Sovereign debt. *Handbook of International Economics 4*.

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics 69*(1), 64–83.

Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review*.

Benigno, P., G. Eggertsson, and F. Romei (2016, March). Dynamic debt deleveraging and optimal monetary policy.

Bocola, L. (2016, forthcoming). The pass-through of sovereign risk. *Journal of Political Economy*.

Cole, H. L. and T. J. Kehoe (2000). Self-Fulfilling Debt Crises. *Review of Economic Studies 67*(1), 91–116.

Eaton, J. and M. Gersovitz (1982). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies 48*(2), 289–309.

Eggertsson, G. and P. Krugman (2012). Debt, deleveraging, and the liquidity trap:a fisher-minsky-koo approach. *Quarterly Journal of Economics* (127(3)), 1469–1513.

Escolano, J. and V. Gaspar (2016). Optimal debt policy under asymmetric risk. IMF Working Paper.

Gali, J. and T. Monacelli (2008). Optimal monetary and fiscal policy in a currency union. *Journal of International Economics 76*, 116–132.

Gennaioli, N., A. Martin, and S. Rossi (2014). Sovereign default, domestic banks, and financial institutions. *Journal of Finance 69*(2), 819–866.

Gourinchas, P.-O., T. Philippon, and D. Vayanos (2016). The analytics of the greek crisis. In *NBER Macroeconomics Annual 2016, Volume 31*, NBER Chapters. National Bureau of Economic Research, Inc.

Huidrom, R., M. A. Kose, J. J. Lim, and F. L. Ohnsorge (2016, June). Do fiscal multipliers depend on fiscal positions? CAMA Working Papers 2016-35, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University.

Huidrom, R., M. A. Kose, and F. L. Ohnsorge (2016, June). Challenges of fiscal policy in emerging and developing economies. Policy Research Working Paper Series 7725, The World Bank.

Leland, H. E. (1998). Agency costs, risk management, and capital structure. *Journal of Finance 53*, 1213–1243.

Lorenzoni, G. and I. Werning (2013, July). Slow moving debt crises.

Martin, P. and T. Philippon (2014). Inspecting the mechanism: Leverage and the great recession in the eurozone. Working Paper NYU.

Perez, D. (2016). Sovereign debt, domestic banks and the provision of public liquidity.

Romei, F. (2015). Need for (the Right) Speed: the Timing and Composition of Public Debt Deleveraging. Economics Working Papers MWP2015/11, European University Institute.

Schmitt-Grohe, S. and M. Uribe (2003). Closing small open economy models. *Journal of International Economics 61*, 163–185.